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Climate Policy and the Optimal Extraction of High- and Low-Carbon Fossil Fuels

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Summary

We study how restricting CO2 emissions affects resource prices and depletion over time. We use a Hotelling-style model with two non-renewable fossil fuels that differ in their carbon content (e.g. coal and natural gas) and in addition are imperfect substitutes in final good production. We show that an economy facing a CO2 flow-constraint may substitute towards the relatively dirty input. As the economy tries to maximise output per unit of emissions it is not only carbon content that matters: productivity matters as well. With an announced constraint the economy first substitutes towards the less productive input such that more of the productive input is available when constrained. Preliminary empirical results suggest that it is cost-effective to substitute away from dirty coal to cleaner oil or gas, but to substitute from natural gas towards the dirtier input oil.

Keywords: Climate Policy, Non-Renewable Resources, Input Substitution

JEL Classification: O13, Q31, Q43

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Climate policy and the optimal extraction of high- and low-carbon fossil fuels

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Abstract

We study how restricting CO₂ emissions affects resource prices and depletion over time. We use a Hotelling-style model with two non-renewable fossil fuels that differ in their carbon content (e.g. coal and natural gas) and in addition are imperfect substitutes in final good production. We show that an economy facing a CO₂ flow-constraint may substitute towards the relatively dirty input. As the economy tries to maximise output per unit of emissions it is not only carbon content that matters: productivity matters as well. With an announced constraint the economy first substitutes towards the less productive input such that more of the productive input is available when constrained. Preliminary empirical results suggest that it is cost-effective to substitute away from dirty coal to cleaner oil or gas, but to substitute from natural gas towards the dirtier input oil.

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1 Introduction

Climate change policies that call for a reduction in CO₂ emissions are likely to have an economy-wide impact by imposing significant cost on most sectors in the economy. Substitution from high-carbon to low-carbon energy sources may allow an economy to reduce carbon dioxide...
(CO₂) emissions at lower cost. For example, a country can build gas-fueled powerplants instead of coal-fueled powerplants. Or the country can expand sectors that rely on low-carbon inputs at the cost of sectors that mainly use high-carbon inputs. The overall cost of climate change policies therefore depends on the behaviour of both energy users and energy suppliers, and important questions in this context are: how should energy users substitute between different energy sources; should they make a transition towards a 'low-carbon economy'; how will resource rents for energy producing countries change; should they leave reserves of high-carbon resources (e.g. coal) unexploited, at least for a while?

In a standard static partial equilibrium setting, a CO₂ emission tax affects the user cost of high-carbon energy more than that of low-carbon energy and substitution will take place towards low-carbon energy. We show that in the more appropriate dynamic setting, with energy coming from non-renewable resource stocks, the results are quite different. Extending the canonical non-renewable resource model with a second resource, we find that a binding CO₂ emission constraint not necessarily calls for substitution towards low-carbon fuels in the short-run, but – depending on a well-defined measure of scarcity of the two resources – may instead call for relatively more intensive high-carbon fuel use in the short-run and less of it in the long-run.

Taking the current global policy regarding global warming as a starting point, we study how a permanent cap on carbon dioxide emissions ('Kyoto forever') affects the composition of energy use, the timing of extraction of different energy resources and their scarcity rents when the government uses a cost-effective instrument. We build a model that is as close as possible to the standard non-renewable resource model and distinguish between two non-renewable resources, for example coal and natural gas, that are imperfect substitutes in production and differ in CO₂ emissions per unit of effective energy.

We build our arguments on the fact that high-carbon and low-carbon inputs are imperfect substitutes at an aggregate level. Substitution between different types of products implies
indirect substitution between energy types and types of fossil fuels. For example, a shift in the transport sector from road transport to rail implies a change in the fossil fuel mix as trucks use oil-based products while the rail sector uses electricity, which can be generated by gas-fueled powerplants. The energy sector can substitute between fossil fuel types when deciding upon investment in new powerplants: although for an individual power plant the choice between coal, oil, and gas is a discrete one, the point of indifference between the three inputs may differ at different locations, leading to imperfect substitution at the aggregate level.

We show that relative extraction in the constrained economy not only depends on the carbon content of the two inputs, but also on their relative productivity and physical scarcity. The best way to cope with an emission constraint is to intertemporally reallocate the extraction of the two given resource stocks such that production per unit of carbon dioxide emissions is relatively high at the time the emission constraint is binding, and low when the constraint no longer (or – in the case of an anticipated constraint – not yet) binds. Hence the constrained economy uses the resource with the lowest amount of emissions per unit of output relatively more intensively, as compared to an unconstrained economy. This resource is not necessarily the resource with lowest amount of carbon per unit of energy: because of diminishing returns to each of the energy inputs, the scarcer a resource relatively is, the higher its marginal productivity per unit of emissions.

Our empirical results suggest that it is cost-effective to substitute away from dirty coal to cleaner oil or gas. However, when it comes to choose between relatively clean natural gas and the dirtier input oil, the paradoxical "dirty-first result" might apply, i.e. there should be substitution from (low-carbon) gas towards (high-carbon) oil, as the latter is found to be relatively more productive per unit of CO₂ emissions.

The option of substituting low-carbon for high-carbon fuels to meet climate targets has been studied analytically in Chakravorty et al. (2006b) and numerically in Chakravorty et al. (1997).
The latter paper develops a numerical integrated assessment model with several non-renewables (oil, coal and natural gas), multiple energy demand sectors, and a clean renewable resource. The authors simulate three scenarios for technical change with optimal climate policy, but do not analytically identify the forces underlying relative extraction patterns. In Chakravorty et al. (2006b), climate policy consists of an exogenous ceiling on the stock of pollution. A high- and a low-carbon fossil fuel, together with a clean backstop technology, are used in energy generation. The optimal order of extraction is studied. This work maintains the assumption that the fossil fuels are perfect substitutes, so that often one resource is exclusively used and at certain points in time there is a complete switch in resource use from one to the other fuel.

Most theoretical papers studying climate policy and fossil fuel extraction use a single (polluting) non-renewable resource. Withagen (1994) extends the standard Hotelling (1931) model with stock externalities from resource use and studies the optimal extraction path. Grimaud and Rougé (2005) treat pollution as a flow and extend the model with endogenous technological change and growth.

A second branch of theoretical papers has both a polluting non-renewable and a non-polluting backstop technology. Tahvonen (1997) extends Withagen’s model with extraction costs and a backstop and shows that, if the initial stock of externalities is low enough, the extraction path of the non-renewable may have an inverted U-shape form. In a related paper, Chakravorty et al. (2006a) study the effects of an exogenous ceiling on the stock of emissions on the use of the non-renewable resource and the backstop technology during and after the period that the constraint is binding.

Few papers study imperfect substitution between non-renewable resources. Exceptions are Beckmann (1974) and Hartwick (1978), but these early studies are not concerned with carbon emissions.

In the remainder of the paper, we first present our model in Section 2, and we study the
economy without any form of climate policy in Section 2. In Section 4 we study an unexpected and initially binding constant CO₂ emission ceiling, and show that it might be optimal to use relatively more of the high-carbon input. In Section 5 we study the empirical relevance of this paradoxical “dirty-first” result. Section 6 presents the effects of an announced constraint, and in Section 7 we look at the robustness of our results with respect to alternative policies and technological change. We conclude in Section 8.

2  The model

The representative consumer derives utility from final good \( Y \) and faces an intertemporal budget constraint:

\[
dV(t)/dt = r(t) V(t) - Y(t).
\]

Here \( V(t) \) is wealth and \( r(t) \) is the market interest rate, at time \( t \). The consumer maximizes intertemporal utility:

\[
U(t) = \int_t^{\infty} \ln Y(\tau) \cdot e^{-\rho \tau} d\tau,
\]

where \( \rho \) is the utility discount rate. Maximizing (1) subject to the intertemporal budget constraint implies the following Ramsey rule:

\[
\hat{Y}(t) = r(t) - \rho.
\]

where, as in the remainder of this paper, the hat denotes the growth rate \( (\hat{Y} = d\ln Y/dt) \).

The competitive final goods industry produces \( Y \) from two fossil fuel inputs, \( H \) and \( L \), both scaled to units of energy, according to the following constant returns to scale CES technology (we suppress the time argument when no confusion arises):

\[
Y = A \left( \eta_H R_H^{\sigma-1} + \eta_L R_L^{\sigma-1} \right)^{\sigma/(\sigma-1)},
\]

5
where $A$ is the level of total factor productivity, $R_i$ is the amount extracted of resource $i \in \{H, L\}$, $\eta_H$ and $\eta_L$ are positive technology parameters and $\sigma \in (0, \infty)$ is the constant elasticity of substitution. The use of fossil fuels causes emissions of carbon dioxide. The two inputs differ in their CO$_2$ emission intensity per unit of energy and we denote the (constant) CO$_2$ emission coefficients of $H$ and $L$ by $\varepsilon_H$ and $\varepsilon_L$ respectively, with $\varepsilon_H > \varepsilon_L$ so that $H$ is the relatively dirty or high-carbon input. The total amount of emissions is denoted by $Z$.\(^1\) If the economy is subject to an emissions constraint, total emissions cannot exceed a maximally allowed amount $\bar{Z}$, according to the following constraint:

$$\varepsilon_H R_H (t) + \varepsilon_L R_L (t) = Z (t) \leq \bar{Z}. \tag{4}$$

As we are interested in the reaction of the economy to the constraint rather than in optimal climate policy itself, we assume that the constraint $\bar{Z}$ is exogenous. The government allocates tradable emission permits over producers in the final goods industry, who trade them at a market price $p_Z$ and buy resources of type $i$ at price $p_{Ri}$.\(^2\) The price of the final good is normalized to one for every period. Firms maximize profits and the first order conditions for resource use read (from (3) and (4)):

$$A^{\frac{\sigma-1}{\sigma}} \eta_i \left( \frac{Y}{R_i} \right)^{\frac{1}{\sigma}} = p_{Ri} + \varepsilon_i p_Z. \tag{5}$$

This equation states that the marginal revenue from resource input $i$ (the marginal product at the left-hand side) equals its marginal cost (the user price at the right-hand side), which

\(^1\)Our notation is consistent with the measurement of $R_i$ in units of energy and $Z$ in units of carbon. By rescaling $R_i$ and $Z$ it is possible to normalize - without loss of generality - three of the four parameters $\varepsilon_L$, $\varepsilon_H$, $\eta_L$, and $\eta_H$, to unity. However, to facilitate interpretation and comparison to the data, we do not apply this normalisation.

\(^2\)Although we present the results for the decentralized economy with regulation through tradable pollution permits, it can be shown that a planner who maximizes utility subject to the exogenous emission constraint chooses exactly the same allocation. Hence, the setting we study is one of cost-effective environmental regulation.
consists of the price of the resource augmented with the cost of pollution in case the constraint is binding.\(^3\)

The two fossil fuels are extracted from stocks of non-renewable resources, \(S_H\) and \(S_L\) respectively, according to

\[
dS_i/dt = -R_i, \tag{6}
\]

\[
\int_0^\infty R_i dt \leq S_{i0},
\]

where \(S_{i0}\) is the initial stock of resource \(i\). Resource owners maximize the net present value of profits from exploiting the non-renewable resource stocks, taking resource price \(p_{Ri}\) as given. Extraction costs are assumed to be zero so the resource price is a pure scarcity rent. For each of the resources this results in the familiar Hotelling rule:

\[
\hat{p}_{Ri}(t) = r(t). \tag{7}
\]

From this we see that the relative resource rent \(p_{RH}/p_{RL}\) will be constant over time, as both rents grow at the same rate.

We are now ready to study extraction of the two resources. We first study extraction in an economy without a CO\(_2\) emission constraint and then move to a constrained but otherwise identical economy.

\(^3\)Note that we will always have an interior solution. If \(R_i = 0\) we would have \(Y = 0\) for \(\sigma \leq 1\), while \(\partial Y/\partial R_i = A^{\frac{1}{1+\sigma}} \eta_i (Y/R_i)^{1/\sigma} \to \infty\) for \(\sigma > 1\) which violates (5) for finite \(p_{Ri}\) and \(p_Z\).
3 The economy without (the prospect of) climate policy

Suppose that from some instant $T$ (possibly equal to 0) on the economy is unconstrained and does not expect future climate policy. In this case the economy is described by a pure depletion or cake eating model from $t = T$ on (see e.g. Heal, 1993). Time differentiating (5) (with $p_Z = 0$) and substituting (7), we find that both inputs grow at the same rate. Combining the results with (3), we find that the two scarcity rents grow at rate $\hat{p}_{R_i} = r = \hat{A}$. Finally, substituting (2), we find that extraction and emissions decrease at a rate equal to the utility discount rate:

$$\hat{R}_H = \hat{R}_L = -\rho \forall t \geq T,$$

(8)

After integrating (8) and imposing the constraint that forward-looking resource owners anticipate that eventually all reserves will be sold, we find that the extraction rates of the two resources can be expressed as:

$$R_i(t) = \rho S_i(t) \forall t \geq T.$$  

(9)

Consequently total emissions equal

$$Z(t) = \rho \cdot (\varepsilon_H S_H(t) + \varepsilon_L S_L(t)) \forall t \geq T$$

(10)

(see (4)). According to (8) and (9), relative extraction is constant over time and equal to instant $T$’s relative stock:

$$\frac{R_H(t)}{R_L(t)} = \frac{S_H(T)}{S_L(T)} \forall t \geq T.$$  

(11)

From the first order conditions (5) and equilibrium relative extraction (11) we find the equilib-
rium relative scarcity rent:

$$\frac{p_{RH}(t)}{p_{RL}(t)} = \frac{\eta_H}{\eta_L} \left( \frac{S_H(T)}{S_L(T)} \right)^{-1/\sigma} \forall t. \quad (12)$$

These results reveal that as long as the economy is unconstrained and does not expect future climate policy, relative extraction in the unconstrained economy is constant and equals relative stocks at each point in time. Since conservation of both resource stocks requires that resource owners earn the same return on the two resources, both resource prices grow at the common rate \( r \) in equilibrium. Hence, the relative price is constant over time and the constant-returns-to-scale production function then implies that relative demand is constant as well. As resource owners want to fully exploit the available reserves, stock dynamics require relative extraction to equal relative stocks which implies that the initial relative scarcity rent in an unconstrained economy is determined by initial availability of the resources.

4 An unexpected emission constraint

We now introduce the constraint on emissions. The constraint is unexpectedly introduced at time \( t = 0 \) and is binding by then. It will stay at the level \( \bar{Z} \) forever, which is known by all agents. The constraint will not bind forever, though, since resource stocks, from which emissions stem, are depleted over time (cf. (10)). In particular, we derive the following result:

**Lemma 1.** Define \( T \) as the instant from which onward emissions cease to be constrained. If constraint \( \bar{Z} \) is introduced unexpectedly at \( t = 0 \), then:

$$T = \frac{\epsilon_H S_{H0} + \epsilon_L S_{L0}}{\bar{Z}} - \frac{1}{\rho}. \quad (13)$$
Proof. The total amount of CO$_2$ that will be emitted from $t = 0$ on can be written as $\varepsilon_H S_{H0} + \varepsilon_L S_{L0} = [\varepsilon_H (S_{H0} - S_H(T)) + \varepsilon_L (S_{L0} - S_L(T))] + [\varepsilon_H S_H(T) + \varepsilon_L S_L(T)]$. The first term in square brackets represents total emissions in the period that the economy is constrained, so this term equals $T \bar{Z}$. For any $t \geq T$, we can use (4) and (9), from which we find that the second term in square brackets equals $\bar{Z}/\rho$. Combining results, we find (13). \hfill \Box

Clearly, a larger initial stock or a stricter environmental policy implies a longer period of being restricted. A lower discount rate, and hence more patient consumers, implies that the economy is suffering the constraint for a shorter period as the economy tends to extract and pollute less (see (10)).

To meet the emissions constraint, (4), resource use can be reduced equi-proportionally, or its composition can be changed (relative to the period before $t = 0$). In the latter case, emissions per unit of output will change:

**Lemma 2.** Define $\bar{S} \equiv (\eta_H \varepsilon_L/\eta_L \varepsilon_H)^{\sigma}$. Emissions intensity $Z/Y$ reaches a minimum for $R_H/R_L = \bar{S}$ and increases in $|R_H/R_L - \bar{S}|$.

**Proof.** From (3) and (4) we find that $Z/Y$ is a function of $R_H/R_L$ only. Taking the first order derivative $d(Z/Y) / d(R_H/R_L)$, we find the result. \hfill \Box

Because of imperfect substitutability, a very high or very low level of one of the resource inputs – while still meeting the emission constraint – results in relatively little output and a high emission intensity. The more polluting one input relatively is (as indicated by a relatively large $\varepsilon_i$), the less intensively this input must be used should one want to minimize emissions intensity. Similarly, if one input is much more productive than the other one (as indicated by the $\eta_i$’s), intensive use of this input results in relatively high output and low emission intensity.

In equilibrium, the development of relative extraction in the constrained economy with an unannounced emission constraint can be summarized by the following proposition:
Proposition 1. Suppose a CO\textsubscript{2} emission constraint is unexpectedly introduced. Then

1. if the high-carbon input (low-carbon input) is relatively scarce, that is if \( S_{H0}/S_{L0} < (>)\tilde{S}, \)

   (a) the relative scarcity rent \( p_{RH}/p_{RL} \) jumps up (down) on impact;

   (b) relative extraction \( R_{H}/R_{L} \) jumps up (down) on impact, but decreases (increases) over time as long as the economy is constrained;

   (c) relative extraction stays above (below) the level of the relative stocks \( S_{H}/S_{L} \) as long as the economy is constrained, but equals relative stocks when the constraint ceases to be binding;

   (d) the high-carbon resource stock declines faster (less fast) than the low-carbon resource stock as long as the economy is constrained;

2. if the high- and low-carbon input are equally scarce (that is, if \( S_{H0}/S_{L0} = \tilde{S} \)), the relative scarcity rent, relative extraction and relative stocks do not change after the imposition of the emission constraint;

3. if the two inputs are not equally scarce, emissions per unit of output jump down but increase over time to a higher level compared to the period before the constraint was imposed; they remain constant after the constraint ceases to be binding.

Proof. See Appendix.

The proposition states that at the instant on which emissions become unexpectedly constrained, substitution takes place towards the relatively scarce input, that is towards input \( i \) for which \( S_{i0}/S_{j0} < \tilde{S} \), where \( \tilde{S} \equiv (\eta_{H}\varepsilon_{L}/\eta_{L}\varepsilon_{H})^{\sigma} \) see lemma 2. The increase in the relative use of the scarce input implies that over time this input will become even scarcer, since the relative stock \( S_{i}/S_{j} \) decreases over time (part 1(d) of the proposition). This explains the jump in the relative scarcity rent (part 1(a) of the proposition).
We illustrate the paths of extraction, for the case in which $S_{H0}/S_{L0} < \bar{S}$, by the thick arrows in Figure 1. The constrained economy moves along line $\bar{Z}$, at which emissions are at the imposed ceiling and which is defined by $R_H = (\bar{Z} - \varepsilon_L R_L)/\varepsilon_H$. Since over time the economy moves to lower production isoquants, pollution per unit of GDP gradually increases over time. The unconstrained economy, which according to (9) extracts a constant fraction of each available stock, moves down along a ray from the origin with slope $S_{H0}/S_{L0}$.

Two basic forces drive the evolution of relative energy use: physical scarcity and marginal productivity per unit of pollution. The emission constraint induces the economy to save on pollution per unit of GDP. If relative energy use, $R_H/R_L$, was equal to $\bar{S}$, output per unit of emissions would be maximized; the closer relative use approaches $\bar{S}$, the higher output per unit of emissions. As the unconstrained economy aligns relative resource use with resource supply, as measured by relative stocks, it uses relatively little of the relatively scarce resource, while
this resource might have the highest marginal product per unit of CO$_2$. Once the constraint is imposed, the economy starts to use more of the resource that has highest marginal productivity per unit of pollution, and hence relative extraction jumps closer to $\bar{S}$. However, relative use cannot deviate too much from relative stocks, since at the time the constraint no longer binds (time $T$), relative resource use and available stocks have to be aligned again. Therefore the pollution constraint makes the economy intertemporally reallocate the extraction of resources, such that output per unit of pollution is high when the pollution constraint is most binding, and then gradually substitutes towards the resource with lower productivity per unit of pollution as the constraint becomes less binding. Eventually, once the constraint does not bind anymore, the economy smoothly ends up at the point where resource use and supply are aligned.

The implication is that the high-carbon input might be used intensively first. This ”dirty-first result” arises when the high-carbon resource is physically relatively scarce, such that resource use in line with relative stocks implies that the high-carbon input has higher productivity per unit of CO$_2$. For future reference it is useful to formalize this ”dirty-first condition” as:

$$\frac{S_{H0}}{S_{L0}} < \left(\frac{\eta_H}{\eta_L}\right)^\sigma \equiv \bar{S}. \quad (14)$$

To further explain why relative resource use changes over time and intertemporal substitution between high- and low-carbon resources takes place in the constrained economy, we divide (5) for the low-carbon input by that for the high-carbon input and rewrite the result, to derive the following expression:

$$\frac{\eta_H}{\eta_L} \left(\frac{R_H}{R_L}\right)^{-1/\sigma} = (1 - \zeta) \frac{p_{RH}}{p_{RL}} + \zeta \frac{\varepsilon_H}{\varepsilon_L}, \quad (15)$$

where $\zeta = p_{Z\varepsilon_L}/(p_{RL} + p_{Z\varepsilon_L})$ is the share of pollution costs in the user price of low-carbon resources. This equation reveals that relative demand for energy sources depends on the relative
user price, which is a weighted average of relative scarcity rents and relative pollution costs. Relative scarcity rents \((p_{RH}/p_{RL})\) and pollution costs \((\varepsilon_H/\varepsilon_L)\) are constant over time (see (12)). However, the share of pollution cost in the user price \(\zeta\) gradually falls, since scarcity rents increase and the price of pollution permits falls. As a result, the relative user price of high-carbon resources changes over time, thus inducing intertemporal substitution.

Whether the relative user price rises or falls depends on the sign of \(\varepsilon_H/\varepsilon_L - p_{RH}/p_{RL}\) (see (15)). If \(\varepsilon_H/\varepsilon_L < p_{RH}/p_{RL}\), the relative user price of high-carbon resources increases over time. Intuitively, with this inequality the high-carbon resource is relatively costly mainly because of scarcity cost rather than pollution cost, and this resource benefits the least from lower pollution costs. Users then gradually substitute towards the low-carbon resource during the period that the emissions constraint is binding. This case arises if the inequality in (14) is satisfied.\(^4\) In the opposite situation, with \(\varepsilon_H/\varepsilon_L > p_{RH}/p_{RL}\) and (14) holding with reverse inequality, the high-carbon resource mainly benefits from pollution price reductions and users gradually substitute to the high-carbon resource.

We conclude this section by a comparative static result. As climate change agreements typically specify fixed-term installments of pollution reduction and are subject to renegotiation, it is relevant to study the effects of a change in the stringency of the pollution cap. If the emission constraint becomes tighter, pollution costs become a more important determinant in the cost of resource use as compared to scarcity rents, ceteris paribus. As a consequence the relative extraction rate jumps closer towards \(\bar{S}\) (where \(\bar{S}\) is the level that would apply if scarcity did not matter), as is stated by the following proposition:

**Proposition 2.** Suppose a binding \(\text{CO}_2\) constraint is unexpectedly further tightened, and let input \(i\) be the relatively scarce input: \(S_{i0}/S_{j0} < (\eta_i\varepsilon_j/\eta_j\varepsilon_i)\). Then, compared to the case with

\(^4\)If \(S_{H0}/S_{L0} < \bar{S}\), we have \(R_{H}/R_{L} < \bar{S}\), from (A.5) in the appendix, and then \(\varepsilon_H/\varepsilon_L < p_{RH}/p_{RL}\), from (A.6).
the initial (looser) constraint,

1. the economy is constrained for a longer period;

2. relative extraction jumps further towards the relatively scarce input;

3. \( S_i/S_j \) is lower at the instant the constraint ceases to be binding, and hence relative extraction \( R_i/R_j \) will be lower when unconstrained;

4. the relative scarcity rent \( p_{R_i}/p_{R_j} \) jumps further upwards;

5. the carbon-intensity of output jumps further downwards.

Proof. See Appendix.

With a more stringent constraint, fewer resources can be extracted so that it takes longer before unconstrained emissions are below the level of the ceiling and the economy is constrained for a longer period. Furthermore, the tighter constraint induces the economy to further increase the productivity per unit of emissions. The resulting relative extraction rate and relative resource rent are closer to the level (viz. \( \bar{S} \)) that would apply in an economy in which pollution only (rather than scarcity) would matter.

5 The empirical relevance of the “dirty-first condition”

The necessary condition for the relative use of high-carbon inputs to go up (our “dirty-first result”) is, as given in inequality condition (14), that the high-carbon input is relatively scarce in a physical sense, but relatively productive in terms of its marginal contribution to output per unit of CO\(_2\) emissions. We now want to explore whether this inequality could hold in reality. We use data on prices, consumption, and stocks of coal, oil and gas, for the period 1984-2005.
(1987-2005 for coal due to availability of data on coal prices), to see for which fuels the inequality (14) holds.\(^5\)

Productivity parameters \(\eta_i\) in (14) cannot be directly observed, but can be derived from observed equilibrium prices and quantities: assuming the data reflect a zero pollution tax and using the firms’ optimality conditions (5) (with \(p_Z = 0\)) to eliminate \(\eta_i\), we can rewrite (14) as

\[
\frac{S_H(t)}{S_L(t)} < \frac{R_H(t)}{R_L(t)} \frac{\left(\frac{p_{RH}(t)}{\varepsilon_H} / \frac{p_{RL}(t)}{\varepsilon_L}\right)^\sigma}{}. \tag{16}
\]

A first look at the data shows that roughly the following relations hold: \(S_{\text{coal}}(t) \gg S_{\text{oil}}(t) \approx S_{\text{gas}}(t)\); \(R_{\text{oil}}(t) \gg R_{\text{coal}}(t) > R_{\text{gas}}(t)\); \(\frac{p}{\varepsilon_{\text{oil}}}(t) \approx \frac{p}{\varepsilon_{\text{gas}}}(t) > \frac{p}{\varepsilon_{\text{coal}}}(t)\). First we consider the combination with \(H = \text{coal}\) and \(L = \text{oil}\): the left-hand side of the inequality in (16) exceeds unity and the right-hand side is smaller than unity (for any \(\sigma \geq 0\)). Hence, the inequality (the ”dirty-first condition”) does not hold and we conclude that, according to the data, climate policy will induce substitution from high-carbon coal to low-carbon oil. If we make the same comparison for \(H = \text{coal}\) and \(L = \text{gas}\), we see that with \(\sigma \geq 0\) the inequality is again likely to be violated. Hence the data suggest that, after the introduction of a ceiling on the amount of \(\text{CO}_2\) emitted, there will be substitution from high-carbon coal towards low-carbon gas. With \(H = \text{oil}\) and \(L = \text{gas}\), however, the inequality in (16) is likely to hold. That is, the data suggest that climate policy induces substitution from low-carbon gas to high-carbon oil.

In the next step, we looked at the inequality in (16) for individual years. With a production function with coal and oil as inputs, we then find that the inequality is indeed violated for any \(\sigma \geq 0\), for all years, and the pattern of substitution is towards the low-carbon input oil. The same result holds when \(H = \text{coal}\) and \(L = \text{gas}\): climate policy induces substitution from

\(^5\)We used data from the 2006 BP Statistical Review of World Energy, available at http://www.bp.com/statisticalreview. We converted all data in Million Tonnes of Oil Equivalents. We use relative emission coefficients that are compatible with US and German data. An appendix with further details on data collection, the calibration, and regressions, is available from the authors upon request.
high-carbon input coal towards the low-carbon input gas. However, when $H = \text{oil}$ and $L = \text{gas}$, the results are indecisive. For 11 out of our 22 observations we find that the result depends on the size of $\sigma$, while for the other half of our observations the inequality holds for any $\sigma \geq 0$ (hence substitution towards the high-carbon input). In the former case the inequality holds for values of $\sigma$ that are not too large, where the critical value of $\sigma$ ranges from 0.6 to 17.5.

As a final exploration, we used our data to estimate the elasticity of substitution between oil and gas. We used both country-level panel data and world-level time series data to estimate productivity parameters and both short-run and long-run elasticities of substitution. All regressions that report a positive value for the elasticity of substitution, and for which we cannot reject the null hypothesis of no autocorrelation, report an elasticity of substitution between oil and gas that is sufficiently low for the inequality in (16) to hold. Hence, the regressions suggest that, following from proposition 1, with a ceiling on carbon dioxide emissions, it is optimal to substitute from low-carbon gas towards high-carbon oil.

In sum: Our data suggest that both oil and gas are more productive per unit of CO$_2$ than scarce, relative to coal, and hence climate policy is likely to induce substitution from the high-carbon fuel coal to the low(er)-carbon inputs oil and gas. However, according to our data the marginal productivity of carbon coming from the use of oil is higher than the marginal productivity of carbon coming from gas, while the two resources are roughly equally scarce in a physical sense. As our theory suggests, this would make it optimal to substitute from gas towards oil when climate policy constrains CO$_2$ emissions, and the ”dirty-first result” might be of more than just theoretical interest.
6 Announcement effects

We now investigate how the economy reacts to an emission constraint in the case that agents anticipate the actual implementation of the policy.\textsuperscript{6} In particular, we study the path of resource extraction for the situation in which the carbon constraint starts to be effective at time $t_K > 0$, but is announced at time $t = 0$, so that preparations can be made over the period $t \in (0, t_K)$.

Agents maximize the same objective functions subject to the same constraints as in the previous section, with the only difference that the constraint (4) is now binding from $t = t_K$ instead of $t = 0$. The resulting path of relative extraction can be characterized by the following proposition:

**Proposition 3.** Suppose a CO\textsubscript{2} emission constraint is announced before it is actually implemented. Then,

1. if $S_{H0}/S_{L0} < (> ) S$,

   (a) relative extraction $R_H/R_L$ (i) jumps down (up) at the announcement, (ii) stays constant until actual implementation, (iii) jumps up (down) at actual implementation and (iv) gradually declines (increases) until the pollution constraint ceases to be binding, attaining the level it had before implementation;

   (b) the high-carbon resource stock (i) gets depleted less fast (faster) than the low-carbon resource stock between announcement and start of implementation; (ii) the opposite happens when the pollution constraint is binding;

   (c) the level of emissions as well as emissions per unit of GDP jump down at implementation;

\textsuperscript{6}Kennedy (2002) also studies the effect of an announced emission constraint. Using a two-period model without resources he shows that it may be optimal for a small country to reduce emissions before the 2008-2012 commitment period, either because of co-benefits (e.g. reductions in emissions of other pollutants than CO\textsubscript{2} that go together with a reduction in fossil fuel combustion) or because early investments in physical capital help reducing adjustment costs.
2. If $S_{H0}/S_{L0} = \bar{S}$, (a) relative extraction and relative stocks remain constant forever, and 
(b) neither emissions nor emissions per unit of GDP jump at the time of implementation.

Proof. See Appendix.

The proposition implies that the announcement of an emission constraint at a future date immediately causes a drop in the rate of extraction of the relatively more productive resource (in terms of GDP per unit of emissions) and a rush on resources that will be used less after implementation. As a consequence the constrained period starts with (relatively) more of the productive resource, and resource owners of the other resource face a smaller loss (i.e. a smaller drop in scarcity rent), as compared to the situation without announcement. At the instant the constraint becomes binding the extraction rate of the productive input jumps up, and from then on relative extraction develops as would be the case with an unanticipated constraint.

We illustrate the extraction paths for the case where $S_{H0}/S_{L0} < \bar{S}$ in Figure 2 by the thick arrows. For the same case, Figure 3 illustrates the development of relative extraction and relative stocks over time. Initially relative extraction is below relative stocks, causing an increase in the latter, while after the introduction of the constraint relative extraction jumps up to a level higher than that of the relative stocks, and hence the latter decline until relative extraction and relative stocks are equal at the instant that the constraint ceases to be binding (part 1 of proposition 3).

At the time the constraint is implemented, the economy substitutes towards the more productive resource (in terms of GDP per unit of CO2), while keeping output constant (a jump in relative extraction along the production isoquant from $R_{H}(0)/R_{L}(0)$ to $R_{H}(t_K)/R_{L}(t_K)$ in Figure 2). As a consequence, the economy’s pollution intensity $Z/Y$ decreases. Since the introduction of the constraint is expected and fully anticipated, consumption cannot jump and substitution takes place along a production isoquant, changing emissions but not the level of
output of the final good. This is in contrast with the case without announcement in which both emissions and output jump at the instant the constraint is introduced.

7 Alternative policies and technical change

In this section we check whether our results, and particularly the possibility of a “dirty-first” result, are robust with respect to alternative policies (a stock constraint and an emission intensity constraint) and to the introduction of technological change in the model.

7.1 Stock and emission intensity constraints

The emissions reduction policy studied so far constrained the flow of pollution, as the simplest interpretation of the Kyoto protocol. However, it is widely recognized that not the flow but the stock of cumulative emissions, or CO$_2$ concentration levels, should be the criterion of sound
climate change policy. Moreover, even a flow constraint can be combined with a flexibility provision that firms could “bank” emission permits, allowing them to keep permits for later use or borrow against the future. To check how our results could change with an emissions concentration target or banking policy, we study how a permanent constraint on cumulative emissions affects relative extraction of high- and low-carbon resources.

We denote cumulative emissions by $X$, so that $\dot{X} = Z$. The policy that is announced and implemented at time zero caps cumulative emissions, $X(t) \leq \bar{X}$, at any point in time. The amount of pollution permits introduced at time 0 in the market equals $\bar{X} - X(0) > 0$; the permits are bankable and tradable. We assume that the constraint is binding at introduction, which requires that cumulative pollution from unconstrained resource use exceeds the amount of permits, i.e. $X(0) + \varepsilon_H S_H(0) + \varepsilon_L S_L(0) \geq \bar{X}$.

Each unit of CO$_2$ emissions reduces the remaining stock of permits. Hence, the stock of
permits is like a non-renewable resource and the permit price, $p_Z$, must grow at rate $r$ to make owners of permits indifferent between selling now or selling in future. Now the users of permits, the producers, face resource price as well as pollution prices growing at the same rate $r$, so that the user price, at the right-hand side of first-order condition (5), grow at rate $r$ as well and the relative user price of the two resources stays constant over time. Hence, while the flow constraint induced substitution over time, the stock constraint fixes relative resource use over time. The question now is whether the relative use of high-carbon inputs could be higher under the stock constraint than in the unconstrained economy, in which case we would find again the “dirty-first result”.

With both resource inputs growing at the same rate, we find – as demonstrated already above for the unconstrained economy – that $\widehat{R}_H = \widehat{R}_L = \hat{\bar{Y}} - \hat{A} = (r - \rho) - r = -\rho$ so that cumulative extraction of resource $i$ and cumulative pollution from resource $i$ after $t = 0$ equal $R_i(0)/\rho$ and $\varepsilon_i R_i(0)/\rho$, respectively. In addition, relative resource extraction $R_H/R_L$ is constant over time, even though the economy is constrained. The market sector now chooses levels of resource inputs so as to maximize net present value of output under the constraints that cumulative extraction does not exceed available resources, and cumulative pollution equals available emissions permits. Using our solutions for the growth rates of resource input and interest rate, we can write the maximization problem as a static one (at time $t = 0$; we omit this time indicator):

$$\text{Max. } AF(R_H, R_L)/\rho, \text{ s.t. } R_i \leq \rho S_i, \sum_i \varepsilon_i R_i = \rho (\bar{X} - X)$$  \hspace{1cm} (17)

where $F(.)$ is the CES function in (3). From the solution of (17) we derive the following:

**Proposition 4.** Suppose a binding stock constraint is unexpectedly introduced. Then

1. relative extraction $R_H/R_L$ jumps up (down) if $S_{H0}/S_{L0} < (>\bar{S})$, and
2. leaves relative extraction unaffected if $S_{H0}/S_{L0} = \bar{S}$.

**Proof.** See Appendix.

Hence, under exactly the same conditions as under the flow constraint, $S_{H0}/S_{L0} < \bar{S}$, also the stock constraint induces the economy to use relatively more of dirty input.

Note that a constraint on cumulative emissions is not equivalent to an emissions concentration target, since it abstracts from decay of the emissions stock in the atmosphere that comes from ocean CO$_2$ uptake and other carbon sinks. If we model the change in CO$_2$ concentrations, $C$, in the simplest possible way as the balance between emissions and proportional decay, viz. $\dot{C} = Z - \delta C$, and assume a policy that caps emissions forever by imposing $C(t) \leq \bar{C}$, the equilibrium path for relative extraction has features of both the stock-constraint path and the flow-constraint path. Initially, $C(t) < \bar{C}$, so the concentration level can increase but a rising pollution price reflects that the ceiling is being approached, like in the stock constraint case. Once concentrations hit the ceiling, the flow of pollution is restricted to total decay ($Z = \delta C$) until resource stocks are so small that unrestricted resource use results in low pollution levels and declining concentrations ($Z = \rho \sum \varepsilon_i S_i < \delta \bar{C}$), like in the flow constraint case. Again, the dirty-first result will appear for $S_{H0}/S_{L0} < \bar{S}$.

As an alternative route to mitigate climate change, one that is claimed to be politically more attractive, there have been proposals to set targets for emissions intensity (in particular in the USA when it voted down the Kyoto Protocol and in Canada recently). In our model this implies an upper bound on $Z/Y$. Recall that $R_H/R_L = \bar{S}$ minimizes $Z/Y$ and that, because of the linear homogeneity of the production function, $Z/Y$ increases with $|R_H/R_L - \bar{S}|$. Hence, the equilibrium relative extraction rate must be close enough to $\bar{S}$ under an intensity constraint. Starting from an unconstrained equilibrium in which high-carbon inputs have the highest productivity per unit of CO$_2$ ($S_{H0}/S_{L0} < \bar{S}$), the economy will satisfy a (binding)
intensity constraint by increasing relative high-carbon use. Hence, our dirty-first result shows up under the same conditions as with flow or stock constraint.

In sum, we find that however pollution is constrained (as a flow, stock, atmospheric concentration, or per unit of GDP alike), the economy starts using more of the resource input that has the highest marginal productivity per unit of pollution. This input is the one with high CO\textsubscript{2} emissions per unit of energy if its physical scarcity (relative to productivity) forces unconstrained use of it to be small (i.e. if $S_{H}/S_{L0} < \bar{S}$).

### 7.2 Technological change

One could wonder whether technological change affects the “dirty-first” result that it might be optimal to substitute towards the high-carbon input, after the introduction of climate policy. While we saw that neutral technological change, $\hat{A}$, has no impact on the relative use of the two resources, this changes with non-neutral or biased technical change, to be modeled by different rates of increase in $\eta_H$ and $\eta_L$. An increase in $\eta_H/\eta_L$ implies dirty-input-using technological change: the prospect of higher relative productivity of the high-carbon input in the future induces users to postpone use of this resource. Compared to the situation with neutral technological change, dirty-input-using technological change would shift the use of the high carbon input to the future and would partly offset any dirty-first effect of a emissions constraint. However, if technological change has a high-carbon-saving bias (causing $\eta_H/\eta_L$ to decrease), the opposite would happen: frontloading of the high-carbon input, as compared to the neutral technological change case, and reinforcing any dirty-first result.

The interesting question is therefore whether high-carbon-using or high-carbon-saving technological change (i.e. an increase in $\eta_H/\eta_L$) is the likely equilibrium outcome after the introduction of climate policy. To answer this question we need a model of endogenous innovation, for example along the lines of the model of directed technological change by Acemoglu (2002).
Although the full development of such a model is left for future research, we can try to use the following general insight from Acemoglu’s model without natural resources: when the use of factor $x$ increases relative to factor $y$, innovation tends to be factor-$x$ using (see also Di Maria and Smulders, 2004). This suggests that if users tend to shift to high-carbon inputs in immediate reaction to the emissions constraint (our dirty-first result), innovation becomes high-carbon-using. However, later on relative use of the high-carbon input must be necessarily lower than without the emissions constraint, which will trigger high-carbon-saving technological change. As a result, the productivity of the high-carbon input will be higher especially in the medium-run, but not in the short-run (innovation takes time) and not in the long-run, when innovation becomes pollution-saving (all in comparison to the unconstrained case). The optimal reaction is then to concentrate extraction and use of the high-carbon resource in the medium-run, rather than the short-run and the long-run, as compared to the case without endogenous biased technological change. We therefore expect that endogenous technological change mitigates the reaction of relative extraction to the emissions constraint ($R_H/R_L$ stays closer to the unconstrained level), but that the direction of the change in relative use as well as the conditions for a dirty-first result are not affected.

8 Concluding remarks

In reaction to a ceiling on the amount of carbon dioxide emissions an economy may want to substitute between high-carbon and low-carbon fuels. We have shown that in the standard Hotelling model extended with a second, imperfectly substitutable resource, the economy optimally decreases $CO_2$ intensity of GDP. However, this is not always obtained through substitution of low-carbon for high-carbon inputs (e.g. natural gas for oil). Since producers want to maximize output, given the emission constraint, resource users initially substitute towards the input
which, at the margin, has the highest level of output per unit of carbon dioxide. This may be
the input with most CO₂ emissions per unit of energy, in particular when this input is physically
relatively scarce: it is then used in production at relatively low levels and hence diminishing
returns cause its productivity to be relatively high. With an anticipated constraint, the reaction
is more complex: the economy switches towards the less productive input (in terms of GDP
per unit of carbon) before the constraint becomes binding and jumps towards a relatively more
intensive use of the more productive input when the emission ceiling becomes binding.

A preliminary empirical investigation indicates that it is optimal to substitute away from
coal towards gas and oil, but also at the same time to substitute away from low-carbon input
gas towards high-carbon input oil. Hence, in order to cope with climate change, energy policies
should not necessarily be directed to a fast transition to low-carbon energy sources. In addition
to relative pollution content, scarcity of resources as well as their productivity differences, as
shaped by substitution possibilities, should be taken into account.

The general insight from our analysis is that incorporating scarcity and intertemporal sub-
stitution in extraction into the analysis of pollution constraints may revert conclusions from the
usual static models. The limited substitution between energy resources in production plays an
essential role as well: demand factors are crucial in determining to which resource the economy
should substitute to minimize the cost of climate change policy. These factors include the sec-
toral composition of the economy and the degree to which technologies of energy users is biased
to a particular type of energy.

For future research it is interesting to consider the role of induced technological change in
more detail, as well as that of extraction costs, uncertainty, and strategic supply reactions from
monopolistic resource owners. A more detailed calibration or estimation of the model then
becomes possible as well.
References


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A Appendix

We simplify notation using variables without subscripts to denote high-carbon to low-carbon ratios: \( R(t) \equiv R_H(t)/R_L(t) \), \( S(t) \equiv S_H(t)/S_L(t) \) and \( p(t) \equiv p_{RH}(t)/p_{RL}(t) \), and similarly \( \eta \equiv \eta_H/\eta_L \), \( \varepsilon \equiv \varepsilon_H/\varepsilon_L \), and \( S_0 \equiv S_{H0}/S_{L0} \). For any variable \( x \) we define \( x(\tau^-) \equiv \lim_{t\uparrow\tau} x(t) \) and \( x(\tau^+) \equiv \lim_{t\downarrow\tau} x(t) \).

Before proving the propositions, we present and prove the following lemma, which summarizes the dynamics of relative extraction \( R \) over three relevant time periods: when the constraint is announced but not yet effective, when the constraint binds, when the constraint is not binding anymore.

**Lemma 3.** Let \( t = 0 \) be the instant at which the constraint is announced, \( t_K \) be the instant at which the constraint becomes binding, and \( t_U \) the instant at which the constraint ceases to be binding. Then without further shocks

\[
R(t) = S(t_U), \quad \forall \ t \in (0,t_K)
\]  

(A.1)
\[ R(t) = R(T_U^-), \forall t \geq T_U. \]  

(A.2)

\[ \frac{dR}{dt} = f(R) \left[ R^{1/\sigma} - \eta/\varepsilon \right], \forall t \in (t_K, T_U), \]  

(A.3)

where \( f \) is a function of \( R \) and parameters with \( f > 0 \) and \( \partial f / \partial \bar{Z} = 0 \) for all \( R > 0 \),

\[
\int_{t}^{T_U} \left( \frac{1}{1 + \varepsilon R(\tau)} - \frac{1}{1 + \varepsilon S(t)} \right) d\tau + \left( \frac{1}{1 + \varepsilon R(T_U)} - \frac{1}{1 + \varepsilon S(t)} \right) \frac{1}{\rho} = 0, \forall t \in [t_K, T_U) \quad (A.4)
\]

\[ \frac{dR(t)}{dt} \preceq 0 \iff (\eta/\varepsilon)^{\sigma/\varepsilon} \preceq R(t) \preceq S(t) \preceq R(T_U), \forall t \in (t_K, T_U). \]  

(A.5)

Proof. For all \( t \in [0, t_K) \cup [T_U, \infty) \) we have \( p_Z = 0 \) and, from (5), \( p(t) = \eta (R(t))^{-1/\sigma} \). For all \( t \geq T_U \), we have, from (11), \( R(t) = S(t) \). Since \( p \) is constant over time (see (7)), we find 
\[ p(t) = \eta (S(T_U))^{-1/\sigma} \forall t; \] this proves (A.1).

Prices cannot jump in absence of unexpected events due to arbitrage. Then \( R \) can only jump if output \( Y \) jumps (see (5)), which is ruled out by the concavity of the utility function. This proves (A.2).

To derive (A.3), substitute one of the first-order conditions (5) into the other to eliminate \( p_Z \), and rewrite:

\[ \frac{A^{(\sigma - 1)/\sigma} \eta_L}{p_L} \left( \frac{Y}{R_L} \right)^{1/\sigma} = \frac{1 - p/\varepsilon}{1 - R^{-1/\sigma} \eta/\varepsilon}. \]  

(A.6)

Time differentiate and substitute (7) and (2) to replace \( \widehat{p_L} \) by \( \widehat{Y} + \rho \): 

\[ (\sigma - 1)\widehat{A} - \sigma \left( \widehat{Y} + \rho \right) + \widehat{Y} - \widehat{R}_L = \frac{1}{1 - R^{1/\sigma} \varepsilon/\eta} \widehat{R}. \]  

(A.7)

Define \( \theta_L = (1 + \eta R^{1-1/\sigma})^{-1} \) and \( \lambda_L = (1 + \varepsilon R)^{-1} \), which are the production elasticity and
share in total pollution of the low-carbon input, respectively. This implies:

\[
\frac{1}{1 - R^{1/\sigma_\mathcal{E}/\eta}} = \frac{\lambda_L (1 - \theta_L)}{\lambda_L - \theta_L}.
\]  

(A.8)

Time differentiating the binding emission constraint (4), we find \( \tilde{R}_H = \lambda_L \tilde{R} \) and \( \tilde{R}_L = -(1 - \lambda_L) \tilde{R} \).

Time differentiating the production function and inserting the two expressions from the emission constraint, we find:

\[
\tilde{Y} = \tilde{A} + (\lambda_L - \theta_L) \tilde{R}.
\]  

(A.9)

Substituting (A.8) and (A.9) into (A.7) and rearranging, we find:

\[
\tilde{R} = \frac{(\theta_L - \lambda_L) \sigma \rho}{(\theta_L - \lambda_L)^2 \sigma + \theta_L (1 - \theta_L)}.
\]  

(A.10)

The left-hand side of (A.6) is positive, so that \( \text{sign}(\varepsilon - p) = \text{sign}[R - (\eta/\varepsilon)^\sigma] \). Since \( p \) and \( \varepsilon \) are constant over time, \( [R - (\varepsilon/\eta)^\sigma] \) cannot switch sign. Since, from (A.8), \( \text{sign}(\theta_L - \lambda_L) = \text{sign}[R - (\eta/\varepsilon)^\sigma] \), we can write (A.10) as in (A.3). This proves (A.3).

To derive (A.4), we note that the definitions of \( Z, R \) and \( S \) imply \( \frac{\varepsilon L S_L + \varepsilon H S_H}{\varepsilon L S_L + \varepsilon H S_H} Z \left( \frac{1}{1 + \varepsilon R} - \frac{1}{1 + \varepsilon S} \right) = \frac{R_H}{S_L} - \frac{R_H}{S_H} \). Evaluating \( Z \) and \( R \) at time \( \tau \) and \( S \) at time \( t \), and integrating over \( \tau \) from \( t \) to infinity, the right-hand side becomes zero because of full depletion, so that, after dividing out a positive term, we may write \( \int_t^\infty Z(\tau) \left[ \frac{1}{1 + \varepsilon R(\tau)} - \frac{1}{1 + \varepsilon S(t)} \right] d\tau = 0 \). For \( \tau > t_K \), \( Z(\tau) = \tilde{Z} \) up till \( T_U \) and \( Z(\tau) = \tilde{Z} e^{\sigma (T_U - \tau)} \) after \( T_U \) and \( R \) is constant after \( T_U \) and continuous at \( T_U \), according to (A.2). Then the above integral can be rewritten as in (A.4).

To proof (A.5), note that (A.4) implies that if \( R \) monotonically decreases over time, then \( R(t) \) must first exceed, but eventually fall short of \( S(t) \). More generally, for \( \forall t \in (t_K, T_U) \), we have: if \( dR(\tau)/d\tau \leq 0, \forall \tau \in (t, T_U) \), then \( R(t) \geq S(t) \geq R(T_U) \). Equation (A.3) shows that,
indeed, \( dR/dt \) cannot switch sign between \( t_K \) and \( T_U \). Hence we have (A.5).

\[ \square \]

### A.1 Proof of proposition 1

Prior to the unexpected constraint \( (t < t_K = 0) \), the economy acts like the unconstrained economy, so that, from (11), \( R(0^-) = S(0^-) = S(0) \). Then part 1(b) follows from (A.5) with \( t_K = 0 \). Part 1(c) follows from (A.2) and (A.5). Part 1(a) follows from 1(c) and (12). From stock dynamics (6) we derive

\[
\frac{dS}{dt} = \frac{R_L}{S_L} (S - R) .
\]

(A.11)

Combined with part 1(b) of the proposition, this proves 1(d). This completes the proof of part 1 of proposition 1. The proof of part 2 is analogous.

Finally we prove part 3 using lemma 2. From 1(c) and (11), we have \(|R(0^-) - \bar{S}| = |S_0 - \bar{S}| > |R(0^+) - \bar{S}|\) and with lemma 2 this proves the downward jump. The ”increase over time” follows from 1(b). From 1(c) and (11), we have \(|R(0^-) - \bar{S}| = |S_0 - \bar{S}| < |R(T) - \bar{S}|\). With lemma 2, this proves the higher end-level. The last part follows from (11).

### A.2 Proof of proposition 2

Denote by \( \bar{Z}^o \) the ”old” constraint that is introduced at \( t = 0 \) and which would, in the absence of shocks, cease to bind at \( T^o \). Denote by \( \bar{Z}^n \) the ”new” constraint that at time \( t^n \) unexpectedly replaces \( \bar{Z}^o \), where \( \bar{Z}^o > \bar{Z}^n \), and ceases to bind at \( T^n \).

We prove part 1 by using the procedure we used for the proof of lemma 1 and derive \( T^n \)
from (9), (10), and (A.2) in the following way:

\[
\varepsilon_H S_{H0} + \varepsilon_L S_{L0} = [\varepsilon_H (S_{H0} - S_H(t^n)) + S_H(t^n) - S_H(T^n)] \\
+ \varepsilon_L (S_{L0} - S_L(t^n)) + S_L(t^n) - S_L(T^n)] \\
+ \varepsilon_H S_H(T^n) + \varepsilon_L S_L(T^n) \\
= t^n \tilde{Z}^o + (T^n - t^n) \tilde{Z}^n + \varepsilon_H \frac{R_H(T^n)}{\rho} + \varepsilon_L \frac{R_L(T^n)}{\rho}
\]

\[
\frac{\varepsilon_H S_{H0} + \varepsilon_L S_{L0}}{Z^o} - \frac{1}{\rho} = t^n + (T^n - t^n) \frac{\tilde{Z}^n}{Z^o} + \frac{Z^n}{Z^o} \frac{1}{\rho} - \frac{1}{\rho}
\]

\[
T^n - T^o = \left(t^n - T^n - \frac{1}{\rho}\right) \frac{\tilde{Z}^o - \tilde{Z}^n}{Z^o}
\]

This explicitly solves for \(T^n\). Since by assumption the new constraint is binding when introduced, we must have \(t^n < T^n\), and hence \(T^n \leq T^o \iff \tilde{Z}^o \leq \tilde{Z}^n\), which proves part 1.

We prove parts 2-4 for \(S_{H0}/S_{L0} < (\eta/\varepsilon)^\sigma \equiv \bar{S}\) only; the other cases are analogous. We continue the notation of the proof of proposition 1. Since \(\partial f/\partial \tilde{Z} = 0\) in (A.3), a decline in \(\tilde{Z}\) affects the equilibrium path of \(R(t)\) only through an increase in \(T_U\). Write \(R^o(t)\) and \(R^n(t)\) for relative extraction with the old and the new value for \(\tilde{Z}\) respectively. Suppose the unexpected change in the constraint would not on impact change relative extraction, i.e. \(R^n(t^n+) = R^o(t^n+)\).

Then, from (A.3), \(R^n(t) = R^o(t) \forall t \in (t^n, T^n]\), but \(R^n(t) < R^o(t) \forall t \in (T^n, T^n]\) and the integral at the left-hand side of (A.4) with \(R = R^n\), \(t = t^n\) and \(T_U = T^n\) exceeds the integral with \(R = R^o, t = t^n\) and \(T_U = T^o\). But this violates the equality in (A.4) for the new path. If \(R^n(t^n+) < R^o(t^n+)\), then the integral for the new path is positive a fortiori. Hence, we must have \(R^n(t^n+) > R^o(t^n+)\), which proves part 2 of the proposition.

We prove part 3 in a similar way. Suppose \(R^n(T^n) = R^o(T^o)\), then \(R^n(t) = R^o(t - T^n + T^o)\) for \(t \in (t^n + T^n - T^o, T^n]\) and \(R^n(t) > R^o(t^n)\) for \(t \in (t^n, t^n + T^n - T^o]\). But then (A.4) is violated on the new path since the integral becomes negative. A fortiori (A.4) is violated with \(R^n(T^n) > R^o(T^o)\). Hence we must have \(R^n(T^n) < R^o(T^o)\). From (11) it follows that
\[ S^n(T^n) < S^o(T^o) \], which proves part 3.

Combining the results in part 3 with (A.1), we find \[ R^o(T^o) = S^o(T^o) > S^n(T^n) = R^n(T^n). \]

From (12), we then have \[ p^o(T^o) < p^n(T^n). \]

Part 5 directly follows from part 3 of proposition 1.

### A.3 Proof of proposition 3

Suppose the constraint is announced at \( t = 0 \), becomes binding at \( t = t_K > 0 \) and ceases to be binding at \( t = T_A \).

Assume that \( S(t_K) < (\eta/\varepsilon)^\sigma \equiv \bar{S} \). Then, from part 1 of proposition 1 and (A.11), we have

\[ S(t_K) > S(T_A). \]  \hspace{1cm} (A.12)

Suppose \( S_0 \leq R(0^+) \). Then from (A.1), (A.2), and (A.11) the relative stock has to jump up at \( t = t_K \) for (A.12) to hold, which violates continuity of stocks. So \( S_0 > R(0^+) = S(T_A) \). It follows from (A.11) that \( dS/dt > 0 \ \forall \ t \in (0, t_K) \) so that \( S_0 < S(t_K) \). Since we started from the assumption \( S(t_K) < \bar{S} \), we must have \( S_0 < \bar{S} \). The reasoning for the cases \( S(t_K) \geq \bar{S} \) are analogous. This proves parts (i) and (ii) of part 1(a), part (i) of part 1(b), and part 2(a) of the proposition; parts 1(a) (iv) and 1(b)(ii) follow from part 1 of proposition 1.

Arbitrage prevents resource rents to jump, so from (7) \( r \) is finite and from (2) income cannot jump at any \( t > 0 \). Since the emissions constraint starts to be binding at \( t = t_K > 0 \) by construction, either emissions jump down and hence emissions per unit of income jump down at \( t = t_K \), or emissions do not jump and neither do emissions per unit of output. The latter requires that relative extraction does not jump, which requires \( S_0 = \bar{S} \), see part 2 (a) of this proposition. The former, a jump down in emissions with continuous income, requires a jump along a production isoquant. Hence \( Z/Y \) jumps down and \( R_L \) and \( R_H \) jump in opposite
directions. Continue with the case $S_0 < \bar{S}$ (the proof for the other case is similar). Then from part 1(a), we have $R(0^+) = R(t_K^-) = S(T_A) < \bar{S}$, and $R(t_K^+) > S(t_K) > S(T_A)$, so that $R(t_K^+) > R(t_K^-)$. That is, at time $t_K$, $R$ jumps up, so that $R_H$ jumps up and $R_L$ jumps down. This proves parts 1(b)(iii), 1(c), and 2(b).

A.4 Proof of proposition 4

First, a binding constraint implies $\varepsilon_H S_H(t) + \varepsilon_L S_L(t) > \bar{X} - X(t)$, where the rate of change of both the left-hand side and the right-hand side equals $Z(t)$, so that if the inequality holds at $t = 0$, it holds at all $t > 0$. This allows us to drop the time indicator. Second, in section 7 we have shown that with the pollution constraint binding, we have $\varepsilon_H R_H + \varepsilon_L R_L = \rho (\bar{X} - X)$. Combining both results, we may write:

$$R_H = \rho S_H \iff R > S; R_L = \rho S_L \iff R < S. \quad (A.13)$$

Now we define $R_L = \rho (\bar{X} - X(0)) / (\varepsilon_L S) = R_H \equiv R_H S$, where $\{R_H, R_L\}$ are the extraction rates that give the highest possible level of output when constrained. Then the solution to (17) reads and implies:

1. if $S_H > R_H / \rho$ and $S_L > R_L / \rho$ then $R_H = R_H$ and $R_L = R_L$ so that $R = \bar{S}$;
2. if $S_H < R_H / \rho$ then $R_H = \rho S_H < R_H$ and $R_L = R_L + \varepsilon (R_H - \rho S_H) > R_L$ so that, given (A.13), $S < R < \bar{S}$;
3. if $S_L < R_L / \rho$ then $R_L = \rho S_L < R_L$ and $R_H = R_H + \varepsilon (R_L - \rho S_L) > R_H$ so that, given (A.13), $S > R > \bar{R}$.

Line 1 (line 2 and 3) proves the statements in part 2 (1) of the proposition.
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