Monte-Carlo Simulations Revised: A Reply to Arqus

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Abstract

This contribution revises Monte-Carlo based simulation techniques as used in Business Taxation and Accounting literature, most prominently proposed by fellows of arqus. Therefore, we focus on a methodically orientated discussion. Our results suggest that the standard approach leads to biased estimates of expected discounted tax payments assuming cash flow uncertainty and incomplete loss-offset regulations. We built up our critique by giving an analytical expression for expected discounted tax payments by formulating the structure of probability weighted tax states in the future conditional on cash flows above or below zero. Consequently, this enables us to evaluate the results obtained from the standard Monte-Carlo approach within a numerical analysis.

JEL Code: G31, H25, K34.
Keywords: Arqus, cash flow uncertainty, limited loss-offset, Monte-Carlo simulations.
1 Introduction

In accounting and taxation literature, it has become a common approach to employ a simulation based framework towards the economic analysis of tax law under uncertainty. The adequacy of this methodical approach is contended by the intrinsic non-linearity of real-world loss-offset rules and the state-dependency of tax payments.\(^1\) As a conclusion drawn from this reasoning, analytical formulations are prejudged as too complex and numerical cash flow statements are applied to extract model solutions under cash flow uncertainty.\(^2\) In fact, there exists an emerging literature characterized by applying Monte-Carlo simulations to the analysis of real-world tax systems.\(^3\)

We dedicate this contribution’s aim to a revision of this methodical approach, that has actually become quite a prominent tool within the economic analysis of tax law under uncertainty. Roughly delineated, the thereby induced critique lies in assessing the reliability of results obtained from the standard approach, since reliability is inextricably linked with the derivation of an analytical expression for expected tax payments.

In this, our reasoning follows from a parsimonious reproduction of the standard assumptions made in literature as sketched in the following: For our considerations, we refer to the incorporation of uncertainty in the gross tax base by making distributional assumptions about one-period cash flows contributing to the shareholder value. We consider tax law as a deterministic function applied to the resulting stochastic tax base. Therefore, we point out that a certain tax base’s probability to occur is well defined due to the standard assumption of normally distributed one-period cash flows. Accordingly, the presence of a limited loss-offset implies that tax law in fact applies to conditional one-period cash flows, i.e. one-period cash flows are truncated at zero. Hence, we show that those two considerations make an analytical expression for state-dependent discounted tax payments easily feasible.

\(^1\) i.e. Niemann (2004)
\(^2\) Ibid.
\(^3\) For instance, Niemann (2004) analyzes the impact of particular loss offset limitations on intrastate and cross-border investment decisions. As an other example, Niemann and Treisch (2005) examine the combined impact of the 2005 modification of the Austrian group taxation regime and loss-offset limitations on cross-border investment decisions of domestic corporations. As a further example, Sureth and Dahle (2008) analyze the impact of various minimum taxation concepts on corporate investment decisions.
In contrast, the commonly used approach obtains a tax transformed data set by applying tax law to Monte-Carlo generated data. Tax effects are estimated by calculating sample averages, and hence, both cash flow truncation at zero is neglected and probability weights are assumed to be equal among different tax states. Hence, state-dependency is actually misspecified. Consequently, the commonly used approach leads to biased estimates of expected discounted tax payments, since this strategy towards estimation does not fit the assumptions lying beyond.

In order to illustrate the adequacy of our critique, this paper is organized as follows: Section (2) compactly summarizes the basic assumptions and presents a parsimonious version of the standard inter-temporal simulation setting. For clarification, by this we do not refer to a simplification of the standard setting, but drop further potential components of the tax base, that come in, in a deterministic fashion, as exogenous income streams, constant depreciations and further lump sum parameters of that kind. Section (3) is dedicated to a more detailed examination of expected discounted tax payments. Therefore a step wise derivation of the proposed analytical expression is given. Given those considerations, we outline our main results obtained from a Monte-Carlo analysis within section (4). Finally, section (5) summarizes and concludes.

2 The Model

As previously announced, we now formulate the simulation model in terms of a parsimonious reproduction of the existing literature. Therefore, we need the two assumptions presented in the following. The first assumption is about one-period cash flows contributing to the shareholder value. The second assumption summarizes the structuring of the tax system. We refer to the underlying stochastic process determining one-period cash flows within assumption A.1, and refer to the tax system within assumption A.2. Hence, for our considerations, we assume that:

A.1: One-period cash flows contributing to the shareholder value (previous to any tax considerations) follow a first differenced random walk with constant drift $\mu$. The innovations within every period $t = 1, \ldots, T$ are normally distributed with zero expected value and constant variance, i.e. $\epsilon_t \sim N(0, \sigma^2)$. 

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A.2: Tax payments for each period are generated by a tax function. Tax payments are denoted by $\psi_t$. The tax payment generating function applies the statutory tax rate $\tau$ to a deterministic non-continuous transformation of one-period cash flows, i.e. there is no subsidy received for current period losses, but within every period when a profit occurs, losses from previous periods, accumulated within the loss deposit $l_{t-1}$, are feasible for deduction. Therefore, the amount of loss-offset is limited by the extend of the profit.

No other potential components of the tax base are considered.

Given assumptions A.1 and A.2, we formulate the present value of the firm over a finite time horizon $T$ as the discounted sum of one-period cash flows, denoted as $\Delta v_t$, net of tax payments to be paid within a certain period, $\psi_t$. We therefore denote the (tax adjusted) constant discount factor by $\delta$ with $0 < \delta < 1$. Hence, the discounted value of the firm $V_0$ is equal to:

$$V_0 = \sum_{t=1}^{T} (\Delta v_t - \psi_t)\delta^t \quad \text{with} \quad \Delta v_t \sim N(\mu, \sigma^2)$$

(1)

According to A.1, $\Delta v_t$ follows a first differenced random walk with constant drift $\mu$, and constant variance $\sigma^2$.

As previously stated within assumption A.2, loss-offset regulations are incomplete, and taxes to be paid for each period $t$ are obtained by applying the statutory (flat) tax rate $\tau$ to the tax base. We therefore formulate A.2 within the following expressions:

$$\psi_t = \begin{cases} \tau \max((\Delta v_t|\Delta v_t > 0) + l_{t-1}, 0) & \text{if} \quad \Delta v_t > 0 \\ 0 & \text{otherwise.} \end{cases}$$

(2)

Thus, only current period profits net of losses, feasible for deduction at time $t$, denoted as $l_{t-1}$, are subject to tax immediately. A current period loss is subject to tax in the future via loss deduction. Consequently, the subsequent expression states the update equation for the loss deposit available from the previous period, according to equation (2):

$$l_{t-1} = l_{t-2} + \begin{cases} \min(\Delta v_{t-1}|\Delta v_{t-1} > 0, -l_{t-2}) & \text{if} \quad \Delta v_{t-1} > 0 \\ \Delta v_{t-1}|\Delta v_{t-1} \leq 0 & \text{otherwise.} \end{cases}$$

(3)
As we state the loss deposit in a recursive fashion, we put losses from the starting period, $l_0$, feasible for deduction within period $t = 1$ at $l_0 = 0$.

### 3 Expected Discounted Value of Tax Payments

Given those considerations, we now concentrate on an analytical expression for the expected value of discounted tax payments according to the (standard) assumptions given in the previous section. A true value of the size we are interested in, in terms of expectations, serves as a useful benchmark for evaluating the simulation based approach. This is just the convenient way to obtain a measure for the reliability of an estimator.

We therefore rewrite expression (1) in terms of expectations $E(\cdot)$ and obtain the provisional formulation:

$$E(V_0) = \mu \delta(1 - \delta^T) \frac{1}{1 - \delta} - E(\Psi_0) \quad (4)$$

The first part of the previous statement on the right hand side, denotes the expected value of one-period cash flows contributing to the shareholder value over some finite horizon of $T$ periods, which by assumption A.1 is trivial anyway. So in the following, we will concentrate on the second right hand side part of expression (4), which denotes the expected value of discounted tax payments by $E(\Psi_0)$.

By our two assumptions, the tax base is generated by a deterministic transformation of the underlying stochastic one-period cash flows. Firstly, just considering the tax base before deducting losses from former periods, this implies the tax function basically being defined on two scenarios about one-period cash flows, one accounting for $\Delta v_t|\Delta v_t > 0$, occurring if $\Delta v_t > 0$, and the other scenario accounting for the case that $\Delta v_t|\Delta v_t \leq 0$, occurring if $\Delta v_t \leq 0$. Given this, one may define an auxiliary binary variable $d_t$ as:

$$d_t = \begin{cases} 1 & \text{if } \Delta v_t > 0 \\ 0 & \text{otherwise.} \end{cases} \quad (5)$$
Hence, one may rewrite one-period cash flows contributing to the shareholder value as:

\[ \Delta v_t = d_t(\Delta v_t | \Delta v_t > 0) + (1 - d_t)(\Delta v_t | \Delta v_t \leq 0) \tag{6} \]

Taking expectations of this expression, leads to the preliminary formulation:

\[ E(\Delta v_t) = E(d_t)E(\Delta v_t | \Delta v_t > 0) + (1 - E(d_t))E(\Delta v_t | \Delta v_t \leq 0) \tag{7} \]

Given assumption A.1, such that we have \( \Delta v_t \sim \mathcal{N}(\mu, \sigma^2) \), this implies that

\[
\begin{align*}
E(d_t) &= E(\Delta v_t > 0) = 1 - \Phi(z) \\
E(\Delta v_t | \Delta v_t > 0) &= \mu + \sigma \lambda(z) \\
E(\Delta v_t | \Delta v_t \leq 0) &= \mu - \sigma \lambda(-z)
\end{align*}
\]

Where we use \( z = -\mu/\sigma \). Further, \( \Phi(z) \) denotes the value of the standard normal cumulative distribution and \( \lambda(z) \) stands for the standard normal hazard function, such that \( \lambda(z) = \frac{\phi(z)}{1 - \Phi(z)} \) with \( \phi(z) \) the standard normal probability distribution function.

Given those expressions, we can plug into expression (7), and verify:

\[
E(\Delta v_t) = (1 - \Phi(z))(\mu + \sigma \lambda(z)) + \Phi(z)(\mu - \sigma \lambda(-z)) \\
= (1 - \Phi(z))(\mu + \sigma \frac{\phi(z)}{1 - \Phi(z)}) + \Phi(z)(\mu - \sigma \frac{\phi(z)}{\Phi(z)}) \\
= \mu(1 - \Phi(z) + \Phi(z)) + \sigma \phi(z) - \sigma \phi(z) \\
= \mu
\]

And hence, this allows us to split up one-period expectations about \( \Delta v_t \) into two scenarios, just as required by equation (2). \( \blacksquare \)

So far, we clarified that equation (2) refers to conditional expectations of one-period cash flows contributing to the shareholder value. This follows from assumption A.1 together with taking account of incomplete loss deduction. In this context, we state that for every \( t \) we have a one-period probability \( 1 - \Phi(z) \) of
having a profit in amount of $\mu + \sigma \lambda(z)$ at time $t + 1$, and a probability of $\Phi(z)$ for a loss in amount of $\mu - \sigma \lambda(-z)$, respectively. Hence, this implies that given a starting period $t = 0$, the all over probability of having had a profit (loss) for a $t - j$ times ($j$ times) at time $t$ is equal to $(1 - \Phi(z))^{t-j}\Phi(z)^j$.

Next, we consider losses feasible for deduction at time $t$, as we formulated within expression (3) in the previous section. The very additional point to be taken note of, lies in the fact that due to loss deduction, every tax path over time has to be considered as a unique state. This follows from assumption A.2, since the period when a loss occurs matters for determining tax payments in subsequent periods, but not vice versa (i.e., there is no loss carry-back and we have opportunity costs entering by $\delta$). Hence, within every period $t$ a number of $2^t$ different tax states have to be distinguished. Finally, given a deterministic tax payment generating function by assumption, we formulate the expected discounted value of tax payments over a finite time horizon $T$ as

$$E(\Psi_0) = \sum_{t=1}^{T} \omega'_t \begin{pmatrix} E(\psi^1_t) \\ \vdots \\ E(\psi^{2^t}_t) \end{pmatrix} \delta^t$$

with $\omega_t$ the vector summarizing the probability weights assigned to each tax state for a certain period $t$ (with typical element $(1 - \Phi(z))^{t-j}\Phi(z)^j$ for $j = 0, \ldots, t$).

Thus, the previous result enables us to evaluate the standard simulation approach. This is done within the Monte-Carlo analysis presented next.

### 4 Monte-Carlo Analysis

To give a demonstration of our results, we consider a numerical example. We therefore investigate estimation of the discounted value of tax payments in respect of the analytical expression as stated by equation (8). We consider discounted tax payments as a function of the parameters $\mu$ and $\sigma$. Other parameters are assumed as fixed. We therefore chose $T = 3$ and assume the statutory tax rate to equal $\tau = 0.25$. Further we assume the (tax adjusted) discount factor to equal $\delta = 0.9$.

In order to ensure robustness of our results, we consider a parameter space determined by the values $\mu = -0.3, -0.2, -0.1, 0, 0.1, 0.2, 0.3$ and $\sigma = 0.05, 0.15,$
0.25, 0.35, 0.45, 0.55, 0.65, 0.75, 0.85, 0.95. Thus, plugging into equation (8), leads to the corresponding results for the expected discounted value of tax payments. Given the declared analytical results, we run a Monte-Carlo repeatedly sampling a 1000 firms a 10000 times. (Note, we therefore refer to the sample size as $N$ and the number of repetitions as $M$.) Hence, over each sample drawn, we estimate discounted tax payments and finally obtain 10000 estimates.

We employ two types of estimators: The one is a pure means estimator, that simply averages over all tax paths observed from the Monte-Carlo generated data. This is the commonly used approach. As an alternative, we reveal using (consistent) estimates of $\mu$ and $\sigma$, $\hat{\mu}$ and $\hat{\sigma}$. Hence, we obtain estimates $\hat{z}$ for the regarding value of the standard-normal probability distribution function and the standard-normal cumulative distribution function, respectively. Given those results, we can easily construct estimates for the truncated at zero normal distribution and the probability weights, $\hat{\omega}_t$ for $t = 1, 2, 3$. Finally, plugging into expression (8), leads to an estimated counterpart of the values for expected discounted tax payments. Given the estimates, we compare them with their analytical counterparts. We refer to the pure means estimates by $\hat{\Psi}_0$ and the estimates obtained from the suggested alternative approach by $\hat{\Psi}_0^*$. The regarding results are summarized within table 1 and table 2. We therefore report relative percentage differences of the estimates form the actual parameter value. Standard errors are reported in parenthesis, given right below the according estimate.

The results reported for $\hat{\Psi}_0$ given in table 1, suggest that the pure means estimator, an hence a pure averaging over calculated tax payments according to the paths of the synthetical data observed, performs weakly. For every parameter constellation $(\mu, \sigma)$ assumed, the standard approach as proposed by arqus is outperformed by the probability weighted truncated means estimation approach and the resulting estimates $\hat{\Psi}_0^*$, by far. The latter ones are reported within table 2. Further, we report the bias proportion of the mean squared error for both estimators in table 3. Sub-table (a) contains the results obtained for the pure means estimator $\hat{\Psi}_0$, and sub-table (b) reports the mean squared error bias proportion for the alternative approach, i.e. the estimator $\hat{\Psi}_0^*$. Neither, those results are very surprising: While almost all of the mean squared error is due to variance for the results obtained from the alternative approach, the standard approach leads to severely biased estimates of expected discounted tax payments.
<table>
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<tr>
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Table 1: Monte-Carlo results for $\hat{\Psi}_0$, $\frac{\hat{\Psi}_0 - E(\Psi_0)}{E(\Psi_0)}$ reported, standard errors in parenthesis.

$(M = 10.000, N = 1000, T = 3, \delta = 0.9, \tau = 0.25)$
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<td>(0.0056)</td>
<td>(0.0064)</td>
<td>(0.0071)</td>
<td>(0.0077)</td>
<td>(0.0083)</td>
</tr>
</tbody>
</table>

Table 2: Monte-Carlo results for $\hat{\Psi}_0^*$, $\frac{\hat{\Psi}_0^* - E(\Psi_0)}{E(\Psi_0)}$ reported, standard errors in parenthesis

($M = 10,000$, $N = 1000$, $T = 3$, $\delta = 0.9$, $\tau = 0.25$)
<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>0.05</th>
<th>0.15</th>
<th>0.25</th>
<th>0.35</th>
<th>0.45</th>
<th>0.55</th>
<th>0.65</th>
<th>0.75</th>
<th>0.85</th>
<th>0.95</th>
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<tbody>
<tr>
<td>-0.3</td>
<td>0.05</td>
<td>100.000%</td>
<td>99.999%</td>
<td>99.934%</td>
<td>99.309%</td>
<td>96.450%</td>
<td>81.095%</td>
<td>14.533%</td>
<td>25.537%</td>
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<tr>
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<td>100.000%</td>
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<td>87.722%</td>
<td>0.905%</td>
<td>55.973%</td>
<td>75.494%</td>
<td>82.592%</td>
<td>85.905%</td>
<td>87.469%</td>
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<tr>
<td>-0.1</td>
<td>0.25</td>
<td>99.999%</td>
<td>96.364%</td>
<td>25.018%</td>
<td>79.978%</td>
<td>86.271%</td>
<td>88.758%</td>
<td>89.750%</td>
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<td>90.538%</td>
<td>90.945%</td>
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<td>91.776%</td>
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<td>77.881%</td>
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<td>62.189%</td>
<td>76.578%</td>
<td>82.027%</td>
<td>84.487%</td>
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<td>87.224%</td>
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<td>98.366%</td>
<td>89.276%</td>
<td>56.468%</td>
<td>29.243%</td>
<td>54.637%</td>
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<td>78.597%</td>
<td>45.560%</td>
<td>4.128%</td>
<td>9.784%</td>
<td>34.276%</td>
<td>51.923%</td>
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</tbody>
</table>

<table>
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<tr>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>0.05</th>
<th>0.15</th>
<th>0.25</th>
<th>0.35</th>
<th>0.45</th>
<th>0.55</th>
<th>0.65</th>
<th>0.75</th>
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</tr>
</thead>
<tbody>
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<td>0.010%</td>
<td>0.008%</td>
<td>0.012%</td>
<td>0.010%</td>
<td>0.064%</td>
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<td>0.122%</td>
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<td>0.005%</td>
<td>0.095%</td>
<td>0.003%</td>
<td>0.013%</td>
<td>0.079%</td>
<td>0.004%</td>
<td>0.010%</td>
<td>0.049%</td>
<td>0.009%</td>
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<td>0.164%</td>
<td>0.012%</td>
<td>0.334%</td>
<td>0.433%</td>
<td>0.487%</td>
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<td>0.279%</td>
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<td>0.007%</td>
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<td>0.004%</td>
<td>0.010%</td>
<td>0.009%</td>
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<td>0.2</td>
<td>0.55</td>
<td>0.004%</td>
<td>0.006%</td>
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<td>0.024%</td>
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<td>0.010%</td>
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<td>0.009%</td>
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<tr>
<td>0.3</td>
<td>0.65</td>
<td>0.004%</td>
<td>0.003%</td>
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<td>0.009%</td>
<td>0.000%</td>
<td>0.052%</td>
<td>0.007%</td>
<td>0.008%</td>
<td>0.007%</td>
<td>0.003%</td>
</tr>
</tbody>
</table>

Table 3: MSE bias proportions for $\hat{\Psi}_0$ (a) and $\hat{\Psi}_0^*$ (b)

$(M = 10.000, N = 1000, T = 3, \delta = 0.9, \tau = 0.25)$
5 Summary and Conclusions

This paper analyzes whether the commonly used Monte-Carlo approach as most prominently proposed by fellows of arqus, leads to reliable estimates of tax effects, assuming cash flow uncertainty and incomplete loss-offset. We give evidence that it is not, since state-dependency, as well as implied probability weights are misspecified. Our line of argument drawing to this conclusion is based on straightforward analytical results for expected discounted tax payments. The commonly assumed characteristics of the stochastic process determining cash flows and the non-linearity of the tax function therefore suggest understanding expected tax payments as the probability weighted sum of a deterministic tax payment generating function applied to truncated at zero expectations about one period cash flows.

This draws to the conclusion that solutions extracted from the standard approach are contrary to the assumptions they rely on. And hence, from a methodical point of view, it turns out to be a hasty decision to exclusively give attention to an unverified estimation approach. This, particularly with regard to the feasibility of an analytical approach to solution serving as a benchmark. However, this contribution’s aim is to promote methodical advancement for further applied studies. And therefore, as a necessity within every subject, we point out the need to admit more methodical discussion in the future.
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