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Working Paper

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Diskussionsbeitrag, No. 494

Provided in Cooperation with:
School of Economics and Management, University of Hannover

Suggested Citation: Dalgaard, Carl-Johan; Strulik, Holger (2012) : Physiology and Development: Why the West is Taller than the Rest, Diskussionsbeitrag, No. 494, Leibniz Universität Hannover, Wirtschaftswissenschaftliche Fakultät, Hannover

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Physiology and Development: Why the West is Taller than the Rest

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Leibniz Universität Hannover, Discussion Paper No. 494
ISSN 0949-9962

March 2012

Abstract. We hypothesize that the timing of the fertility transition is an important determinant of comparative physiological development. In support, we provide a model of long-run growth, which elucidates the links between population size, average body size and income during development. Industrialization is shown to be accompanied by a reduction in family size and an intensification of nutrition per child. Early transition countries are therefore expected to be more developed today, economically and physiologically. Empirically, the timing of the fertility transition is strongly correlated with average body size across countries.

Keywords: unified growth theory, body size, fertility, nutrition.


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1. Introduction

Average body size varies to a remarkable extent across the world. As Figure 1 documents, body size measured by average height varies by as much as 20 cm across countries, with the tallest individuals being located in Western Europe. How did these differences come about? Why are the tallest individuals located in the West? These questions are important as stature impacts on income, longevity as well as educational outcomes (e.g., Fogel, 1994; Schultz, 2002; Case and Paxson, 2008; Case et al., 2009; Cinnirella et al., 2011). Put differently, stature appears to influence all of the most commonly used markers of “human development” (Anand and Sen, 1997). In fact, even self-reported happiness appears positively correlated with height (Deaton and Arora, 2009; Carrieri and Paola, 2012). The present paper advances a theory of how the observed differences in stature as well as in prosperity emerged.

Figure 1: Global Height Distribution: Females born ca. 1980

The figure shows a histogram for female body height, 75 countries. Source: see Data Appendix.

Specifically, the proposed theory suggests that during early phases of development, episodes of technological change led to only minute changes in living standards and body size but worked to increase population density.¹ Eventually, however, unbalanced technological change between agriculture and non-agriculture led to rising relative prices of provisions, triggering a reduction in

¹See Kunitz (1987) and Koepke and Baten (2005) on the historical constancy of body size in Europe over the two millennia leading up to the demographic transition. See Ashraf and Galor (2011) on evidence regarding living standards and population density over the same period.
family size and rising nutritional spending per child. This change in the allocation of household resources, away from the quantity of children towards greater quality (in the sense of nutritional intake), enabled a process of rapid growth to occur; a process usually referred to as the take-off into sustained growth. Moreover, the pervasive rise in human body size, which has been observed across countries from the onset of the transition until today, arose as a consequence of an intensification of nutritional investments in children. By extension, in countries where the fertility transition occurred earlier one would expect to see taller citizens today.\(^2\)

In support of the theory we develop a two-sector OLG model within which physiological development can be studied alongside fertility and the evolution of prosperity. Utility of adults is increasing in three arguments: the amount of nutrition they can offer to their children, the number of offspring, and (parental) consumption of non-food goods. These preferences are assumed to fulfill a “hierarchy of needs” principle: in a time of crisis parents will smooth the consumption of their offspring more strongly than their family size, and family size more strongly than the consumption of non-food goods.\(^3\) Moreover, parents are assumed to cover their “subsistence needs”, understood as their basal metabolic rate. Subsistence consumption is endogenous and depends on body size of the parent (predetermined at the time of optimization) as well as on fertility.\(^4\) Finally, better nutrition during childhood increases adult body size.

In a two-sector economy, in which productivity growth is sector specific and due to learning-by-doing, the “hierarchy of needs” principle, along with the physiological elements, has three important implications for the mechanics of development. First, fertility is declining in the relative price of food. This captures Malthus’ “preventive check hypothesis”: the tendency for fertility to be negatively associated with the relative price of provisions. Second, nutrition per child is increasing in the price of food. Third, “Engel’s law” holds, whereby the consumption of food as a fraction of total expenditure declines (and consumption of non-food goods increases) with income. These elements combine so as to create a development trajectory capturing four

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\(^2\)We focus on the nutrition channel behind contemporary height differences. But this does not rule out that genetic differences could also influence contemporary height differences. Indeed, the model features deep physiological parameters, which might well have been influenced by evolutionary pressure and turn out to influence long-run body size. In practise, therefore, both genetic selection and the take-off mechanism have probably helped shape the global distribution of body size. We return to this issue in the context of the empirical testing of the theory.

\(^3\)The notion of a hierarchy of needs goes back to the work of Maslow (1943) and is supported by evidence in psychology, as well as micro studies on household behavior in times of crises, as detailed below.

\(^4\)These assumptions are supported by evidence from biology and the nutrition sciences. Indeed the mappings from fertility and body size to basal metabolism (i.e., “subsistence consumption”) can be given strong micro foundations all the way down to functional forms and associated parameter values.
major transitions in human history: the demographic transition; the take-off in living standards; the take-off in body size and the secular shift in employment out of agriculture.

The individual elements interact in the following way. During early stages of development most of the population is preoccupied with food procurement; i.e., employment is mainly in agriculture. As a result, productivity growth is faster within agriculture than outside agriculture. The relative price of agricultural goods is thus declining, favoring investments in the quantity of children by parents. Rising fertility raises overall productivity growth, due to the positive influence of population growth on learning-by-doing generated knowledge. While fertility thus is increasing, quality investments are modest, leading to a (weakly) declining body size of the representative individual. However, as a consequence of Engel’s law and productivity growth in both sectors, employment is gradually being shifted towards non-food activities. Eventually, productivity growth in the non-agricultural sector exceeds that of agriculture. As a result, the relative price of agricultural goods begins to rise. This shifts the balance between quantity and quality investments at the level of the household; parents respond to the changing relative prices by investing more heavily in quality, and less in the quantity of children. As a consequence average body size rises, whereas fertility declines. Hence, while fertility overall follows an inverse U-shaped path, the path for body size is U-shaped. The gradual acceleration of technological progress, and declining fertility allows income per capita growth to take off.

These results are broadly consistent with the historical evolution of fertility and body size in many developed countries. In particular, the causes of a U-shaped path for body size along with an inverted U-shaped path for fertility, as depicted in Figure 2 for the case of England, have long been debated in the anthropometric economic history literature (see e.g., Komlos, 1993, 1998). The model delivers this outcome, while at the same time being able to account for the path of fertility, structural change and the path of income per capita during the same period (see Strulik and Weisdorf, 2008 for evidence). Moreover, if the fertility transition was accompanied by an intensification of nutritional investments per child, as the present research maintains, it follows that early transition countries should be places inhabited by physiologically bigger individuals today. The differential timing of the take-off should be a major reason why “the West is taller than the rest”.

In an effort to gauge the empirical relevance of the model we provide several independent checks. First, we calibrate the model, and proceed to show that it accounts well for the English
Figure 2: Crude Birth Rate and Average Height 1745-1960: England

Sources: Wrigley and Schofield (1989) for crude birth rates 1790-1865. The crude birth rate for 1960 is from World Development Indicators (2005); rates in between calculated by simple linear interpolation (marked by dotted line). Komlos (1993, Table 6) for height data 1745-1855; army enlists 20-23 year-olds. Height data for 1875-1925, 1955 and 1960 is taken from Boyer (2004, Table 11.5); army enlists 20-24 year-olds. Notes: The two height series have been combined by assuming height was constant between 1855 and 1875 (marked by dotted line). The constancy between 1855 and 65 is supported by an actual constancy of height in the 18-19 year-old age bracket from 1845 to 1865 (Komlos, 1993, Table 6). The further constancy between 1865 and 1875 is a leap of faith. The period from 1925 to 1955 is a linear interpolation (marked by dotted line).

take-off in terms of population growth and average body size, but also TFP growth in industry as well as nutritional consumption per capita.

Second, with the calibrated model in hand, we perturb initial conditions so as to produce “counterfactual” dates for the fertility transition and thus take-off to growth. For each simulated transition we record the predicted body size as of the year 2000. This exercise reveals that the model is fairly effective in capturing the currently observed variation in body size across Europe, given realistic variation in transition dates, and that it predicts a positive influence from time since take-off on current body size. Hence, the simulation motivates a regression-based empirical assessment of the link between the timing of the take-off and current body size, which we conduct below.
Third, we explore the validity of the proposed quantity-quality trade-off by way of regression analysis as this element is key to our theoretical results. Using panel data for 13 European countries, 1856-1980, we find a robust negative correlation between adult body size and prevailing birth rates at the time the cohort was born, consistent with the proposed quantity-quality trade-off. This correlation is robust to simultaneous control for income, mortality and more. In previous work (Dalgaard and Strulik, 2011) we surveyed evidence from the fields of biology and anthropology (in addition to economics) which supports a fundamental size-number trade-off at the household level. The evidence presented below thus complements existing knowledge in the area by providing new economy-wide evidence of the size-number trade-off.

Fourth, we explore whether the timing of the take-off to growth is significantly correlated with body size in a cross-section of countries, in the manner predicted by the theory. Consistent with the theory, and our simulations, our regression analysis reveals that early take-off countries systematically are inhabited by bigger individuals today. This correlation is robust to rigorous control for geographic confounders as well as income per capita and mortality during childhood.

The present research is related to the literature on growth in the very long-run (see Galor, 2006, for a survey). At the mechanical level the present paper is most closely related to the study by Dalgaard and Strulik (2011), which examines the pre take-off dynamics of body size and population size, and the work of Strulik and Weisdorf (2008), which develops a unified growth theory of the take-off, accounting for the fertility transition, the growth acceleration and structural change. Also related is Abdus and Rangazas (2011) who develop a model of long-run growth in income and body size. In contrast to the present study, however, the authors do no examine the fertility/body size nexus, nor do they address comparative physiological development.

The work of Weir (1993) should also be singled out, since Weir conceptualizes a size-number trade-off akin to the one formalized below, and uses it to explain the French fertility transition. Our empirical analysis of the quantity-quality trade-off follows closely Schneider (1996), as detailed below.

Finally, the present research is broadly related to a large anthropometric economic history literature (see Steckel, 2009, for a survey). This literature is rich on hypothesis’ regarding the forces that shaped the path of body size before, during and after the take-off. We acknowledge
that there are many forces that may have shaped the path of body size, aside from the mechanisms highlighted here. The question is whether such alternative accounts are simultaneously reconcilable with the other fundamental transitions that occurred around the same time, involving output growth, fertility and a changing sector composition of output and employment. The present paper demonstrates that a mechanism involving relative food prices, in the presence of a size-number quantity-quality trade-off, is.

The paper proceeds as follows. The next section develops the model and studies the comparative statics of the general equilibrium. Section 3 adds endogenous technological change and studies the entire development trajectory and the models’ cross-sectional implications. Section 4 tests the proposed quantity-quality trade-off and Section 5 examines the empirical cross-country link between years since the fertility transition and contemporary body size. Section 6 concludes.

2. The Model

Consider an economy in the process of development. The economy is closed, and time is discrete. It comprises two sectors, each producing a unique good: agricultural goods and non-agricultural goods, respectively. The only accumulable input of production is labor; labor is fully mobile between the two sectors. People live for two periods: childhood (after weaning) and adulthood. In the model we ignore matching problems; adults are thus assumed to reproduce asexually. The remaining details of the model are given below.

2.1. The Evolution of Body Size Across Generations. Recently West et al. (2001) have established and tested a theory of ontogenetic growth, built up from first principles, which we draw on in order to derive the law of motion of body tissue across generations. Following Dalgaard and Strulik (2011) we integrate the work of West et al. into the present discrete time OLG structure in the following way.

The starting point is an energy conservation equation:

\[ E_t^c = b_c N_t + e_c (N_{t+1}' - N_t) \] (1)

in which \( E_t^c \) is energy consumption during childhood after weaning (prior consumption is covered by adult metabolic needs, as explained below), \( N_t \) denotes the number of human cells after weaning, \( N_{t+1}' \) is the number of cells of the child as a grown up, \( b_c \) is the metabolic energy a
cell requires during childhood for maintenance and replacement, and $e_c$ the energy required to create a new cell. Hence the left hand side is energy “input”, and the right hand side captures energy use. Solving (1) for $N_{t+1}'$ we obtain the number of cells of an adult as a function of the number of cells of a child after weaning and energy intake during childhood:

$$N_{t+1}' = \frac{E_c^t}{e_c} + \left(1 - \frac{b_c}{e_c}\right) N_t.$$ (2)

To proceed we insert the fact that body mass consists of the mass of a single cell $\bar{m}$ times the number of cells. This implies for the size of an adult that $m_{t+1} = \bar{m} N_{t+1}'$. Moreover, using the fact that after weaning the size of a child equals $\mu$ times the size of the mother (Charnov, 1991, 1993) we have $\bar{m} N_t = \mu m_t$.\textsuperscript{5} Substituting $N_t$ and $N_{t+1}'$ in (5) and and solving for $m_{t+1}$, gives (3):

$$m_{t+1} = \frac{\bar{m}}{e_c} E_c^t + \left(1 - \frac{b_c}{e_c}\right) \mu m_t.$$ (3)

This intergenerational law of motion for body size has a simple interpretation: The size of the adult, $m_{t+1}$ is determined by energy consumption during childhood, $E_c^t$, plus initial size, $\mu m_t$, adjusted for energy needs during childhood, $-(b_c/e_c)\mu m_t$. Finally, denote by $c_t$ the consumption of a child, to be determined below from optimization. Then total energy intake during childhood is $c_t \cdot \epsilon = E_c^t$, where $\epsilon$ converts units of goods into calories. Inserting this into (3) leaves us with a law of motion for body size across generations:

$$m_{t+1} = a \cdot \epsilon \cdot c_t + (1 - d) \cdot \mu \cdot m_t$$ (4)

where $a \equiv \bar{m}/e_c$ and $d \equiv b_c/e_c$. This is the intergenerational law of motion for body size, which we will invoke below.

2.2. Metabolic Needs of Adults. Once the individual reaches adulthood we assume her body size remains constant for the remainder of her life; we are thus focusing on the irreversible component of body size. At the time when individuals make economic choices, the body size is therefore predetermined.\textsuperscript{6}

\textsuperscript{5}A physiological justification for this assumption is that child development until weaning depends on energy consumption in utero and during the breastfeeding phase. Since larger mothers consume absolutely more energy the offspring should be larger at this point as it receives a fraction thereof. With this interpretation the linearity should be seen as a simplification. It has no substantive implications for our main results if the linearity is relaxed except for reduced tractability.

\textsuperscript{6}We could add food demand of adults beyond metabolic needs without implications for our main results. Since height is fixed for adults, excess food demand would lead to overweight and obesity. Investigating the evolution
During adulthood individuals are subject to subsistence requirements which depend on body size and on fertility. In particular, we use the fact that rearing up a child from conception to weaning requires a fraction $\rho$ of the mother’s metabolic energy (Prentice and Whitehead, 1987; Sadurskis et al., 1988). With $E_t$ denoting total energy consumed by adults and $B_t$ denoting the basal metabolic rate of the mother we thus have $E_t = (1 + \rho n_t) \cdot B_t$.

Finally, the basal metabolic rate of the mother is determined by Kleiber’s law, $B_t = B_0 m_t^b$ (Kleiber, 1932). Kleiber’s law states that energy consumption increases with body mass at factor $b \approx 3/4$; this association has by now been verified for (virtually) all animals and living organisms. In addition it can be derived theoretically as a manifestation of how energy is diffused and absorbed in a biological organism under optimal conditions, as would emerge through natural selection (West et al, 1997). Hence, this mapping can be given rigorous micro foundations.

Inserting it into the above energy constraint for adults provides: $7$

$$E_t = (1 + \rho n_t) \cdot B_t = (1 + \rho n_t) \cdot B_0 m_t^b, \quad B_0 > 0, \ b \approx 3/4.$$  

(5)

2.3. Individual’s Optimization. Facing the energy constraint of their own metabolic needs adults derive utility from the number of children $n_t$, nutritional expenditure for each child $c_t$, and consumption of manufactured goods $q_t$. Specifically, we assume the following utility function:

$$u_t = q_t + \gamma \log(n_t) + \left( \frac{c_t n_t}{1 - \sigma} \right)^{1-\sigma}, \quad \sigma > 1. \tag{6}$$

where the compound $c_t n_t$ is total child expenditure or “child quality” in the sense of Becker (1960).

While perhaps unfamiliar looking the utility function is not of arbitrary structure. In order to generate meaningful results we rely on a “hierarchy of needs” principle. The principle adapted is that nourishment of offsprings (and, implicitly, of the adult via subsistence requirements) is a first priority activity, family planning comes next, and expenditure on manufactured goods, i.e. goods for convenience and entertainment, comes last. To capture this, preferences are described by a utility function in which the elasticity of marginal utility is highest for child nutrition,

---

7In principle, we could distinguish between energy expended at rest and in activity. In a representative agent study, however, this just complicates the algebra without new insights. The distinction between metabolic needs at rest and in activity “only” becomes interesting in a heterogenous agent framework, as shown in Dalgaard in Strulik (2011b) in the context of efficiency wages.
second highest for fertility and lowest for consumption of manufactured goods. This means that in times of crises parents try to smooth nutrition of their offsprings relatively stronger than their family size, and family size stronger than consumption of manufactured goods.

The basic idea of a hierarchy of needs goes back to the work of the psychologist Abraham Maslow (1943). In particular, Maslow argued that physiological needs and safety needs take precedent over the establishment of a family. While the notion that such universal needs exist – and can be ranked – has been controversial in psychology, recent empirical work has provided some corroborating evidence in favor of Maslow’s needs hierarchy (Tay and Diener, 2011). Moreover, empirical studies of household behavior in times of crises show that households try particularly hard to smooth calorie consumption, while allowing other consumption items to adjust (e.g. Frankenberg et al, 2003; Stillman and Thomas, 2008). In particular, periods of crisis have been shown to be associated with deferred marriage, lower marital fertility, or both (Lee, 1990; Caldwell, 2004). The above utility function is designed to replicate this type of behavior.\(^8\)

Turning next to the budget constraint, let \(w_t\) denote income (measured in units of the manufactured good) and \(p_t\) the relative price of food. Then the budget constraint of a parent is given by the following equation:

\[
w_t = q_t + p_t c_t n_t + p_t (1 + p_t) \cdot \frac{B_0 m_t^b}{\epsilon},
\]

where subsistence needs have been converted from units of energy (kcal. per period) into units of goods by applying the energy exchange rate introduced above.

The individuals’ problem consist of choosing family size \(n_t\), expenditure per child \(c_t\), and non-food consumption \(q_t\) so as to maximize (6) subject to (7). From the first order conditions we obtain:

\[
n_t = \frac{\gamma \epsilon}{p_t \rho B_0 m_t^b}, \quad c_t = \frac{\rho B_0 m_t^b}{\gamma \epsilon} \cdot p_t^{(\sigma - 1)/\sigma}.
\]

Observe the influence from the relative price of food: both fertility $n_t$ and (combining the two first order conditions) total quality investments $c_t n_t = p_t^{-1/\sigma}$ decrease when food (and thus children) become relatively more expensive. Nutrition per child, however, improves as food prices go up. Hence, industrialization, associated with rising $p_t$, will imply higher expenditure on manufactured goods, smaller families, less food expenditure, and better nutrition per child.

2.4. Production. Food is produced in an agricultural sector (indexed by $A$) and non-food is produced in an industrial sector (indexed by $Q$). The production functions are

\[ Y_t^A = A_t (L_t^A)^\alpha \]
\[ Y_t^Q = Q_t L_t^Q. \]  

(10)  
(11)

Here, $L_t^A$ and $L_t^Q$ denote sectoral labor inputs and $A_t$ and $Q_t$ denote sector specific productivity. Agriculture may be subject to decreasing returns originating from limited land (normalized to one), i.e. $\alpha \leq 1$.  

9

Let $L_t$ denote the number of adults at time $t$. Following Galor and Weil and suppressing, for simplicity, child mortality the adult population (workforce) evolves according to

\[ L_{t+1} = n_t L_t. \]  

(12)

This completes the model.

2.5. General Equilibrium. In order to derive the static general equilibrium consider the non-arbitrage condition associated with the decision to work in the $A$ and $Q$ sector, respectively. Assuming labor is compensated by the average product in agriculture the condition is that

\[ w_t = p_t A_t \left( L_t^A \right)^{\alpha-1} = Q_t. \]

(13)

Let $v_t$ denote total food demand per family in period $t$, $v_t = p_t (c_t n_t + (1 + \rho n_t) B_0 m_t^b / \epsilon)$. After inserting (8) and (9) we get food demand as a function of food prices and body size.

\[ v_t = \left[ \gamma + p_t^{\sigma-1} + p_t B_0 m_t^b / \epsilon \right] / p_t. \]

(14)

9In Dalgaard and Strulik (2011) we allow $m$ to enter the production function, which has important implications in a pre take-off setting. In the present context, however, the productive influence from body size will quickly “vanish” once technology takes off. Hence, in the interest of simplicity we leave it out.
Food market clearing requires that demand equals supply, \( Y^A_t = v_t L_t \), that is after inserting (10) and (14):

\[
A_t(L_t^A)^\alpha = \left[ \gamma + p_t \frac{\sigma - 1}{\sigma} + p_t B_0 m_t^b / \epsilon \right] L_t / p_t.
\] (15)

Finally, substituting \( L_t^A \) from (13) provides an implicit function for prices in equilibrium.

\[
0 = f(p_t) = p_t^{1/(1-\alpha)} \left( \gamma + p_t \frac{\sigma - 1}{\sigma} + p_t B_0 m_t^b / \epsilon \right) \cdot L_t - \left( \frac{A_t}{Q_t^a} \right)^{1/(1-\alpha)}.
\] (16)

Note that population size \( L_t \) and body size \( m_t \) as well as technology in agriculture and manufacturing, \( A_t \) and \( Q_t \), are state variables, which are predetermined in period \( t \).

**Proposition 1.** *(Existence of general equilibrium).* A general equilibrium exist, and it is unique.

**Proof.** It is easy to verify that \( f(p_t) \) is monotonically increasing in \( p_t \) with \( \lim_{p_t \to 0} f(p) < 0 \) and \( \lim_{p_t \to \infty} f(p) = \infty \); (16) therefore pins down the unique equilibrium price associated with clearing in the markets for \( A \)-goods, labor and thus also \( Q \)-goods. \( \square \)

**Corollary 1.** Equilibrium food prices are a positive function of population density \( (L) \), of body size \( (m) \), of productivity in manufacturing \( (Q) \), and the weight for children in parental utility \( (\gamma) \). They are a negative function of the level of agricultural productivity \( (A) \) and of the energy exchange rate \( (\epsilon) \).

The proof follows from (16) and the implicit function theorem.

These results are quite intuitive. For instance, a larger number of people to feed and a larger number of body cells per person to feed drive up food demand and thus, given supply, equilibrium prices for food. In short, for given land, diet, and level of technology, our model predicts food prices to be higher in more densely populated areas and for societies consisting of, on average, physically larger individuals. The remaining comparative statics are straightforward to interpret via their impact on supply or demand. However, they only relate to the short run, where the size of population and average body size of the citizen’s are given. In the long-run changes in these parameters may affect the economy through fertility and body size. To clarify their overall impact we need to understand the dynamic properties of the model.

**2.6. Stationary State.** At a stationary state, knowledge in agriculture and manufacturing does not change, \( A_t = A_{t+1} = A \) and \( Q_t = Q_{t+1} = Q \). Inserting equations (8) and (9) into (3) and
(12) we obtain the law of motion for body size and population size, respectively:

\[
m_{t+1} = a(\rho/\gamma)B_0 \cdot m_t^b \cdot p_t^{(\sigma-1)/\sigma} + (1-d)\mu m_t \tag{17}
\]

\[
L_{t+1} = \frac{\gamma \epsilon}{\rho p_t B_0 m_t^b} \cdot L_t. \tag{18}
\]

**Proposition 2.** (Existence of a steady state.) The exists a unique steady state \((m^*, L^*)\) at

\[
m^* = \left( \frac{a \epsilon^{\sigma-1} (\rho B_0 / \gamma)^{1/\sigma}}{1 - (1-d)\mu} \right)^{1-b/\sigma} \tag{19}
\]

\[
L^* = \frac{(p^*)^{[1+\alpha(\sigma-1)]/[\sigma(1-\alpha)]}}{1 + (1 + \rho) \cdot \gamma / \rho \cdot (p^*)^{1-\sigma} \cdot \left( \frac{A}{Q^{\alpha}} \right)^{1/(1-\alpha)}}, \tag{20}
\]

where the relative price of agricultural goods is given by:

\[
p^* = \left[ (\gamma / \rho B_0) \cdot a^{-b} \cdot (1 - (1-d)\mu)^{b} \right]^{1/\sigma} \cdot \epsilon^{(1-b)/\sigma-b}. \tag{21}
\]

**Proof.** Since \(L_t\) is constant at the steady-state, \(n_t = 1\), implying the equilibrium price of food \(p_t = p^* = \gamma \epsilon / (\rho B_0 m^b)\). Taller people in equilibrium cause, ceteris paribus, a higher relative price for food, since more body cells have to be fed. Inserting this information into (17) we get equilibrium body size. Finally, inserting equilibrium prices and equilibrium body size in (18) and solving this equation at the steady-state one gets, after some additional manipulations, the expression for equilibrium population size. \(\square\)

2.7. **Comparative Statics.** Since \(1-b/\sigma > 0\) the following comparative statics can be straightforwardly read off equation (19) and (20).

**Proposition 3.** (Determinants of Long-Run Body Size.)

(I) Steady state body size increases if: (a) the weight of children in utility \(\gamma\) declines, (b) the energy costs of creating a cell or maintaining a cell declines (smaller \(d\) or larger \(a\)), or, (c) the energy exchange rate \((\epsilon)\) rises.

(II) Steady state body size is independent of: (a) the level of productivity \((A, Q)\) and (b) the nature of the production technology \(\alpha\).

Most of the above results are also found in the simpler one-sector setting (Dalgaard and Strulik, 2011, where we also show stability of the steady-state). In particular, body size responses similarly to the changes of the technological environment \((A, Q, \alpha)\), preferences \((\gamma)\), and the
physiological parameters ($a$ and $d$). Hence the reader is referred to Dalgaard and Strulik (2011) for a discussion of these results.

The influence from the energy exchange rate on $m^*$, however, is new to the present analysis: When more calories can be extracted from one unit of food expenditure, i.e. when $\epsilon$ rises, parents provide more nutrition for their children in the long-run, for which reason steady state body size rises.

The interpretation is as follows. Starting at steady state, imagine $\epsilon$ increases permanently; say, as a consequence of the discovery of a new crop. At the time of impact (i.e., for $L$ and $m$ given) the increase in $\epsilon$ lowers the price of food (cf. Corollary 1). This induces parents to have more children, but to provide less nutrition per child. In the long-run, however, the temporary increase in fertility translates into a permanent increase in population (as will be seen momentarily; Proposition 4), which works to raise the steady state relative price on agricultural goods (see (21)). Hence, in the new stationary steady state, due to the hierarchy of needs principle, parents are investing slightly more in nutrition per child, leading to greater stature. Notice, however, that the independence of long-run body size from the energy exchange rate, obtained in Dalgaard and Strulik (2011) can be recovered in the present framework as well. In Dalgaard and Strulik (2011), $\sigma = 1$, which violates the hierarchy of needs principle, adopted in the present analysis. As can readily be confirmed, in this case $m^*$ is independent of $\epsilon$.

Inspection of equations (21) and (20) reveals the following result:

**Proposition 4. (Determinants of Long-Run Population Size.)** Steady state population size increases if: (a) the weight of children in utility $\gamma$ increases, (b) the energy costs of creating a cell or maintaining a cell increases (smaller $d$ or larger $a$), (c) the energy exchange rate ($\epsilon$) increases, (d) the level of productivity in agriculture ($A$) rises, (e) the level of productivity in non-food production ($Q$) falls.

These results are all generalizations of the comparative statics obtained in Dalgaard and Strulik (2011) and Strulik and Weisdorf (2008) where similar qualitative results are obtained.

3. Balanced growth: The take-off and beyond

In this section we introduce endogenous growth of knowledge in agriculture and manufacturing. Since market financed R&D is a relatively new phenomenon (Mokyr, 2005), we follow Strulik and Weisdorf (2008) and assume learning-by-doing drives productivity. That is,
\( \Delta A_t \equiv A_{t+1} - A_t = \nu (Y_t^A)^\xi \) and \( \Delta Q_t \equiv Q_{t+1} - Q_t = \delta (Y_t^Q)^\phi \). Reasonably, we assume decreasing returns of learning-by-doing, that is \( 0 < \xi < 1 \) and \( 0 < \phi < 1 \).

Inserting the production technologies (10) and (11) and solving for sectoral growth rates of knowledge we obtain:

\[
g_{t+1}^A \equiv \frac{A_{t+1} - A_t}{A_t} = \nu (A_t)^{\xi-1} (L_t^A)^{\alpha \xi} - 1, \quad g_{t+1}^Q \equiv \frac{Q_{t+1} - Q_t}{Q_t} = \delta (Q_t)^{\phi-1} (L_t^Q)^{\phi} - 1. \tag{22}
\]

The specific adjustment dynamics are determined via calibration. But it is useful to consider the basic elements that drive the dynamics upfront. For this purpose we inspect the relative price of food derived from (13) and the labor share in agriculture derived from (15) and (14).

\[
p_t = \frac{Q_t L_t^{1-\alpha}}{A_t} \tag{23}
\]

\[
\frac{L_t^A}{L_t} = \left( p_t^{(\sigma - 1) / \sigma} + p_t B_0 m_t^b / \epsilon + \gamma \right) \cdot \frac{1}{Q_t}. \tag{24}
\]

Consider an economically backward society. The level of productivity in both sectors is very low, for which reason wages are low. \( Q \) is particularly low, since as a reasonable initial condition, most of the labor force is employed in the A sector. As a consequence, manufactured goods are relatively expensive, and its demand is therefore modest.

Most of the population is working in agriculture; \( L_t^A / L_t \) is high (see (24)). As a result of this allocation of labor the speed of knowledge creation is faster in agriculture which generates a tendency of food prices to fall (see (23)). Given falling food prices people reproduce at higher rates and productivity growth is (almost completely) compensated by population growth such that income remains (almost) constant. Yet, continual population growth and food production gradually raise the speed of learning in agriculture such that \( p_t \) falls. With rising productivity in agriculture, the sector releases labor for production of manufactured goods. The process is slow at first, but gradually gains momentum. The shifting output structure influences the productivity growth gap. Eventually a point is reached where the relative price of food begins to fall and fertility falls. This process also affects the quantity-quality trade-off: with rising \( p_t \) and falling fertility nutritional expenditure per child begins to rise. A quantity-quality trade-off sets in during industrialization, leading to larger body size.

The take-off of economic growth and ontogenetic growth, however, are phenomena of transitional dynamics and another interesting question is whether a time path exists along which body
size stays constant. From (17) we see immediately that this requires the price of food to be constant. Otherwise, parents would have an incentive to further increase (or reduce) food provision per child. From (18) we see that a constant price implies constant population growth because the preferred number of offspring stays constant. Let a balanced growth path be defined as a time path along which both sectors grow at a common rate (possibly zero) such that neither the production of food nor the production of manufactured goods vanishes asymptotically. From (24) it then follows that only a stationary population supports balanced growth. Otherwise there would be Malthusian dynamics operating towards increasing food prices if the population continues to grow or decreasing food prices if the population shrinks. With a stationary population, however, there are no scale effects of learning-by-doing, and the (exponential) power of knowledge growth peters out. The following proposition (proven in the Appendix) shows that the balanced growth path does not only exist and is unique but that it is also stable under a mild condition.

Proposition 5. (Existence of a balanced growth path) If $\phi/(1 - \phi) + 1 - \alpha < \alpha \xi/(1 - \xi)$, the economy converges towards a balanced growth path of constant prices, constant body size, zero population growth, and zero (exponential) economic growth.

The condition rules out explosive growth, that is a path where agricultural productivity and population size are jointly growing at perpetually increasing rates. Intuitively, along such a path people learn new techniques in agriculture so quickly (compared to learning in manufacturing) that the prices for agricultural goods decrease perpetually and fuel further population growth. This counter-factual scenario can always be avoided by choosing a sufficiently small learning parameter for agricultural knowledge.

3.1. Calibration. In order to evaluate the mechanics of the model along the adjustment path towards the long-run steady-state, we set out with a calibration for England. We begin by setting $B_0 = 70$ and $b = 0.75$ according to Kleiber's (1932) law. Following Prentice and Whitehead (1987) we put $\rho = 0.15$ implying that a woman pregnant with one child must consume 1.15 times the energy of a non-pregnant woman. According to the WHO (2006) a female grown up weighs on average $m = 59$ kg and a female child weighs on average 9 kg after weaning, assuming weaning after 12 months. This implies that, on average, $\mu = 0.15$. With respect to body size we have to confront the problem that there are no detailed long-run historical time series for
females and that the available series for males provides body size in terms of height rather than weight. We deal with this problem by converting body size into height (stature), denoted by $s_t$, using a body mass index (BMI) of 21.5 and the formula $s_t = \sqrt{mt / (BMI)}$. Since we have one degree of freedom with the physiological parameters we set $d = 0.5$ and adjust $a$ such that equilibrium height is 1.82 meter, 5 cm above the current average height of 20 year old males in England.

The specific size of the physiological parameters is not decisive for the shape of transitional dynamics. These are crucially determined by technologies and preferences. We begin with setting $1 - \alpha$ to 0.2 according to Clarke’s (2007) average estimate of the share of land in agriculture. We set parameters such that the predicted time series for population growth, body size, and productivity growth in manufacturing approximate best the historical times series for England and Wales before, during, and after the industrial revolution and the demographic transition. This leads to the estimates $\xi = 0.4$, $\phi = 0.95$, $\nu = 24$, $\delta = 2.4$, $\sigma = 1.25$ and $\epsilon = 1.2$. The estimates of $\xi$ and $\phi$ imply a relatively flat learning curve in agriculture and a relatively steep learning curve in manufacturing. They produce a long phase of low productivity growth in agriculture which is mainly translated into population growth such that economic development is slow. During this Malthusian phase most workers are occupied with agriculture such that learning in manufacturing is slow as well. The gradual movement of labor into manufacturing, however, eventually leads to relatively high productivity growth in this sector, which is then further fueled by fast learning-by-doing and not slowed down (as in agriculture) by decreasing return with respect to labor input. Thus, once initiated, the industrial revolution is relatively quick compared to the slow evolution in the centuries before. The estimate of $\sigma$ implies a price elasticity of food demand (beyond metabolic needs of the parent) of 0.2, a value close to Clark’s (2007b) estimate of the price elasticity of food near subsistence consumption.

3.2. Long-Run Adjustment Dynamics and Implications. Figure 4 shows the long–run dynamics predicted by our baseline calibration. In order to compare results with the historical time series we have assumed that a generation takes 25 years and converted generational growth rates into annual ones. We have started the economy at year 0 AD close to the subsistence steady-state and adjusted initial values of the state variables such that the fertility transition and the rise of body size are set in motion in the mid 19th and early 20th century, respectively. The Figure focusses on the most interesting time window around the Industrial Revolution and the

Take-off of body size from year 1700 to 2000. As can be seen, net fertility (population growth) peaks around 1850, agricultural productivity peaks around 1920 and industrial productivity peaks in the late 20th century.
The predicted trajectories approximate the historical time series for net fertility (data from Reher, 2004), TFP growth (data from Madsen et al., 2010) and stature (data from Komlos, 1993, and Hatton and Bray, 2010) reasonably well.\(^{10}\) The model correctly predicts that body size goes down during the early phase of industrialization. Eventually, however, the Industrial Revolution brings about a previously unseen increase in body size. At the end of the 20th century average male height is predicted to be 1.76 meters, a value that matches the actual height of 16 to 24 year old men in England in the year 2000 (NHS, 2011) as well as the height of men born 1976-1980 reported in Hatton et al. (2010). During the 21st century height is predicted to increase further until it settles down with overshooting behavior at its steady-state level of 1.82 when the economy approaches the balanced growth path.

Given the simple structure of the model, the growth rate of TFP in manufacturing coincides with growth of income per capita. As can be seen by the middle panel in Figure 4, the physiological evolution is thus consistent with the historical observation of relatively low rates of TFP growth and income growth during the Industrial Revolution.

The last panel shows the predicted consumption of calories per capita. Since 1800 the trajectory is roughly consistent with the estimates of Floud et al. (2011) and the FAO (2012, data for 1960-2000). For the 18th century the trajectory lies above Floud et al.’s estimates and below Allen’s (2005) estimate.\(^{11}\)

3.3. **Simulated Cross-Sectional Implications.** The motivation for the present study is the desire to try to understand contemporary comparative physiological development. Hence, in an effort to study the models’ implications in this regard, we next investigate the impact of the onset of the fertility transition on (average) body size today.

Specifically, we keep all the parameters values from the calibration above constant, and perturb the initial conditions for agricultural productivity and population size such that the fertility transition takes place (in most cases) later than in England. We then, for each simulation, record the year of the fertility transition and the predicted body size of adult males in the year

\(^{10}\)We took from Komlos (1993) the data on height of 20 to 29 year old British soldiers. Komlos provides the data as index numbers. In order to compare with the Hatton et al. data we used conversion rate of an index number of 98.6 (indexed height in 1850) into absolute height of 1.66 meter (predicted height in 1850). Madsen et al. (2010) provided estimates of aggregate TFP in Britain on an annual basis obtained from growth accounting as Solow residuals. Naturally, there is huge fluctuation on annual TFP growth rates, which we smoothed by using 50 year moving averages as in Strulik (2011).

\(^{11}\)The fit of the calorie trajectory could be improved by relaxing the assumption that the energy exchange rate stays constant and take into account technological progress in food processing, that is by imposing an increasing rate of energy extracted per unit of food.
2000 (i.e. the cohort born in 1975). Figure 5 shows the result. In countries where the fertility transition occurs during the mid 19th century, predicted body size a century and a half later is about 1.80 m. In contrast, for latecomers that underwent the fertility decline, say in the 1920s, average body size is predicted to be about 1.70 m.

According to Reher (2004), Sweden was the forerunner in terms of the fertility decline, seeing its onset in the late 1850’s/ early 1860s; the latecomer in a European context was Portugal with the onset of the fertility decline occurring in the 1920s. According to Hatton and Bray (2010) the average size of Swedes born in 1980 was 180.4 cm, with the comparable number being 172.3 for Portugal. Hence the model does a reasonably good job at capturing the range in body size across Europe, as of today.

4. **Empirics I: On the Size-Number Trade-off**

A critical element of the model developed above is the invoked size-number trade-off; a trade-off involving fertility and average body size of the offspring. In previous work we surveyed evidence from the fields of biology and anthropology (in addition to economics) that testify to the existence of this sort of a quantity-quality trade-off at the household level, as well as in non-human populations. We also presented new evidence to its existence at the aggregate level, using cross-section data pertaining to non-transition countries as of the end of the 20th century.
(See Dalgaard and Strulik, 2011, Section 2 & 5). In the following we add to this aggregate evidence by examining the historical fertility/body size nexus across Europe.

In a perhaps somewhat neglected contribution Schneider (1996) provided the first formal evidence in favor of the quantity-quality trade-off emphasized in the present paper, by examining the link between fertility and average height in a panel of European countries. Below we revisit the issue. In so doing we improve upon Schneider’s study in terms of data coverage. Schneider examined nine European countries over the period 1750-1920 for a total of 101 observations, whereas we are able to examine 13 European countries from 1856-1980 (divided into five year epochs), for a total of 251 country-year observations. Hence, the following represents a meaningful out-of-sample check of Schneider’s original findings. The data used below are described in the Data Appendix.

4.1. **Specification.** Formally, the empirical model for body size that Schneider proposed has the following form:

\[
\log(m_{it}) = \beta n_{it} + X_{it} \gamma + c_i + \theta_t + u_{it},
\]

where \(i\) is a country index, and \(t\) refers to time; \(c_i\) is a country fixed effect, \(\theta_t\) is a time varying intercept and \(u_{it}\) is noise. The left hand side variable \(m_{it}\) is adult body size, measured by average height, for individuals born around time \(t\). \(n_{it}\) is fertility, which we proxy by the crude birth rate. \(X_{it}\) includes additional time varying controls. Following Schneider we introduce mortality, measured by the crude death rate (CDR); prosperity, measured by GDP per capita and a dummy for World War I. It is important to stress that the right hand side variables all speak to the conditions around the birth year of the cohort. Accordingly, reverse causality (from body size to per capita income, say) can be ruled out. The object of interest is \(\beta\), and the prediction is that \(\hat{\beta} < 0\).

How well does the proposed theoretical model and the above empirical specification line up? In some respects the match is admittedly not perfect. In particular, our theoretical model does not leave room for mortality to play a role in shaping human body size. This omission, of course, is not meant to deny that the disease environment during childhood could influence adult body size. Rather, the omission of mortality from the theoretical model serves to simplify the analysis and highlight the mechanism we expect was important to the transition: the quantity-quality trade-off between family size and nutrition per child (thus: adult body size). Hence, on
pragmatic grounds CDR should enter the empirical model. At the same time it serves to check the robustness of the advocated quantity-quality mechanism. Similarly, GDP per capita is not a direct determinant of adult stature in our model. However, GDP per capita and the relative price of provisions (for which we have no data) should in theory be highly (and positively) correlated; by extension, GDP per capita and adult body size would become positively correlated. As a result, one may think of income as acting as a stand-in for movements in $p$.\footnote{This is demonstrated in the Appendix, using the calibrated model from Section 4.}

In other respects the match between Schneider’s empirical model and the theoretical structure above is better. Most obviously, our theory provides a formal foundation for a link between fertility and adult body size. In addition, our model provides a clear motivation for the inclusion of country fixed effects. As shown in Proposition 3, long-run body size should be influenced by the diet. To the extent that the nature of the diet differs across Europe the country fixed effect would pick it up. Moreover, if the diet changed secularly in Europe, during the period in question, time fixed effects (/a time trend) should pick it up. Another separate reason why country fixed effects are warranted has to do with evolutionary processes. As seen from Proposition 3, body size is influenced by deep physiological parameters ($a$ and $d$), which are likely to have been influenced by selective pressure. The argument is the following. Within biology there is considerable evidence that human body shape has been influenced by evolutionary pressure, leading to shorter limbs in colder climates, so as to limit heat loss. Now, equation (3) does not allow for heat loss, which means that heat loss is implicit in the parameters of the energy conservation equation. Specifically, less energy costs of running and maintaining a body cell would capture less heat loss. In the model this is equivalent to a smaller value for $b_c$ and thus $d$. The evolutionary argument above could thus be interpreted as saying that individuals with smaller values for $d$ should have had a selective advantage in cooler climate zones. Note that lower cell costs in this way, according to the model, would produce larger asymptotic body size. As a result, in areas where $d$ is larger (i.e., in colder environments) individuals should be larger, ceteris paribus.\footnote{It should be noted, however, that a slight modification of the theoretical model would ensure that income plays a direct role: if we admit a time cost of child rearing to be present, and introduce a minimum fertility level in the utility function, greater income will stimulate investments in nutrition, and serve to lower fertility.}

\footnote{In biology, the link between climate and length of limbs is known as “Allen’s rule”, whereas the climate gradient in body mass of mammal’s is known as “Bergman’s rule”. Both are viewed as a consequence of evolutionary pressures. See Ruff (2002) for a survey.}
4.2. **Results.** Figure 6 provides a simple cross-plot between fertility and adult body size, pooling all country-year observations. Consistent with the proposed quantity-quality trade-off a clear negative correlation is apparent. Yet this simple correlation may be spurious, driven instead by various omitted factors. Hence, for a more adequate assessment of the correlation formal regression analysis is warranted.

**Figure 6: Body size vs. Fertility: 13 European countries 1856-1980**

![Graph showing the correlation between adult body size (cm) and fertility (crude birth rate in birth year) pooling cross-section and time series information.](image)

The figure shows the pure correlation between adult body size (cm) and fertility (crude birth rate in birth year) pooling cross-section and time series information.

Data: See Data Appendix

Consequently, Table 1 reports the results from estimating equation (25). In the first column the correlation from Figure 6 is reexamined when country fixed effects are taken into account, and column 2 further adds time fixed effects. The latter has a marked impact on the point estimate for \( n \), which drops considerably in absolute value. However, introducing the additional controls proposed by Schneider (1996) has little impact on the economic and statistical significance of fertility.

Column 6 replicates the “full specification” adopted by Schneider, which involves a time trend rather than time fixed effects. The results are comparable to those recovered by Schneider: fertility as well as income per capita turn out to be significant, along with the time trend and the world war I dummy: cohorts born during the first world war grew up to be slightly shorter than what would be expected based on the controls. In light of our bigger sample column 7 explores adding indicators for the second world war, and the great depression. Interestingly, the great
Table 1: Adult Body Size and Birth Rates: 13 European Countries 1856-1980

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Standard errors are clustered at the country level; *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The crude birth rate, death rate and average income refers to conditions in the birth year of the cohort. Hence, measurement the right hand side variables (CBR, CDR, GDP per capita) are lagged 20 years compared with the year of observation for body size.

depression also seems to have left a dent in European body size, comparable in magnitude to that of World War I. However, whereas Schneider finds mortality to be -marginally- significant, the crude death rate is insignificant in our expanded sample. Moreover, if the time trend is replaced by time dummies, income per capita is also rendered insignificant (column 5). In contrast, the crude birth rate continues to be significant at the five percent level of confidence. As can be seen from the $R^2$ the model appears to do a good job at spanning the variation in the data.

Reverse causality is, as noted above, unlikely to be important in the present setting. While omitted variable bias cannot be ruled out, it is clear that the model accounts well for the variation in the data, thus leaving only modest room for further improvements in the overall fit. In any case, it is not clear that potentially omitted variables necessarily would bias the results in a direction favorable to the hypothesis under examination.

The out-of-sample check largely corroborate Schneider’s original findings, yet suggests that mortality may have played a slightly more modest role in accounting for rising body size than what one might have expected based on Schneider (1996). By extension, the results reported in Table 1 provide corroborating evidence in favor of the key quantity-quality trade-off highlighted
by the proposed theory: greater fertility rates during childhood are associated with lower adult body size.

5. Empirics II: Comparative Physiological Development and the Fertility Decline

In this section we take a first pass at examining the key prediction of the model. Namely, that the timing of the fertility transition is an important determinant of contemporary cross-country differences in body size.

5.1. Data. The key dependent variable is body size of females, measured in kg. From the point of view of the theoretical model, this is an ideal measure of body size. The data largely originates from the Demographic Health Surveys (DHS), and reflect surveys conducted around the year 2000. In addition, we obtained a few additional data points for a set of European countries, as detailed in the appendix. These additional data are useful, as DHS mainly survey low or middle income countries.

The other central variable for the empirical analysis is the “timing of the take-off”. Since the timing of the fertility transition, in theory, is a key marker of the take-off to growth we use this as our measure for the “take-off date”. The study by Reher (2004) provides transition dates for a large number of countries. When we combine our data on body size and the timing of the fertility decline, we are left with 67 country observations. Figure 7 depicts a scatter plot of years since the transition (YST = 2000 - transition year,) and body size circa 2000.

Qualitatively the picture conforms with expectations: early take-off countries are populated by women of greater physiological stature around the turn of the 21st century, in keeping with the theory. But at a more subtle level, it is clear that the model does not capture all details fully, as the mapping between the two variables is concave, rather than convex (Figure 5). This feature of the model is also visible in the simulation (Figure 4), where the model underestimates the speed at which body size rises right after the take-off. However, at the end of the period examined the model and the data “converge”. From an empirical standpoint, then, the time series data and the cross-sectional data seem mutually consistent. That is, during transition body size increases rapidly in the immediate aftermath of the transition, yet slows down as the economy matures. This would produce a concave mapping between YST and body size, as
The figure shows Years since fertility decline, defined as 2000 - year of onset, versus current (ca. 2000) body size (kg). The sample includes 67 countries in total. See Appendix for details on data.

found in Figure 7. Hence, these patterns are consistent with the overall hypothesis: early take-off is linked to greater current body size. For present purposes we will allow the data to speak “freely” in the estimations, and thus adopt a non-linear specification for YST, while leaving the interesting task of mimicking the said non-linearity theoretically for future research to resolve.

5.2. **Specification.** The specification we take to the cross-section data has the following form:

\[
\log(m_i) = \beta_1 YST_i + \beta_2 YST_i^2 + X_i'\gamma + u_i, \quad (26)
\]

where \(X_i\) contains potential confounders, and \(u_i\) is noise.

Whereas reverse causality cannot account for a positive correlation between YST and current body size, one may obviously worry about omitted variables. Hence, the first set of additional controls are motivated by the fact that body size likely has been molded by climate-related evolutionary pressures (e.g., Ruff, 2002). As explained above, there is good biological reason to expect that larger bodies had an evolutionary advantage in colder climate zones. Accordingly, we add a set of variables that might pick up this kind of variation: average frost days, distance to the equator, tropical area and continental fixed effects. Naturally, these geography controls may also pick up cross-country differences in diet, which the model predicts could influence body size (cf. Proposition 3).
Another evolutionary theory holds, that the disease environment could prompt selective pressure, affecting body size (Migliano et al., 2007). The theory is that in high disease environments it is an advantage to reach sexual maturity early in life. But earlier sexual maturity comes at a cost of premature cessation of body growth. Hence, in areas with a “hostile” climate, the theory proposes, one would expect to see smaller bodies being selected. Naturally, the aforementioned climate variables may also act as controls for this channel. Since the climate tends to be more hostile towards humans in the tropics (from the point of view of diseases), one would expect to see individuals near the equator to be smaller, ceteris paribus. But to capture the disease channel more specifically we also introduce malaria ecology.\(^{15}\)

As a third check we also introduce income per capita and child mortality around the year of birth of the cohorts captured by our data on body size. As noted above, body size is recorded around 2000. The DHS data stems from the maternal health module, which means that the women surveyed were between 15 and 49 years of age in 2000. As a result, we employ GDP per capita and mortality rates in 1980 as a proxy for childhood circumstances. While GDP per capita and mortality obviously are endogenous, we hope to limit the scope of bias by lagging them by 20 years compared to the year of observation of body size.\(^{16}\)

Finally, since body size is not always measured \textit{exactly} in 2000, we add a control for the year of observation. In addition, we add a dummy which takes on the value of 1 if the observation derives from DHS, and zero otherwise. In this manner we hope to eliminate any bias resulting from the different sources of the data on body size. All data sources are listed in the Data Appendix.

5.3. \textbf{Results.} Table 2 reports the results from our checks of the basic correlation depicted in Figure 7. In column 1 we introduce YST and YST squared. The visually obvious correlation from Figure 6 is reflected in highly significant point estimates. Together with our continental controls, and source indicators, the model accounts for roughly 2/3 of the variation body size. In the next five columns we add the climate controls sequentially, and together. The introduction

\(^{15}\)Ideally one would of course like to have data on climate 50,000 or more years ago to better approximate the climate conditions that might have set the evolutionary processes in motion. In the present case we rely on the assumption that current climate conditions are sufficiently highly correlated, in a cross-country context, with past conditions to be a meaningful proxy.

\(^{16}\)One might also contemplate education as a determinant. But education of the parents cannot have any \textit{direct} physiological impact on adult body size of the offspring; at most it can influence the more proximate determinants of the adult size of children: income and child health. Besides, the strong correlation between education and income and health is well known. As a result, education is not part of the control set on a priori grounds.
of these variables has only a limited effect on the point estimates for YST and YST squared. In the end only average frost days retains significance (Column 6), with a point estimate that is consistent with the evolutionary arguments laid out above.

In Columns 7-9 we add childhood environmental controls in the form of income and mortality. Both are individually significant (column 7 and 8), with signs that conform with expectations: higher income and lower mortality during childhood is associated with greater adult stature. In Column 10 we add all of the controls at the same time. In the full specification we account for about 80% of the variation in body size. As can be seen, the linear term for YST looses significance (but only just, with a p-value of 0.11). However, YST and YST squared are jointly significant (p-value of about 0.07). Hence, even when we control rigorously for geography as well as income and mortality, YST remains a significant determinant of current differences in body size across the world. In contrast, the climate controls (except malaria ecology) turn insignificant in column 10. This is unsurprising as we simultaneously are controlling for income and mortality, which undoubtedly span much of the same variation. But these results do serve to illustrate the non-triviality of the significance of the historical correlate: YST.

What is the economic significance of the estimates? Taken at face value a point estimate of about 0.2 for YST, and -0.001 for YST squared implies that body size (conditional on income etc.) is maximized after about a century post transition, contributing with - in total - roughly 10 kg worth of extra body size. This amounts to about 40% of the gap in physiological development in our world-wide sample (cf. Figure 7). From this perspective “history” appears to hold a substantial explanatory power vis-á-vis global inequality in physiological development around the year 2000.

6. Conclusion

In the present study we have taken on the task of understanding the global variation in physiological development, measured by human body size. Understanding physiological development is important, as stature appears to be causally related to many proximate sources of welfare: education, longevity and even self-reported happiness. Concretely, we advance the hypothesis that the current variation in physiological development has important historical roots in the differential timing of the fertility transition and the take-off to growth.
Table 2: Comparative Physiological Development

<table>
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<th>(4)</th>
<th>(5)</th>
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<td>Body size (kg)</td>
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<tr>
<td>Years since fertility decline</td>
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<td>0.238***</td>
<td>0.199**</td>
<td>0.228***</td>
<td>0.294***</td>
<td>0.235***</td>
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<tr>
<td>Years since fertility decline, squared</td>
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<td>-0.001***</td>
<td>-0.001***</td>
<td>-0.001***</td>
<td>-0.001***</td>
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<td>Tropical area (% total area)</td>
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<td>Child mortality (below 5), 1980</td>
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<td>-0.033***</td>
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<td>R-squared</td>
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<td>0.763</td>
<td>0.701</td>
<td>0.768</td>
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Robust standard errors in parentheses. *** p < 0.01, ** p < 0.05, *p < 0.1. All regressions include a constant term. Body size is measured as weight in kg.
The proposed mechanism behind the take-off is that differential technological progress across sectors eventually led to an increasing price of provisions, triggering a quantity-quality trade-off according to which family size shrunk, and nutrition per child increased. As fertility declined, growth took off and average body size started increasing secularly.

There is strong evidence in favor of the quantity-quality trade-off adopted in the present research, both inside and outside of the field of economics. In particular, the trade-off appears to transcend the human species, which suggest it is likely fundamental. In the analysis above we have provided further evidence of the trade-off by studying the evolution of European body size during roughly the last 130 years. Consistent with the existence of such a trade-off we find that, conditional on income and mortality, there is a strong negative correlation between adult body size and the fertility rates prevailing at the time the cohort was born.

Our initial checks also provide evidence in favor of the fundamental hypothesis in focus: conditional on contemporary income and mortality, as well as obvious geography confounders, the timing of the take-off remains a significant determinant of body size. Hence, simply put: “The West is taller than the rest” because it went through the fertility transition earlier. The same process paved the way to prosperity, as captured by the model.

From the point of view of the present study, several future lines of research may be fruitful. With regards to the advanced hypothesis stronger evidence is needed in favor of the link between the timing of the fertility transition and current body size. In spite of our best efforts to control for likely confounders, doubt may legitimately linger as for identification.

Many different hypotheses have been advanced in the anthropometric economic history literature regarding the long-run evolution of human body size. It remains an open question which of these alternative accounts simultaneously are consistent with observed changes in fertility, sectoral allocations and income. But a reasonable first approximation can be the proposed theory involving changing relative prices of provisions in the presence of a size-number quantity-quality trade-off.
6.1. **Proof of Proposition 5.** Let $g^x_t$ denote the growth rate of a variable $x_t$ and $g^x$ the associated balanced growth rate. Lagging (23) one period we observe:

$$1 + g^p_t = \frac{(1 + g^Q_t)(1 + g^L_t)^{1-\alpha}}{1 + g^A_t}.$$  \hspace{1cm} (A.1)

As usual a balanced growth path is defined by constant rates of growth and constant sectoral labor shares. In addition, balanced growth requires that the relative price of food is constant to ensure that fertility and body size are constant. This requires that

$$(1 + g^A) = (1 + g^Q)(1 + g^L)^{1-\alpha}.$$  \hspace{1cm} (A.2)

Imposing $g_p = 0$ in (A.1) and inserting (A.2) we obtain that constant productivity growth in both sectors requires that

$$(1 + g^A)^{\xi-1} = (1 + g^L)^{\alpha \xi}, \quad (1 + g^Q)^{\phi-1} = (1 + g^L)^{\phi}.$$  \hspace{1cm} (A.3)

Inserting (A.3) into (A.2) provides the condition

$$(1 + g^L)^{\phi/(\phi-1)}(1 + g^L)^{1-\alpha} = (1 + g^L)^{\alpha \xi/(1-\xi)}.$$  \hspace{1cm} (A.3)

From this we infer that balanced growth requires either no population growth or the knife edge condition $\phi/(1 - \phi) + 1 - \alpha = \alpha \xi/(1 - \xi)$. Ignoring the degenerate knife-edge case, we establish uniqueness of the stationary solution by assuming that $\phi/(1 - \phi) + 1 - \alpha < \alpha \xi/(1 - \xi)$. This rules out all unbalanced trajectories (along which the population is exploding or permanently shrinking) as long-run solutions. The balanced growth path remains as the only feasible solution.

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**Figure A1: Stature and Fertility and Income: 1700 – 2000**

Blue solid line: stature (left axis). Red dashed line: fertility (right axis). Income per capita ($Q$) is normalized such that $Q(1700) = 1$. Parameters as for Figure 1.
6.2. **The reduced form correlation between income and body size: simulation.** In the model there is no direct (partial) effect of income on fertility and offspring nutrition. Yet, it is worth noting that the model does provide a clear income/body size correlation via prices; from an empirical standpoint GDP per capita may then serve as a stand-in for the relative price, as noted in Section 5.

To see this, Figure A.1 shows the link between income, body size and fertility, according to the simulation conducted in Section 4. As can be seen, body size and income are positively correlated from the time of the take-off and until today.

6.3. **Data Appendix.**

6.3.1. **Panel data set.**

- The data on **height** is from Hatton and Bray (2010). It consists mainly of army recruits of about 20 years of age.
- The data on **GDP per capita** is due to Angus Maddison, [http://www.ggdc.net/MADDISON/oriindex.htm](http://www.ggdc.net/MADDISON/oriindex.htm).
- The data on vital rates (**Crude birth rate and crude death rate**) is taken from Reher (2004).
- The dummy WWI takes on the value 1 in the (two) periods encapsulating 1914-1918; WWII is the same for 1940-45. Finally Great Depression involves the years 1929-1935.

There is a slight inconsistency between the height data and the data on vital rates. Height data comes as five year observations with the demarcations 1856-1860, 1861-1865 etc, whereas the vital rates come in the time undervalues 1855-59, 1860-1864 etc. In the panel exercise this difference is finessed by assuming that average vital rates for 1855-59 (etc) are identical to those of 1856-1860 (etc). While unattractive, it seems doubtful l that this minute mismatch influences the results.

6.3.2. **Cross Country data.** Data on the timing of the **fertility decline** is due to Reher (2004).

For most of the countries in the sample data on **female weight** is from Demographic Health Surveys (DHS); extracted by StatCompiler ([http://www.statcompiler.com/](http://www.statcompiler.com/)) on 24.1.2012. Survey’s close to the year 2000 were selected. Since mothers are singled out in the DHS, implying women are in the age interval 15-49; the sampled individuals were therefore born in 1985 or earlier.

We supplemented these data by data on height for 9 European countries. The data also concerns women, born in 1980. The source is Garcia and Quinta-Domeque (2007), supplemented by Herpin (2003, p. 73) for France. In the latter case the data concerns 20-29 year-olds, observed in 2001 (implying they were born around 1980). The French figure is adjusted down by 0.8 cm to correct for self reporting (see Hatton and Bray, 2010). Finally, in order to generate data on weight we employed data on female BMI (in 2000) from the study by Finucane et al (2011, Supplementary material p. 60 ff). Using these BMI numbers along with the height data (for the 9 country European sample), weight data is constructed using the formula $BMI = w/h^2$ (height in m).

**GDP per capita and mortality, 1980**: World development indicators (available online at: http://data.worldbank.org/)

**Frost (average of days per year)**: Source: Masters and McMillan (2001).


