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## Depletion of Non-Renewable Resources and Endogenous Technical Change

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# Depletion of Non-Renewable Resources and Endogenous Technical Change

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## **Abstract**

Non-renewable resources are an obstacle for positive long run growth if they are essential for production, households solve an intertemporal Ramsey problem and population is growing. Modern growth models predict that growth is positively related to growth in production factors. Hence, there are opposing forces at work if labor as one factor is growing and the use of the non-renewable resource as another factor is shrinking. The paper develops a semi-endogenous growth model with one labor and one resource using sector and derives conditions for stable positive long run growth in per capita production and consumption.

*Keywords:* Non-renewable resources, semi-endogenous growth

*JEL Classification Number:* Q32, O31, O33

# 1 Introduction

Non-renewable resources necessary for production are a problem if consumers maximize intertemporal utility over an infinite time horizon and the rate of time preference is positive. This is true as long as there is no technological progress which augments production factors. Due to the impatience of the consumers capital accumulation is too low and the resource usage is too high so that consumption per capita declines over time. Capital can not substitute for the non-renewable resource in the long run even if one abstracts from depreciation of capital (Groth 2007).

Another problem in the context of non-renewable resources is population growth. A growing population has, *ceteris paribus*, a growing need for using the resource. If one defines a sustainable long run growth path to yield at least a constant per capita consumption profile then it becomes clear that in the light of the preceding paragraph a growing population adds to the problem.

In general technological progress is a possible solution to this problem. By augmenting the production factors, an ongoing reduction in the use of the resource is possible without reducing per capita consumption. This is true e.g. for the case of purely exogenous technological progress which yields a constant growth rate in the factor augmenting technology. However, technology is certainly not exogenous, but is from a macroeconomic perspective driven by incentives that are determined by market forces .

Groth (2007) gives an excellent overview over the literature on (semi-)endogenous growth theory in the context of non-renewable resources.<sup>1</sup> The models in this literature generally have a two sector structure where an R&D sector is involved in innovation and thereby supplying a final goods sector with new technologies. Groth (2007) groups these model according to the assumptions made in order to set up the growth mechanism, i.e. the R&D sector. If this sector does not use the non-renewable resource, then positive growth in per capita figures as consumption is feasible in the long run. If the resource is used in the growth producing process of

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<sup>1</sup>Contributions that are relevant in this area are Suzuki (1976), Jones and Manuelli (1997), Aghion and Howitt (1998, chap. 5), Scholz and Ziemes (1999), Schou (2000, 2002), Grimaud and Rougé (2003) and Groth and Schou (2007).

innovating the conclusions are more pessimistic, i.e. a strong “standing on shoulders of giants” effect is needed to yield long run positive growth.

Because the above mentioned studies are using endogenous growth models to motivate technical change, some thoughts about endogenous growth theory are in order. As Jones (1999) points out, there exist endogenous growth models of the first and the second generation. First generation growth models yield the doubtful result that the growth rate of e.g. per capita production is positively related to the size of the economy measured by e.g. the labor force. Models of this type are e.g. the ones in Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992). Jones (1995) shows that this is not in accordance with empirical observations. This has led the profession to develop growth models of the second generation type with results that the above mentioned growth rate does not have this “strong” scale effect, as it was termed by Jones (1999), at least on the balanced growth path. However, what these models of the second generation type have in common is a “weak” scale effect. This scale effect shows up in per capita terms like per capita production. A larger economy should have, *ceteris paribus*, a higher per capita production. Jones (2005) summarizes the empirical literature on growth theory which at least partially analyzes the relationship between per capita production and the size of economies. He concludes that there is some support for this “weak” scale effect, but it must be noted that the studies cited in Jones (2005) were not originally targeted to test the hypothesis of “weak” scale effects.

At this point another strand of the economic literature gains importance. Acemoglu (1998), Kiley (1999), Acemoglu (2001) and Acemoglu und Zilibotti (2001) are using the argument of directed technical change to explain the behavior of wage inequality between high and low skilled workers. The key argument in these studies is that the rise in the supply of high skilled workers has enlarged the market for technologies directed to them. This gives rise to higher incentive to develop new technologies favoring high skilled workers which can lead under certain circumstances to rising relative wages for the high skilled. Although the economic problem behind these studies is different from the present one, they are giving an analytical framework

which is directly relevant to the topic of this paper. In a recent contribution Di Maria and Valente (2006) are using the theory of directed technical change to elaborate on the endogenous bias of capital and resource saving technical change.

The discussion up to this point is relevant in the present context because the theory of directed technical change heavily made use of endogenous growth models of the first generation type. The driving force behind the results is the scale effect in the growth rates. The question that must be answered is, can the argument of directed technical change still be used if one switches to second generation growth models as the literature demands. Antony (2007) shows that the “weak” scale effect can substitute for the “strong” scale effect in explaining the above mentioned wage inequality without changing the major results. Empirical evidence for the existence of such scale effects in an open economy context is also found.

This gives the framework for the analysis in this paper. A two sector semi-endogenous growth model of the second generation type is developed with one sector using labor and the other using the non-renewable resource. Capital is used to build differentiated intermediate input factors specific to each sector and is accumulated by foregone consumption of the households. The change in the degree of differentiation of input factors gives the change in technology and is endogenously determined by profit maximizing firms. There are two R&D sectors, one innovating for the labor using and one for the resource using sector, the resource is an essential input for the latter R&D sector. The “weak” scale effect gives rise to the following opposing forces. A growing population and work force leads the market to design more and more differentiated input factors devoted to the labor using sector, i.e. labor augmenting technical change. The limited supply of the non-renewable resource demands an ever decreasing usage of the resource giving rise to a negative “weak” scale effect. Population now has an ambiguous impact. On the one hand it creates new technologies through a scale effect, on the other hands it implies, *ceteris paribus*, a higher usage of the resource to meet people’s needs. Through a decreasing use of the resource the resource augmenting technology is negatively affected. The major result of the theoretical analysis below is that as long the non-renewable resource is

not too important, in terms of an output elasticity, there exists a saddle path stable balanced growth path on which per capita production grows at a positive constant rate.

## 2 The Model

A model is developed containing the main arguments of the preceding section. The model economy consists of two sectors, one producing with the non-renewable resource and one with labor. Production technologies in both sectors are given by the standard Romer technology with differentiated input factors. The degree of differentiation is interpreted as the level of technology for the specific sector and is endogenous. Final output in the economy is given by a Cobb-Douglas aggregate of the production of the two sectors, hence the resource is a necessary production factor. Households are assumed to optimize an intertemporal utility function over an infinite time horizon, time is continuous. A representative household can save a part of its income, consisting of wages, interest payments and revenues from holding the non-renewable resource. Savings are transformed into a capital stock which is used to form differentiated input factors for the two sectors of the economy. Capital can depreciate at a constant rate.

The main findings of the model are as follows. Households optimality conditions are well known to be the Keynes-Ramsey and the Hotelling rule. Technology, or the degree of differentiation in the two sectors, is determined by the extent of the market, i.e. it is directly proportionate to the usage of the resource and labor. This result is due to a zero profit condition in the market for differentiated input factors. Due to a Cobb-Douglas aggregation the capital stocks in both sectors of the economy are directly proportionate to each other, i.e. they grow with the same rate. An important point is the existence of a stable steady state in the economy with possibly a positive growth rate of the economy. It turns out that this depends on the output share of the resource sector which is given due to the Cobb-Douglas specification as a constant. This constant has in any event to be smaller than  $\frac{1}{2}$ , the higher the population growth rate is, the closer to this threshold the share can

be. If this is the case, there exists a locally saddle path stable equilibrium where per capita production and consumption grow with positive constant rates. This rate is positively related to the population growth rate. The usage of the resource shrinks in equilibrium at a rate equal to the time preference rate of the representative household.

## 2.1 Households

The economy is populated by  $L_t$  consumers at time  $t$ ,  $L_t$  grows with a constant rate  $\frac{\dot{L}_t}{L_t} = n$  at every instant of time. The representative consumer maximizes intertemporal utility given by

$$U = \int_0^{\infty} \ln(c_t)e^{-\rho t} dt,$$

subject to the relevant budget constraint.

This leads to the two optimality conditions

$$\begin{aligned} \frac{\dot{c}_t}{c_t} &= r_t - \rho - n, \\ \frac{\dot{q}_t}{q_t} &= r_t, \end{aligned}$$

where  $q_t$  denotes the price of the resource and  $r_t$  is the net interest rate in the economy. These conditions are of course known as the Keynes-Ramsey and the Hotelling rule.

## 2.2 Production

It is assumed that the economy consists of two sectors, one employing labor and one employing the non-renewable resource. Both combine their specific production factors with differentiated additional input factors. The degree of differentiation gives the state of technology in the economy which will be endogenized later on. The production technology in each sector is thus identical to the one in Romer (1987). It has constant returns to scale with respect to the rivalrous input factors. The environment for each firm active in one of the two sectors is assumed to be perfectly



competitive. The analysis below is therefore conducted for a representative firm element of a unit mass of firms. The time subscript is suppressed in the following, all figures correspond to the current time  $t$ .

Production in two sectors is given by

$$Y_L = (L_p)^\alpha \int_0^{N_L} x_i^{1-\alpha} di, \quad (1)$$

$$Y_R = R_p^\alpha \int_0^{N_R} x_j^{1-\alpha} dj, \quad (2)$$

where  $L_p$  and  $R_p$  denote labor and the non-renewable resource employed by the two sectors.  $N_L$  is the set of differentiated input factors that can be used together with labor,  $N_R$  is analogously defined for the sector using the non-renewable resource,  $x_i$  and  $x_j$  are the used quantities associated with these factors.  $N_L$  and  $N_R$  give the state of technology in the two sectors of the economy.

The final good in the economy is produced from  $Y_L$  and  $Y_R$  according to

$$Y = Y_R^\beta Y_L^{1-\beta},$$

and  $p_R$  and  $p_L$  will denote the prices associated with  $Y_L$  and  $Y_R$ .

Demand for the differentiated input factors comes from the optimality condition that the marginal product equals the price of these input factors. The demand functions are given by

$$x_i = (1 - \alpha)^{\frac{1}{\alpha}} \left( \frac{\xi_L}{p_L} \right)^{-\frac{1}{\alpha}} L_p, \quad (3)$$

$$x_j = (1 - \alpha)^{\frac{1}{\alpha}} \left( \frac{\xi_R}{p_R} \right)^{-\frac{1}{\alpha}} R_p, \quad (4)$$

$$(5)$$

where  $\xi_L$  and  $\xi_R$  are the prices for the respective variants of differentiated input factors.

Differentiated input factors are produced by the inventors of these factors. In order to enter the market for these factors with a specific variant, the producer has to incur

a fixed cost  $f$  at every instance in time<sup>2</sup>. These fixed costs are in terms of the primary production factor of the sector for which he provides differentiated input factors, i.e. either labor of the non-renewable resource. Production of the differentiated input factors is then accomplished by use of capital goods which are produced linearly from investment goods created from foregone consumption of the final goods by the household sector. The rental price for these capital goods is therefore given by the gross interest rate,  $r_g = r + \delta$ , where  $\delta$  is the rate of capital good's depreciation. It is assumed that the original developer of one variant of the differentiated input factor has a comparative advantage in marginal cost. A potential competitor can provide the market with the same variant without incurring the fixed cost at a higher marginal cost in terms of capital goods usage. Let the relative difference between these marginal costs be  $\gamma$ , then the original developer is assumed to set a limit price of  $\xi_R = \xi_L = \gamma r_g$ . Instantaneous profits for the developers of differentiated input factors are thus given by

$$\begin{aligned}\pi_i &= (\gamma - 1)r_g\gamma^{-\frac{1}{\alpha}}(1 - \alpha)^{\frac{1}{\alpha}}\left(\frac{r_g}{pL}\right)^{-\frac{1}{\alpha}}L_p, \\ \pi_j &= (\gamma - 1)r_g\gamma^{-\frac{1}{\alpha}}(1 - \alpha)^{\frac{1}{\alpha}}\left(\frac{r_g}{pR}\right)^{-\frac{1}{\alpha}}R_p.\end{aligned}$$

Since there are no intertemporal effects in the production structure of intermediate input factors, profit maximization implies that entry in the market for differentiated input factors occurs as long profits exceed the fixed cost of entry. In equilibrium net profits are zero

$$\begin{aligned}\pi_i &= fw, \\ \pi_j &= fq.\end{aligned}$$

These zero net profit conditions give the set of differentiated input factors available

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<sup>2</sup>Of course, these fixed costs have to be infinitesimally small since there is a continuum of firms and instances in time (see e.g. Grossman and Helpman 2002 on this point).

to both sectors of the economy

$$N_L = \frac{\gamma - 1}{\gamma} \frac{1 - \alpha}{\alpha} \frac{1}{f} L_p, \quad (6)$$

$$N_R = \frac{\gamma - 1}{\gamma} \frac{1 - \alpha}{\alpha} \frac{1}{f} R_p. \quad (7)$$

$$(8)$$

This result is very important since it implies a scale effect in the reduced form of the production function in each sector. The pure extent of the set of intermediate input factors is a non-rivalrous input in these production functions. Because production has already constant returns to scale in the rivalrous input factors, the reduced forms exhibit increasing returns to scale, giving rise to growth in per capita terms.

With these results it can be computed how  $L$  and  $R$ , the total employment of labor and the resource, split into production and fixed cost

$$\begin{aligned} L_p &= \frac{\gamma\alpha}{\gamma + \alpha - 1} L, \\ R_p &= \frac{\gamma\alpha}{\gamma + \alpha - 1} R, \\ L_f &= \frac{(\gamma - 1)(1 - \alpha)}{\gamma + \alpha - 1} L, \\ R_f &= \frac{(\gamma - 1)(1 - \alpha)}{\gamma + \alpha - 1} R. \end{aligned}$$

Capital is used in production of differentiated input factors. Denote the capital stock used in sector  $L$  and  $R$  by  $K_L$  and  $K_R$ , then

$$\int_0^{N_L} x_i di = K_L, \quad (9)$$

$$\int_0^{N_R} x_j dj = K_R, \quad (10)$$

and

$$(11)$$

$$\int_0^{N_L} x_i^{1-\alpha} di = N_L^\alpha K_L^{1-\alpha},$$

$$\int_0^{N_R} x_j^{1-\alpha} dj = N_R^\alpha K_R^{1-\alpha}.$$

With this result production in the two sectors can be written as

$$\begin{aligned} Y_L &= (N_L L_p)^\alpha K_L^{1-\alpha}, \\ Y_R &= (N_R R_p)^\alpha K_R^{1-\alpha}. \end{aligned}$$

Relative prices satisfy, due to the Cobb-Douglas aggregation of sector production,

$$\begin{aligned} \frac{Y_R}{Y_L} &= \frac{\beta}{1-\beta} \frac{p_L}{p_R}, \\ p_R &= \left( \frac{q}{\alpha N_R} \right)^\alpha \left( \frac{\gamma r_g}{1-\alpha} \right)^{1-\alpha}, \\ p_L &= \left( \frac{w}{\alpha N_L} \right)^\alpha \left( \frac{\gamma r_g}{1-\alpha} \right)^{1-\alpha}. \end{aligned}$$

Substituting the demand functions (3) and (4) and the technology conditions (6) and (7) into the production functions (1) and (2) yields

$$\frac{Y_R}{Y_L} = \left( \frac{p_R}{p_L} \right)^{\frac{\alpha}{1-\alpha}} \left( \frac{R}{L} \right)^2.$$

Using (3), (4), (9) and (10) leads to

$$\frac{K_R}{K_L} = \left( \frac{p_R}{p_L} \right)^{\frac{1}{\alpha}} \left( \frac{R}{L} \right)^2.$$

Using these results it turns out that

$$\frac{K_R}{K_L} = \frac{\beta}{1-\beta}.$$

With the constraint  $K = K_R + K_L$ , this implies that  $K_R = \beta K$  and  $K_L = (1-\beta)K$ .

Aggregate production of final goods can therefore be written as

$$\begin{aligned} Y &= \eta R^{2\alpha\beta} L^{2\alpha(1-\beta)} K^{1-\alpha}, \\ \eta &= \beta^{\beta(1-\alpha)} (1-\beta)^{(1-\beta)(1-\alpha)} \left( \frac{\gamma-1}{\gamma} \frac{1-\alpha}{\alpha} \frac{1}{f} \right)^\alpha \left( \frac{\gamma\alpha}{\gamma+\alpha-1} \right)^{2\alpha}. \end{aligned} \tag{12}$$

where again increasing returns to scale show up due to increasing returns in the sector production functions. Since these increasing returns to scale have a micro

foundation each factor earns its marginal product and final good producers earn zero profits. The surplus due to the increasing returns to scale covers the fixed costs involved in providing differentiated intermediate input factors.

### 2.3 Dynamics

The dynamics of the economy are completely characterized by the two variables  $x = \frac{C}{K}$  and  $r_g$ . Usually in the context of non-renewable resources the variable  $z = \frac{R}{S}$ , the depletion rate, is also considered, where  $S$  denotes the stock of the resource. This of course could be done here as well. However, if  $z$  is added to the system of  $x$  and  $r_g$  it turns out that  $\dot{z}$  is influenced only by  $x$  and neither  $\dot{x}$  nor  $\dot{r}_g$  are influenced by  $z$ . Therefore it is sufficient to concentrate on  $x$  and  $r_g$ .

Resource prices are given by the marginal product condition

$$q = \alpha\beta \frac{Y}{R_p},$$

implying a dynamic behavior according to

$$\begin{aligned} \frac{\dot{q}}{q} &= \frac{\dot{Y}}{Y} - \frac{\dot{R}_p}{R_p}, \\ &= 2\alpha\beta \frac{\dot{R}_p}{R_p} + 2\alpha(1-\beta)n + (1-\alpha)\frac{\dot{K}}{K} - \frac{\dot{R}_p}{R_p}, \end{aligned}$$

Capital accumulates according to

$$\frac{\dot{K}}{K} = \frac{Y}{K} - \frac{C}{K} - \delta,$$

where  $C$  is aggregate consumption spending. Together with the Hotelling rule this gives

$$\frac{\dot{R}_p}{R_p} = \frac{\dot{R}}{R} = -\frac{2\alpha(1-\beta)}{2\alpha\beta-1}n + \frac{1-\alpha}{2\alpha\beta-1}x + \frac{1-\alpha}{2\alpha\beta-1}\delta,$$

Then using  $C = Lc$  and the Keynes-Ramsey rule

$$\begin{aligned}\frac{\dot{x}}{x} &= \frac{\dot{C}}{C} - \frac{\dot{K}}{K}, \\ &= -\frac{\alpha}{1-\alpha}r_g + x - \rho.\end{aligned}\tag{13}$$

The gross interest rate  $r_g$  is determined by the marginal product of capital

$$r_g = (1 - \alpha)\frac{Y}{K},$$

and behaves according to

$$\frac{\dot{r}_g}{r_g} = 2\alpha\beta\frac{\dot{R}}{R} + 2\alpha(1 - \beta)(n + h) - \alpha\frac{\dot{K}}{K}.$$

With the results so far this gives

$$\frac{\dot{r}_g}{r_g} = -\frac{2\alpha(1 - \beta)}{2\alpha\beta - 1}n + \frac{\alpha(2\beta - 1)}{2\alpha\beta - 1}x - \frac{\alpha}{1 - \alpha}r_g - \frac{\alpha}{2\alpha\beta - 1}\delta.\tag{14}$$

## 2.4 Steady State

In the steady state both, the interest rate and the consumption capital ratio, are constant, i.e.  $\frac{\dot{x}}{x} = \frac{\dot{r}_g}{r_g} = 0$ . This gives

$$\begin{aligned}r_g^* &= 2(1 - \beta)n - (2\beta - 1)\rho + \delta, \\ x^* &= \frac{2\alpha(1 - \beta)}{1 - \alpha}n - \frac{2\alpha\beta - 1}{1 - \alpha}\rho + \frac{\alpha}{1 - \alpha}\delta.\end{aligned}$$

Linearizing the above system (13) and (14) around the steady state gives the Jacobian

$$J = \begin{pmatrix} -\frac{\alpha}{1-\alpha}r_g^* & \frac{\alpha(2\beta-1)}{2\alpha\beta-1}r_g^* \\ -\frac{\alpha}{1-\alpha}x^* & x^* \end{pmatrix}$$

The eigenvalues of  $J$  are given by

$$\begin{aligned}\nu_1 &= \frac{1}{2} \left( \rho + \sqrt{\rho^2 - 4 \frac{\alpha}{2\alpha\beta - 1} r_g^* x^*} \right), \\ \nu_2 &= \frac{1}{2} \left( \rho - \sqrt{\rho^2 - 4 \frac{\alpha}{2\alpha\beta - 1} r_g^* x^*} \right).\end{aligned}$$

For  $\alpha\beta < \frac{1}{2}$  the system has two real eigenvalues, one positive and one negative, and hence the system is locally saddle path stable. In the case  $\alpha\beta > \frac{1}{2}$  the system is either generally unstable or oscillating unstable.

On the balanced growth path consumption and wages grow with rates

$$\frac{\dot{c}}{c} = \frac{\dot{w}}{w} = (1 - 2\beta)n - 2\beta\rho.$$

Aggregate output and the capital stock grow with rate

$$\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = 2(1 - \beta)n - 2\beta\rho.$$

The usage of the non-renewable resource grows with rate

$$\frac{\dot{R}}{R} = -\rho$$

and its price develops according to

$$\frac{\dot{q}}{q} = 2(1 - \beta)n - (2\beta - 1)\rho.$$

For saddle path stable positive growth in per capita production and consumption there are two conditions to be satisfied. First  $\alpha\beta < \frac{1}{2}$  and  $\beta < \frac{1}{2} \frac{n}{n+\rho}$ . The first condition is best interpreted when looking at the reduced form of final goods production (12). The resource needs not to be too important in terms of its output elasticity. The second condition refers to the importance of the resource sector in final goods production. the share of the resource in sector production has to be small and in any event smaller than  $\frac{1}{2}$ . The range for possible values for  $\beta$  is increasing in

the population growth rate  $n$  but is bounded from above by  $\frac{1}{2}$ . If this condition is fulfilled, the first mentioned stability condition is met automatically.

### 3 Conclusion

A model has been developed that demonstrates the importance of scale effects in the process of generating growth in per capita production and consumption. Despite the presence of population growth, capital depreciation and a non-renewable resource, stable positive long run growth might be feasible. The condition for this to be true is that the labor using sector is more important than the resource using sector in the economy.

Growth in this model is essentially driven by population growth as in almost all semi-endogenous growth models. Hence, population growth is good for growth. This result is not new to the literature concerning non-renewable resources. As Groth (2007) notes when summarizing the growth literature with respect to scale effects in production, population growth always increases growth in per capita consumption when the dynamic system is stable. However, this result stems from an analysis where rather the social than the private return on capital is used in the Keynes-Ramsey rule, when increasing returns stem from capital usage. In the present model increasing returns stem from the production factors labor and the non-renewable resource and only the private returns to these factors are used in the analysis.

The above model might be criticized because of the Cobb-Douglas aggregation function which implies an elasticity of substitution of exactly one between labor and resource using sectors. However, the Cobb-Douglas assumption is heavily used in the literature on exhaustible resources. Therefore the results can be more easily compared to the literature. Additionally this case is interesting because the resource is essential for production in this case and at the same time the average product of the resource is not bounded from above (Groth 2007).



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