

Vannetelbosch, Vincent; Tercieux, Olivier

**Working Paper**

## A Characterization of Stochastically Stable Networks

Nota di Lavoro, No. 48.2005

**Provided in Cooperation with:**

Fondazione Eni Enrico Mattei (FEEM)

*Suggested Citation:* Vannetelbosch, Vincent; Tercieux, Olivier (2005) : A Characterization of Stochastically Stable Networks, Nota di Lavoro, No. 48.2005, Fondazione Eni Enrico Mattei (FEEM), Milano

This Version is available at:

<http://hdl.handle.net/10419/73945>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

## **A Characterization of Stochastically Stable Networks**

Olivier Tercieux and Vincent Vannetelbosch

NOTA DI LAVORO 48.2005

**APRIL 2005**

CTN – Coalition Theory Network

Olivier Tercieux, *CentER, Tilburg University*  
Vincent Vannetelbosch, *FNRS and CORE, University of Louvain*

This paper can be downloaded without charge at:

The Fondazione Eni Enrico Mattei Note di Lavoro Series Index:  
<http://www.feem.it/Feem/Pub/Publications/WPapers/default.htm>

Social Science Research Network Electronic Paper Collection:  
<http://ssrn.com/abstract=700605>

# A Characterization of Stochastically Stable Networks

## Summary

Jackson and Watts [J. of Econ. Theory 71 (2002), 44-74] have examined the dynamic formation and stochastic evolution of networks. We provide a refinement of pairwise stability,  $p$ -pairwise stability, which allows us to characterize the stochastically stable networks without requiring the "tree construction" and the computation of resistance that may be quite complex. When a  $1/2$ -pairwise stable network exists, it is unique and it coincides with the unique stochastically stable network. To solve the inexistence problem of  $p$ -pairwise stable networks, we define its set-valued extension with the notion of  $p$ -pairwise stable set. The  $1/2$ -pairwise stable set exists and is unique. Any stochastically stable network is included in the  $1/2$ -pairwise stable set. Thus, any network outside the  $1/2$ -pairwise stable set must be considered as a nonrobust network. We also show that the  $1/2$ -pairwise stable set can contain no pairwise stable network and we provide examples where a set of networks is more "stable" than a pairwise stable network.

**Keywords:** Network formation, Pairwise stability, Stochastic stability

**JEL Classification:** C70, D20

*Vincent Vannetelbosch is Research Associate of the National Fund for Scientific Research (FNRS), Belgium. We would like to thank Paul Belleflamme, Francesco Feri, Matt Jackson, Ana Mauleon, Philippe Solal and Anne van den Nouweland for helpful comments or discussions. Financial support from the research project BEC 2003-02084 (Universidad del Pais Vasco) funded by the Spanish government, from the CNRS project GW/SCSHS/SH/2003-41, from the Belgian French Community's program Action de Recherches Concertée 03/08-302 (UCL) and from the French Ministry of Research (Action Concertée Incitative) is gratefully acknowledged. The research of Olivier Tercieux has been made possible by a FSR fellowship from UCL.*

*This paper was presented at the 9<sup>th</sup> Coalition Theory Workshop on "Collective Decisions and Institutional Design" held in Barcelona, Spain, on 30-31 January 2004 and organised by the Universitat Autònoma de Barcelona.*

*Address for correspondence:*

Vincent Vannetelbosch  
CORE, Université Catholique de Louvain  
Voie du Roman Pays 34  
B-1348 Louvain-la-Neuve  
Belgium  
Phone: 003210474142  
Fax: 003210474301  
E-mail: vannetelbosch@core.ucl.ac.be

# 1 Introduction

The organization of individual agents into networks and groups or coalitions has an important role in the determination of the outcome of many social and economic interactions.<sup>1</sup> There are many possible approaches to model network formation. One is simply to model it explicitly as a non-cooperative game (see e.g. Aumann and Myerson, 1988). A different approach is to analyze the networks that one might expect to emerge in the long run and to examine a sort of stability requirement that individuals not benefit from altering the structure of the network. This is the approach that was taken by Jackson and Wolinsky (1996) when defining pairwise stable networks. A network is pairwise stable if no player benefits from severing one of their links and no other two players benefit from adding a link between them, with one benefiting strictly and the other at least weakly. Another approach is to analyze the process of network formation in a dynamic framework.<sup>2</sup> Jackson and Watts (2002) have proposed a dynamic process in which individuals form and sever links based on the improvement that the resulting network offers them relative to the current network. This deterministic dynamic process may end at stable networks or in some cases may cycle. To explore whether some networks might be regarded as more reasonable than others, Jackson and Watts (2002) add to this deterministic process random perturbations and examine the distribution over networks as the level of random perturbations vanishes.

Exploiting the tree construction of Freidlin and Wentzel (1984), Jackson and Watts (2002) have shown that the outcome of their selection process (called stochastically stable networks) can be fully characterized in terms of resistances. However, these results are not always helpful in determining the outcome, because the required computation for resistances and the tree construction may be quite complex. To be more precise, this problem is known to be NP-complete in complexity theory.<sup>3</sup> Thus we do not have much knowledge on which network will arise in these processes in general. In order to extend the applicability of these results, more succinct criteria are needed to determine the outcome of this selection theory. One goal of the paper is to find a criterion for network selection that is free from the computation of resistances and the tree construction.<sup>4</sup>

---

<sup>1</sup>Jackson (2003, 2004) has provided a survey of models of network formation.

<sup>2</sup>Watts (2001) has extended the Jackson and Wolinsky model to a dynamic process but she has limited attention to the specific contest of the connections model and a particular deterministic dynamic.

<sup>3</sup>See Garey and Johnson (1979, p.206). We know that for NP-complete problems, all *known* algorithms to solve the problem require time which is exponential in the problem size (for instance in the number of individuals considered).

<sup>4</sup>In noncooperative games Young (1993), Ellison (1993), Kandori, Mailath and Rob (1993) among others have applied the Freidlin and Wentzell (1984) techniques in order to provide evolutionary models that select among (strict) Nash equilibria. But these results are submitted to the same criticism than

We propose a new concept,  $p$ -pairwise stability, which is a refinement of the notion of pairwise stability. A network is said to be  $p$ -pairwise stable if when we add a set of links to this network (or sever a set of links), then if we allow players to successively create or delete links, they will come back to the initial network. The parameter  $p \in [0, 1]$  indicates the "number" of links that can be modified:  $p = 0$  means that all links may be modified,  $p = 1$  means that no link may be added or severed. Thus, 1-pairwise stability reverts to Jackson and Wolinsky (1996) pairwise stability concept. Also, a network is said to be  $\frac{1}{2}$ -pairwise stable if when we add a set of links to this network (or sever a set of links) such that the number of changes is less than half the total of possible changes, then if we allow players to successively create or delete links, they will come back to the initial network.

We show that when a  $\frac{1}{2}$ -pairwise stable network exists, it is unique. Moreover it is the only stochastically stable network in Jackson and Watts (2002) stochastic evolutionary process. But while our notion of a  $\frac{1}{2}$ -pairwise stable network leads to a unique selection when it exists, it does not always exist. Therefore, we define its set-valued extension with the notion of  $\frac{1}{2}$ -pairwise stable set of networks that is proved to exist and to coincide with the  $\frac{1}{2}$ -pairwise stable network when it exists. We also show that if a network is stochastically stable then it belongs to the  $\frac{1}{2}$ -pairwise stable set of networks. Thus, any network outside the  $\frac{1}{2}$ -pairwise stable set must be considered as a non-robust network. Interestingly, the  $\frac{1}{2}$ -pairwise stable set of networks can contain no pairwise stable network. We see this as a drawback of pairwise stability, and we provide examples where a set of networks is more "stable" than a pairwise stable network.

The paper is organized as follows. In Section 2 we define the notion of  $p$ -pairwise stable network and we study its properties. In Section 3 we propose a set-valued extension, the  $p$ -pairwise stable set of networks. In Section 4 we provide an evolutionary foundation to the  $\frac{1}{2}$ -pairwise stable set of networks. In Section 5 we conclude.

## 2 $p$ -Pairwise Stable Networks

Let  $N = \{1, \dots, n\}$  be the finite set of players who are connected in some network relationship. The network relationships are reciprocal and the network is thus modeled as a Jackson and Watts (2002) and so they are not always helpful in determining the selected action profiles. Then, some authors have looked for criteria for equilibrium (or non-equilibrium) selection that are free from the computation of resistances and the tree construction. For instance, Young (1993) has shown that in a two player, two action game, only the risk-dominant equilibrium (in the sense of Harsanyi and Selten (1988)) is stochastically stable. This result was generalized by Maruta (1997) and Durieu, Solal and Tercieux (2003) to two players finite games.

non-directed graph.<sup>5</sup> Individuals are the nodes in the graph and links indicate bilateral relationships between individuals. Thus, a network  $g$  is simply a list of which pairs of individuals are linked to each other. If we are considering a pair of individuals  $i$  and  $j$ , then  $\{i, j\} \in g$  indicates that  $i$  and  $j$  are linked under the network  $g$ . For simplicity, we write  $ij$  to represent the link  $\{i, j\}$ , and so  $ij \in g$  indicates that  $i$  and  $j$  are linked under the network  $g$ . Let  $g^N$  be the set of all subsets of  $N$  of size 2.  $G^N$  denotes the set of all possible networks or graphs on  $N$ , with  $g^N$  being the complete network. The network obtained by adding link  $ij$  to an existing network  $g$  is denoted  $g + ij$  and the network obtained by deleting link  $ij$  from an existing network  $g$  is denoted  $g - ij$ . For any network  $g$ , let  $N(g) = \{i \mid \exists j \text{ such that } ij \in g\}$  be the set of players who have at least one link in the network  $g$ .

Different network configurations lead to different values of overall production or overall utility to players. These various possible valuations are represented via a value function. A *value function* is a function  $v : G^N \rightarrow \mathbb{R}$ . The set of all possible value functions is denoted  $\mathcal{V}$ . A value function only keeps track of how the total societal value varies across different networks. We also wish to keep track of how that value is allocated or distributed among the players forming a network. An allocation rule is a function  $Y : G^N \times \mathcal{V} \rightarrow \mathbb{R}^N$  such that  $\sum_{i \in N} Y_i(g, v) = v(g)$  for all  $v$  and  $g$ . It is important to note that an allocation rule depends on both  $g$  and  $v$ . This allows an allocation rule to take full account of a player  $i$ 's role in the network. This includes not only what the network configuration is, but also and how the value generated depends on the overall network structure.

In evaluating societal welfare, we may take various perspectives.<sup>6</sup> A network  $g$  is *Pareto efficient* relative to  $v$  and  $Y$  if there does not exist any  $g' \subseteq G^N$  such that  $Y_i(g', v) \geq Y_i(g, v)$  for all  $i$  with strict inequality for some  $i$ . This definition of efficiency of a network takes  $Y$  as fixed, and hence can be thought of as applying to situations where no intervention is possible. A network  $g \subseteq G^N$  is *strongly efficient* relative to  $v$  if  $v(g) \geq v(g')$  for all  $g' \subseteq G^N$ . This is a strong notion of efficiency as it takes the perspective that value is fully transferable.

A simple way to analyze the networks that one might expect to emerge in the long run is to examine a sort of equilibrium requirement that agents not benefit from altering the structure of the network. A weak version of such condition is the pairwise stability notion defined by Jackson and Wolinsky (1996). A network is pairwise stable if no player

---

<sup>5</sup>Bala and Goyal (2000) have studied network formation in directed networks. See also Dutta and Jackson (2000).

<sup>6</sup>Throughout the paper we use the notation  $\subseteq$  for weak inclusion and  $\subsetneq$  for strict inclusion. We also use the symbols  $\vee$  and  $\wedge$  which mean "or" and "and", respectively. Finally,  $\#$  will refer to the notion of cardinality.

benefits from severing one of their links and no other two players benefit from adding a link between them, with one benefiting strictly and the other at least weakly.

**Definition 1** *A network  $g$  is pairwise stable with respect to value function  $v$  and allocation rule  $Y$  if*

(i) *for all  $ij \in g$ ,  $Y_i(g, v) \geq Y_i(g - ij, v)$  and  $Y_j(g, v) \geq Y_j(g - ij, v)$ , and*

(ii) *for all  $ij \notin g$ , if  $Y_i(g, v) < Y_i(g + ij, v)$  then  $Y_j(g, v) > Y_j(g + ij, v)$ .*

Let us say that  $g'$  is adjacent to  $g$  if  $g' = g + ij$  or  $g' = g - ij$  for some  $ij$ . A network  $g'$  defeats  $g$  if either  $g' = g - ij$  and  $Y_i(g', v) > Y_i(g, v)$ , or if  $g' = g + ij$  with  $Y_i(g', v) \geq Y_i(g, v)$  and  $Y_j(g', v) \geq Y_j(g, v)$  with at least one inequality holding strictly. Pairwise stability is equivalent to saying that a network is pairwise stable if it is not defeated by another (necessarily adjacent) network. The following example shows the main insight of the stability requirement we will introduce. In particular, the example shows that a network that is both pareto-dominant and pairwise stable can be "less stable" than another network.

**Example 1.** Consider a situation where four players can form links. The payoffs they obtained from the different network configurations are (see Figure 1): for a non-empty network  $g$ ,  $Y_i(g) = \#(g)$  if  $i \in N(g)$  with  $\#(g)$  being the number of links in  $g$ ,  $Y_i(g) = 0$  if  $i \notin N(g)$ , and  $Y_i(g) = 10$  if  $g$  is the empty network. Both the empty network and the complete network are pairwise stable networks. The empty network is also the efficient network.

Suppose that at least two links are added to the empty network to form  $g'$ . Then, from  $g'$  all "undefeated" improving paths go to the complete network and none goes back to the empty network. An improving path is a sequence of networks that can emerge when players form or sever links based on the improvement the resulting network offers relative to the current network. Each network in the sequence differs by one link from the previous one. If a link is added, then the two players involved must both agree to its addition, with at least one of the two strictly benefiting from the addition of the link. If a link is deleted, then it must be that at least one of the two players involved in the link strictly benefits from its deletion. By an "undefeated" improving path, we mean that the final network in the sequence of the improving path is not defeated. Suppose now that at most four links are deleted from the complete network to form  $g''$ . Then, from  $g''$  all "undefeated" improving paths go back to the complete network. Thus, we say that the empty network (while being the efficient network) is "less stable" than the complete network, while both are pairwise stable.

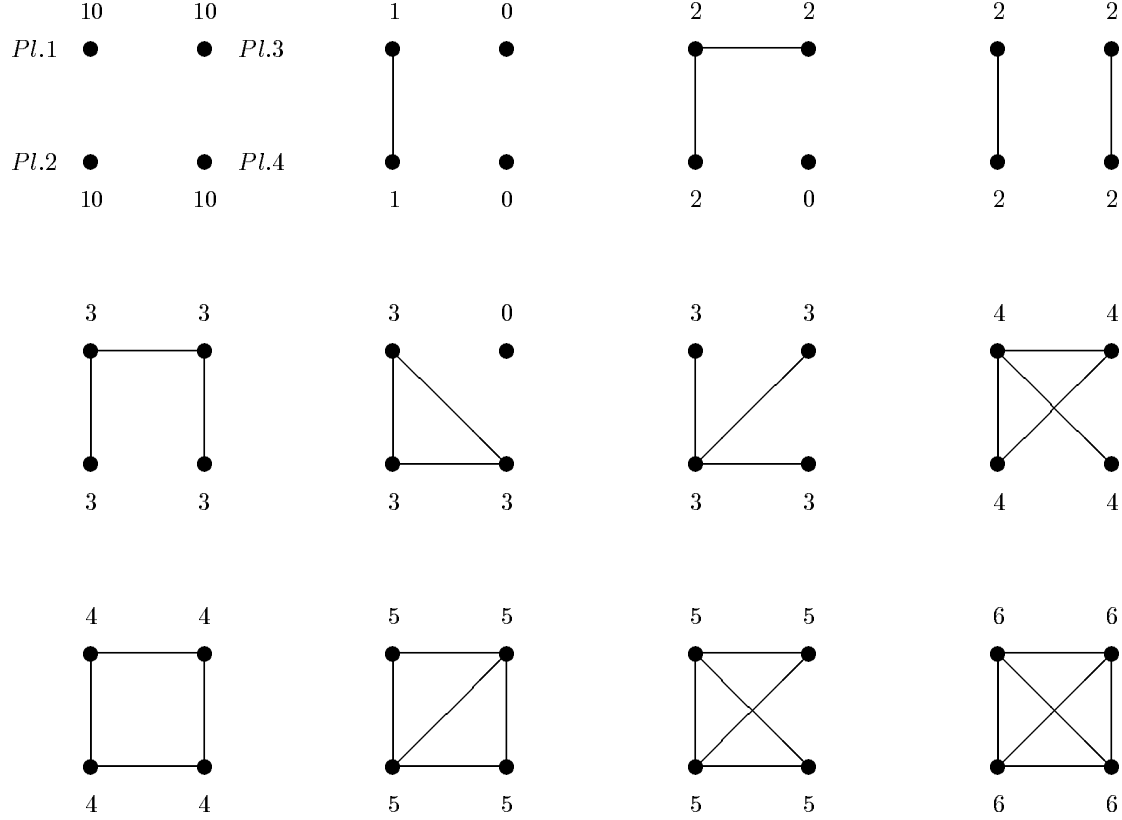


Figure 1: The empty and complete networks are pairwise stable (Example 1).

In order to formalize such refinement of pairwise stability, we first define a notion of distance between two networks. For  $g, g' \subseteq g^N$  we denote by

$$d(g, g') \equiv \frac{\#\{ij \in g^N \mid (ij \in g \wedge ij \notin g') \vee (ij \notin g \wedge ij \in g')\}}{\#g^N}$$

the *distance* between  $g$  and  $g'$ . That is,  $d(g, g')$  is the number of links that  $g$  does have while  $g'$  does not, plus the number of links that  $g$  does not have while  $g'$  does, the total being divided by the maximum number of links. Thus,  $0 \leq d(g, g') \leq 1$ . The formal definition of an improving path is due to Jackson and Watts (2002). An improving path from a network  $g$  to a network  $g'$  is a finite sequence of graphs  $g_1, \dots, g_K$  with  $g_1 = g$  and  $g_K = g'$  such that for any  $k \in \{1, \dots, K - 1\}$  either:

- (i)  $g_{k+1} = g_k - ij$  for some  $ij$  such that  $Y_i(g_k - ij) > Y_i(g_k)$ , or
- (ii)  $g_{k+1} = g_k + ij$  for some  $ij$  such that  $Y_i(g_k + ij) > Y_i(g_k)$  and  $Y_j(g_k + ij) \geq Y_j(g_k)$ .



The length of an improving path is  $K - 1$ ,  $K \geq 2$ . If there exists an improving path from  $g'$  to  $g$ , then as Jackson and Watts (2002) we use the symbol  $g' \rightarrow g$ . For a given network,  $g$ , let  $im(g) = \{g' \subseteq g^N \mid g' \rightarrow g\}$ . This is the set of networks for which there is an improving path leading from  $g'$  to  $g$ . An improving path from  $g'$  to  $g$  is of maximum length if  $g_K$  is not defeated by any  $g'' \subseteq g^N$ . For  $g' \neq g$ , we write  $g' \mapsto g$  if:

- (i) all improving paths of maximum length from  $g'$  go to  $g$ ,
- (ii) there does not exist an infinite improving path from  $g'$ .<sup>7</sup>

We write  $g \mapsto g$  if from  $g$  there is no improving path. For a given network  $g$ , let  $IM(g) = \{g' \subseteq g^N \mid g' \mapsto g\}$ . In the sequel, we note  $\phi(p)$  the largest number smaller or equal to  $p$  such that  $\phi(p) \cdot \#g^N$  is an integer. This notation will be useful in defining our notion of  $p$ -pairwise stable networks.

**Definition 2** *Let  $p \in [0, 1]$ . A network  $g$  is  $p$ -pairwise stable with respect to allocation rule  $Y$  and value function  $v$  if for all  $g' \subseteq g^N$  such that  $d(g', g) \leq 1 - \phi(p)$ , we have  $g' \in IM(g)$ .*

Any network  $g$  that is  $p$ -pairwise stable is  $p'$ -pairwise stable for  $p' \geq p$ . The notion of  $p$ -pairwise stability is a refinement of pairwise stability in the following sense. A network  $g$  is pairwise stable if and only if it is 1-pairwise stable. Thus, any network  $g$  that is  $p$ -pairwise stable is pairwise stable.

**Proposition 1** *Let  $p \leq \frac{1}{2}$ . A  $p$ -pairwise stable network is unique when it exists.*

**Proof.** We proceed by contradiction. Let us assume that  $g^1$  and  $g^2$  are two distinct  $p$ -pairwise stable networks where  $p \leq \frac{1}{2}$ . Then, they are  $\frac{1}{2}$ -pairwise stable. If  $d(g^1, g^2) \leq 1 - \phi(\frac{1}{2})$ , we have a straightforward contradiction. (Since we must have  $g^1 \in IM(g^2)$ , i.e.  $g^1 \mapsto g^2$  which is not possible since  $g^1 (\neq g^2)$  is pairwise stable (or indifferently 1-pairwise stable).

Assume now that  $d(g^1, g^2) > 1 - \phi(\frac{1}{2})$ . Pick  $g^1$  and delete elements in  $\{(ij \in g^1 \wedge ij \notin g^2)\}$  and add elements in  $\{(ij \notin g^1 \wedge ij \in g^2)\}$  so that we obtain a network  $g'$  satisfying  $d(g', g^1) = 1 - \phi(\frac{1}{2})$ . By construction, this network  $g'$  satisfies  $d(g', g^2) \leq \phi(\frac{1}{2}) \leq 1 - \phi(\frac{1}{2})$ . Then, since  $g^1$  and  $g^2$  are  $\frac{1}{2}$ -pairwise stable, we have that  $g' \in IM(g^1)$ , i.e.  $g' \mapsto g^1$ ,

---

<sup>7</sup>An infinite improving path from  $g$  is an infinite sequence of graphs  $g_1, g_2, \dots$  such that for any  $k \in \{1, 2, 3, \dots\}$  either (i)  $g_{k+1} = g_k - ij$  for some  $ij$  such that  $Y_i(g_k - ij) > Y_i(g_k)$ , or (ii)  $g_{k+1} = g_k + ij$  for some  $ij$  such that  $Y_i(g_k + ij) > Y_i(g_k)$  and  $Y_j(g_k + ij) \geq Y_j(g_k)$ . Thus, no network is not defeated in an infinite improving path.

and  $g' \in IM(g^2)$ , i.e.  $g' \mapsto g^2$ , which is not possible since  $g^2 \neq g^1$ . ■

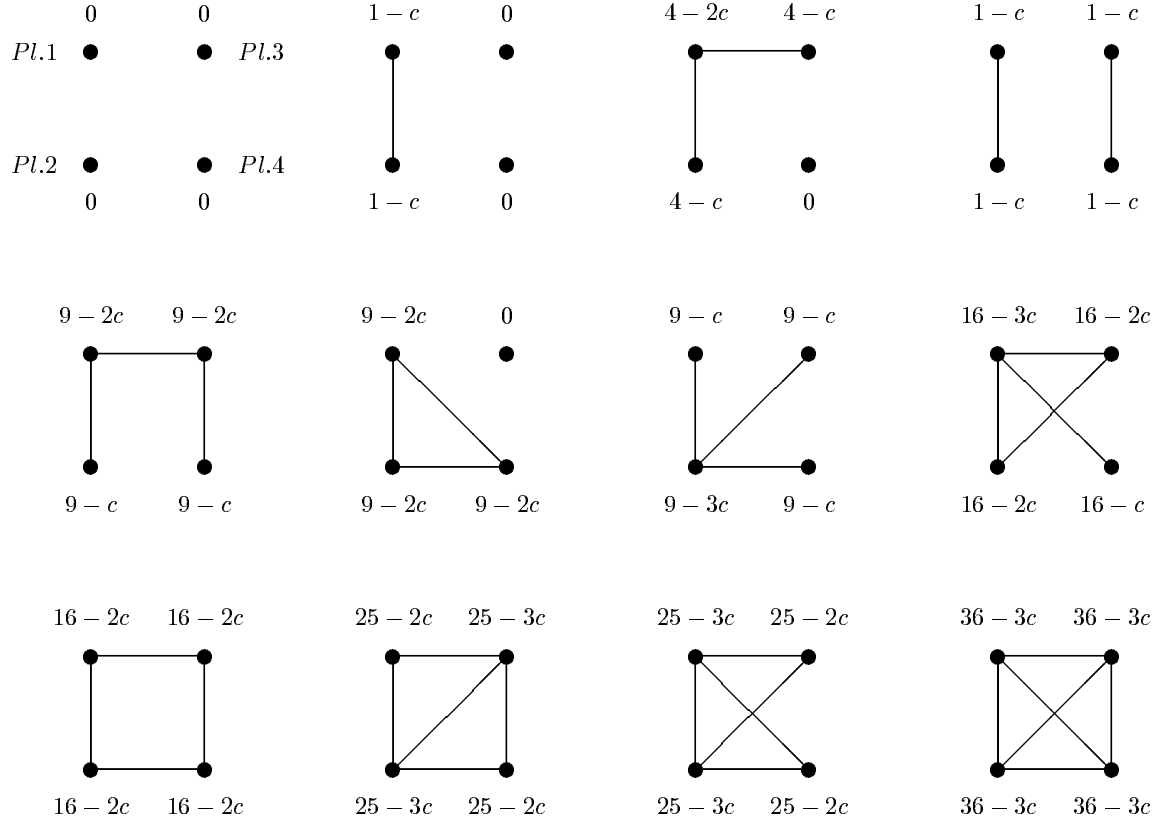
In Example 1, the empty network is pairwise stable and is the unique strongly stable network.<sup>8</sup> However, the complete network is the unique  $\frac{1}{2}$ -pairwise stable network. The reason is that from any network  $g'$  with  $\#(g') \geq 3$  (or  $d(g', g^N) \leq \frac{1}{2}$ ) any "undefeated" improving paths go to the complete network  $g^N$ , but none goes to the empty network.<sup>9</sup> The next two examples show that a  $\frac{1}{2}$ -pairwise stable network may fail to exist while a pairwise stable network exists. In the first example, none of the two pairwise stable networks is  $\frac{1}{2}$ -pairwise stable, because there exists a network at mid distance from which there are improving paths going to both pairwise stable networks. In the second example, the unique pairwise stable is not  $\frac{1}{2}$ -pairwise stable because improving paths are enclosed in a cycle.

**Example 2.** Consider a situation where four players can form links. The payoffs they obtained from the different network configurations are (see Figure 2):  $Y_i(g) = [\#(g)]^2 - c \cdot \#\{j \in N \text{ such that } ij \in g\}$  if  $i \in N(g)$ ,  $Y_i(g) = 0$  if  $i \notin N(g)$ , (and so,  $Y_i(g) = 0$  if  $g$  is the empty network). The parameter  $c > 0$  is the individual cost of forming a link. For  $c < 11$  the complete network is pairwise stable, for  $c > 1$  the empty network is pairwise stable. For  $c < 5$  our refinement will select the complete network which is the unique  $\frac{1}{2}$ -pairwise stable network. For  $c > 7$  the empty network is the unique  $\frac{1}{2}$ -pairwise stable network. But, if  $5 < c < 7$  then a  $\frac{1}{2}$ -pairwise stable network fails to exist. The reason is that at  $g' = \{12, 13, 34\}$  players 2 and 4 have incentives to form the link 24 but at the same time players 1 or 3 has an incentive to sever the link he has with 2 or 4. So, from  $g'$  some improving paths go to the empty network, while others go to the complete network. It follows that no  $\frac{1}{2}$ -pairwise stable network exists.

**Example 3.** Suppose that five players can form links. In the complete network,  $Y_i(g) = 8$  for all  $i$ . In any network  $g$  players  $i \notin N(g)$  have a payoff  $Y_i(g) = 0$ . In networks  $g$  such that  $\#(g) \in [3, 9]$ , we have  $Y_i(g) = 9 - \#(g)$  if  $i \in N(g)$ . In any  $g$  such that  $\#(g) = 1$  or 2 and players 4 or 5 belong to  $N(g)$  then  $Y_i(g) = 0$  for all  $i$ . In any  $g$  such that  $\#(g) = 2$  and players 4 and 5 do not belong to  $N(g)$ , we have that  $Y_i(g) = 7$  for  $i \in N(g)$ .

<sup>8</sup>Jackson and van den Nouweland (2004) have introduced the notion of strongly stable networks. A strongly stable network is a network which is stable against changes in links by any coalition of individuals. Strongly stable networks are Pareto efficient and maximize the overall value of the network if the value of each component of a network is allocated equally among the members of that component.

<sup>9</sup>Note that in all examples of the paper, we will choose the number of players  $N$  so that  $\#g^N = \frac{N(N-1)}{2}$  is even. This will allow us to have  $\phi(\frac{1}{2}) = \frac{1}{2}$ .



Finally, let  $Y_1(\{12\}) = Y_3(\{13\}) = Y_2(\{23\}) = 6$ ,  $Y_2(\{12\}) = Y_1(\{13\}) = Y_3(\{23\}) = 8$ . Figure 3 presents some of these network configurations. In this example there is a unique pairwise stable network, the complete network. But, there does not exist a  $\frac{1}{2}$ -pairwise stable network. Indeed, from any  $g'$  such that  $d(g', g^N) \geq \frac{1}{5}$ , no improving path goes to  $g^N$ .

Thus, a  $\frac{1}{2}$ -pairwise stable network does not always exist. In the spirit of Tercieux (2004) we aim to solve the problem of non-existence of  $\frac{1}{2}$ -pairwise stable networks by providing a set-valued extension. Interestingly, such an approach will put into relief that a set of networks that are not pairwise stable can be more "stable" than a pairwise stable network.

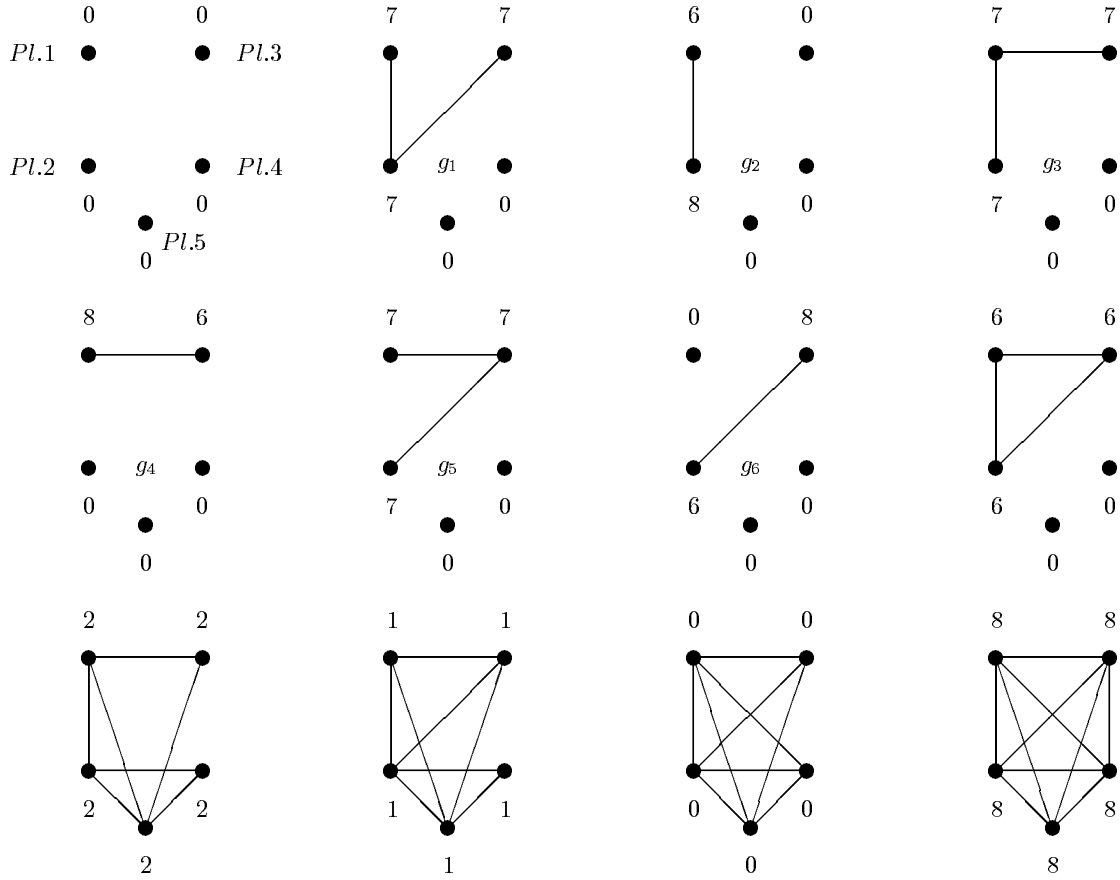


Figure 3: Another example of non-existence of  $\frac{1}{2}$ -stable networks (Example 3).

### 3 $p$ -Pairwise Stable Sets of Networks

Let us first restate the definition of an improving path. An improving path from a network  $g$  to a set of networks  $G \subseteq G^N$  is a finite sequence of graphs  $g_1, \dots, g_K$  with  $g_1 = g$  and  $g_K \in G$  such that for any  $k \in \{1, \dots, K-1\}$  either:

- (i)  $g_{k+1} = g_k - ij$  for some  $ij$  such that  $Y_i(g_k - ij) > Y_i(g_k)$ , or
- (ii)  $g_{k+1} = g_k + ij$  for some  $ij$  such that  $Y_i(g_k + ij) > Y_i(g_k)$  and  $Y_j(g_k + ij) \geq Y_j(g_k)$ .

The length of an improving path is  $K-1$ ,  $K \geq 2$ . An improving path from  $g'$  to  $G \subseteq G^N$  is of maximum length if  $g_K$  is not defeated by any  $g'' \notin G$ . For  $g' \notin G$ , we write  $g' \mapsto G$  if:

- (i) all improving paths of maximum length from  $g'$  go to  $G$ ,

- (ii) for any infinite improving path from  $g'$ , there exists  $K$  such that for all  $k \geq K$ ,  $g_k \in G$ .

We write  $g \mapsto G$  if from  $g \in G$  there is no improving path going to  $g' \notin G$ . For a given set of network  $G$ , let  $IM(G) = \{g' \subseteq g^N \mid g' \mapsto G\}$ . Note that, in the following, for  $G \subseteq G^N$ , and  $g' \subseteq g^N$  we will note  $d(g', G) \leq 1 - \phi(p)$  if  $d(g', g) \leq 1 - \phi(p)$  for some  $g \in G$ .

**Definition 3** Let  $p \in [0, 1]$ . A set of networks  $G \subseteq G^N$  is  $p$ -pairwise stable with respect to allocation rule  $Y$  and value function  $v$  if

- (1) for all  $g' \subseteq g^N$  such that  $d(g', G) \leq 1 - \phi(p)$ , we have  $g' \in IM(G)$ ,  
(2) there does not exist  $G' \subsetneq G$  such that  $G'$  satisfies (1).

**Remark 1** The set  $G^N$  (trivially) satisfies (1) in Definition 3 for any  $p \in [0, 1]$ .

**Proposition 2** Let  $p \in [0, 1]$ . Two (distinct)  $p$ -pairwise stable set of networks must be disjoint.

**Proof.** We proceed by contradiction. Assume that  $G$  and  $G'$  are two (distinct)  $p$ -pairwise stable sets of networks and  $G \cap G' \neq \emptyset$ . Then, for all  $g' \subseteq g^N$  such that  $d(g', G \cap G') \leq 1 - \phi(p)$ , we have  $g' \in IM(G)$ . But since this assertion is also true for  $G'$ , we have that for all  $g' \subseteq g^N$  such that  $d(g', G \cap G') \leq 1 - \phi(p)$ ,  $g' \in IM(G \cap G')$ . Thus  $G \cap G'$  satisfies (1) in Definition 3, contradicting the fact that  $G$  (and  $G'$ ) are  $p$ -pairwise stable sets, i.e. the minimality is violated (point (2) in Definition 3 of  $p$ -pairwise stable sets). ■

As underlined earlier, the main drawback of our definition of  $\frac{1}{2}$ -pairwise stable networks is that existence may fail also when a pairwise stable network exists. We now show that our set-valued notion of  $\frac{1}{2}$ -pairwise stable set always exists. As will become clear (for instance through Example 3), when there does not exist any  $\frac{1}{2}$ -pairwise stable network, our notion allows to eliminate many possibilities. Moreover, it is possible that the  $\frac{1}{2}$ -pairwise stable set of networks does not contain any pairwise stable network (see Example 3). We claim that this last point is important and underlines an important drawback of pairwise stability. The selection result we will introduce in the next section will give a foundation to this informal argument since we will prove that any network outside the  $\frac{1}{2}$ -pairwise stable set is not robust in a precise sense.

**Proposition 3** *Let  $p \in [0, 1]$ . There always exists at least one  $p$ -pairwise stable set of networks.*

**Proof.** Let us proceed by contradiction. Let  $p \in [0, 1]$  and assume that there does not exist any set of networks  $G \subseteq G^N$  that is  $p$ -pairwise stable. This means that for any  $G^0 \subseteq G^N$  that satisfies (1) in Definition 3 (there always exist such a  $G^0$ , see Remark 1), we can find a proper subset  $G^1$  that satisfies (1). But again for  $G^1$ , we can find a proper subset  $G^2$  that satisfies (1). Iterating the reasoning we can build an infinite (decreasing) sequence  $\{G^k\}_{k \geq 0}$  of **distinct** elements of  $G^N$  satisfying (1). But since  $\#G^N < \infty$ , this is not possible; so the proof is completed. ■

Note first that if  $g$  is a  $\frac{1}{2}$ -pairwise stable network then  $\{g\}$  is a  $\frac{1}{2}$ -pairwise stable set of networks. What our next result shows in particular is that  $\{g\}$  is the only  $\frac{1}{2}$ -pairwise stable set of networks and thus the two notions coincide in that special case.

**Proposition 4** *Let  $p \leq \frac{1}{2}$ . There always exists a unique  $p$ -pairwise stable set of networks.*

**Proof.** We proceed by contradiction. Assume that  $G^1$  and  $G^2$  are two distinct  $p$ -pairwise stable networks where  $p \leq \frac{1}{2}$ . Then, they satisfy (1) in Definition 3 for  $p = \frac{1}{2}$ .

If  $d(G^1, G^2) \leq 1 - \phi(\frac{1}{2})$ . Then we have a straightforward contradiction. (Since from  $g \in G^1$  we must have  $g \in IM(G^1)$ , i.e.  $g \mapsto G^1$  and  $g \in IM(G^2)$ , i.e.  $g \mapsto G^2$  which is not possible since  $G^1 \cap G^2 = \emptyset$ .)

If  $d(G^1, G^2) > 1 - \phi(\frac{1}{2})$ , we take  $g^1 \in G^1$  and  $g^2 \in G^2$ . Then, pick  $g^1$  and delete elements in  $\{(ij \in g^1 \wedge ij \notin g^2)\}$  and add elements in  $\{(ij \notin g^1 \wedge ij \in g^2)\}$  so that we obtain a network  $g'$  satisfying  $d(g', G^1) = 1 - \phi(\frac{1}{2})$ . By construction, this network  $g'$  satisfies  $d(g', G^2) \leq \phi(\frac{1}{2}) \leq 1 - \phi(\frac{1}{2})$ . Then, since  $G^1$  and  $G^2$  are  $p$ -pairwise stable for  $p \leq \frac{1}{2}$  (i.e. they both satisfy (1) in Definition 3 for  $p = \frac{1}{2}$ ), we have that  $g' \in IM(G^1)$ , i.e.  $g' \mapsto G^1$  and  $g' \in IM(G^2)$ , i.e.  $g' \mapsto G^2$  which again is not possible since  $G^1 \cap G^2 = \emptyset$ . ■

In Example 2 we have that, for  $5 < c < 7$ , there does not exist a  $\frac{1}{2}$ -pairwise stable network, but the set formed by the complete and empty networks is the  $\frac{1}{2}$ -pairwise stable set of networks. In Example 3, the complete network is the unique pairwise stable network and there is no  $\frac{1}{2}$ -pairwise stable network. However, the  $\frac{1}{2}$ -pairwise stable set of networks is  $G' = \{g_1, g_2, g_3, g_4, g_5, g_6\}$  (see Figure 3), which does not include the complete network, because there is a cycle  $g_1 \rightarrow g_2 \rightarrow g_3 \rightarrow g_4 \rightarrow g_5 \rightarrow g_6 \rightarrow g_1$  and all "undefeated" improving paths from any  $g'$  such that  $d(g', G') \leq \frac{1}{2}$  go to  $G'$  and stay in  $G'$ . By an

"undefeated" improving path, we mean that the final network in the sequence of the improving path is not defeated by a network that does not belong to  $G'$ .

Our set-valued notion generalizes many existing concepts of the literature. We can easily link this to two definitions, the first one is the well-known definition of pairwise stable networks of Jackson and Wolinsky (1996). The second one is the one of closed cycle provided by Jackson and Watts (2002). The following straightforward proposition is stated without proof.

**Proposition 5**  *$\{g\}$  is a  $p$ -pairwise stable set if and only if  $g$  is a  $p$ -pairwise stable network. And so,  $\{g\}$  is a  $1$ -pairwise stable set if and only if it is a pairwise stable network.*

The following definition is due to Jackson and Watts (2002, p.273). A set of networks  $G$ , form a *cycle* if for any  $g \in G$  and  $g' \in G$ , there exists an improving path connecting  $g$  to  $g'$ . A cycle  $G$  is a *closed cycle* if no network in  $G$  lies on an improving path leading to a network that is not in  $G$ .

**Proposition 6**  *$G$  is a  $1$ -pairwise stable set if and only if it is a closed cycle.*

**Proof.** The proof can be found in Appendix A. ■

## 4 Evolutionary Selection

In this section, we show that our notion of  $\frac{1}{2}$ -pairwise stable networks (and  $\frac{1}{2}$ -pairwise stable set of networks) is relevant in the stochastic evolutionary process proposed by Jackson and Watts (2002).

### 4.1 The Process

Let us recall first the Jackson and Watts (2002)'s process. At a discrete set of times,  $\{1, 2, 3, \dots\}$  decisions to add or sever a link are made. At each date, a pair of players  $ij$  is randomly identified with probability  $p(ij) > 0$ . The (potential) link between these two players is the only link that can be altered at that time. If the link is already in the network, then the decision is whether to sever it, and otherwise the decision is whether to add the link. The players involved act myopically, adding the link if it makes each at least as well off and one strictly better off, and severing the link if its deletion makes either player better off. After the action is taken, there is some small probability  $\varepsilon > 0$  that a

mutation (or tremble, or mistake) occurs and the link is deleted if it is present, and added if it is absent.<sup>10</sup>

The above process defines a (finite) Markov chain with states being the network in place at the end of a given period. Note that with mutations as part of the process, each state of the system is reachable with positive probability from every other state. The Markov chain is said to be irreducible and aperiodic, and thus has a unique corresponding stationary distribution (see Freidlin and Wentzel, 1984). As  $\varepsilon$  goes to zero, the stationary distribution converges to a unique limiting stationary distribution. A network that is in the support of the limiting (as  $\varepsilon$  goes to zero) stationary distribution of the above-described Markov process is said to be *stochastically stable*. Intuitively, a stochastically stable network is one that is observed infinitely many more times than others when the probability of mutations is infinitely small. Jackson and Watts (2002) provides a characterization of stochastically stable networks using the tree construction of Freidlin and Wentzell (1984). In the following, we prove that our concept can be used to avoid this complex construction.

## 4.2 Relationship between $p$ -Pairwise Stability and Stochastic Stability

The following theorem shows that under the process we have just described, the only networks that will arise with a significant frequency in the long run (i.e., the stochastically stable one) are in the  $\frac{1}{2}$ -pairwise stable set.

**Theorem 1** *Let  $G$  be the  $\frac{1}{2}$ -pairwise stable set of networks. The set of stochastically stable networks is included in  $G$ .*

**Proof.** See Appendix B. ■

Thus any network outside the  $\frac{1}{2}$ -pairwise stable set must be considered as a non-robust network. To be more precise, the stochastic process presented above can be thought of as a check on the robustness of pairwise networks or cycles. Although a number of networks may be pairwise stable, they can differ in how resilient they are to the random mutations. For instance, it may be relatively hard to leave and easy to get back to some networks, our above theorem tells us that such networks are included in the  $\frac{1}{2}$ -pairwise stable set of networks. This result also tells us that any network that is not in the  $\frac{1}{2}$ -pairwise stable set is relatively easy to leave and hard to get back.

---

<sup>10</sup>Mutations may be due to exogenous unmodeled factors that are beyond player's control. Alternatively, players may simply make errors in calculating whether adding or severing a link is beneficial. Finally, we could think to players having a limited information. Thus they occasionally experiment to see if adding or severing a link will make them better off (endogenous mutations have been formalized in several papers, see for instance van Damme and Weibull (2002) or Maruta (2002)).



In order to understand these points, note that once the process has reached the  $\frac{1}{2}$ -pairwise stable set of networks  $G$ , it cannot leave it without further mutations. On the first hand, in order to get off that set, it is necessary that strictly more than  $\frac{\#g^N}{2}$  mutations occur (notice that in order to give the intuition of our result, we skip some technical points in assuming that  $N$  is such that  $\frac{\#g^N}{2}$  is an integer). If it is not the case, the process will come back to  $G$  with no further mutation. On the other hand, as it will become clear, if the process has reached a network that is outside  $G$ , it is sufficient that less than  $\frac{\#g^N}{2}$  mutations occur to allow the process to reach a network that belong to  $G$ . In order to see why it is so, note that from a network  $g'$  that does not belong to  $G$ , with (less than)  $\frac{\#g^N}{2}$  mutations, one can reach a network  $\bar{g}$  such that  $d(g, \bar{g}) \leq \frac{1}{2}$  where  $g$  belongs to  $G$ . Thus, by definition, the process will move to  $G$  without any further mutations. To see how we can build  $\bar{g}$ , we just have to add links to  $g'$  that belong to  $g$  and not to  $g'$  or to delete links that do not belong to  $g$  but belong to  $g'$ . By repeating this procedure less than  $\frac{\#g^N}{2}$  times, we can reach such a  $\bar{g}$ . Thus there exist networks in  $G$  which are the easiest to reach from other networks, where - again - "easiest" is interpreted as requiring the fewest mutations. These networks are stochastically stable. The formal argument is given in the appendix.

Of course, we would like to have a full characterization of the set of stochastically stable networks. In order to do so, we provide several sufficient conditions that go in that sense. These results are corollaries of Theorem 1. The first one shows that if there exists a  $\frac{1}{2}$ -pairwise stable network then it must be the unique stochastically stable network. Note that this result can be seen as a parallel to the one of Young (1993) [Theorem 3, p.72] in noncooperative games.

**Corollary 1** *Assume that a network  $g$  is the  $\frac{1}{2}$ -pairwise stable network. Then  $g$  is the unique stochastically stable network.*

The following two corollaries directly come from the fact that if  $g$  is stochastically stable then  $g$  is part of a 1-pairwise stable set of networks. Furthermore, if  $g \in G$  is stochastically stable and  $G$  is a 1-pairwise stable set then all  $g' \in G$  are stochastically stable (this follows from Lemma 2 in Jackson and Watts (2002) together with our Proposition 6 that establishes the equivalence between a 1-pairwise stable set and a closed cycle).

**Corollary 2** *Let  $G$  be the  $\frac{1}{2}$ -pairwise stable set of networks. If  $G$  is 1-pairwise stable then  $G$  is the set of stochastically stable networks.*

**Corollary 3** *Let  $G$  be the  $\frac{1}{2}$ -pairwise stable set of networks. If  $G' \subseteq G$  is the unique 1-pairwise stable set in  $G$  then  $G'$  is the set of stochastically stable networks.*

**Example 4.** Suppose that three players can form links (see Figure 4). In the complete network,  $Y_i(g) = 3$  for all  $i$ . In any network  $g$  players  $i \notin N(g)$  have a payoff  $Y_i(g) = 0$ . In any  $g$  such that  $\#(g) = 2$ ,  $Y_i(g) = 2$  if  $i \in N(g)$ . Finally, let  $Y_1(\{12\}) = Y_3(\{13\}) = Y_2(\{23\}) = 1$ ,  $Y_2(\{12\}) = Y_1(\{13\}) = Y_3(\{23\}) = 4$ . In this example there is a unique pairwise stable network, the complete network. There does not exist a  $\frac{1}{2}$ -pairwise stable network,  $\{g^N\}$  is the 1-pairwise stable set, and all networks except the empty one belong to the  $\frac{1}{2}$ -pairwise stable set of networks.

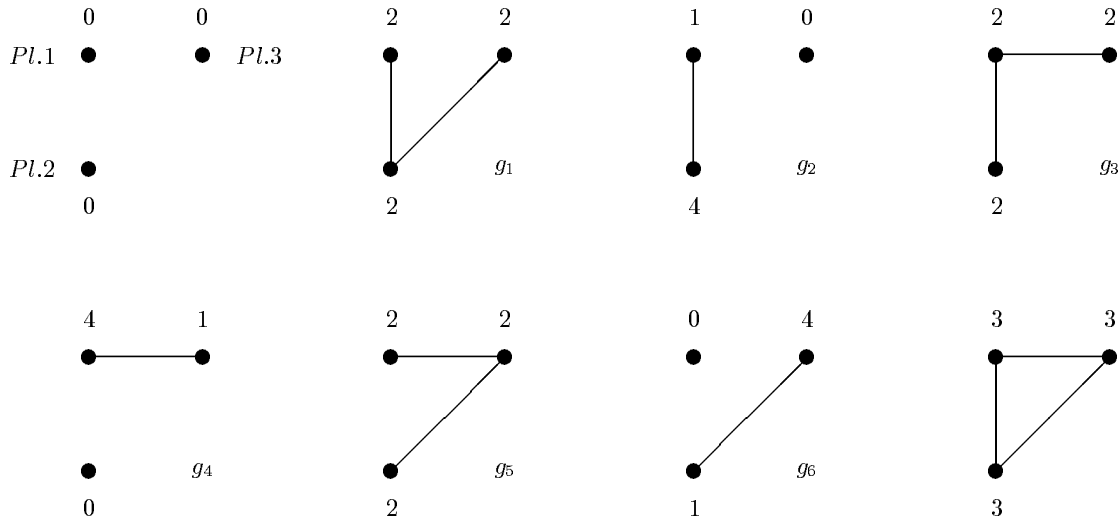


Figure 4:  $\frac{1}{2}$ -pairwise stable set and stochastically stable networks (Example 4).

In Example 4, the complete network is the unique pairwise stable network and there is no  $\frac{1}{2}$ -pairwise stable network because of the cycle  $g_1 \rightarrow g_2 \rightarrow g_3 \rightarrow g_4 \rightarrow g_5 \rightarrow g_6 \rightarrow g_1$ . The  $\frac{1}{2}$ -pairwise stable set of networks is  $G' = \{g_1, g_2, g_3, g_4, g_5, g_6, g^N\}$  but this set is not 1-pairwise stable. Indeed,  $\{g^N\}$  is the unique 1-pairwise stable set and so by corollary 3 is the unique stochastically stable network.

The next example shows that our sufficient conditions are quite tight in the following sense: a  $p$ -pairwise stable network with  $p = \frac{1}{2} + \varepsilon$  ( $\varepsilon$  small) may not be a stochastically stable network.

**Example 5.** Suppose that fifty players can form links. For  $\#(g) \leq 611$ , let  $Y_i(g) = 611 - \#(g)$  if  $i \in N(g)$  and  $Y_i(g) = 0$  otherwise. For  $\#(g) \geq 612$ , let  $Y_i(g) = \#(g) - 611$  if  $i \in N(g)$  and  $Y_i(g) = 0$  otherwise. The empty network is a  $p$ -pairwise stable network for  $p \geq (615/1225) \simeq 0.502$ , but the empty network is not stochastically stable. The unique stochastically stable network is the complete one, which is also the unique  $\frac{1}{2}$ -pairwise

stable network.

## 5 Conclusion

In this paper, we have defined a refinement of pairwise stability:  $p$ -pairwise stability. When a  $\frac{1}{2}$ -pairwise stable network exists, we have shown that it is unique and that it coincides with the unique stochastically stable network. To solve the inexistence problem of  $p$ -pairwise stable networks, we have defined its set-valued extension with the notion of  $p$ -pairwise stable set. We have shown that  $\frac{1}{2}$ -pairwise stable set exists and is unique. In addition, any stochastically stable networks is included in the  $\frac{1}{2}$ -pairwise stable set.

## Appendix

### A Proof of Proposition 6

In this part we prove Proposition 6 that establishes the equivalence between our notion of 1-pairwise stability and the notion of a closed cycle proposed by Jackson and Watts (2002). In order to do so, we first state and prove some useful lemmas. The following lemma is stated without proof.<sup>11</sup>

**Lemma 1** *If  $G$  is such that for all  $g \in G$ ,  $g \in IM(G)$  (note that this is (1) in Definition 3 of a 1-pairwise stable set) then there exists  $C \subseteq G$  that is a closed cycle.*

Our next lemma provides a first step in establishing a link between 1-pairwise stability and closed cycles.

**Lemma 2** *If  $C$  is a closed cycle then there exists  $G \subseteq C$  that is 1-pairwise stable.*

**Proof.** Since  $C$  is a closed cycle, we know that for all  $g \in C$ ,  $g \in IM(C)$ . Then  $C$  satisfies (1) of Definition 3 of a 1-pairwise stable set. Now assume that there does not exist any  $G \subseteq C$  that is 1-pairwise stable. Then any  $G \subseteq C$  has a proper subset that satisfies (1) in the definition of 1-pairwise stable sets. Now, as in the proof of Proposition 3, this implies that there exists an infinite decreasing sequence  $\{G^k\}_{k \geq 0}$  where  $G^0 = C$  and  $G^{k+1} \subsetneq G^k$  for all  $k \geq 0$ . But since  $\#G^N < \infty$ , this is not possible; so the proof is completed. ■

Now we are ready to complete the proof of Proposition 6. We first prove the "if" part. Suppose that  $G$  is a closed cycle but  $G$  is not 1-pairwise stable and show that

---

<sup>11</sup>A complete proof would mimic the proof of Lemma 1 in Jackson and Watts (2002, p.273).

this lead to a contradiction. This last point can be due to the violation of **(1)** or **(2)** in the definition of a 1-pairwise stable set. Assume first that **(1)** is violated. Such a violation implies in particular that there exists  $g \in G$  and  $g' \notin G$  such that  $g \rightarrow g'$ . Which contradicts the definition of a closed cycle. Assume now that **(2)** is violated. This means that there exists  $G' \subsetneq G$  that satisfies **(1)** in the definition of a 1-pairwise stable set i.e., for all  $g' \in G'$ ,  $g' \in IM(G')$ . But by Lemma 1, we know that there exists a closed cycle  $C \subseteq G' \subsetneq G$ . Then, we have the following: first, because  $G$  is a (closed) cycle, we have that for all  $g, g' \in G$ ,  $g \rightarrow g'$ . But we also have, because  $C$  is a closed cycle, that for all  $g \in C(\subsetneq G)$  and  $g' \in G - C$ ,  $g \rightarrow g'$  is wrong. Thus we obtain a contradiction.

We now prove the **"only if" part**. We know by Lemma 1 that since  $G$  is 1-pairwise stable, there exists  $C \subseteq G$  that is a closed cycle. We must prove that  $C = G$ . So let us proceed by contradiction and assume that  $C \subsetneq G$ . We know by Lemma 2 that there exists  $G' \subseteq C \subsetneq G$  that is 1-pairwise stable. This leads to a straightforward contradiction since it contradicts **(2)** (the minimality) in the 1-pairwise stability of  $G$ . This completes the proof of Proposition 6.

## B Proof of Theorem 1

In order to prove Theorem 1, we first introduce some useful definitions and notations.

### B.1 Definitions

For a given network  $g$ , remember that  $im(g) = \{g' \subseteq g^N \mid \text{there exists an improving path from } g' \text{ to } g\}$ . A path  $\mathbf{p} = \{g_1, \dots, g_K\}$  is a sequence of adjacent networks. The resistance of a path  $\mathbf{p} = \{g_1, \dots, g_K\}$  from  $g'$  to  $g$ , denoted  $r(\mathbf{p})$ , is computed by  $r(\mathbf{p}) = \sum_{i=1}^{K-1} I(g_i, g_{i+1})$ , where

$$I(g_i, g_{i+1}) = \begin{cases} 0 & \text{if } g_i \in im(g_{i+1}) \\ 1 & \text{otherwise} \end{cases}.$$

Resistance keeps track of how many mutations must occur along a special path to follow that path from one network to another. A mutation is necessary to move from one network to an adjacent one whenever it is not in the relevant player's interests to sever or add the link that distinguishes the two adjacent networks.

Let  $r(g', g) = \min\{r(\mathbf{p}) \mid \mathbf{p} \text{ is a path from } g' \text{ to } g\}$  and set  $r(g, g) = 0$ . Note that  $r(g', g) = 0$  iff  $g' \in im(g)$  or  $g' = g$ . Thus (by proposition 6) if  $g, g' \in G$  where  $G$  is 1-pairwise stable, then  $r(g', g) = 0$ .

Given a network  $g$ , a  $g$ -tree is a directed graph which has as vertices all networks and has a unique directed path leading from each  $g'$  to  $g$ . Let  $T(g)$  denote all the  $g$ -trees, and

represent a  $t \in T(g)$  as a collection of ordered pairs of networks, so that  $g'g'' \in t$  if and only if there is a directed edge connecting  $g'$  to  $g''$  in the  $g$ -tree  $t$ .

The resistance of a network  $g$  is computed as  $r(g) = \min_{t \in T(g)} \sum_{g'g'' \in t} r(g', g'')$ . The main result of Jackson and Watts (2002) that is closely related to the technics developed in Young (1993) is that the set of stochastically stable networks is the set  $\{g \mid r(g) \leq r(g') \text{ for all } g'\}$ . We will use this characterization in order to prove our main results.

## B.2 The Proof

The proof is divided into two parts: (1) we bound the resistances of paths that begin at  $g \in G$  and the resistances of paths ending at  $g \in G$ ; (2) we show that some  $g \in G$  minimizes stochastic potential.

(1) We give a lower bound on the resistance of the transitions that begin at  $g \in G$  and end at any  $g' \notin G$ . Recall that by definition of  $p$ -pairwise stability for  $p \leq \frac{1}{2}$ ,  $r(g, g') > (1 - \phi(\frac{1}{2})) \cdot \#g^N \geq \frac{\#g^N}{2}$ .

We give now an upper bound on the resistance of paths that begin at any  $g' \notin G$  and end in  $G$ . Pick  $g' \notin G$ . (Note that if  $d(g', G) \leq 1 - \phi(\frac{1}{2})$  then, by definition of  $G$ ,  $g' \in IM(G)$  i.e. no mutation is necessary to go to  $G$ . Thus we will implicitly assume that  $d(g', G) > 1 - \phi(\frac{1}{2})$ .) We delete elements in  $\{(ij \in g' \wedge ij \notin g)\}$  and add elements in  $\{(ij \notin g' \wedge ij \in g)\}$  so that we obtain a network  $\bar{g}$  satisfying  $d(\bar{g}, g') = 1 - \phi(\frac{1}{2})$ . By construction, this network  $\bar{g}$  satisfies  $d(\bar{g}, g) \leq \phi(\frac{1}{2}) \leq 1 - \phi(\frac{1}{2})$  where  $g \in G$ . But  $G$  is a  $p$ -pairwise stable set of networks for  $p \leq \frac{1}{2}$ ; therefore with less than  $(1 - \phi(\frac{1}{2})) \cdot \#g^N$  mutations, we will reach a network in  $G$  (note that once the process has reached  $G$ , we cannot leave it without mutations). Therefore,  $r(g', \bar{g}) \leq (1 - \phi(\frac{1}{2})) \cdot \#g^N$  for some  $\tilde{g} \in G$ . We will note such a  $\tilde{g}$  by  $\Psi(g')$ . Thus for every  $g' \notin G$ ,  $r(g', \Psi(g')) \leq (1 - \phi(\frac{1}{2})) \cdot \#g^N$ .

(2) Suppose by contradiction that  $g' \notin G$  is stochastically stable. Denote by  $t'$  (one of) the  $g'$ -tree(s) ( $t' \in T(g')$ ) that minimizes resistance.

We know that there is a sequence  $g_1, \dots, g_n$  with  $g_1 = \Psi(g') (\in G)$  and  $g_n = g'$  such that:

- $g_l g_{l+1} \in t'$  for every  $l = 1, \dots, n - 1$
- there is a  $k \in \{1, \dots, n - 1\}$  such that  $g_k \in G$  and  $g_{k+1} \notin G$ . Delete this edge and add one from  $g'$  to  $\Psi(g')$ . We obtain a tree  $t'' \in T(g_k)$  where  $g_k \in G$ .

By construction,  $r(g_k) = r(g') - r(g_k, g_{k+1}) + r(g', \Psi(g'))$ . But as proved above, we have  $r(g_k, g_{k+1}) > (1 - \phi(\frac{1}{2})) \cdot \#g^N \geq r(g', \Psi(g'))$ . Hence,  $r(g_k) < r(g')$ . This contradicts the fact that  $g'$  minimizes stochastic potential. This completes the proof.

## References

- [1] Aumann, R. and R. Myerson, "Endogenous Formation of Links between Players and Coalitions: an Application of the Shapley Value," in *The Shapley value*, A. Roth Ed., Cambridge University Press, 175-191 (1988).
- [2] Bala, V. and S. Goyal, "Self-Organization in Communication Networks," *Econometrica* 68, 1131-1230 (2000).
- [3] van Damme, E. and J. Weibull, "Evolution in Games with Endogenous Mistake Probabilities," *Journal of Economic Theory* 106, 296-315 (2002).
- [4] Durieu, J. Solal, P. and O. Tercieux, "Adaptive Learning and Curb Set Selection," Mimeo. (2003).
- [5] Dutta, B. and M.O. Jackson, "The Stability and Efficiency of Directed Communication Networks," *Review of Economic Design* 5, 251-272 (2000).
- [6] Ellison, G., "Learning, Local interaction, and Coordination," *Econometrica* 61, 1047-1071 (1993).
- [7] Freidlin, M. and A. Wentzell, *Random Perturbations of Dynamical Systems*. Springer-Verlag, New York (1984).
- [8] Garey, R.G. and D.S. Johnson, *Computers and Intractability. A Guide to the Theory of NP-Completeness*. W. H Freeman and Company (1979)
- [9] Harsanyi, J. and R. Selten, *A General Theory of Equilibrium in Games*, Cambridge: MIT Press (1988).
- [10] Jackson, M.O., "The Stability and Efficiency of Economic and Social Networks," in *Networks and Groups: Models of Strategic Formation*, edited by B. Dutta and M.O. Jackson, Springer-Verlag: Heidelberg (2003).
- [11] Jackson, M.O., "A Survey of Models of Network Formation: Stability and Efficiency," in *Group Formation in Economics: Networks, Clubs and Coalitions*, edited by G. Demange and M. Wooders, Cambridge University Press (2004).
- [12] Jackson, M.O. and A. van den Nouweland, "Strongly Stable Networks," forthcoming in *Games and Economic Behavior* (2004).
- [13] Jackson, M.O. and A. Watts, "The Evolution of Social and Economic Networks," *Journal of Economic Theory* 71, 44-74 (2002).

- [14] Jackson, M.O. and A. Wolinsky, "A Strategic Model of Social and Economic Networks," *Journal of Economic Theory* 71, 44-74 (1996).
- [15] Kandori, M., G. Mailath, and R. Rob "Learning, Mutation, and Long Run Equilibria in Games," *Econometrica*, 61, pp.29-56 (1993).
- [16] Maruta, T., "On the Relationship Between Risk-Dominance and Stochastic Stability," *Games and Economic Behavior* 19, 221-234 (1997).
- [17] Maruta, T., "Binary Games with State Dependent Stochastic Choice," *Journal of Economic Theory* 103, 351-376 (2002).
- [18] Tercieux, O, " $p$ -Best Response Set," forthcoming in *Journal of Economic Theory*.
- [19] Watts, A., "A Dynamic Model of Network Formation," *Games and Economic Behavior* 34, 331-341 (2001).
- [20] Young, P., "The Evolution of Conventions," *Econometrica* 61, 57-64 (1993).

## NOTE DI LAVORO DELLA FONDAZIONE ENI ENRICO MATTEI

### Fondazione Eni Enrico Mattei Working Paper Series

Our Note di Lavoro are available on the Internet at the following addresses:

<http://www.feem.it/Feem/Pub/Publications/WPapers/default.html>

<http://www.ssrn.com/link/feem.html>

<http://www.repec.org>

### NOTE DI LAVORO PUBLISHED IN 2004

IEM	1.2004	<i>Anil MARKANDYA, Suzette PEDROSO and Alexander GOLUB: <u>Empirical Analysis of National Income and So2 Emissions in Selected European Countries</u></i>
ETA	2.2004	<i>Masahisa FUJITA and Shlomo WEBER: <u>Strategic Immigration Policies and Welfare in Heterogeneous Countries</u></i>
PRA	3.2004	<i>Adolfo DI CARLUCCIO, Giovanni FERRI, Cecilia FRALE and Ottavio RICCHI: <u>Do Privatizations Boost Household Shareholding? Evidence from Italy</u></i>
ETA	4.2004	<i>Victor GINSBURGH and Shlomo WEBER: <u>Languages Disenfranchisement in the European Union</u></i>
ETA	5.2004	<i>Romano PIRAS: <u>Growth, Congestion of Public Goods, and Second-Best Optimal Policy</u></i>
CCMP	6.2004	<i>Herman R.J. VOLLEBERGH: <u>Lessons from the Polder: Is Dutch CO2-Taxation Optimal</u></i>
PRA	7.2004	<i>Sandro BRUSCO, Giuseppe LOPOMO and S. VISWANATHAN (lxv): <u>Merger Mechanisms</u></i>
PRA	8.2004	<i>Wolfgang AUSENNEGG, Pegaret PICHLER and Alex STOMPER (lxv): <u>IPO Pricing with Bookbuilding, and a When-Issued Market</u></i>
PRA	9.2004	<i>Pegaret PICHLER and Alex STOMPER (lxv): <u>Primary Market Design: Direct Mechanisms and Markets</u></i>
PRA	10.2004	<i>Florian ENGLMAIER, Pablo GUILLEN, Loreto LLORENTE, Sander ONDERSTAL and Rupert SAUSGRUBER (lxv): <u>The Chopstick Auction: A Study of the Exposure Problem in Multi-Unit Auctions</u></i>
PRA	11.2004	<i>Bjarne BRENDSTRUP and Harry J. PAARSCH (lxv): <u>Nonparametric Identification and Estimation of Multi-Unit, Sequential, Oral, Ascending-Price Auctions With Asymmetric Bidders</u></i>
PRA	12.2004	<i>Ohad KADAN (lxv): <u>Equilibrium in the Two Player, k-Double Auction with Affiliated Private Values</u></i>
PRA	13.2004	<i>Maarten C.W. JANSSEN (lxv): <u>Auctions as Coordination Devices</u></i>
PRA	14.2004	<i>Gadi FIBICH, Arieh GAVIOUS and Aner SELA (lxv): <u>All-Pay Auctions with Weakly Risk-Averse Buyers</u></i>
PRA	15.2004	<i>Orly SADE, Charles SCHNITZLEIN and Jaime F. ZENDER (lxv): <u>Competition and Cooperation in Divisible Good Auctions: An Experimental Examination</u></i>
PRA	16.2004	<i>Marta STRYSZOWSKA (lxv): <u>Late and Multiple Bidding in Competing Second Price Internet Auctions</u></i>
CCMP	17.2004	<i>Slim Ben YOUSSEF: <u>R&amp;D in Cleaner Technology and International Trade</u></i>
NRM	18.2004	<i>Angelo ANTOCI, Simone BORGHESI and Paolo RUSSU (lxvi): <u>Biodiversity and Economic Growth: Stabilization Versus Preservation of the Ecological Dynamics</u></i>
SIEV	19.2004	<i>Anna ALBERINI, Paolo ROSATO, Alberto LONGO and Valentina ZANATTA: <u>Information and Willingness to Pay in a Contingent Valuation Study: The Value of S. Erasmo in the Lagoon of Venice</u></i>
NRM	20.2004	<i>Guido CANDELA and Roberto CELLINI (lxvii): <u>Investment in Tourism Market: A Dynamic Model of Differentiated Oligopoly</u></i>
NRM	21.2004	<i>Jacqueline M. HAMILTON (lxvii): <u>Climate and the Destination Choice of German Tourists</u></i>
NRM	22.2004	<i>Javier Rey-MAQUIEIRA PALMER, Javier LOZANO IBÁÑEZ and Carlos Mario GÓMEZ GÓMEZ (lxvii): <u>Land, Environmental Externalities and Tourism Development</u></i>
NRM	23.2004	<i>Pius ODUNGA and Henk FOLMER (lxvii): <u>Profiling Tourists for Balanced Utilization of Tourism-Based Resources in Kenya</u></i>
NRM	24.2004	<i>Jean-Jacques NOWAK, Mondher SAHLI and Pasquale M. SGRO (lxvii): <u>Tourism, Trade and Domestic Welfare</u></i>
NRM	25.2004	<i>Riaz SHAREEF (lxvii): <u>Country Risk Ratings of Small Island Tourism Economies</u></i>
NRM	26.2004	<i>Juan Luis EUGENIO-MARTÍN, Noelia MARTÍN MORALES and Riccardo SCARPA (lxvii): <u>Tourism and Economic Growth in Latin American Countries: A Panel Data Approach</u></i>
NRM	27.2004	<i>Raúl Hernández MARTÍN (lxvii): <u>Impact of Tourism Consumption on GDP. The Role of Imports</u></i>
CSRM	28.2004	<i>Nicoletta FERRO: <u>Cross-Country Ethical Dilemmas in Business: A Descriptive Framework</u></i>
NRM	29.2004	<i>Marian WEBER (lxvi): <u>Assessing the Effectiveness of Tradable Landuse Rights for Biodiversity Conservation: an Application to Canada's Boreal Mixedwood Forest</u></i>
NRM	30.2004	<i>Trond BJORN DAL, Phoebe KOUNDOURI and Sean PASCOE (lxvi): <u>Output Substitution in Multi-Species Trawl Fisheries: Implications for Quota Setting</u></i>
CCMP	31.2004	<i>Marzio GALEOTTI, Alessandra GORIA, Paolo MOMBRINI and Evi SPANTIDAKI: <u>Weather Impacts on Natural, Social and Economic Systems (WISE) Part I: Sectoral Analysis of Climate Impacts in Italy</u></i>
CCMP	32.2004	<i>Marzio GALEOTTI, Alessandra GORIA, Paolo MOMBRINI and Evi SPANTIDAKI: <u>Weather Impacts on Natural, Social and Economic Systems (WISE) Part II: Individual Perception of Climate Extremes in Italy</u></i>
CTN	33.2004	<i>Wilson PEREZ: <u>Divide and Conquer: Noisy Communication in Networks, Power, and Wealth Distribution</u></i>
KTHC	34.2004	<i>Gianmarco I.P. OTTAVIANO and Giovanni PERI (lxviii): <u>The Economic Value of Cultural Diversity: Evidence from US Cities</u></i>
KTHC	35.2004	<i>Linda CHAIB (lxviii): <u>Immigration and Local Urban Participatory Democracy: A Boston-Paris Comparison</u></i>



KTHC	36.2004	<i>Franca ECKERT COEN and Claudio ROSSI</i> (Ixviii): <u>Foreigners, Immigrants, Host Cities: The Policies of Multi-Ethnicity in Rome. Reading Governance in a Local Context</u>
KTHC	37.2004	<i>Kristine CRANE</i> (Ixviii): <u>Governing Migration: Immigrant Groups' Strategies in Three Italian Cities – Rome, Naples and Bari</u>
KTHC	38.2004	<i>Kiflemariam HAMDE</i> (Ixviii): <u>Mind in Africa, Body in Europe: The Struggle for Maintaining and Transforming Cultural Identity - A Note from the Experience of Eritrean Immigrants in Stockholm</u>
ETA	39.2004	<i>Alberto CAVALIERE</i> : <u>Price Competition with Information Disparities in a Vertically Differentiated Duopoly</u>
PRA	40.2004	<i>Andrea BIGANO and Stef PROOST</i> : <u>The Opening of the European Electricity Market and Environmental Policy: Does the Degree of Competition Matter?</u>
CCMP	41.2004	<i>Micheal FINUS</i> (Ixix): <u>International Cooperation to Resolve International Pollution Problems</u>
KTHC	42.2004	<i>Francesco CRESPI</i> : <u>Notes on the Determinants of Innovation: A Multi-Perspective Analysis</u>
CTN	43.2004	<i>Sergio CURRARINI and Marco MARINI</i> : <u>Coalition Formation in Games without Synergies</u>
CTN	44.2004	<i>Marc ESCRHUELA-VILLAR</i> : <u>Cartel Sustainability and Cartel Stability</u>
NRM	45.2004	<i>Sebastian BERVOETS and Nicolas GRAVEL</i> (Ixvi): <u>Appraising Diversity with an Ordinal Notion of Similarity: An Axiomatic Approach</u>
NRM	46.2004	<i>Signe ANTHON and Bo JELLES MARK THORSEN</i> (Ixvi): <u>Optimal Afforestation Contracts with Asymmetric Information on Private Environmental Benefits</u>
NRM	47.2004	<i>John MBURU</i> (Ixvi): <u>Wildlife Conservation and Management in Kenya: Towards a Co-management Approach</u>
NRM	48.2004	<i>Ekin BIROL, Ágnes GYOVAI and Melinda SMALE</i> (Ixvi): <u>Using a Choice Experiment to Value Agricultural Biodiversity on Hungarian Small Farms: Agri-Environmental Policies in a Transition al Economy</u>
CCMP	49.2004	<i>Gernot KLEPPER and Sonja PETERSON</i> : <u>The EU Emissions Trading Scheme. Allowance Prices, Trade Flows, Competitiveness Effects</u>
GG	50.2004	<i>Scott BARRETT and Michael HOEL</i> : <u>Optimal Disease Eradication</u>
CTN	51.2004	<i>Dinko DIMITROV, Peter BORM, Ruud HENDRICKX and Shao CHIN SUNG</i> : <u>Simple Priorities and Core Stability in Hedonic Games</u>
SIEV	52.2004	<i>Francesco RICCI</i> : <u>Channels of Transmission of Environmental Policy to Economic Growth: A Survey of the Theory</u>
SIEV	53.2004	<i>Anna ALBERINI, Maureen CROPPER, Alan KRUPNICK and Nathalie B. SIMON</i> : <u>Willingness to Pay for Mortality Risk Reductions: Does Latency Matter?</u>
NRM	54.2004	<i>Ingo BRÄUER and Rainer MARGGRAF</i> (Ixvi): <u>Valuation of Ecosystem Services Provided by Biodiversity Conservation: An Integrated Hydrological and Economic Model to Value the Enhanced Nitrogen Retention in Renaturated Streams</u>
NRM	55.2004	<i>Timo GOESCHL and Tun LIN</i> (Ixvi): <u>Biodiversity Conservation on Private Lands: Information Problems and Regulatory Choices</u>
NRM	56.2004	<i>Tom DEDEURWAERDERE</i> (Ixvi): <u>Bioprospection: From the Economics of Contracts to Reflexive Governance</u>
CCMP	57.2004	<i>Katrin REHDANZ and David MADDISON</i> : <u>The Amenity Value of Climate to German Households</u>
CCMP	58.2004	<i>Koen SMEKENS and Bob VAN DER ZWAAN</i> : <u>Environmental Externalities of Geological Carbon Sequestration Effects on Energy Scenarios</u>
NRM	59.2004	<i>Valentina BOSETTI, Mariaester CASSINELLI and Alessandro LANZA</i> (Ixvii): <u>Using Data Envelopment Analysis to Evaluate Environmentally Conscious Tourism Management</u>
NRM	60.2004	<i>Timo GOESCHL and Danilo CAMARGO IGLIORI</i> (Ixvi): <u>Property Rights Conservation and Development: An Analysis of Extractive Reserves in the Brazilian Amazon</u>
CCMP	61.2004	<i>Barbara BUCHNER and Carlo CARRARO</i> : <u>Economic and Environmental Effectiveness of a Technology-based Climate Protocol</u>
NRM	62.2004	<i>Elissaios POPYRAKIS and Reyer GERLAGH</i> : <u>Resource-Abundance and Economic Growth in the U.S.</u>
NRM	63.2004	<i>Györgyi BELA, György PATAKI, Melinda SMALE and Mariann HAJDÚ</i> (Ixvi): <u>Conserving Crop Genetic Resources on Smallholder Farms in Hungary: Institutional Analysis</u>
NRM	64.2004	<i>E.C.M. RUIJGROK and E.E.M. NILLESEN</i> (Ixvi): <u>The Socio-Economic Value of Natural Riverbanks in the Netherlands</u>
NRM	65.2004	<i>E.C.M. RUIJGROK</i> (Ixvi): <u>Reducing Acidification: The Benefits of Increased Nature Quality. Investigating the Possibilities of the Contingent Valuation Method</u>
ETA	66.2004	<i>Giannis VARDAS and Anastasios XEPAPADEAS</i> : <u>Uncertainty Aversion, Robust Control and Asset Holdings</u>
GG	67.2004	<i>Anastasios XEPAPADEAS and Constadina PASSA</i> : <u>Participation in and Compliance with Public Voluntary Environmental Programs: An Evolutionary Approach</u>
GG	68.2004	<i>Michael FINUS</i> : <u>Modesty Pays: Sometimes!</u>
NRM	69.2004	<i>Trond BJØRNDAL and Ana BRASÃO</i> : <u>The Northern Atlantic Bluefin Tuna Fisheries: Management and Policy Implications</u>
CTN	70.2004	<i>Alejandro CAPARRÓS, Abdelhakim HAMMOUDI and Tarik TAZDAÏT</i> : <u>On Coalition Formation with Heterogeneous Agents</u>
IEM	71.2004	<i>Massimo GIOVANNINI, Margherita GRASSO, Alessandro LANZA and Matteo MANERA</i> : <u>Conditional Correlations in the Returns on Oil Companies Stock Prices and Their Determinants</u>
IEM	72.2004	<i>Alessandro LANZA, Matteo MANERA and Michael MCALEER</i> : <u>Modelling Dynamic Conditional Correlations in WTI Oil Forward and Futures Returns</u>
SIEV	73.2004	<i>Margarita GENIUS and Elisabetta STRAZZERA</i> : <u>The Copula Approach to Sample Selection Modelling: An Application to the Recreational Value of Forests</u>

CCMP	74.2004	<i>Rob DELLINK and Ekko van IERLAND</i> : <u>Pollution Abatement in the Netherlands: A Dynamic Applied General Equilibrium Assessment</u>
ETA	75.2004	<i>Rosella LEVAGGI and Michele MORETTO</i> : <u>Investment in Hospital Care Technology under Different Purchasing Rules: A Real Option Approach</u>
CTN	76.2004	<i>Salvador BARBERÀ and Matthew O. JACKSON</i> (lxx): <u>On the Weights of Nations: Assigning Voting Weights in a Heterogeneous Union</u>
CTN	77.2004	<i>Àlex ARENAS, Antonio CABRALES, Albert DÍAZ-GUILERA, Roger GUIMERA and Fernando VEGA-REDONDO</i> (lxx): <u>Optimal Information Transmission in Organizations: Search and Congestion</u>
CTN	78.2004	<i>Francis BLOCH and Armando GOMES</i> (lxx): <u>Contracting with Externalities and Outside Options</u>
CTN	79.2004	<i>Rabah AMIR, Effrosyni DIAMANTOUDI and Licun XUE</i> (lxx): <u>Merger Performance under Uncertain Efficiency Gains</u>
CTN	80.2004	<i>Francis BLOCH and Matthew O. JACKSON</i> (lxx): <u>The Formation of Networks with Transfers among Players</u>
CTN	81.2004	<i>Daniel DIERMEIER, Hülya ERASLAN and Antonio MERLO</i> (lxx): <u>Bicameralism and Government Formation</u>
CTN	82.2004	<i>Rod GARRATT, James E. PARCO, Cheng-ZHONG QIN and Amnon RAPOPORT</i> (lxx): <u>Potential Maximization and Coalition Government Formation</u>
CTN	83.2004	<i>Kfir ELIAZ, Debraj RAY and Ronny RAZIN</i> (lxx): <u>Group Decision-Making in the Shadow of Disagreement</u>
CTN	84.2004	<i>Sanjeev GOYAL, Marco van der LEIJ and José Luis MORAGA-GONZÁLEZ</i> (lxx): <u>Economics: An Emerging Small World?</u>
CTN	85.2004	<i>Edward CARTWRIGHT</i> (lxx): <u>Learning to Play Approximate Nash Equilibria in Games with Many Players</u>
IEM	86.2004	<i>Finn R. FØRSUND and Michael HOEL</i> : <u>Properties of a Non-Competitive Electricity Market Dominated by Hydroelectric Power</u>
KTHC	87.2004	<i>Elissaios PAPHAKIS and Reyer GERLAGH</i> : <u>Natural Resources, Investment and Long-Term Income</u>
CCMP	88.2004	<i>Marzio GALEOTTI and Claudia KEMFERT</i> : <u>Interactions between Climate and Trade Policies: A Survey</u>
IEM	89.2004	<i>A. MARKANDYA, S. PEDROSO and D. STREIMIKIENE</i> : <u>Energy Efficiency in Transition Economies: Is There Convergence Towards the EU Average?</u>
GG	90.2004	<i>Rolf GOLOMBEK and Michael HOEL</i> : <u>Climate Agreements and Technology Policy</u>
PRA	91.2004	<i>Sergei IZMALKOV</i> (lxx): <u>Multi-Unit Open Ascending Price Efficient Auction</u>
KTHC	92.2004	<i>Gianmarco I.P. OTTAVIANO and Giovanni PERI</i> : <u>Cities and Cultures</u>
KTHC	93.2004	<i>Massimo DEL GATTO</i> : <u>Agglomeration, Integration, and Territorial Authority Scale in a System of Trading Cities. Centralisation versus devolution</u>
CCMP	94.2004	<i>Pierre-André JOUVET, Philippe MICHEL and Gilles ROTILLON</i> : <u>Equilibrium with a Market of Permits</u>
CCMP	95.2004	<i>Bob van der ZWAAN and Reyer GERLAGH</i> : <u>Climate Uncertainty and the Necessity to Transform Global Energy Supply</u>
CCMP	96.2004	<i>Francesco BOSELLO, Marco LAZZARIN, Roberto ROSON and Richard S.J. TOL</i> : <u>Economy-Wide Estimates of the Implications of Climate Change: Sea Level Rise</u>
CTN	97.2004	<i>Gustavo BERGANTIÑOS and Juan J. VIDAL-PUGA</i> : <u>Defining Rules in Cost Spanning Tree Problems Through the Canonical Form</u>
CTN	98.2004	<i>Siddhartha BANDYOPADHYAY and Mandar OAK</i> : <u>Party Formation and Coalitional Bargaining in a Model of Proportional Representation</u>
GG	99.2004	<i>Hans-Peter WEIKARD, Michael FINUS and Juan-Carlos ALTAMIRANO-CABRERA</i> : <u>The Impact of Surplus Sharing on the Stability of International Climate Agreements</u>
SIEV	100.2004	<i>Chiara M. TRAVISI and Peter NIJKAMP</i> : <u>Willingness to Pay for Agricultural Environmental Safety: Evidence from a Survey of Milan, Italy, Residents</u>
SIEV	101.2004	<i>Chiara M. TRAVISI, Raymond J. G. M. FLORAX and Peter NIJKAMP</i> : <u>A Meta-Analysis of the Willingness to Pay for Reductions in Pesticide Risk Exposure</u>
NRM	102.2004	<i>Valentina BOSETTI and David TOMBERLIN</i> : <u>Real Options Analysis of Fishing Fleet Dynamics: A Test</u>
CCMP	103.2004	<i>Alessandra GORIA e Gretel GAMBARELLI</i> : <u>Economic Evaluation of Climate Change Impacts and Adaptability in Italy</u>
PRA	104.2004	<i>Massimo FLORIO and Mara GRASSEN</i> : <u>The Missing Shock: The Macroeconomic Impact of British Privatisation</u>
PRA	105.2004	<i>John BENNETT, Saul ESTRIN, James MAW and Giovanni URGA</i> : <u>Privatisation Methods and Economic Growth in Transition Economies</u>
PRA	106.2004	<i>Kira BÖRNER</i> : <u>The Political Economy of Privatization: Why Do Governments Want Reforms?</u>
PRA	107.2004	<i>Pehr-Johan NORBÄCK and Lars PERSSON</i> : <u>Privatization and Restructuring in Concentrated Markets</u>
SIEV	108.2004	<i>Angela GRANZOTTO, Fabio PRANOVI, Simone LIBRALATO, Patrizia TORRICELLI and Danilo MAINARDI</i> : <u>Comparison between Artisanal Fishery and Manila Clam Harvesting in the Venice Lagoon by Using Ecosystem Indicators: An Ecological Economics Perspective</u>
CTN	109.2004	<i>Somdeb LAHIRI</i> : <u>The Cooperative Theory of Two Sided Matching Problems: A Re-examination of Some Results</u>
NRM	110.2004	<i>Giuseppe DI VITA</i> : <u>Natural Resources Dynamics: Another Look</u>
SIEV	111.2004	<i>Anna ALBERINI, Alistair HUNT and Anil MARKANDYA</i> : <u>Willingness to Pay to Reduce Mortality Risks: Evidence from a Three-Country Contingent Valuation Study</u>
KTHC	112.2004	<i>Valeria PAPPONETTI and Dino PINELLI</i> : <u>Scientific Advice to Public Policy-Making</u>
SIEV	113.2004	<i>Paulo A.L.D. NUNES and Laura ONOFRI</i> : <u>The Economics of Warm Glow: A Note on Consumer's Behavior and Public Policy Implications</u>
IEM	114.2004	<i>Patrick CAYRADE</i> : <u>Investments in Gas Pipelines and Liquefied Natural Gas Infrastructure What is the Impact on the Security of Supply?</u>
IEM	115.2004	<i>Valeria COSTANTINI and Francesco GRACCEVA</i> : <u>Oil Security. Short- and Long-Term Policies</u>

IEM	116.2004	<i>Valeria COSTANTINI and Francesco GRACCEVA: <u>Social Costs of Energy Disruptions</u></i>
IEM	117.2004	<i>Christian EGENHOFER, Kyriakos GIALOGLOU, Giacomo LUCIANI, Maroeska BOOTS, Martin SCHEEPERS, Valeria COSTANTINI, Francesco GRACCEVA, Anil MARKANDYA and Giorgio VICINI: <u>Market-Based Options for Security of Energy Supply</u></i>
IEM	118.2004	<i>David FISK: <u>Transport Energy Security. The Unseen Risk?</u></i>
IEM	119.2004	<i>Giacomo LUCIANI: <u>Security of Supply for Natural Gas Markets. What is it and What is it not?</u></i>
IEM	120.2004	<i>L.J. de VRIES and R.A. HAKVOORT: <u>The Question of Generation Adequacy in Liberalised Electricity Markets</u></i>
KTHC	121.2004	<i>Alberto PETRUCCI: <u>Asset Accumulation, Fertility Choice and Nondegenerate Dynamics in a Small Open Economy</u></i>
NRM	122.2004	<i>Carlo GIUPPONI, Jaroslaw MYSLAK and Anita FASSIO: <u>An Integrated Assessment Framework for Water Resources Management: A DSS Tool and a Pilot Study Application</u></i>
NRM	123.2004	<i>Margaretha BREIL, Anita FASSIO, Carlo GIUPPONI and Paolo ROSATO: <u>Evaluation of Urban Improvement on the Islands of the Venice Lagoon: A Spatially-Distributed Hedonic-Hierarchical Approach</u></i>
ETA	124.2004	<i>Paul MENSINK: <u>Instant Efficient Pollution Abatement Under Non-Linear Taxation and Asymmetric Information: The Differential Tax Revisited</u></i>
NRM	125.2004	<i>Mauro FABIANO, Gabriella CAMARSA, Rosanna DURSI, Roberta IVALDI, Valentina MARIN and Francesca PALMISANI: <u>Integrated Environmental Study for Beach Management: A Methodological Approach</u></i>
PRA	126.2004	<i>Irena GROSFELD and Iraj HASHI: <u>The Emergence of Large Shareholders in Mass Privatized Firms: Evidence from Poland and the Czech Republic</u></i>
CCMP	127.2004	<i>Maria BERRITTELLA, Andrea BIGANO, Roberto ROSON and Richard S.J. TOL: <u>A General Equilibrium Analysis of Climate Change Impacts on Tourism</u></i>
CCMP	128.2004	<i>Reyer GERLAGH: <u>A Climate-Change Policy Induced Shift from Innovations in Energy Production to Energy Savings</u></i>
NRM	129.2004	<i>Elissaios POPYRAKIS and Reyer GERLAGH: <u>Natural Resources, Innovation, and Growth</u></i>
PRA	130.2004	<i>Bernardo BORTOLOTTI and Mara FACCIO: <u>Reluctant Privatization</u></i>
SIEV	131.2004	<i>Riccardo SCARPA and Mara THIENE: <u>Destination Choice Models for Rock Climbing in the Northeast Alps: A Latent-Class Approach Based on Intensity of Participation</u></i>
SIEV	132.2004	<i>Riccardo SCARPA Kenneth G. WILLIS and Melinda ACUTT: <u>Comparing Individual-Specific Benefit Estimates for Public Goods: Finite Versus Continuous Mixing in Logit Models</u></i>
IEM	133.2004	<i>Santiago J. RUBIO: <u>On Capturing Oil Rents with a National Excise Tax Revisited</u></i>
ETA	134.2004	<i>Ascensión ANDINA DÍAZ: <u>Political Competition when Media Create Candidates' Charisma</u></i>
SIEV	135.2004	<i>Anna ALBERINI: <u>Robustness of VSL Values from Contingent Valuation Surveys</u></i>
CCMP	136.2004	<i>Gernot KLEPPER and Sonja PETERSON: <u>Marginal Abatement Cost Curves in General Equilibrium: The Influence of World Energy Prices</u></i>
ETA	137.2004	<i>Herbert DAWID, Christophe DEISSENBERG and Pavel ŠEVČIK: <u>Cheap Talk, Gullibility, and Welfare in an Environmental Taxation Game</u></i>
CCMP	138.2004	<i>ZhongXiang ZHANG: <u>The World Bank's Prototype Carbon Fund and China</u></i>
CCMP	139.2004	<i>Reyer GERLAGH and Marjan W. HOFKES: <u>Time Profile of Climate Change Stabilization Policy</u></i>
NRM	140.2004	<i>Chiara D'ALPAOS and Michele MORETTO: <u>The Value of Flexibility in the Italian Water Service Sector: A Real Option Analysis</u></i>
PRA	141.2004	<i>Patrick BAJARI, Stephanie HOUGHTON and Steven TADELIS (lxxi): <u>Bidding for Incomplete Contracts</u></i>
PRA	142.2004	<i>Susan ATHEY, Jonathan LEVIN and Enrique SEIRA (lxxi): <u>Comparing Open and Sealed Bid Auctions: Theory and Evidence from Timber Auctions</u></i>
PRA	143.2004	<i>David GOLDREICH (lxxi): <u>Behavioral Biases of Dealers in U.S. Treasury Auctions</u></i>
PRA	144.2004	<i>Roberto BURGNET (lxxi): <u>Optimal Procurement Auction for a Buyer with Downward Sloping Demand: More Simple Economics</u></i>
PRA	145.2004	<i>Ali HORTACSU and Samita SAREEN (lxxi): <u>Order Flow and the Formation of Dealer Bids: An Analysis of Information and Strategic Behavior in the Government of Canada Securities Auctions</u></i>
PRA	146.2004	<i>Victor GINSBURGH, Patrick LEGROS and Nicolas SAHUGUET (lxxi): <u>How to Win Twice at an Auction. On the Incidence of Commissions in Auction Markets</u></i>
PRA	147.2004	<i>Claudio MEZZETTI, Aleksandar PEKEČ and Ilia TSETLIN (lxxi): <u>Sequential vs. Single-Round Uniform-Price Auctions</u></i>
PRA	148.2004	<i>John ASKER and Estelle CANTILLON (lxxi): <u>Equilibrium of Scoring Auctions</u></i>
PRA	149.2004	<i>Philip A. HAILE, Han HONG and Matthew SHUM (lxxi): <u>Nonparametric Tests for Common Values in First-Price Sealed-Bid Auctions</u></i>
PRA	150.2004	<i>François DEGEORGE, François DERRIEN and Kent L. WOMACK (lxxi): <u>Quid Pro Quo in IPOs: Why Bookbuilding is Dominating Auctions</u></i>
CCMP	151.2004	<i>Barbara BUCHNER and Silvia DALL'OLIO: <u>Russia: The Long Road to Ratification. Internal Institution and Pressure Groups in the Kyoto Protocol's Adoption Process</u></i>
CCMP	152.2004	<i>Carlo CARRARO and Marzio GALEOTTI: <u>Does Endogenous Technical Change Make a Difference in Climate Policy Analysis? A Robustness Exercise with the FEEM-RICE Model</u></i>
PRA	153.2004	<i>Alejandro M. MANELLI and Daniel R. VINCENT (lxxi): <u>Multidimensional Mechanism Design: Revenue Maximization and the Multiple-Good Monopoly</u></i>
ETA	154.2004	<i>Nicola ACOCELLA, Giovanni Di BARTOLOMEO and Wilfried PAUWELS: <u>Is there any Scope for Corporatism in Stabilization Policies?</u></i>
CTN	155.2004	<i>Johan EYCKMANS and Michael FINUS: <u>An Almost Ideal Sharing Scheme for Coalition Games with Externalities</u></i>
CCMP	156.2004	<i>Cesare DOSI and Michele MORETTO: <u>Environmental Innovation, War of Attrition and Investment Grants</u></i>

CCMP	157.2004	<i>Valentina BOSETTI, Marzio GALEOTTI and Alessandro LANZA: <u>How Consistent are Alternative Short-Term Climate Policies with Long-Term Goals?</u></i>
ETA	158.2004	<i>Y. Hossein FARZIN and Ken-Ichi AKAO: <u>Non-pecuniary Value of Employment and Individual Labor Supply</u></i>
ETA	159.2004	<i>William BROCK and Anastasios XEPAPADEAS: <u>Spatial Analysis: Development of Descriptive and Normative Methods with Applications to Economic-Ecological Modelling</u></i>
KTHC	160.2004	<i>Alberto PETRUCCI: <u>On the Incidence of a Tax on PureRent with Infinite Horizons</u></i>
IEM	161.2004	<i>Xavier LABANDEIRA, José M. LABEAGA and Miguel RODRÍGUEZ: <u>Microsimulating the Effects of Household Energy Price Changes in Spain</u></i>

#### NOTE DI LAVORO PUBLISHED IN 2005

CCMP	1.2005	<i>Stéphane HALLEGATTE: <u>Accounting for Extreme Events in the Economic Assessment of Climate Change</u></i>
CCMP	2.2005	<i>Qiang WU and Paulo Augusto NUNES: <u>Application of Technological Control Measures on Vehicle Pollution: A Cost-Benefit Analysis in China</u></i>
CCMP	3.2005	<i>Andrea BIGANO, Jacqueline M. HAMILTON, Maren LAU, Richard S.J. TOL and Yuan ZHOU: <u>A Global Database of Domestic and International Tourist Numbers at National and Subnational Level</u></i>
CCMP	4.2005	<i>Andrea BIGANO, Jacqueline M. HAMILTON and Richard S.J. TOL: <u>The Impact of Climate on Holiday Destination Choice</u></i>
ETA	5.2005	<i>Hubert KEMPF: <u>Is Inequality Harmful for the Environment in a Growing Economy?</u></i>
CCMP	6.2005	<i>Valentina BOSETTI, Carlo CARRARO and Marzio GALEOTTI: <u>The Dynamics of Carbon and Energy Intensity in a Model of Endogenous Technical Change</u></i>
IEM	7.2005	<i>David CALEF and Robert GOBLE: <u>The Allure of Technology: How France and California Promoted Electric Vehicles to Reduce Urban Air Pollution</u></i>
ETA	8.2005	<i>Lorenzo PELLEGRINI and Reyer GERLAGH: <u>An Empirical Contribution to the Debate on Corruption Democracy and Environmental Policy</u></i>
CCMP	9.2005	<i>Angelo ANTOCI: <u>Environmental Resources Depletion and Interplay Between Negative and Positive Externalities in a Growth Model</u></i>
CTN	10.2005	<i>Frédéric DEROLAN: <u>Cost-Reducing Alliances and Local Spillovers</u></i>
NRM	11.2005	<i>Francesco SINDICO: <u>The GMO Dispute before the WTO: Legal Implications for the Trade and Environment Debate</u></i>
KTHC	12.2005	<i>Carla MASSIDDA: <u>Estimating the New Keynesian Phillips Curve for Italian Manufacturing Sectors</u></i>
KTHC	13.2005	<i>Michele MORETTO and Gianpaolo ROSSINI: <u>Start-up Entry Strategies: Employer vs. Nonemployer firms</u></i>
PRCG	14.2005	<i>Clara GRAZIANO and Annalisa LUPORINI: <u>Ownership Concentration, Monitoring and Optimal Board Structure</u></i>
CSRM	15.2005	<i>Parashar KULKARNI: <u>Use of Ecolabels in Promoting Exports from Developing Countries to Developed Countries: Lessons from the Indian LeatherFootwear Industry</u></i>
KTHC	16.2005	<i>Adriana DI LIBERTO, Roberto MURA and Francesco PIGLIARU: <u>How to Measure the Unobservable: A Panel Technique for the Analysis of TFP Convergence</u></i>
KTHC	17.2005	<i>Alireza NAGHAVI: <u>Asymmetric Labor Markets, Southern Wages, and the Location of Firms</u></i>
KTHC	18.2005	<i>Alireza NAGHAVI: <u>Strategic Intellectual Property Rights Policy and North-South Technology Transfer</u></i>
KTHC	19.2005	<i>Mombert HOPPE: <u>Technology Transfer Through Trade</u></i>
PRCG	20.2005	<i>Roberto ROSON: <u>Platform Competition with Endogenous Multihoming</u></i>
CCMP	21.2005	<i>Barbara BUCHNER and Carlo CARRARO: <u>Regional and Sub-Global Climate Blocs. A Game Theoretic Perspective on Bottom-up Climate Regimes</u></i>
IEM	22.2005	<i>Fausto CAVALLARO: <u>An Integrated Multi-Criteria System to Assess Sustainable Energy Options: An Application of the Promethee Method</u></i>
CTN	23.2005	<i>Michael FINUS, Pierre v. MOUCHE and Bianca RUNDSHAGEN: <u>Uniqueness of Coalitional Equilibria</u></i>
IEM	24.2005	<i>Wietze LISE: <u>Decomposition of CO2 Emissions over 1980–2003 in Turkey</u></i>
CTN	25.2005	<i>Somdeb LAHIRI: <u>The Core of Directed Network Problems with Quotas</u></i>
SIEV	26.2005	<i>Susanne MENZEL and Riccardo SCARPA: <u>Protection Motivation Theory and Contingent Valuation: Perceived Realism, Threat and WTP Estimates for Biodiversity Protection</u></i>
NRM	27.2005	<i>Massimiliano MAZZANTI and Anna MONTINI: <u>The Determinants of Residential Water Demand Empirical Evidence for a Panel of Italian Municipalities</u></i>
CCMP	28.2005	<i>Laurent GILOTTE and Michel de LARA: <u>Precautionary Effect and Variations of the Value of Information</u></i>
NRM	29.2005	<i>Paul SARFO-MENSAH: <u>Exportation of Timber in Ghana: The Menace of Illegal Logging Operations</u></i>
CCMP	30.2005	<i>Andrea BIGANO, Alessandra GORIA, Jacqueline HAMILTON and Richard S.J. TOL: <u>The Effect of Climate Change and Extreme Weather Events on Tourism</u></i>
NRM	31.2005	<i>Maria Angeles GARCIA-VALIÑAS: <u>Decentralization and Environment: An Application to Water Policies</u></i>
NRM	32.2005	<i>Chiara D'ALPAOS, Cesare DOSI and Michele MORETTO: <u>Concession Length and Investment Timing Flexibility</u></i>
CCMP	33.2005	<i>Joseph HUBER: <u>Key Environmental Innovations</u></i>
CTN	34.2005	<i>Antoni CALVÓ-ARMENGOL and Rahmi İLKILIÇ (Ixxii): <u>Pairwise-Stability and Nash Equilibria in Network Formation</u></i>
CTN	35.2005	<i>Francesco FERI (Ixxii): <u>Network Formation with Endogenous Decay</u></i>
CTN	36.2005	<i>Frank H. PAGE, Jr. and Myrna H. WOODERS (Ixxii): <u>Strategic Basins of Attraction, the Farsighted Core, and Network Formation Games</u></i>

CTN	37.2005	<i>Alessandra CASELLA and Nobuyuki HANAOKI</i> (lxxii): <u>Information Channels in Labor Markets. On the Resilience of Referral Hiring</u>
CTN	38.2005	<i>Matthew O. JACKSON and Alison WATTS</i> (lxxii): <u>Social Games: Matching and the Play of Finitely Repeated Games</u>
CTN	39.2005	<i>Anna BOGOMOLNAIA, Michel LE BRETON, Alexei SAVVATEEV and Shlomo WEBER</i> (lxxii): <u>The Egalitarian Sharing Rule in Provision of Public Projects</u>
CTN	40.2005	<i>Francesco FERI</i> : <u>Stochastic Stability in Network with Decay</u>
CTN	41.2005	<i>Aart de ZEEUW</i> (lxxii): <u>Dynamic Effects on the Stability of International Environmental Agreements</u>
NRM	42.2005	<i>C. Martijn van der HEIDE, Jeroen C.J.M. van den BERGH, Ekko C. van IERLAND and Paulo A.L.D. NUNES</i> : <u>Measuring the Economic Value of Two Habitat Defragmentation Policy Scenarios for the Veluwe, The Netherlands</u>
PRCG	43.2005	<i>Carla VIEIRA and Ana Paula SERRA</i> : <u>Abnormal Returns in Privatization Public Offerings: The Case of Portuguese Firms</u>
SIEV	44.2005	<i>Anna ALBERINI, Valentina ZANATTA and Paolo ROSATO</i> : <u>Combining Actual and Contingent Behavior to Estimate the Value of Sports Fishing in the Lagoon of Venice</u>
CTN	45.2005	<i>Michael FINUS and Bianca RUNDSHAGEN</i> : <u>Participation in International Environmental Agreements: The Role of Timing and Regulation</u>
CCMP	46.2005	<i>Lorenzo PELLEGRINI and Reyer GERLAGH</i> : <u>Are EU Environmental Policies Too Demanding for New Members States?</u>
IEM	47.2005	<i>Matteo MANERA</i> : <u>Modelling Factor Demands with SEM and VAR: An Empirical Comparison</u>
CTN	48.2005	<i>Olivier TERCIEUX and Vincent VANNETELBOSCH</i> (lxx): <u>A Characterization of Stochastically Stable Networks</u>

(lxv) This paper was presented at the EuroConference on “Auctions and Market Design: Theory, Evidence and Applications” organised by Fondazione Eni Enrico Mattei and sponsored by the EU, Milan, September 25-27, 2003

(lxvi) This paper has been presented at the 4th BioEcon Workshop on “Economic Analysis of Policies for Biodiversity Conservation” organised on behalf of the BIOECON Network by Fondazione Eni Enrico Mattei, Venice International University (VIU) and University College London (UCL), Venice, August 28-29, 2003

(lxvii) This paper has been presented at the international conference on “Tourism and Sustainable Economic Development – Macro and Micro Economic Issues” jointly organised by CRENoS (Università di Cagliari e Sassari, Italy) and Fondazione Eni Enrico Mattei, and supported by the World Bank, Sardinia, September 19-20, 2003

(lxviii) This paper was presented at the ENGIME Workshop on “Governance and Policies in Multicultural Cities”, Rome, June 5-6, 2003

(lxix) This paper was presented at the Fourth EEP Plenary Workshop and EEP Conference “The Future of Climate Policy”, Cagliari, Italy, 27-28 March 2003

(lxx) This paper was presented at the 9<sup>th</sup> Coalition Theory Workshop on "Collective Decisions and Institutional Design" organised by the Universitat Autònoma de Barcelona and held in Barcelona, Spain, January 30-31, 2004

(lxxi) This paper was presented at the EuroConference on “Auctions and Market Design: Theory, Evidence and Applications”, organised by Fondazione Eni Enrico Mattei and Consip and sponsored by the EU, Rome, September 23-25, 2004

(lxxii) This paper was presented at the 10<sup>th</sup> Coalition Theory Network Workshop held in Paris, France on 28-29 January 2005 and organised by EUREQua.

**2004 SERIES**

<b>CCMP</b>	<i>Climate Change Modelling and Policy</i> (Editor: Marzio Galeotti )
<b>GG</b>	<i>Global Governance</i> (Editor: Carlo Carraro)
<b>SIEV</b>	<i>Sustainability Indicators and Environmental Valuation</i> (Editor: Anna Alberini)
<b>NRM</b>	<i>Natural Resources Management</i> (Editor: Carlo Giupponi)
<b>KTHC</b>	<i>Knowledge, Technology, Human Capital</i> (Editor: Gianmarco Ottaviano)
<b>IEM</b>	<i>International Energy Markets</i> (Editor: Anil Markandya)
<b>CSRM</b>	<i>Corporate Social Responsibility and Sustainable Management</i> (Editor: Sabina Ratti)
<b>PRA</b>	<i>Privatisation, Regulation, Antitrust</i> (Editor: Bernardo Bortolotti)
<b>ETA</b>	<i>Economic Theory and Applications</i> (Editor: Carlo Carraro)
<b>CTN</b>	<i>Coalition Theory Network</i>

**2005 SERIES**

<b>CCMP</b>	<i>Climate Change Modelling and Policy</i> (Editor: Marzio Galeotti )
<b>SIEV</b>	<i>Sustainability Indicators and Environmental Valuation</i> (Editor: Anna Alberini)
<b>NRM</b>	<i>Natural Resources Management</i> (Editor: Carlo Giupponi)
<b>KTHC</b>	<i>Knowledge, Technology, Human Capital</i> (Editor: Gianmarco Ottaviano)
<b>IEM</b>	<i>International Energy Markets</i> (Editor: Anil Markandya)
<b>CSRM</b>	<i>Corporate Social Responsibility and Sustainable Management</i> (Editor: Sabina Ratti)
<b>PRCG</b>	<i>Privatisation Regulation Corporate Governance</i> (Editor: Bernardo Bortolotti)
<b>ETA</b>	<i>Economic Theory and Applications</i> (Editor: Carlo Carraro)
<b>CTN</b>	<i>Coalition Theory Network</i>