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Working Paper

The Computational Difficulty of Bribery in Qualitative Coalitional Games

Nota di Lavoro, Fondazione Eni Enrico Mattei, No. 100.2007

Provided in Cooperation with:
Fondazione Eni Enrico Mattei (FEEM)

Suggested Citation: Dowell, Andrew; Wooldridge, Michael; McBurney, Peter (2007) : The Computational Difficulty of Bribery in Qualitative Coalitional Games, Nota di Lavoro, Fondazione Eni Enrico Mattei, No. 100.2007

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NOTA DI LAVORO 100.2007

November 2007
CTN – Coalition Theory Network

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Summary

Qualitative coalitional games (QCG) are representations of coalitional games in which self interested agents, each with their own individual goals, group together in order to achieve a set of goals which satisfy all the agents within that group. In such a representation, it is the strategy of the agents to find the best coalition to join. Previous work into QCGs has investigated the computational complexity of determining which is the best coalition to join. We plan to expand on this work by investigating the computational complexity of computing agent power in QCGs as well as by showing that insincere strategies, particularly bribery, are possible when the envy-freeness assumption is removed but that it is computationally difficult to identify the best agents to bribe.

Keywords: Bribery, Coalition Formation, Computational Complexity

JEL Classification: C63, C78

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The Computational Difficulty of Bribery in Qualitative Coalitional Games *

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November 26, 2007

Abstract
Qualitative coalitional games (QCG) are representations of coalitional games in which self interested agents, each with their own individual goals, group together in order to achieve a set of goals which satisfy all the agents within that group. In such a representation, it is the strategy of the agents to find the best coalition to join. Previous work into QCGs has investigated the computational complexity of determining which is the best coalition to join. We plan to expand on this work by investigating the computational complexity of computing agent power in QCGs as well as by showing that insincere strategies, particularly bribery, are possible when the envy-freeness assumption is removed but that it is computationally difficult to identify the best agents to bribe.

1 Introduction
Coalition formation is an important aspect of group decision making within multi-agent systems and as such is a key issue in multi-agent research [18, 17]. Efforts have been undertaken regarding coalitional formation in an attempt to circumvent some of the computational problems associated with computing agent strategy, for example, computing the Shapley Value and the core solution. Such efforts have involved computing these concepts under different representations of these games. Consequently, there exist many representations of coalitional games including: marginal contribution nets [12]; multi-attribute coalitional games [13]; and weighted threshold games [6]. Marginal contribution nets are a compact representation for coalitional games which consist of rules of the syntactic form: pattern → value, where a rule is said to apply to a group of agents if these agents meet the requirement of the Pattern. Under this representation, there exist algorithms for computing the Shapley value and the core. The Shapley value, in particular, can be computed in time linear in the size of the input.

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Multi-attribute coalitional games are games where the value of a coalition is measured by the attributes of the agents who belong to it. They are represented as a tuple \( \langle Ag, M, A, w \rangle \) where \( Ag \) is the set of agents, \( M \) is the set of attributes, \( A \in \mathbb{R}^{m \times n} \) is the attribute matrix (where entry \( A_{i,j} \) denotes the value of attribute \( i \) for agent \( a_j \)), \( a : \mathbb{R}^{m \times n} \times 2^N \rightarrow \mathbb{R}^m \) is the set of aggregators which take, as input, both a row of the attribute matrix and a coalition \( C \subseteq Ag \) and outputs a vector of values and \( w \) is the aggregate value function which takes, as input, a vector of values and outputs a single real value \( (w : \mathbb{R}^m \rightarrow \mathbb{R}) \). This representation induces a coalitional game \( \langle Ag, v \rangle \) where the value function is defined by \( v(C) = w(a(A, S)) \) for any \( C \subseteq Ag \). There exist positive results for this representation also, including that it does not require as much space as other representations do.

Weighted threshold games are coalitional games given by a set of agents \( Ag \), their non-negative weights \( W \) and a threshold \( T \in \mathbb{Z}^+ \) such that any coalition \( C \subseteq Ag \) is winning (has value 1) if the sum of the weights of the agents in that coalition are greater than or equal to \( T \) and is losing (has value 0) otherwise. They are represented in the form of a tuple \( \langle Ag, W, T \rangle \). Under this representation, determining if the core is non-empty can be done in polynomial time, where as computing the Shapley value is computationally difficult.

In cooperative goal-orientated multi-agent systems (GOMAS), where each individual agent has as set of goals they would like to achieve, agents may believe that they are more likely to achieve their goals by cooperating with other agents and forming coalitions, such that if the agents in the coalition were to act in a similar manner then each one of them should achieve at least one of their goals. Thus, in such a system, it is the strategy of each agent to find the best coalition to join. Qualitative coalitional games (QCG from now on) are representations of GOMAS and for this framework Wooldridge & Dunne [21] investigate how hard it is for an individual agent to compute which is the best coalition to join. These authors construct several decision problems which compute whether coalitions satisfy certain desirable criteria and compute the computational complexity of these problems. However, one criteria that they do not consider is the power of the agents within the coalitions, that is, the ability of the agents to influence the decision of the group they belong to. An agent can measure if they can influence the outcome within the group by computing if they display critical defection. An agent displays critical defection if the coalition they belong to is successful or winning but the same coalition with them removed is unsuccessful. In this way, it maybe the case that the agents may want to join the coalitions in which they are more powerful since they can influence the overall decision making process of that group. Useful measures of an agents power are given by \textit{The Banzhaf Power Index} and \textit{The Banzhaf Score} [9]. The Banzhaf Score for an agent \( a_j \) (written \( \theta_j \)) is the number of successful coalitions for which agent \( a_j \) is pivotal and The Banzhaf Index for an agent \( a_j, BZ_j \) is the ratio of the agent’s Banzhaf Score to the sum of The Banzhaf Scores of every agent in the game. Intuitively, if an agent’s Banzhaf Index equals zero then they contribute nothing to the achievement of any goals at all in the groups they belong to and can not influence the decision within them at all. Conversely if an agent’s Banzhaf Index is exactly one then they are the only agent who contributes anything toward achieving any goals at all within the set of agents.

In addition to this, we will also show that, through extending the QCG framework
such that we can no longer assume envy-freeness, agents can adopt other insincere strategies regarding coalition formation. Through constructing new decision problems as well as using our complexity results from agent power we will then investigate how difficult it is for an agent to compute these insincere strategies. The rest of this paper will take the following form. In Section 2, we will define the QCG framework as well as Banzhaf Power before extending the QCG framework and defining the problems that compute these strategies and measuring their computational complexity. These results will be given in Section 4. We will then discuss other work that has been done which is related to our work as well as conclude in Section 5.

2 Qualitative Coalitional Games (QCG)

We begin by introducing the framework for the model of a qualitative coalitional game (QCG). This is defined as a \((n + 3)\)-tuple \(\Gamma = (G, Ag, G_1, \ldots, G_n, V)\), where:

- \(G = \{g_1, \ldots, g_m\}\) is a set of possible goals,
- \(Ag = \{a_1, \ldots, a_n\}\) is a set of agents,
- \(G_i \subseteq G\) is a set of goals for each agent \(a_i \in Ag\) with the interpretation being that any of the goals \(G_i\) would satisfy \(a_i\) but \(a_i\) is indifferent between the members of \(G_i\), and
- \(V : 2^{Ag} \rightarrow 2^{2^G}\) is a characteristic function, which for every coalition \(C \subseteq Ag\) determines a set \(V(C)\) of choices, the intended interpretation being that if \(G' \in V(C)\), then one of the choices available to \(C\) is to bring about all the goals in \(G'\) simultaneously.

To ease the burden of calculating the set of choices \(V(C)\) we adopt an approach, whereby function \(V\) is represented as a formula \(\Psi\) of propositional logic over the propositional variables \(Ag\) and \(G\) such that \(\Psi[C, G'] = \top \iff G' \in V(C)\). Clearly, this computation can be performed in deterministic polynomial time. For such a function, we say that:

- A set of goals \(G'\) satisfies agent \(a_i\) if \(G' \cap G_i \neq \emptyset\) (where \(\emptyset\) is the empty set),
- \(G'\) satisfies coalition \(C\) (\(C \subseteq Ag\)) if it satisfies every member of \(C\).
- \(G'\) is feasible for coalition \(C\) if \(\Psi[C, G'] = \top\).
- For a set of goals \(G'\), coalition \(C\) is successful if and only if \(\Psi[C, G'] = \top\) and \(G'\) satisfies every agent in coalition \(C\).

In terms of QCGs, we define an agent to display critical defection in the following manner:

An agent \(a_j\) is said to display critical defection if for a set of goals \(G' \subseteq G\) the coalition \(C \cup \{a_j\}\) is successful but the coalition \(C\) is not successful.
A measure of an agent’s critical defection is given by the Banzhaf Score [9].

**Definition 1** The Banzhaf Score for an agent $a_j$ (written $\theta_j$) is the number of successful coalitions for which agent $a_j$ is pivotal.

**Definition 2** The Banzhaf Index [20] for an agent $a_j$, $BZ_j$, is the ratio of the agent’s Banzhaf Score to the sum of The Banzhaf Scores of every agent in the game. Mathematically, this is expressed as

$$BZ_i = \frac{\theta_i}{\sum_{j=1}^{n} \theta_j}.$$  

Associated with The Banzhaf Index, is another measure of power - The Banzhaf Measure [9].

**Definition 3** The Banzhaf Measure for an agent $a_j$, $BZ^*_j$ is the ratio of its Banzhaf Score to the number of coalitions to which it does not belong to. Mathematically this is expressed as:

$$BZ^*_j = \frac{\theta_j}{2^n - 1}.$$  

### 3 When We Lose The Envy Freeness Assumption

It is assumed that the QCG system is envy-free, that is, each individual agent only wants to achieve any one of the goals in their individual goal set and achieving any one of these goals is not affected by the achievement of other goals not in their individual goal set. Thus, it is assumed that each agent adopts the strategy: “Which is the best coalition to join?”

However, we shall show that one can not always assume envy-freeness. We shall consider a situation where the accomplishment of goals in an agents individual goal set is influenced by the accomplishment of other goals not in this set.

To emphasize this point, we consider the following motivational example.

**Example.**

Suppose there exists a set of agents $Ag = \{a_1, \ldots, a_n\}$ who have encountered an area of land at co-ordinates $(x, y)$. Suppose that a group of these agents wish to build a tower at $(x, y)(B)$, another group of agents desire to dig a ditch at $(x, y)(D)$, and another group of agents desire to farm the land at $(x, y)(F)$. Thus, this QCG has 3 goals $G = \{B, D, F\}$. These 3 goals are *incompatible* and can not all be achieved. If a tower is built then the land can not be farmed or dug. Conversely, if the land is farmed, then it can not be dug or build upon or if the land is dug, it can’t be built or farmed upon. Thus, an agent who wants to achieve goal $B$ can be thought of as wanting to achieve $B$ and not wanting to see goals $F$ and $D$ achieved. Conversely, an agent who wants to either farm or build on the land can be
thought of as wanting to achieve goals $B$ or $F$ and not wanting to see goal $D$ achieved.

An agent $a_i$ who wants achieve $B$ knows if $D$ or $F$ are accomplished then $B$ can not be accomplished since the goals $B, D$ and $F$ are incompatible. The agent could, rather than ask themselves: “Which is the best coalition to join to achieve goal $B$,” adopt the strategy: “Which is the best coalition to join to achieve goal $B$ and how do I stop the goals $D$ and $F$ from also being achieved?” Thus, in addition to trying to find the best coalition to join, agents may try to stop or hinder the formation of successful coalitions which achieve goals that are incompatible with theirs. One way of accomplishing this is through bribery. This involves offering an agent who belongs to a successful coalition which achieves a particular goal set an incentive to leave this coalition.

In this way, when the goals are incompatible, one can not always assume envy-freeness in QCGs and when one can’t always assume envy-freeness, insincere strategies can be adopted by the agents in the QCG.

The approaches to the problem of bribery in other work (see the section 5) ask the question: Can bribery happen? That is, how hard is it to bribe someone so that the outcome is affected? In this paper we ask the question: Who can be bribed? That is, how hard is it to identify agents whom bribing would change the outcome in the favour of the manipulator?

We propose that the manipulator agent $a$ will target agents who display the following criteria:

- Agents who display critical defection [20]. That is, for a set of goals $G'$, the agents who belong to a successful coalition, such that, if they were to leave that coalition then it would be rendered unsuccessful. Conversely, the manipulator may want to identify agents who are not free riders. By avoiding free-riders, the manipulator will avoid agents who don’t have the ability to influence the outcome of the group decision.

- Agents who are veto players. A veto player is a player who, for the set of goals $G'$, is present in every successful coalition that achieves at least one of the goals in $G'$. By identifying the veto players for a set of goals $G'$, they could identify the agents who belong to every successful coalition for $G'$ and so veto players could be useful targets for bribery.

In this way, we can use our complexity results from computing agent power for computing if certain agents are appropriate to bribe.

In addition to the above criteria, an insincere agent may also wish to identify a successful coalition for a goal set $G'$ in which every agent in that coalition displays critical defection. That is, every strict subset of this successful coalition is not successful. Wooldridge and Dunne proved that the problem of computing if there exists such a coalition is $\text{NP}$-complete [21].

Wooldridge and Dunne also proved that, for 2 agents $a_i, a_j \in Ag$, determining if agent $a_j$ is a veto player for the set of goals $G_i$ is $\text{co-NP}$ complete [21]. If agent $a_j$ is a
veto player for the goal set $G_i$, then one can say that $a_i$ is dependent on $a_j$ to achieve the goals in $G_i$. Consequently, one can construct a directed graph, with the agents represented by vertices, such that, if agent $a_i$ is dependent on agent $a_j$ then there is an edge directed from vertex $i$ to vertex $j$. Such a graph is called a dependence graph.

If a manipulator knows that the goal set of agent $a_i$ consists of exactly the set of goals which are incompatible with theirs, then such a graph, could be useful for a manipulator agent to find the veto players for that goal set. With this in mind, we shall investigate the following decision problems.

**Problem 1 Critical Defection.**

**Question:** For a set of goals $G' \subseteq G$, does there exist a coalition $C \cup \{a_i\}$ (where $C \subseteq A \setminus \{a_i\}$) such that $C \cup \{a_i\}$ is a successful coalition but $C$ is not.

**Input:** $\Gamma, G' \subseteq G$, agent $a_i$.

**Output:** ‘Yes’ if there exists a coalition $C \cup \{a_i\}$ such that $G' \cap G_j \neq \emptyset, \forall a_j \in C \cup \{a_i\}$ then $\Psi[C \cup \{a_i\}, G'] = \top$ and $\Psi[C, G'] = \bot$.

‘No’ otherwise.

**Problem 2 Free Rider.**

**Question:** For a set of goals $G'$, is it the case that for every successful coalition $C \cup \{a_i\}$, the defection of agent $a_i$ is never critical?

**Input:** $\Gamma, G' \subseteq G$, agent $a_i$.

**Output:** “Yes” if for all coalitions $C \cup \{a_i\}$ such that $G' \cap G_j \neq \emptyset, \forall a_j \in C \cup \{a_i\}$ then it is never the case that $\Psi[C \cup \{a_i\}, G'] = \top$ and $\Psi[C, G'] = \bot$.

“No” Otherwise.

**Problem 3 Banzhaf Score**

**Question:** What is the value of the Banzhaf score for agent $a_i$ - that is how many successful coalitions $C \cup \{a_i\}$ exist such that the defection of agent $a_i$ is critical?

**Input:** $\Gamma$, agent $a_i$.

**Output:** a numerical value.

**Problem 4 Banzhaf measure:**

**Question:** What is the value of the Banzhaf measure for agent $a_i$, that is what is the value of $\frac{\theta_i}{m}$?

**Input:** $\Gamma$, agent $a_i$.

**Output:** a numerical value.

**Problem 5 Banzhaf Index:**

**Question:** What is the value of The Banzhaf Index for agent $a_i$, that is what is the value of $\sum_{j=1}^{n} \frac{\theta_j}{\theta_i}$?

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Input: $\Gamma$, agent $a_i$.
Output: a numerical value.

Problem 6 Dependence Graph.
Input: $\Gamma$, directed graph $G = (V, E)$ where $|V| = |Ag|$, an injective mapping $F$ from $V$ to $Ag$.
Question: For the injective mapping $F$ is $G$ a dependence graph for $\Gamma$?
Output: “Yes” if $\forall (i, j) \in E$, agent $F^{-1}(i)$ is dependent on $F^{-1}(j)$ (that is, $F^{-1}(j)$ is a veto player for $F^{-1}(i)$).
“No” Otherwise.

4 Computational Complexity Results.

We begin with our result for the Critical Defection problem.

Theorem 1 The Critical Defection problem is NP-complete.

Proof: Membership: The following non-deterministic algorithm can solve this problem: “Guess $C \subseteq Ag \setminus \{X\}$ and Verify that $G' \cap G_j \neq \emptyset$ for all agents $j \in C \cup \{X\}$ and that both $\Psi[C \cup \{X\}, G'] = \top$ and $\Psi[C, G'] = \bot$.” Since verifying that $G' \cap G_j \neq \emptyset$ for all agents $j \in C \cup \{X\}$ and that both $\Psi[C \cup \{X\}, G'] = \top$ and $\Psi[C, G'] = \bot$ can be performed in deterministic polynomial time then the Critical Defection problem can be solved using NP computation.

Hardness: Reduction from SAT [11]. We proceed as follows:

- $Ag = \{x_1, \ldots, x_n, X\}$ where $X$ does not appear in $\Phi(x_1, \ldots, x_n)$.
- $X = a_i$
- $G = G_{x_i} = G_X = G' = \{g\}$ and,
- $\Psi = \Phi \wedge g \wedge X$.

We claim that

$\Phi$ is satisfiable $\iff$ $X$ is pivotal for some coalition $C$

($\Rightarrow$) Assume that $\Phi(x_1, \ldots, x_n)$ is satisfiable. Then there exists a valuation $Z \subseteq \{x_1, \ldots, x_n\}$ such that $\Phi(x_1, \ldots, x_n)[Z] = \top$. Now, consider the valuation $Z_2 = Z \cup \{g, X\}$. By construction $\Psi[Z_2] = \top$; note that this tells us that $\{x_1, \ldots, x_n, X\}$ is a successful coalition, since they have a feasible goal (i.e., $g$) which satisfies all members of the coalition. Now, $\Psi[Z_2 \setminus \{X\}] = \bot$; hence $\{x_1, \ldots, x_n\}$ are not successful. Hence $X$ displays critical defection in the coalition $\{x_1, \ldots, x_n, X\}$ and $X$ is a good target for bribery.
Assume that \( X \) is pivotal for some coalition \( C \subseteq Ag \). Then \( \Psi[C, g] = \top \), and since \( \Psi = \Phi(x_1, \ldots, x_n) \land g \land X \), then \( \Phi \) is satisfiable.

Thus, it is computationally difficult for an agent to identify a suitable agent to bribe. Consequently, an agent may also desire to identify who not to bribe. This was addressed in the Free Rider problem, for which we have the following complexity result:

**Theorem 2** The problem of Free Rider is co-NP-complete.

**Proof:** We can prove co-NP-completeness by observing that this problem is the complement of the problem Critical Defection, which we proved to be NP-complete.

Thus, it is also computationally difficult to identify the “good” targets and eliminate the “bad” targets for bribery in a QCG.

**Theorem 3** The problems of computing the Banzhaf Score, Measure and Index are \#P-complete.

**Proof:** Banzhaf Score - counting problem associated with a NP-complete problem - \#P-complete by definition.

Recall that the formula for The Banzhaf measure is given by:

\[
BZ_i^* = \frac{\xi_i}{2^{n-1}},
\]

i.e. it is value of The Banzhaf Score divided by a constant \( 2^{n-1} \). Calculating the value of the Banzhaf Score is \#P-complete and dividing this value by a constant can be done in polynomial time. Thus, to show \#P-completeness for this problem, we are required to show that \#P' = \#P, which is trivial since both \#P' \subseteq \#P and \#P \subseteq \#P'.

Recall that the formula for calculating The Banzhaf Index is given by:

\[
BZ_i = \frac{\theta_i}{\sum_{j=1}^{n} \theta_j}.
\]

Recall that computing the Banzhaf Score for agents \( a_i \) is \#P-complete and that computing the Banzhaf score for every individual agent \( a_j \in Ag \) is also \#P-complete. This means that as well as being as hard as a known \#P-complete problem, this problem also belongs to the complexity class \#P. Thus, a non-deterministic Turing Machine \( M' \) can compute if each agent displays critical defection and the number of successful computations of \( M' \) can be counted. Suppose \( M' \) had access to a P oracle which could in one step compute the summation and division in the formula, then \( M'' \) could compute the Banzhaf Index for agent \( a_i \).

Therefore, since \#P' = \#P, and since computing the Banzhaf Score is \#P-complete, then computing the Banzhaf Index is also \#P-complete.

Suppose now we are given the graph \( G \), the function \( F \) and the graph \( \Gamma \) (as defined in the input to problem 3), the question we ask ourselves is: How are \( G \), \( F \) and \( \Gamma \) related? Wooldridge et al. [22] suggest 2 ways: Soundness and completeness.
1. **Soundness:** $G$ is sound with respect to the dependence relation if and only if for all vertices $(v_i, v_j) \in E$ then $\mathcal{F}(v_j)$ is a veto player for $\mathcal{F}(v_i)$.

2. **Completeness:** $G$ is complete with respect to the dependence relation if and only if for all $a_i, a_j \in Ag$ such that $a_i$ is a veto player for $a_j$ then $(\mathcal{F}^{-1}(a_j), \mathcal{F}^{-1}(a_i)) \in E$.

**Theorem 4** Given $G, F$ and $\Gamma$ the problem of computing if $G$ is sound for $\Gamma$ and $F$ with respect to the dependence relation is co-NP complete.

**Proof:**

The complement problem to soundness asks: “Does there exist a pair of vertices $(v_i, v_j)$ such that $\mathcal{F}(v_j)$ is not a veto player for $\mathcal{F}(v_i)$?” This problem can be solved using the following non-deterministic algorithm:

“For every pair of vertices $(v_i, v_j)$, guess a coalition $C \subseteq Ag \setminus \{\mathcal{F}(v_j)\}$ and a goal set $G' \subseteq G$. Verifying that $(v_i, v_j) \in E$, and either:

- $G' \cap G_{\mathcal{F}(v_j)} = \emptyset$ or,
- $\Psi[C, G_{\mathcal{F}(v_j)}] = \bot$, or
- $\mathcal{F}(v_i)$ is not in $C$,

can be done in deterministic polynomial time.”

Thus the complement problem can be solved using NP computation. Since the complement problem to the problem of soundness belongs to NP, then the problem of soundness belongs to the class co-NP.

For each directed edge $(v_i, v_j) \in E$ computing $\mathcal{F}(v_i)$ and $\mathcal{F}(v_j)$ can be done in deterministic polynomial time. Thus the complexity of the problem of computing soundness is as hard as the problem of computing if $\mathcal{F}(v_j)$ is a veto player for $\mathcal{F}(v_i)$. This is co-NP complete from [21].

Therefore, the problem of computing soundness is co-NP-complete.

**Theorem 5** Given $G, F$ and $\Gamma$ the problem of computing if $G$ is complete for $\Gamma$ and $F$ with respect to the dependence relation is co-NP complete.

**Proof:** For a pair of agents $(a_i, a_j) \in Ag$ the problem of computing if $a_j$ is a veto player for $a_i$ is co-NP complete from [21]. For these agents, computing $(\mathcal{F}^{-1}(a_i), \mathcal{F}^{-1}(a_j))$ can be done in deterministic polynomial time, as can verifying if $(\mathcal{F}^{-1}(v_i), \mathcal{F}^{-1}(v_j)) \in E$.

Since co-NP$^P = \text{co-NP}$ [14] then the problem of computing completeness is also co-NP complete.

Both theorems 4 and 5 allow us to construct the following theorem:

**Theorem 6** *The Dependence Graph* problem is co-NP complete.
Proof: Showing that $G$ is the dependence graph is equivalent to showing that $G$ is both sound and complete. Let $L_1$ be the co-NP complete language that computes if $G$ is sound and let $L_2$ the co-NP complete language that computes if $G$ is complete. Consequently, $L = L_1 \cap L_2$ is the language which computes if $G$ is both sound and complete and therefore the dependence graph. Therefore this problem belongs to the class co-NP.

We prove completeness by observing that for an edge $v_i, v_j \in E$ then the problem of computing if agent $F(v_j)$ is a veto player for agent $F(v_i)$ is co-NP complete. Since there are a polynomial number of edges then we can conclude that computing the Dependence Graph problem is as hard as computing the co-NP complete problem veto player.

Therefore, the Dependence Graph problem is co-NP complete.

In this way, it is computationally difficult for an agent to compute if bribery is possible.

5 Related Work

Much work has been undertaken into investigating insincere strategies adopted by agents within multi-agent systems ([3, 4, 5, 7, 8]). Much of this work has focused on manipulation of voting procedure, that is, agents acting selfishly to achieve an outcome that is most desirable to them and not necessarily to the other agents or to the system as a whole.

Surprisingly, there has been very little work addressing the computational issues regarding bribery in voting procedures and much of the work focuses on whether bribery is possible. Faliszewski et al. [8] address the issue of bribery by asking the question: Can one agent change the outcome by changing (at most) the preference lists of $k$ agents. Proccacia et al. [16] address bribery from the point of view of unregistered voters entering and changing the outcome when the procedure was applied to registered voters only.

Regarding bribery in coalition formation, much work has not been directly undertaken into looking at bribery. Abbink et al. [1] construct an experimental bribery game which investigates the influence of certain characteristics over corruption (e.g. bribery). One result they conclude is that through enforcing high penalties on those caught acting insincerely reduces the level of corruption. Additional work regarding bribery includes investigating into coalition formation in the presence of transfers and externalities, however, we have found no existing literature directly linking bribery and externalities. Much of the work into coalition formation in the presence of transfers and, that is known to the authors, focuses on computing stability and equilibrium in such environments [19, 2]. Surprisingly little work has also been undertaken looking at the computational complexity of computing insincere strategy in coalition formation, particularly bribery, leaving a lot of scope for future work on the topic.
6 Conclusions

Our results show that for an individual agent, computing the best coalition to join, in terms of agent power, is computationally intractable. This, unfortunately, is consistent with the results in [21] where they also show that, generally, it is computationally difficult to find the best coalition to join. Thus, as further work, one could look into constructing algorithms which make the process of finding the best coalitions to join less difficult.

Our results also show that one can not always assume envy-freeness in the QCG framework but that, in the situations where this condition fails to hold, computing if insincere strategy is possible (particularly the process of identifying suitable agents to bribe) is computationally difficult. Compared to other domains, for example, in weighted threshold games the process of computing if there is an agent who displays critical defection is NP-complete and computing the Banzhaf Index is \#P-complete [15]. Thus, it is just as hard to compute these on a weighted threshold domain as it is on a QCG domain. However, there are some computational problems with computing the Banzhaf index on a weighted threshold domain, for example, the computational process displays many logical paradoxes and fails to satisfy certain postulates [10]. This tarnishes the credibility of using this index in a weighted threshold domain. No such computational complexity results are known yet for marginal contribution net and multi-attribute coalitional game representations.

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(lxxxii) This paper was presented at the Workshop on “Climate Mitigation Measures in the Agro-Forestry Sector and Biodiversity Futures”, Trieste, 16-17 October 2006 and jointly organised by The Ecological and Environmental Economics - EEE Programme, The Abdus Salam International Centre for Theoretical Physics - ICTP, UNESCO Man and the Biosphere Programme - MAB, and The International Institute for Applied Systems Analysis - IIASA.

(lxxxiii) This paper was presented at the 12th Coalition Theory Network Workshop organised by the Center for Operation Research and Econometrics (CORE) of the Université Catholique de Louvain, held in Louvain-la-Neuve, Belgium on 18-20 January 2007.

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