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## Cost-Optimal Power System Extension under Flow-Based Market Coupling

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# Cost-Optimal Power System Extension under Flow-based Market Coupling

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## Abstract

Electricity market models, implemented as dynamic programming problems, have been applied widely to identify possible pathways towards a cost-optimal and low carbon electricity system. However, the joint optimization of generation and transmission remains challenging, mainly due to the fact that different characteristics and rules apply to commercial and physical exchanges of electricity in meshed networks. This paper presents a methodology that allows to optimize power generation and transmission infrastructures jointly through an iterative approach based on power transfer distribution factors (PTDFs). As PTDFs are linear representations of the physical load flow equations, they can be implemented in a linear programming environment suitable for large scale problems. The algorithm iteratively updates PTDFs when grid infrastructures are modified due to cost-optimal extension and thus yields an optimal solution with a consistent representation of physical load flows. The method is first demonstrated on a simplified three-node model where it is found to be robust and convergent. It is then applied to the European power system in order to find its cost-optimal development under the prescription of strongly decreasing CO<sub>2</sub> emissions until 2050.

*Keywords:* Power system planning, Power generation and transmission, Iterative linear optimization, PTDF, Electricity market model, Power flow model, Flow-based market coupling

JEL classification: C61, H54, L94, Q40

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## 1. Introduction

Motivated by ambitious emission reduction and renewable energy integration targets, the European power system is expected to undergo substantial changes. Electricity market models, implemented as a

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dynamic programming problem, have been applied widely to identify possible pathways (e.g. in ECF (2010) or EC (2011)). However, these models mostly lack an appropriate representation of the physical grid which represents the backbone of today’s power system. Specifically, a joint optimization of generation and transmission is difficult, mainly due to the fact that different characteristics and rules apply to commercial and physical exchanges of electricity in meshed networks. This is specifically true when dealing with an intermeshed alternating current (AC) transmission network like the European power system. According to Kirchhoff’s circuit law, multiple paths are taken by the physical flows when settling trades from one point to another via the intermeshed grid (so-called loop flows), such that a large number of lines may be impacted.

Many studies have dealt with the problem of transmission system expansion. Comprehensive literature surveys for the general problem of transmission system expansion and corresponding modeling issues are provided in Groschke et al. (2009) and Wu et al. (2006). As stated in Wu et al. (2006), the problem comprises economic and engineering considerations, which can easily be confirmed when analyzing the corresponding fields of research.

From an engineering perspective, early approaches to transmission system expansion can be found in Garver (1970) or Villasana et al. (1985) that both formulate linear load flow equations in order to find overloaded lines in snapshots of the future power system. Later works also deploy other optimization methods, such as mixed-integer linear programming (Alguacil et al. (2003), de la Torre et al. (2008)), Benders decomposition (Binato et al. (2001)) or heuristic methods (de Oliveira et al. (2005)).

The second stream analyzing transmission system extensions is mostly based on economic considerations: In Hogan et al. (2010) the analytical model uses PTDF in order to integrate loop flows that were previously found to have a significant impact on the efficiency of the market outcome in meshed networks by Bushnell and Stoft (1997). A very similar modeling framework is applied by Rosellón and Weigt (2011) to analyze an incentive mechanism for transmission expansion with a profit-maximizing transmission system company and a competitive wholesale market based on nodal pricing, and in Rosellón et al. (2012) to specifically analyze the impact of different cost functions for transmission grid extensions.

Most studies test their approach on simple test systems, and some of them apply their methodology to some small-scale real-world problem. The application to large-scale problems, however, remains challenging. Furthermore, the problem gets even more complicated when transmission grid extensions are optimized jointly with the extension of generation facilities. One of the approaches pointing in this direction was presented by Fürsch et al. (2013) who analyze the cost-efficient achievement of renewable energy targets in Europe until 2050. They use an iterative algorithm that builds on net transfer capacities (NTC), that is,

however, not tested for its robustness. Furthermore, NTC-based congestion management has been found to entail inefficiencies in the market outcome compared to alternative approaches (Hogan (1992), Chao and Peck (1996)). In fact, flow-based transmission rights and congestion management are now being widely introduced in liberalized power markets in order to promote market integration, facilitate the European market functioning, and improve the European social welfare while guaranteeing the security of supply Aguado et al. (2012). For a general discussion of different regimes, the interested reader is referred to Chao et al. (2000). Analyses of different congestion management regimes in the European context and possible increases in market efficiency were published in Ehrenmann and Smeers (2005), Jullien et al. (2012), Neuhoff et al. (2011a) and Neuhoff et al. (2011b). Practical feasibility of the concept is currently proven in the Central Western European (CWE) Region, as discussed in Aguado et al. (2012).

This paper presents a methodology that couples an electricity market model with a power flow model to jointly optimize both power generation and transmission grid infrastructures under flow-based market coupling using an iterative approach based on power transfer distribution factors (PTDFs). The objective of the proposed method is to find the overall cost-optimal solution for serving electricity to the consumers, and thus to optimize social welfare. PTDFs are linear representations of the load flow equations which can be used to calculate physical active power flows in the power network given market transactions. As such, they can be implemented in a linear programming (LP) environment determining the cost-optimal development of power system infrastructures under certain restrictions. However, PTDFs change with each alteration of the grid configuration, since they depend (nonlinearly) on the impedances of the transmission lines. Hence, we suggest a method whereby the PTDFs are updated and fed back every time the grid is modified. With the idea of linearizing the nonlinear optimization problem and solving the resulting LP in an iterate manner, the methodology is conceptually close to the successive linear programming (SLP) approach that was introduced by Griffith and Stewart (1961) and has since then been applied extensively for large-scale nonlinear optimization problems, especially in the petrochemical industry (e.g. Baker and Lasdon (1985)).

The developed methodology is demonstrated on a simplified three-node model where the iterative optimization algorithm is found to be robust and convergent. In a case study the method is then applied to find an optimal power system infrastructure development under the prescription of strongly decreasing CO<sub>2</sub> emissions in Europe until 2050 by using a European electricity market model and a European transmission network model. The Market Model covers all EU-27 countries plus Norway and Switzerland and the aggregated Network Model represents these by over 200 nodes and 450 lines.

The remainder of the paper is structured as follows: Section 2 presents the methodology developed to jointly optimize power generation and transmission grid infrastructures in an iterative manner based on PTDFs. The algorithm is applied to a simple three node network in Section 3, whereas Section 4 presents the large-scale application to the European power system. Section 5 concludes.

## 2. Methodology

This section is subdivided into two parts. First, starting from the most general formulation of the load flow equations in an intermeshed AC grid, a linear PTDF representation is derived suitable for being integrated in a large scale linear optimization problem. Then, a model is presented focusing on the problem of integrating load flow calculations in an economic optimization framework with the objective to find the cost-optimal grid infrastructure in a multi-node network with different load and generation characteristics.

### 2.1. AC load flow equations and PTDF representation

As noted in most electrical engineering books (e.g. Andersson (2011)), the most general form of the network equations in an AC power system can be written as follows:

$$\begin{aligned}
P_i &= U_i \sum_{j \in I} U_j (g_{ij} \cos(\delta_i - \delta_j) + b_{ij} \sin(\delta_i - \delta_j)) \\
Q_i &= U_i \sum_{j \in I} U_j (g_{ij} \sin(\delta_i - \delta_j) - b_{ij} \cos(\delta_i - \delta_j)) \\
P_{ij} &= U_i^2 g_{ij} - U_i U_j g_{ij} \cos(\delta_i - \delta_j) - U_i U_j b_{ij} \sin(\delta_i - \delta_j) \\
Q_{ij} &= -U_i^2 (b_{ij} + b_{ij}^{sh}) + U_i U_j b_{ij} \cos(\delta_i - \delta_j) - U_i U_j g_{ij} \sin(\delta_i - \delta_j)
\end{aligned} \tag{1}$$

In the above equations,  $P_i$  and  $Q_i$  represent the active and reactive power infeed at node  $i$ , whereas  $P_{ij}$  and  $Q_{ij}$  stand for the active and reactive power flow on line  $ij$  connecting node  $i$  and  $j$ , respectively.  $I$  is the set of nodes the network consists of. As can be seen, voltage levels  $U$  and phase angles  $\delta$  of the nodes as well as series conductances  $g$  and series susceptances  $b$  of the transmission lines are determining active and reactive power flows. Noticeably, the above equations are highly nonlinear.

For the purpose of implementing load flow calculations in a linear optimization environment, as presented in this paper, a linear representation of the above equations has to be found. To this end, the following assumptions are made:

- All voltages are set to 1 p.u., meaning that there are no voltage drops.
- Reactive power is neglected, i.e.  $Q_i$  and  $Q_{ij}$  is set to zero.

- Losses are neglected, and line reactance is much larger than the resistance:  $X \gg R \approx 0$ .
- Voltage angle differences are small, such that  $\sin(\delta_i - \delta_j) \approx \delta_i - \delta_j$ .

By making these assumptions, the AC load flow equations can be simplified to a linear relationship:

$$P_{ij} = b_{ij}(\delta_i - \delta_j) = \frac{x_{ij}}{x_{ij}^2 + R_{ij}^2}(\delta_i - \delta_j) \approx \frac{1}{x_{ij}}(\delta_i - \delta_j) \quad (2)$$

According to Kirchoff's power law, the active power injection at bus  $i$  is then given by

$$P_i = \sum_{j \in \Omega_i} \frac{1}{x_{ij}}(\delta_i - \delta_j) = \left( \sum_{j \in \Omega_i} \frac{1}{x_{ij}} \right) \delta_i + \sum_{j \in \Omega_i} \left( -\frac{\delta_j}{x_{ij}} \right) \quad (3)$$

with  $\Omega_i$  being the set of buses adjacent to  $i$ . For a system with multiple ( $N$ ) branches, Equation (3) can be written in matrix form as

$$\mathbf{P}_{nodal} = \mathbf{B} \cdot \boldsymbol{\Theta} \quad (4)$$

where  $\mathbf{P}_{nodal}$  is the vector containing the net active power injections  $P_i$ ,  $\boldsymbol{\Theta}$  the vector of phase angles and  $\mathbf{B}$  is the nodal admittance matrix with the following entries:

$$B_{ij} = -\frac{1}{x_{ij}} \quad (5)$$

$$B_{ii} = \sum_{j \in \Omega_i} \frac{1}{x_{ij}} \quad (6)$$

Due to the fact that  $\mathbf{B}$  is singular, the row and column belonging to the reference bus is deleted (thus assuming a zero reference angle at this bus). The resulting vectors and matrix are named  $\mathbf{B}'$ ,  $\boldsymbol{\Theta}'$  and  $\mathbf{P}'_{node}$ . We can now solve Equation (4) for  $\boldsymbol{\Theta}'$ :

$$\boldsymbol{\Theta}' = \mathbf{B}'^{-1} \cdot \mathbf{P}'_{nodal} \quad (7)$$

Next, we consider the dependency between the load flow on line  $ij$  and the phase angle over the same line according to Equation (2) and find the matrix representation to be:

$$\mathbf{P}_{branch} = \mathbf{H} \cdot \boldsymbol{\Theta}' \quad (8)$$

with  $\mathbf{P}_{branch}$  the vector of the net active power flows  $P_{ij}$  and  $H_{ki} = 1/x_{ij}$ ,  $H_{kj} = -1/x_{ij}$  and  $H_{km} = 0$  for  $m \neq i, j$  (note that  $k$  runs over the branches  $ij$ ).  $\boldsymbol{\Theta}'$  can then be inserted in Equation (8) to give:

$$\mathbf{P}_{branch} = \mathbf{H} \cdot \boldsymbol{\Theta}' = \mathbf{H} \cdot \mathbf{B}'^{-1} \cdot \mathbf{P}'_{nodal} = \mathbf{PTDF} \cdot \mathbf{P}'_{nodal} \quad (9)$$

The elements of  $\mathbf{PTDF}$  are the power transfer distribution factors, constituting the linear relationship



between the load flows on the lines and nodal power balances. It is important to note that **PTDF** depends via **H** and  $\mathbf{B}'^{-1}$  in a nonlinear way on the line impedances  $x_{ij}$ , which is why changes to the grid capacities cannot be incorporated directly in the linear cost optimization.

In the next step, a market model will be introduced that simulates the dispatch of different power plants in different market regions and thus nodal power balances in a cost-minimizing manner. Power flows can then be calculated using the PTDF approach as it is introduced in this section, and an additional restriction ensures that line flows stay below thermal limits. Furthermore, the model will be implemented such that thermal limits (i.e. transmission capacity) can be increased when contributing to the cost-optimal solution.

## 2.2. Model for the cost optimal expansion of grid infrastructures

The above deduced linear power flow representation can be embedded in a linear electricity market model minimizing the costs of electricity supply. Herein, an exogenously given demand shall be supplied at least cost by the various technological options of generation and transmission. Electricity market models are commonly modeled as a linear optimization problem which is well suited for most applications, especially when large systems with high technological, spatial and temporal resolution are analyzed (e.g. Müsgens (2006) or Nagl et al. (2011)). With the methodology deduced in the previous section, load flow calculations and grid extensions can explicitly be included in such a linear program, such that generation and grid can be optimized jointly while respecting the underlying physics of the power flow problem.

Moreover, the methodology presented in this section is also able to account for possible DC grid extensions. Compared to the AC system, flows on the DC lines are much easier to deal with when assuming point-to-point connections that are equipped with converter stations (which is the common technological approach). This technical equipment makes it possible to perfectly control the flows on the corresponding line, such that trades can directly be settled via those lines (in other words, trades directly translate into physical flows).

Suppose that the level of demand in market  $i$  at time  $t$ ,  $D_i^t$ , is an exogenous parameter entering the optimization problem. The power that can be generated in market  $i$  at time  $t$  by technology  $a$  at costs of  $c_{i,a}^t$  is denoted by  $G_{i,a}^t$ . Furthermore, transmission capacities between  $i$  and  $j$  are denoted in vector-form by  $\mathbf{P}_{max}$  and can be built up at costs of  $\lambda$ . All quantities can vary with respect to space and time.

Within this framework, the following linear program formalizes the optimization problem for the cost-efficient supply of electricity including generation as well as AC and DC transmission expansion.

$$\min C_{tot} = \sum_{i \in I} \sum_{a \in A} \sum_{t \in T} G_{i,a,t} c_{i,a,t} + \lambda^{AC} \cdot \mathbf{P}_{max}^{AC} + \lambda^{DC} \cdot \mathbf{P}_{max}^{DC} \quad (10)$$

s.t.

$$\sum_{a \in A} G_{i,a,t} + \sum_{j \in J} T_{j,i,t} = D_{i,t} \quad (11)$$

$$T_{i,j} = T_{i,j}^{AC} + T_{i,j}^{DC} \quad (12)$$

$$\mathbf{P}^{AC} = \mathbf{PTDF} \cdot \mathbf{T}^{AC} \quad (13)$$

$$\mathbf{P}^{DC} = \mathbf{T}^{DC} \quad (14)$$

$$-\mathbf{P}_{max}^{AC} \leq \mathbf{P}^{AC} \leq \mathbf{P}_{max}^{AC} \quad (15)$$

$$-\mathbf{P}_{max}^{DC} \leq \mathbf{P}^{DC} \leq \mathbf{P}_{max}^{DC} \quad (16)$$

Equation (10), being the objective function, states that total costs for electricity supply shall be minimized. Costs arise from producing electricity on the one hand and costs related to transmission grid extensions on the other.<sup>1</sup> The equilibrium condition (Equation (11)) ensures that supply equals demand in each market region  $i$  at every instant in time  $t$ . Electricity can be supplied either by generation in the local market ( $G_{i,a,t}$ ) or by imports from other markets ( $T_{j,i,t}$ ). Trades can be settled via AC ( $T_{i,j}^{AC}$ ) or DC ( $T_{i,j}^{DC}$ ) grid infrastructures, as stated in Equation (12). Physical flows can be calculated as follows: For the AC grid, we use the methodology based on PTDFs as introduced in Section 2.1 and recaptured in Equation (13). For the DC grid, trades directly translate into physical flows (Equation (14)). The last two Equations (15) and (16) restrict the resulting flows to the line capacities  $\mathbf{P}_{max}$  that are currently installed. Line capacities in turn are subject to optimization.

As shown in Section 2.1, the PTDF matrix depends nonlinearly on the physical characteristics of the AC grid, especially on line reactances. When AC grid capacities are altered, the PTDF matrix will change, too. Thus, whenever the optimal solution includes increasing or decreasing line capacities, the underlying PTDF matrix that was used to deduce the optimum is no longer a valid one for the resulting system. Consequently, a new PTDF matrix has to be calculated based on the new grid infrastructure, and updated within the above optimization problem. The problem therefore has to be solved iteratively while updating the PTDF matrix every time the market model has found an optimal solution.

As the optimization problem formulated by Equations (10) to (16) may not have a bounded solution, or

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<sup>1</sup>For the sake of simplicity the expansion of generation capacity is not included at this stage. This condition can easily be relaxed, as done in the large-scale application presented in Section 4.

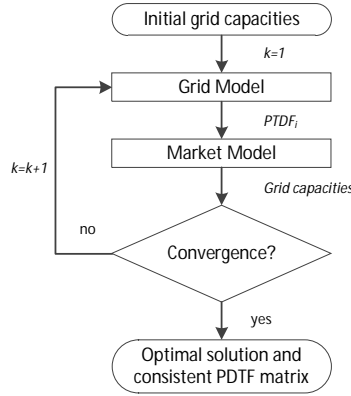
the changes in design may become too large, thus invalidating the linear approximations, move limits must additionally be imposed (Arora (2011)). These can be expressed by

$$-\Delta^{AC}(k) \leq \mathbf{P}_{max}^{AC}(k) - \mathbf{P}_{max}^{AC}(k-1) \leq \Delta^{AC}(k) \quad (17)$$

$$-\Delta^{DC}(k) \leq \mathbf{P}_{max}^{DC}(k) - \mathbf{P}_{max}^{DC}(k-1) \leq \Delta^{DC}(k) \quad (18)$$

where  $\Delta^{AC}(k)$  and  $\Delta^{DC}(k)$  denote the maximum allowed change in the optimization variables between the  $k-1$ th and the  $k$ th iteration step. Note that the optimization problem remains linear when introducing Equations (17) and (18).

A schematic representation of the resulting process is shown in Figure 1.



**Figure 1:** Schematic representation of the iterative process

The method described essentially solves a nonlinear optimization problem by iteratively linearizing and solving the resulting LP. Hence, it is conceptually close to the successive linear programming (SLP) approach that was introduced by Griffith and Stewart (1961). As an important feature of this class of algorithms solving nonlinear optimization problems, it can be shown that if the iteration converges to a point  $\mathbf{x}$  with  $\mathbf{x}(k) - \mathbf{x}(k-1) = 0$ , then  $\mathbf{x}$  is a Karush-Kuhn-Tucker (KKT) point (Bazaraa et al. (2006)).<sup>2</sup>

Note that an alternative approach to the process described in Figure 1 would be the calculation of the PTDF matrix directly in the market model according to Equation (9). However, as the elements of  $\mathbf{PTDF}$  depend on the line capacities  $\mathbf{P}_{max}^{AC}$ , this would make Equation (13) non-linear and the optimization problem difficult to solve, especially in large-scale applications. In fact, algorithms for non-linear optimization problems are either not effective or only find a local instead of the global optimum (see e.g. Boyd and Van-

<sup>2</sup>A KKT point is a feasible point of the optimization problem that satisfies the KKT conditions. The KKT conditions, in turn, are the first order necessary conditions for a solution in nonlinear programming to be optimal.

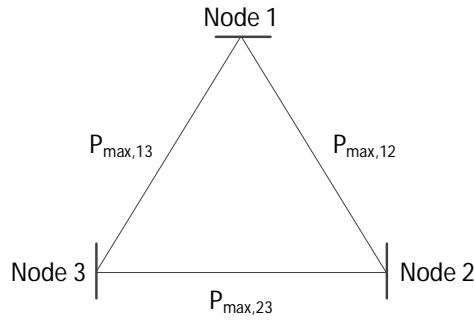
denberghe (2004)). In contrast, there are very effective methods for solving linear programming problems, such as the Simplex algorithm (see e.g. Murty (1983) or Todd (2002)).

### 3. Three node network

In this section the methodology developed in Section 2 shall be applied to a simple example. A three node network was chosen as this is the easiest setting with loop flows playing a role. As introduced in Equations (10) to (16), we only consider grid extensions at this stage in order to keep the example easy to follow. The cost-optimal solution for a full electricity supply shall be found that potentially involves transmission grid extensions, based on a given demand level as well as fixed available generation capacities and costs structures.

#### 3.1. Setting of the three node network example

The setting of the three node network considered in this part of the analysis is shown in Figure 2.



**Figure 2:** Three-node network considered in this section

We assume a system initially consisting of two nodes (1 and 2) with demand and generation levels of 300 MW each. In order to increase the security of supply, a line of 50 MW capacity has been built that connects these two nodes. In reaction to an increase in load of 500 MW in Node 1, an additional generation unit is built, characterized by a maximum nominal power of 600 MW and cheaper generation costs. However, it is located at a third Node 3 and thus needs grid connection. We consider a 10-year planning horizon for which the grid shall be optimized. Generation costs at Node 1 and 2 are 20 Euro/MWh or 1.752 Mio. Euro/MW supplied for 10 years, and 15 Euro/MWh or 1.314 Mio. Euro/(MW\*10a) at Node 3. Costs for grid upgrades amount to 1000 Euro per MW and km, with distances of 300 km between all nodes.

Table 1 summarizes load level  $P_{load}$ , available generation capacity  $P_{gen}$  and generation costs  $C_{gen}$  at each of the three nodes, as well as the costs for grid upgrades.

Parameter	Unit	Node 1	Node 2	Node 3
$P_{load}$	MW	800	300	0
$P_{gen}$	MW	300	300	600
$C_{gen}$	Mio.Euro/(MW*10a)	1.752	1.752	1.314
		Line 1-2	Line 1-3	Line 2-3
$C_{tran}^{AC}$	Mio.Euro/MW	0.3	0.3	0.3
$C_{tran}^{DC}$	Mio.Euro/MW	1.5	1.5	1.5

**Table 1:** Assumptions for the three node network example

Based on Equation (9), the transaction-based PTDF matrix for this network can be calculated as shown in Equation (19). Each entry of the matrix is labeled with the corresponding transaction  $T_{i,j}$  and impact on line  $L_{i,j}$  in order to facilitate reading.

$$PTDF = \frac{1}{x_{12} + x_{13} + x_{23}} \begin{matrix} & \begin{matrix} L_{13} \\ L_{12} \\ L_{23} \end{matrix} & \begin{matrix} T_{13} & T_{23} & T_{12} \\ \begin{bmatrix} x_{12} + x_{23} & x_{23} & x_{12} \\ x_{13} & -x_{23} & x_{13} + x_{23} \\ x_{13} & x_{13} + x_{12} & -x_{12} \end{bmatrix} \end{matrix} \end{matrix} \quad (19)$$

As in Hogan et al. (2010), we apply the law of parallel circuits when adding line capacities, such that the reactances' dependency on line capacity takes the following form:

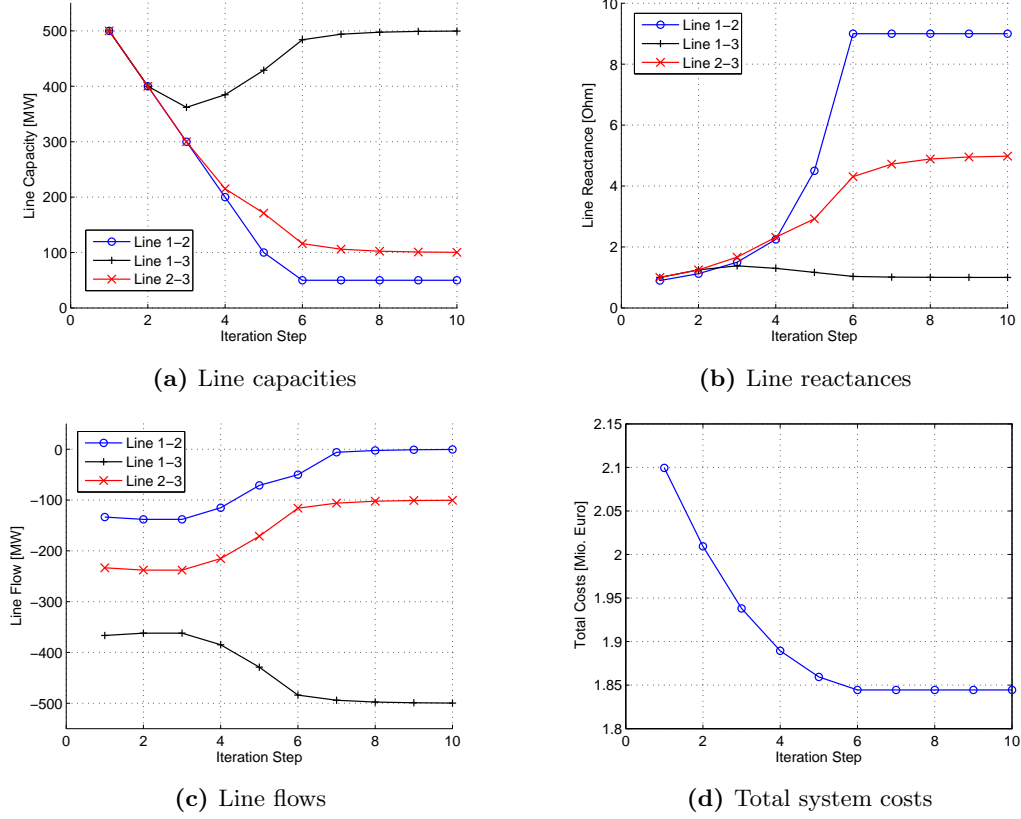
$$x_{ij}(k) = \frac{x_{ij}(1)}{P_{max,ij}(k)/P_{ij}(1)}. \quad (20)$$

Note that the algorithm needs a starting point for the iteration (i.e.  $x_{ij,1}$  and  $P_{ij,1}$ ). Starting from values for line capacities and corresponding line reactances the algorithm iteratively searches for optimal grid capacities while updating line reactances according to Equation (20).

Observing from Table 1 that Node 1 lacks 500 MW of generation that needs to be imported from outside, we set the initial value of all lines to 500 MW in order to start with a feasible solution. Furthermore, we assume corresponding line capacities of 1 Ohm on each line. For the move limit, we set  $\Delta^{AC} = \Delta^{DC} = 100MW$  to avoid unbounded solutions and changes in line capacities that become too large.

### 3.2. Results of the three node network example

Based on the setting presented in the previous section we run the model as it was developed in Section 2 to find the cost-optimal grid extensions when all three nodes shall be connected through an AC and/or DC line. The results are presented in Figure 3, showing all endogenous system properties that are subject to change when running the iterative simulation. Note that in this example, DC grid extensions are not part



**Figure 3:** Results of the three node network

of the optimal solution due to higher investment costs compared to AC transmission grids.

As line reactances are initially set to 1 Ohm each, a power transfer of  $x$  from Node A to B results in power flows of  $2/3 \cdot x$  on Line A-B and  $1/3 \cdot x$  on Lines A-C and C-B. Node 1 is lacking 500 MW of power generation that needs to be imported from Node 3, hence resulting in a flow of  $2/3 \cdot 500$  MW on Line 1-3 and  $1/3 \cdot 500$  MW via Node 2 (i.e. on Line 1-2 and Line 2-3). Furthermore, the optimal solution includes the usage of the full capacity available at lower costs in Node 3. Hence, 100 MW are supplied from Node 3 to Node 2, resulting in a flow of  $2/3 \cdot 100$  MW on Line 2-3 and  $1/3 \cdot 100$  MW via Node 1. This yields flows of  $133 \frac{1}{3}$  MW on Line 1-2,  $366 \frac{2}{3}$  MW on Line 1-3 and  $233 \frac{1}{3}$  on Line 2-3. In iteration step 1, total costs sum up to 2.0994 Mio. Euro. As the three lines are not deployed to their full capacity, they are downgraded by the maximum amount the move limit allows (100 MW) during the next step ( $k = 2$ ). Line flows remain equal as the reactances relative to each other do not change.

During the next iteration ( $k = 3$ ), necessary upgrades on Line 1-2 and 2-3 further decrease by another 100 MW, whereas Line 1-3 needs to remain greater than  $366 \frac{2}{3}$  MW (see above). As a consequence,

reactances on Lines 1-2 and 2-3 increase further than on Line 1-3, resulting in more power flowing on Line 1-3. Hence, in the following iterations, capacity on Line 1-3 increases, whereas the other two lines are further downgraded. This continues until iteration 6 in which Line 1-2 reaches its initial value of 50 MW and does not change any further. Capacities on Line 1-3 and 2-3 level off at 500 and 100 MW, respectively, and total system costs remain unchanged at 1.8444 Mio. Euro after  $k = 6$ . Even though line capacities, reactances and flows still change slightly after that, convergence can be assumed at this stage.

#### 4. Large-scale application

This section presents an application and extension of the previously developed method to the European power system. Specifically, an electricity market model and a transmission network model both covering the European power system are coupled via PTDFs in order to find the cost-optimal power system infrastructure development under the prescription of strongly decreasing CO<sub>2</sub> emissions in Europe until 2050. The following two main questions shall be answered within this framework:

- What does a cost-optimized European power system (both generation and grid) look like in 2030 (medium term) and 2050 (long term)?
- What is the cost-optimal system development to reach a 90% CO<sub>2</sub> reduction target if grid extensions are avoided as much as possible and what is the impact on the generation mix?

##### 4.1. Scenario definition

We define two scenarios to approach the questions that were raised. Years of reference included in the analysis are 2011, 2020, 2030, 2050. In both scenarios, political targets include the fulfillment of the National Renewable Energy Action Plans (NREAP), as well as a European CO<sub>2</sub> emission reduction quota that increases linearly up to 90% compared to 1990 levels in 2050. Nuclear power is assumed not to be an option for the cost-optimal investment decision in the model. However, all nuclear power stations that are currently under construction or already planned are included as exogenous extensions. Demand levels are taken from ENTSO-E for 2011 and extrapolated according to region-specific GDP growth (see Table A.5 in the Appendix). Furthermore, both scenarios assume line capacities for the year 2011 and 2020 not to be optimized. We argue that optimized grid extensions would not be realistic within this timeframe, due to the long planning and permission procedures of such projects. For the year 2011, the model represents current line capacities while for 2020 a number of mid-term grid extensions are included as reported in the Ten Year Network Development Plan (TYNDP) (ENTSO-E (2012)).

The difference between the scenarios refers to the conditioning of the transmission grid extensions. In *Scenario 1*, the transmission grid is cost-optimally extended. *Scenario 2* sheds light on the cost-optimal development of the European power system if grid extensions are avoided as much as possible – while still reaching ambitious CO<sub>2</sub> targets. *Scenario 2* thus deals with the issue of hardly being able to extend the power grid, e.g. due to long permission procedures or low social acceptance. Technically, in *Scenario 2* the market model is modified so that grid extension costs are assumed to be prohibitively high. The optimization algorithm is thus forced to search for the cost-optimal solution comprising as few grid extensions as possible. The main settings of *Scenario 1* and *2* are listed in Table 2.

	<b>Scenario 1</b>	<b>Scenario 2</b>
Grid expansion 2020-2050	Optimal	Avoided
Grid expansion until 2020	Limited to planned projects	
CO <sub>2</sub> reduction quota	90% in 2050 (compared to 1990)	
Renewable Energies	Realization of NREAP targets	
Nuclear power	Limited to planned projects (TYNDP)	

**Table 2:** Main settings for the scenario analysis

#### 4.2. European transmission network model

To analyze the power flows in the European transmission network, a detailed model of the high voltage grid is used. This model was developed with DIgSILENT’s power system calculation tool PowerFactory and covers all ENTSO-E members. It consists of a total of over 200 nodes, representing generation and load centers within Europe, 450 high voltage AC (HVAC) lines and all the high voltage DC (HVDC) lines within the ENTSO-E area. The grid model is built for AC load flow calculations and can thus be used not just for active power flows, but also to calculate losses within the network, reactive power flows and the necessary compensation to maintain network stability.

As a starting point for the iterations two versions of the grid model were prepared: one representing the European grid as it was in 2011 and another for the predicted state of the network in 2020. For the 2020 projection, it was assumed that all projects in mid-term planning from ENTSO-E’s Ten Year Network Development Plan will be built. In total 83 GVA of extra capacity was added for new HVAC lines and 15 GW for new HVDC lines between 2011 and 2020.

In 2011 and 2020 the majority of DC lines lie between the different synchronous zones of the ENTSO-E area, such as the undersea connection between France and Great Britain. To allow the market model the choice of extending the HVDC network, an Overlay Network of DC lines was constructed for 2030 and 2050 with DC connections permitted between all neighboring market regions. To take account of the effect of DC



transfers on the AC grid, a PTDF for DC transactions was calculated. This DC PTDF linearizes the effect of DC transfers on the AC network, capturing for example the power flows to and from the DC connection points.

Whereas in the market model all the load and generation is aggregated for each market region (i.e. assuming a copper plate with no internal power transfers), the grid model consists of multiple nodes per market region. In order to be able to determine nodal power balances and calculate line flows within the market model which change depending on the dispatch of generation technologies at any given time, the distribution of demand and generation assets across the nodes within each market region is determined using allocation keys, which are based on factors such as population density, siting of heavy industry as well as location of conventional and renewable energy power plants.<sup>3</sup> The allocation keys ( $\mathbf{K}$ ) were directly incorporated into the PTDF.  $\mathbf{K}^D$ ,  $\mathbf{K}^G$  and  $\mathbf{K}^{DC}$  denote the keys for demand, generation technologies and DC connection points, respectively. In this way the nodal power balances within the model can be determined for any dispatch situation, with the power flows then following directly from the usual PTDF. Thus Equation (13) is reformulated as follows:

$$\mathbf{P}^{AC} = \mathbf{PTDF} \cdot (\mathbf{K}^D \cdot \mathbf{D} - \mathbf{K}^G \cdot \mathbf{G} - \mathbf{K}^{DC} \cdot \mathbf{T}^{DC}) \quad (21)$$

#### 4.3. European electricity market model

The electricity market model used in this analysis is an extended version of the long term investment and dispatch model for conventional, renewable, storage and transmission technologies as presented in Richter (2011). It covers 29 countries (EU27 plus Norway and Switzerland) at an aggregated level (i.e. 16 market regions).<sup>4</sup> Endogenous investments in renewable energy technologies have recently been added to the model (Fürsch et al. (2013)).<sup>5</sup>

The model determines possible paths of how the installed capacities will develop and how they are operated until 2050, assuming that the European markets will achieve the cost-minimizing mix of different technologies - a market result that is achieved under perfect competition. The objective of the model is to minimize accumulated discounted total system costs<sup>6,7</sup> while being subject to several techno-economic

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<sup>3</sup>The network as well as the allocation keys were validated by comparing cross-border flows in the model against publicly available data from ENTSO-E, after which the impedances and allocation keys were optimized to ensure good agreement across several snapshots of the network.

<sup>4</sup>The aggregation was done due to very long computational times.

<sup>5</sup>Earlier versions included investments in renewable energy technologies as an exogenous parameter.

<sup>6</sup>Accumulated discounted total system cost include investment costs, fixed operation and maintenance costs, variable production costs and costs due to ramping thermal power plants as well as grid extension costs. Investment costs are annualized with a 5% interest rate for the depreciation time.

<sup>7</sup>Minimizing accumulated discounted total system costs implies a cost-based competition of electricity generation and perfect foresight.

restrictions (such as the hourly matching of supply and demand, fuel availabilities or potential space for renewable energies), and politically implied restrictions (such as an EU-wide CO<sub>2</sub> emission reduction target or limited nuclear power deployment). The dispatch is calculated for eight typical days per year on an hourly basis (scaled to 8760 hours), representing variations in electricity demand as well as in solar and wind resources along with their multivariate interdependencies (ENSTO-E (2012), EuroWind (2011)). To account for local weather conditions, the model considers several wind and solar power regions (subregions) within market regions based on hourly meteorological wind speed and solar radiation data (EuroWind (2011)). Previous model versions have been applied e.g. in Nagl et al. (2011) or Paulus and Borggreffe (2011), whereas a detailed model description is contained in Fürsch et al. (2013). The main market model assumptions as they were used in this study are listed in Appendix A.

For the study presented in this paper, the market model has been extended to include the grid optimization algorithm as presented in Equations (10) to (16) and (21). Note that a flow-based congestion management is modeled by introducing these equations.

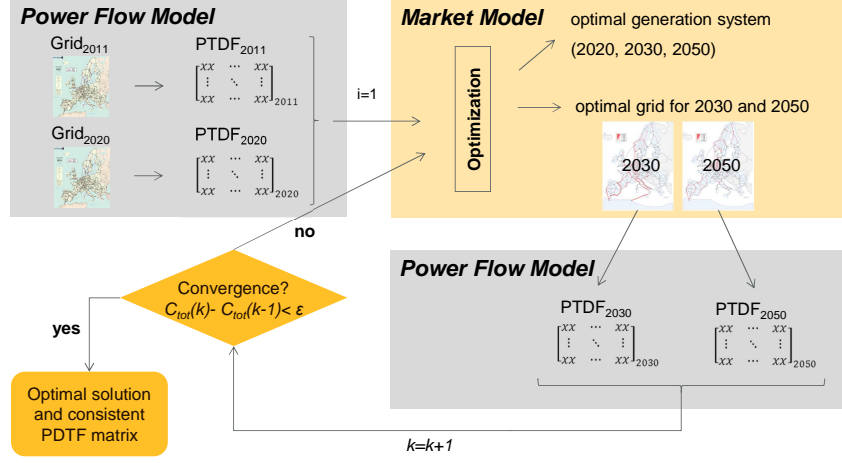
#### 4.4. Iteration between the models

As described in Section 2, the interface between the transmission network model and the electricity market model is defined by the PTDF matrix. As the corresponding grid infrastructures enter the transmission network model exogenously, PTDF matrices can directly be calculated from the power flow model for the years 2011 and 2020 in a consistent manner.

For later years, however, grid extensions are optimized, leading to variations in line capacities with respect to initial values. These extensions alter the impedances within the transmission network model, which in turn change the PTDF matrix. Since the way the PTDF changes is non-linear, it cannot be incorporated directly into the linear optimization problem, so instead the PTDF is updated iteratively until it converges on the optimal consistent solution. As a starting point for the 2030 and 2050 networks, the 2020 PTDF is used.

The convergence is analyzed with respect to the difference in the accumulated (i.e. 2011-2050) discounted total system costs (discount rate is assumed as 5%) between two iteration steps  $k$  and  $k + 1$ . As a criterion to stop the iterations, a change in total system costs between iterations of  $\epsilon = 0.5$  bn. Euro was used.

On running the optimization in *Scenario 2*, it is found that the grid extensions in *Scenario 2* are only marginal (see below). In fact, they are so small that no iteration of the PTDFs is necessary in *Scenario 2*. Hence, the results of the iterative process that are presented hereafter correspond to *Scenario 1* only.

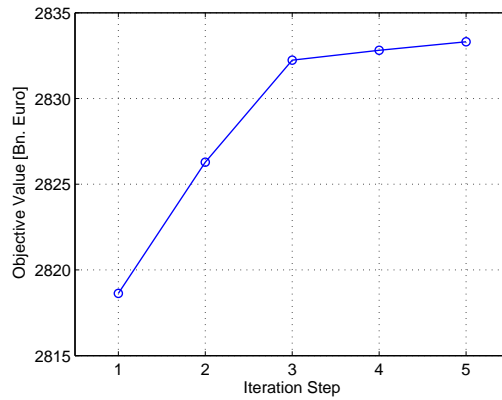


**Figure 4:** Iteration between the market and the power flow model of the European power system

#### 4.5. Results of the large scale application

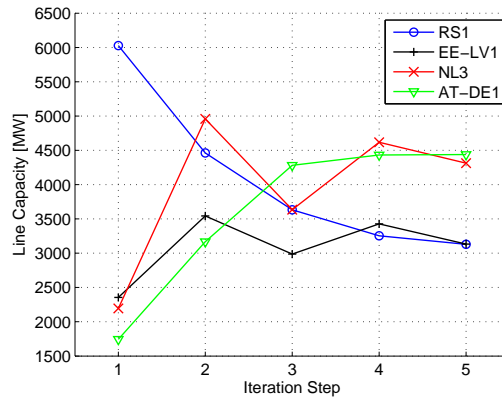
##### 4.5.1. Development of endogenous model results during the iteration in Scenario 1

While iteratively running the electricity market and the transmission network model in *Scenario 1*, we trace total system costs and check for convergence. The development of total system costs is reported in Figure 5. By comparing the total system costs of two subsequent iteration steps we find the results converging after step 5 as  $C_{tot}(5) - C_{tot}(4) = 0.495 < 0.5$ . Note the monotonic change in total system costs that was also observed in the three node network. However, in the three node network we have started with an initial guess ( $k = 1$ ) that included an excess amount of transmission capacity and hence costs that were reduced during the iterative solution of the optimization problem. In contrast, the large-scale application uses the grid configuration of 2020 as an initial guess which is well below the extent of grid capacities in the optimal solution and thus causes the objective value to grow during the iterative process.



**Figure 5:** Development of accumulated discounted total system costs during the iteration in Scenario 1

Whereas the change in total system costs is low in relative terms, optimal line capacities change more substantially, as can be seen in Figure 6 where the development of four exemplary lines is presented. Compared to total system costs, not all of the optimal line capacities show monotonic change; there is also zigzagging behavior, however with decreasing amplitude. Overall, optimal line capacities as well as other results that are subject to change during the iteration (e.g. generation capacities) are observed to converge. We can thus conclude that the methodology developed in Section 2 is applicable and robust not only for small and simple test systems, but also for extremely large scale problems, such as the one presented in this section.



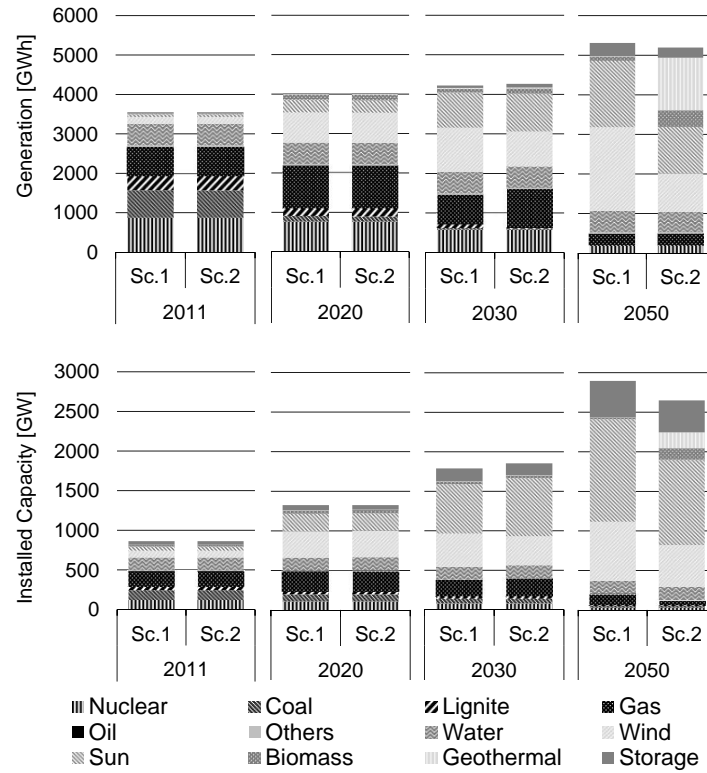
**Figure 6:** Development of optimal line capacities during the iteration in Scenario 1 (exemplary)

For *Scenario 2*, no iteration was necessary as line capacities and thus PTDF matrices only change marginally (see below). Consequently, the initial PTDF matrices are consistent with the results obtained during the optimization.

#### 4.5.2. Scenario results

The aggregated European capacity and generation mix as determined in the two scenarios are reported in Figure 7. In both scenarios, an equally increasing demand has to be covered by an increasingly renewable-based generation mix. The switch from conventional to renewable based generation is driven by a restrictive CO<sub>2</sub> quota that implies a 90% reduction compared to 1990 levels in 2050 and the restrictive use of nuclear power. As a large share of the electricity generation is provided by technologies deploying fluctuating forms of renewable energy sources (namely wind and solar power) with unstable and low utilization rates, the system calls for technological options offering flexible and securely available power. These are provided by different means in the two scenarios. In both scenarios natural gas power stations as well as storage devices are deployed. However, the usage of gas power plants is restricted by emission reduction targets,

and storage facilities are assumed to be comparatively expensive, even in the long term. As another mean of integrating flexible and securely available power, *Scenario 1* substantially extends the grid infrastructures to take advantage of balancing effects between regions (arising from different load as well as wind and solar power generation profiles). In contrast, *Scenario 2* which is forced to avoid grid extensions as much as possible, geothermal power is deployed as it is available close to load centers and offers high utilization rates as well as flexible and securely available power.



**Figure 7:** Development of the European generation and capacity mix in Scenarios 1 and 2

The amount of grid extensions is presented in Table 3. By assumption, the grid is not optimized but equally extended until 2020 in both scenarios. As can be observed, *Scenario 1* is characterized by extensive grid extensions during the entire time frame considered in the optimization. As the optimization algorithm avoids grid extensions as much as possible in *Scenario 2*, very little grid capacity is added to the system – thus favoring the alternative of building near-load generation.

Figure 8 presents both the location of the optimized grid extensions between 2020 and 2050 (left hand side) as well as the final line capacities in 2050 (right hand side) for *Scenario 1*.<sup>8</sup> As presented, the overlay

<sup>8</sup>In *Scenario 2*, the transmission grid infrastructure remains almost equal to the 2020 level

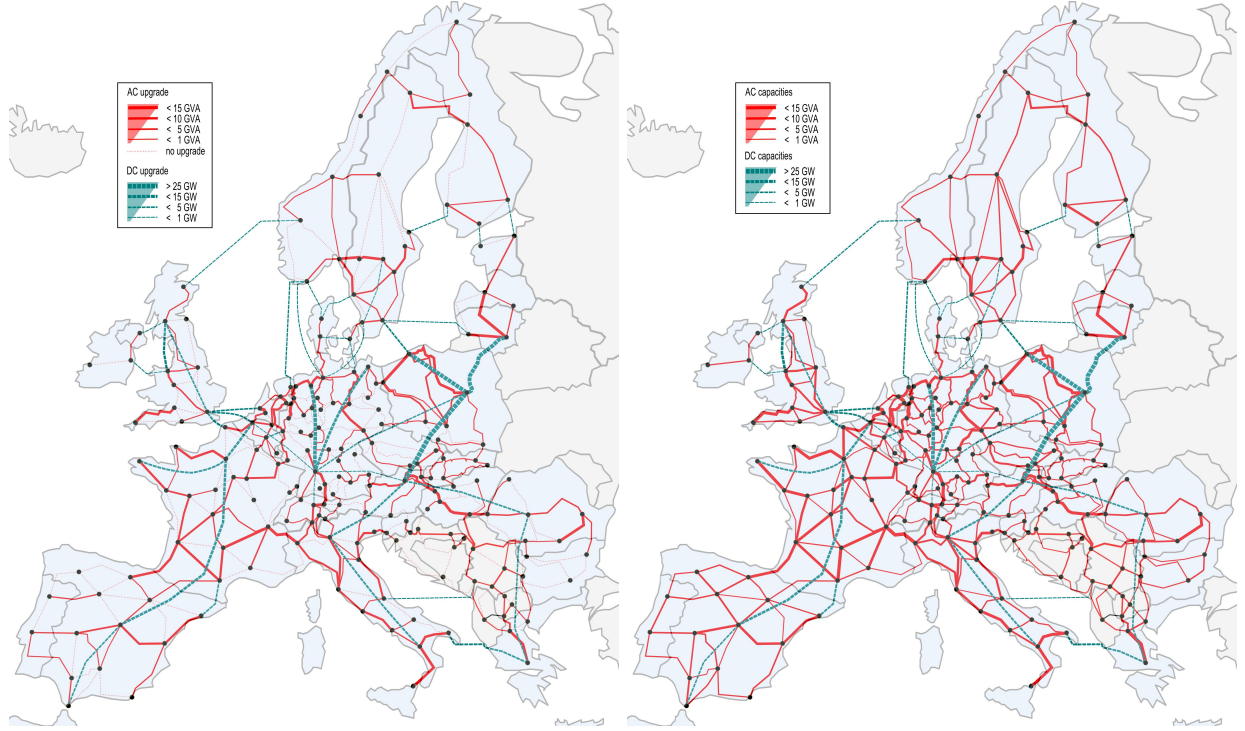
	<b>Scenario 1</b>		<b>Scenario 2</b>	
	AC Grid [GVA]	DC Grid [GW]	AC Grid [GVA]	DC Grid [GW]
Installed capacity 2011	967.9	9.3	967.9	9.3
Capacity added 2011-2020	82.8	14.9	82.8	14.9
Capacity added 2020-2030	331.0	30.2	0.0	0.0
Capacity added 2030-2050	611.8	181.1	6.2	0.0
Installed capacity 2050	1993.5	235.5	1056.8	24.2

**Table 3:** AC [GVA] and DC [GW] grid extensions in *Scenario 1* and *2*

DC grid is mainly used to transport power from (remote) renewable production sites to load centers, e.g. wind power from the Northern coasts of Great Britain to London, and further towards the continent (i.e. France, Belgium and the Netherlands), wind power from Northern Germany, Northern Poland and the Baltic towards load centers in Central Europe or solar power from Southern Spain to Madrid and further towards France. The optimization model includes DC grid extensions in the cost-efficient solution due to its capability of providing point-to-point transfers over large distances. These avoid large-scale AC extensions (including those that become necessary due to parallel loop flows) as they are direct corridors between the two points of connection. Furthermore, a limit of 15 GVA was set for each of the aggregated AC transmission lines in the model, representing the maximum amount that can be built and transported in a single power corridor.

When comparing *Scenario 1* and *2*, three main effects can be identified that are caused by avoiding grid extensions as much as possible:

- An increasing quantity of renewable energy supply is curtailed due to lower grid capacities. These are insufficient to transport the entire amount of available electricity production to the consumer during high infeed hours. In 2050, the shares of curtailed energy in *Scenario 2* reach 4.8, 14.9 and 29.8% for PV, onshore and offshore wind, respectively, compared to only 0.4, 2.7 and 6.4% in *Scenario 1*.
- In *Scenario 2*, renewable energy technologies cannot be deployed at sites where the underlying resource availability is best. This can be observed in the average utilization rates of the PV facilities and wind turbines. Whereas PV installations achieve on average 1296 full load hours in *Scenario 1*, the value drops to 1089 h in *Scenario 2*. The decrease is even more drastic for wind power, with the average number of full load hours dropping from 2837 to 1833 h. Note that the reduction in utilization rates also stems from increasing needs of curtailment due to grid congestions.
- Alternative renewable energy sources are deployed as wind and solar power area potentials close to load centers are exhausted. In order to avoid grid extensions, alternative sources of renewable energy



**Figure 8:** Grid upgrades between 2020 and 2050 (left hand side) and grid capacities in 2050 (right hand side) in Scenario 1

need to be tapped that are available close to where the electricity is consumed. This applies to biomass and geothermal power. Note that in contrast to fluctuating wind and solar power, these two forms of renewable energy are dispatchable and thus comparatively flexible and securely available, resulting in lowered storage capacities in *Scenario 2*, although they are also relatively expensive according to the cost assumptions of the model.

The above mentioned effects lead to higher total system costs in *Scenario 2* than in *Scenario 1*. In fact, the difference between the total system costs in the two scenarios considered in this paper can be interpreted as the economic value of grid extensions for the European power system until 2050. Total system costs are reported and compared in Table 4. The additional costs of restricting grid extensions to a minimum amount to 591 bn. Euro, thus representing a 20.9% increase compared to a scenario with optimal grid extensions.

	Scenario 1	Scenario 2
Total system costs [bn. Euro]	2833	3424
Cost difference [bn. Euro]	591 (20.9%)	

**Table 4:** Comparison of accumulated discounted total system costs until 2050 in *Scenario 1* and *2*

## 5. Conclusions

Joint optimization of generation and transmission assets in power systems is an important yet difficult task, mainly due to the fact that different characteristics and rules apply to commercial and physical exchanges of electricity in meshed networks. In this paper a method is developed based on an iterative PTDF calculation that is suitable for determining the cost-optimal extension of large-scale power systems - such as the European interconnected network - including investment and dispatch of generation, storage and grid facilities. An interface is implemented based on PTDF matrices that couples electricity market and transmission network models and combines the inherent advantages of both model types. Specifically, the algorithm is formulated as a linear optimization problem that can be solved efficiently in an iterative manner. It is tested for a simple three node network where it is found to be robust and convergent.

The method is then applied in a large-scale case study to find the cost-optimal power system infrastructure development under the prescription of strongly increasing CO<sub>2</sub> emission reduction targets in Europe until 2050 by using a linear European electricity market model and a European transmission network model. Results that are subject to change during the iteration are found to converge rapidly after only 5 iteration steps. We thus conclude that the methodology developed is applicable and robust not only for small and simple test systems, but also for extremely large scale problems. Cost-optimal transmission grid extensions in the European power system (*Scenario 1*) are found to be substantial, as they help to deploy renewable energy sources at sites characterized by high resource availability (mainly wind and solar) as well as exploiting balancing potentials (particularly with respect to load as well as wind and solar power generation) on a European scale and hence in a cost-efficient manner. In contrast, by avoiding transmission grid extensions as much as possible (*Scenario 2*), power has to be generated close to load centers. Hence, less favorable sites with respect to wind and solar resources are deployed. Moreover, restrictive area potentials for wind and solar technologies entail the need to additionally deploy other, more expensive types of renewable energies, such as geothermal and biomass energy. Overall, the inefficiency induced by avoiding transmission grid extensions as much as possible amounts to 591 bn. Euro by 2050, thus representing a 20.9% increase compared to the cost-efficient solution.

The approach presented in this paper could be further developed in various directions. The market model could be formulated as a mixed-integer problem that can still be solved effectively and that would allow restricting grid extensions to multiples of available line configurations. Increasing the spatial disaggregation towards a nodal-pricing regime would be another interesting extension. Furthermore, the methodology could be applied to answer a number of relevant questions in the context of power system planning and market



design, e.g. by analyzing the optimal trade-off between grid extensions, storage capacities and renewable energy curtailment or by determining the gains in social welfare that could be created when switching from NTC to flow-based market coupling.

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## Appendix A. Market model assumptions

Country	2011	2020	2030	2050
Belgium	102.60	116.13	116.13	130.24
Bulgaria	36.91	41.83	41.83	52.32
Czech Republic	73.71	87.96	99.00	123.83
Denmark	37.20	41.45	41.45	46.49
Germany	605.49	612.05	630.66	630.66
Estonia	9.79	11.06	11.06	13.83
Ireland	29.37	34.16	34.16	38.31
Greece	62.20	71.86	71.86	94.43
Spain	297.91	416.54	416.54	547.36
France	555.80	598.95	642.85	720.96
Italy	347.83	407.45	469.37	616.79
Latvia	7.01	10.00	10.00	12.51
Lithuania	10.93	14.00	14.00	17.51
Luxembourg	6.47	7.00	7.00	7.85
Hungary	44.21	52.40	52.40	65.55
Netherlands	124.94	135.85	135.85	152.36
Austria	66.40	77.53	77.53	86.95
Poland	155.84	202.36	202.36	253.10
Portugal	55.22	66.54	66.54	87.43
Romania	66.41	86.52	86.52	108.21
Slovenia	14.14	15.61	15.61	19.52
Slovakia	30.27	35.55	35.55	44.47
Finland	90.37	101.65	101.65	114.00
Sweden	160.30	174.18	174.18	195.35
United Kingdom	372.16	397.75	397.75	446.07
Switzerland	57.49	65.42	65.42	73.37
Norway	104.34	118.73	118.73	133.15
<b>Sum</b>	<b>3525.31</b>	<b>4000.51</b>	<b>4136.00</b>	<b>4832.61</b>

**Table A.5:** Assumptions for the gross electricity demand [TWh]

<b>Grid Technology</b>	<b>Extension costs</b>
AC overhead line incl. compensation	445 Euro/(MVA*km)
DC overhead line	400 Euro/(MW*km)
DC underground	1250 Euro/(MW*km)
DC submarine	1100 Euro/(MW*km)
DC converter pair	150000 Euro/MW

**Table A.6:** Assumptions for the grid extension costs

<b>Technology</b>	<b>2011</b>	<b>2020</b>	<b>2030</b>	<b>2050</b>
CCGT	1250	1250	1250	1250
CCGT CHP	1500	1500	1500	1500
CCGT CHP CCS	x	x	1700	1600
Hard Coal	1500	1500	1500	1500
Hard Coal CHP	2650	2650	2275	2050
Hard Coal CHP CCS	x	x	2875	2600
Lignite	1850	1850	1850	1850
Lignite CCS	x	x	2550	2450
Nuclear	3157	3157	3157	3157
OCGT	700	700	700	700
Oil	800	800	800	800
Biomass gas chp	2600	2597	2595	2590
Biomass gas lc	2400	2398	2395	2390
Biomass solid	3300	3297	3293	3287
Biomass solid chp	3500	3497	3493	3486
CAES	850	850	850	850
CSP	x	3989	3429	2805
Enhanced geothermal system	15000	10504	9500	9026
Geothermal high enthalpy	1500	1050	950	903
Hydro storage	x	x	x	x
Pump storage	x	x	x	x
PV ground	1532	1167	842	661
PV roof	1702	1297	935	734
Run of river	x	x	x	x
Wind Offshore	3100	2200	1900	1700
Wind Onshore	1250	1200	1150	1050

**Table A.7:** Assumptions for the generation technology investment costs [Euro/kW]

<b>Fuel type</b>	<b>2011</b>	<b>2020</b>	<b>2030</b>	<b>2050</b>
Nuclear	3.6	3.7	3.7	3.9
Lignite	1.4	1.45	1.45	1.45
Oil	60.4	99	110	116
Coal	11.8	12.5	12.8	13.1
Gas	18.2	25.2	28.3	31.3

**Table A.8:** Assumptions for the gross fuel prices [Euro/MWh<sub>th</sub>]