

Xepapadeas, Anastasios; Brock, William

**Working Paper**

## Optimal Control and Spatial Heterogeneity: Pattern Formation in Economic-Ecological Models

Nota di Lavoro, No. 96.2005

**Provided in Cooperation with:**

Fondazione Eni Enrico Mattei (FEEM)

*Suggested Citation:* Xepapadeas, Anastasios; Brock, William (2005) : Optimal Control and Spatial Heterogeneity: Pattern Formation in Economic-Ecological Models, Nota di Lavoro, No. 96.2005, Fondazione Eni Enrico Mattei (FEEM), Milano

This Version is available at:

<https://hdl.handle.net/10419/74266>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

**Optimal Control and Spatial  
Heterogeneity: Pattern Formation  
in Economic-Ecological Models**

William Brock and Anastasios Xepapadeas

NOTA DI LAVORO 96.2005

**JULY 2005**

ETA – Economic Theory and Applications

William Brock, *Department of Economics, University of Wisconsin*  
Anastasios Xepapadeas, *Department of Economics, University of Crete*

This paper can be downloaded without charge at:

The Fondazione Eni Enrico Mattei Note di Lavoro Series Index:  
<http://www.feem.it/Feem/Pub/Publications/WPapers/default.htm>

Social Science Research Network Electronic Paper Collection:  
<http://ssrn.com/abstract=773925>

# Optimal Control and Spatial Heterogeneity: Pattern Formation in Economic-Ecological Models

## Summary

This paper extends Turing analysis to standard recursive optimal control frameworks in economics and applies it to dynamic bioeconomic problems where the interaction of coupled economic and ecological dynamics under optimal control over space creates (or destroys) spatial heterogeneity. We show how our approach reduces the analysis to a tractable extension of linearization methods applied to the spatial analog of the well known costate/state dynamics. We explicitly show the existence of a non-empty Turing space of diffusive instability by developing a linear-quadratic approximation of the original non-linear problem. We apply our method to a bioeconomic problem, but the method has more general economic applications where spatial considerations and pattern formation are important. We believe that the extension of Turing analysis and the theory associated with the dispersion relationship to recursive infinite horizon optimal control settings is new.

**Keywords:** Spatial analysis, Pattern formation, Turing mechanism, Turing space, Pontryagin's principle, Bioeconomics

**JEL Classification:** Q2, C6

*This paper was presented at the 3<sup>rd</sup> workshop on Spatial-Dynamic Models of Economics and Ecosystems held in Trieste on 13-15 April 2005 and organised by Ecological and Environmental Economics - EEE Programme, a joint three-year programme of ICTP - The Abdus Salam International Centre for Theoretical Physics, FEEM - Fondazione Eni Enrico Mattei, and The Beijer International Institute of Ecological Economics.*

*William Brock thanks NSF Grant SES-9911251 and the Vilas Trust. Anastasios Xepapadeas thanks the Secretariat for Research, University of Crete, Research Program 1266.*

*Address for correspondence:*

Anastasios Xepapadeas  
Department of Economics  
University of Crete  
University Campus  
74 100 Rethymno Crete  
Greece  
Phone: +30 2831 0 77419  
Fax: +30 2831 0 77406  
E-mail: xepapad@econ.soc.uoc.gr

## 1 Introduction

In economics the importance of space has long been recognized in the context of location theory,<sup>1</sup> although as noted by Krugman (1998) there has been some neglect in the systematic analysis of spatial economics, associated mainly with difficulties in developing tractable models of imperfect competition which are essential in the analysis of location patterns. After the early 1990s there was a renewed interest in spatial economics, mainly in the context of *new economic geography*,<sup>2</sup> which concentrates on issues such as the determinants of regional growth and regional interactions, or the location and size of cities (e.g. Krugman, 1993).

In environmental and resource management problems the majority of the analysis has been concentrated on taking into account the temporal variation of the phenomena, and has been focused on issues such as the transition dynamics towards a steady state, or the steady-state stability characteristics. However, it is clear that when renewable and especially biological resources are analyzed, the spatial variation of the phenomenon is also important. Biological resources tend to disperse in space under forces promoting “spreading”, or “concentrating” (Okubo, 2001); these processes along with intra and inter species interactions induce the formation of spatial patterns for species. In the management of economic-ecological problems, the importance of introducing the spatial dimension can be associated with a few attempts to incorporate spatial issues, such as resource management in patchy environments (Sanchirico and Wilen, 1999, 2001; Sanchirico, 2004; Brock and Xepapadeas, 2002), the study of control models for interacting species (Lenhart and Bhat, 1992; Lenhart et al., 1999), the control of surface contamination in water bodies (Bhat et al. 1999), or the creation of marine reserves (Neubert, 2003).

---

<sup>1</sup>See for example Alfred Weber (1909), Harold Hotelling (1929), Walter Christaller (1933), and August Lösch (1940) for early analysis.

<sup>2</sup>Krugman (1998) attributes this new research to: the ability to model monopolistic competition using the well known model of Dixit and Stiglitz (1977); the proper modeling of transaction costs; the use of evolutionary game theory; and the use of computers for numerical examples.

In the economic-ecological context, a central issue that this paper attempts to explore is under what conditions interacting processes characterizing movements of biological resources, and economic variables which reflect human actions on the resource (e.g. harvesting effort), could generate steady-state spatial patterns for the resource and the associated economic variables. That is, a steady-state concentration of the resource and the economic variable which is different at different points in a given spatial domain. We will call this formation of spatial patterns *spatial heterogeneity*, in contrast to *spatial homogeneity* which implies that the steady state concentration of the resource and the economic variable is the same at all points in a given spatial domain.<sup>3</sup>

As stated by Levin (2002) pattern formation and the emergence of robust patterns as asymptotic outcomes of dynamical systems is the first aspect of the two main processes characterizing complex adaptive systems,<sup>4</sup> the other being evolution. A common framework for studying pattern formation is the use of the concept of diffusion as central concept in modelling the movements in space-time of populations of species, chemicals or other state variables, which are interacting locally and redistribute via random movements.. Diffusion is thus defined as a process whereby the microscopic irregular movement of particles such as cells, bacteria, chemicals, or animals results in some macroscopic regular motion of the group (Okubo and Levin, 2001; Murray, 1993, 2003). Biological diffusion is based on random walk models, which when coupled with population growth equations, lead to general reaction-diffusion systems.<sup>5</sup> As stated by Okubo, et al. (2001, p. 348),

“In general a diffusion process in an ecosystem tends to give rise to a uniform density of population in space, [that is spatial homogeneity]. As a consequence it may be expected that

---

<sup>3</sup>Trivially all dynamic models where spatial characteristics and dispersal are ignored lead to spatial homogeneity.

<sup>4</sup>Following Levin (1999) complex adaptive systems can be defined by three properties: (i) diversity and individuality of components; (ii) localized interactions among those components; and (iii) an autonomous process that uses the outcomes of those interactions to select a subset of those components for replication or enhancement.

<sup>5</sup>When only one species is examined the coupling of classical diffusion with a logistic growth function leads to the so-called Fisher-Kolmogorov equation.

diffusion, when it occurs, plays the general role of increasing stability in a system of mixed populations and resources. ...However there is an important exception known as *diffusion induced instability*, or *diffusive instability*. This exception might not be a rare event especially in aquatic systems.”

It was Alan Turing (1952) who suggested that under certain conditions reaction-diffusion systems, which have an asymptotically stable equilibrium in the absence of diffusion can generate spatially heterogeneous patterns under diffusion. This is the so-called *Turing mechanism* or *Turing effect* for generating diffusion instability.<sup>6</sup> The Turing effect implies that an initially spatially homogeneous state can be transformed into a stable patterned state under perturbations induced by diffusion.

Levin (2002) presents other mechanisms that can act as pattern generators, although the Turing mechanism has a central part in his discussion. However Levin, and as far as we know other researchers in the field, do not treat optimal management of a Turing dynamical mechanism as we do this in the current paper. We use the classical problem of optimal harvesting of a renewable resource as a leading example, but we believe our paper will help in formulating an analytically tractable approach to the optimal management of general complex adaptive systems as discussed by Levin.

In this context the purpose of this paper is to explore the impact of the Turing mechanism on the emergence of diffusive instability in optimal control problems in space-time using as a leading example a unified economic/ecological model of optimal resource management. This is a different approach to the one most commonly used to address spatial issues, which is the use of metapopulation models in discrete patchy environments with dispersal among patches (e.g. Sanchirico and Willen, 1999; Sanchirico, 2004). The use of the Turing mechanism allows us to analyze in detail conditions under which diffusion could produce spatial

---

<sup>6</sup>It should also be noted that the emergence of spatial homogeneity or not depends on boundary conditions associated with the spatial domain. If there is no flux on the boundary of the spatial domain (zero flux conditions), then spatial homogeneity might be expected, although as it will be shown, the Turing mechanism under appropriate conditions can generate spatial heterogeneity with zero flux conditions.

heterogeneity and generation of spatial patterns, or spatial homogeneity. Thus the Turing mechanism can be used to reveal conditions which generate spatial heterogeneity in models where ecological variables interact with economic variables. When spatial heterogeneity emerges the concentration of variables of interest (e.g. resource stock or harvesting effort) in a steady state, are different in different locations of a given spatial domain.

The importance of the Turing mechanism in spatial economics has been recognized by Fujita et al. (1999, chapter 6) in the analysis of core-periphery models. Our analysis extends this approach mainly by explicit introduction of diffusion processes governing interacting economic and ecological variables in continuous time space in optimal management models, and by developing the ideas for the emergence of spatial heterogeneity in an optimizing context by an appropriate modification of Pontryagin's maximum principle.

In particular we consider the emergence of spatial heterogeneity in the context of an optimizing model, where the objective of a social planner is to maximize a welfare criterion subject to resource dynamics that include a diffusion process. We present a suggestion for extending Pontryagin's maximum principle to the optimal control of diffusion. Although conditions for the optimal control of partial differential equations have been derived either in abstract settings (e.g. Lions 1971) or for specific problems,<sup>7</sup> our derivation not only makes the paper self contained, but it is also close to the optimal control formalism used by economists, so it can be used for analyzing other types of economic problems, where state variables are governed by diffusion processes. Furthermore, the Pontryagin principle developed in this paper allows for an extension of the Turing mechanism for generation of spatial patterns, to the optimal control of systems under diffusion.

A new - to our knowledge - characteristic of our continuous space-time approach is that we are able to embed Turing analysis in an optimal control recursive infinite horizon approach in a way that allows us to

---

<sup>7</sup>See for example Lenhart and Bhat (1992); Lenhart et al. (1999); Bhat et al. (1999); Raymond and Zidani (1998, 1999).

locate sufficient conditions on parameters of the system (for example, the discount rate on the future, and interaction terms in the dynamics) for diffusive instability to emerge even in systems that are being optimally controlled. This mathematically challenging problem becomes tractable by exploiting the recursive structure of the utility and the dynamics in our continuous space/time framework in contrast to the more traditional approach of discrete patch optimizing models. This is so because the symmetries in the spatial structure coupled with the recursivity in the temporal structure of our framework reduce the potentially very large number of state and costate variables to a pair of “sufficient” variables that describe the dynamics of the whole system. We believe that our framework will be quite easily adaptable to other applications, including an extension of the classical Ramsey-Solow growth model to include spatial externalities. Colin Clark’s classic volume (1990), as well as the work of Sanchirico and Wilen (1999, 2001), is very suggestive, but they do not contain the unification of Turing analysis with infinite horizon temporally recursive optimal control problems that we present here.<sup>8</sup>

Here, we use our methodology to study an optimal fishery management problem under biomass diffusion. For the fishery problem, our results suggest that diffusion could alter the usual saddle point characteristics of the spatially homogeneous steady state as defined by the modified Hamiltonian dynamic system. In an analog to the Turing mechanism for an optimizing system, spatial heterogeneity in a steady state could be the result of optimal management. In particular we locate conditions for the Turing set of parameters inducing diffusive instability to be non-empty in the case where we have, under a positive discount rate, a saddle point steady state when diffusion is zero. On the other hand diffusion could stabilize, in the saddle point sense, an unstable steady state of an optimal control problem.

---

<sup>8</sup>We would note again that the Turing mechanism is not the only source of spatial heterogeneity in resource management models. As shown by Neubert (2003), a spatially heterogeneous steady state emerges in the temporal equilibrium of a bioeconomic model of optimal harvesting and marine reserve design, where the associated Hamiltonian function is linear in harvesting effort.



## 2 On the Optimal Control of Diffusion: An Extension of Pontryagin's Principle

In this section we explicitly introduce optimization and we analyze the effects of the optimal control of diffusion processes in the emergence of spatial heterogeneity through diffusion driven instability.

We start by considering an optimal control problem defined in the spatial domain  $z \in Z = [z_0, z_1]$  and the time domain  $t \in [t_0, t_1]$ . Let  $x(t, z), u(t, z)$  be the scalar state and control variables respectively at time  $t$  and spatial point  $z$ , taking values in compact sets  $\mathcal{X}$  and  $\mathcal{U}$ . Let  $f(x(t, z), u(t, z))$  be a net benefit function satisfying standard concavity assumptions and consider the following optimal control problem:

$$\max_{\{u(t,z)\}} \int_{z_0}^{z_1} \int_{t_0}^{t_1} f(x(t, z), u(t, z)) dt dz \quad (1)$$

$$\text{s.t. } \frac{\partial x(t, z)}{\partial t} = g(x(t, z), u(t, z)) + D \frac{\partial^2 x(t, z)}{\partial z^2} \quad (2)$$

$$x(t_0, z) \text{ given, } \left. \frac{\partial x(t, z)}{\partial z} \right|_{z=z_0} = \left. \frac{\partial x(t, z)}{\partial z} \right|_{z=z_1} = 0 : \text{ zero flux} \quad (3)$$

$$x(t, z_0) = x(t, z_1) = 0 : \text{ hostile boundary, } x(t_0, z) \text{ } z \in (z_0, z_1) \text{ given} \quad (4)$$

In the above problem the transition equation (2) states that the rate of change of the state variable, e.g the concentration of a biological resource or some other stock, at a given spatial point is determined by a general growth function  $g(x(t, z), u(t, z))$  which reflects the kinetics of the state variable, and by dispersion reflected by  $D \frac{\partial^2 x(t, z)}{\partial z^2}$ . In (2)  $D > 0$  is diffusivity or the diffusion coefficient and the basic assumptions regarding diffusion are those of the classical approach (or Ficksian diffusion), stating that the flux of the resource is proportional to the gradient of the resource concentration and that the movement is from high to low concentration. The first part of (3) provides initial conditions, while the second part is a zero flux condition. By zero flux condition it is assumed that there is no external biomass or effort input on the

boundary of the spatial domain.<sup>9</sup> Conditions (4) are an alternative set of boundary conditions indicating that the exterior of the spatial domain  $(z_0, z_1)$  is completely hostile to the resource (e.g. Murray, 2003, Vol II, p. 120; Neubert, 2003). So if  $x$  denotes a species, (4) imply that individuals that cross the boundary die.

Problem (1) is an optimal control problem in fixed and finite time and spatial domains. The zero flux terminal condition (3) corresponds to a “free endpoint problem” for the state variable, since the terminal value of the state variable is not *a priori* specified at terminal time or terminal space. The hostile boundary condition (4) can be associated with a type of a “fixed endpoint problem” for the state variable, since the terminal value of the state variable is zero at terminal space for all  $t$ . These terminal conditions will be used to specify the appropriate transversality conditions for the problem.

Problem (1) to (4) has been analyzed in more general forms (e.g. Lions, 1971). We however choose to present here an extension of Pontryagin’s principle for this problem, because it is in the spirit of optimal control formalism used by economists, and thus can be used for other applications, but also because it makes the whole analysis in the paper self contained.<sup>10</sup> Furthermore, as noted in the introduction, the use of Pontryagin’s principle in continuous time space allows for a drastic reduction in the dimensionality of the dynamic system describing the phenomenon and makes the problem tractable. Our results are presented below, with proofs in the Appendix.

**Maximum Principle under diffusion: Necessary Conditions - Finite time horizon (MPD-FT).** *Let  $u^* = u^*(t, z)$  be a choice of instrument that solves problem (1) to (4) and let  $x^* = x^*(t, z)$  be the associate path for the state variable. Then there exists a function  $\lambda(t, z)$  such that for each  $t$  and  $z$ .*<sup>11</sup>

---

<sup>9</sup>The zero flux boundary conditions is imposed so that the organizing pattern is self-organizing and not driven by boundary conditions (Murray 2003, Vol II, p.82).

<sup>10</sup>Similar conditions have been derived for other cases. such as the control of parabolic equations (Raymond and Zidani,1998, 1999), boundary control (Lenhart et al., 1999), or distributed parameter control (Dean Carlson et al., 1991; Lenhart and Bhat, 1992).

<sup>11</sup>In some cases in order to simplify notation, and when no confusion arises, sub-

1.  $u^* = u^*(t, z)$  maximizes the generalized Hamiltonian function

$$\mathbf{H}(x(t, z), u(t, z), \lambda(t, z)) = f(x(t, z), u(t, z)) + \lambda(t, z) \left[ g(x(t, z), u(t, z)) + D \frac{\partial^2 x(t, z)}{\partial z^2} \right]$$

or under appropriate concavity assumptions:

$$f_u + \lambda(t, z) g_u = 0 \quad (5)$$

- 2.

$$\frac{\partial \lambda(t, z)}{\partial t} = - \frac{\partial \mathbf{H}}{\partial x} - D \frac{\partial^2 \lambda(t, z)}{\partial z^2} = - \left( f_x + \lambda(t, z) g_x + D \frac{\partial^2 \lambda(t, z)}{\partial z^2} \right) \quad (6)$$

$$\frac{\partial x(t, z)}{\partial t} = g(x(t, z), u^*(t, z)) + D \frac{\partial^2 x(t, z)}{\partial z^2} \quad (7)$$

evaluated at  $u^* = u^*(x(t, z), \lambda(t, z))$ .

3. The following transversality conditions hold

$$\int_{z_0}^{z_1} \lambda(t_1, z) x(t_1, z) dz = 0, \Rightarrow \lambda(t_1, z) = 0, z \in [z_0, z_1] \quad (8)$$

For zero flux boundary conditions (3) it also holds that

$$\frac{\partial \lambda(t, z_1)}{\partial z} = \frac{\partial \lambda(t, z_0)}{\partial z} = 0 \quad (9)$$

The result can also be extended to infinite time horizon problems

---

scripts associated with functions denote partial derivatives.

with discounting. In this case the problem is:

$$\int_{z_0}^{z_1} \int_{t_0}^{\infty} e^{-\rho t} f(x(t, z), u(t, z)) dt dz, \rho > 0 \quad (10)$$

$$\text{s.t } \frac{\partial x}{\partial t} = g(x(t, z), u(t, z)) + D \frac{\partial^2 x(t, z)}{\partial z^2} \quad (11)$$

$$x(t_0, z_0) \text{ given, } \left. \frac{\partial x(t, z)}{\partial z} \right|_{z=z_0} = \left. \frac{\partial x(t, z)}{\partial z} \right|_{z=z_1} = 0 : \text{ zero flux} \quad (12)$$

$$x(t, z_0) = x(t, z_1) = 0 : \text{ hostile boundary, } x(t_0, z), z \in (z_0, z_1) \text{ given} \quad (13)$$

**Maximum Principle under diffusion: Necessary Conditions - Infinite time horizon (MPD-IT).** Let  $u^* = u^*(t, z)$  be a choice of instrument that solves problem (10) to (13) and let  $x^* = x^*(t, z)$  be the associate path for the state variable. Then there exists a function  $\lambda(t, z)$  such that for each  $t$  and  $z$

1.  $u^* = u^*(t, z)$  maximizes the generalized current value Hamiltonian function

$$\mathbf{H}(x(t, z), u, \lambda(t, z)) = f(x, u) + \lambda(t, z) \left[ g(x(t, z), u(t, z)) + D \frac{\partial^2 x(t, z)}{\partial z^2} \right],$$

or under appropriate concavity assumptions:

$$f_u + \lambda(t, z) g_u = 0 \quad (14)$$

- 2.

$$\frac{\partial \lambda(t, z)}{\partial t} = \rho \lambda(t, z) - \frac{\partial \mathbf{H}}{\partial x} - D \frac{\partial^2 \lambda(t, z)}{\partial z^2} = \quad (15)$$

$$\rho \lambda(t, z) - \left( f_x + \lambda(t, z) g_x + D \frac{\partial^2 \lambda(t, z)}{\partial z^2} \right) \quad (16)$$

$$\frac{\partial x(t, z)}{\partial t} = g(x(t, z), u^*(t, z)) + D \frac{\partial^2 x(t, z)}{\partial z^2}$$

evaluated at  $u^* = u^*(x(t, z), \lambda(t, z))$

3. *Transversality conditions at infinity are part of the sufficient conditions given below.*

It is clear that conditions (5)-(9) or (14)-(16) can characterize the whole dynamic system in continuous time space. It is interesting to note that (15) - (16) is a modified dynamic Hamiltonian system defined in continuous space time. In this system the diffusion coefficient for the costate variable is negative, and it is the opposite of the state variable's diffusion coefficient. Since the costate variable can be interpreted as the shadow value of the resource stock, negative diffusion implies that the movement in space is from low shadow values to higher shadow values. Furthermore, the opposite signs of the diffusion coefficient for the state and the costate variable imply that time 'runs backward' in the state variable and 'runs forward' in the costate variable which is a *forward capitalization* type variable in capital theoretic terms.

The conditions derived above are essentially necessary conditions. Sufficiency conditions can also be derived by extending sufficiency theorems of optimal control. Proofs are provided in the Appendix.

**Maximum Principle under diffusion: Sufficient conditions - Finite time horizon**

*Assume that functions  $f(x, u)$  and  $g(x, u)$  are concave differentiable functions for problem (1) to (4) and suppose that functions  $x^*(t, z)$ ,  $u^*(t, z)$  and  $\lambda(t, z)$  satisfy necessary conditions (5)-(9) for all  $t \in [t_0, t_1]$ ,  $z \in [z_0, z_1]$  and that  $x(t, z)$  and  $\lambda(t, z)$  are continuous with*

$$\lambda(t, z) \geq 0 \text{ for all } t \text{ and } z. \quad (17)$$

*Then the functions  $x^*(t, z)$ ,  $u^*(t, z)$  solve the problem (1) to (4). That is, the necessary conditions (5) - (9) are also sufficient.*

The result can also be extended along the lines of Arrow's sufficiency theorem. We state here the infinite horizon case.

**Maximum Principle under diffusion: Sufficient conditions - Infinite time horizon**

*Let  $\mathbf{H}^0$  denote the maximized Hamiltonian, or  $\mathbf{H}^0(x, \lambda) = \max_u \mathbf{H}(x, u, \lambda)$ . If the maximized Hamiltonian is a concave function of  $x$  for given  $\lambda$ , then*

functions  $x^*(t, z)$ ,  $u^*(t, z)$  and  $\lambda(t, z)$  that satisfy conditions (14)-(16) for all  $z \in [z_0, z_1]$  and the transversality conditions

$$\lim_{t \rightarrow \infty} e^{-\rho t} \int_{z_0}^{z_1} \lambda(t, z) dz \geq 0, \quad \lim_{t \rightarrow \infty} e^{-\rho t} \int_{z_0}^{z_1} \lambda(t, z) x(t, z) dz = 0 \quad (18)$$

or

$$\lim_{t \rightarrow \infty} e^{-\rho t} \lambda(t, z) x(t, z) = 0 \text{ when } (\lambda(t, z), x(t, z)) \geq 0 \quad \forall t, z \quad (19)$$

solve the problem (10) to (13).

### 3 Optimal Harvesting under Biomass Diffusion

Having established the optimality conditions, we are interested in the implications of diffusion on optimally controlled systems regarding mainly the possibility of emergence of spatial heterogeneity under optimal control, but also the possibility of diffusion acting as a stabilizing force for unstable steady states under optimal control. To illustrate our approach we use a classical case from ecological economics, namely the optimal harvesting of a renewable biological resource (e.g. fishery). Let  $x(t, z)$  denote the concentration of the biomass of a renewable resource (e.g. fish) at spatial point  $z \in \mathcal{Z}$ , at time  $t$ , with  $x$  taking non-negative values in a compact set  $\mathcal{X}$ , and  $\mathcal{Z}$  a one-dimensional spatial domain such that  $0 \leq z \leq a$ . Boundary conditions could be either zero flux at  $z = 0$  and  $z = a$ , that is,  $\left. \frac{\partial x(t, z)}{\partial z} \right|_{z=0} = \left. \frac{\partial x(t, z)}{\partial z} \right|_{z=a} = 0$ , or of the hostile type that is,  $x(t, 0) = x(t, a) = 0$ , implying that fish do not survive outside the spatial domain. Biomass grows according to a standard concave growth function  $F(x)$  and disperses in space with a constant diffusion coefficient  $D$ , or

$$\frac{\partial x(t, z)}{\partial t} = F(x(t, z)) - H(t, z) + D \frac{\partial^2 x(t, z)}{\partial z^2}$$

Harvesting  $H(t, z)$  of the resource is determined as  $H(t, z) = qx(t, z)E(t, z)$ , where  $E(t, z)$  denotes harvesting effort (e.g. boats) at spatial point  $z$  and time  $t$ , taking non-negative values in a compact set  $\mathcal{E}$ , and  $q > 0$  is the catchability coefficient. The total cost of applying effort  $E(t, z)$  at location  $z$  is given by an increasing and convex function  $c(E(t, z))$  in effort. Let benefits from harvesting at each point in space be given by

an increasing and concave function  $S(H(t, z))$ . The optimal harvesting problem in space-time is then defined as:

$$\max_{E(t,z)} \int_0^\infty \int_{\mathcal{Z}} e^{-\rho t} [S(H(t, z)) - c(E(t, z))] dz dt \quad (20)$$

$$\text{s.t. } \frac{\partial x(t, z)}{\partial t} = F(x(t, z)) - qx(t, z)E(t, z) + D \frac{\partial^2 x(t, z)}{\partial z^2} \quad (21)$$

$$x(0, z) \text{ given, and zero flux on } 0, a, \text{ or} \quad (22)$$

$$x(t, 0) = x(t, a) = 0, \quad x(0, z), \quad z \in (0, a) \text{ given} \quad (23)$$

Following the results of the previous section, **MPD-IT** implies that the optimal control maximizes the generalized current value Hamiltonian for each location  $z$ ,

$$\begin{aligned} \mathcal{H} = & S(H(t, z)) - c(E(t, z), z) + \\ & \mu(t, z) \left[ x(t, z)(s - rx(t, z)) - qx(t, z)E + D \frac{\partial^2 x(t, z)}{\partial z^2} \right] \end{aligned} \quad (24)$$

Setting  $S'(H(t, z)) = p(z) > 0$ , necessary conditions for the **MPD-IT**, omitting  $t$  to simplify notation, imply

$$\frac{\partial \mathcal{H}}{\partial E(z)} = 0 \text{ or } (p(z) - \mu(z))qx(z) = c'(E(z)) \quad (25)$$

$$E^0(z) = E(x(z), \mu(z)), \quad E^0(z) \geq 0, \text{ if } p(z) - \mu(z) \geq 0, \quad (26)$$

$$\frac{\partial E}{\partial x} = \frac{(p - \mu)q}{c''} > 0, \quad \frac{\partial E}{\partial \mu} = -\frac{qx}{c''} < 0 \text{ for all } z \quad (27)$$

Then, the Hamiltonian system in space time becomes:

$$\frac{\partial x}{\partial t} = F(x) - qx E(x, \mu) + D \frac{\partial^2 x}{\partial z^2} = G_1(x, \mu) + D \frac{\partial^2 x}{\partial z^2} \quad (28)$$

$$\frac{\partial \mu}{\partial t} = \left[ \rho - F'(x) + qE(x, \mu) \right] \mu - pqE(x, \mu) - D \frac{\partial^2 \mu}{\partial z^2} = G_2(x, \mu) - D \frac{\partial^2 \mu}{\partial z^2} \quad (29)$$

The Hamiltonian system (28) - (29) indicates that in the optimally controlled system the resource's biomass moves from high concentration to low concentration, while the biomass shadow value moves in space

from points of low value to points of high value. The purpose of our analysis is to examine conditions under which the *optimally controlled* diffusion system (28) - (29) could either produce a spatially heterogeneous pattern that will persist in the steady state, in the sense that the biomass concentration and the biomass shadow value will be different in different points of the spatial domain, or that the system will settle to a spatially homogeneous, or ‘flat’, state where the biomass concentration and the biomass shadow value are the same in every point of the spatial domain. We will explore the possibility of the Turing mechanism acting as a driver for inducing spatial heterogeneity.

### 3.1 The Turing mechanism in optimally controlled systems

The Turing mechanism for generating diffusion instability in reaction diffusion systems relies on the analysis of the stability of a spatially homogeneous (or ‘flat’) steady state of the associated dynamical system under perturbations induced by diffusion. In the optimally controlled system this implies that the Turing effect should be examined in association with the stability of the spatially homogeneous steady state of the Hamiltonian system (28) - (29). A “flat” steady state  $(x^*, \mu^*)$  for this system is determined as the solution of  $\frac{\partial x}{\partial t} = \frac{\partial \mu}{\partial t} = 0$  for  $D = 0$ . Given the nonlinear nature of (28) - (29), although it is possible to derive general conditions for the emergence of Turing instability, it not possible to derive closed form solutions and verify whether the conditions for the emergence of Turing instability are satisfied in a non-empty parameter set.

Since we feel it is important at this stage to verify the emergence of Turing instability in an optimally controlled system under diffusion, a task which to our knowledge has not been performed, we will replace the non-linear control problem with its linear quadratic approximation and verify the emergence of Turing instability for the linear quadratic model. In this way we can derive precise conditions under which Turing instability can emerge in linear quadratic models or models that can be formulated in terms of their linear quadratic approximations.



We start by replacing problem (20) - (23) with its linear quadratic approximation. In doing so we extend the method developed by Fleming (1971), and Magill (1977)<sup>12</sup> - by which a non-linear optimal stochastic control problem is replaced by a simpler linear quadratic optimal stochastic control problem - to the case in which a deterministic control problem (such as a resource management problem), where the transition of the system is described by a partial differential equation with a diffusion term, and not by an ordinary differential equation, is replaced by a linear quadratic approximation.

**Proposition 1** *Let  $(x^*, \mu^*)$  be a flat steady state of the Hamiltonian system (28) - (29) satisfying the optimality conditions (14)-(16). Let  $E^*$  be the corresponding steady-state effort, and  $H^* = qE^*x^*$ . Then under certain conditions problem (20) - (23) can be replaced by the linear quadratic (LQ) problem:*

$$\max_{u(t,z)} \int_0^\infty \int_{\mathcal{Z}} e^{-\rho t} \left[ -\frac{Q}{2}y^2 - \frac{R}{2}u^2 \right] dz dt \quad Q, R, \rho > 0 \quad (30)$$

$$s.t. \frac{\partial y(t,z)}{\partial t} = Sy(t,z) - Gu(t,z) + D \frac{\partial^2 y(t,z)}{\partial z^2}, S, G > 0 \quad (31)$$

$$y(0,z) \text{ given, and zero flux on } 0 \text{ and } a, \text{ or} \quad (32)$$

$$\text{hostile boundary } y(t,0) = y(t,a) = 0, y(0,z), z \in (0,a) \text{ given} \quad (33)$$

where

$$(y(t,z), \gamma(t,z), p(t,z)) = \quad (34)$$

$$(x(t,z) - x^*, E(t,z) - E^*, \mu(t,z) - \mu^*)$$

$$\text{and } u(t,z) = \gamma(t,z) + \frac{N}{B}y(t,z), N \gtrless 0, B < 0 \quad (35)$$

and the initial state  $x_0 = x(0,z)$  is close to  $x^*$  for all  $z \in \mathcal{Z}$ .

For the derivation and the definitions of the parameters of the LQ problem see Appendix.

Following the results of the previous section, **MPD-IT** implies that optimal control maximizes the generalized current value Hamiltonian for

---

<sup>12</sup>See also Judd (1996) for a similar approach.

the LQ problem for each location  $z$ ,

$$\mathcal{H} = -\frac{Q}{2}y^2 - \frac{R}{2}u^2 + p(t, z) \left[ Sy - Gu + D\frac{\partial^2 y}{\partial z^2} \right] \quad (36)$$

The necessary conditions for the **MPD-IT**, omitting  $t$  to simplify notation, imply

$$\frac{\partial \mathcal{H}}{\partial E(z)} = -Ru - pG = 0 \Rightarrow u^0 = -\frac{G}{R}p \quad (37)$$

$$\text{with } E = u^0 + E^* - N/B \geq 0 \quad (= 0 \text{ if } u^0 + E^* < N/B) \quad (38)$$

Then, the Hamiltonian system in space time becomes:

$$\frac{\partial y(t, z)}{\partial t} = Sy(t, z) - \frac{G}{R}p(t, z) + D\frac{\partial^2 y(t, z)}{\partial z^2} \quad (39)$$

$$\frac{\partial p(t, z)}{\partial t} = [\rho - S]p(t, z) + Qx(t, z) - D\frac{\partial^2 p(t, z)}{\partial z^2} \quad (40)$$

### 3.1.1 Existence of the Turing mechanism in optimally controlled LQ system

The flat steady state for  $(y^*, p^*)$  for the LQ problem is determined as the solution of  $\frac{\partial y}{\partial t} = \frac{\partial p}{\partial t} = 0$  of (39) - (40) for  $D = 0$ . It is clear by the homogeneity of the flat system (39) - (40) that the origin is the steady state, or  $(y^*, p^*) = (0, 0)$ . The stability of this steady state depends on the Jacobian matrix

$$J = \begin{bmatrix} S & G/R \\ Q & \rho - S \end{bmatrix}$$

Therefore for the flat steady state we have  $\text{tr}(J) = \rho > 0$  and  $\det J = (\rho - S)S - G/R$ . Hence, if  $\det J > 0$  the steady state is unstable, while if  $\det J < 0$  the steady state has the local saddle point property. In the saddle point case there is a one-dimensional manifold such that for any initial value of  $y$  there is an initial value for  $p$ , such that the system converges to the origin along the manifold.

The idea behind the Turing mechanism for diffusion driven instability and pattern formation is that an asymptotically stable, in the absence of diffusion, spatially homogeneous steady state, can be destabilized locally

by perturbations induced by diffusion. The result of this instability could be the emergence of a regular stable patterned distribution of biomass and its shadow value across the spatial domain.

To analyze the impact of diffusion consider the Jacobian of the full Hamiltonian system (39) - (40), to obtain:

$$\mathbf{w}_t = J\mathbf{w} + \tilde{D}\mathbf{w}_{zz}, \quad (41)$$

$$\mathbf{w} = \begin{pmatrix} y(t, z) \\ p(t, z) \end{pmatrix}, \quad \mathbf{w}_t = \begin{pmatrix} \partial y / \partial t \\ \partial p / \partial t \end{pmatrix}, \quad \mathbf{w}_{zz} = \begin{pmatrix} \partial^2 y / \partial z^2 \\ \partial^2 p / \partial z^2 \end{pmatrix}, \quad \tilde{D} = \begin{pmatrix} D & 0 \\ 0 & -D \end{pmatrix} \quad (42)$$

Following Murray (2003) we consider the time-independent solution of the spatial eigenvalue problem

$$\mathbf{W}_{zz} + k^2\mathbf{W} = 0, \quad \mathbf{W}_z = 0, \quad \text{for } z = 0, a \quad (43)$$

where  $k$  is the eigenvalue. For the one-dimensional domain  $(0, a)$  we have solutions for (43) which are of the form

$$\mathbf{W}_k(z) = \mathbf{A}_n \cos\left(\frac{n\pi z}{a}\right), \quad n = \pm 1, \pm 2, \dots, \quad (44)$$

where  $\mathbf{A}_n$  are arbitrary constants. Solution (44) satisfies the zero flux condition at  $z = 0$  and  $z = a$ .<sup>13</sup> The eigenvalue is  $k = n\pi/a$ , and  $1/k = a/n\pi$  is a measure of the wave-like pattern. The eigenvalue  $k$  is called the *wavenumber* and  $1/k$  is proportional to the wavelength  $\omega$ :  $\omega = 2\pi/k = 2a/n$ . Let  $\mathbf{W}_k(z)$  be the eigenfunction corresponding to the wavenumber  $k$ . We then look for solutions of (41) of the form

$$\mathbf{w}(t, z) = \sum_k c_k e^{\lambda t} \mathbf{W}_k(z) \quad (45)$$

Substituting (45) into (41), using (43) and canceling  $e^{\lambda t}$  we obtain for

---

<sup>13</sup>If we are to use the hostile boundary conditions (4) then the solution would be of the form  $\mathbf{W}_k(z) = \mathbf{A}_n \sin\left(\frac{n\pi z}{a}\right)$ ,  $n = \pm 1, \pm 2, \dots$ , so that boundary conditions are satisfied at 0 and  $a$ .

each  $k$  or equivalently each  $n$ , that  $\lambda \mathbf{W}_k = J \mathbf{W}_k - Dk^2 \mathbf{W}_k$ . Since we require non-trivial solutions for  $\mathbf{W}_k$ ,  $\lambda$  must solve

$$\left| \lambda I - J + \tilde{D}k^2 \right| = 0$$

Then the eigenvalue  $\lambda(k)$  as a function of the wavenumber is obtained as the roots of

$$\lambda^2 - \rho\lambda + h(k^2) = 0 \quad (46)$$

$$h(k^2) = -D^2k^4 + D(2S - \rho)k^2 + \det J \quad (47)$$

where the roots are given by:

$$\lambda_{1,2}(k^2) = \frac{1}{2} \left( \rho \pm \sqrt{\rho^2 - 4h(k^2)} \right)$$

It should be noted that the flat (no diffusion) case corresponds to  $k^2 = 0$ , so that  $h(k^2 = 0) = \det J$ , and  $\lambda_{1,2} = \frac{1}{2} \left( \rho \pm \sqrt{\rho^2 - 4 \det J} \right)$ . We examine the implication of diffusion in the case where the spatially homogeneous steady state is a saddle point, that is  $\lambda_2 < 0 < \lambda_1$  for  $k^2 = 0$ , and diffusion generates spatial heterogeneity through the Turing mechanism.

In this case  $\det J < 0$ . Since  $\text{tr } J > 0$  the spatially homogeneous system converges to the flat steady state  $(x^*, p^*) = (0, 0)$  along the stable manifold. On this manifold and in the neighborhood of the steady state, for any initial value of  $y$  there is an initial value of  $p$  such that the spatially homogeneous system converges to the flat steady state. For the optimally-controlled system the optimal solution in the neighborhood of the steady state is such that

$$\begin{pmatrix} y^0(t, z) \\ p(t, z) \end{pmatrix} = C_2 \mathbf{v}_2 e^{\lambda_2 t}, \text{ for all } z \quad (48)$$

where  $C_2$  is a constant determined by initial conditions on  $y$  and transversality conditions, and  $\mathbf{v}_2$  is the eigenvector corresponding to  $\lambda_2$ . In particular for the linearized system the transversality condition at infinity,  $\lim_{t \rightarrow \infty} e^{-\rho t} \int_0^\infty p(t, z) y^0(t, z) = 0$  for all  $z$ , forces the constant  $C_1$  associated with positive root  $\lambda_1$  to be zero. Thus by choosing  $C_2$  such

that initial conditions on  $y$  and transversality conditions at infinity are satisfied, the initial conditions for  $p$  are selected such that the linearized system ends on the stable manifold. The corresponding path for the optimal control  $u$  is given by  $u^0 = (-G/R)p(t, z)$  for all  $z$ . Solution (48) can be used to define the stable manifold as a function  $p = \phi(y)$ , and the associated optimal policy function  $u^0 = \psi(y)$ . By choosing appropriate values for  $y$  in the neighborhood of the steady state, such that  $y_L < y^* < y_U$ , the stable manifold can be represented by the set

$$M_S = \{(y, p) : p = \phi(y), y \in (y_L, y_U)\} \quad (49)$$

For any point along the manifold the state-costate system converges to the spatially homogeneous steady state.

We consider now the impact of a perturbation induced by diffusion. Under diffusion the smallest root  $\lambda_2$  is given by

$$\lambda_2(k^2) = \frac{1}{2} \left( \rho - \sqrt{\rho^2 - 4h(k^2)} \right), k^2 = \frac{n^2 \pi^2}{a^2} \quad (50)$$

Then,

- If  $0 < h(k^2) < \rho^2/4$  for some  $k$ , then  $\lambda_2$  becomes real and positive.
- If  $h(k^2) > \rho^2/4$  for some  $k$ , then both roots corresponding to  $\lambda_2$  are complex with positive real parts.

In both cases above, the linearly stable steady state  $(y^*, p^*) \in M_S$  becomes unstable to *spatial* disturbances. Therefore if  $h(k^2) > 0$  for some  $k$ , then  $\lambda_2(k^2) > 0$  and the optimally controlled Hamiltonian system becomes unstable to spatial perturbations, in the neighborhood of the flat steady state and along the stable manifold. From (47) the quadratic function  $h(k^2)$  is concave, and therefore has a maximum. Furthermore,  $h(0) = \det J < 0$  and  $h'(0) = (2S - \rho)$ . Then  $h(k^2)$  has a maximum for

$$k_{\max}^2 : h'(k_{\max}^2) = 0, \text{ or } k_{\max}^2 = \frac{(2S - \rho)}{2D} > 0, \text{ for } (2S - \rho) > 0 \quad (51)$$

If  $h(k_{\max}^2) > 0$  or  $-D^2k_{\max}^4 + D(2S - \rho)k_{\max}^2 + \det J > 0$ , and  $2S - \rho > 0$ , then there exist two positive roots  $k_1^2 < k_2^2$  such that  $h(k^2) > 0$  and  $\lambda_2(k^2) > 0$  for  $k^2 \in (k_1^2, k_2^2)$ . Using (51) the existence of two positive roots  $k_1^2 < k_2^2$  requires

$$\frac{(2S - \rho)^2}{4} + \det J > 0, \text{ or} \quad (52)$$

$$\frac{(2S - \rho)^2}{4} + (\rho - S)S - \frac{QG}{R} = \frac{\rho^2}{4} - \frac{QG}{R} > 0 \quad (53)$$

The interval  $(k_1, k_2)$  determines the range of the unstable modes associated with the spatial heterogeneous solution, while  $h(k^2)$  is the dispersion relationship associated with the optimal control problem.<sup>14</sup> Diffusion driven instability in the optimally controlled system emerges if the maximum of the dispersion relationship is in the positive quadrant along with the negative condition on the Jacobian of the flat system. These conditions are summarized below.

$$(\rho - S)S - \frac{QG}{R} < 0 \quad (54)$$

$$2S - \rho > 0 \quad (55)$$

$$\frac{\rho^2}{4} - \frac{QG}{R} > 0 \quad (56)$$

with

$$k_{1,2}^2 = \frac{(2S - \rho) \pm \sqrt{(\rho^2 - 4QG/R)}}{2} > 0 \quad (57)$$

The set of parameters for which (54)-(56) is satisfied is the *Turing space*. It is clear that for  $\rho = 0$  the Turing space is empty and diffusion driven instability does not emerge. However for higher discount rates and for appropriate values of  $Q, G, S$  and  $R$ , the Turing space need not be empty. This is shown in figure 1 where the Turing space is defined in the  $(\rho, R)$  space for given values of  $Q, G, S$ .

[Figure 1]

The inequality (54) is satisfied above line  $BB$ , the inequality (55)

---

<sup>14</sup>For a detailed analysis of the dispersion relationship in problems without optimization, see Murray (2003).

is satisfied below the line  $2SCD$ , while the inequality (56) is satisfied above the line  $AA$ . Thus the Turing space is the area  $DCB$ .

Assume that for a parameter constellation  $(\rho, S, R, Q, G)$  the Turing set is not empty. Then the optimal spatially heterogeneous solution, under zero flux boundary conditions emerging from (44) and (45), is the sum of unstable modes or

$$\mathbf{w}^0(t, z) \sim \sum_{n_1}^{n_2} \mathbf{B}_n \exp \left[ \lambda_2 \left( \frac{n^2 \pi^2}{a^2} \right) t \right] \cos \frac{n\pi z}{a}, k^2 = \left( \frac{n\pi}{a} \right)^2$$

where  $\lambda_2(k^2) > 0$  for  $k^2 \in (k_1^2, k_2^2)$ ,  $n_1$  is the smallest integer greater or equal to  $ak_1/\pi$  and  $n_2$  is the largest integer less than or equal to  $ak_2/\pi$ , and the wavenumbers  $k_1$  and  $k_2$  are such that  $h(k^2) > 0$ . Since  $\lambda_2(k^2) > 0$  for  $k^2 \in (k_1^2, k_2^2)$  only these modes grow with time; all the remaining modes for which  $\lambda_2(k^2) < 0$  tend to zero exponentially. Assume that the spatial domain is such that there is only one unstable wave number, or  $n = 1$ . Then the only unstable mode is  $\cos(\pi z/a)$ , and the growing instability is determined by

$$\mathbf{w}^0(t, z) \sim \mathbf{B}_1 \exp \left[ \lambda_2 \left( \frac{\pi^2}{a^2} \right) t \right] \cos \frac{\pi z}{a} \quad (58)$$

where the vector of constants  $\mathbf{B}_1$  is determined by initial conditions. Since the instability occurs on the stable manifold of the linearized system (49) it would be natural to choose initial conditions for  $y$  and  $p$  on this manifold. Take  $\mathbf{B}_1 = (\epsilon_x, \epsilon_p)$ , then using the definition of  $\mathbf{w}$  from (42) we have that the optimal spatially heterogeneous solution evolves approximately as:

$$y^0(t, z) \sim \epsilon_x \exp \left[ \lambda_2 \left( \frac{\pi^2}{a^2} \right) t \right] \cos \frac{\pi z}{a}, \frac{\pi^2}{a^2} = k^2 \quad (59)$$

$$p(t, z) \sim \epsilon_p \exp \left[ \lambda_2 \left( \frac{\pi^2}{a^2} \right) t \right] \cos \frac{\pi z}{a} \quad (60)$$

Solutions (59) - (60) indicate that diffusion causes the spatially homogeneous steady state to be transformed into a wave-like pattern as  $t$  increases. This of course is spatial heterogeneity since the biomass and

its shadow value will, at any given point in time, have different values in different spatial points. Then the path for optimal effort in the neighborhood of the flat steady state will be determined as  $u^0(t, z) = (-G/R)p(t, z)$ , while the spatially heterogeneous optimal effort is determined, using (34) and (35) as:

$$E^0(t, z) = E^* + u^0(t, z) - \frac{N}{B} = E^* - \frac{G}{R}p(t, z) - \frac{N}{B}, \quad E^0(t, z) \geq 0 \quad (61)$$

Furthermore a conjecture can be stated. For the optimal paths  $(y^0(t, z), u^0(t, z))$  of the solution to the LQ problem, an analog in time-space of a Michel-type transversality condition (Michel, 1982) is verified. This transversality conditions implies that the maximum of the Hamiltonian of the LQ problem for every spatial point is zero when  $t$  goes to infinity. Following Michel (1982) the maximum of the Hamiltonian should verify for every  $(t, z)$  that:

$$\begin{aligned} \mathcal{H}^{\max}(t, z) = e^{-\rho t} & \left[ -\frac{Q}{2} (y^0(t, z))^2 - \frac{R}{2} (u^0(t, z))^2 + \right. \\ & \left. p(t, z) \left( S y^0(t, z) - G u^0(t, z) + D \frac{\partial^2 y(t, z)}{\partial z^2} \right) \right] \end{aligned} \quad (62)$$

Substituting (59) - (60) into (62), taking the limit as  $t \rightarrow \infty$  and noting that, by the definition of a steady state, for all  $z$  the term

$$p(t, z) \left[ S y^0(t, z) - G u^0(t, z) + D \frac{\partial^2 y(t, z)}{\partial z^2} \right] \quad (63)$$

is zero as  $t \rightarrow \infty$ , we obtain

$$\begin{aligned} \lim_{t \rightarrow \infty} \mathcal{H}^{\max}(t, z) = & \quad (64) \\ \lim_{t \rightarrow \infty} & \left[ -\frac{1}{2} e^{(2\lambda_2 - \rho)t} \left[ Q \epsilon_x^2 + \frac{G^2}{R} \epsilon_p^2 \right] \cos^2 \left( \frac{\pi z}{a} \right) \right] = 0 \end{aligned}$$

since, as can be seen from (50),  $2\lambda_2 < \rho$ .

The value function of the LQ problem

$$V(y(0, z), 0, 0) = \sup \int_0^\infty \int_Z e^{-\rho t} \left[ -\frac{Q}{2} y^2 - \frac{R}{2} u^2 \right] dz dt \quad (65)$$



should verify that:

$$V(y^0(0, z), 0, 0) = \tag{66}$$

$$\int_0^\infty \int_{\mathcal{Z}} e^{-\rho t} \left[ -\frac{Q}{2} (y^0(t, z))^2 - \frac{R}{2} (u^0(t, z))^2 \right] dz dt =$$

$$-\frac{1}{2} \int_0^\infty e^{(2\lambda_2 - \rho)t} \int_{\mathcal{Z}} e^{(2\lambda_2 - \rho)t} \left[ Q\epsilon_x^2 + \frac{G^2}{R}\epsilon_p^2 \right] \cos^2\left(\frac{\pi z}{a}\right) dt dz \tag{67}$$

which is finite since  $2\lambda_2 < \rho$ , indicating that the LQ problem is well posed. These results can be summarized in the following proposition.

**Proposition 2** *For an optimal harvesting system of an LQ form or for a non-linear system that can be adequately approximated by an LQ system, which exhibits the saddle point property at a steady state in the absence of diffusion, it is optimal, under biomass diffusion and for a certain set of parameter values, to have emergence of diffusive instability, induced by the Turing mechanism. Diffusive instability leads to a spatially heterogeneous optimal path where the biomass and its shadow value will, at any given point in time, have different values in different spatial points.*

The significance of this proposition, which extends the concept of the Turing mechanism to the optimal control of diffusion, is that spatial heterogeneity and pattern formation, resulting from diffusive instability, might be an optimal outcome under certain circumstances. For regulation purposes and for the harvesting problem examined above, it is clear that the spatially heterogeneous steady-state shadow value of the resource stock, and the corresponding harvesting effort, can be used to define *optimal regional* fees or quotas. Although the full characterization of the spatially heterogeneous steady state is outside the purpose of this paper, since our target is to show the existence of the Turing mechanism in optimally controlled systems, there are some inferences that can be heuristically made from the results obtained by the LQ problem.

If the LQ approximation is an adequate one for the non-linear system, it is expected that a saddle point steady state of the non-linear system  $(x^*, \mu^*)$  will also be destabilized by perturbations caused by diffusion

through the Turing effect.<sup>15</sup> With a non-empty Turing space, spatially heterogenous solutions similar to (59) - (60) grow exponentially. This however cannot be valid for all  $t$ , since then exponential growth would imply that  $(x, \mu) \rightarrow \infty$  at  $t \rightarrow \infty$ . However, the kinetics of the Hamiltonian system (28) - (29) and the transversality conditions at infinity (18) should bound the solution in the positive quadrant.<sup>16</sup> This implies that for a subset of the spatial domain the resource stock and its shadow value are above the flat steady-state levels and for another subset they are below the flat steady-state levels, in a wave-like pattern. In this case an ultimate steady-state spatially heterogeneous solution for the optimally controlled system will emerge.<sup>17</sup> This steady state can be characterized by taking the steady state of (28) - (29) and defining the dynamic system in the spatial domain  $[0, a]$ .

$$0 = F(x) - qx E(x, \mu) + D \frac{\partial^2 x}{\partial z^2}, \text{ or } -G_1(x, \mu) = D \frac{\partial^2 x}{\partial z^2} \quad (68)$$

$$0 = \left[ \rho - F'(x) + qE(x, \mu) \right] \mu - pqE(x, \mu) - D \frac{\partial^2 \mu}{\partial z^2}, \quad (69)$$

$$\text{or } G_2(x, \mu) = D \frac{\partial^2 \mu}{\partial z^2} \quad (70)$$

Setting  $v = \frac{\partial x}{\partial z}$ ,  $u = \frac{\partial \mu}{\partial z}$ , we obtain the first-order system

$$-G_1(x, \mu) = D \frac{\partial v}{\partial z} \quad (71)$$

$$v = \frac{\partial x}{\partial z} \quad (72)$$

$$G_2(x, \mu) = D \frac{\partial u}{\partial z} \quad (73)$$

$$u = \frac{\partial \mu}{\partial z} \quad (74)$$

Under zero flux boundary conditions the boundary conditions for this system are  $v(0) = v(a) = 0$ , and  $u(0) = u(a) = 0$  from zero flux,

---

<sup>15</sup>It should be noticed that it was around this steady state that the LQ approximation was carried out.

<sup>16</sup>See Murray (2003, Vol II, pp. 93-94) for this type of argument.

<sup>17</sup>In this context it may be shown (Segel and Levin, 1976) that the destabilized spatially homogeneous pattern is replaced asymptotically by a stable spatially heterogeneous solution.

while under hostile boundary conditions we have  $x(0) = x(a) = 0$  and  $\mu(0) = \mu(a) = 0$ .

### 3.1.2 Diffusion as a stabilizer

We examine now the case where the spatially homogeneous steady state is unstable, that is  $\text{Re } \lambda_{1,2} > 0$  for  $k^2 = 0$ , and diffusion acts as a stabilizing form. Since  $\text{tr } J > 0$ , this implies that  $\det J > 0$ . Let  $\Delta_D = \rho^2 - 4[\det J] > 0$  so that we have two positive real roots at the flat steady state. Diffusion can stabilize the system in the sense of producing a negative root. For the smallest root to turn negative or  $\lambda_2 < 0$ , it is sufficient that  $h(k^2) < 0$ . The quadratic function (47) is concave, and therefore has a maximum. Furthermore  $h(0) = \det J > 0$  and if  $h'(0) = (2F - \rho) > 0$  there is a root  $k_2^2 > 0$ , as shown in figure 2, such that for  $k^2 > k_2^2$ , we have  $\lambda_2 < 0$ . The solutions for  $y(t, z)$  and  $p(t, z)$  will be determined by the sum of exponentials of  $\lambda_1$  and  $\lambda_2$ . Since we want to stabilize the system we set the constant associated with the positive root  $\lambda_1$  equal to zero. Then the solution will depend on the sum of unstable and stable modes associated with  $\lambda_2$ .

[Figure 2]

Following the previous procedure the solutions for  $y$  and  $p$  will be of the form:

$$\begin{pmatrix} y(t, z) \\ p(t, z) \end{pmatrix} \sim \sum_0^{n_2} \mathbf{C}_{\hat{n}} \exp \left[ \lambda_2 \left( \frac{n^2 \pi^2}{a^2} \right) t \right] \cos \frac{n\pi z}{a} + \sum_{n_2}^N \mathbf{C}_n \exp \left[ \lambda_2 \left( \frac{n^2 \pi^2}{a^2} \right) t \right] \cos \frac{n\pi z}{a}, \quad (75)$$

where  $n_2$  is the smallest integer greater or equal to  $ak_2^2/\pi$  and  $N > n_2$ . Since  $\lambda_2 \left( \frac{n^2 \pi^2}{a^2} \right) < 0$  for  $n > n_2$ , all the modes of the second term of (75) decay exponentially. So to converge to the steady state we need to set  $\mathbf{C}_{\hat{n}} = 0$ , then the spatial patterns corresponding to the second term of (75) will die out with the passage of time and the system will converge to the spatially homogeneous steady state  $(y^*, p^*) = (0, 0)$ .

This result can be summarized in the following proposition.

**Proposition 3** *For an optimal harvesting system of an LQ form or for a nonlinear system that can be adequately approximated by an LQ system, with an unstable steady state in the absence of diffusion, it is optimal, under biomass diffusion and for a certain set of parameter values, to stabilize the steady state. Stabilization is in the form of saddle point stability where spatial patterns decay and the system converges along one direction to the previously unstable spatially homogeneous steady state.*

The significance of this proposition is that it shows that under diffusion it is optimal to stabilize a steady state which was unstable under spatial homogeneity.

## 4 Concluding Remarks

The present paper seeks to provide a conceptual framework for studying pattern formation in optimally controlled systems associated with economic applications. Considering the Turing mechanism as the pattern generator we develop the optimal control of a dynamical system under diffusion by appropriately extending Pontryagin's maximum principle. Using as our leading example the classical problem of harvesting of a renewable resource (fishery) we show that, when we have a saddle point equilibrium with zero diffusion for a positive discount rate, then there exists a non-empty parameter set such that the Turing mechanism acting on the associated Hamiltonian Dynamic System implies that the optimal choice of control (harvesting effort) in time-space leads to the emergence of a spatial pattern for both the resource stock (state variable) and its corresponding shadow value (costate variable). In the same context we show that, when we have an unstable steady state with zero diffusion, then the presence of diffusion in the optimal harvesting problem can, in certain cases, stabilize an unstable spatially homogeneous steady state.

The methodological approach developed in this paper can be linked to further research in the optimal management and the design of optimal policies for general complex adaptive systems arising in economics, where self organizing aspects reflected in notions such as 'the invisible hand' or Pareto optimality are complemented by policy interactions aiming at directing the system to a desired outcome (Levin, 2002). The

spatial and pattern formation aspect of these complex adaptive systems, with the Turing mechanism acting as a pattern generator, when coupled with policy interventions produce the type of optimal control problem in space-time studied in this paper.

In more general terms the Turing mechanism is one pattern generator that can be used in the study of socio-economic systems in the context of developing statistical mechanics approaches aiming at exploring how individual microscopic interactions give rise to macroscopic phenomena (Durlauf, 1997). It should be noted that the application of pattern generators to complex socio-economic systems has yet to overcome tractability issues, although there are some exceptions such as the Large Type Limit concept (Brock et al., 2005) and its generalization (Diks and Vanderweide, 2003) that provide an analytically tractable pattern generator for stock market applications. The use of the Turing mechanism as pattern generator in recursive infinite horizon optimal control developed in this paper, apart from its usefulness in studying other economic applications and pattern formation in time-space, can also be useful as a basis for extending the analysis to general pattern generating systems where patterns emerge from individual agent heterogeneity into macroscopic dynamics and macroscopic patterns.

## Appendix

### Extension of Pontryagin's Principle: Necessary conditions

We develop a variational argument along the lines of Kamien and Schwartz (1981, pp. 115-116). Problem (1) to (4) can be written as:

$$J = \int_{z_0}^{z_1} \int_{t_0}^{t_1} f(x(t, z), u(t, z)) dt dz = \int_{z_0}^{z_1} \int_{t_0}^{t_1} \left\{ f(x(t, z), u(t, z)) \right. \\ \left. \lambda(t, z) \left[ g(x(t, z), u(t, z)) + D \frac{\partial^2 x}{\partial z^2} - \frac{\partial x}{\partial t} \right] \right\} dt dz \quad (76)$$

We integrate by parts the last two terms of (76). The  $\lambda(t, z) \frac{\partial x}{\partial t}$  term becomes

$$(-1) \int_{z_0}^{z_1} \int_{t_0}^{t_1} \lambda(t, z) \frac{\partial x}{\partial t} dt = \\ \int_{z_0}^{z_1} \left[ -\lambda(t_1) x(t_1) + \lambda(t_0) x(t_0) + \int_{t_0}^{t_1} x(t, z) \frac{\partial \lambda}{\partial t} dt \right] dz \quad (77)$$

The term  $\lambda(t, z) D \frac{\partial^2 x}{\partial z^2}$  becomes

$$D \int_{z_0}^{z_1} \int_{t_0}^{t_1} \lambda(t, z) \frac{\partial^2 x(t, z)}{\partial z^2} = \\ D \int_{t_0}^{t_1} \left[ \lambda(t, z_1) \frac{\partial x(t, z)}{\partial z} \Big|_{z=z_1} - \lambda(t, z_0) \frac{\partial x(t, z_0)}{\partial z} \Big|_{z=z_0} - \int_{z_0}^{z_1} \frac{\partial x(t, z)}{\partial z} \frac{\partial \lambda(t, z)}{\partial t} dz \right] dt = \\ - D \int_{t_0}^{t_1} \left[ \int_{z_0}^{z_1} \frac{\partial x(t, z)}{\partial z} \frac{\partial \lambda(t, z)}{\partial t} dz \right] dt \quad (78)$$

by the zero flux conditions (3) on the state variable, or by setting  $\lambda(z_1) = \lambda(z_0) = 0$  if we use the hostile boundary conditions  $x(z_1) = x(z_0) = 0$ .

Integrating by parts once more we have

$$\begin{aligned}
& (-1) D \int_{t_0}^{t_1} \left[ \int_{z_0}^{z_1} \frac{\partial x(t, z)}{\partial z} \frac{\partial \lambda(t, z)}{\partial t} dz \right] dt = \\
& D \int_{t_0}^{t_1} \left[ -\frac{\partial \lambda(t, z_1)}{\partial z} x(t, z_1) + \frac{\partial \lambda(t, z_0)}{\partial z} x(t, z_0) \right. \\
& \left. + \int_{z_0}^{z_1} x(t, z) \frac{\partial^2 \lambda(t, z)}{\partial z^2} \right] dz dt \tag{79}
\end{aligned}$$

Thus (76) becomes

$$\begin{aligned}
& \int_{z_0}^{z_1} \int_{t_0}^{t_1} f(x(t, z), u(t, z)) dt dz = \\
& \int_{z_0}^{z_1} \int_{t_0}^{t_1} \left[ f(x(t, z), u(t, z)) + \lambda(t, z) g(x(t, z), u(t, z)) \right. \\
& \left. + x(t, z) \frac{\partial \lambda(t, z)}{\partial t} + x(t, z) D \frac{\partial^2 \lambda(t, z)}{\partial z^2} \right] dt dz \\
& + \int_{z_0}^{z_1} [-\lambda(t_1, z) x(t_1, z) + \lambda(t_0, z) x(t_0, z)] dz + \\
& D \int_{t_0}^{t_1} \left[ -\frac{\partial \lambda(t, z_1)}{\partial z} x(t, z_1) + \frac{\partial \lambda(t, z_0)}{\partial z} x(t, z_0) \right] dt \tag{80}
\end{aligned}$$

We consider a one parameter family of comparison controls  $u^*(t, z) + \epsilon \eta(t, z)$ , where  $u^*(t, z)$  is the optimal control,  $\eta(t, z)$  is a fixed function and  $\epsilon$  is a small parameter. Let  $y(t, z, \epsilon)$ ,  $t \in [t_0, t_1]$ ,  $z \in [z_0, z_1]$  be the state variable generated by (2) and (3) or (4) with control  $u^*(t, z) + \epsilon \eta(t, z)$ ,  $t \in [t_0, t_1]$ ,  $z \in [z_0, z_1]$ . We assume that  $y(t, z, \epsilon)$  is a smooth function of all its arguments and that  $\epsilon$  enters parametrically. For  $\epsilon = 0$  we have the optimal path  $x^*(t, z)$ ; furthermore all comparison paths must satisfy initial and zero flux or hostile boundary conditions. Thus,

$$y(t, z, 0) = x^*(t, z), \quad y(t_0, z, \epsilon) = x(t_0, z) \text{ fixed} \tag{81}$$

$$\left. \frac{\partial y(t, z)}{\partial z} \right|_{z=z_0} = \left. \frac{\partial y(t, z)}{\partial z} \right|_{z=z_1} = 0, \quad \text{zero flux} \tag{82}$$

$$y(t, z_1, \epsilon) = y(t, z_0, \epsilon) = 0 \text{ hostile boundary} \tag{83}$$

When the functions  $u^*$ ,  $x^*$  and  $\eta$  are held fixed, the value of (1) evaluated

along the control function  $u^*(t, z) + \epsilon\eta(t, z)$  and the corresponding state function  $y(t, z, \epsilon)$  depend only on the single parameter  $\epsilon$ . Therefore,

$$J(\epsilon) = \int_{z_0}^{z_1} \int_{t_0}^{t_1} [f(y(t, z, \epsilon), u^*(t, z) + \epsilon\eta(t, z))] dt dz$$

or using (80)

$$\begin{aligned} J(\epsilon) &= \int_{z_0}^{z_1} \int_{t_0}^{t_1} [f(y(t, z, \epsilon), u^*(t, z) + \epsilon\eta(t, z)) \\ &\quad + \lambda(t, z) g(y(t, z, \epsilon), u^*(t, z) + \epsilon\eta(t, z)) \\ &\quad + y(t, z, \epsilon) \frac{\partial \lambda(t, z)}{\partial t} + Dy(t, z, \epsilon) \frac{\partial^2 \lambda(t, z)}{\partial z^2}] dt dz \\ &\quad + \int_{z_0}^{z_1} [-\lambda(t_1, z) y(t_1, z, \epsilon) + \lambda(t_0, z) y(t_0, z, \epsilon)] dz \\ &\quad + D \int_{t_0}^{t_1} \left[ -\frac{\partial \lambda(z_1)}{\partial z} y(t, z_1, \epsilon) + \frac{\partial \lambda(z_0)}{\partial z} y(t, z_0, \epsilon) \right] dt \end{aligned} \quad (84)$$

Since  $u^*$  is a maximizing control the function  $J(\epsilon)$  assumes the maximum when  $\epsilon = 0$ . Thus  $\left. \frac{dJ(\epsilon)}{d\epsilon} \right|_{\epsilon=0}$  or

$$\begin{aligned} \left. \frac{dJ(\epsilon)}{d\epsilon} \right|_{\epsilon=0} &= \\ &\int_{z_0}^{z_1} \int_{t_0}^{t_1} \left[ \left( f_x + \lambda g_x + \frac{\partial \lambda(t, z)}{\partial t} + D \frac{\partial^2 \lambda(t, z)}{\partial z^2} \right) y_\epsilon + (f_u + \lambda g_u) \eta(t, z) \right] dt dz + \\ &\int_{z_0}^{z_1} [-\lambda(t_1, z) y_\epsilon(t_1, z, \epsilon) + \lambda(t_0, z) y_\epsilon(t_0, z, \epsilon)] dz + \\ &D \int_{t_0}^{t_1} \left[ -\frac{\partial \lambda(z_1)}{\partial z} y_\epsilon(t, z_1, \epsilon) + \frac{\partial \lambda(z_0)}{\partial z} y_\epsilon(t, z_0, \epsilon) \right] dt = 0 \end{aligned} \quad (85)$$

- In (85)  $y_\epsilon(t_0, z, \epsilon) = 0$ , since  $y(t_0, z, \epsilon) = x(t_0, z)$  fixed by initial conditions. Next we impose the condition

$$\int_{z_0}^{z_1} \lambda(t_1, z) \beta(t_1, z) = 0 \quad (86)$$

for all  $\beta(t_1, z)$  piecewise continuous functions in  $[z_0, z_1]$ . It follows,



using Athans and Falb's (1996, p260) fundamental lemma that

$$\lambda(t_1, z) = 0, z \in [z_0, z_1] \quad (87)$$

Furthermore if we impose zero flux conditions on  $\lambda$ , then,

$$\frac{\partial \lambda(t, z_1)}{\partial z} = \frac{\partial \lambda(t, z_0)}{\partial z} = 0 \quad (88)$$

Conditions (86) or (87) and (88) can be used as transversality conditions. Then we obtain from ((85))

$$\frac{\partial \lambda}{\partial t} = - \left( f_x + \lambda g_x + D \frac{\partial^2 \lambda}{\partial z^2} \right) \quad (89)$$

$$f_u + \lambda g_u = 0 \quad (90)$$

- If we use hostile boundary conditions then from (83),  $y(t, z_1, \epsilon) = y(t, z_0, \epsilon) = 0$  fixed, and  $y_\epsilon(t_0, z, \epsilon) = y_\epsilon(t, z_1, \epsilon) = 0$  in (85). Then (89) and (90) are obtained by imposing transversality conditions (86) or (87).

So if we define a generalized Hamiltonian function

$$\mathbf{H} = f(x, u) + \lambda \left[ g(x, u) + D \frac{\partial^2 x}{\partial z^2} \right]$$

then by (89) and (90) optimality conditions become conditions (5) - (15), along with the appropriate transversality conditions.

The infinite horizon case with discounting is obtained by following the same approach and using Arrow and Kurz (1970, Chapter II.6).  $\square$

### **Extension of Pontryagin's Principle: Sufficiency**

Suppose that  $x^*(t, z)$ ,  $u^*(t, z)$ ,  $\lambda(t, z)$  satisfy conditions (5) and (15) and let  $x(t, z)$ ,  $u(t, z)$  functions satisfy (2). Let  $f^*, g^*$  denote functions evaluated along  $(x^*(t, z), u^*(t, z))$  and let  $f, g$  denote functions evaluated along the feasible path  $(x(t, z), u(t, z))$ . To prove sufficiency we

need to show that

$$W \equiv \int_{z_0}^{z_1} \int_{t_0}^{t_1} (f^* - f) dt dz \geq 0$$

From the concavity of  $f$  it follows that

$$f^* - f \geq (x^*(t, z) - x(t, z)) f_x^* + (u^*(t, z) - u(t, z)) f_u^* \quad (91)$$

Then

$$W \geq \int_{z_0}^{z_1} \int_{t_0}^{t_1} [(x^*(t, z) - x(t, z)) f_x^* + (u^*(t, z) - u(t, z)) f_u^*] dt dz \quad (92)$$

$$= \int_{z_0}^{z_1} \int_{t_0}^{t_1} \left[ (x^*(t, z) - x(t, z)) \left( -\frac{\partial \lambda(t, z)}{\partial t} - \lambda(t, z) g_x^* - D \frac{\partial^2 \lambda(t, z)}{\partial z^2} \right) \right. \quad (93)$$

$$\left. + (u^*(t, z) - u(t, z)) (-\lambda(t, z) g_u^*) \right] dt dz \\ = \int_{z_0}^{z_1} \int_{t_0}^{t_1} \lambda [g^* - g - (x^*(t, z) - x(t, z)) g_x^* - (u^*(t, z) - u(t, z))] g_u^* dt dz \geq 0 \quad (94)$$

Condition (93) follows from (92) by using conditions (5) and (15) to substitute for  $f_x^*$  and  $f_u^*$ . Condition (94) is derived by integrating first by parts the terms involving  $\frac{\partial \lambda}{\partial t}$ , substituting for  $\frac{\partial x}{\partial t}$  from (2), and using the transversality conditions, as has been done above, then by integrating twice the terms involving  $\frac{\partial^2 \lambda}{\partial z^2}$  and using again the zero flux or the hostile boundary conditions. The non-negativity of the integral in (94) follows from (17) and the concavity of  $g$ .

The result can be easily extended along the lines of Arrow's sufficiency theorem (Arrow and Kurz, 1970, Chapter II.6) with a transversality condition at infinity.

$$\lim_{t \rightarrow \infty} e^{-\rho t} \int_{z_0}^{z_1} \lambda(t, z) dz \geq 0, \quad \lim_{t \rightarrow \infty} e^{-\rho t} \int_{z_0}^{z_1} \lambda(t, z) x(t, z) dz = 0, \quad \text{or} \quad (95)$$

$$\lim_{t \rightarrow \infty} e^{-\rho t} \lambda(t, z) x(t, z) = 0 \quad \text{when} \quad (\lambda(t, z), x(t, z)) \geq 0 \quad \text{for all } t, z \quad (96)$$

□

### Linear - Quadratic Approximation of the Optimal Control Problem under Diffusion

First we derive the LQ approximation for the general problem and then we apply it to the problem of optimal harvesting.

Consider the general optimal control problem under diffusion.

$$\max_{\{u(t,z)\}} \int_{z_0}^{z_1} \int_{t_0}^{t_1} f(x(t,z), u(t,z)) dt dz \quad (97)$$

$$\text{s.t. } \frac{\partial x(t,z)}{\partial t} = g(x(t,z), u(t,z)) + D \frac{\partial^2 x(t,z)}{\partial z^2} \quad (98)$$

$$x(t_0, z) \text{ given, } \left. \frac{\partial x(t,z)}{\partial z} \right|_{z=z_0} = \left. \frac{\partial x(t,z)}{\partial z} \right|_{z=z_1} = 0, \text{ zero flux} \quad (99)$$

$$x(t, z_0) = x(t, z_1) = 0, \text{ hostile boundary } x(t_0, z), z \in (z_0, z_1) \text{ given} \quad (100)$$

with the Hamiltonian function

$$\mathbf{H}(x(t,z), u(t,z), \lambda(t,z)) = f(x, u) + \lambda \left[ g(x, u) + D \frac{\partial^2 x}{\partial z^2} \right] \quad (101)$$

For problem (97) - (100) let  $(x^*, u^*, \lambda^*)$  be a flat optimal steady state associated with the Hamiltonian system (5)-(6) for  $D = 0$ . This optimal steady state satisfies the optimality conditions (5)-(9). Our approach is to extend the method developed by Fleming (1971) and Magill (1977), by which a non-linear optimal stochastic control problem is replaced by a simpler linear quadratic optimal stochastic control problem, to the case of a deterministic control problem, such as (97) - (100) where the transition of the system is described by a PDE with a diffusion term and not by a stochastic ODE. Assume that the diffusion process (98) starts close to the steady state or that  $x_0 = x(0, z)$  starts close to  $x^*$  for all  $z \in Z$ , and let  $(y(t, z), \gamma(t, z), p(t, z)) = (x(t, z) - x^*, u(t, z) - u^*, \lambda(t, z) - \lambda^*)$ . Perturb the control  $u$  by letting

$$u(t, z) = u^* + \varepsilon (u(t, z) - u^*) = u^* + \varepsilon \gamma(t, z) \quad (102)$$

For a control of the form (102) we adapt Athans and Falb (1966 page 261) to focus on perturbations of the form below,

$$x(t, z) = x^* + \varepsilon y(t, z) + \varepsilon^2 \xi(t, z) + o(\varepsilon^2, t, z) \quad (103)$$

where  $y$  and  $\xi$  are first and second order state perturbations respectively and  $o(\varepsilon^2, t, z) \rightarrow 0$  as  $\varepsilon^2 \rightarrow 0$  uniformly in  $(t, z)$ .

Athans and Falb (1966, pp. 254-265) show that control perturbations of the form (102) lead to state perturbations of the form (103) under appropriate regularity conditions for the case where  $\mathcal{Z}$  is one point. We proceed heuristically here. Substituting (103) and (102) into (98), the  $g(x, u)$  function describing the kinetic of the state variable, we obtain

$$g(x^* + \varepsilon y(t, z) + \varepsilon^2 \xi(t, z) + o(\varepsilon^2, t, z), u^* + \varepsilon \gamma(t, z)) \quad (104)$$

Substituting also for  $x(t, z)$  in the derivative  $\frac{\partial x(t, z)}{\partial t}$  and  $\frac{\partial^2 x(t, z)}{\partial z^2}$ , using (103) and expanding as a Taylor series around  $(x^*, u^*)$ , we obtain<sup>18</sup>

$$\begin{aligned} \varepsilon \frac{\partial y(t, z)}{\partial t} + \varepsilon^2 \frac{\partial \xi(t, z)}{\partial t} &= g(x^*, u^*) + g_x(\varepsilon y + \varepsilon^2 \xi) + g_u(\varepsilon \gamma) + w' W w + \\ + \varepsilon D \frac{\partial^2 y(t, z)}{\partial z^2} + \varepsilon^2 D \frac{\partial^2 \xi(t, z)}{\partial z^2} &+ \text{higher order terms} \end{aligned} \quad (105)$$

$$w = (\varepsilon y + \varepsilon^2 \xi, \varepsilon \gamma)', \quad W = \begin{pmatrix} g_{xx} & g_{xu} \\ g_{ux} & g_{uu} \end{pmatrix}$$

where all derivatives are evaluated at the flat steady state. Divide (105) by  $\varepsilon$  and then take the limit as  $\varepsilon \rightarrow 0$ , and note that  $g(x^*, u^*) = 0$  because  $(x^*, u^*)$  is a steady state, to obtain the linear approximation of (98) around the flat steady state as

$$\frac{\partial y(t, z)}{\partial t} = +g_x y(t, z) + g_u \gamma(t, z) + D \frac{\partial^2 y(t, z)}{\partial z^2} \quad (106)$$

$$\text{with } y(t_0, z) = 0 \text{ for all } z. \quad (107)$$

If, using the equality of the  $\varepsilon$ -terms in (105) we cancel these terms, divide by  $\varepsilon^2$  and then take the limit  $\varepsilon^2 \rightarrow 0$ , we obtain a differential equation

---

<sup>18</sup>Subscripts denote derivatives.

in the second-order state perturbation

$$\frac{\partial \xi(t, z)}{\partial t} = g_x \xi(t, z) + g_{xx} \xi(t, z) + g_{uu} \gamma(t, z) + 2g_{xu} \xi(t, z) \gamma(t, z) + D \frac{\partial^2 \xi(t, z)}{\partial z^2} \quad (108)$$

$$\text{with } \xi(t_0, z) = 0 \text{ for all } z. \quad (109)$$

Write the performance functional (97) using the Hamiltonian function (101) with  $x(t, z)$  and  $u(t, z)$  given by the perturbations (103) and (102) and with  $\lambda(t, z)$  evaluated along the optimal path  $\lambda^*(t, z)$ , as

$$J(u) = \int_{z_0}^{z_1} \int_{t_0}^{t_1} \left[ \mathbf{H}(x(t, z), u(t, z), \lambda^*(t, z)) - \lambda^*(t, z) \frac{\partial x(t, z)}{\partial t} \right] dt dz \quad (110)$$

Write the performance functional along an optimal path as

$$J(u^*) = \int_{z_0}^{z_1} \int_{t_0}^{t_1} \left[ \mathbf{H}(x^*(t, z), u^*(t, z), \lambda^*(t, z)) - \lambda^*(t, z) \frac{\partial x^*(t, z)}{\partial t} \right] dt dz \quad (111)$$

then

$$\begin{aligned} J(u) - J(u^*) = & \int_{z_0}^{z_1} \int_{t_0}^{t_1} \left[ \mathbf{H}(x(t, z), u(t, z), \lambda^*(t, z)) - \mathbf{H}(x^*(t, z), u^*(t, z), \lambda^*(t, z)) \right. \\ & \left. - \lambda^*(t, z) \frac{\partial (x(t, z) - x^*(t, z))}{\partial t} \right] dt dz \end{aligned} \quad (112)$$

By expanding around the optimal steady state  $(x^*, u^*, \lambda^*)$  we obtain,

with derivatives evaluated at the optimal steady state,<sup>19</sup>

$$J(u) - J(u^*) = \int_{z_0}^{z_1} \int_{t_0}^{t_1} \left[ \mathbf{H}_x \varepsilon y(t, z) + \mathbf{H}_u \varepsilon \gamma(t, z) + \frac{1}{2} \mathbf{v}' \mathbf{Q} \mathbf{v} + o(\varepsilon^2, t, z) + \lambda^*(t, z) \frac{\partial(x(t, z) - x^*(t, z))}{\partial t} \right] dt dz \quad (113)$$

$$\mathbf{v} = (\varepsilon y(t, z), \varepsilon \gamma(t, z))' \quad (114)$$

$$\mathbf{Q}(y, \gamma) = \begin{pmatrix} H_{xx} & H_{xu} \\ H_{ux} & H_{uu} \end{pmatrix} = \begin{pmatrix} f_{xx} & f_{xu} \\ f_{ux} & f_{uu} \end{pmatrix} + \lambda^* \begin{pmatrix} g_{xx} & g_{xu} \\ g_{ux} & g_{uu} \end{pmatrix} \quad (115)$$

In (113) integrate by parts the term  $\lambda^*(t, z) \frac{\partial(x(t, z) - x^*(t, z))}{\partial t}$  as in (77) to obtain

$$\int_{z_0}^{z_1} [-\lambda(t_1, z) [x(t_1, z) - x^*(t_1, z)] + \lambda(t_0, z) [x(t_0, z) - x^*(t_0, z)]] + \int_{t_0}^{t_1} [x(t, z) - x^*(t, z)] \frac{\partial \lambda^*(t, z)}{\partial t} dt \Big] dz \quad (116)$$

In (116) the first term in the bracket is zero by transversality conditions, the second term is zero by initial conditions on the state perturbation, while the third term under the integral can be written, using (103) and the optimality conditions, as:

$$-H_x \frac{\partial(x^* + \varepsilon y(t, z) + \varepsilon^2 \xi(t, z) + o(\varepsilon^2, t, z) - x^*(t, z))}{\partial t} \quad (117)$$

Furthermore  $H_u = 0$  by the optimality conditions. Substituting into (113) dividing by  $\varepsilon$  and taking limits we obtain

$$J(u) - J(u^*) = \int_{z_0}^{z_1} \int_{t_0}^{t_1} \frac{1}{2} \mathbf{v}' \mathbf{Q} \mathbf{v} dt dz \quad (118)$$

Therefore a “good approximation” of problem (97) - (100) can be obtained if we replace in problem (97) - (100) the function  $f(x(t, z), u(t, z))$  with  $\frac{1}{2} \mathbf{v}' \mathbf{Q} \mathbf{v}$  and the transition equation (98) with the linearized diffusion equation (106).

---

<sup>19</sup>See Athans and Falb (1966) for such an expansion in the context of deriving necessary conditions for standard control problems without diffusion.

The same substitution can be made in an infinite horizon problem by replacing the transversality condition used to simplify (116) with the requirement that we require controls that produce solutions for the state variable that grow by less than discounting. This approximation is similar to the one produced by Magill (1977) for the optimal stochastic control problem. It should be noted that since the diffusion coefficient  $D$  is independent of the state and the control, this term drops out from the approximation of the objective, but enters the problem through the linearized diffusion equation. It is clear that extra terms including the diffusion coefficient should be added into the approximating matrix  $\mathbf{Q}$  in the general case where  $D = D(x, u)$ .

*Application to the Optimal Harvesting Problem*

We apply this result to the optimal harvesting problem (20) - (23). Let  $(x^*, E^*, \mu^*)$  be the flat steady state for this problem, and define  $U(x, E) = [S(H(t, z)) - c(E(t, z))]$ . Following our results above we obtain

$$\frac{1}{2} \mathbf{v}' \mathbf{Q} \mathbf{v} = L^0(y, \gamma) = \frac{1}{2} \begin{bmatrix} y \\ \gamma \end{bmatrix}^T \begin{bmatrix} A & N \\ N & B \end{bmatrix} \begin{bmatrix} y \\ \gamma \end{bmatrix} = \quad (119)$$

$$\frac{1}{2} [Ay^2 + 2Ny\gamma + B\gamma^2] \quad (120)$$

$$A = U_{xx}(x^*, E^*) = S''(H^*)(H^*)^2 + \mu^* F''(x^*) \quad (121)$$

$$N = U_{xE}(x^*, E^*) = q \left[ S''(H^*) H^* + S'(H^*) \right] \quad (122)$$

$$B = U_{EE}(x^*, E^*) = S''(H^*)(qx^*)^2 - c''(E^*) \quad (123)$$

$$H^* = qE^*x^* \quad (124)$$

where,  $A < 0$  by the concavity of benefit and growth functions and the fact that the shadow value of the resource is non-negative at the steady state,  $N \geq 0$ , and  $B < 0$  by the concavity of the benefit function and the convexity of the cost function.

Following Brock and Malliaris (1989) we make a change in units, so that

$$u = \gamma + \frac{N}{B}y \quad (125)$$

Then we have

$$L^0(y, u) = \frac{1}{2} \left[ \left( A - \frac{N^2}{B} \right) y^2 + Bu^2 \right] \quad (126)$$

Furthermore, the linearized transition equation becomes

$$\frac{\partial y(z, t)}{\partial t} = \left[ F'(x^*) - qE^* \right] y(z, t) - qx^*\gamma(z, t) + D \frac{\partial y^2(z, t)}{\partial z^2} \quad (127)$$

$$Fy(z, t) - G\gamma(z, t) + D \frac{\partial y^2(z, t)}{\partial z^2}, \text{ or} \quad (128)$$

$$\frac{\partial y(t, z)}{\partial t} = \left( F - \frac{GN}{B} \right) y(t, z) - Gu(t, z) + D \frac{\partial^2 y(t, z)}{\partial z^2} \quad (129)$$

where  $G = qx^* > 0$  for a positive steady state for the resource. In order to have a well posed LQ problem with a concave net benefit function and a transition equation with positive growth for the resource, we assume:

$$\left( A - \frac{N^2}{B} \right) = -Q < 0 \quad (130)$$

$$B = -R < 0 \quad (131)$$

$$\left( F - \frac{GN}{B} \right) = S > 0 \quad (132)$$

The LQ approximation around the flat steady state of the original spatial problem is

$$\max_{u(t, z)} \int_0^\infty \int_{\mathcal{Z}} e^{-\rho t} \left[ -\frac{Q}{2} x^2 - \frac{R}{2} u^2 \right] dt dz \quad Q, R, \rho > 0 \quad (133)$$

$$\text{s.t. } \frac{\partial y(t, z)}{\partial t} = Sy(t, z) - Gu(t, z) + D \frac{\partial^2 y(t, z)}{\partial z^2}, F, G > 0 \quad (134)$$

$$y(0, z) \text{ given, and zero flux on } 0, a, \text{ or} \quad (135)$$

$$y(t, 0) = y(t, a) = 0, y(0, z), z \in (0, a) \text{ given} \quad (136)$$

□



## References

- [1] ATHANS, M. AND P. FALB (1966): *Optimal Control*, New York: McGraw-Hill.
- [2] ARROW, K. AND M. KURZ (1970): *Public Investment, the Rate of Return, and Optimal Fiscal Policy*, Baltimore: Johns Hopkins University Press for Resources for the Future.
- [3] BHAT, M., R. FISTER, AND S. LENHART (1999): “An Optimal Control Model for the Surface Runoff Contamination of a Large River Basin,” *Ecological Applications*, 3, 518-530.
- [4] BROCK, W., and A. MALLIARIS (1989): *Differential Equations, Stability and Chaos in Dynamic Economics*, Amsterdam: North-Holland.
- [5] BROCK, W. AND A. XEPAPADEAS (2002): “Optimal Ecosystem Management when Species Compete for Limiting Resources,” *Journal of Environmental Economics and Management*, 44(2), 189-230.
- [6] BROCK, W., C. HOMMES, AND F. WAGENER (2005): “Evolutionary Dynamics in Markets with Many Traders Types,” *Journal of Mathematical Economics*, 41, 7-42.
- [7] CARLSON, D., A. HAURIE, AND A. LEIZAROWITZ (1991): *Infinite Horizon Optimal Control*, Second Edition, Berlin: Springer-Verlag.
- [8] CRISTALLER, W. (1933): *Central Places in Southern Germany*, Fischer: Jena.
- [9] CLARK, C. (1990): *Mathematical Bioeconomics: The Optimal Management Of Renewable Resources*, Second Edition, New York: Wiley.
- [10] DIKS, C. AND R. VAN DER WEIDE (2003): “Heterogeneity as a Natural Source of Randomness,” *CeNDEF*, University of Amsterdam, and Tinbergen Institute.
- [11] DIXIT, A. AND J. STIGLITZ (1977): “Monopolistic Competition and Optimum Product Diversity,” *American Economic Review*, 67(3), 297-308.
- [12] DURLAUF, S. (1997): “Statistical Mechanics Approaches to Socioeconomic Behavior,” in *The Economy as a Complex Evolving*

- System II*, ed. by W. Arthur, S. Durlauf and P. Lane, Redwood City: Addison-Wesley.
- [13] FLEMING, W. (1971): “Stochastic Control for Small Noise Intensities,” *SIAM J Control*, 9, 473-517.
- [14] FUJITA, M., P. KRUGMAN, AND VENABLES, A. (1999): *The Spatial Economy*, Cambridge, Massachusetts: The MIT Press.
- [15] HOTELLING, H. (1929): “Stability in Competition,” *Economic Journal*, 1929, 39, pp. 41-57.
- [16] KAMIEN, M. AND N. SCHWARTZ, (1981): *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management*, Second Edition, New York, North-Holland.
- [17] JUDD, K. (1996): “Approximation, Perturbation, and Projection Methods in Economic Analysis,” in *Handbook of Computational Economics*, ed. by H. Amman, D. Kendrick, and J. Rust, Amsterdam: Elsevier.
- [18] KRUGMAN, P. (1993): “On the Number and Location of Cities.” *European Economic Review*, 37, 293-298.
- [19] ————. (1998): “Space: The Final Frontier,” *Journal of Economic Perspectives*, 2(2), 161-174.
- [20] LENHART, S. AND M. BHAT (1992): “Application of Distributed Parameter Control Model in Wildlife Damage Management,” *Mathematical Models and Methods in Applied Sciences*, 4, 423-439.
- [21] LENHART, S., M. LIANG, AND V. PROTOPOPESCU (1999): “Optimal Control of Boundary Habitat Hostility for Interacting Species,” *Mathematical Models in the Applied Sciences*, 22, 1061-1077.
- [22] LEVIN, S. (1999): *Fragile Dominion: Complexity and the Commons*, Reading MA: Perseus Books.
- [23] ————. (2002): “Complex Adaptive Systems: Exploring the Known the Unknown and the Unknowable,” *Bulletin (New Series) of the American Mathematical Society*, 40, 3-19.
- [24] LIONS, J. - L. (1971): *Optimal Control of Systems Governed by Partial Differential Equations*, New York: Springer - Verlag.
- [25] LÖSCH, A. (1940): *The Economics of Location*, Jena: Fisher.

- [26] MAGILL, M. (1977): “A Local Analysis of N-Sector Capital Accumulation under Uncertainty,” *Journal of Economic Theory*, 25, 211-219.
- [27] MICHEL, P. (1982): “On the Transversality Condition in Infinite Horizon Optimal Problems,” *Econometrica*, 50, 975-986.
- [28] MURRAY, J. (1993): *Mathematical Biology*, Second Edition, Berlin: Springer.
- [29] ————— (2003): *Mathematical Biology*, Third Edition, Berlin: Springer.
- [30] NEUBERT, M. (2003): “Marine Reserves and Optimal Harvesting,” *Ecology Letters*, 6, 843-849.
- [31] OKUBO, A. (2001): “Introduction: The Mathematics of Ecological Diffusion,” in *Diffusion and Ecological Problems: Modern Perspectives*, ed. by A. Okubo and S. Levin, 2nd Edition, Berlin: Springer.
- [32] OKUBO, A., HASTINGS, A., AND POWELL, T. (2001): “Population Dynamics in Temporal and Spatial Domains,” in *Diffusion and Ecological Problems: Modern Perspectives*, ed. by A. Okubo and S. Levin, 2nd Edition, Berlin: Springer.
- [33] OKUBO, A. AND LEVIN, S. (2001): “The Basics of Diffusion.” in *Diffusion and Ecological Problems: Modern Perspectives*, ed. by A. Okubo and S. Levin, 2nd Edition, Berlin: Springer.
- [34] RAYMOND, J.-P. AND H. ZIDANI (1998): “Pontryagin’s Principle for State-Constrained Control Problems Governed by Parabolic Equations with Unbounded Controls,” *SIAM Journal of Control and Optimization*, 36(6), 1853-1879.
- [35] ————— (1999): “Hamiltonian Pontryagin’s Principles for Control Problems Governed by Semilinear Parabolic Equations.” *Applied Mathematics Optimization*, 39, 143-177.
- [36] SANCHIRICO, J. (2004): “Additivity Properties of Metapopulation Models: Implications for the Assessment of Marine Reserves,” *Resources for the Future*, Discussion Paper 02-66, *Journal of Environmental Economics and Management*, In Press, Available online 21 August 2004.
- [37] SANCHIRICO, J. AND J. WILEN (1999): “Bioeconomics of

- Spatial Exploitation in a Patchy Environment,” *Journal of Environmental Economics and Management*, 37(2), 129-150.
- [38] SANCHIRICO, J. AND J. WILEN (2001): “A Bioeconomic Model of Marine Reserve Creation,” *Journal of Environmental Economics and Management*, 42(3), 257-276.
- [39] SEGEL, L.A., AND S. LEVIN (1976): “Application of Non Linear Stability Theory to the Study of the Effects of Diffusion on Predator Prey Interactions,” in *Topics in Statistical Mechanics and Biophysics: A Memorial to Julius L. Jackson*, ed. by R.A. Piccirelli, New York: American Institute of Physics.
- [40] TURING, A. (1952): “The Chemical Basis of Morphogenesis”, *Philosophical Transactions of the Royal Society of London*, 237, 37-72.
- [41] WEBER, A. (1909): *Theory of the Location of Industries*, Chicago: University of Chicago Press.

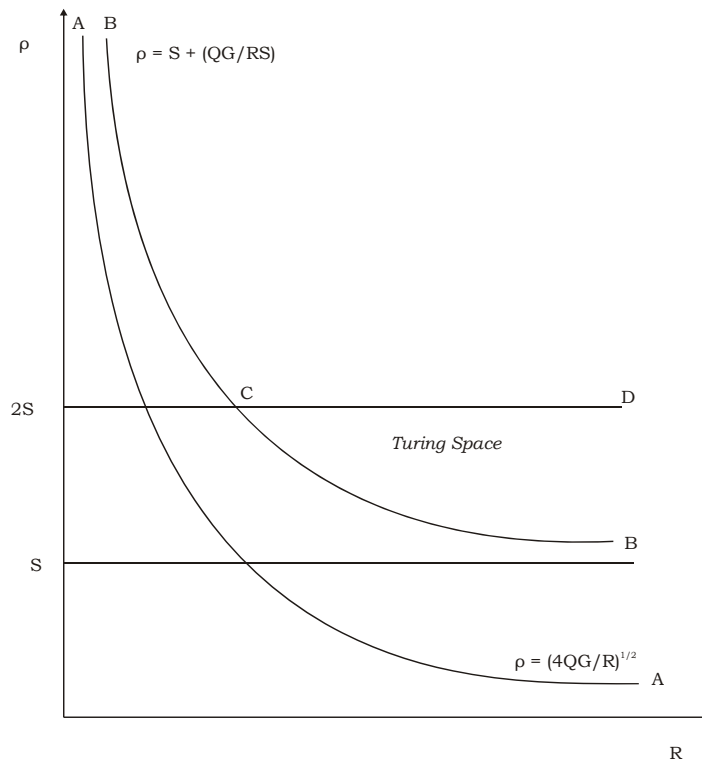


Figure 1: Turing Space

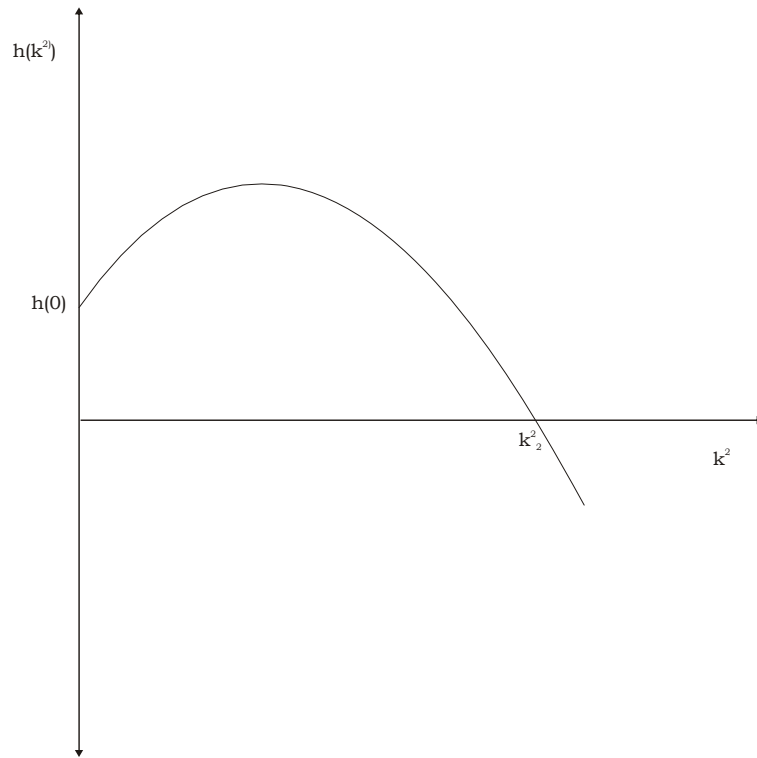


Figure 2: Diffusion as Stabilizer

## NOTE DI LAVORO DELLA FONDAZIONE ENI ENRICO MATTEI

### Fondazione Eni Enrico Mattei Working Paper Series

Our Note di Lavoro are available on the Internet at the following addresses:

<http://www.feem.it/Feem/Pub/Publications/WPapers/default.html>

<http://www.ssrn.com/link/feem.html>

<http://www.repec.org>

### NOTE DI LAVORO PUBLISHED IN 2004

IEM	1.2004	<i>Anil MARKANDYA, Suzette PEDROSO and Alexander GOLUB: <u>Empirical Analysis of National Income and So2 Emissions in Selected European Countries</u></i>
ETA	2.2004	<i>Masahisa FUJITA and Shlomo WEBER: <u>Strategic Immigration Policies and Welfare in Heterogeneous Countries</u></i>
PRA	3.2004	<i>Adolfo DI CARLUCCIO, Giovanni FERRI, Cecilia FRALE and Ottavio RICCHI: <u>Do Privatizations Boost Household Shareholding? Evidence from Italy</u></i>
ETA	4.2004	<i>Victor GINSBURGH and Shlomo WEBER: <u>Languages Disenfranchisement in the European Union</u></i>
ETA	5.2004	<i>Romano PIRAS: <u>Growth, Congestion of Public Goods, and Second-Best Optimal Policy</u></i>
CCMP	6.2004	<i>Herman R.J. VOLLEBERGH: <u>Lessons from the Polder: Is Dutch CO2-Taxation Optimal</u></i>
PRA	7.2004	<i>Sandro BRUSCO, Giuseppe LOPOMO and S. VISWANATHAN (lxv): <u>Merger Mechanisms</u></i>
PRA	8.2004	<i>Wolfgang AUSENNEGG, Pegaret PICHLER and Alex STOMPER (lxv): <u>IPO Pricing with Bookbuilding, and a When-Issued Market</u></i>
PRA	9.2004	<i>Pegaret PICHLER and Alex STOMPER (lxv): <u>Primary Market Design: Direct Mechanisms and Markets</u></i>
PRA	10.2004	<i>Florian ENGLMAIER, Pablo GUILLEN, Loreto LLORENTE, Sander ONDERSTAL and Rupert SAUSGRUBER (lxv): <u>The Chopstick Auction: A Study of the Exposure Problem in Multi-Unit Auctions</u></i>
PRA	11.2004	<i>Bjarne BRENDSTRUP and Harry J. PAARSCH (lxv): <u>Nonparametric Identification and Estimation of Multi-Unit, Sequential, Oral, Ascending-Price Auctions With Asymmetric Bidders</u></i>
PRA	12.2004	<i>Ohad KADAN (lxv): <u>Equilibrium in the Two Player, k-Double Auction with Affiliated Private Values</u></i>
PRA	13.2004	<i>Maarten C.W. JANSSEN (lxv): <u>Auctions as Coordination Devices</u></i>
PRA	14.2004	<i>Gadi FIBICH, Arieh GAVIOUS and Aner SELA (lxv): <u>All-Pay Auctions with Weakly Risk-Averse Buyers</u></i>
PRA	15.2004	<i>Orly SADE, Charles SCHNITZLEIN and Jaime F. ZENDER (lxv): <u>Competition and Cooperation in Divisible Good Auctions: An Experimental Examination</u></i>
PRA	16.2004	<i>Marta STRYSZOWSKA (lxv): <u>Late and Multiple Bidding in Competing Second Price Internet Auctions</u></i>
CCMP	17.2004	<i>Slim Ben YOUSSEF: <u>R&amp;D in Cleaner Technology and International Trade</u></i>
NRM	18.2004	<i>Angelo ANTOCI, Simone BORGHESI and Paolo RUSSU (lxvi): <u>Biodiversity and Economic Growth: Stabilization Versus Preservation of the Ecological Dynamics</u></i>
SIEV	19.2004	<i>Anna ALBERINI, Paolo ROSATO, Alberto LONGO and Valentina ZANATTA: <u>Information and Willingness to Pay in a Contingent Valuation Study: The Value of S. Erasmo in the Lagoon of Venice</u></i>
NRM	20.2004	<i>Guido CANDELA and Roberto CELLINI (lxvii): <u>Investment in Tourism Market: A Dynamic Model of Differentiated Oligopoly</u></i>
NRM	21.2004	<i>Jacqueline M. HAMILTON (lxvii): <u>Climate and the Destination Choice of German Tourists</u></i>
NRM	22.2004	<i>Javier Rey-MAQUIEIRA PALMER, Javier LOZANO IBÁÑEZ and Carlos Mario GÓMEZ GÓMEZ (lxvii): <u>Land, Environmental Externalities and Tourism Development</u></i>
NRM	23.2004	<i>Pius ODUNGA and Henk FOLMER (lxvii): <u>Profiling Tourists for Balanced Utilization of Tourism-Based Resources in Kenya</u></i>
NRM	24.2004	<i>Jean-Jacques NOWAK, Mondher SAHLI and Pasquale M. SGRO (lxvii): <u>Tourism, Trade and Domestic Welfare</u></i>
NRM	25.2004	<i>Riaz SHAREEF (lxvii): <u>Country Risk Ratings of Small Island Tourism Economies</u></i>
NRM	26.2004	<i>Juan Luis EUGENIO-MARTÍN, Noelia MARTÍN MORALES and Riccardo SCARPA (lxvii): <u>Tourism and Economic Growth in Latin American Countries: A Panel Data Approach</u></i>
NRM	27.2004	<i>Raúl Hernández MARTÍN (lxvii): <u>Impact of Tourism Consumption on GDP. The Role of Imports</u></i>
CSRM	28.2004	<i>Nicoletta FERRO: <u>Cross-Country Ethical Dilemmas in Business: A Descriptive Framework</u></i>
NRM	29.2004	<i>Marian WEBER (lxvi): <u>Assessing the Effectiveness of Tradable Landuse Rights for Biodiversity Conservation: an Application to Canada's Boreal Mixedwood Forest</u></i>
NRM	30.2004	<i>Trond BJORN DAL, Phoebe KOUNDOURI and Sean PASCOE (lxvi): <u>Output Substitution in Multi-Species Trawl Fisheries: Implications for Quota Setting</u></i>
CCMP	31.2004	<i>Marzio GALEOTTI, Alessandra GORIA, Paolo MOMBRINI and Evi SPANTIDAKI: <u>Weather Impacts on Natural, Social and Economic Systems (WISE) Part I: Sectoral Analysis of Climate Impacts in Italy</u></i>
CCMP	32.2004	<i>Marzio GALEOTTI, Alessandra GORIA, Paolo MOMBRINI and Evi SPANTIDAKI: <u>Weather Impacts on Natural, Social and Economic Systems (WISE) Part II: Individual Perception of Climate Extremes in Italy</u></i>
CTN	33.2004	<i>Wilson PEREZ: <u>Divide and Conquer: Noisy Communication in Networks, Power, and Wealth Distribution</u></i>
KTHC	34.2004	<i>Gianmarco I.P. OTTAVIANO and Giovanni PERI (lxviii): <u>The Economic Value of Cultural Diversity: Evidence from US Cities</u></i>
KTHC	35.2004	<i>Linda CHAIB (lxviii): <u>Immigration and Local Urban Participatory Democracy: A Boston-Paris Comparison</u></i>

KTHC	36.2004	<i>Franca ECKERT COEN and Claudio ROSSI</i> (Ixviii): <u>Foreigners, Immigrants, Host Cities: The Policies of Multi-Ethnicity in Rome. Reading Governance in a Local Context</u>
KTHC	37.2004	<i>Kristine CRANE</i> (Ixviii): <u>Governing Migration: Immigrant Groups' Strategies in Three Italian Cities – Rome, Naples and Bari</u>
KTHC	38.2004	<i>Kiflemariam HAMDE</i> (Ixviii): <u>Mind in Africa, Body in Europe: The Struggle for Maintaining and Transforming Cultural Identity - A Note from the Experience of Eritrean Immigrants in Stockholm</u>
ETA	39.2004	<i>Alberto CAVALIERE</i> : <u>Price Competition with Information Disparities in a Vertically Differentiated Duopoly</u>
PRA	40.2004	<i>Andrea BIGANO and Stef PROOST</i> : <u>The Opening of the European Electricity Market and Environmental Policy: Does the Degree of Competition Matter?</u>
CCMP	41.2004	<i>Micheal FINUS</i> (Ixix): <u>International Cooperation to Resolve International Pollution Problems</u>
KTHC	42.2004	<i>Francesco CRESPI</i> : <u>Notes on the Determinants of Innovation: A Multi-Perspective Analysis</u>
CTN	43.2004	<i>Sergio CURRARINI and Marco MARINI</i> : <u>Coalition Formation in Games without Synergies</u>
CTN	44.2004	<i>Marc ESCRHUELA-VILLAR</i> : <u>Cartel Sustainability and Cartel Stability</u>
NRM	45.2004	<i>Sebastian BERVOETS and Nicolas GRAVEL</i> (Ixvi): <u>Appraising Diversity with an Ordinal Notion of Similarity: An Axiomatic Approach</u>
NRM	46.2004	<i>Signe ANTHON and Bo JELLES MARK THORSEN</i> (Ixvi): <u>Optimal Afforestation Contracts with Asymmetric Information on Private Environmental Benefits</u>
NRM	47.2004	<i>John MBURU</i> (Ixvi): <u>Wildlife Conservation and Management in Kenya: Towards a Co-management Approach</u>
NRM	48.2004	<i>Ekin BIROL, Ágnes GYOVAI and Melinda SMALE</i> (Ixvi): <u>Using a Choice Experiment to Value Agricultural Biodiversity on Hungarian Small Farms: Agri-Environmental Policies in a Transition al Economy</u>
CCMP	49.2004	<i>Gernot KLEPPER and Sonja PETERSON</i> : <u>The EU Emissions Trading Scheme. Allowance Prices, Trade Flows, Competitiveness Effects</u>
GG	50.2004	<i>Scott BARRETT and Michael HOEL</i> : <u>Optimal Disease Eradication</u>
CTN	51.2004	<i>Dinko DIMITROV, Peter BORM, Ruud HENDRICKX and Shao CHIN SUNG</i> : <u>Simple Priorities and Core Stability in Hedonic Games</u>
SIEV	52.2004	<i>Francesco RICCI</i> : <u>Channels of Transmission of Environmental Policy to Economic Growth: A Survey of the Theory</u>
SIEV	53.2004	<i>Anna ALBERINI, Maureen CROPPER, Alan KRUPNICK and Nathalie B. SIMON</i> : <u>Willingness to Pay for Mortality Risk Reductions: Does Latency Matter?</u>
NRM	54.2004	<i>Ingo BRÄUER and Rainer MARGGRAF</i> (Ixvi): <u>Valuation of Ecosystem Services Provided by Biodiversity Conservation: An Integrated Hydrological and Economic Model to Value the Enhanced Nitrogen Retention in Renaturated Streams</u>
NRM	55.2004	<i>Timo GOESCHL and Tun LIN</i> (Ixvi): <u>Biodiversity Conservation on Private Lands: Information Problems and Regulatory Choices</u>
NRM	56.2004	<i>Tom DEDEURWAERDERE</i> (Ixvi): <u>Bioprospection: From the Economics of Contracts to Reflexive Governance</u>
CCMP	57.2004	<i>Katrin REHDANZ and David MADDISON</i> : <u>The Amenity Value of Climate to German Households</u>
CCMP	58.2004	<i>Koen SMEKENS and Bob VAN DER ZWAAN</i> : <u>Environmental Externalities of Geological Carbon Sequestration Effects on Energy Scenarios</u>
NRM	59.2004	<i>Valentina BOSETTI, Mariaester CASSINELLI and Alessandro LANZA</i> (Ixvii): <u>Using Data Envelopment Analysis to Evaluate Environmentally Conscious Tourism Management</u>
NRM	60.2004	<i>Timo GOESCHL and Danilo CAMARGO IGLIORI</i> (Ixvi): <u>Property Rights Conservation and Development: An Analysis of Extractive Reserves in the Brazilian Amazon</u>
CCMP	61.2004	<i>Barbara BUCHNER and Carlo CARRARO</i> : <u>Economic and Environmental Effectiveness of a Technology-based Climate Protocol</u>
NRM	62.2004	<i>Elissaios POPYRAKIS and Reyer GERLAGH</i> : <u>Resource-Abundance and Economic Growth in the U.S.</u>
NRM	63.2004	<i>Györgyi BELA, György PATAKI, Melinda SMALE and Mariann HAJDÚ</i> (Ixvi): <u>Conserving Crop Genetic Resources on Smallholder Farms in Hungary: Institutional Analysis</u>
NRM	64.2004	<i>E.C.M. RUIJGROK and E.E.M. NILLESEN</i> (Ixvi): <u>The Socio-Economic Value of Natural Riverbanks in the Netherlands</u>
NRM	65.2004	<i>E.C.M. RUIJGROK</i> (Ixvi): <u>Reducing Acidification: The Benefits of Increased Nature Quality. Investigating the Possibilities of the Contingent Valuation Method</u>
ETA	66.2004	<i>Giannis VARDAS and Anastasios XEPAPADEAS</i> : <u>Uncertainty Aversion, Robust Control and Asset Holdings</u>
GG	67.2004	<i>Anastasios XEPAPADEAS and Constadina PASSA</i> : <u>Participation in and Compliance with Public Voluntary Environmental Programs: An Evolutionary Approach</u>
GG	68.2004	<i>Michael FINUS</i> : <u>Modesty Pays: Sometimes!</u>
NRM	69.2004	<i>Trond BJØRNDAL and Ana BRASÃO</i> : <u>The Northern Atlantic Bluefin Tuna Fisheries: Management and Policy Implications</u>
CTN	70.2004	<i>Alejandro CAPARRÓS, Abdelhakim HAMMOUDI and Tarik TAZDAÏT</i> : <u>On Coalition Formation with Heterogeneous Agents</u>
IEM	71.2004	<i>Massimo GIOVANNINI, Margherita GRASSO, Alessandro LANZA and Matteo MANERA</i> : <u>Conditional Correlations in the Returns on Oil Companies Stock Prices and Their Determinants</u>
IEM	72.2004	<i>Alessandro LANZA, Matteo MANERA and Michael MCALEER</i> : <u>Modelling Dynamic Conditional Correlations in WTI Oil Forward and Futures Returns</u>
SIEV	73.2004	<i>Margarita GENIUS and Elisabetta STRAZZERA</i> : <u>The Copula Approach to Sample Selection Modelling: An Application to the Recreational Value of Forests</u>



CCMP	74.2004	<i>Rob DELLINK and Ekko van IERLAND</i> : <u>Pollution Abatement in the Netherlands: A Dynamic Applied General Equilibrium Assessment</u>
ETA	75.2004	<i>Rosella LEVAGGI and Michele MORETTO</i> : <u>Investment in Hospital Care Technology under Different Purchasing Rules: A Real Option Approach</u>
CTN	76.2004	<i>Salvador BARBERÀ and Matthew O. JACKSON</i> (lxx): <u>On the Weights of Nations: Assigning Voting Weights in a Heterogeneous Union</u>
CTN	77.2004	<i>Àlex ARENAS, Antonio CABRALES, Albert DÍAZ-GUILERA, Roger GUIMERA and Fernando VEGA-REDONDO</i> (lxx): <u>Optimal Information Transmission in Organizations: Search and Congestion</u>
CTN	78.2004	<i>Francis BLOCH and Armando GOMES</i> (lxx): <u>Contracting with Externalities and Outside Options</u>
CTN	79.2004	<i>Rabah AMIR, Effrosyni DIAMANTOUDI and Licun XUE</i> (lxx): <u>Merger Performance under Uncertain Efficiency Gains</u>
CTN	80.2004	<i>Francis BLOCH and Matthew O. JACKSON</i> (lxx): <u>The Formation of Networks with Transfers among Players</u>
CTN	81.2004	<i>Daniel DIERMEIER, Hülya ERASLAN and Antonio MERLO</i> (lxx): <u>Bicameralism and Government Formation</u>
CTN	82.2004	<i>Rod GARRATT, James E. PARCO, Cheng-ZHONG QIN and Amnon RAPOPORT</i> (lxx): <u>Potential Maximization and Coalition Government Formation</u>
CTN	83.2004	<i>Kfir ELIAZ, Debraj RAY and Ronny RAZIN</i> (lxx): <u>Group Decision-Making in the Shadow of Disagreement</u>
CTN	84.2004	<i>Sanjeev GOYAL, Marco van der LEIJ and José Luis MORAGA-GONZÁLEZ</i> (lxx): <u>Economics: An Emerging Small World?</u>
CTN	85.2004	<i>Edward CARTWRIGHT</i> (lxx): <u>Learning to Play Approximate Nash Equilibria in Games with Many Players</u>
IEM	86.2004	<i>Finn R. FØRSUND and Michael HOEL</i> : <u>Properties of a Non-Competitive Electricity Market Dominated by Hydroelectric Power</u>
KTHC	87.2004	<i>Elissaios PAPHAKIS and Reyer GERLAGH</i> : <u>Natural Resources, Investment and Long-Term Income</u>
CCMP	88.2004	<i>Marzio GALEOTTI and Claudia KEMFERT</i> : <u>Interactions between Climate and Trade Policies: A Survey</u>
IEM	89.2004	<i>A. MARKANDYA, S. PEDROSO and D. STREIMIKIENE</i> : <u>Energy Efficiency in Transition Economies: Is There Convergence Towards the EU Average?</u>
GG	90.2004	<i>Rolf GOLOMBEK and Michael HOEL</i> : <u>Climate Agreements and Technology Policy</u>
PRA	91.2004	<i>Sergei IZMALKOV</i> (lxv): <u>Multi-Unit Open Ascending Price Efficient Auction</u>
KTHC	92.2004	<i>Gianmarco I.P. OTTAVIANO and Giovanni PERI</i> : <u>Cities and Cultures</u>
KTHC	93.2004	<i>Massimo DEL GATTO</i> : <u>Agglomeration, Integration, and Territorial Authority Scale in a System of Trading Cities. Centralisation versus devolution</u>
CCMP	94.2004	<i>Pierre-André JOUVET, Philippe MICHEL and Gilles ROTILLON</i> : <u>Equilibrium with a Market of Permits</u>
CCMP	95.2004	<i>Bob van der ZWAAN and Reyer GERLAGH</i> : <u>Climate Uncertainty and the Necessity to Transform Global Energy Supply</u>
CCMP	96.2004	<i>Francesco BOSELLO, Marco LAZZARIN, Roberto ROSON and Richard S.J. TOL</i> : <u>Economy-Wide Estimates of the Implications of Climate Change: Sea Level Rise</u>
CTN	97.2004	<i>Gustavo BERGANTIÑOS and Juan J. VIDAL-PUGA</i> : <u>Defining Rules in Cost Spanning Tree Problems Through the Canonical Form</u>
CTN	98.2004	<i>Siddhartha BANDYOPADHYAY and Mandar OAK</i> : <u>Party Formation and Coalitional Bargaining in a Model of Proportional Representation</u>
GG	99.2004	<i>Hans-Peter WEIKARD, Michael FINUS and Juan-Carlos ALTAMIRANO-CABRERA</i> : <u>The Impact of Surplus Sharing on the Stability of International Climate Agreements</u>
SIEV	100.2004	<i>Chiara M. TRAVISI and Peter NIJKAMP</i> : <u>Willingness to Pay for Agricultural Environmental Safety: Evidence from a Survey of Milan, Italy, Residents</u>
SIEV	101.2004	<i>Chiara M. TRAVISI, Raymond J. G. M. FLORAX and Peter NIJKAMP</i> : <u>A Meta-Analysis of the Willingness to Pay for Reductions in Pesticide Risk Exposure</u>
NRM	102.2004	<i>Valentina BOSETTI and David TOMBERLIN</i> : <u>Real Options Analysis of Fishing Fleet Dynamics: A Test</u>
CCMP	103.2004	<i>Alessandra GORIA e Gretel GAMBARELLI</i> : <u>Economic Evaluation of Climate Change Impacts and Adaptability in Italy</u>
PRA	104.2004	<i>Massimo FLORIO and Mara GRASSENI</i> : <u>The Missing Shock: The Macroeconomic Impact of British Privatisation</u>
PRA	105.2004	<i>John BENNETT, Saul ESTRIN, James MAW and Giovanni URGA</i> : <u>Privatisation Methods and Economic Growth in Transition Economies</u>
PRA	106.2004	<i>Kira BÖRNER</i> : <u>The Political Economy of Privatization: Why Do Governments Want Reforms?</u>
PRA	107.2004	<i>Pehr-Johan NORBÄCK and Lars PERSSON</i> : <u>Privatization and Restructuring in Concentrated Markets</u>
SIEV	108.2004	<i>Angela GRANZOTTO, Fabio PRANOVI, Simone LIBRALATO, Patrizia TORRICELLI and Danilo MAINARDI</i> : <u>Comparison between Artisanal Fishery and Manila Clam Harvesting in the Venice Lagoon by Using Ecosystem Indicators: An Ecological Economics Perspective</u>
CTN	109.2004	<i>Somdeb LAHIRI</i> : <u>The Cooperative Theory of Two Sided Matching Problems: A Re-examination of Some Results</u>
NRM	110.2004	<i>Giuseppe DI VITA</i> : <u>Natural Resources Dynamics: Another Look</u>
SIEV	111.2004	<i>Anna ALBERINI, Alistair HUNT and Anil MARKANDYA</i> : <u>Willingness to Pay to Reduce Mortality Risks: Evidence from a Three-Country Contingent Valuation Study</u>
KTHC	112.2004	<i>Valeria PAPPONETTI and Dino PINELLI</i> : <u>Scientific Advice to Public Policy-Making</u>
SIEV	113.2004	<i>Paulo A.L.D. NUNES and Laura ONOFRI</i> : <u>The Economics of Warm Glow: A Note on Consumer's Behavior and Public Policy Implications</u>
IEM	114.2004	<i>Patrick CAYRADE</i> : <u>Investments in Gas Pipelines and Liquefied Natural Gas Infrastructure What is the Impact on the Security of Supply?</u>
IEM	115.2004	<i>Valeria COSTANTINI and Francesco GRACCEVA</i> : <u>Oil Security. Short- and Long-Term Policies</u>

IEM	116.2004	<i>Valeria COSTANTINI and Francesco GRACCEVA: <u>Social Costs of Energy Disruptions</u></i>
IEM	117.2004	<i>Christian EGENHOFER, Kyriakos GIALOGLOU, Giacomo LUCIANI, Maroeska BOOTS, Martin SCHEEPERS, Valeria COSTANTINI, Francesco GRACCEVA, Anil MARKANDYA and Giorgio VICINI: <u>Market-Based Options for Security of Energy Supply</u></i>
IEM	118.2004	<i>David FISK: <u>Transport Energy Security. The Unseen Risk?</u></i>
IEM	119.2004	<i>Giacomo LUCIANI: <u>Security of Supply for Natural Gas Markets. What is it and What is it not?</u></i>
IEM	120.2004	<i>L.J. de VRIES and R.A. HAKVOORT: <u>The Question of Generation Adequacy in Liberalised Electricity Markets</u></i>
KTHC	121.2004	<i>Alberto PETRUCCI: <u>Asset Accumulation, Fertility Choice and Nondegenerate Dynamics in a Small Open Economy</u></i>
NRM	122.2004	<i>Carlo GIUPPONI, Jaroslav MYSLAK and Anita FASSIO: <u>An Integrated Assessment Framework for Water Resources Management: A DSS Tool and a Pilot Study Application</u></i>
NRM	123.2004	<i>Margaretha BREIL, Anita FASSIO, Carlo GIUPPONI and Paolo ROSATO: <u>Evaluation of Urban Improvement on the Islands of the Venice Lagoon: A Spatially-Distributed Hedonic-Hierarchical Approach</u></i>
ETA	124.2004	<i>Paul MENSINK: <u>Instant Efficient Pollution Abatement Under Non-Linear Taxation and Asymmetric Information: The Differential Tax Revisited</u></i>
NRM	125.2004	<i>Mauro FABIANO, Gabriella CAMARSA, Rosanna DURSI, Roberta IVALDI, Valentina MARIN and Francesca PALMISANI: <u>Integrated Environmental Study for Beach Management: A Methodological Approach</u></i>
PRA	126.2004	<i>Irena GROSFELD and Iraj HASHI: <u>The Emergence of Large Shareholders in Mass Privatized Firms: Evidence from Poland and the Czech Republic</u></i>
CCMP	127.2004	<i>Maria BERRITTELLA, Andrea BIGANO, Roberto ROSON and Richard S.J. TOL: <u>A General Equilibrium Analysis of Climate Change Impacts on Tourism</u></i>
CCMP	128.2004	<i>Reyer GERLAGH: <u>A Climate-Change Policy Induced Shift from Innovations in Energy Production to Energy Savings</u></i>
NRM	129.2004	<i>Elissaios POPYRAKIS and Reyer GERLAGH: <u>Natural Resources, Innovation, and Growth</u></i>
PRA	130.2004	<i>Bernardo BORTOLOTTI and Mara FACCIO: <u>Reluctant Privatization</u></i>
SIEV	131.2004	<i>Riccardo SCARPA and Mara THIENE: <u>Destination Choice Models for Rock Climbing in the Northeast Alps: A Latent-Class Approach Based on Intensity of Participation</u></i>
SIEV	132.2004	<i>Riccardo SCARPA Kenneth G. WILLIS and Melinda ACUTT: <u>Comparing Individual-Specific Benefit Estimates for Public Goods: Finite Versus Continuous Mixing in Logit Models</u></i>
IEM	133.2004	<i>Santiago J. RUBIO: <u>On Capturing Oil Rents with a National Excise Tax Revisited</u></i>
ETA	134.2004	<i>Ascensión ANDINA DÍAZ: <u>Political Competition when Media Create Candidates' Charisma</u></i>
SIEV	135.2004	<i>Anna ALBERINI: <u>Robustness of VSL Values from Contingent Valuation Surveys</u></i>
CCMP	136.2004	<i>Gernot KLEPPER and Sonja PETERSON: <u>Marginal Abatement Cost Curves in General Equilibrium: The Influence of World Energy Prices</u></i>
ETA	137.2004	<i>Herbert DAWID, Christophe DEISSENBERG and Pavel ŠEVČIK: <u>Cheap Talk, Gullibility, and Welfare in an Environmental Taxation Game</u></i>
CCMP	138.2004	<i>ZhongXiang ZHANG: <u>The World Bank's Prototype Carbon Fund and China</u></i>
CCMP	139.2004	<i>Reyer GERLAGH and Marjan W. HOFKES: <u>Time Profile of Climate Change Stabilization Policy</u></i>
NRM	140.2004	<i>Chiara D'ALPAOS and Michele MORETTO: <u>The Value of Flexibility in the Italian Water Service Sector: A Real Option Analysis</u></i>
PRA	141.2004	<i>Patrick BAJARI, Stephanie HOUGHTON and Steven TADELIS (lxxi): <u>Bidding for Incomplete Contracts</u></i>
PRA	142.2004	<i>Susan ATHEY, Jonathan LEVIN and Enrique SEIRA (lxxi): <u>Comparing Open and Sealed Bid Auctions: Theory and Evidence from Timber Auctions</u></i>
PRA	143.2004	<i>David GOLDREICH (lxxi): <u>Behavioral Biases of Dealers in U.S. Treasury Auctions</u></i>
PRA	144.2004	<i>Roberto BURGNET (lxxi): <u>Optimal Procurement Auction for a Buyer with Downward Sloping Demand: More Simple Economics</u></i>
PRA	145.2004	<i>Ali HORTACSU and Samita SAREEN (lxxi): <u>Order Flow and the Formation of Dealer Bids: An Analysis of Information and Strategic Behavior in the Government of Canada Securities Auctions</u></i>
PRA	146.2004	<i>Victor GINSBURGH, Patrick LEGROS and Nicolas SAHUGUET (lxxi): <u>How to Win Twice at an Auction. On the Incidence of Commissions in Auction Markets</u></i>
PRA	147.2004	<i>Claudio MEZZETTI, Aleksandar PEKEČ and Ilia TSETLIN (lxxi): <u>Sequential vs. Single-Round Uniform-Price Auctions</u></i>
PRA	148.2004	<i>John ASKER and Estelle CANTILLON (lxxi): <u>Equilibrium of Scoring Auctions</u></i>
PRA	149.2004	<i>Philip A. HAILE, Han HONG and Matthew SHUM (lxxi): <u>Nonparametric Tests for Common Values in First-Price Sealed-Bid Auctions</u></i>
PRA	150.2004	<i>François DEGEORGE, François DERRIEN and Kent L. WOMACK (lxxi): <u>Quid Pro Quo in IPOs: Why Bookbuilding is Dominating Auctions</u></i>
CCMP	151.2004	<i>Barbara BUCHNER and Silvia DALL'OLIO: <u>Russia: The Long Road to Ratification. Internal Institution and Pressure Groups in the Kyoto Protocol's Adoption Process</u></i>
CCMP	152.2004	<i>Carlo CARRARO and Marzio GALEOTTI: <u>Does Endogenous Technical Change Make a Difference in Climate Policy Analysis? A Robustness Exercise with the FEEM-RICE Model</u></i>
PRA	153.2004	<i>Alejandro M. MANELLI and Daniel R. VINCENT (lxxi): <u>Multidimensional Mechanism Design: Revenue Maximization and the Multiple-Good Monopoly</u></i>
ETA	154.2004	<i>Nicola ACOCELLA, Giovanni Di BARTOLOMEO and Wilfried PAUWELS: <u>Is there any Scope for Corporatism in Stabilization Policies?</u></i>
CTN	155.2004	<i>Johan EYCKMANS and Michael FINUS: <u>An Almost Ideal Sharing Scheme for Coalition Games with Externalities</u></i>
CCMP	156.2004	<i>Cesare DOSI and Michele MORETTO: <u>Environmental Innovation, War of Attrition and Investment Grants</u></i>

CCMP	157.2004	<i>Valentina BOSETTI, Marzio GALEOTTI and Alessandro LANZA: <u>How Consistent are Alternative Short-Term Climate Policies with Long-Term Goals?</u></i>
ETA	158.2004	<i>Y. Hossein FARZIN and Ken-Ichi AKAO: <u>Non-pecuniary Value of Employment and Individual Labor Supply</u></i>
ETA	159.2004	<i>William BROCK and Anastasios XEPAPADEAS: <u>Spatial Analysis: Development of Descriptive and Normative Methods with Applications to Economic-Ecological Modelling</u></i>
KTHC	160.2004	<i>Alberto PETRUCCI: <u>On the Incidence of a Tax on PureRent with Infinite Horizons</u></i>
IEM	161.2004	<i>Xavier LABANDEIRA, José M. LABEAGA and Miguel RODRÍGUEZ: <u>Microsimulating the Effects of Household Energy Price Changes in Spain</u></i>

#### NOTE DI LAVORO PUBLISHED IN 2005

CCMP	1.2005	<i>Stéphane HALLEGATTE: <u>Accounting for Extreme Events in the Economic Assessment of Climate Change</u></i>
CCMP	2.2005	<i>Qiang WU and Paulo Augusto NUNES: <u>Application of Technological Control Measures on Vehicle Pollution: A Cost-Benefit Analysis in China</u></i>
CCMP	3.2005	<i>Andrea BIGANO, Jacqueline M. HAMILTON, Maren LAU, Richard S.J. TOL and Yuan ZHOU: <u>A Global Database of Domestic and International Tourist Numbers at National and Subnational Level</u></i>
CCMP	4.2005	<i>Andrea BIGANO, Jacqueline M. HAMILTON and Richard S.J. TOL: <u>The Impact of Climate on Holiday Destination Choice</u></i>
ETA	5.2005	<i>Hubert KEMPF: <u>Is Inequality Harmful for the Environment in a Growing Economy?</u></i>
CCMP	6.2005	<i>Valentina BOSETTI, Carlo CARRARO and Marzio GALEOTTI: <u>The Dynamics of Carbon and Energy Intensity in a Model of Endogenous Technical Change</u></i>
IEM	7.2005	<i>David CALEF and Robert GOBLE: <u>The Allure of Technology: How France and California Promoted Electric Vehicles to Reduce Urban Air Pollution</u></i>
ETA	8.2005	<i>Lorenzo PELLEGRINI and Reyer GERLAGH: <u>An Empirical Contribution to the Debate on Corruption Democracy and Environmental Policy</u></i>
CCMP	9.2005	<i>Angelo ANTOCI: <u>Environmental Resources Depletion and Interplay Between Negative and Positive Externalities in a Growth Model</u></i>
CTN	10.2005	<i>Frédéric DEROLAN: <u>Cost-Reducing Alliances and Local Spillovers</u></i>
NRM	11.2005	<i>Francesco SINDICO: <u>The GMO Dispute before the WTO: Legal Implications for the Trade and Environment Debate</u></i>
KTHC	12.2005	<i>Carla MASSIDDA: <u>Estimating the New Keynesian Phillips Curve for Italian Manufacturing Sectors</u></i>
KTHC	13.2005	<i>Michele MORETTO and Gianpaolo ROSSINI: <u>Start-up Entry Strategies: Employer vs. Nonemployer firms</u></i>
PRCG	14.2005	<i>Clara GRAZIANO and Annalisa LUPORINI: <u>Ownership Concentration, Monitoring and Optimal Board Structure</u></i>
CSRM	15.2005	<i>Parashar KULKARNI: <u>Use of Ecolabels in Promoting Exports from Developing Countries to Developed Countries: Lessons from the Indian LeatherFootwear Industry</u></i>
KTHC	16.2005	<i>Adriana DI LIBERTO, Roberto MURA and Francesco PIGLIARU: <u>How to Measure the Unobservable: A Panel Technique for the Analysis of TFP Convergence</u></i>
KTHC	17.2005	<i>Alireza NAGHAVI: <u>Asymmetric Labor Markets, Southern Wages, and the Location of Firms</u></i>
KTHC	18.2005	<i>Alireza NAGHAVI: <u>Strategic Intellectual Property Rights Policy and North-South Technology Transfer</u></i>
KTHC	19.2005	<i>Mombert HOPPE: <u>Technology Transfer Through Trade</u></i>
PRCG	20.2005	<i>Roberto ROSON: <u>Platform Competition with Endogenous Multihoming</u></i>
CCMP	21.2005	<i>Barbara BUCHNER and Carlo CARRARO: <u>Regional and Sub-Global Climate Blocs. A Game Theoretic Perspective on Bottom-up Climate Regimes</u></i>
IEM	22.2005	<i>Fausto CAVALLARO: <u>An Integrated Multi-Criteria System to Assess Sustainable Energy Options: An Application of the Promethee Method</u></i>
CTN	23.2005	<i>Michael FINUS, Pierre v. MOUCHE and Bianca RUNDSHAGEN: <u>Uniqueness of Coalitional Equilibria</u></i>
IEM	24.2005	<i>Wietze LISE: <u>Decomposition of CO2 Emissions over 1980–2003 in Turkey</u></i>
CTN	25.2005	<i>Somdeb LAHIRI: <u>The Core of Directed Network Problems with Quotas</u></i>
SIEV	26.2005	<i>Susanne MENZEL and Riccardo SCARPA: <u>Protection Motivation Theory and Contingent Valuation: Perceived Realism, Threat and WTP Estimates for Biodiversity Protection</u></i>
NRM	27.2005	<i>Massimiliano MAZZANTI and Anna MONTINI: <u>The Determinants of Residential Water Demand Empirical Evidence for a Panel of Italian Municipalities</u></i>
CCMP	28.2005	<i>Laurent GILOTTE and Michel de LARA: <u>Precautionary Effect and Variations of the Value of Information</u></i>
NRM	29.2005	<i>Paul SARFO-MENSAH: <u>Exportation of Timber in Ghana: The Menace of Illegal Logging Operations</u></i>
CCMP	30.2005	<i>Andrea BIGANO, Alessandra GORIA, Jacqueline HAMILTON and Richard S.J. TOL: <u>The Effect of Climate Change and Extreme Weather Events on Tourism</u></i>
NRM	31.2005	<i>Maria Angeles GARCIA-VALIÑAS: <u>Decentralization and Environment: An Application to Water Policies</u></i>
NRM	32.2005	<i>Chiara D'ALPAOS, Cesare DOSI and Michele MORETTO: <u>Concession Length and Investment Timing Flexibility</u></i>
CCMP	33.2005	<i>Joseph HUBER: <u>Key Environmental Innovations</u></i>
CTN	34.2005	<i>Antoni CALVÓ-ARMENGOL and Rahmi İLKILIÇ (Ixxii): <u>Pairwise-Stability and Nash Equilibria in Network Formation</u></i>
CTN	35.2005	<i>Francesco FERI (Ixxii): <u>Network Formation with Endogenous Decay</u></i>
CTN	36.2005	<i>Frank H. PAGE, Jr. and Myrna H. WOODERS (Ixxii): <u>Strategic Basins of Attraction, the Farsighted Core, and Network Formation Games</u></i>

CTN	37.2005	<i>Alessandra CASELLA and Nobuyuki HANAOKI</i> (lxxii): <u>Information Channels in Labor Markets. On the Resilience of Referral Hiring</u>
CTN	38.2005	<i>Matthew O. JACKSON and Alison WATTS</i> (lxxii): <u>Social Games: Matching and the Play of Finitely Repeated Games</u>
CTN	39.2005	<i>Anna BOGOMOLNAIA, Michel LE BRETON, Alexei SAVVATEEV and Shlomo WEBER</i> (lxxii): <u>The Egalitarian Sharing Rule in Provision of Public Projects</u>
CTN	40.2005	<i>Francesco FERI</i> : <u>Stochastic Stability in Network with Decay</u>
CTN	41.2005	<i>Aart de ZEEUW</i> (lxxii): <u>Dynamic Effects on the Stability of International Environmental Agreements</u>
NRM	42.2005	<i>C. Martijn van der HEIDE, Jeroen C.J.M. van den BERGH, Ekko C. van IERLAND and Paulo A.L.D. NUNES</i> : <u>Measuring the Economic Value of Two Habitat Defragmentation Policy Scenarios for the Veluwe, The Netherlands</u>
PRCG	43.2005	<i>Carla VIEIRA and Ana Paula SERRA</i> : <u>Abnormal Returns in Privatization Public Offerings: The Case of Portuguese Firms</u>
SIEV	44.2005	<i>Anna ALBERINI, Valentina ZANATTA and Paolo ROSATO</i> : <u>Combining Actual and Contingent Behavior to Estimate the Value of Sports Fishing in the Lagoon of Venice</u>
CTN	45.2005	<i>Michael FINUS and Bianca RUNDSHAGEN</i> : <u>Participation in International Environmental Agreements: The Role of Timing and Regulation</u>
CCMP	46.2005	<i>Lorenzo PELLEGRINI and Reyer GERLAGH</i> : <u>Are EU Environmental Policies Too Demanding for New Members States?</u>
IEM	47.2005	<i>Matteo MANERA</i> : <u>Modeling Factor Demands with SEM and VAR: An Empirical Comparison</u>
CTN	48.2005	<i>Olivier TERCIEUX and Vincent VANNETELBOSCH</i> (lxx): <u>A Characterization of Stochastically Stable Networks</u>
CTN	49.2005	<i>Ana MAULEON, José SEMPERE-MONERRIS and Vincent J. VANNETELBOSCH</i> (lxxii): <u>R&amp;D Networks Among Unionized Firms</u>
CTN	50.2005	<i>Carlo CARRARO, Johan EYCKMANS and Michael FINUS</i> : <u>Optimal Transfers and Participation Decisions in International Environmental Agreements</u>
KTHC	51.2005	<i>Valeria GATTAI</i> : <u>From the Theory of the Firm to FDI and Internalisation: A Survey</u>
CCMP	52.2005	<i>Alireza NAGHAVI</i> : <u>Multilateral Environmental Agreements and Trade Obligations: A Theoretical Analysis of the Doha Proposal</u>
SIEV	53.2005	<i>Margaretha BREIL, Gretel GAMBARELLI and Paulo A.L.D. NUNES</i> : <u>Economic Valuation of On Site Material Damages of High Water on Economic Activities based in the City of Venice: Results from a Dose-Response-Expert-Based Valuation Approach</u>
ETA	54.2005	<i>Alessandra del BOCA, Marzio GALEOTTI, Charles P. HIMMELBERG and Paola ROTA</i> : <u>Investment and Time to Plan: A Comparison of Structures vs. Equipment in a Panel of Italian Firms</u>
CCMP	55.2005	<i>Gernot KLEPPER and Sonja PETERSON</i> : <u>Emissions Trading, CDM, JI, and More – The Climate Strategy of the EU</u>
ETA	56.2005	<i>Maia DAVID and Bernard SINCLAIR-DESGAGNÉ</i> : <u>Environmental Regulation and the Eco-Industry</u>
ETA	57.2005	<i>Alain-Désiré NIMUBONA and Bernard SINCLAIR-DESGAGNÉ</i> : <u>The Pigouvian Tax Rule in the Presence of an Eco-Industry</u>
NRM	58.2005	<i>Helmut KARL, Antje MÖLLER, Ximena MATUS, Edgar GRANDE and Robert KAISER</i> : <u>Environmental Innovations: Institutional Impacts on Co-operations for Sustainable Development</u>
SIEV	59.2005	<i>Dimitra VOUVAKI and Anastasios XEPAPADEAS</i> (lxxiii): <u>Criteria for Assessing Sustainable Development: Theoretical Issues and Empirical Evidence for the Case of Greece</u>
CCMP	60.2005	<i>Andreas LÖSCHEL and Dirk T.G. RÜBBELKE</i> : <u>Impure Public Goods and Technological Interdependencies</u>
PRCG	61.2005	<i>Christoph A. SCHALTEGGER and Benno TORGLER</i> : <u>Trust and Fiscal Performance: A Panel Analysis with Swiss Data</u>
ETA	62.2005	<i>Irene VALSECCHI</i> : <u>A Role for Instructions</u>
NRM	63.2005	<i>Valentina BOSETTI and Gianni LOCATELLI</i> : <u>A Data Envelopment Analysis Approach to the Assessment of Natural Parks' Economic Efficiency and Sustainability. The Case of Italian National Parks</u>
SIEV	64.2005	<i>Arianne T. de BLAEIJ, Paulo A.L.D. NUNES and Jeroen C.J.M. van den BERGH</i> : <u>Modeling 'No-choice' Responses in Attribute Based Valuation Surveys</u>
CTN	65.2005	<i>Carlo CARRARO, Carmen MARCHIORI and Alessandra SGOBBI</i> : <u>Applications of Negotiation Theory to Water Issues</u>
CTN	66.2005	<i>Carlo CARRARO, Carmen MARCHIORI and Alessandra SGOBBI</i> : <u>Advances in Negotiation Theory: Bargaining, Coalitions and Fairness</u>
KTHC	67.2005	<i>Sandra WALLMAN</i> (lxxiv): <u>Network Capital and Social Trust: Pre-Conditions for 'Good' Diversity?</u>
KTHC	68.2005	<i>Asimina CHRISTOFOROU</i> (lxxiv): <u>On the Determinants of Social Capital in Greece Compared to Countries of the European Union</u>
KTHC	69.2005	<i>Eric M. USLANER</i> (lxxiv): <u>Varieties of Trust</u>
KTHC	70.2005	<i>Thomas P. LYON</i> (lxxiv): <u>Making Capitalism Work: Social Capital and Economic Growth in Italy, 1970-1995</u>
KTHC	71.2005	<i>Graziella BERTOCCHI and Chiara STROZZI</i> (lxxv): <u>Citizenship Laws and International Migration in Historical Perspective</u>
KTHC	72.2005	<i>Elsbeth van HYLCKAMA Vlieg</i> (lxxv): <u>Accommodating Differences</u>
KTHC	73.2005	<i>Renato SANSA and Ercole SORI</i> (lxxv): <u>Governance of Diversity Between Social Dynamics and Conflicts in Multicultural Cities. A Selected Survey on Historical Bibliography</u>
IEM	74.2005	<i>Alberto LONGO and Anil MARKANDYA</i> : <u>Identification of Options and Policy Instruments for the Internalisation of External Costs of Electricity Generation. Dissemination of External Costs of Electricity Supply Making Electricity External Costs Known to Policy-Makers</u> <u>MAXIMA</u>

IEM	75.2005	<i>Margherita GRASSO and Matteo MANERA: <u>Asymmetric Error Correction Models for the Oil-Gasoline Price Relationship</u></i>
ETA	76.2005	<i>Umberto CHERUBINI and Matteo MANERA: <u>Hunting the Living Dead A “Peso Problem” in Corporate Liabilities Data</u></i>
CTN	77.2005	<i>Hans-Peter WEIKARD: <u>Cartel Stability under an Optimal Sharing Rule</u></i>
ETA	78.2005	<i>Joëlle NOAILLY, Jeroen C.J.M. van den BERGH and Cees A. WITHAGEN (lxxvi): <u>Local and Global Interactions in an Evolutionary Resource Game</u></i>
ETA	79.2005	<i>Joëlle NOAILLY, Cees A. WITHAGEN and Jeroen C.J.M. van den BERGH (lxxvi): <u>Spatial Evolution of Social Norms in a Common-Pool Resource Game</u></i>
CCMP	80.2005	<i>Massimiliano MAZZANTI and Roberto ZOBOLI: <u>Economic Instruments and Induced Innovation: The Case of End-of-Life Vehicles European Policies</u></i>
NRM	81.2005	<i>Anna LASUT: <u>Creative Thinking and Modelling for the Decision Support in Water Management</u></i>
CCMP	82.2005	<i>Valentina BOSETTI and Barbara BUCHNER: <u>Using Data Envelopment Analysis to Assess the Relative Efficiency of Different Climate Policy Portfolios</u></i>
ETA	83.2005	<i>Ignazio MUSU: <u>Intellectual Property Rights and Biotechnology: How to Improve the Present Patent System</u></i>
KTHC	84.2005	<i>Giulio CAINELLI, Susanna MANCINELLI and Massimiliano MAZZANTI: <u>Social Capital, R&amp;D and Industrial Districts</u></i>
ETA	85.2005	<i>Rosella LEVAGGI, Michele MORETTO and Vincenzo REBBA: <u>Quality and Investment Decisions in Hospital Care when Physicians are Devoted Workers</u></i>
CCMP	86.2005	<i>Valentina BOSETTI and Laurent GILOTTE: <u>Carbon Capture and Sequestration: How Much Does this Uncertain Option Affect Near-Term Policy Choices?</u></i>
CSRM	87.2005	<i>Nicoletta FERRO: <u>Value Through Diversity: Microfinance and Islamic Finance and Global Banking</u></i>
ETA	88.2005	<i>A. MARKANDYA and S. PEDROSO: <u>How Substitutable is Natural Capital?</u></i>
IEM	89.2005	<i>Anil MARKANDYA, Valeria COSTANTINI, Francesco GRACCEVA and Giorgio VICINI: <u>Security of Energy Supply: Comparing Scenarios From a European Perspective</u></i>
CCMP	90.2005	<i>Vincent M. OTTO, Andreas LÖSCHEL and Rob DELLINK: <u>Energy Biased Technical Change: A CGE Analysis</u></i>
PRCG	91.2005	<i>Carlo CAPUANO: <u>Abuse of Competitive Fringe</u></i>
PRCG	92.2005	<i>Ulrich BINDSEIL, Kjell G. NYBORG and Ilya A. STREBULAEV (lxv): <u>Bidding and Performance in Repo Auctions: Evidence from ECB Open Market Operations</u></i>
CCMP	93.2005	<i>Sabrina AUCI and Leonardo BECCHETTI: <u>The Stability of the Adjusted and Unadjusted Environmental Kuznets Curve</u></i>
CCMP	94.2005	<i>Francesco BOSELLO and Jian ZHANG: <u>Assessing Climate Change Impacts: Agriculture</u></i>
CTN	95.2005	<i>Alejandro CAPARRÓS, Jean-Christophe PEREAU and Tarik TAZDAÏT: <u>Bargaining with Non-Monolithic Players</u></i>
ETA	96.2005	<i>William BROCK and Anastasios XEPAPADEAS (lxxvi): <u>Optimal Control and Spatial Heterogeneity: Pattern Formation in Economic-Ecological Models</u></i>

- (lxv) This paper was presented at the EuroConference on “Auctions and Market Design: Theory, Evidence and Applications” organised by Fondazione Eni Enrico Mattei and sponsored by the EU, Milan, September 25-27, 2003
- (lxvi) This paper has been presented at the 4<sup>th</sup> BioEcon Workshop on “Economic Analysis of Policies for Biodiversity Conservation” organised on behalf of the BIOECON Network by Fondazione Eni Enrico Mattei, Venice International University (VIU) and University College London (UCL), Venice, August 28-29, 2003
- (lxvii) This paper has been presented at the international conference on “Tourism and Sustainable Economic Development – Macro and Micro Economic Issues” jointly organised by CRENoS (Università di Cagliari e Sassari, Italy) and Fondazione Eni Enrico Mattei, and supported by the World Bank, Sardinia, September 19-20, 2003
- (lxviii) This paper was presented at the ENGIME Workshop on “Governance and Policies in Multicultural Cities”, Rome, June 5-6, 2003
- (lxix) This paper was presented at the Fourth EEP Plenary Workshop and EEP Conference “The Future of Climate Policy”, Cagliari, Italy, 27-28 March 2003
- (lxx) This paper was presented at the 9<sup>th</sup> Coalition Theory Workshop on "Collective Decisions and Institutional Design" organised by the Universitat Autònoma de Barcelona and held in Barcelona, Spain, January 30-31, 2004
- (lxxi) This paper was presented at the EuroConference on “Auctions and Market Design: Theory, Evidence and Applications”, organised by Fondazione Eni Enrico Mattei and Consip and sponsored by the EU, Rome, September 23-25, 2004
- (lxxii) This paper was presented at the 10<sup>th</sup> Coalition Theory Network Workshop held in Paris, France on 28-29 January 2005 and organised by EUREQua.
- (lxxiii) This paper was presented at the 2nd Workshop on "Inclusive Wealth and Accounting Prices" held in Trieste, Italy on 13-15 April 2005 and organised by the Ecological and Environmental Economics - EEE Programme, a joint three-year programme of ICTP - The Abdus Salam International Centre for Theoretical Physics, FEEM - Fondazione Eni Enrico Mattei, and The Beijer International Institute of Ecological Economics
- (lxxiv) This paper was presented at the ENGIME Workshop on “Trust and social capital in multicultural cities” Athens, January 19-20, 2004
- (lxxv) This paper was presented at the ENGIME Workshop on “Diversity as a source of growth” Rome November 18-19, 2004
- (lxxvi) This paper was presented at the 3rd Workshop on Spatial-Dynamic Models of Economics and Ecosystems held in Trieste on 11-13 April 2005 and organised by the Ecological and Environmental Economics - EEE Programme, a joint three-year programme of ICTP - The Abdus Salam International Centre for Theoretical Physics, FEEM - Fondazione Eni Enrico Mattei, and The Beijer International Institute of Ecological Economics

#### 2004 SERIES

<b>CCMP</b>	<i>Climate Change Modelling and Policy</i> (Editor: Marzio Galeotti )
<b>GG</b>	<i>Global Governance</i> (Editor: Carlo Carraro)
<b>SIEV</b>	<i>Sustainability Indicators and Environmental Valuation</i> (Editor: Anna Alberini)
<b>NRM</b>	<i>Natural Resources Management</i> (Editor: Carlo Giupponi)
<b>KTHC</b>	<i>Knowledge, Technology, Human Capital</i> (Editor: Gianmarco Ottaviano)
<b>IEM</b>	<i>International Energy Markets</i> (Editor: Anil Markandya)
<b>CSRM</b>	<i>Corporate Social Responsibility and Sustainable Management</i> (Editor: Sabina Ratti)
<b>PRA</b>	<i>Privatisation, Regulation, Antitrust</i> (Editor: Bernardo Bortolotti)
<b>ETA</b>	<i>Economic Theory and Applications</i> (Editor: Carlo Carraro)
<b>CTN</b>	<i>Coalition Theory Network</i>

#### 2005 SERIES

<b>CCMP</b>	<i>Climate Change Modelling and Policy</i> (Editor: Marzio Galeotti )
<b>SIEV</b>	<i>Sustainability Indicators and Environmental Valuation</i> (Editor: Anna Alberini)
<b>NRM</b>	<i>Natural Resources Management</i> (Editor: Carlo Giupponi)
<b>KTHC</b>	<i>Knowledge, Technology, Human Capital</i> (Editor: Gianmarco Ottaviano)
<b>IEM</b>	<i>International Energy Markets</i> (Editor: Anil Markandya)
<b>CSRM</b>	<i>Corporate Social Responsibility and Sustainable Management</i> (Editor: Sabina Ratti)
<b>PRCG</b>	<i>Privatisation Regulation Corporate Governance</i> (Editor: Bernardo Bortolotti)
<b>ETA</b>	<i>Economic Theory and Applications</i> (Editor: Carlo Carraro)
<b>CTN</b>	<i>Coalition Theory Network</i>