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# **Stochastic Stability in Network with Decay**

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# Stochastic Stability in Network with Decay

## Summary

This paper considers a simple communication network characterized by an endogenous architecture and an imperfect transmission of information. We analyze the process of network formation in a dynamic framework where self – interested individuals can form or delete links and, occasionally, are doing mistakes. Then, using stochastic stability, we identify which network structures the formation process will converge to.

**Keywords:** Network, Decay, Strategical interaction

**JEL Classification:** A14, D20, J00

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## 1. Introduction

In recent years there has been a growing interest in social and economic networks. Given the large evidence that network structure affects economic outcomes, a very important issue is to know which network structure will form. Indeed many authors have examined the evolution of the interaction structure in different economic contexts. A very common characteristic of the networks is the presence of decay, that is the value an individual receives from another is a decreasing function of the number of links of the shorter path among them. Decay could be considered as the effect of generic frictions in the relations among agents, for example noise or delay, that are inevitable in the real world. But with the introduction of decay the network models become very complicated and it is very difficult to provide a complete description of all possible equilibria. In this paper, under the assumption that agents do mistakes, we show a way to provide a full description of all stochastically stable equilibria without the need to know all possible equilibria when agents do not make errors.

We consider the two-way flow model with decay described in Bala and Goyal [1]. It considers a setting in which each individual is a source of benefits, agents unilaterally<sup>1</sup> form (costly) links to access those benefits and in a link the benefits flow in both directions (two-way flow) without distinction of who supports the link cost. Thus individual links generate externalities whose value depends on the associated level of decay. This model does not provide a complete description of all possible equilibria for a large range of the model's parameters and does not provide any result regarding the dynamic process. The contribution of our paper is that, even

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<sup>1</sup> One player doesn't need another player's permission to form a link with him and the costs of link formation are supported only by the agent who initiates the link.

though we are not able to describe completely all equilibria system converges, we can provide a full description of a special subset of them: all stochastically stable equilibria in networks with a sufficiently large number of agents. To do this we need a little change in the model's assumptions: in the strategy revision process we allow only nonsimultaneous revisions. With this modification the model becomes more manageable remaining almost identical to that described in Bala and Goyal [1]. Of course, all strict Nash equilibria are the same in both models. To single out stochastically stable equilibria, we have to assume that agents are making mistakes in their decision process; then we study the limit of the invariant distribution as the probability of mistakes goes to zero. The result is a full characterization of equilibria for all possible values of parameters; the equilibrium network architectures are very simple for a sufficiently large number of agents: complete, star or empty network according to link cost.

Another paper that studies the network formation in presence of decay is that of Watts [21]. It considers the dynamics of network formation in the case of the connection model of Jackson and Wolinski [14] and shows that the resulting network structures are path-dependent. However, their approach differs significantly from ours because they restrict attention to models where, to form links, the consensus of both implied players is necessary <sup>2</sup>. Other papers related to network formation are those of Jackson and Watts [12] and Goyal and Vega Redondo [9]. Jackson and Watts study the network formation in a setting where players can form and sever links to play a coordination game and, occasionally, make mistakes; then stochastic stability is used to identify the limiting networks. Goyal and Vega Redondo study a similar model with the difference that links are one-sided. They

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<sup>2</sup> The link cost is supported by both implied players

consider both cases where the benefit derives only from directly linked players, and those where the benefit derives from directly and indirectly linked players.<sup>3</sup>

The paper is organized in the following way: In section 2 we describe the model. Section 3 contains the main result. Section 4 discuss the possibility of generalizing the result in different contests. Section 5 concludes the discussion and provides possible directions for further research.

## 2. The Model

Let  $N = \{1, 2, \dots, n\}$  be a set of agents where  $n \geq 3$ . We assume that every agent is endowed with one unit of private information of value 1 as well as of a quantity of information derived from other agents in the network.

Each agent can choose a subset of other players with whom to establish links. Let  $g_i = (g_{i,1}, \dots, g_{i,i-1}, g_{i,i+1}, \dots, g_{i,n})$  be the set of links formed by player  $i$  where  $g_{ij} \in \{0, 1\}$  for each  $j \in N \setminus \{i\}$ . We say agent  $i$  forms a link with agent  $j$  if  $g_{ij} = 1$ . The set of all players' link decisions, denoted by  $g = (g_1, g_2, \dots, g_n)$ , defines a direct graph  $\{N, \Gamma\}$  called network. The network will be denoted by  $g$  and the set of all possible network will be denoted by  $G$ . Specifically, the network  $g$  has the set of players  $N$ , as its set of vertices, and its set of arrows,  $\Gamma \subset N \times N$ , is defined as follows:

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<sup>3</sup> On the same issue the paper of Bramoulle, Goyal, Lopez Pintado and Vega Redondo [3] considers the link formation in a setting where agents form links to play an anti-coordination game. Droste, Gilles, Johnson [4] consider the dynamic of link formation in a population where players have a fixed location.

$$(2.1) \quad \Gamma = \{(i, j) \in N \times N : g_{ij} = 1\}$$

Given a network  $g$ , we say that 2 players are directly linked if at least one of them has established a link with the other one, i.e.  $\max\{g_{ji}, g_{ij}\} = 1$ . To describe the direct links with no regard to who supports them, we define the *closure*  $\bar{g}_{ij} = \max\{g_{ij}, g_{ji}\}$ . Let  $\bar{g}_i = (\bar{g}_{i,1}, \dots, \bar{g}_{i,i-1}, \bar{g}_{i,i+1}, \dots, \bar{g}_{i,n})$  be the set of direct links of agent  $i$ . Then  $\bar{g} = (\bar{g}_1, \bar{g}_2, \dots, \bar{g}_n)$  describes the graph with no regard to who supports the links.

Let  $N^d(i; g) \equiv \{j \in N : g_{i,j} = 1\}$  be the set of players in network  $g$  with whom player  $i$  has established links, while  $v^d(i; g) \equiv |N^d(i; g)|$  is its cardinality. In a similar way, let  $N^d(i; \bar{g}) \equiv \{j \in N : \bar{g}_{i,j} = 1\}$  be the set of players in network  $g$  with whom player  $i$  is connected, while  $v^d(i; \bar{g}) \equiv |N^d(i; \bar{g})|$  is its cardinality.

We say there is a *path* in  $g$  between  $i$  and  $j$  if  $\bar{g}_{ij} = 1$  or there exists a set of agents  $\{j_1, j_2, \dots, j_m\} \in N \setminus \{i, j\}$  such that  $\bar{g}_{ij_1} = \bar{g}_{j_1 j_2} = \dots = \bar{g}_{j_m j} = 1$ . By  $T_{ij}$  we denote the set of all paths between agents  $i$  and  $j$ .

In  $g$  the distance between agents  $i$  and  $j$ , denoted by  $d(i, j; g)$ , is defined as the number of links of the shorter path in  $T_{ij}$ <sup>4</sup>. A sub-network  $g' \subset g$  is called a *component* of  $g$  if for all  $i, j \in g'$ ,  $i \neq j$ , there exists a path in  $g'$  connecting  $i$  and  $j$ , and there does not exist a path between an agent in  $g'$  and one in  $g \setminus g'$ . A network with only one component is called *connected*.

Given any  $g$ , the notation  $g + ij$  denotes the network obtained with the formation of a new link between agents  $i$  and  $j$ ; similarly,  $g - ij$  refers to the network obtained deleting the link between agents  $i$  and  $j$ .

By *minimally connected* we mean a connected network  $g$  such that  $g - ij$  is not connected for all  $i, j \in g$  characterized by  $g_{ij} = 1$ .

A network is called *essential* if  $g_{ij} \cdot g_{ji} = 0$  for  $\forall i, j \in N$ ; *empty* and denoted by  $g^e$  if  $\bar{g}_{i,j} = 0$  for  $\forall i, j \in N$ ; *complete* and denoted by  $g^c$  if  $\bar{g}_{i,j} = 1$  for  $\forall i, j \in N$ ; *star* and denoted by  $g^s$  if there exists some  $i \in N$  such that  $\bar{g}_{i,j} = 1$  and  $\bar{g}_{k,j} = 0$  for all  $k, j \in N \setminus \{i\}$  and  $j \neq k$ ; among the star networks we denote by  $g^{cs}$  the star with all links supported by the central agent, by  $g^{ps}$  the star with all links supported by peripheral agents and by  $g^{ms}$  all the intermediate cases. Finally we define the following sets of networks:  $G^c$  is the set of all essential  $g^c$ ;  $G^s$  is the set of all essential  $g^s$ ;  $G^{ps}$  is the set of all essential  $g^{ps}$ ;  $G^{cs}$  is the set of all essential  $g^{cs}$ .

The links are costly: every agent pays a cost  $k > 0$  for each link she supports. In our model, as in Bala and Goyal [1] or Goyal and Vega Redondo [9], link formation is one-sided and non-cooperative: the formation of a link requires only the consensus of the supporting player.

In our model decay is exogenous. Let  $\mathbf{d}$  be the share of information that a player receives from another directly linked player.

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<sup>4</sup> The shorter path is that with the lower number of direct links and if a path between  $i$  and  $j$  does not exist we assume  $d(i, j; g) = \infty$ .



For a generic agent  $i$  the strategy space is identified by  $G_i$ , that is, the set of possible link decisions. In the following we consider that  $G_i = G_j \forall i, j \in N$ . Then, given the strategies of other players,  $g_{-i} = (g_1, \dots, g_{i-1}, g_{i+1}, \dots, g_n)$ , the payoff to a player  $i$ , from her participation to the game playing some strategy  $g_i$ , is given by:

$$(2.2) \quad \Pi(g_i, g_{-i}) = \sum_{j \in N \setminus i} d^{d(i,j;g)} - k \cdot v^d(i;g)$$

Time is modeled discretely;  $t = 1, 2, 3, \dots$ . At time  $t$  the state of the system will be given by strategy profile  $g(t)$  specifying the links established by each player. At every period  $t$  one agent is randomly chosen to revise her strategy. When an agent receives this opportunity, she selects a best-response to the strategy profile in the previous periods:

$$(2.3) \quad g_i(t) \in \operatorname{argmax}_{g_i \in G_i} \Pi[g_i, g_{-i}(t-1)];$$

If there are several best-responses, any one of them is chosen with equal probability. This strategy revision process defines a Markov chain on  $G \equiv G_1 \times G_2 \times \dots \times G_n$ . In the following we denote this process by *unperturbed dynamic or selection mechanism*. As we will see, in our framework, this Markov chain could be characterized by several absorbing sets<sup>5</sup>. Then, the equilibria depend upon the initial conditions.

To select among all possible equilibria, we employ the standard techniques used by Kandory, Mailath and Rob [15] and Young [21]<sup>6</sup>. We

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<sup>5</sup> A nonempty set of networks is *absorbing* if it is a minimal set with the property that, under the selection mechanism, the probability to exit from it is zero; moreover we note that an absorbing set may contain many networks or may contain only a single network, in this case we call it *absorbing network*.

<sup>6</sup> To see other applications Ellison [5 and 6], Robson and Vega Redondo [18].

suppose, conditional on the chance to revise their strategy, players make mistakes. In this case, the player chooses her strategy at random with some small probability  $\epsilon > 0$ . For any  $\epsilon > 0$ , the process defines an aperiodic and irreducible Markov chain that has a unique invariant probability distribution  $\mathbf{m}_\epsilon$ . We analyze the structure of  $\mathbf{m}_\epsilon$  as the probability of mistakes  $\epsilon$  converges to zero. A network  $g$  is called *stochastically stable* if  $\hat{\mathbf{m}}(g) > 0$  where  $\hat{\mathbf{m}} = \lim_{\epsilon \rightarrow 0} \mathbf{m}_\epsilon$  and the set of stochastically stable networks is defined as  $\hat{G} \equiv \{g \in G : \hat{\mathbf{m}}(g) > 0\}$ .

### 3. Results

In this section we describe the equilibria of the model and show the proofs. These results are strictly related to those in section 5 in Bala and Goyal [1]. In proposition 5.3, these authors give a complete description of strict Nash equilibria only for link cost of  $k < \mathbf{d} - \mathbf{d}^2$ . But for different link costs, a multiplicity of network architectures can exist that can be strict Nash equilibria and that could be, given the initial conditions, equilibrium states the system will converge to. Bala and Goyal give a complete description of strict Nash and prove the convergence to strict Nash equilibria in all parameter regions only for  $n = 4$ . But for a general value of  $n$  they only show how, from every initial configuration, the dynamic process converges to  $g^c$  when  $k < \mathbf{d} - \mathbf{d}^2$  and to  $g^e$  when  $k > \mathbf{d} + (n-2) \cdot \mathbf{d}^2$ . So they leave open the question on which network configurations will form in the interval  $\mathbf{d} - \mathbf{d}^2 < k < \mathbf{d} + (n-2) \cdot \mathbf{d}^2$ . We do not solve this problem but

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<sup>7</sup> Table IV in Bala and Goyal [1] shows the results of numerical simulation for different values of  $n$ ; they show that, with a positive probability, the unperturbed dynamics can generate networks that are different from a star, like double stars, limit cycles and other architectures.

we are able to provide a complete description of a special subset of equilibria using the concept of stochastic stability. But the model remains sufficiently complex to require a further simplification to carry on the analysis: in each period we only permit one agent to revise the strategy. The interpretation is that strategy revisions are governed by a Poisson process; in other words there exist sufficiently small intervals of time in which only one revision can take place. This modification in the revision process permits us to rule out the possibility of simultaneous revisions producing a simpler dynamic analysis and it does not change the static results described in Bala and Goyal [1]. Indeed the strict Nash equilibria are identical in the two models. The following theorem describes all stochastically stable states.

**Theorem 1:** Let  $0 < d < 1$ .

- I) If  $k < d - d^2$ , then  $\hat{G} = G^c$
- II) If  $d - d^2 < k < d$ , there exists  $n'(k, d)$  such that if  $n > n'(k, d)$  then  $\hat{G} = G^s$ .
- III) If  $k > d$ , there exists  $\hat{k}(d, n)$  such that if  $k < \hat{k}(d, n)$ ,  $\hat{G} = G^{ps} \cup \{g^e\}$  while if  $k > \hat{k}(d, n)$   $\hat{G} = \{g^e\}$ .

To prove this theorem we use the notion of a recurrent set in the sense of Definition 7.4 in Samuelson [19]: a recurrent set  $X$  is a collection of absorbing set with the following two properties: a) it is impossible, for a single mutation, followed by unperturbed dynamic, to lead the system in an absorbing set not belonging to  $X$ ; b) given two absorbing sets  $s'$  and  $s''$  in  $X$ , we can find a sequence of absorbing sets in  $X$ ,  $s_1, \dots, s_k, \dots, s_K$  with  $s_1 = s'$  and  $s_K = s''$ , such that for any  $k \in [2, K]$  is possible to move from  $s_{k-1}$  to  $s_k$  by a transition that includes a single mutation followed by unperturbed dynamics

(minimality condition). In the following we denote this kind of sequence as *path of one step mutations*. Finally we use the result of proposition 7.7 in Samuelson [19]: if a state is stochastically stable, then it is contained in a recurrent set and all states in the same recurrent set are stochastically stable.

In the proof we show that only one recurrent set exists for each interval of link cost. In part I, given that the unperturbed dynamic always leads the system in a  $g \in G^c$ , it is sufficient to demonstrate that property  $b$  is satisfied by  $G^c$ . In part II we show that, starting from any initial state, a single mutation followed by an unperturbed dynamic is enough to lead the system in a  $g \in G^s$ ; this evidence provides a sufficient condition for the existence of only recurrent sets containing star networks. Then we show that property  $b$  is satisfied by  $G^s$ ; this evidence provides a sufficient condition that all star networks are in the same recurrent set. Finally we show that, starting from any  $g^s$ , a single mutation followed by an unperturbed dynamic is never sufficient to lead the system in a different network structure; this evidence is enough to prove that only star networks are in the unique recurrent set. The proof of part III follows similar arguments with the difference that when the link cost is (relatively) small  $g^e \cup G^{ps}$  is the unique recurrent set; otherwise only  $g^e$  is in the unique recurrent set.

**Proof.** Consider part I ( $k < \mathbf{d} - \mathbf{d}^2$ ). In this case the dynamic process converges always to  $g \in G^c$  from any network's configuration. Indeed in this interval of link cost, the best-response (for all agents) is to replace all indirect links with direct links. Then, we have to demonstrate that all types of  $g \in G^c$  are in the same recurrent set, i.e. for any couple  $g', g'' \in G^c$  a path of one-step mutations, that leads from  $g'$  to  $g''$ , exists in  $G^c$ .

Consider an agent  $i_1 \in g'$  that changes strategy by choosing her corresponding strategy in  $g''$ . Then, if  $i_1$  obtains the chance to revise her strategy after the other agents have revised, we obtain another type of  $g \in G^c$  where player  $i_1$  has the same profile of link decisions as in  $g''$ . Indeed all agents  $j$  such that  $\bar{g}_{i_1 j} = 0$  will form a link with agent  $i_1$  and all agents  $j$  such that  $g_{i_1 j} = 1$  and  $g_{j, i_1} = 1$  will sever link  $j i_1$ . We note that this new network is a strict Nash equilibrium and denote it by  $g_1$ . Consider an agent  $i_2 \in g_1 / \{i_1\}$  that changes strategy by choosing her corresponding strategy in  $g''$ . We note that the link between  $i_1$  and  $i_2$  does not change with this mutation because after the previous stage this link is supported as in  $g''$ . If  $i_2$  obtains the chance to revise its strategy after the other agents have done so, we obtain another type of  $g \in G^c$ , denoted by  $g_2$ , where players  $i_1$  and  $i_2$  have the same profile of link decisions as in  $g''$ . Consider an agent  $i_3 \in g_2 / \{i_1, i_2\}$  that changes strategy by choosing her corresponding strategy in  $g''$ . The links between  $i_1$ ,  $i_2$  and  $i_3$  do not change with this mutation because in the previous stages these links are supported as in  $g''$ . If  $i_3$  obtains a chance to revise her strategy after the other agents have revised, we obtain another type of  $g \in G^c$  where players  $i_1$ ,  $i_2$  and  $i_3$  have the same profile of link decisions as in  $g''$ . In this way, we can find a path of one step mutations, which produce the transition between two generic types of  $g^c$ . The rest of the proof derives from the result of proposition 7.7 in Samuelson [19].

Consider part II ( $d - d^2 < k < d$ ). In this proof we need the following three lemma.

**Lemma 1:** Suppose  $\mathbf{d} - \mathbf{d}^2 < k < \mathbf{d}$ . To induce a transition from any network architecture to a  $g \in G^{cs}$  it is sufficient to have one mutation followed by an unperturbed dynamic.

The proof is in the appendix.

**Lemma 2:** Suppose  $\mathbf{d} - \mathbf{d}^2 < k < \mathbf{d}$ . For any couple  $g', g'' \in G^s$ , a path of one-step mutations, that leads from  $g'$  to  $g''$ , exists in  $G^s$ .

The proof is in the appendix.

**Lemma 3:** Let be  $\mathbf{d} - \mathbf{d}^2 < k < \mathbf{d}$  and suppose any  $g \in G^s$ . Then exists  $n'(k, \mathbf{d})$  such that after a single mutation followed by an unperturbed dynamic, the system converges to any  $g \in G^s$  if  $n > n'(k, \mathbf{d})$ .

The proof is in the appendix.

The proof moves in three steps. In the first we show that recurrent sets without a  $g \in G^{cs}$  cannot exist. To prove this statement the result in lemma 1 is sufficient: assume a recurrent set that does not contain a  $g \in G^{cs}$ ; a single mutation, followed by an unperturbed dynamic, is sufficient to move the system in a  $g \in G^{cs}$ . This fact violates the property *a* for a recurrent set. In the second step we show that all  $g \in G^s$  are contained in the same recurrent set. To show this statement the result in lemma 2, satisfying property *b* for a recurrent set, is sufficient: assume that  $G^s$  is split into two or more subsets and that each subset is contained in a separate recurrent set. The result stated in lemma 2 is in contradiction with property *a* of the recurrent set. Hence these two results together tell us that only one recurrent set containing  $G^s$  exists. In the third step we show that, for values of  $n$

sufficiently large, the unique recurrent set contains only  $G^s$ . To prove this the result stated in lemma 3 is sufficient. The rest of the proof derives from the result of proposition 7.7 in Samuelson [19].

Proof of part *III* of Theorem 1 ( $k > \mathbf{d}$ ) uses similar arguments. We need the following three lemma.

**Lemma 4:** *Suppose  $k > \mathbf{d}$ . To induce a transition from any network architecture to  $g^e$  it is sufficient to have one mutation followed by an unperturbed dynamic.*

The proof is in the appendix.

**Lemma 5:** *Let be  $k > \mathbf{d}$  and suppose  $g = g^e$ . Then after any single mutation followed by unperturbed dynamic the system converges to:*

- a.  $g^e$  if  $k > e + (n-1) \cdot e^2 / 2$
- b.  $g \in G^{ps} \cup \{g^e\}$  if  $k < e + (n-1) \cdot e^2 / 2$ .

The proof is in the appendix.

**Lemma 6:** *Let be  $\mathbf{d} < k < e + (n-1) \cdot e^2 / 2$  and suppose any  $g \in G^{ps}$ . Then after any single mutation followed by an unperturbed dynamic the system converges to any  $g \in G^{ps} \cup \{g^e\}$ .*

The proof is in the appendix.

In the first step we show that recurrent sets without  $g^e$  cannot exist. To prove this statement the result in lemma 4 is sufficient: assume a recurrent set that does not contain  $g^e$ ; a single mutation, followed by an unperturbed dynamic, is sufficient to move the system into  $g^e$ . This fact violates

property  $a$  for a recurrent set. In the second step we show that, for sufficiently large values of  $k$ , the unique recurrent set contains only  $g^e$ . To prove this, the result stated in lemma 5, part  $a$ , is sufficient. In the third step we show that for values relatively small of  $k$  the unique recurrent set consists of  $G^{ps} \cup \{g^e\}$ . To prove this statement we have to check the conditions  $a$  and  $b$  for a recurrent set. The results stated in lemma 6 and lemma 5, part  $b$ , tell us that the minimality condition (property  $b$ ) is verified: the system can move from  $g^e$  to any  $g \in G^{ps}$  and vice versa by a single mutation followed by an unperturbed dynamic; then for any couple  $g', g'' \in G^{ps} \cup \{g^e\}$ , a path of one-step mutations, that leads from  $g'$  to  $g''$ , exists in  $G^{ps} \cup \{g^e\}$ . Moreover these results are enough to verify property  $a$  for a recurrent set: a single mutation followed by an unperturbed dynamic is not enough to move the system in a network  $g \notin G^{ps} \cup \{g^e\}$ . We note that starting from  $g^e$  this is verified in lemma 5 and, starting from any  $g \in G^{ps}$ , it is verified in lemma 6. The rest of the proof derives from the result of proposition 7.7 in Samuelson [19]. QED

#### 4. Discussion of the results

The key points that permit the results stated in theorem 1 are the setting of the revision process and the link-cost structure.

A change in the revision process affects the number of the perturbations that are necessary to pass from one state to another and consequently the stochastic potential of each equilibrium state. Therefore the set of stochastically stable states could be different. For example, suppose a setting where in every period the chance to revise the strategy pertains to



only one link (potential or effective) in the network<sup>8</sup>. Lemma 1 no longer holds, indeed, starting from any network  $g \in G$  to cause a transition toward a network  $g \in G^{cs}$ , one mutation is no more enough, more mutations are necessary. Then, the sufficient condition for the existence of only recurrent sets containing star networks is not satisfied and the result as stated in part II of theorem 1 no longer holds. With similar reasoning, we find that the result of part III of theorem 1 changes too. Indeed lemma 4 no longer holds because, starting from any state, to cause a transition toward an empty network more than one mutation is necessary. Therefore, the sufficient condition for the existence of only one recurrent set containing empty networks is no more satisfied. It is likely that the results in lemma 1 and lemma 4 are true for all revision processes that permit revisions of a sufficiently high number of links at the same time. In this case, a single perturbation affecting a large number of links could be sufficient to promote the transitions as stated in lemma 1 and 4.<sup>9</sup>

Consider now the implications of the link-cost structure. The polar case is one where both players implied in a link have to pay a cost. This change requires a different framework of link revision because the players receiving the proposal have to be able either to accept or reject the formation of new links. Suppose that when a player proposes the formation of new links, all

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<sup>8</sup> This kind of revision process is a polar case compared with that we use in the model.

<sup>9</sup> Lemma 1 would be true if a single perturbation permits to one player to form a sufficient number of links such that all indirectly linked players have an incentive to link directly with him and to delete every other link; a similar reasoning can be applied to lemma 4 to demonstrate that if a single perturbation permits one player to form a sufficient number of links, the network goes into a state characterized by a positive probability of going into a empty network.

implied players have to accept or not using a myopic behavior<sup>10</sup>. The results as stated in part II and III of theorem 1 change<sup>11</sup>. Lemma 1 is no longer true; for example, consider a double star network in which a central agent tries to form new links with any peripheral agent of the opposite star. It could be that the best-response of these peripheral agents is not to accept the new links if the proponent is directly linked with (relatively) few agents. Moreover, lemma 3 is not true; indeed, starting from a star, after a single mutation brings the system into a state with one or more disconnected players, there is a positive probability that the system converges to a double star (for example, when after the mutation, the first revising player is a peripheral one). In this state the best-response of both central players is not to accept the formation of new direct links given that indirect links are preferred to the direct ones. Therefore, the sufficient conditions for the existence of only recurrent sets with only star networks are not satisfied. But if we assume the possibility that link costs are asymmetrical, it is likely that theorem 1 part II is still valid for a large set of link-cost structures: all those where the cost to receive a link is lower than  $d - d^2$ . The intuition is that in this range of (receiving) cost, the best-response of all players is to accept the links proposed by an indirectly linked player. As for the part III of theorem 1, we note that the star is not a stable network because the center has an incentive to cut the links. In this case the candidates for stochastically stable networks are the empty network and the networks where each node is connected with more links. Moreover, we are not able to demonstrate that lemma 4 (or an equivalent statement) is true. Therefore, a sufficient condition for the existence of only one recurrent set containing  $g^e$  is not

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<sup>10</sup> i.e. they accept the new link if this increases their net payoff considering the state in the previous period.

satisfied. Like for part II of the theorem 1, assuming the cost to receive a link is lower than  $d - d^2$ , it is likely that these results are still valid.

Finally, we consider the role of  $n$ . Indeed the result stated in part II of theorem 1 is true only for sufficiently large values of  $n$ . What happens for small values of  $n$  is that lemma 3 is no more true in the sense that, starting from a star, a single mutation can induce the system to move to a different architecture<sup>12</sup>. Then the unique recurrent set (therefore the set of stochastically stable networks) can contain not only star networks but different network architectures too.

## 5. Conclusion

In this paper we have analyzed the formation of social networks characterized by an exogenous decay such as that described in Bala and Goyal [1]. Using the concept of stochastic stability, we can produce a reduction in the number of possible equilibrium network architectures in a large range of link costs. Indeed, these authors for intermediate link costs, using dynamic selection, do not provide a full characterization of equilibria. Instead, using our refinement, we find very simple equilibrium networks for intermediate link costs: the star networks. Then we note that this kind of network architecture is very common in many fields: commerce, communication, industrial organization, transport, relationships and so on.

Further development can be made in many directions. First, we might use a framework where small deviations from the best-response are more probable than the large ones. Second, we might study more general conditions that permit the existence of only one recurrent set. Third, we can

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<sup>11</sup> Linked star is a network with two stars joined by the central agents.

<sup>12</sup> for example in a double star.

model the decay as endogenous using social games that can be more respondent to empirical situations. Finally, we could study applications regarding the diffusion of technologies and the hierarchical and social structure in the enterprises and firms.

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## Appendix

### Proof of lemma 1

Consider any network  $g \in G$  and suppose an agent  $i$  switches to a strategy where she forms links with all others. When other agents have the chance to revise their strategy, they delete all their links. If agent  $i$  has the chance to revise the strategy after all other agents have revised, then the state will be a  $g \in G^{cs}$  (with agent  $i$  at the center). QED.

### Proof of lemma 2

Consider a  $g \in G^{cs}$  and denote by  $c$  the central agent and by  $p$ -agents the peripherals. If  $c$  deletes all links, the first agent with the chance to revise her strategy forms links with all others. The result will be a  $g \in G^{cs}$  with a different identity of  $c$ . A mutation of  $c$ , who deletes one link, could induce a transition toward a  $g \in G^{ms}$  with  $n-2$  links supported by  $c$  and one supported by a  $p$ -agent<sup>13</sup>. From this state a similar mutation of  $c$  could bring the system to a  $g \in G^{ms}$  with  $n-3$  links supported by  $c$ . By similar single mutations the system could transit to a  $g \in G^{ps}$ . Now consider a  $g \in G^{ps}$  as initial state. A mutation of a  $p$ -agent, who deletes her link, could induce a transition towards a  $g \in G^{ms}$  with one link supported by  $c$ . From this state a similar single mutation could induce a transition toward a  $g \in G^{ms}$  with two links supported by  $c$ . Then, by similar single mutations the system could

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<sup>13</sup> If the no-connected agent has the chance to revise immediately after the mutation.

transit to a  $g \in G^{cs}$ . Then for any couple  $g', g'' \in G^s$  we can find in  $G^s$  a path of one-step mutations, that leads from  $g'$  to  $g''$ . QED.

### Proof of lemma 3

We define two network structures: the *quasi linked stars*, denoted by  $g^{qls}$ , and the *quasi linked stars 2*, denoted by  $g^{qls2}$ .

The  $g^{qls}$  is described as follows: fix 2 agents ( $c$  and  $m$ ) characterized by  $\bar{g}_{cm} = 1$  and partition the remaining agents into the following sets:

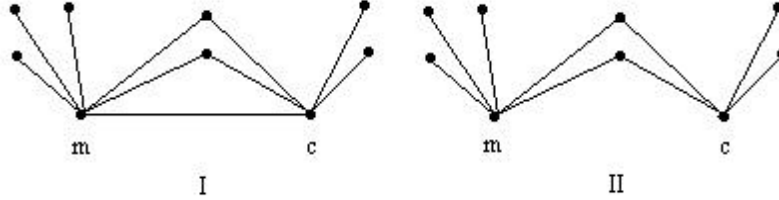
$$N_c \equiv \{i \in N : \bar{g}_{ic} = 1 \wedge \bar{g}_{ij} = 0 \forall j \in N / c\}$$

$$N_m \equiv \{i \in N : \bar{g}_{im} = 1 \wedge \bar{g}_{ij} = 0 \forall j \in N / m\},$$

$$N_{cm} \equiv \{i \in N : \bar{g}_{ic} = 1 \wedge \bar{g}_{im} = 1 \wedge \bar{g}_{ij} = 0 \forall j \in N / \{c, m\}\};$$

$\#x = |N_x|$  is the cardinality of the set  $N_x$ ; the set of all possible  $g^{qls}$  is denoted by  $G^{qls}$ . The  $g^{qls2}$  is described as the  $g^{qls}$  with the difference  $\bar{g}_{cm} = 0$  and  $\#cm \geq 1$ . The set of all possible  $g^{qls2}$  is denoted by  $G^{qls2}$ . Figure 1 shows two examples.

Figure 1:  $g^{qls}$  (I) and  $g^{qls2}$  (II) with  $n=9$



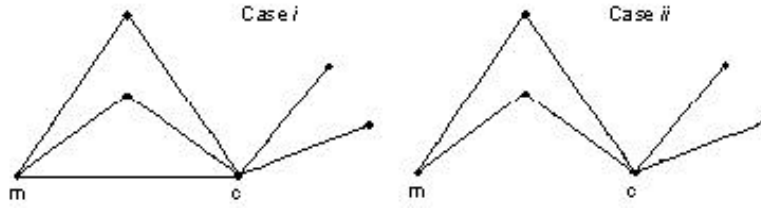
Note that when  $d > k$  all strategies that are best response have to produce a connected network.

Consider any  $g \in G^s$  with  $n$  agents. We can distinguish 2 kinds of agents: the central agent, denoted by  $c$ , and the peripheral agents, named *p-agents*. Suppose a mutation of  $c$ . She may sever some supported link and/or form new links; the resulting network may be: i) a connected but no essential  $g^s$ ; ii) a  $g^s$  with some no connected agent; iii) a  $g^e$ . The first agent with the chance to revise will form links with all no connected agents (if any) and severs all no essential links. Then the system goes in a  $g \in G^s$  if at least one of the following conditions is true: i) after the mutation the network is either

a connected  $g^s$  or a  $g^e$ ; ii) agent  $c$  has the chance to revise before all other agents; otherwise, if both conditions  $i$  and  $ii$  are false, the system goes in a  $g^{qls}$  with  $\#cm = 0$ .

Suppose a mutation of a  $p$ -agent and denote it by  $m$ ; agent  $m$  may choose to support new links and/or delete the link with  $c$  (if  $g_{mc} = 1$ ). (Figure 2 shows two examples). We consider the networks that are obtainable with a mutation of  $p$ -agent as cases of  $g^{qls}$  and  $g^{qls2}$  characterized by  $\#m = 0$ . Therefore we prove that, when the system is characterized by any  $g \in \{G^{qls} \cup G^{qls2}\}$ , it converges in a  $g \in G^s$  with probability 1.

Figure 2: possible network structures after a mutation of a  $p$ -agent in a  $g^{qls}$  with  $n=6$



Consider a  $g \in G^{qls}$  with  $n$  agents. For all  $i \in N_m$  to support a link with any  $i \in N_{cm}$  is not a best-response because it reduces her payoff of  $d^2 - (d - k)$ ; to support one link with a  $i \in N_c$  is dominated by the formation of one link with  $c$ : in the first case the payoff changes for an amount of  $d - k - d^3$ , while in the second case for an amount of  $d - d^2 + \#c(d^2 - d^3) - k$ ; we note that  $d - d^2 + \#c(d^2 - d^3) - k \geq d - k - d^3$  if  $\#c \geq 1$ , with strictly inequality if  $\#c > 1$ ; then for any  $i \in N_m$ , to support a new link with  $c$  is a best-response if the payoff increases, that is when:

$$(1.1) \quad \#c \geq [k - (d - d^2)] / (d^2 - d^3)$$

In this case the best response of any  $i \in \{i \in N_{cm} \cup m : g_{ic} = 1\}$  is to continue to support link  $ic$ <sup>14</sup>. Using the same arguments we can demonstrate that if:

$$(1.2) \quad \#m \geq [k - (d - d^2)] / (d^2 - d^3),$$

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<sup>14</sup> Note that: a) if any  $i \in N_{cm}$  severs the link with  $c$ , she is going into the set  $N_m$  and  $d(i, c) = 2$ ; b) if  $m$  severs the link  $mc$ , then  $d(m, c) = 2$ .



the best-response of any  $i \in N_c$  is to form a new link  $im$ , while that one of any  $i \in \{i \in N_{cm} \cup c : g_{im} = 1\}$  is to continue to support link  $im$ . Note that if:

$$(1.3) \quad n \geq 2 + 2[k - (d - d^2)] / (d^2 - d^3)$$

the set  $g \in G^{qls} : \#cm \leq n - 2 - 2[k - (d - d^2)] / (d^2 - d^3)$ , where all networks satisfy at least one condition among (1.1) and (1.2), is not empty.

Consider a network  $g \in G^{qls}$  with  $n$  agents, where (1.1) and (1.2) are not satisfied and (1.3) is true. Without loss of generality, suppose that  $\#c > \#m$ . In this state there are the following best responses:

- i) for any  $i \in N_c \cup \{i \in N_m : g_{im} = 0\} \cup \{i \in N_{cm} : g_{ic} = 0 \wedge g_{im} = 0\}$ , it is not to change strategy;
- ii) for any  $i \in \{i \in N_m : g_{im} = 1\}$ , it is to sever link  $im$  and support a new link  $ic$ ;
- iii) for any  $i \in \{i \in N_{cm} : g_{ic} = g_{im} = 1\}$ , it is to sever the link  $im$ ;
- iv) for any  $i \in \{i \in N_{cm} : (g_{ic} = 1 \wedge g_{im} = 0) \vee (g_{ic} = 0 \wedge g_{im} = 1)\}$ , it is to sever the supported link;
- v) for  $c$  and  $m$ , it is to sever all supported links with agents  $i \in N_{cm}$  and, sometimes, link  $cm$ <sup>15</sup>.

Therefore the unperturbed dynamics brings the system either in a network  $g^{qls2}$  (if link  $cm$  is severed) or in networks  $g \in G^{qls}$  characterized by less and less  $\#cm$ . In this second case the system converges in a network  $g \in G^{qls}$  where at least a one condition among (1.1) and (1.2) is verified.

Now suppose a network  $g \in G^{qls}$  where only one condition among (1.1) and (1.2) is verified, for example let be true condition (1.1); note that in this state  $\#c > \#m$ . The best response for any  $i \in N$  is to have a direct link with  $c$ . For any sequence of strategy's revisions, if (1.2) does not become true<sup>16</sup> and the network does not become<sup>17</sup> a  $g^{qls2}$ , the system goes in a  $g \in G^{qls}$

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<sup>15</sup> For agent  $c$  to sever link  $cm$  is a best response if  $\exists i \in N_{cm} : g_{ic} = 1$  and  $\#m + |\{i \in N_{cm} : g_{ci} = 1\}| < [k - (d - d^2)] / (d^2 - d^3)$ ; note the right side is the number of agents in the set  $N_m$  after the revision of  $c$  and it is the same as right side of (1.1) and (1.2). For agent  $m$  we can use similar arguments.

<sup>16</sup> it may happen if  $c$  deletes many links with  $i \in N_{cm}$

<sup>17</sup> it may happen if  $c$  severs the link with  $m$ .

with  $\#m = 0$  and, from this state, in a  $g^s$ . Indeed in a state where all agents are directly linked with  $c$ , all links  $ij$  where  $i, j \in N/c$  are severed as soon as possible; if  $c$  has the chance to revise, she severs all direct links with agents that are indirectly linked, but given condition (1.1), these agents will form a direct link with  $c$  as soon as possible and the network goes back in  $g \in G^{qls}$  with  $\#m = 0$ . When both conditions (1.1) and (1.2) are verified, the best-response of all agent  $i \in N$  is to have direct links with  $c$  and  $m$ . Then, in the revision process,  $\#c$  and  $\#m$  are decreasing. Therefore the system goes in a state where only one condition among (1.1) and (1.2) is verified.

Now suppose that a  $g \in G^{qls2}$  with  $n$  agents. Using the same arguments for a  $g^{qls}$  we find that for any  $i \in N_m$  to support a new link with  $c$  is a best-response if:

$$(1.4) \quad \#c > [k - (d - d^3)] / (d^2 - d^4)$$

In this case the best response of any  $i \in \{i \in N_{cm} : g_{ic} = 1\}$  is to continue to support the link with  $c$ . (See arguments for condition (1.1)). Similarly, if the condition:

$$(1.5) \quad \#m > [k - (d - d^3)] / (d^2 - d^4)$$

is verified the best response of any  $i \in N_c$  is to form a new link with  $m$ , while that one of any  $i \in \{i \in N_{cm} : g_{im} = 1\}$  is to continue to support the link with  $m$ . We note that right side of (1.4) and (1.5) is smaller than right side of (1.1) and (1.2). Therefore may happen that (1.1) and (1.2) are not satisfied while (1.4) and (1.5) are satisfied. When  $d - d^2 < k < d - d^3$ , conditions (1.4) and (1.5) are always true given that the right side is negative. The following condition assures the existence of a no empty subset<sup>18</sup> of  $G^{qls2}$ , in which all networks satisfy at least one condition among (1.4) and (1.5):

$$(1.6) \quad n \geq 3 + 2[k - (d - d^3)] / (d^2 - d^4)$$

Note that cases of  $g \in G^{qls2}$  with at least one true condition among (1.1) and (1.2) can be treated like a  $g \in G^{qls}$  (see above); indeed the best response of  $m$  and/or  $c$  is to support a link  $cm$ . Then we consider only the cases of  $g \in G^{qls2}$  where (1.1) and (1.2) are no true.

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<sup>18</sup>  $g \in G^{qls2} : \#cm \leq n - 3 - 2[k - (d - d^3)] / (d^2 - d^4)$

Consider a network  $g \in G^{qls2}$  where (1.4) and (1.5) are not satisfied and (1.7) is true. Without loss of generality, suppose that  $\#c \geq \#m$ . Remember that this case is possible only if  $d - d^3 < k < d$ . In this state we have the following best responses:

i) for any agent  $i \in N/c, m$ , it is the same that in a  $g^{qls}$  when (1.1) and (1.2) are not true but with the following difference: for any  $i \in N_{cm}$  to sever one link is a best response only if  $\#cm > 1$ ;

ii) for  $m$ , it is to sever all supported links with agents  $i \in N_{cm}$  and, if one of the following conditions is true:

a)  $g_{mi} = 1 \wedge g_{im} = 0 \forall i \in N_{cm}$ ;

b)  $\#c + |\{i \in N_{cm} : g_{mi} = 1\}| \geq [k - (d - d^2)]/(d^2 - d^3)$ ,

to support a new link with  $c$ .

iii) for  $c$ , it is to sever all supported links with agents  $i \in N_{cm}$  and, if one of the following conditions is true:

a)  $g_{ci} = 1 \wedge g_{ic} = 0 \forall i \in N_{cm}$ ;

b)  $\#m + |\{i \in N_{cm} : g_{ci} = 1\}| \geq [k - (d - d^2)]/(d^2 - d^3)$ ,

to support a new link with  $m$ . Therefore the unperturbed dynamics brings the system in a network  $g^{qls}$  (if a link  $cm$  is created) or in networks  $g \in G^{qls2}$  characterized by less and less  $\#cm$ . In this second case the system converges in a network  $g \in G^{qls2}$  where at least a one condition among (1.4) and (1.5) is verified.

Now consider a network  $g_1 \in G^{qls2}$  where only one condition among (1.4) and (1.5) is true and no one among (1.1) and (1.2) is satisfied; without loss of generality suppose that (1.4) is true. In this case note that  $\#c > \#m$  and it is possible only if  $d - d^3 < k < d$ . In this state there are the following best responses:

i) for  $c$  and  $m$  it is the same than previous case;

ii) for any  $i \in N_c \cup \{i \in N_{cm} : g_{im} = 0\}$  it is not to change strategy;

iii) for any  $i \in N_m$  it is to support a new link with  $c$  and, if  $g_{im} = 1$ , to sever link  $im$ ;

iv) for any  $i \in \{i \in N_{cm} : g_{im} = 1\}$  it is to sever the link  $im$  if  $\#cm > 1$ ;

Therefore, if only condition (1.4) remains true and the network does not become a  $g^{qls}$ , the system goes in a  $g \in G^{qls2}$  where  $\#m = 0$  and  $|\{i \in N_{cm} : g_{im} = 1\}| = 0$ ; if  $m$  revises her strategy, the resulting network will be a  $g \in G^s$  (she substitutes all supported links with one link with  $c$ ), otherwise, if  $c$  is revising, the resulting network will be  $g^{qls2}$  with  $\#m > 0$ ; but given condition (1.4), all agents  $i \in N_m$  will form a direct link with  $c$  as soon as possible and the network goes back in  $g \in G^{qls2}$  with  $\#m = 0$ .

When conditions (1.4) and (1.5) are true, the best-response of all agents  $i \in N/c, m$  is to have direct links with  $c$  and  $m$ . Then, in the revision process,  $\#c$  and  $\#m$  are decreasing. We have to distinguish two sub-cases: when  $d - d^3 < k < d$  and when  $d - d^2 < k < d - d^3$ . If  $d - d^3 < k < d$  the system goes in the state where only one condition among (1.4) and (1.5) is verified and, from this state in a  $g \in G^s$ . When  $d - d^3 < k < d$  both conditions (1.4) and (1.5) are true for all possible values of  $\#m$  and  $\#c$ . Then, if the network does not become a  $g^{qls}$ , the system could go in a  $g \in G^{qls2}$  where  $\#c = \#m = 0$ ,  $\#cm = n - 2$  and  $g_{ci} = g_{mi} = 0 \forall i \in N_{cm}$ . It is possible to demonstrate that, from this network structure, the system goes in a star network with probability 1 in a finite time

Using similar arguments we can treat cases where are true conditions (1.1) and (1.5) as well as cases where (1.2) and (1.4) are satisfied to show the convergence in a state where only one condition remains true. QED.

#### Proof of lemma 4

Consider any  $g \in G$  and suppose a mutation in which an agent  $m$  forms links with all others. When other agents have the chance to revise, they delete their links; then the network becomes a  $g^{cs}$ . In this state when agent  $m$  has the chance to revise, she severs all links because  $k > d$ . Consequently, the state will be a  $g^e$ . QED.

#### Proof of lemma 5

Note that the best-response of an agent supporting links with agents that are not linked with any other, is to sever such links. Consider a  $g^e$  with  $n$  agents and suppose a mutation in which an agent  $i$  switches to a strategy where she forms  $x$  ( $0 < x < n - 1$ ) links. All no-connected agents have as best-response to form a link with  $i$  if:

$$(1.8) \quad x > (k - e)/e^2$$

otherwise their best-response is no to form links and, when agent  $i$  has a new chance to revise, the network transit to  $g^e$ . If (1.8) is true and agent  $i$  has the chance to revise her strategy after that  $y$  no-connected agents have done ( $0 < y < n - x$ ), the state will be a  $g^{ps}$  with  $y$  agents (agent  $i$  deletes all direct links to single agents). Follows that best-response of no-connected agents is to support a link with  $i$  if:

$$(1.9) \quad y > (k - e)/e^2$$

Then if (1.9) is true the resulting network will be a  $g^{ps}$  with  $n$  agents, otherwise the network will be  $g^e$ <sup>19</sup>; indeed in this second case the best-response of all agents is no to support links. Arranging conditions (1.8) and (1.9), we find that when the network is a  $g^e$  and:

$$(1.10) \quad k > e + (n - 1) \cdot e^2 / 2$$

after one mutation followed by unperturbed dynamic the system always converges in  $g^e$ , otherwise, if (1.10) is false, the system converges in  $g^e$  or in a  $g^{ps}$ . QED.

#### Proof of lemma 6

We use the same notation than in proof of lemma 3. Consider any  $g \in G^{ps}$  and suppose a mutation of  $c$ . If  $c$  forms new links with any other the network will be a no essential star. In this state  $p$ -agents with the chance to revise will sever all no essential links and network will be a  $g^{ms}$ . When  $c$  has the chance to revise, she will severs all supported link with peripheral agents. Therefore the system goes in a state with  $g^{ps}$  and  $x$  no connected agents. If the number of agents connected in  $g^{ps}$  ( $n - x$ ) is sufficiently large the best-response of all agents is to be tied with  $c$  and the system converges in  $g^{ps}$ ; otherwise the system converges in  $g^e$ . Now we focus our attention on the mutation of a peripheral agent and denote it by  $m$ . After this mutation the resulting networks can be summarized in two cases: *a*)  $m$  supports new links with any (from 1 to  $n - 2$ )  $p$ -agents; *b*)  $m$  supports new links with any (from 0 to  $n - 2$ )  $p$ -agents and severs the link with  $c$  (See fig. 2).

Consider case *a*. The network is a  $g^{qls}$  (See fig. 1) where  $c$  does not support any link,  $m$  supports all direct links with  $i$ -agents and  $\#m = 0$ . The best-response of any  $i \in N_{cm}$  is to be tied with  $c$  if condition (1.1) is true,

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<sup>19</sup> After that all peripheral agents have revised their strategy.

otherwise their best-response is to sever the link with  $c$ . In the first case only  $m$  has incentive to change strategy. Therefore when  $m$  has the chance to revise the system goes in the initial state. If condition (1.1) is false and any  $i \in N_{cm}$  has the chance to revise before of  $m$ , the system goes in a  $g \in G^{qls}$  with  $\#m > 0$ . Then we discuss the following sub-cases:

1.  $\#c - 1 > \#m$  and following condition is true:

$$(1.11) \quad d + (\#cm + \#c - 1) \cdot d^2 + \#m \cdot d^3 > k$$

In this case only  $i \in N_{cm}$  and  $m$  have incentive to change strategy. If any  $i \in N_{cm}$  revises the system goes in a  $g \in G^{qls}$  with a smaller  $\#cm$  and larger  $\#m$ . When  $m$  has the chance to revise, she severs all links supported with  $i \in N_m \cup N_{cm}$  and, if the following it is true:

$$(1.12) \quad d + (\#cm + \#c) \cdot d^2 < k$$

$m$  severs the link with  $c$ . In this case the system goes in  $g^e$  because no agents have an incentive to be tied with  $c$ . If (1.12) is false,  $c$  maintains the link with  $m$ , as well as the best-response of all no-connected agents is to support a link with  $c$ ; therefore the system goes in  $g \in G^{ps}$ .

2. Condition (1.11) is true and  $\#c - 1 < \#m$ . Differently from previous case the best-response of any  $i \in N_c$  is to sever the link with  $c$  and to be tied with  $m$ . When  $m$  has the chance to revise, she severs all supported links with  $i \in N_{cm} \cup N_m$ ; if condition (1.12) is true  $m$  severs the link with  $c$  otherwise not. If  $m$  severs the link with  $c$  the system goes in a state with two components, each one is a  $g^{ps}$  with  $c$  and  $m$  central agents. In this state if  $\#m$  is sufficiently large the best-response of all agents is to be tied with  $m$  and the system goes in a  $g \in G^{ps}$ , otherwise the system goes in  $g^e$ . If (1.12) is false ( $m$  remains tied to  $c$ ) the system goes in a  $g \in G^{qls}$  with  $\#cm = 0$  where all  $i \in N_c$  support the link with  $c$  and all  $i \in N_m$  support the link with  $m$ . In this network the best-response of any  $i \in N_m \cup N_c$  is to be tied with  $c$  or  $m$  depending on which one provides the larger payoff. Therefore the network will go in a  $g^{ps}$ .

3. If condition (1.11) is false any  $i \in N_c$  will sever the links with  $c$  and, eventually, will support one with  $m$  if  $\#m$  is sufficiently large<sup>20</sup>. When  $m$

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<sup>20</sup> that is if  $d + (\#cm + \#m - 1) \cdot d^2 + \#c \cdot d^3 > k$ .

has the chance to revise, she severs all supported links. Therefore given that (1.11) is false the system goes in  $g^e$  in any case.

Consider case *b*. The network is a  $g^{qls2}$  (see fig. 1) where  $c$  does not support any link,  $m$  supports all direct links with  $i \in N_{cm}$  and  $\#m = 0$ . The best-response of any  $i \in N_{cm}$  is to be tied with  $c$  if condition (1.4) is true, otherwise their best-response is to sever the supported link with  $c$  if exists at least another  $i \in N_{cm}$ . In the first case only  $m$  has incentive to change strategy. Therefore when  $m$  has the chance to revise the system goes in the initial state. If condition (1.4) is false and any  $i \in N_{cm}$  has the chance to revise before of  $m$ , the system goes in a  $g \in G^{qls2}$  with  $\#m > 0$ . Then we discuss the following sub-cases:

1.  $\#c - I > \#m$  and following condition is true:

$$(1.13) \quad d + (\#cm + \#c - 1) \cdot d^2 + d^3 + \#m \cdot d^4 > k$$

In this case only  $i \in N_{cm}$  and  $m$  have incentive to change strategy. If any  $i \in N_{cm}$  revises the system goes in a  $g \in G^{qls2}$  with a smaller  $\#cm$  and larger  $\#m$ . When  $m$  has the chance to revise, she severs all links supported with  $i \in N_{cm} \cup N_m$  and, if (1.12) is true, no agent has best-response to be tied with  $c$ . Therefore the system goes in a  $g^e$ . If (1.12) is false,  $m$  will support a link with  $c$  as well as all no-connected agents when have the chance to revise. Then the network will go in a  $g \in G^{ps}$ .

2. Condition (1.13) is true and  $\#c - I < \#m$ . The best-response of any  $i \in N_c$  is to sever the link with  $c$  and to be tied with  $m$ . When  $m$  has the chance to revise, if condition (1.12) is true she severs all supported links with  $i \in N_{cm} \cup N_m$  and the system goes in a state with two components, each one is a  $g^{ps}$  with  $c$  and  $m$  central agents. In this state if  $\#m$  is sufficiently large the best-response of all agents is to be tied with  $m$  and the system goes in  $g \in G^{ps}$ , otherwise the system goes in  $g^e$ . If (1.12) is false, when has the chance to revise,  $m$  severs all supported links with  $i \in N_{cm} \cup N_m$  and supports a new link with  $c$ . Given that the resulting network will be a  $g^{qls}$  the proof follows case *a*.

3. If condition (1.13) is false, any  $i \in N_c$  will sever the links with  $c$  and, eventually, will support one with  $m$  if  $\#m$  is sufficiently large<sup>21</sup>. When  $m$

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<sup>21</sup> that is if  $d + (\#cm + \#m - 1) \cdot d^2 + d^3 + \#c \cdot d^4 > k$ .

has the chance to revise, she severs all supported links. Therefore given that (1.13) is false the system goes in  $g^e$  in any case. QED.



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