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Dynamic Duopoly with Inattentive Firms

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Dynamic Duopoly with Inattentive Firms

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Dynamic Duopoly with Inattentive Firms ^{*}

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Preliminary Version

Abstract

This paper analyzes an infinite horizon dynamic duopoly with stochastic demand in which firms face costs of absorbing and processing information. Our main result is that the structure of dates at which firms choose to absorb information differ starkly between price and quantity competition. Firms synchronize their actions under price competition whereas they plan sequentially and in an alternating manner under quantity competition. The reason is that under quantity competition the planning firm reduces the uncertainty in the residual demand curve of the inattentive firm which renders planning less attractive for that firm. The opposite holds true under price competition.

Keywords: Inattentiveness, Price Competition, Quantity Competition, Synchronization

JEL Classification: C73, D43, D83

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1 Introduction

It is a heavily discussed question in economics if competing firms choose to synchronize their actions or if they adjust their processes at different times. Since the seminal papers about dynamic duopoly by Maskin and Tirole (1987, 1988a, 1988b) this question attracted considerable attention by researchers. The literature that addressed this question evolved along two lines. The first strand assumes that there is some kind of physical friction that hinders firms from adjusting their plans each period. Thus firms are exogenously equipped with commitment power and the mechanism that drives the results concerning synchronization versus non-synchronization is rooted in the assumption of strategic complementarity or substitutability of the firms' strategy variable. The second strand abstracts from physical frictions but allows firms to choose to be committed for some time period.

In this paper we propose a different perspective to look at this question. Instead of focussing on physical frictions we set out from the assumption that it is costly for firms to absorb and process the information they need in order to make the right decisions. As e.g. Radner (1992) points out this assumption is quite realistic since absorbing and processing the relevant information for decision-making is an important goal of managerial occupation. Thus the associated cost are not negligible. For several industries this formulation is probably the more important one since changing prices hardly involves any costs for most products and sometimes even quantities can be easily changed. Yet, it is often much harder to determine the optimal new price or quantity in an uncertain environment.¹

In order to incorporate the feature that information processing is costly into our analysis we assume that each time a firm wants to act on new information it has to bear a finite positive fix cost. As a consequence, firms will absorb and process information only if the cost of doing so equals its expected benefit. If a firm rationally refrains from acting on information during a certain time period we say that it stays rationally inattentive.

As a result we obtain that the timing pattern at which firms choose to plan differs between price and quantity competition. Firms synchronize their actions under price competition while they plan sequentially and in an alternating manner under quantity competition. The intuition for this result is mainly driven by the following effects. If a firm chooses to plan under price competition it optimally increases its price in a good demand

¹Recent empirical work also seems to contradict the finding the prices are adjusted only infrequently, see e.g. Bils and Klenow (2004).

state and lowers it in a bad demand state. Since we assume goods to be substitutes it thereby exerts a positive externality on the other firm in the good state and a negative one in the bad state. This increases the variance of the demand for the other firm and renders planning more profitable for the other firm. As a result both firms plan at the same time. Under quantity competition exactly the opposite holds true and so firms plan sequentially.

To be more precise we consider an infinite horizon continuous time model of competition between two firms who produce a differentiated good facing a stochastic demand function. Since firms face costs of absorbing and processing information they choose to plan only at some points in time and stay inattentive in the meantime. During the inattentiveness period uncertainty builds up in the system and firms choose their next planning dates via balancing the cost of planning and the gain obtained by having a re-optimized plan. At its planning date a firm simultaneously decides about its next planning date and about the path that its strategy variable follows during its inattentiveness period. At its planning date it observes the current realization of the shock, the whole history of shocks and strategy variables but it does not observe the current value of its opponent's strategy variable. So at every instant firms play a one-shot Bertrand or Cournot game with potentially different and imperfect information. The assumption that while planning the attentive firm does not observe the current price or quantity of the other firm implies that there is no commitment possibility for both firms. Thus by assuming this kind of information structure we switch off the "commitment effect" that is crucial for the results obtained by the literature so far. What matters for our results concerning synchronization or non-synchronization of plans is merely how the decision to plan of one firm affects the other firm's advantage of planning. As pointed out before this differs between price and quantity competition and we derive the result that synchronization of plans is the unique Markov Perfect Equilibrium under Bertrand competition, whereas choosing to plan in an asynchronous and alternating manner is the unique Markov Perfect Equilibrium under Cournot competition. Moreover we completely characterize the paths that the strategy variables follow in between two consecutive planning dates.

Our analysis relates to two different strands of the literature. The first strand is concerned with incorporating the assumption of costly information processing into economic models. The paper which is closely related to our approach of modeling inattentiveness is Reis (2006).² He analyzes the optimal length of a monopolist's inattentiveness period and derives an

²See also Reis (2005) or Mankiw and Reis (2006).

approximate solution in a general setting. He tests the models' predictions using US-inflation data and finds that his recursively state-dependent approach fits the data better than previous state-contingent models do. Due to the monopolistic setup of his model he does not address the issue of synchronization or non-synchronization of firms.³

The second strand analyzes the question of synchronization versus non-synchronization of firms' decision-making. This question was first dealt with in a macroeconomic context by e.g. Taylor (1979, 1980) and remained to be an important question in macroeconomics ever since. Maskin and Tirole (1987, 1988a, 1988b) address this question in a series of influential papers from a microeconomic perspective. They analyze a dynamic duopoly model in which firms either compete in prices or quantities. In their analysis they exogenously assume that firms make staggered decisions. They propose two ways to endogenize this assumption. Yet, these two proposals lead to conflicting answers if it comes to quantity competition which leads them to note that "... a more detailed study of the micro-foundations of timing in firms' decision-making is called for, an ambitious task that will have to be deferred to the future." (1987, p. 962). Lau (2001) extends the Maskin and Tirole idea via allowing players to choose whether their commitment lasts for one or two periods. He shows that non-synchronization is the outcome of a game in which the variables are strategic complements.⁴ Unfortunately, he does not consider strategic complements in his analysis. Bhaskar (2002) provides a model with two industries that interact with each other. Firms in each industry are atomistic and do not act strategically but there is aggregate strategic complementarity across industries. He shows that this can lead to staggered price setting. All of these papers focus on the possibility of commitment and therefore it is always crucial for synchronization whether firms' strategy variables are strategic complements or substitutes.⁵ As pointed out before, our paper completely abstracts from any device that could give rise to commitment power of the firms and our results are not driven by the fact that strategy variables are strategic complements or substitutes. What matters in our context is how the action of a planning firm affects the other firm's demand uncertainty and so the only aspect that is important is whether a

³There are other approaches for modeling inattentiveness. For example, Sims (2003, 2005) proposes an approach in which agents are attentive in all periods but can only absorb parts of the incoming information. This approach is used e.g. by Moscarini (2004).

⁴A model that is similar in spirit but looks at wage setting is provided by de Fraja (1993).

⁵For a recent treatment of dynamic duopoly, that does not focus on synchronization, see Jun and Vives (2004).

planning firm's choice exerts a positive externality on the other firm (as it is the case in price competition) or a negative one (as it is the case in quantity competition).

The rest of the paper is organized as follows. Section 2 sets out the model. In Section 3 we characterize the strategy variables' optimal paths. Section 4 analyzes the equilibrium structure of planning dates under Cournot and Bertrand competition. We discuss two possible extensions in Section 5 and Section 6 concludes.

2 The Model

There are two firms denoted by $i = 1, 2$. Each firm i produces a differentiated perishable good at zero marginal cost.⁶ Each firm faces an inverse demand curve

$$p_t^i = \alpha\theta_t - \frac{\beta}{\theta_t}q_t^i - \frac{\gamma}{\theta_t}q_t^j, \quad i \neq j \quad (1)$$

at date t , with $\alpha > 0$, and $\beta > \gamma \geq 0$. Fluctuations in market demand are represented by θ_t whose evolution is governed by a geometric Brownian motion

$$d\theta_t = \sigma\theta_t dz_t, \quad \text{with } \theta_0 = 1, \quad (2)$$

The drift rate is zero and dz_t is a standard Wiener process. Therefore θ_t has an expected value of θ_0 and a variance of $\theta_0^2(e^{\sigma^2 t} - 1)$, with $0 < \sigma^2 < \infty$. The chosen representation of the inverse demand curve reflects the effects that fluctuations of the market sentiment have: An increase in θ increases market size (it flattens the demand curve) and consumers' willingness to pay (increases the intercept) and vice versa.⁷ Firms compete for an infinite period of time and we distinguish two cases, namely either price competition or quantity competition.

As mentioned before it is costly to absorb and process information and so a firm decides to remain uninformed about the true state of the world θ_t in some time intervals. Whenever a firm updates her information and adjusts her price or quantity to the new information she faces a finite adjustment cost of $K \geq 0$. A firm decides at which time periods she plans in order to

⁶As Singh and Vives (1984) show, the analysis would not change if firms faced positive constant marginal costs c because this would only lower the effective intercept from α to $a = \alpha - c$.

⁷Our results hold as long as we assume that shocks are positively correlated. We assume the shocks to be perfectly positively correlated in order to simplify the following analysis.

adjust her price or quantity path. We denote planning dates by $D^i(k)$, with $D^i(k) : \mathbf{N}_0 \rightarrow \mathbf{R}$. If at date $D^i(k-1)$ a firm plans then the time that elapses up to its next adjusting date is $d^i(k) = D^i(k) - D^i(k-1)$ and we call $d^i(k)$ the inattentiveness period of firm i .

If firm i decides to plan at date $D^i(k)$ it observes the current shock $\theta_{D^i(k)}$. Moreover, it also learns the whole history of the shock realizations θ_t from $D^i(k-1)$ to $D^i(k)$ and all the rival's prices p_t^j or quantities q_t^j that it has set until date $D^i(k)$. But it does not observe the current price $p_{D^i(k)}^j$ or quantity $q_{D^i(k)}^j$ of its rival. It can only make an inference about the price or quantity that its rival sets at $D^i(k)$.

What matters for this inference is illustrated in a simple example. If both firms plan simultaneously at $D^i(k-1)$ and $D^i(k)$, then $E[s_t^j]$, where $s_t \in \{p, q\}$, with $D^i(k) > t > D^i(k-1)$, is conditional on $\theta_{D^i(k-1)}$ the state of the world that both firms observe while planning and on the time that elapses since the last planning date $t - D^i(k-1)$.⁸ Firms need both pieces of information to calculate the expected value of θ_t .

At its planning date $D^i(k)$ firm i makes two decisions: Firstly, it determines the complete path of prices or quantities from today until its next planning date, namely either $\{p_t^i(\theta_{D^i(k)}, E[p_t^j], t - D^i(k))\}_{t=D^i(k)}^{D^i(k+1)}$ or $\{q_t^i(\theta_{D^i(k)}, E[q_t^j], t - D^i(k))\}_{t=D^i(k)}^{D^i(k+1)}$. Secondly, it decides about its next planning date because it will not gain any further information in between.

Thus if the last planning date of firm i was $D^i(k)$ then the expected profit that firm i obtains at some time $t \in [D^i(k), D^i(k+1)[$ is given by

$$\pi^i(\theta_{D^i(k)}, t - D^i(k), p_t^i, E[p_t^j]) = \max_{p_i} E \left[p_i \left(\frac{\theta_t((\alpha\theta_t)(\beta - \gamma) - \beta p_i + \gamma p_j)}{\beta^2 - \gamma^2} \right) \right], \quad (3)$$

if firms compete in prices. If firms compete in quantities, instead, firm i 's expected profit obtained at some time $t \in [D^i(k), D^i(k+1)[$ is given by

$$\pi^i(\theta_{D^i(k)}, t - D^i(k), q_t^i, E[q_t^j]) = \max_{q_i} E \left[q_i \left(\alpha\theta_t - \frac{\beta q_i + \gamma q_j}{\theta_t} \right) \right]. \quad (4)$$

The equilibrium concept we employ is Markov Perfect Equilibrium.

⁸In the following we will denote the expectation conditional on the information at the current planning date by $E[\cdot]$.

3 Characterization of Optimal Paths

Before analyzing the planning decisions of firms we must determine the optimal (price or quantity) path that a firm chooses in between planning dates. In this section we characterize the path under the assumption that firms synchronize their actions.⁹ This is instructive since it reveals that the paths differ starkly between price and quantity competition although firms gain no further information in between planning dates.

Proposition 1. *If firms plan simultaneously and observe θ_0 at $D(0) = 0$, then the path of firm i 's strategy variable until its next planning date is either:*

$$p_t^i = \frac{\alpha(\beta - \gamma)\theta_0 e^{\sigma^2 t}}{2\beta - \gamma}, \text{ or} \quad (5)$$

$$q_t^i = \frac{\alpha\theta_0^2}{e^{\sigma^2 t}(\beta + \gamma)}, \quad (6)$$

if firm j does not plan in the meantime.

Proof: Suppose that firms plan simultaneously at $D(0) = 0$, and observe θ_0 . Further assume, that both plan simultaneously at some $D(1) > 0$. Now firms have to determine the path that their strategy variable should follow during the inattentiveness period. At date 0 firm i solves either (3) or (4) for each instant t in this inattentiveness interval conditional on the information that both firms plan simultaneously at 0 and $D(1)$. Moreover, it knows that both firms observe θ_0 , and that at t the information about θ_0 is outdated by t periods for both firms. This implies that firms form symmetric expectations about θ_t and the strategy chosen by the opponent. Thus at each instant during their inattentiveness period firms end up playing the static Cournot and Bertrand equilibrium with imperfect but symmetric information. Maximization of (3) and (4) using that $E[\theta_t^2 | \theta_0, t - 0] = \theta_0^2 e^{\sigma^2 t}$ and $E\left[\frac{1}{\theta_t} | \theta_0, t - 0\right] = \frac{1}{\theta_0} e^{\sigma^2 t}$ yields (5) and (6). ■

The intuition behind the result, that the quantity path is decreasing in the variance whereas the price path is increasing in the variance can be easily described in a stylized setting in which the realization of the shock is

⁹The description of the strategy variables' optimal paths for the case in which one firm plans in the meantime is provided in Section 4.2.1.

either high or low with equal probability. A positive shock shifts the demand curve outward and it becomes flatter, whereas the reverse holds true for a negative shock. With fixed prices the profit of firm i increases in a good demand state by more than it decreases in a bad demand state. Thus the size of the market increases on average in the variance of the shock. Due to our assumption about the stochastic process the variance increases linearly in time and so the expected size of the market increases in the time that passed since the last planning date. Price-setting firms react optimally to that increase by choosing an increasing price path.

With fixed quantities the reverse holds true, since the profit of firm i increases in a good demand state by less than it decreases in a bad demand state. Thus the expected size of the market decreases in the time that elapsed since the last planning date. Thus quantity-setting firms react optimally to this shrinking by reducing its quantity.

4 Characterization of Planning Dates

4.1 The Optimality Condition

Now we turn to the characterization of optimal planning dates. Each firm i maximizes its expected present discounted (at the rate $r > 0$) value of profits including planning costs

$$E[\pi^i] = E\left[\sum_{k=0}^{\infty} \left\{ \int_{D^i(k)}^{D^i(k+1)} e^{-rt} \pi^1(s_t^i, s_t^j, t - D^i(k), \theta_t) dt - e^{-rD^i(k+1)} K \right\}\right], \quad (7)$$

via choosing an infinite sequence of planning dates $\{D^i(k)\}_{k=1}^{\infty}$. Note that if the costs of planning are zero, firm i chooses to be always attentive.

This problem has a recursive structure between planning dates. Let D^i and $D^{i'}$ denote the current and the next planning date of firm i , then we can restate the problem for any $D^i, D^{i'}, i \in \{1, 2\}$ that satisfy $D^i \leq D^j < D^{i'} \leq D^{j'}$ as

$$V^1(D^1, D^2, \theta_{D^1}) = \max_{D^{1'}} E\left[\int_{D^1}^{D^2} e^{-rt} \pi^1(s_t^1, s_t^2, t - D^1, \theta_t) dt + e^{-rD^2} W^1(D^{1'}, D^{2'}, \theta_{D^2})\right], \quad (8)$$

$$W^1(D^{1'}, D^{2'}, \theta_{D^2}) = E \left[\int_{D^2}^{D^{1'}} e^{-rt} \pi^1(s_t^1, s_t^2, t - D^1, \theta_t) dt \right. \\ \left. - e^{-rD^{1'}} K + e^{-rD^{1'}} V^1(D^{1'}, D^{2'}, \theta_{D^{1'}}) \right], \quad (9)$$

and similarly for firm 2. $V^1(\cdot)$ is firm 1's value function if it is about to decide and if the next planning date of firm 2 is D^2 . Similarly $W^1(\cdot)$ is firm 1's value function if firm 2 is about to decide and firm 1 moves next at $D^{1'}$.

Standard results imply that the differentiability of the payoff functions carries over to the value functions $V^1(\cdot)$ and $W^1(\cdot)$. Thus the first-order condition of the optimization problem can be characterized as:

$$E \left[\frac{\partial}{\partial D^{1'}} \left(e^{-rD^2} W^1(D^{1'}, D^{2'}, \theta_{D^2}) \right) \right] = 0. \quad (10)$$

From (10) it is evident that we need to derive a simple equation for $W^1(\cdot)$ in order to get an analytical expression of the problem's first order condition. From (8) and (9) we obtain

$$W^1(D^{1'}, D^{2'}, \theta_{D^2}) = E \left[\int_{D^2}^{D^{1'}} e^{-rt} \pi^1(s_t^1, s_t^2, t - D^1, \theta_t) dt \right. \\ \left. + \int_{D^{1'}}^{D^{2'}} e^{-rt} \pi^1(s_t^1, s_t^2, t - D^{1'}, \theta_{D^{1'}}) dt - e^{-rD^{1'}} K \right. \\ \left. + e^{-rD^{2''}} W^1(D^{1''}, D^{2''}, \theta_{D^{2''}}) \right] \quad (11)$$

Differentiating (11) with respect to $D^{1'}$ yields:

$$E \left[e^{-rD^{1'}} \left(\pi^1(s_{D^{1'}}^1, s_{D^{1'}}^2, D^{1'} - D^1, \theta_{D^1}) - \pi^1(s_{D^{1'}}^1, s_{D^{1'}}^2, 0, \theta_{D^1}) \right) + r e^{-rD^{1'}} K \right. \\ \left. + \int_{D^{1'}}^{D^{2''}} e^{-rt} \frac{\partial \pi^1(s_t^1, s_t^2, t - D^{1'}, \theta_{D^1})}{\partial D^{1'}} dt + \frac{\partial}{\partial D^{1'}} \left(e^{-rD^{2''}} W^1(D^{1''}, D^{2''}, \theta_{D^{2''}}) \right) \right]. \quad (12)$$

Substituting (10) and

$$E \left[\frac{\partial}{\partial D^{1'}} \left(e^{-rD^{2''}} W^1(D^{1''}, D^{2''}, \theta_{D^{2''}}) \right) \right] = 0,$$

in (12) yields:

$$\begin{aligned}
& e^{-rD^{1'}} E \left[\pi^1(s_{D^{1'}}^1, s_{D^{1'}}^2, 0, \theta_{D^1}) - \pi^1(s_{D^{1'}}^1, s_{D^{1'}}^2, D^{1'} - D^1, \theta_{D^1}) - rK \right] \\
& = E \left[\int_{D^{1'}}^{D^{2'}} e^{-rt} \frac{\partial \pi^1(s_t^1, s_t^2, t - D^{1'}, \theta_{D^1})}{\partial D^{1'}} dt \right].
\end{aligned} \tag{13}$$

The intuition contained in the first order condition can be nicely presented in the framework of a discrete time approximation. Each instant firm i trades off whether it should plan today or postpone planning to the next instant. If it plans today it instantaneously reaps the profit from having a fresh plan and incurs planning cost. Furthermore it re-optimizes the plan that its strategy variable takes until its next planning date. Due to the recursive structure of the problem it merely considers the expected discounted profits from now until the consecutive planning date of its opponent. From the perspective of firm i 's last adjustment date the expected profits of planning today can be represented as:

$$e^{-rD^{1'}} E \left[\pi^1(s_{D^{1'}}^1, s_{D^{1'}}^2, 0, \theta_{D^1}) - K + \int_{D^{1'}}^{D^{2'}} e^{-r(t-D^{1'})} \pi^1(s_t^1, s_t^2, t - D^{1'}, \theta_{D^1}) dt \right]. \tag{14}$$

If it did not plan today but waits for Δt periods, it instantaneously gets the profits from following an outdated plan. At the next instant it earns the expected profit from having a fresh plan and incurs planning costs. After re-optimizing its plan it also considers the expected discounted profits from now until the consecutive planning date of its opponent. From the perspective of firm i 's last adjustment date the expected profits of postponing planning by Δt can be represented as:

$$\begin{aligned}
& e^{-rD^{1'}} E \left[\pi^1(s_{D^{1'}}^1, s_{D^{1'}}^2, D^{1'} - D^1, \theta_{D^1}) + e^{-r\Delta t} \left(\pi^1(s_{D^{1'}+\Delta t}^1, s_{D^{1'}+\Delta t}^2, 0, \theta_{D^{1'}+\Delta t}) - K \right) \right. \\
& \left. + \int_{D^{1'}+\Delta t}^{D^{2'}} e^{-r(t-D^{1'}+\Delta t)} \pi^1(s_t^1, s_t^2, t - D^{1'}+\Delta t, \theta_{D^{1'}+\Delta t}) dt \right].
\end{aligned} \tag{15}$$

Subtracting (14) from (15) and letting $\Delta t \rightarrow 0$ yields (13).

Thus the left hand side of (13) captures the discounted expected value of planning

$$e^{-rD^{1'}} E \left[\pi^1(s_{D^{1'}}^1, s_{D^{1'}}^2, 0, \theta_{D^1}) - \pi^1(s_{D^{1'}}^1, s_{D^{1'}}^2, D^{1'} - D^1, \theta_{D^1}) \right],$$

net of adjustment cost. The sign of the value of planning is positive since it is better to act on precise information. The term on the right hand side of (13) warrants some discussion as well. As pointed out in the discrete time approximation the derivative represents the difference in expected profits between planning an instant later and planning today at each instant between the current planning date of firm i and the next planning date of firm j . Since planning tomorrow implies that the plan that firm i sets is at each instant less outdated than the plan that it would set if it plans today, we expect this difference to be positive too.

The basic trade-off between planning today or waiting until tomorrow is always the same for both firms irrespective of the mode of competition and the opponent's decision whether it wants to plan at the same instant or not. However both aspects influence heavily how the structure of planning dates will look like in equilibrium. Before we characterize the structure of planning dates under Bertrand and Cournot competition in section 4.3 we turn to the analytical derivation of the terms contained in (13) in the next section.

4.2 Analytical derivation

4.2.1 Value of planning

In this section we characterize the value of planning under Bertrand and Cournot competition. Before being able to do so we establish a general Lemma for calculating the expectation about the product of two random variables at different points in time which turns out to be important for this proof. This is done in the Appendix in Lemma 1. We look at the value of planning for two scenarios. In the first scenario both firms plan simultaneously whereas in the second firms plan sequentially. Proposition 2 first states the result obtained under Bertrand competition.

Proposition 2 *Consider the situation in which both firms compete in prices. Firm i 's value of planning is higher if firm j plans at the same instant than if firm j does not plan at the same instant.*

Proof:

Consider the situation in which both firms compete in prices, plan simultaneously for the first time at $D(0) = 0$ and observe θ_0 .

First we determine the expected instantaneous profit of having a fresh plan. If firm i plans for the second time at some future instant u it observes

the true state of the world and maximizes its profit

$$\frac{p_u^i \theta_u (\gamma(p_u^j + \alpha\theta_u) + \beta(\alpha\theta_u - p_u^i))}{\beta^2 - \gamma^2}, \quad (16)$$

by choosing the optimal p_u^i . Thus the best response of firm i is given by

$$p_u^i = \frac{\gamma(p_u^j - \alpha\theta_u) + \alpha\beta\theta_u}{2\beta}. \quad (17)$$

If firm j plans for the second time at u as well, then

$$p_u^i = p_u^j = \frac{\alpha\theta_u(\beta - \gamma)}{2\beta - \gamma}.$$

Thus the corresponding expected profit equals

$$\frac{\beta(\beta - \gamma)\theta_0^3 e^{3\sigma^2 u} \alpha^2}{(\beta + \gamma)(2\beta - \gamma)^2}, \quad (18)$$

if both firms plan simultaneously.

If firm j plans for the second time at $s < u$ whereas it does not plan at u it solves the following problem in order to determine the optimal p_u^2 . Firstly, it forms expectations about the best response of firm i in u conditional on the state of the world that it observed while planning for the last time (θ_u) and the time that elapsed between its last planning date and u , i.e. $u - s$. Thus firm j expects the best response of firm i at u to be

$$E[p_u^i] = \frac{\gamma(p_u^j - \alpha\theta_u) + \alpha\beta\theta_u}{2\beta}. \quad (19)$$

Secondly, it has to determine its own best response. In order to do that firm j forms expectations about market demand at u and thereby about its expected profit

$$E \left[\frac{p_u^j \theta_t (\gamma(p_u^i + \alpha\theta_u) + \beta(\alpha\theta_u - p_u^j))}{\beta^2 - \gamma^2} \text{Bigg} \right],$$

which equals

$$\frac{p_u^j \theta_s (p_u^j \beta + \theta_s e^{\sigma^2(u-s)} \beta \alpha + \gamma E[p_u^i] - \theta_s e^{\sigma^2(u-s)} \gamma \alpha)}{\beta^2 - \gamma^2}.$$

Maximizing this with respect to p_u^j yields that firm j 's best response is

$$p_u^j = \frac{\alpha\theta_s e^{\sigma^2(u-s)}(\beta - \gamma) + \gamma E[p_u^i]}{2\beta}. \quad (20)$$

Solving (19) and (20) for p_u^j and $E[p_u^i]$ yields that

$$p_u^j = \frac{\alpha(\beta - \gamma)\theta_s(2e^{\sigma^2(u-s)}\beta + \gamma)}{4\beta^2 - \gamma^2}. \quad (21)$$

Since firm i knows $u - s$ and observes θ_s while planning at u , it can forecast the best response of firm j in u . This implies that it determines its own best reply by using (21) in (17). Thus p_u^i is given by

$$p_u^i = \frac{\alpha(\beta - \gamma)(\gamma^2\theta_s - \gamma^2\theta_u + 2\theta_s e^{\sigma^2(u-s)}\beta\gamma + 4\theta_u\beta^2)}{2\beta(4\beta^2 - \gamma^2)}. \quad (22)$$

Plugging (22) and (21) in (16) yields that the expected profit of firm i is given by

$$\frac{\alpha^2\theta_u(\beta - \gamma)(\gamma^2(\theta_s - \theta_u) + 2\theta_s e^{\sigma^2(u-s)}\beta\gamma + 4\theta_u\beta^2)}{4(\beta + \gamma)\beta(4\beta^2 - \gamma^2)^2}.$$

Thus the expected profit from having a fresh plan if the other firm does not plan at the same instant is equal to

$$\begin{aligned} & \frac{\alpha^2\theta_0^3(\beta - \gamma)}{e^{2\sigma^2s}4(\beta + \gamma)\beta(4\beta^2 - \gamma^2)^2} \left((4\beta^2 - \gamma^2)^2 e^{\sigma^2(3u+2s)} \right. \\ & \left. + 4\beta\gamma(4\beta^2 + \gamma\beta - \gamma^2)e^{\sigma^2(3s+2u)} + 2\gamma^2(\beta^2 + 2\gamma - \gamma^2)e^{\sigma^2(4s+u)} + \gamma^4 e^{\sigma^2 5s} \right), \end{aligned} \quad (23)$$

where (23) is derived by using the result stated in Lemma 1 that $E[\theta_u\theta_s^2|\theta_0] = \theta_0^3 \exp(3\sigma^2s)$, $E[\theta_u^2\theta_s|\theta_0] = \theta_0^3 \exp(\sigma^2(2s + u))$, and $E[\theta_u^3|\theta_0] = \theta_0^3 \exp(3\sigma^2u)$.

Now we determine the expected instantaneous profit of having an outdated plan. If firm i does not plan at u it maximizes its expected profit

$$\frac{p_u^i\theta_0(p_u^i\beta + \theta_s e^{\sigma^2u}\beta\alpha + \gamma p_u^j - \theta_0 e^{\sigma^2u}\gamma\alpha)}{\beta^2 - \gamma^2}, \quad (24)$$

by choosing the optimal p_u^i . Thus the best response of firm i is given by

$$p_u^i = \frac{\gamma E[p_u^j] + \alpha\theta_0 e^{\sigma^2u}(\beta - \gamma)}{2\beta}. \quad (25)$$

If firm j plans for the second time at u , then i expects the best response of firm j to be

$$E[p_u^j] = \frac{\gamma p_u^i + \alpha \theta_0 (\beta - \gamma)}{2\beta}. \quad (26)$$

Solving (25) and (26) for $E[p_u^j]$ and p_u^i yields:

$$p_u^i = \frac{\alpha \theta_0 (\beta - \gamma) (2\beta e^{\sigma^2 u} + \gamma)}{(4\beta^2 - \gamma^2)}, \quad (27)$$

$$E[p_u^j] = \frac{\alpha \theta_0 (\beta - \gamma) (2\beta + \gamma e^{\sigma^2 u})}{(4\beta^2 - \gamma^2)}. \quad (28)$$

Plugging (27) and (28) in (24) yields that the expected profit of following an outdated plan if the opponent firm plans at this instant is given by

$$\frac{\theta_0^3 \alpha^2 \beta (2\beta e^{\sigma^2 u} + \gamma)^2 (\beta - \gamma)}{(\beta + \gamma) (4\beta^2 - \gamma^2)^2}. \quad (29)$$

Now consider the situation in which firm j does plan at date $s < u$ but does not plan at u . Then i expects the best response of firm j at u to be

$$E[p_u^j] = \frac{\gamma p_u^i + \alpha \theta_0 e^{\sigma^2 u} (\beta - \gamma)}{2\beta}. \quad (30)$$

Solving (25) and (30) for $E[p_u^j]$ and p_u^i yields:

$$p_u^i = \frac{\alpha \theta_0 (\beta - \gamma) (2\beta e^{\sigma^2 u} + \gamma e^{\sigma^2 u})}{(4\beta^2 - \gamma^2)}, \quad (31)$$

$$E[p_u^j] = \frac{\alpha \theta_0 (\beta - \gamma) (2\beta e^{\sigma^2 u} + \gamma e^{\sigma^2 u})}{(4\beta^2 - \gamma^2)}. \quad (32)$$

Plugging (31) and (32) in (24) yields that the expected profit of following an outdated plan if the opponent firm does not plan at this instant is given by

$$\frac{\theta_0^3 \alpha^2 \beta (2\beta e^{\sigma^2 u} + \gamma e^{\sigma^2 u})^2 (\beta - \gamma)}{(\beta + \gamma) (4\beta^2 - \gamma^2)^2}. \quad (33)$$

Now we are in the position to state the expected value of planning for the considered scenarios. The expected value of planning at some future instant u for firm i is given by the difference between (23) and (33), which

is equal to

$$\begin{aligned} & \frac{\alpha^2 \theta_0^3 (\beta - \gamma)}{4(\beta + \gamma)\beta(4\beta^2 - \gamma^2)^2} \left(16\beta^4 (e^{3\sigma^2 u} - e^{2\sigma^2 u}) + 16\beta^3 \gamma (e^{\sigma^2(2u+s)} - e^{\sigma^2(2u-s)}) \right. \\ & + 4\beta^2 \gamma^2 (e^{\sigma^2(2u+s)} + 2e^{\sigma^2(u+2s)} - 2e^{3\sigma^2 u} - e^{2\sigma^2(u-s)}) \\ & \left. + 4\beta \gamma^3 (e^{\sigma^2(u+2s)} - e^{\sigma^2(2u+s)}) + \gamma^4 (e^{3\sigma^2 s} + e^{3\sigma^2 u} - 2e^{\sigma^2(u+2s)}) \right) > 0, \end{aligned} \quad (34)$$

if firm j does not plan at the same instant. Whereas if firm j also plans at u firm i 's expected value of planning is given by the difference between (29) and (18), which is equal to

$$\frac{\beta(\beta - \gamma)\theta_0^3 \alpha^2}{(\beta + \gamma)(4\beta^2 - \gamma^2)^2} \left(4\beta^2 (e^{3\sigma^2 u} - e^{2\sigma^2 u}) + 4\beta \gamma (e^{3\sigma^2 u} - e^{\sigma^2 u}) + \gamma^2 (e^{3\sigma^2 u} - 1) \right) > 0. \quad (35)$$

Subtracting (34) from (35) yields

$$\begin{aligned} & \frac{\alpha^2 \theta_0^3 (\beta - \gamma) \gamma}{4(\beta + \gamma)\beta(4\beta^2 - \gamma^2)^2} \left(16\beta^3 (e^{3\sigma^2 u} + e^{\sigma^2(2u-s)} - e^{\sigma^2(2u+s)} - e^{\sigma^2 u}) \right. \\ & + 4\beta^2 \gamma (3e^{3\sigma^2 u} + e^{2\sigma^2(u-s)} - 2e^{\sigma^2(u+2s)} - e^{\sigma^2(2u+s)} - 1) \\ & \left. + 4\beta \gamma^2 (e^{\sigma^2(2u+s)} - e^{\sigma^2(u+2s)}) + \gamma^3 (2e^{\sigma^2(u+2s)} - e^{3\sigma^2 u} - e^{3\sigma^2 s}) \right), \end{aligned} \quad (36)$$

which is positive if $u > s \geq 0$. ■

Proposition 3 states the result which is obtained if firms compete in quantities.

Proposition 3 *Consider the situation in which both firms compete in quantities. Firm i 's value of planning is higher if firm j does not plan at the same instant than if firm j plans at the same instant.*

Proof: The derivation of expected profits follows the same logic as presented in the Proof of Proposition 3 and is therefore omitted. Here we merely state the analytical expressions.

Firm i 's expected instantaneous profit of having a fresh plan at some future instant u is given by

$$\begin{aligned} & \frac{\alpha^2 \theta_0^3}{2\beta(4\beta^2 - \gamma^2)^2 e^{2\sigma^2(u+s)}} \left(e^{\sigma^2(5u+2s)} (8\beta^4 - 4\beta^2 \gamma^2 + \gamma^4) \right. \\ & \left. + e^{\sigma^2(u+6s)} \beta \gamma (2\beta \gamma + 2\gamma^2 - 8\beta^2) + e^{\sigma^2(3u+4s)} \gamma^2 (4\beta^2 - 2\beta \gamma - \gamma^2) \right), \end{aligned} \quad (37)$$

if firm j does not plan at the same instant. Whereas if firm j also plans at u , firm i 's expected instantaneous profit of having a fresh plan is

$$\frac{\alpha^2 \theta_0^3 e^{3\sigma^2 u} \beta}{(2\beta + \gamma)^2}. \quad (38)$$

Firm i 's expected instantaneous profit of having an outdated plan at some future instant u is given by

$$\frac{\alpha^2 \theta_0^3 \beta (2\beta e^{\sigma^2(u-s)} - \gamma e^{\sigma^2(u+s)})^2}{e^{\sigma^2(3u-2s)} (4\beta^2 - \gamma^2)^2}, \quad (39)$$

if firm j does not plan at the same instant. Whereas if firm j plans at u then firm i 's expected instantaneous profit of having an outdated plan is

$$\frac{\alpha^2 \theta_0^3 \beta (2\beta - \gamma e^{2\sigma^2 u})^2}{(4\beta^2 - \gamma^2)^2 e^{\sigma^2 u}}. \quad (40)$$

Firm i 's expected value of planning at some future instant u is therefore given by

$$\begin{aligned} & \frac{\alpha^2 \theta_0^3}{2\beta(4\beta^2 - \gamma^2)^2} \left(8\beta^4 (e^{3\sigma^2 u} - e^{-\sigma^2 u}) + 8\beta^3 \gamma (e^{-\sigma^2(u-2s)} - e^{-\sigma^2(u-4s)}) \right. \\ & + 4\beta^2 \gamma^2 (e^{\sigma^2(u+2s)} - e^{3\sigma^2 u}) + 2\beta \gamma^3 (e^{-\sigma^2(u-4s)} - e^{\sigma^2(u+2s)}) \\ & \left. + \gamma^4 (e^{3\sigma^2 u} - e^{\sigma^2(u+2s)}) \right) > 0, \end{aligned} \quad (41)$$

if firm j does not plan at the same instant. Whereas if firm j plans at u then firm i 's expected value of planning at this instant is

$$\frac{\alpha^2 \theta_0^3 \beta^2 ((\beta - \gamma) e^{3\sigma^2 u} - \beta e^{-\sigma^2 u} + \gamma e^{\sigma^2 u})}{(4\beta^2 - \gamma^2)^2} > 0. \quad (42)$$

Subtracting (42) from (41) yields

$$\begin{aligned} & \frac{\alpha^2 \theta_0^3 \gamma}{2\beta(4\beta^2 - \gamma^2)^2} \left(8\beta^3 (e^{3\sigma^2 u} - e^{\sigma^2 u} - e^{-\sigma^2(u-4s)} + e^{-\sigma^2(u-2s)}) \right. \\ & + 4\beta^2 \gamma (e^{\sigma^2(u+2s)} - e^{3\sigma^2 u}) + 2\beta \gamma^2 (e^{-\sigma^2(u-4s)} - e^{\sigma^2(u+2s)}) \\ & \left. + \gamma^3 (e^{3\sigma^2 u} - e^{\sigma^2(u+2s)}) \right), \end{aligned} \quad (43)$$

which is positive if $u > s \geq 0$. ■

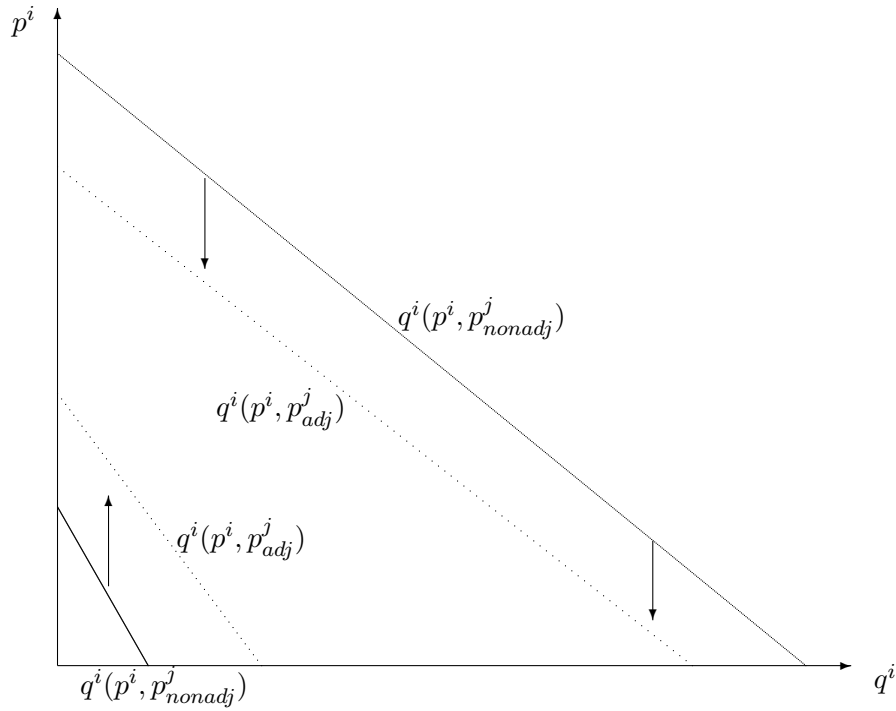


Figure 1: Comparison of the value of planning under Cournot competition

We provide the intuition behind both results in a stylized setting in which market demand is either high or low with equal probability.

First consider the situation in which firms compete in quantities. Both firms plan simultaneously at date 0 and decide about their next planning date. The value of planning of firm 1 at some future date t given that firm 2 does not plan at the same instant is depicted by the distance between the solid lines in Figure 1. Now suppose, that firm 2 would plan at this instant. If demand decreased firm 2 chooses to produce a smaller quantity than without planning. As a consequence prices increase compared to the situation in which firm 2 does not plan and thus the decrease in profits of firm 1 is ameliorated. The converse holds true if the shock to market demand would have been positive. In total, however, by planning firm 2 reduces the expected incorrectness of firm 1's plan, which is depicted by the distance between the dotted lines in Figure 1. This in turn implies, that the losses from being inattentive for firm 1 decrease. Therefore it has an incentive to postpone planning if firm 2 plans at the same instant.

Now consider the situation in which firms compete in prices, both firms plan simultaneously at date 0 and decide about their next planning date. The value of planning of firm 1 at some future date given that firm 2 does not plan at the same instant is depicted by the difference between the solid lines in Figure 2. Now suppose, that firm 2 would plan at this instant. If there was a negative shock to market demand firm 2 would set a lower price. This amplifies the negative consequence on firm 1 since it sells even less than

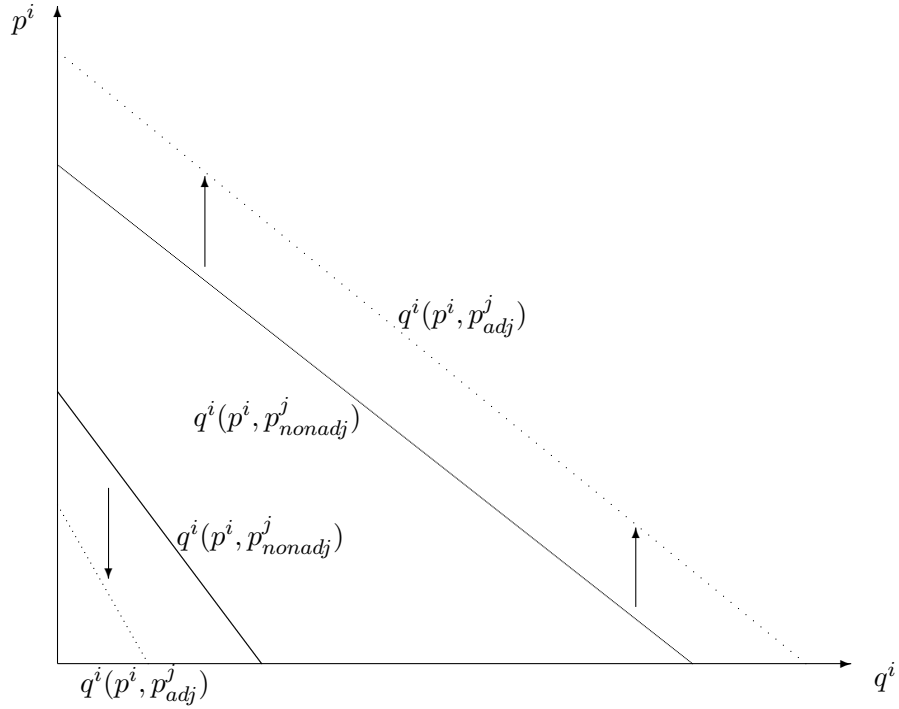


Figure 2: Comparison of the value of planning under Bertrand competition

without planning of firm 2. In the case of a positive demand shock firm 2 would amplify the positive consequences for firm 1. In total, however, by planning firm 2 increases the expected incorrectness of firm 1's plan, which is depicted by the distance between the dotted lines in Figure 2. This in turn implies, that the losses from being inattentive for firm 1 increase. Therefore it has an even higher incentive to plan at date t if firm 2 also plans at this instant.

The intuition underlying the Proof of Proposition 2 and 3 immediately implies the following result that completes the characterization of optimal paths.

Corollary *Consider the situation in which firm i plans at u whereas firm j stays inattentive at this instant. The path of the strategy variable that the inattentive firm sets jumps downward at u , irrespective of the mode of competition.*

Proof Consider the situation in which both firms plan simultaneously at $D(0) = 0$ and observe some θ_0 . If firms compete in prices and if firm i plans at some future instant u then the optimal price of firm j is

$$p_u^j = \frac{\alpha\theta_0(\beta - \gamma)(2\beta e^{\sigma^2 u} + \gamma)}{(4\beta^2 - \gamma^2)}, \quad (44)$$

whereas the optimal price of firm j is given by

$$p_u^j = \frac{\alpha\theta_0(\beta - \gamma)(2\beta + \gamma)e^{\sigma^2 u}}{(4\beta^2 - \gamma^2)}, \quad (45)$$

if firm i does not plan at u . Clearly (45) exceeds (44).

If firms compete in quantities and if firm i plans at some future instant u then the optimal quantity of firm j is

$$q_u^j = \frac{\alpha\theta_0^2(2\beta - e^{2\sigma^2 u}\gamma)}{(4\beta^2 - \gamma^2)e^{\sigma^2 u}}, \quad (46)$$

whereas the optimal quantity is given by

$$q_u^j = \frac{\alpha\theta_0^2}{e^{\sigma^2 u}(\beta + \gamma)}, \quad (47)$$

if firm i does not plan at u . Clearly (47) exceeds (46). ■

The intuition behind this result is the following. It follows from Proposition 1 that the quantity path of an inattentive firm decreases in the incorrectness of its plan. Now if one firm plans while the other one remains inattentive, the planning firm reduces the expected incorrectness of the non-planning firm's plan. This effect leads to an increase in the quantity set by the inattentive firm. However, there is a second countervailing effect. Since the attentive firm observes the true state of the world it chooses a quantity that is higher than the one it would have chosen without planning. As shown in Proposition 1, this is the case because the optimal action under Cournot competition is to set a larger quantity with full information than with imperfect information. As quantities are strategic substitutes this effect leads to a decrease in the quantity set by the inattentive firm. Since the second effect dominates the first one, we have that the quantity set by the inattentive firm is lower if the other firm is attentive compared to the situation in which it would remain inattentive as well.

The second statement contained in Proposition 1 states that the price path of an inattentive firm increases in the variance. Now if one firm plans while the other firm remains inattentive, the planning firm increases the expected incorrectness of the non-planning firm's plan. This effect leads to an increase in the price set by the inattentive firm. Again the second effect works in the other direction. As shown in Proposition 1 the attentive firm observes the true state of the world and so charges a smaller price than it would have chosen without full information. Thus the inattentive firm

expects the attentive firm to reduce its price. Since prices are strategic substitutes the inattentive firm reduces its price as well. Again the second effect dominates the first one, which delivers the result that the inattentive firm's price is smaller if the other firm is attentive compared to the situation in which the other firm remains inattentive as well.

After having derived analytical expressions for the instantaneous profits we turn to the analysis of the intertemporal effects in the subsequent section.

4.2.2 Intertemporal effects

First consider the situation in which both firms compete in prices. The following Lemma presents the analytical representation of the intertemporal effects. We determine the gain in the future from postponing planning by one instant. As mentioned before this gain accrues because the firm has a plan that is an instant closer to the optimum in all future periods till its next planning date. We look at this gain in two different scenarios, namely simultaneous and sequential planning.

Lemma 2 *Consider the situation in which both firms compete in prices. Under sequential planning firm i 's difference in expected profits per instant is represented by*

$$\begin{aligned}
& - \frac{\alpha^2 \theta_0^3 \sigma^2 (\beta - \gamma)}{4\beta(2\sigma^2 - r)(4\beta^2 - \gamma^2)(\beta + \gamma)} \left(16\beta^4 (e^{u(3\sigma^2 - r)} - e^{\sigma^2 u + (2\sigma^2 - r)v}) \right. \\
& + 16\beta^3 \gamma (e^{(3\sigma^2 - r)u} - e^{\sigma^2 u + (2\sigma^2 - r)v}) + 8\beta^2 \gamma^2 (e^{\sigma^2 u + (2\sigma^2 - r)v} - e^{(3\sigma^2 - r)u}) \\
& + 4\beta \gamma^3 (e^{2\sigma^2 s - \sigma^2 u + (2\sigma^2 - r)v} + e^{\sigma^2 u + (2\sigma^2 - r)v} - e^{(\sigma^2 - r)u + 2\sigma^2 s} - e^{(3\sigma^2 - r)u}) \\
& \left. + \gamma^4 (e^{(3\sigma^2 - r)u} + 2e^{3\sigma^2 s - 2\sigma^2 u + (2\sigma^2 - r)v} - 2e^{(3\sigma^2 - r)u} - e^{\sigma^2 u + (2\sigma^2 - r)v}), \right.
\end{aligned} \tag{48}$$

where v , with $u < v$, denotes the next planning date of firm j . Under simultaneous planning the difference in expected profits per instant of firm i is represented by

$$\frac{\alpha^2 \theta_0^3 \sigma^2 (\beta - \gamma) (e^{\sigma^2 u + (2\sigma^2 - r)v} - e^{(3\sigma^2 - r)u})}{4(2\sigma^2 - r)\beta(\beta + \gamma)}. \tag{49}$$

Proof:

Consider the following situation. Firm i plans at some future instant u and firm j 's current planning date is s and it plans next at some future

instant v , with $s < u < v$. Then the expected instantaneous profit of firm i at some instant τ , with $u < \tau \leq v$ is given by

$$\begin{aligned} & \frac{\alpha^2 \theta_0^3 (\beta - \gamma) e^{2\sigma^2(\tau - u - s)}}{4\beta(\beta + \gamma)(4\beta^2 - \gamma^2)^2} \left(16\beta^4 e^{\sigma^2(3u+2s)} + 16\beta^3 \gamma e^{\sigma^2(3u+2s)} \right. \\ & + 4\beta^2 \gamma^2 (3e^{\sigma^2(2u+3s)} - 2e^{\sigma^2(3u+2s)}) + 4\beta \gamma^3 (e^{\sigma^2(u+4s)} - e^{\sigma^2(3u+2s)}) \quad (50) \\ & \left. + \gamma^4 (e^{\sigma^2(3u+2s)} + e^{5\sigma^2 s} - 2e^{\sigma^2(2u+3s)}) \right). \end{aligned}$$

If firm i would instead postpone planning for one instant and plans at $u' = u + \Delta t$, then its expected instantaneous profit at some instant τ , with $u' < \tau \leq v$ is given by

$$\begin{aligned} & \frac{\alpha^2 \theta_0^3 (\beta - \gamma) e^{2\sigma^2(\tau - (u + \Delta t) - s)}}{4\beta(\beta + \gamma)(4\beta^2 - \gamma^2)^2} \left(16\beta^4 e^{\sigma^2(3(u + \Delta t) + 2s)} + 16\beta^3 \gamma e^{\sigma^2(3(u + \Delta t) + 2s)} \right. \\ & + 4\beta^2 \gamma^2 (3e^{\sigma^2(2(u + \Delta t) + 3s)} - 2e^{\sigma^2(3(u + \Delta t) + 2s)}) + 4\beta \gamma^3 (e^{\sigma^2((u + \Delta t) + 4s)} - e^{\sigma^2(3(u + \Delta t) + 2s)}) \\ & \left. + \gamma^4 (e^{\sigma^2(3(u + \Delta t) + 2s)} + e^{5\sigma^2 s} - 2e^{\sigma^2(2(u + \Delta t) + 3s)}) \right). \quad (51) \end{aligned}$$

Subtracting (50) from (51), dividing the difference by Δt , letting $\Delta t \rightarrow 0$, and integrating the resulting expression over τ from u to v yields (48).

Consider now the following situation. Firm i and firm j plan simultaneously at some future instant u . Firm j plans next at some future instant v , with $u < v$. Then the expected instantaneous profit of firm i at some instant τ , with $u < \tau \leq v$ is given by

$$\frac{\alpha^2 \theta_0^3 \beta (\beta - \gamma) e^{\sigma^2(2\tau + u)}}{(2\beta - \gamma)^2 (\beta + \gamma)}. \quad (52)$$

If firm i would instead postpone planning for one instant and plans at $u' = u + \Delta t$, then its expected instantaneous profit at some instant τ , with $u' < \tau \leq v$ is given by

$$\begin{aligned} & \frac{\alpha^2 \theta_0^3 (\beta - \gamma) e^{2\sigma^2 \tau}}{4\beta(\beta + \gamma)(2\beta - \gamma)^2 e^{2\sigma^2(u + \Delta t)}} \left(4\beta^2 e^{3\sigma^2(u + \Delta t)} + 4\beta \gamma (e^{\sigma^2(3u + 2\Delta t)} - e^{3\sigma^2(u + \Delta t)}) \right. \\ & \left. + \gamma^2 (e^{\sigma^2(3u + 2\Delta t)} + e^{3\sigma^2(u + \Delta t)} - 2e^{\sigma^2(3u + 2\Delta t)}) \right). \quad (53) \end{aligned}$$

Subtracting (52) from (53), dividing the difference by Δt and letting $\Delta t \rightarrow 0$ and integrating the resulting expression over τ from u to v yields (49). ■

Now consider the situation in which both firms compete in quantities. The following Lemma presents the analytical representation of the intertemporal effects in the scenarios of simultaneous and sequential planning.

Lemma 3 *Consider the situation in which both firms compete in quantities. Under sequential planning firm i 's difference in expected profits per instant is represented by*

$$\frac{\alpha^2 \theta_0^3 \sigma^2 (e^{\sigma^2 v} - e^{(\sigma^2+r)u-rv}) e^{(3\sigma^2-r)u-\sigma^2 v}}{(r + \sigma^2) \beta (4\beta^2 - \gamma^2)^2} \times \left(16\beta^4 e^{2\sigma^2 u} + 4\beta^2 \gamma^2 (e^{2\sigma^2 s} - 2e^{2\sigma^2 u}) - 2\beta \gamma^3 e^{2\sigma^2 s} + \gamma^4 (2e^{2\sigma^2 u} - e^{2\sigma^2 s}) \right), \quad (54)$$

where v , with $u < v$ denotes the next planning date of firm j . Under simultaneous planning the difference in expected profits per instant of firm i is represented by

$$\frac{\alpha^2 \theta_0^3 \sigma^2 (8\beta^3 + 4\beta^2 \gamma - \gamma^3) (e^{\sigma^2 v} - e^{(\sigma^2+r)u-rv}) e^{(3\sigma^2-r)u-\sigma^2 v}}{(r + \sigma^2) \beta (8\beta^3 + 4\beta^2 \gamma - 2\beta \gamma^2 - \gamma^3)}. \quad (55)$$

Proof:

Consider the following situation. Firm i plans at some future instant u and firm j 's current planning date is s and it plans next at some future instant v , with $s < u < v$. Then the expected instantaneous profit of firm i at some instant τ , with $u < \tau \leq v$ is given by

$$\begin{aligned} & \frac{\alpha^2 \theta_0^3}{4\beta(4\beta^2 - \gamma^2)^2} \left(8\beta^4 e^{\sigma^2(4u-\tau)} + 8\beta^3 \gamma e^{\sigma^2(4s-\tau)} \right. \\ & + 2\beta^2 \gamma^2 (2e^{\sigma^2(2u+2s-\tau)} - 2e^{\sigma^2(4u-\tau)} + e^{\sigma^2(4s-\tau)}) \\ & \left. + 2\beta \gamma^3 (e^{\sigma^2(4s-\tau)} - e^{\sigma^2(2u+2s-\tau)}) + \gamma^4 (e^{\sigma^2(4u-\tau)} - e^{\sigma^2(2u+2s-\tau)}) \right). \end{aligned} \quad (56)$$

If firm i would instead postpone planning for one instant and plans at $u' = u + \Delta t$, then its expected instantaneous profit at some instant τ , with $u' <$

$\tau \leq v$ is given by

$$\begin{aligned}
& \frac{\alpha^2 \theta_0^3}{2\beta(4\beta^2 - \gamma^2)^2 e^{\sigma^2((u+\Delta t)+2s+\tau)}} \left(8\beta^4 e^{\sigma^2(5(u+\Delta t)+2s)} - 8\beta^3 \gamma e^{\sigma^2((u+\Delta t)+6s)} \right. \\
& + 2\beta^2 \gamma^2 (2e^{\sigma^2(3(u+\Delta t)+4s)} + e^{\sigma^2((u+\Delta t)+6s)} - 2e^{\sigma^2(5(u+\Delta t)+2s)}) \\
& + 2\beta \gamma^3 (e^{\sigma^2((u+\Delta t)+6s)} - e^{\sigma^2(3(u+\Delta t)+4s)}) \\
& \left. + \gamma^4 (e^{\sigma^2(5(u+\Delta t)+2s)} - e^{\sigma^2(3(u+\Delta t)+4s)}) \right). \tag{57}
\end{aligned}$$

Subtracting (56) from (57), dividing the difference by Δt , letting $\Delta t \rightarrow 0$, and integrating the resulting expression over τ from u to v yields (54).

Consider now the following situation. Firm i and firm j plan simultaneously at some future instant u . Firm j plans next at some future instant v , with $u < v$. Then the expected instantaneous profit of firm i at some instant τ , with $u < \tau \leq v$ is given by

$$\frac{\alpha^2 \theta_0^3 \beta e^{3\sigma^2 u}}{(2\beta + \gamma)^2 e^{\sigma^2(\tau-u)}}. \tag{58}$$

If firm i would instead postpone planning for one instant and plans at $u' = u + \Delta t$, then its expected instantaneous profit at some instant τ , with $u' < \tau \leq v$ is given by

$$\begin{aligned}
& \frac{\alpha^2 \theta_0^3 e^{2\sigma^2 u}}{4\beta(4\beta^2 - \gamma^2)^2 e^{\sigma^2(\tau+\Delta t)}} \left(16\beta^4 e^{\sigma^2(3u+5\Delta t)} - 16\beta^3 \gamma e^{\sigma^2(3u+\Delta t)} \right. \\
& + 4\beta^2 \gamma^2 (e^{\sigma^2(3u+\Delta t)} - 2e^{\sigma^2(3u+5\Delta t)} + 2e^{3\sigma^2(u+\Delta t)}) \\
& \left. + 4\beta \gamma^3 (e^{\sigma^2(3u+\Delta t)} - e^{3\sigma^2(u+\Delta t)}) + \gamma^4 (e^{\sigma^2(3u+5\Delta t)} - e^{3\sigma^2(u+\Delta t)}) \right). \tag{59}
\end{aligned}$$

Subtracting (58) from (59), dividing the difference by Δt and letting $\Delta t \rightarrow 0$ and integrating the resulting expression over τ from u to v yields (55). ■

After having derived the analytical expressions for the terms contained in the first order condition of the maximization problem the stage is set to present the equilibrium planning decision of firms under Bertrand and Cournot competition.

4.3 Equilibria under Bertrand and Cournot competition

As pointed out before we are interested in the question whether firms plan simultaneously or sequentially. We assume that under both regimes both

firms have to plan simultaneously for the first time at $D(0) = 0$ where they observe θ_0 . If both firms keep on planning jointly we denote the second planning date by u and the third by v . If firms choose to plan sequentially we assume that firm j starts. We denote firm j 's second planning date by s and its third planning date by v . Firm i 's second planning date is denoted by u , with $0 < s < u < v$.

From introspection of (13) it is evident, that firm i 's optimal second planning date u is a function of firm j 's next planning date v and its last planning date, which is either s in the sequential scenario or 0 in the simultaneous scenario. The same holds true for firm j 's optimal second planning date. In other words each firm tries to choose the optimal distance between its current planning date and the prior and next planning dates of its rival.

Due to the recursive structure at each planning date a firm faces the same trade-off in expectations formed at date 0. Moreover, the recursive structure implies that the trade-off that firm j faces in determining its second planning date is from the perspective of period 0 identical to the trade-off that firm i faces when it has to determine its second one. This reasoning implies, that we can draw on the recursive structure of the game in order to determine the equilibrium distance of planning dates.

Thus under the simultaneous regime the equilibrium distance between v and u is given by

$$v - u = u. \quad (60)$$

The reason for this result is the following. If it is optimal for both firms to plan after u periods, then from the perspective of period 0, it should be optimal for them to plan simultaneously after $2u$ periods for the third time.

Under the sequential regime the equilibrium distance between v and u is given by

$$v - u = u - s. \quad (61)$$

The reason is that from the perspective of period 0 firm j 's decision about v at s is identical to the trade-off that firm i faces when it decides about the distance between u and s . Thus if it is optimal for firm i to plan $u - s$ periods after s , then it is, from the perspective of period 0, optimal for firm j to choose the same distance between v and u .

However, before we set out to determine how the structure of equilibrium planning dates will look like under Bertrand and Cournot competition, we have to determine firm j 's optimal second planning date s conditional on firm i planning at u , with $0 < s < u$.

4.3.1 Optimal second planning date under sequential planning

The results are summarized in the following Lemmata.

Lemma 4 *Consider the situation in which both firms compete in prices and are required to plan sequentially. For every second planning date u of firm i there exists a unique optimal second planning date of firm j , denoted by $s^*(u)$, with $0 < s^*(u) < u$. $s^*(u)$ is implicitly defined by:*

$$\begin{aligned}
& e^{-rs} \left(\frac{\alpha^2 \theta_0^3 (\beta - \gamma)}{4(\beta + \gamma)(4\beta^2 - \gamma^2)^2} \left(16\beta^4 (e^{3\sigma^2 s} - e^{2\sigma^2 s}) + 8\beta^2 \gamma^2 (e^{\sigma^2 s} - e^{3\sigma^2 s}) \right. \right. \\
& \left. \left. + 4\beta \gamma^3 (e^{\sigma^2 s} - e^{3\sigma^2 s}) + \gamma^4 (2e^{\sigma^2 s} - e^{3\sigma^2 s} - 1) \right) - rK \right) \\
& = \frac{\alpha^2 \theta_0^3 \sigma^2 (\beta - \gamma)}{4(2\sigma^2 - r)\beta(\beta + \gamma)(4\beta^2 - \gamma^2)^2 e^{(2\sigma^2 + r)s}} \\
& \times \left(16\beta^4 (e^{(2\sigma^2 - r)u + (3\sigma^2 + r)s} - e^{5\sigma^2 s}) \right. \\
& + 8\beta^2 \gamma^2 (e^{5\sigma^2 s} + e^{3\sigma^2 s} - e^{(2\sigma^2 - r)u + (3\sigma^2 + r)s} - e^{(2\sigma^2 - r)u + (\sigma^2 + r)s}) \\
& + 4\beta \gamma^3 (e^{3\sigma^2 s} - e^{(2\sigma^2 - r)u + (\sigma^2 + r)s}) \\
& \left. + \gamma^4 (e^{(2\sigma^2 - r)u + (3\sigma^2 + r)s} + 2e^{(2\sigma^2 - r)u + (\sigma^2 + r)s} + 2e^{2\sigma^2 s} - 2e^{3\sigma^2 s} - e^{5\sigma^2 s} - 2e^{(2\sigma^2 - r)u + rs}) \right). \tag{62}
\end{aligned}$$

Moreover $s^*(u)$ is increasing in u .

Proof Equation (62) is derived by using (48) and (34) in (13). Now we need to show that an optimal s exists. Let the left hand side of (62) be denoted by $\psi(s)$, whereas the right hand side of (62) is denoted by $\kappa(s, u)$. We can show that

$$\begin{aligned}
\lim_{s \rightarrow 0} \psi(s) & = -rK < 0, \\
\lim_{s \rightarrow 0} \kappa(s, u) & = \frac{\alpha^2 \theta_0^3 \sigma^2 (8\beta^3 - 4\beta^2 \gamma - 6\beta \gamma^2 + \gamma^3)(\beta - \gamma)(e^{(2\sigma^2 - r)u} - 1)}{4\beta(2\sigma^2 - r)(2\beta + \gamma)(2\beta - \gamma)^2},
\end{aligned}$$

which is bigger than zero if $\gamma < \gamma'$ and smaller than zero if $\beta > \gamma \geq \gamma'$, where γ' is defined by

$$8\beta^3 - 4\beta^2 \gamma' - 6\beta(\gamma')^2 + (\gamma')^3 = 0$$

Thus we require $|-rK|$ to be sufficiently high such that it is strictly bigger than $\lim_{s \rightarrow 0} \kappa(s, u) \forall \gamma \in (\gamma', \beta)$. Since it can be shown, that

$$\begin{aligned} \frac{\partial \psi(s)}{\partial s} = & \\ & -r\psi(s) + \frac{\alpha^2 \theta_0^3 \sigma^2 (\beta - \gamma) e^{(\sigma^2 - r)s}}{4\beta(4\beta^2 - \gamma^2)^2(\beta + \gamma)} \left(16\beta^4(4e^{2\sigma^2 s} - 4e^{\sigma^2 s}) \right. \\ & \left. + 8\beta^2 \gamma^2(1 - 3e^{2\sigma^2 s}) + 4\beta \gamma^3(1 + e^{\sigma^2 s}) + \gamma^4(3e^{2\sigma^2 s} - 2) \right), \end{aligned} \quad (63)$$

is strictly bigger than zero and that

$$\begin{aligned} \frac{\partial \kappa(s, u)}{\partial s} = & \\ & \frac{\alpha^2 \theta_0^3 \sigma^2 (\beta - \gamma)}{4\beta(2\sigma^2 - r)(4\beta^2 - \gamma^2)^2(\beta + \gamma)e^{(2\sigma^2 - r)s}} \left(16\beta^4 \sigma^2 (e^{((2\sigma^2 - r)u + 3\sigma^2 + r)s} - 4e^{5\sigma^2 s}) + 16r\beta^4 e^{5\sigma^2 s} \right. \\ & + 8\beta^2 \gamma^2 \sigma^2 (3e^{5\sigma^2 s} + e^{3\sigma^2 s} + e^{(2\sigma^2 - r)u + (\sigma^2 + r)s} - e^{(2\sigma^2 - r)u + (3\sigma^2 + r)s}) \\ & - 8\beta^2 \gamma^2 r (e^{3\sigma^2 s} + e^{5\sigma^2 s}) + 4\beta \gamma^3 \sigma^2 (e^{(2\sigma^2 - r)u + (\sigma^2 + r)s} + e^{3\sigma^2 s}) \\ & - 4r\beta \gamma^3 e^{3\sigma^2 s} + r\gamma^4 (2e^{3\sigma^2 s} + e^{5\sigma^2 s} + 2e^{2\sigma^2 s}) \\ & \left. + \sigma^2 \gamma^4 (e^{(2\sigma^2 - r)u + (3\sigma^2 + r)s} - 2e^{(2\sigma^2 - r)u + (\sigma^2 + r)s} + 4e^{(2\sigma^2 - r)u + rs} - 2e^{3\sigma^2 s} + 3e^{5\sigma^2 s}) \right), \end{aligned} \quad (64)$$

is strictly negative, we have shown that for every u there exists a unique s , denoted by $s^*(u)$, that solves (62). Moreover it can be shown that $|\frac{\partial \kappa(s, u)}{\partial s}|$ is bigger than $|\frac{\partial \psi(s)}{\partial s}|$. Thus the second order condition indicates that $s^*(u)$ is a maximum.

Since it can be shown, that

$$\begin{aligned} \frac{\partial(-\chi(s) + \phi(u, s))}{\partial u} = & \\ & \frac{\alpha^2 \theta_0^3 \sigma^2}{4\beta(\beta + \gamma)(4\beta^2 - \gamma^2)^2 e^{(2\sigma^2 + r)s}} \left(16\beta^4 e^{(2\sigma^2 - r)u + (3\sigma^2 + r)s} \right. \\ & - 8\beta^2 \gamma^2 (e^{(2\sigma^2 - r)u + (3\sigma^2 + r)s} - e^{(2\sigma^2 - r)u + (\sigma^2 + r)s}) - 4\beta \gamma^3 e^{(2\sigma^2 - r)u + (\sigma^2 + r)s} \\ & \left. + \gamma^4 (e^{(2\sigma^2 - r)u + (3\sigma^2 + r)s} + 2e^{(2\sigma^2 - r)u + (\sigma^2 + r)s} - 2e^{(2\sigma^2 - r)u + rs}) \right), \end{aligned} \quad (65)$$

is strictly positive it follows from the Implicit Function Theorem that $s^*(u)$ is increasing in u . ■

Lemma 5 Consider the situation in which both firms compete in quantities and are required to plan sequentially. For every second planning date u of firm i there exists a unique optimal second planning date of firm j , denoted by $s^{**}(u)$, with $0 < s^{**}(u) < u$. $s^{**}(u)$ is implicitly defined by:

$$\begin{aligned}
& e^{-rs} \left(\frac{\alpha^2 \theta_0^3}{2\beta(4\beta^2 - \gamma^2)^2 e^{\sigma^2 s}} \left(\beta^4 (e^{4\sigma^2 s} - 1) + 4\beta^2 \gamma^2 (e^{2\sigma^2 s} - e^{4\sigma^2 s}) \right. \right. \\
& \left. \left. + 2\beta\gamma^3 (e^{2\sigma^2 s} - 1) + \gamma^4 (e^{4\sigma^2 s} - e^{2\sigma^2 s}) \right) - rK \right) \\
& = \frac{\alpha^2 \theta_0^3 \sigma^2 (e^{\sigma^2 u} - e^{(\sigma^2 + r)s - ru}) e^{(\sigma^2 - r)s - \sigma^2 u}}{(r + \sigma^2) \beta (4\beta^2 - \gamma^2)^2} \\
& \times \left(16\beta^4 e^{2\sigma^2 s} + 4\beta^2 \gamma^2 (1 - 2e^{2\sigma^2 s}) - 2\beta\gamma^3 - \gamma^4 \right) = 0.
\end{aligned} \tag{66}$$

Moreover $s^{**}(u)$ is increasing in u .

Proof Equation (66) is derived by using (55) and (38) in (13). Now we need to show that an optimal s exists. Let the left hand side of (66) be denoted by $\chi(s)$, whereas the right hand side of (66) is denoted by $\phi(s, u)$. We can show that

$$\begin{aligned}
\lim_{s \rightarrow 0} \chi(s) & = -rK < 0, \\
\lim_{s \rightarrow 0} \phi(s, u) & = \frac{\alpha^2 \theta_0^3 \sigma^2 (8\beta^3 + 4\beta^2 \gamma - \gamma^3) (e^{\sigma^2 u} - e^{-ru})}{\beta (r + \sigma^2) (8\beta^3 + 4\beta^2 \gamma - 2\beta\gamma^2 - \gamma^2 e^{\sigma^2 u})} > 0.
\end{aligned}$$

Since it can be shown, that

$$\begin{aligned}
& \frac{\partial \chi(s)}{\partial s} = \\
& -r\chi(s) + \frac{\alpha^2 \theta_0^3 \sigma^2 e^{-rs}}{4\beta(4\beta^2 - \gamma^2)^2 e^{\sigma^2 s}} \left(16\beta^4 (3e^{4\sigma^2 s} + 1) \right. \\
& \left. + 8\beta^2 \gamma^2 (e^{2\sigma^2 s} - 3e^{5\sigma^2 s}) - 4\beta\gamma^3 (e^{2\sigma^2 s} + 1) + \gamma^4 (6e^{4\sigma^2 s} - 2e^{2\sigma^2 s}) \right),
\end{aligned} \tag{67}$$

is strictly bigger than zero and that

$$\begin{aligned}
\frac{\partial \phi(s, u)}{\partial s} = & \frac{\alpha^2 \theta_0^3 \sigma^2 e^{(\sigma^2 - r)s - \sigma^2 u}}{\beta(r + \sigma^2)(4\beta^2 - \gamma^2)^2} \left(16\beta^4 \sigma^2 (3e^{\sigma^2(u+2s)} - 4e^{(3\sigma^2 + r)s - ru}) - 16r\beta^4 e^{\sigma^2(\sigma^2(u+2s))} \right. \\
& + 4\beta^2 \gamma^2 \sigma^2 (e^{\sigma^2 u} - 2e^{(\sigma^2 + r)s - ru} + 8e^{(3\sigma^2 + r)s - ru} - 6e^{\sigma^2(u+2s)}) \\
& + 4\beta^2 \gamma^2 r (2e^{\sigma^2(u+2s)} - e^{\sigma^2 u}) + 2\beta \gamma^3 \sigma^2 (2e^{(\sigma^2 + r)s - ru} - e^{\sigma^2 u}) \\
& + 2r\beta \gamma^3 e^{\sigma^2 u} + r\gamma^4 (e^{\sigma^2 u} - 2e^{\sigma^2(u+2s)}) \\
& \left. + \sigma^2 \gamma^4 (2e^{(\sigma^2 + r)s - ru} + 6e^{\sigma^2(u+2s)} - 8e^{(3\sigma^2 + r)s - ru} - e^{\sigma^2 u}) \right), \tag{68}
\end{aligned}$$

is strictly negative, we have shown that for every u there exists a unique s , denoted by $s^{**}(u)$, that solves (66). Moreover it can be shown that $|\frac{\partial \phi(s, u)}{\partial s}|$ is bigger than $|\frac{\partial \chi(s)}{\partial s}|$. Thus the second order condition indicates that $s^{**}(u)$ is a maximum.

Since it can be shown, that

$$\begin{aligned}
\frac{\partial(-\chi(s) + \phi(u, s))}{\partial u} = & \frac{\alpha^2 \theta_0^3 \sigma^2 e^{2\sigma^2 s - (\sigma^2 + r)u}}{\beta(4\beta^2 - \gamma^2)^2} \left(16\beta^4 e^{2\sigma^2 s} + 4\beta^2 \gamma^2 (1 - 2e^{2\sigma^2 s}) - 2\beta \gamma^3 + \gamma^4 (2e^{2\sigma^2 s} - 1) \right), \tag{69}
\end{aligned}$$

is strictly positive it follows from the Implicit Function Theorem that $s^{**}(u)$ is increasing in u . ■

Now the stage is set to determine the equilibrium structure of planning dates under Bertrand and Cournot competition in the following section.

4.3.2 Equilibrium structure of planning dates under Bertrand competition

Proposition 4 contains one of the two main results of this analysis.

Proposition 4 *Consider the situation in which firms compete in prices and plan simultaneously at $D(0) = 0$. Then simultaneous planning constitutes the unique equilibrium.*

Proof If both firms set prices and plan simultaneously at some future date u it follows from (49) and (35) that (13) is given by

$$e^{-ru} \left(\frac{\beta(\beta - \gamma)\theta_0^3 \alpha^2}{(\beta + \gamma)(4\beta^2 - \gamma^2)^2} \left(4\beta^2(e^{3\sigma^2 u} - e^{2\sigma^2 u}) + 4\beta\gamma(e^{3\sigma^2 u} - e^{\sigma^2 u}) \right. \right. \\ \left. \left. + \gamma^2(e^{3\sigma^2 u} - 1) \right) - rK \right) = \frac{\alpha^2 \theta_0^3 \sigma^2 (\beta - \gamma) (e^{\sigma^2 u + (2\sigma^2 - r)v} - e^{(3\sigma^2 - r)u})}{4(2\sigma^2 - r)\beta(\beta + \gamma)}. \quad (70)$$

Along similar lines as in the proofs of Lemma 4 and Lemma 5 it can be shown, that for every third planning date v of firm j there exists a unique $u^*(v)$ that is increasing in v . Using that the equilibrium distance between planning dates implies that $v = 2u$ in (70) yields that simultaneous planning on behalf of the firms is indeed an equilibrium.

Now consider the scenario with sequential planning. If firm j plans for the second time at some future instant s and for the third time at some future instant v then the condition that determines firm i 's optimal second planning date u , with $s < u < v$ is given by

$$e^{-ru} \left(\frac{\alpha^2 \theta_0^3 (\beta - \gamma)}{4(\beta + \gamma)\beta(4\beta^2 - \gamma^2)^2} \left(16\beta^4(e^{3\sigma^2 u} - e^{2\sigma^2 u}) + 16\beta^3\gamma(e^{\sigma^2(2u+s)} - e^{\sigma^2(2u-s)}) \right. \right. \\ \left. \left. + 4\beta^2\gamma^2(e^{\sigma^2(2u+s)} + 2e^{\sigma^2(u+2s)} - 2e^{3\sigma^2 u} - e^{2\sigma^2(u-s)}) \right. \right. \\ \left. \left. + 4\beta\gamma^3(e^{\sigma^2(u+2s)} - e^{\sigma^2(2u+s)}) + \gamma^4(e^{3\sigma^2 s} + e^{3\sigma^2 u} - 2e^{\sigma^2(u+2s)}) \right) - rK \right) = \\ - \frac{\alpha^2 \theta_0^3 \sigma^2 (\beta - \gamma)}{4\beta(2\sigma^2 - r)(4\beta^2 - \gamma^2)^2(\beta + \gamma)} \left(16\beta^4(e^{u(3\sigma^2 - r)} - e^{\sigma^2 u + (2\sigma^2 - r)v}) \right. \\ \left. + 16\beta^3\gamma(e^{(3\sigma^2 - r)u} - e^{\sigma^2 u + (2\sigma^2 - r)v}) + 8\beta^2\gamma^2(e^{\sigma^2 u + (2\sigma^2 - r)v} - e^{(3\sigma^2 - r)u}) \right. \\ \left. + 4\beta\gamma^3(e^{2\sigma^2 s - \sigma^2 u + (2\sigma^2 - r)v} + e^{\sigma^2 u + (2\sigma^2 - r)v} - e^{(\sigma^2 - r)u + 2\sigma^2 s} - e^{(3\sigma^2 - r)u}) \right. \\ \left. + \gamma^4(e^{(3\sigma^2 - r)u} + 2e^{3\sigma^2 s - 2\sigma^2 u + (2\sigma^2 - r)v} - 2e^{(3\sigma^2 - r)u} - e^{\sigma^2 u + (2\sigma^2 - r)v}) \right). \quad (71)$$

Again it can be shown along similar lines as in the proofs of Lemma 4 and Lemma 5, that for every second planning date s and every third planning date v of firm j there exists a unique $u^{**}(v, s)$ that is increasing in v and decreasing in s . However, sequential planning can only be an equilibrium if (71) and (62) are jointly satisfied for some (s, u, v) -combination, which fulfills the requirement that the equilibrium distance of planning dates under

sequential planning implies that $v = 2u - s$. It can be shown, that there is no (s, u, v) -combination satisfying $v = 2u - s$ for which (71) and (62) are jointly fulfilled. Thus there exists no equilibrium in which firms plan sequentially under Bertrand competition. ■

The next section determines the equilibrium structure of planning dates under Cournot competition.

4.3.3 Equilibrium structure of planning dates under Cournot competition

Proposition 5 contains the second main result of this analysis.

Proposition 5 *Consider the situation in which firms compete in quantities and plan simultaneously at $D(0) = 0$. Then sequential and alternating planning constitutes the unique equilibrium.*¹⁰

Proof If both firms set quantities and plan simultaneously at some future date u it follows from (55) and (42) that (13) is given by

$$e^{-ru} \left(\frac{\alpha^2 \theta_0^3 \beta^2 ((\beta - \gamma)e^{3\sigma^2 u} - \beta e^{-\sigma^2 u} + \gamma e^{\sigma^2 u})}{(4\beta^2 - \gamma^2)^2} - rK \right) = \frac{\alpha^2 \theta_0^3 \sigma^2 (8\beta^3 + 4\beta^2 \gamma - \gamma^3) (e^{\sigma^2 v} - e^{(\sigma^2 + r)u - rv}) e^{(3\sigma^2 - r)u - \sigma^2 v}}{(r + \sigma^2) \beta (8\beta^3 + 4\beta^2 \gamma - 2\beta \gamma^2 - \gamma^3)}. \quad (72)$$

Along similar lines as in the proofs of Lemma 4 and Lemma 5 it can be shown, that for every third planning date v of firm j there exists a unique $u^*(v)$ that is increasing in v . Using that the equilibrium distance between planning dates implies that $v = 2u$ in (72) yields that simultaneous planning cannot be an equilibrium.

Now consider the scenario with sequential planning. If firm j plans for the second time at some future instant s and for the third time at some future instant v then the condition that determines firm i 's optimal second

¹⁰Unique here refers to the structure of planning dates. Of course, the equilibrium is not unique in determining which of the firms plans first and which one follows

planning date u , with $s < u < v$, is given by

$$\begin{aligned}
& e^{-ru} \left(\frac{\alpha^2 \theta_0^3}{2\beta(4\beta^2 - \gamma^2)^2} \left(8\beta^4(e^{3\sigma^2 u} - e^{-\sigma^2 u}) + 8\beta^3\gamma(e^{-\sigma^2(u-2s)} - e^{-\sigma^2(u-4s)}) \right. \right. \\
& + 4\beta^2\gamma^2(e^{\sigma^2(u+2s)} - e^{3\sigma^2 u}) + 2\beta\gamma^3(e^{-\sigma^2(u-4s)} - e^{\sigma^2(u+2s)}) \\
& \left. \left. + \gamma^4(e^{3\sigma^2 u} - e^{\sigma^2(u+2s)}) \right) - rK \right) = \frac{\alpha^2 \theta_0^3 \sigma^2 (e^{\sigma^2 v} - e^{(\sigma^2+r)u-rv}) e^{(3\sigma^2-r)u-\sigma^2 v}}{(r + \sigma^2)\beta(4\beta^2 - \gamma^2)^2} \\
& \times \left(16\beta^4 e^{2\sigma^2 u} + 4\beta^2\gamma^2(e^{2\sigma^2 s} - 2e^{2\sigma^2 u}) - 2\beta\gamma^3 e^{2\sigma^2 s} + \gamma^4(2e^{2\sigma^2 u} - e^{2\sigma^2 s}). \right. \\
& \left. (73) \right)
\end{aligned}$$

Again it can be shown along similar lines as in the proofs of Lemma 4 and Lemma 5, that for every second planning date s and every third planning date v of firm j , with $s < u < v$ there exists a unique $u^{**}(v, s)$ that is increasing in v and decreasing in s . However, sequential planning can only be an equilibrium if (73) and (66) are jointly satisfied for some (s, u, v) -combination, which fulfills the requirement that the equilibrium distance of planning dates under sequential planning implies that $v = 2u - s$. It can be shown, that there exists a unique (s, u, v) -combination satisfying $v = 2u - s$ for which (73) and (66) are jointly fulfilled. Thus there exists a unique equilibrium in planning dates in which firms plan sequentially and in an alternating manner under Cournot competition. ■

5 Extension

In this section we discuss two possible extensions of our model. The first one is to allow for two different demand shocks and the second extension deals with asymmetric starting dates.

5.1 Different demand shocks

In our model we assumed that both firms' demand curves are hit by the same shock, θ_t . Yet, since firms produce differentiated products this must not necessarily be the case. This idea can be incorporated in our analysis by assuming that firm i 's demand curve at date t is given by

$$p_t^i = \alpha\theta_t^i - \frac{\delta}{\theta_t^i} q_t^i - \frac{\gamma}{\theta_t^i} q_t^j,$$

while that of firm j is given by

$$p_t^j = \alpha\theta_t^j - \frac{\delta}{\theta_t^j}q_t^j - \frac{\gamma}{\theta_t^j}q_t^i,$$

where θ_t^i and θ_t^j are two different shocks that are (less than perfectly) positively correlated. We think that our results will hold qualitatively in this more general setting, since the driving forces of the basic model are still at work here. Still, firms would choose to plan simultaneously under Bertrand competition and sequentially under Cournot competition.

Another restriction in our model is that the shock that enters the intercept and the shock that affects the slope of the inverse demand curve are perfectly positively correlated. However, introducing two different but correlated shocks into the (inverse) demand of a single firm would considerably complicate the problem, without affecting our results qualitatively as long as the correlation between the shocks remains positive. Yet, if the shocks are independent or even negatively correlated it is less obvious whether a planning firm still reduces the uncertainty of its competitor's residual demand under quantity competition or increases it under price competition. The details of such an analysis are left for future research.

5.2 Asymmetric starting dates

We have assumed that firms start at the same time and have perfect and symmetric information when the game starts. This is a realistic description in a setting in which firms enter the market at the same time. Yet, in many industries firms enter the market sequentially, which implies that firms start out with sequential planning dates. In this context the question arises whether sequential planning is an equilibrium under price competition or if firms planning dates finally converge to the simultaneous planning scenario. Extending the model along this line would shed light on the question whether our assumption of a simultaneous first planning date is innocuous or drives the result that firms synchronize their planning decision under Bertrand competition. We are currently exploring that question.

Here we briefly mention some of the mechanisms that are at work in a setting with a sequential first planning date: At the date at which the second firm enters the market it is forced to plan. This decision increases the uncertainty for the incumbent at this instant. If this increase in the expected incorrectness of the beginning firm's plan is sufficiently high, it will plan immediately. It follows from the results obtained in the preceding

analysis that if firms plan simultaneously for once then they will continue to plan simultaneously at all future planning dates.

If, however, the increase in the expected incorrectness of the beginning firm's plan is not high enough to trigger immediate planning, then the beginner chooses to plan at some later date and possibly firms end up in the sequential and alternating scenario. This may happen because by planning the second firm increases the beginner's uncertainty. Thus the beginner chooses to plan earlier compared to the situation in which there is no entrant. But with its planning decision the beginning firm itself increases the expected incorrectness of the entrant's plan and so the entrant itself chooses to plan earlier, too. However, we can currently not rule out that this "convergence" stops at some point before planning dates are synchronous and that there is indeed an equilibrium in which price-setting firms choose to plan sequentially conditional on starting asymmetrically. If firms compete in quantities we do not expect that the assumption of asymmetric starting dates changes the equilibrium structure of planning dates qualitatively. The reason for this conjecture is, that under Cournot competition and a simultaneous start the unique equilibrium structure of planning dates is already sequential. Thus if firms start by planning sequentially firms should remain in this scenario.

6 Conclusion

In this paper we introduced a model of dynamic duopolistic competition in which firms face costs of absorbing and processing information. We have shown that firms choose to synchronize their decisions under Bertrand competition while they plan in an asynchronous and alternating manner under Cournot competition. This result relies mainly on the effect, that by planning the attentive firm re-optimizes its plan and thereby reduces or expands the incorrectness of the other firm's plan. This mechanism is completely unrelated to the fact that quantities are strategic substitutes and that prices are strategic complements. Furthermore our analysis does not rely on any exogenous device that would equip firms with some kind of commitment power. Thus the results derived in this paper add a new argument to the discussion of whether firms synchronize or asynchronize their decision making.

An interesting way for future research might be to incorporate some form of endogenous commitment on the side of firms into our recursively state contingent planning costs model. A unified analysis of these two approaches

would shed light on the question how both effects interact and which one dominates under which circumstances. This approach would add more realism to the analysis and one can possibly derive clear-cut predictions about which effect dominates in which industry.

7 Appendix

Lemma 1. *Let z_t be a standardized Brownian motion. The corresponding geometric Brownian motion, θ_t , with starting value θ_0 and standard deviation $\sigma > 0$ has the following representation:*

$$\theta_t = \exp\left(-\frac{\sigma^2}{2}t + \sigma z_t\right). \quad (74)$$

Now consider two points in time, namely s and u , with $0 < s < u$. It follows that:

$$E[\theta_s^a \theta_u^b | \theta_0] = \theta_0^{a+b} \exp\left(\sigma^2\left(as\left(a + b - \frac{1}{2}\right) + bu\left(\frac{b-1}{2}\right)\right)\right), \quad (75)$$

with $a, b \in \mathbb{Z}$.

Proof: Consider two points in time s and u , with $0 < s < u$. Let z_t be a standardized Brownian motion and let θ_t be the corresponding geometric Brownian motion represented in (74). By using the result that $\exp(x)^a = \exp(ax)$, θ_s^a and θ_u^b can be represented as:

$$\begin{aligned} \theta_s^a &= \exp\left(-\frac{\sigma^2}{2}as + \sigma az_s\right), \text{ and} \\ \theta_u^b &= \exp\left(-\frac{\sigma^2}{2}bu + \sigma bz_u\right). \end{aligned}$$

Therefore

$$\begin{aligned} E[\theta_s^a \theta_u^b | \theta_0] &= \theta_0^{a+b} E\left[\exp\left(-\frac{\sigma^2}{2}as + \sigma az_s - \frac{\sigma^2}{2}bu + \sigma bz_u\right) | \theta_0\right] \\ &= \theta_0^{a+b} \exp\left(-\frac{\sigma^2}{2}as - \frac{\sigma^2}{2}bu\right) E\left[\exp\left(\sigma b(z_u - z_s) + \sigma(a+b)z_s\right) | \theta_0\right]. \end{aligned}$$

Where the random variable

$$Z := \sigma b(z_u - z_s) + \sigma(a+b)z_s,$$

is normally distributed with mean zero and variance

$$v^2 := \sigma^2 \left(b^2(u - s) + (a + b)^2 s \right).$$

Since Z follows a normal distribution with mean zero and variance v^2 it follows that $E \left[\exp(Z) \right] = \exp(v^2/2)$. Simplifying the corresponding expression yields (75). ■

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