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Life Expectancy, Labor Supply, and Long-Run Growth: Reconciling Theory and Evidence*

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Abstract. We set up a three-period overlapping generation model in which young individuals allocate their time to schooling and work, healthy middle aged individuals allocate their time to leisure and work and their income to consumption and savings for retirement, and old age individuals live off their savings. The three period setup allows us to distinguish between longevity and active life expectancy (i.e. the expected length of period 1 and 2). We show that individuals optimally respond to a longer active life by educating more and, if the labor supply elasticity is high enough, by supplying less labor. We calibrate the model to US data and show that the historical evolution of increasing education and declining labor supply can be explained as an optimal response to increasing active life expectancy. We integrate the theory into a unified growth model and reestablish increasing life expectancy as an engine of long-run economic development.

Keywords: longevity, active life expectancy, education, hours worked, economic growth.

JEL: E20, I25, J22, O10, O40.

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70 is the new 50.
(William C. Byham, 2007)

1. Introduction

Over the course of human history we observe a strong positive correlation between income and life expectancy as well as between income and education (Preston, 1975, Bils and Klenow, 2000, Krueger and Lindahl, 2001). These aggregates showed no visible trend for millennia and then, in most developed countries, began to rise jointly and permanently roughly at the same time, for example around the year 1800 in England. The observed positive *correlation* is thus undisputed, constituting basically a stylized fact of successful human development. Yet there exists a lively debate about the interpretation of the correlation.

One popular hypothesis, built upon human capital theory and the life cycle of earnings (Becker, 1962, Ben-Porath, 1967), argues that increasing life expectancy leads to more education and thereby to faster income growth. With contrast to the second link in this chain of causation, which has been debated for quite a while, the first link, the effect of increasing life expectancy on education, was long considered to be self-evident. Recently, however, this link gained scholarly attention as the so called the Ben-Porath mechanism (Hazan, 2009). It is also at the center of the present paper.[1]

In simple words the Ben-Porath mechanism implies that (the expectation of) a longer life leads to more education because it provides a longer working-period during which people can harvest the fruits of their education in form of higher wages. A longer working life makes the opportunity cost of education, stemming from a later entry into the workforce, worthwhile. This line of reasoning seems to suggest that higher education should be associated with more life-time labor supply and, indeed, Hazan (2009) showed, based on a simplified version of the Ben-Porath model, that increasing longevity has a non-negative effect on life-time labor supply. Hazan then continued to show that for male U.S. citizens increasing education was associated

[1] Economic models arguing in favor of the first link, that is an impact of (adult) longevity on economic growth are, among others, proposed by de la Croix and Licandro (1999), Kalemli-Ozcan, Ryder and Weil (2000), Boucekkine, de la Croix and Licandro (2002, 2003), Zhang et al. (2003), Chakraborty (2004), Soares (2005), Cervellati and Sunde (2005, 2011a), Tamura (2006), and Birchenall (2007). Supportive evidence is provided by Lorentzen et al. (2008) and Cervelatti and Sunde (2011b). A prominent article finding a negative impact of life expectancy on income is Acemoglu et al. (2007). Microeconomic studies generally find a much lower impact of life expectancy on income, see e.g. Weil (2007). A recent review of the literature is Bleakley (2011). Health economists usually argue in favor of reverse causation, running from education to health. A prominent study is Jayachandran and Lleras-Muney (2009); see Cutler et al. (2011) for a recent survey.

with *decreasing* life-time labor supply since the early 19th century; that is, basically since the onset of modern economic growth. Accordingly, higher life expectancy seemingly cannot have caused education levels to rise through the Ben-Porath mechanism. Consequently, increasing life expectancy – through this channel – could not have caused economic growth.

It is important to note that the evidence does not *generally* refute increasing longevity as a driver of education and economic development. It just refutes the simple Ben-Porath mechanism, which in particular ignores a preference for leisure and labor supply at the intensive margin. Hazan is careful in clarifying this point in the concluding remarks of his study (p. 1859): there may exist another theory which can explain how higher life expectancy simultaneously causes less life-time labor supply and more education. The present paper proposes such a theory.

The key idea of our theory is that there exists a distinct period at the end of human life, in which the body is too frail for labor supply to be worthwhile. Basically we re-introduce from the simple life-cycle model the period of old age, conceptualized as a period, in which people can no longer participate in the labor market (for a decent wage) but are lively enough to enjoy utility from consumption. The response of labor supply to increasing life expectancy then crucially depends on which period of the life cycle is expected to get longer. If people expect to stay longer in the inactive and potentially frail state, they work harder during the active period of life. If, in contrast, people expect to stay longer in an active and healthy state, they prefer to reduce labor supply per time increment (i.e. per month or week) in the active period and enjoy more leisure.

With respect to education, increasing longevity has a positive impact no matter where in the life cycle it occurs. Because people derive utility from consumption in every period of their life, a longer life generally induces more education, since higher education provides more income and thus more utility from consumption per time increment during the active and inactive period. The theory thus predicts unambiguously more eduction and less labor supply per time increment if people expect a longer active period of life. With respect to *total* labor supply over the life-time the prediction is generally ambiguous because the negative effect of less labor supply on the intensive margin could be offset by a longer active life. In the paper we show that the negative effect dominates if the labor supply elasticity is sufficiently large. In that case the theory predicts that increasing life expectancy causes more education and less life-time labor supply.

We calibrate the model with data for male US citizens and show that the life cycle model explains the historical evolution of life expectancy, education, and labor supply as presented in Hazan's (2009) study quite well. We then develop a unified growth model in which education is the driver of technological progress as, for example, in Galor and Weil (2000), Galor (2005, 2011), and Cervellati and Sunde (2005), and show that the life cycle model explains the historical evolution of TFP growth and GDP growth quite well. Finally we consider an alternative calibration with data for an average (unisex) US American citizen provided by Ramey and Francis (2009) and show that our model – albeit with a much lower labor supply elasticity – provides also a reasonable fit of the historical trajectories suggested in that study. The fact that education and labor supply, in theory as well as in any application, are driven by increasing life expectancy re-establishes this channel as an important driver of long-run economic development.

The theory thus suggests a compromise between studies arguing in favor of life-expectancy as a driver of economic development (as, for example, Cervellati and Sunde, 2005) versus studies emphasizing health or morbidity (as, for example, Hazan and Zoabi, 2006). Here, we argue that it is the interaction between healthy and unhealthy years of life that can take account for the historical evolution of labor supply and education.

In the medical and gerontological literature we find ample evidence for a distinct third period of life. For example, in the year 2000 in the US 27% of the non-institutionalized elderly population reported fair or poor health and 35% reported limitations of activity due to chronic diseases (Rice and Fineman, 2004). With contrast to economic life cycle models, in which aging is mostly conceptualized as the passing of life-time, biologists define aging as the intrinsic, cumulative, progressive, and deleterious loss of function that eventually culminates in death (Arking, 2006). The work by Mitnitski and coauthors (2002, 2005, 2006) documents impressively how human frailty, on average, increases with age. At the individual level, however, the aging process exhibits great plasticity and is only imperfectly captured by chronological age; some 60 year-olds are as fit as some 40 year-olds and vice versa. Chronological age, that is distance from birth, is thus a poor measure of frailty, which is better approximated by distance from death.

Over the last century, the state of health of the elderly improved substantially. Members of later born cohorts can not only expect to live longer but also to live longer in a healthy state. These gains are measured by *healthy life expectancy*, sometimes also called active life expectancy, defined as the average number of years that a person can expect to live in "full health", that is

without disability or injury (WHO, 2012). Manton et al. (2006) estimate that for 65 year old US citizens the ratio between healthy (active) life expectancy and life expectancy rose from 73.9 % in 1935 to 78.5 % in 1999 and predicts the ratio to increase further to 88 % in the year 2080.

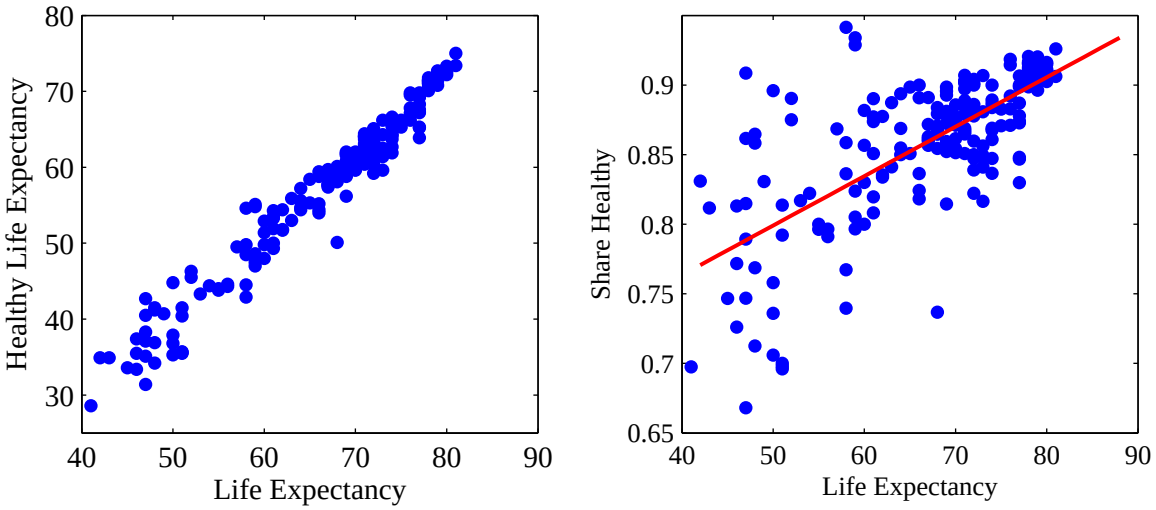
From a gerontological viewpoint it still remains a dream of the future that "70 is the new 50" (Byham, 2007). Nevertheless, in developed countries, older people have already experienced substantial gains in "good" years of life. Baltes and Smith (2003) conclude that the state of health of today's 70 year-old US Americans is comparable to the one of 65 year olds who lived 30 years ago. Naturally, these improvements are not the consequence of genetic mutations but man-made or "manufactured" (Carnes and Olshansky, 1997) and largely driven by education through increasing knowledge about healthy behavior and medical technological progress (Rice and Fineman, 2004, Manton et al., 2006, Skirbekk et al., 2012).

The left hand side of Figure 1 shows that life expectancy and healthy life expectancy are strongly correlated across countries. As life expectancy improves, healthy life expectancy improves "in sync". But what looks like a linear correlation to the naked eye is actually mildly non-linear. This fact is revealed in the right panel of Figure 1. As life expectancy increases, the share of healthy years increases as well, by about 0.35 percent for every year of life expectancy. With improving longevity we get more healthy as well as more unhealthy years but we get a bit more healthy than unhealthy years. That is, healthy or active life expectancy improves relatively to longevity. This is the stylized fact upon which we built our theory.[2]

A related but different proposal to square Hazan's observation with economic rationality has been made by Cervelatti and Sunde (2010, see also Sheshinski, 2009). They show that Hazan's argument rests on the assumption of a rectangular survival curve. Taking age dependent mortality into account, expected years in the workforce are actually increasing and, given a relatively low labor supply of young adults, it can well be that marginal benefits of education exceed marginal costs. Their mechanism, with contrast to ours, is based on survival during working age and, as the authors emphasize, independent from retirement age (length of active life) and longevity (life expectancy). It complements "our" mechanism, which is built on increasing active life length. In their own macro work, Cervelatti and Sunde (2011), have not yet implemented

[2]Ideally, to corroborate our theory we would need data on the evolution of healthy life expectancy within countries. Given that the idea of healthy life expectancy is relatively new, there are, unfortunately, not sufficiently many data available for time series analysis.

Figure 1: Life Expectancy vs. Healthy Life Expectancy Across Countries



Data for 191 countries from WHO (2012). Life expectancy at 5 from year 2000, healthy life expectancy at 5 from year 2002. Share Healthy (SH) is healthy life expectancy divided by life expectancy (LE). Regression line: SH = 0.61+0.0035 LE; 95% confidence interval for coefficient: $[0.0030, 0.0041]$.

their refined view on life expectancy. Our study, with contrast, integrates the life cycle mechanism into a unified growth model and shows how increasing life expectancy explains the onset and gradual increase of economic growth as well the observed decline of labor supply.

Recently, d'Albis et al. (2012) have shown in a continuous time life-cycle model that individuals retire later if mortality declines in old age and earlier if mortality declines at younger ages. However, d'Albis et al. (2012) do neither consider educational choice nor labor supply at the intensive margin. They do also not investigate the quantitative power of their model and abstain from integrating it into a unified growth context.

Finally, Hansen and Loenstrup (2012) propose an alternative channel through which increasing life expectancy may have reverse effects on labor supply and education. It relies on missing capital markets for young people and, like Kalemli-Ozcan and Weil (2010), on uncertainty and missing annuity markets for old people. The mechanism goes as follows. A higher probability to enter old age reduces unintended bequests, a fact that induces middle-aged people to save more. Consumption smoothing individuals, however, prefer to distribute more savings and thus lower consumption on both periods, youth and middle age. With missing capital markets in youth this can only be achieved by spending more time on education. In contrast, our mechanism is built upon the notion of active life expectancy and not on missing markets for annuities or credit. Nevertheless we neglect credit financed consumption in youth in order to simplify the

analysis and to avoid distraction from the main point by adding yet another choice problem and another market.

2. The Model

2.1. **Demographic Structure.** Consider an economy populated by three overlapping generations. In order to focus the analysis on the impact of adult longevity we abstract from endogenous fertility and infant mortality and assume that each time period a new generation of size (measure) one enters life at the beginning of the education period. Any member of the economy lives through three distinct periods of life:

- youth: a period, in which young individuals decide how to allocate their time on working and schooling.
- (healthy) middle age: a period, in which educated individuals decide how to allocate their time on working and leisure and how to allocate their labor income on consumption and savings for old age.
- (frail) old age: a period, in which health and productivity of individuals has deteriorated to such a degree that their labor is no longer in demand.

Without loss of generality we normalize the length of the first period to unity. Later on, in the calibration, we associate the unit length of a period with 20 years. The duration of the second and third period of life is given for any generation but time varying over the course of human history. We denote the expected length of life in middle age by τ_1 and call the term $\Lambda \equiv 1 + \tau_1$ active life expectancy. Likewise we denote the expected time spent in old age and frailty by τ_2 such that $life_e \equiv 1 + \tau_1 + \tau_2$ is (total) life expectancy. Since $\partial\Lambda/\partial\tau_1 = 1$ we say that an increase in τ_1 is an increase in active life expectancy. Finally, we denote by a the actual age of individuals.

Without implications for our qualitative results we could have added an element of uncertainty by imposing transition probabilities from one period of life to the next. For simplicity, we neglected uncertainty issues such that people correctly anticipate the length of their (active) life. Note that our definition of active life expectancy deviates mildly from the use of the word in gerontology where it is defined as years of life in a non-disabled state, that is, as synonymous with healthy life expectancy. Here, active life expectancy defines the years of potential participation in the workforce. At the end of their active life, individuals can no longer participate in the

workforce because their productivity (in a learned occupation) has deteriorated too much. This does not preclude an "active" life outside the workforce in leisurely activities. Furthermore, we allow aging individuals to gradually withdraw from the workforce before the end of their active life according to their taste for leisure.

2.2. **The Decision Problem.** In all three periods of life individuals experience utility from consumption. In the first period, denoted by t, they divide their time between education ϵ_t and work $1-\epsilon_t$. Let w_t be the wage per unit of human capital. Initial human capital is normalized to unity and through education young people acquire human capital $h(\epsilon_t) \geq 1$, $h' > 0$, $h'' > 0$. For simplicity and without loss of generality we ignore credit financed consumption during the first period. Income of the young is then given by $(1-\epsilon_t)w_t$ and period consumption c_t is given by

$$c_t = (1-\epsilon_t)w_t. \tag{1}$$

In the second period individuals divide their time between work l_{t+1} and leisure. To discuss labor supply properly it is useful to conceptualize the period (of length τ_1, for example, 50 years) as divided into time increments (for example, months). At each time increment the individual supplies l_{t+1} units of labor and earns an income $l_{t+1}h(\epsilon_t)w_{t+1}$. During the period individuals spend their income on consumption c_{t+1} and saving for old age s_{t+1}. The period budget constraint is thus given by

$$\tau_1 c_{t+1} + \tau_1 s_{t+1} = \tau_1 l_{t+1} h(\epsilon_t) w_{t+1}. \tag{2}$$

In the third period, retired individuals consume the returns on their savings. Let $R_{t+2} \equiv 1 + r_{t+2}$ denote the gross interest rate. The period budget constraint is then obtained as

$$\tau_2 c_{t+2} = R_{t+2}\tau_1 s_{t+1}. \tag{3}$$

Notice that τ_1 cancels out in (2) but not in (3). A longer working life, keeping saving per time increment constant, leads to more savings available in old age.

Individuals maximize life-time utility. Assuming intertemporal separability, this problem reads

$$\max U = u(c_t) + \tau_1\beta\left[u(c_{t+1}) - v(a, l_{t+1})\right] + \tau_2\gamma u(c_{t+2}). \tag{4}$$

subject to (1)–(3) and $0 \leq \epsilon_t \leq 1$, $0 \leq l_{t+1} \leq 1$. Here, β and γ denote discount factors capturing pure time preference as well as utility weights for consumption experienced in an active and healthy state (β) and in a retired and potentially frail state (γ). In each period utility per time increment is aggregated over time such that a higher weight is attached to longer periods. We assume decreasing marginal utility from consumption, $u' > 0, u'' < 0$, and a well-behaved function for disutility from work, such that the first order conditions provide a maximum. We allow the disutility experienced from labor supply to be potentially increasing with age, $\partial v/\partial a > 0$, which provides a simple device to introduce age-dependent, gradual withdrawal from the labor market. Substituting (1)–(3) into (4), it is straightforward to see that the first order conditions for optimal education, labor supply, and savings require:

$$w_t u'(c_t) = \beta \tau_1 u'(c_{t+1}) h'(\epsilon_t) l_{t+1} w_{t+1} \tag{5}$$

$$u'(c_{t+1}) h(\epsilon_t) w_{t+1} = v'(a, l_{t+1}) \tag{6}$$

$$\beta u'(c_{t+1}) = \gamma R_{t+2} u'(c_{t+2}). \tag{7}$$

2.3. **Education and Labor Supply: The Mechanism in General.** According to (5), optimal education requires that the marginal cost of education in terms of foregone labor income (one unit) evaluated in terms of marginal utility from consumption in youth (the left hand side) equals the marginal benefit in terms of higher income in middle age through the accumulated human capital and the associated skill premium (the term $h'(\epsilon_t) l_{t+1} w_{t+1}$), evaluated in terms of utility (the term $\beta \tau_1 u'(c_{t+1})$). Optimal labor supply, according to (6), requires that the benefit that one unit more of work provides, given by the term $h(\epsilon_t) w_{t+1}$, evaluated in terms of utility, $u'(c_{t+1})$, equals the marginal loss in terms of foregone utility from leisure (i.e. higher disutility from work, $v'(a, l_{t+1})$). Notice that labor supply is a within-period decision. It does not *directly* depend on period length or life expectancy. Optimal saving, according to (7), requires that the utility loss incurred by saving a unit of income more (the left hand side) equals the utility gain in old age that a unit of savings brings about (the right hand side).

Although life expectancy does not affect labor supply directly, it does so in an indirect way because the decisions on savings, labor supply, and education, are non-separable. This is so because education acquired in the first period ϵ_t enters marginal utility from consumption in the

second period, $u'(c_{t+1}) = u'(l_{t+1}h(\epsilon_t)w_{t+1} - s_{t+1})$ from (2). This term appears in all three optimality conditions and makes the decision on all three choice variables interdependent. Because (active) life expectancy enters the optimality conditions for education and savings, it does thus also bear upon the labor supply decision.

The effect of higher life expectancy on labor supply is generally ambiguous. In order to see this assume that higher life expectancy (larger τ_1 or τ_2) leads to more time spent on education. Below we show that this is always the case. More education through more human capital increases income in the second period $h(\epsilon_t)w_{t+1}$. Taken for itself the income effect makes the left hand side of (6) larger. But more income and the higher level of consumption acquired with it reduces the marginal utility from consumption, $u'(l_{t+1}h(\epsilon_t)w_{t+1} - s_{t+1})$, since $u'' < 0$. This substitution effect reduces the left hand side of (6). Moreover, whether individuals react to increasing income by supplying more labor depends on the sign of v'', another gateway for ambiguity. Whether the price effect or the substitution effect dominates depends on the shape of the utility function.

A similarly ambiguous response can be expected with respect to savings. A longer stay in the middle period of life increases the time during which middle age consumption is enjoyed. This entails an income effect that leads to higher consumption per time increment during middle age because the old age period gets relatively shorter. Taken for itself, this effect leads to lower savings. But higher life expectancy also causes more education and more income and consumption per time increment in the middle period. It thus lowers marginal utility form consumption in middle age ($u'(c_{t+1})$ decreases on the left hand side) and thus leads, taken for itself, to more consumption in old age, that is to more savings. Again, the shape of the utility function will tell which effect dominates.

2.4. **Explicit Solution.** In the following we assume that $u(x) = \log(x)$ and $v(a, x) = B(a)x^{1/\eta}$ with $\partial B/\partial a > 0$, and that human capital is accumulated according to Mincer (1974) with a constant return to schooling θ, $h(\epsilon_t) = \omega \cdot \exp(\theta\epsilon_t)$. These parameterizations are general enough to establish our main results but specific enough to obtain an explicit solution of the maximization problem (1)–(4). In the Appendix we show that it is given by (8)–(10).

$$\epsilon_t = \max\left\{0,\ 1 - \frac{1}{\theta(\beta\tau_1 + \gamma\tau_2)}\right\}. \tag{8}$$

$$l_{t+1} = \min\left\{1,\ \left(\frac{B(a)}{\eta}\right)^{-\eta} \cdot \left(\frac{\beta\tau_1}{\beta\tau_1 + \gamma\tau_2}\right)^{-\eta}\right\} \tag{9}$$

$$s_{t+1} = \frac{\gamma\tau_2 h(\epsilon_t) l_{t+1}}{\beta\tau_1 + \gamma\tau_2} \cdot w_{t+1}. \tag{10}$$

Observe from (8) that life can be so short that individuals prefer not to invest in education and remain uneducated. Observe from (9) that active life can be so short that all time in the middle period is allocated to work.

In the following we call η the labor supply elasticity. But note that η does not stand for the labor supply elasticity in the conventional sense, i.e. evaluated with respect to the real wage. As in any available model on growth with endogenous labor supply, labor has to be inelastic with respect to the real wage in order to avoid that individuals stop working in a perpetually growing economy (Prescott, 1986). Here, the supply elasticity is measured with respect to the expected relative length of the middle period in life, $\beta\tau_1/(\beta\tau_1 + \gamma\tau_2)$. It measures by how much labor supply declines when the middle age period gets relatively larger by one percent. Intuitively, individuals prefer to work less when the middle age period of life gets relatively longer because the same level of consumption per time increment in old age can be financed by fewer hours of work per time increment (e.g. per month) in middle age. The elasticity η measures how strong this response is.

With respect to the interior solution we get the following results on comparative statics. All proposition are proved in the Appendix.

Proposition 1 (Education). *The time invested in schooling ϵ_t increases with the period lengths τ_1, τ_2 and the return to schooling θ.*

Intuitively, higher life expectancy motivates more education no matter whether it is caused by a longer middle age or old age because the fruits of education in terms of higher consumption are smoothed over the life cycle and enjoyed in all periods.

Proposition 2 (Labor Supply). *Labor supply per time increment in middle age l_{t+1} decreases with active life expectancy (τ_1) and, if $B'(a) > 0$, with age. It increases with the duration of old age (τ_2).*

The second part of Proposition 2 corresponds with the familiar result from the simple Ben-Porath mechanism: if individuals expect to live longer in the sense of a longer stay in old age, they work harder in middle age. Yet, proposition 2 also shows that if individuals expect to stay longer in middle age, they reduce labor supply per time increment because they expect to

finance the same level of old age consumption with less labor supply per time increment. This result, taken together with Proposition 1, provides a first reconciliation of the evidence presented in Hazan (2009) with rational decision making on human capital formation in a conventional life cycle model: higher life expectancy causes higher education as well as lower labor supply if survival improves in the middle age period, that is, if it is driven by higher *active* life expectancy.

We next turn from individual decisions per time increment to macro-economic aggregates per period. Here, results depend crucially on the labor supply elasticity.

PROPOSITION 3 (Aggregate Labor Supply, ETWH). *Aggregate labor supply of the middle-aged generation $L_{t+1} \equiv \tau_1 l_{t+1}$ decreases with active life expectancy (τ_1) if the labor supply elasticity is sufficiently large, i.e. for $\eta > (\beta\tau_1 + \gamma\tau_2)/(\gamma\tau_2)$.*

Notice that the term $L_{t+1} \equiv \tau_1 l_{t+1}$ does not only measure aggregate labor supply of middle-aged persons but also, since population size has been normalized to unity, the total hours that young individuals expect to work during middle age. It thus captures Hazan's (2009) main variable of interest, expected total working hours (ETWH). The fact that L measures labor supply during middle age (thus ignoring labor supply in youth) squares well with the fact that Hazan's computations assume labor market entry at age 20. The fact that the expectation is built at entry into the education period squares well with the fact that Hazan considers labor supply expected at age 5. Proposition 3 shows that aggregate labor supply, or ETWH, decreases with increasing active life expectancy if labor supply is sufficiently elastic. In that case the negative effect of higher active life expectancy on increasing demand for leisure per time increment in middle age is dominating the positive effect of a longer duration of middle-aged life.

PROPOSITION 4 (Non-Monotonicity). *For $\eta > 1$, L_{t+1} assumes a minimum at $\tau_1 = (\eta - 1)\gamma\tau_2/\beta$.*

Interestingly, a longer expected active life (τ_1) has a non-monotonic effect on aggregate labor supply if the labor supply elasticity is sufficiently large. Originating from a relatively short active life, improving τ_1 has the dominating effect of less labor supply per time increment. If, on the other hand, active life expectancy is already (sufficiently) high, and individuals enjoy already a lot of leisure, further improving active life length has the dominating effect of a longer working life, and total labor supply increases. In this case a period of declining labor supply, as

identified by Hazan for the last century, is predicted to be transitory. If active life expectancy continues to increase, aggregate labor supply, according to the model, will eventually rise again.

PROPOSITION 5 (Savings Rate). *Let $\tilde{s}_{t+1} = s_{t+1}/w_{t+1}$ define the savings rate. The savings rate increases with the return to schooling θ and with the period length of old age τ_2. It decreases with active life expectancy if the labor supply elasticity is sufficiently high, i.e.*

$$\eta > \frac{\beta\tau_1(1-\beta\tau_1-\gamma\tau_2)}{\gamma\tau_2(\beta\tau_1+\gamma\tau_2)}.$$

The aggregate savings rate, $\widetilde{S} \equiv \tau_1\tilde{s}_{t+1}$, increases with active life expectancy (τ_1) if the labor supply elasticity is sufficiently low, i.e. for

$$\eta < \frac{\beta\tau_1+\beta\gamma\tau_1\tau_2+\gamma^2\tau_2^2}{\beta\gamma\tau_1\tau_2+\gamma^2\tau_2^2}.$$

Turning to the impact of life expectancy on savings, the model predicts that people save more when they expect to stay relatively longer in old age, that is if τ_2 increases. The definition of the aggregate savings rate relates to the conventional savings rate obtained in a standard OLG model, in which the length of the working period is typically normalized to unity. It gives the share of the generational wage (per unit of human capital) used to build up the aggregate capital stock. An increasing active life expectancy, taken for itself, has a positive impact on aggregate savings. However, individuals may response to a longer active life by saving less per time increment. Proposition 5 shows that the positive duration effect dominates the substitution effect if labor supply is sufficiently inelastic. In that case more human capital (acquired by the induced higher education) has relatively little effect on leisure. The dominating effect is higher income per time increment in middle age. Because individuals prefer a smooth consumption profile, they transfer some of the additional income to old age. Alternatively, if labor supply is highly elastic, the substitution effect, that is the induced higher demand for leisure, dominates and the aggregate savings rate declines.

3. CALIBRATION

In this section we show that the simple three-period overlapping generations model can explain the correlation between life expectancy and education and between life expectancy and labor supply (ETWH) observed for male U.S. citizens over the last 150 years (as compiled by Hazan, 2009). For that purpose we assume that a unit period corresponds to 20 years and that life

"begins" at age 5. The first period thus lasts until age 25 and if it were completely used for education ($\epsilon_t = 1$) it would basically end with a PhD degree.

As motivated in the introduction we assume that τ_1 and τ_2 improve in sync such that the relative length of active life gets mildly larger with improving life expectancy. Specifically, Let $\underline{\tau}_i$ define lower bounds and $\overline{\tau}_i$ upper bounds for middle and old age, $i = 1, 2$, and let λ denote the factor of proportionality. Life expectancy at 5 is then given by a linear combination of lower and upper bound, that is by

$$life_e = 1 + (1 - \lambda)\underline{\tau}_1 + \lambda\overline{\tau}_1 + (1 - \lambda)\underline{\tau}_2 + \lambda\overline{\tau}_2.$$

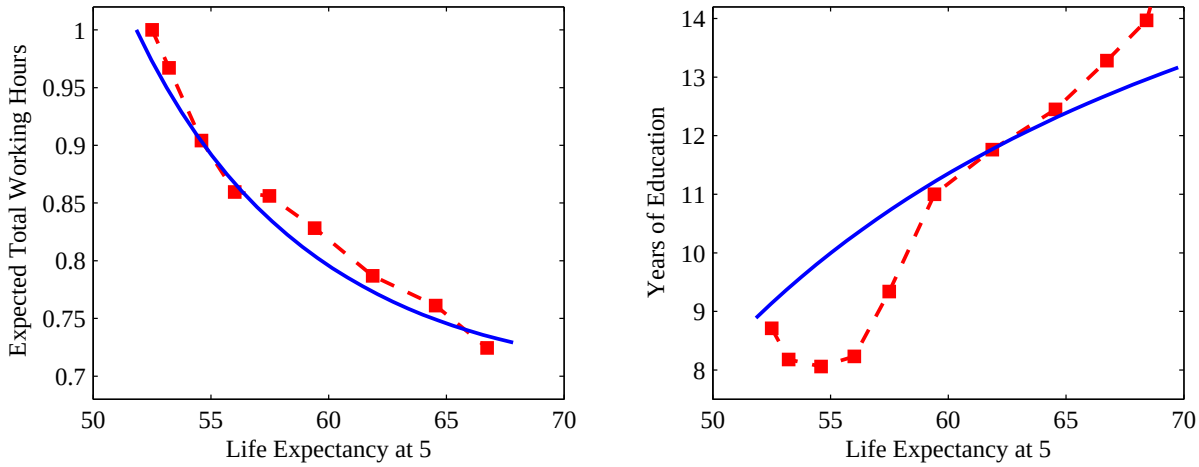
We set $\underline{\tau}_1 = 0.1, \overline{\tau}_1 = 3$ and $\underline{\tau}_2 = 0.3, \overline{\tau}_2 = 0.8$. These values imply that life expectancy at 5 runs from 1.4 to 4.8 units, that is from 28 to 96 years, when λ runs from 0 to 1. The lower bound accords well with life expectancy in ancient and pre-industrial times (Clark, 2007, Chapter 5) and the upper bound coincides with the gerontological estimate of human life-span (Gavrilov and Gavrilova, 1992). At the same time, active life expectancy runs from 1.1 to 4 implying that the share of active (healthy) years rises from 0.69 initially to 0.83, in line with the cross-country observation presented in the Introduction.[3] The historical period investigated by Hazan, in which life expectancy improved from 52.5 years to 70.7 years is covered by λ values between 0.32 and 0.6.

For the return to education there exists a variety of estimates, depending on method and sample, but a consensus value in recent estimates for the average return to education in the US seems to be 0.1 per year of education (Card, 1999, Psacharopoulos and Patrinos, 2004).[4] Since the unit length of the education period lasts for 20 years, a model unit of education $\epsilon_t \in \{0, 1\}$ corresponds with $\{0, 20\}$ years of education. We thus put $\theta = 0.1 \cdot 20 = 2$.

[3] Our estimate of the upper boundary is lower than the highest values observed in the cross-country WHO data. The deviation could be explained by our different notion of active life expectancy. The normal, aging-driven loss of cognitive skills, for example, does not affect the WHO definition of full health. It may thus be that fluid cognitive skills (creativity) have deteriorated to degree that precludes participation in the workforce, particularly in a learned occupation, although the person is otherwise in good shape and and classified as "fully" healthy. Notice also that time discounting amplifies small relative improvements of active life expectancy. The crucial factor $\beta\tau_1/(\beta\tau_1 + \gamma\tau_2)$ improves from 0.24 to 0.85 when λ goes from 0 to 1.

[4] Turner et al. estimate a somewhat larger return to education across US States with values between 0.11 and 0.15 in the period from 1840-2000. We could re-calibrate our model to these values by re-adjusting the time discount factors with insignificant impact on the fit of the historical data.

Figure 3: Model Calibration



Dashed (red) lines: data from Hazan (2009). Solid (blue) lines: model predictions. Expected working hours have been normalized such that expected hours for men born 1840 (when life expectancy is 52.5 years) is equal to unity. Generational results from the model have been converted into years by assuming a unit length for a period of 20 years (i.e., for example $\epsilon_t = 0.4$ is converted to 8 years of education.

With the focus on life-time labor supply we abstain, for simplicity, from introducing age-depending disutility from work. Modeling age-dependent labor supply would add more realism but conceptually it would "only" provide an unequal distribution of life-time labor supply (ETWH) across ages, leaving unaffected the association between ETWH and education, which depends on life expectancy and active life expectancy but not on the distribution of ETWH. On the macro-side, age-dependent labor supply, would severely complicate the aggregation across cohorts and destroy the simplicity of the model. Setting $B(a) = \bar{B}$ we thus estimate $\bar{B}$, β, γ, and η such that the model fits the data on labor supply (ETWH) and years of education compiled by Hazan (2009). This leads to the estimates $\beta = 0.65$, $\gamma = 0.42$, $\eta = 8.47$ and $\bar{B} = 11.8$ and the results shown in Figure 2. Dashed lines display the data from Hazan and solid lines show the predictions of the model. The model fits the historical data quite well. The fit is somewhat better for ETWH than for education because, naturally, the simply Mincerian equation cannot capture the non-monotonicity observed for education for the 1860 to 1880 cohort (when life expectancy was around 55 years). Furthermore the assumed decreasing returns to schooling imply an overall concave correlation of education and life expectancy, which is hardly visible in the data.

For better assessment we normalized the highest value of ETWH to unity. In line with the data the model predicts that ETWH decreases by about 30 percent as life expectancy increases from 52 to 66 years (for obvious reasons the Hazan data ends with the cohort born 1930). At the same time the model predicts that education increases from 9 years to 13 years as life expectancy increases from 52 to 66 years. Overall, it is hard to argue that the predictions from the life-cycle model are inconsistent with the historical data. Increasing life expectancy causes labor supply (ETWH) to fall and years of schooling to rise.

4. General Equilibrium and Long-Run Adjustment Dynamics

4.1. **Setup.** In order to evaluate the model's implications for long-run development we integrate education, labor supply, and savings from the life-cycle model into a simple dynamic general equilibrium setup. We define the unit length of a young generation (20 years) as the unit period for the dynamic macro-economy. This means that at any unit period there are potentially several middle aged generations active on the labor market. In a slight abuse of notation let t now denote the time period as well as the birth year of a cohort whereas j denotes the age of a cohort measured in unit periods. This means that aggregate labor supply (hours worked) in period t is computed as

$$\tilde{L}_t = 1 - \epsilon_t + \sum_{j=0}^{\tau_{1,t}-1} \max\{0,\ \min\{\tau_{1,t} - j,\ 1\}\ \} \cdot l_{t-j}. \tag{11}$$

Likewise, aggregate effective labor supply, or human capital, is computed as

$$H_t = 1 - \epsilon_t + \sum_{j=0}^{\tau_{1,t}-1} \max\{0,\ \min\{\tau_{1,t} - j,\ 1\}\ \} \cdot l_{t-j} \cdot h(\epsilon_{t-j-1}). \tag{12}$$

Here, $\tau_{1,t}$ denotes length of middle age of the generation born in period $t-1$, that is of the generation entering middle age at time t. We assume that individuals correctly predict their effective life expectancy. Allowing for mistakes, for example, by assuming instead adaptive expectation (young individuals expect the active life-length observed for their parents or grandparents) would only mildly modify the predicted adjustment dynamics.

Aggregate savings per period are obtained from savings of the currently alive middle-aged generations as

$$\hat{S}_t = \sum_{j=0}^{\tau_{1,t}-1} \max\{0,\ \min\{\tau_{1,t-j},\ 1\}\ \} \cdot \frac{\gamma\tau_{2,t-j}}{\beta\tau_{1,t-j} + \gamma\tau_{2,t-j}} \cdot l_{t-j} \cdot h(\epsilon_{t-j-1}) \cdot w_t. \tag{13}$$

Following the OLG tradition, we assume that this period's savings are available as aggregate capital stock next period and impose full depreciation of capital within a period (over 20 years), $K_{t+1} = \hat{S}_t$. Aggregate capital and effective labor supply are combined by a Cobb-Douglas production function to produce aggregate output $Y_t = A_t K_t^{\alpha} H_t^{1-\alpha}$, implying that the unit wage is given by $w_t = (1-\alpha)A_t K_t^{\alpha} H_t^{-\alpha}$. The parameter A_t captures total factor productivity (TFP).

Following a core idea of unified growth theory (Galor, 2005, 2011) we assume that the level of education of the currently young generation has a positive impact on state of technology next period. Similar to Cervellati and Sunde (2005) we impose a Cobb-Douglas technology, which also allows for a positive but diminishing external effect from the currently available knowledge to the creation of new knowledge. This means that advances of technology (TFP) are given by

$$A_{t+1} - A_t = \delta \cdot \epsilon_t^{\psi} \cdot A_t^{\phi} \tag{14}$$

with $\delta > 0$, $0 < \phi \leq 1$ and $0 < \psi < 1$.

The final element that closes the model is a feedback effect from the state of economic development to life expectancy. For simplicity, we follow again Cervellati and Sunde and assume a positive impact of the current generation's level of education on next generation's life expectancy. The simplest conceivable way to implement this notion is to utilize the upper and lower bounds introduced in the calibration section, $\underline{\tau}_i$, $\overline{\tau}_i$, $i = 1, 2$, and assume that lower bounds (life expectancy of 28 years) apply without any education in the population and upper bounds (life expectancy equals life-span) apply at maximum education. Actual (active) life expectancy is then determined by the current level of education as a linear combination of the boundaries:

$$\tau_{i,t} = (1 - \epsilon_{t-1})\underline{\tau}_i + \epsilon_{t-1}\overline{\tau}_i, \quad i = 1, 2. \tag{15}$$

This way of modeling preserves the basic idea of life expectancy, $1 + \tau_1 + \tau_2$, and active life expectancy, $1 + \tau_1$, evolving in sync and introduces a simple positive feedback effect of education on longevity. Another implication of the simple form (15) is that the long-run steady-state is

easily assessed. Given convergence towards a constant positive level of education, the remaining dynamics depend on ϕ. For $\phi = 1$ we have the endogenous growth case and the economy approaches a constant positive growth rate of technology and GDP. For $\phi < 1$ we have the semi-endogenous growth case with zero growth along the balanced growth path, that is for time approaching infinity. For the this paper, however, the outlook predicted for the distant future is less interesting than the model's performance in explaining the past and the present.

4.2. **Steady-State.** Although the setup was deliberately simply constructed, it allows for a multitude of steady-states. In particular, if model parameters support a corner solution for education, there may exist a longevity-driven poverty trap without education and a growth path with constant positive education. These locally stable steady-states are separated by an unstable steady-state. In order to see this, insert (15) into (8) to get a first order difference equation for education:

$$\epsilon_t = 1 - \frac{1}{\theta(\beta(\underline{\tau}_1 + \epsilon_{t-1}(\bar{\tau}_1 - \underline{\tau}_1)) + \gamma(\underline{\tau}_2 + \epsilon_{t-1}(\bar{\tau}_2 - \underline{\tau}_2)))}. \tag{16}$$

At a steady-state , education is constant, implying $\epsilon_t = \epsilon_{t-1} = \epsilon^*$. Inserting this into equation (16) we obtain steady-state education.

$$\epsilon^*_{1,2} = \frac{\beta(\bar{\tau}_1 - 2\underline{\tau}_1) + \gamma(\bar{\tau}_2 - 2\underline{\tau}_2) \pm \sqrt{\beta^2\theta\bar{\tau}_1^2 + \gamma\bar{\tau}_2(-4 + \gamma\theta\bar{\tau}_2) + 2\beta(2\underline{\tau}_1 + \bar{\tau}_1(-2 + \gamma\theta\bar{\tau}_2)) + 4\gamma\underline{\tau}_2}/\sqrt{\theta}}{2(\beta(\bar{\tau}_1 - \underline{\tau}_1) + \gamma(\bar{\tau}_2 - \underline{\tau}_2))}.$$

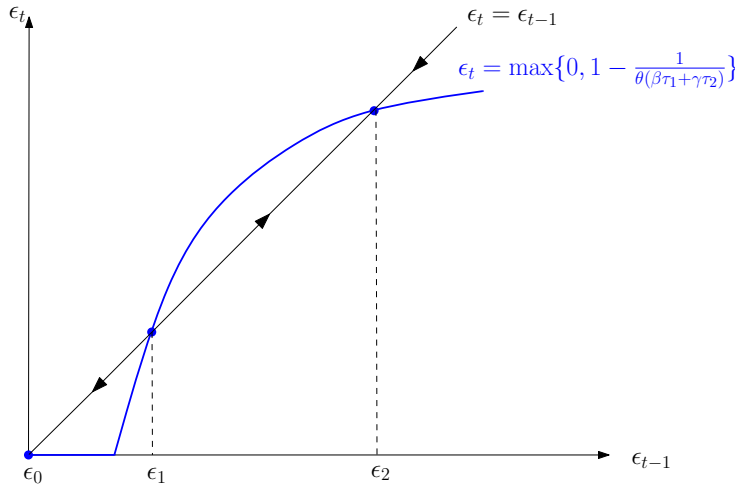
There exist two positive, non-trivial steady states, if the discriminant is positive, that is for

$$\theta > \bar{\theta} \equiv \frac{4(\beta(\bar{\tau}_1 - \underline{\tau}_1) + \gamma(\bar{\tau}_2 - \underline{\tau}_2))}{(\beta\bar{\tau}_1 + \gamma\bar{\tau}_2)^2}. \tag{17}$$

For $\theta < \bar{\theta}$, the discriminant is negative and there exists no positive solution for education. The only remaining solution is $\epsilon^* = 0$. If the return to schooling, θ, is sufficiently low, individuals always prefer to remain uneducated.

Figure 4.2 illustrates the case of three steady states, two positive ones, and a trivial one. The curve for education in period t, according to (16), is concave. Steady-states are observed at the intersections with the identity line. Above the identity line, $\epsilon_t > \epsilon_{t-1}$ and it is easy to see that one positive steady-state is unstable (ϵ_1), separating the two locally stable equilibria (ϵ_0, ϵ_2).

Figure 3: Equilibria for schooling



On the other hand, if the corner solution does not exist, the concave curve according to (16) goes through the origin and intersects the identity line exactly once, implying a unique and globally stable steady-state.

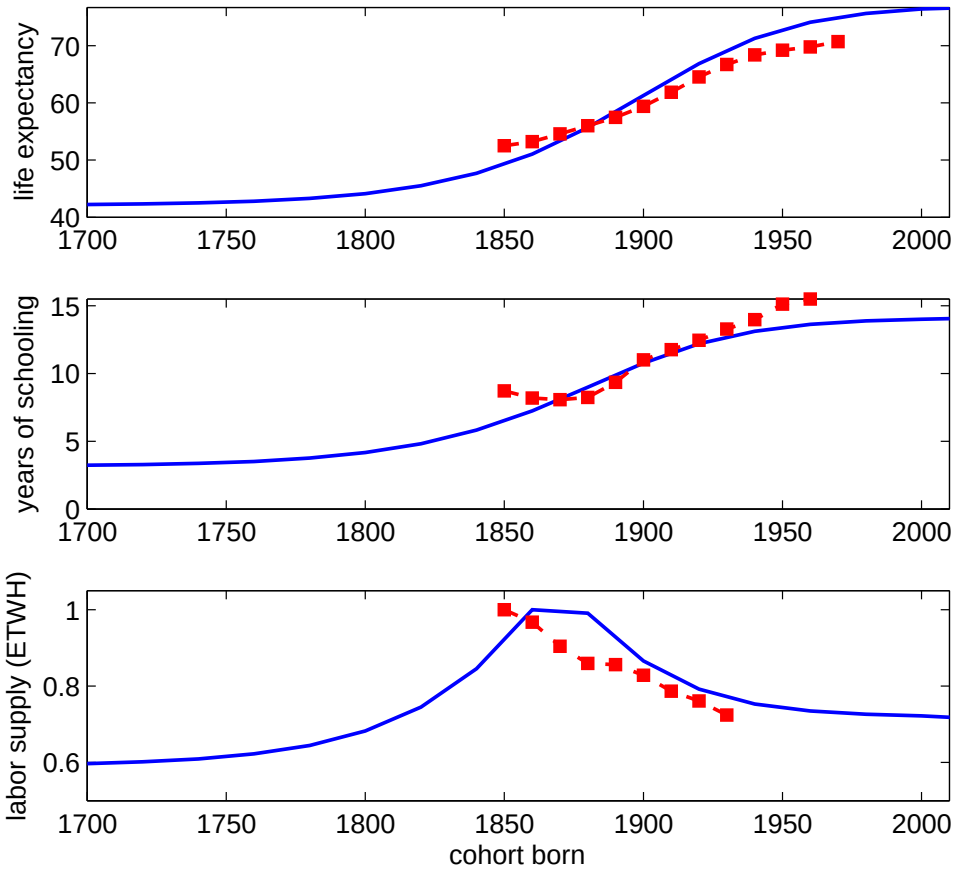
4.3. **Calibration and Results.** For the macro-economy we keep all parameter values from the calibration of the individual life cycle model of Section 3. An implication is that there exists no corner solution. The steady-state is unique. We set $\omega = 0.5$ such that the savings rate (investment rate) approaches 0.17 percent as observed for the US in the late 20th century. Following the growth literature we set the capital share $\alpha = 1/3$. Finally we set the knowledge parameters such that TFP growth reaches a maximum in the 1970s and growth of GDP per worker is about 2 percent in the late 20th century. This leads to the estimates $\delta = 500$, $\psi = 0.95$, and $\phi = 0.87$. After the computation we convert the results per period into annual ones using the assumption that a period lasts for 20 years. The economy starts in the year 0 AD with $A_0 = 1000$.

Figure 3 and 4 present the predicted adjustment dynamics for the most interesting epoch from 1700 to today. Solid lines in Figure 3 show the trajectories for life expectancy, education, and labor supply. Dashed lines show the historical data from Hazan (2009). Increasing life expectancy causes years of schooling to rise and aggregate labor supply (ETWH) first to rise and then to fall. The initial rise of ETWH results solely from increasing active life expectancy because, according to the model, labor supply is at the corner before 1850 and a longer healthy life thus translates

one to one into more aggregate labor supply. After 1850, individuals increasingly enjoy leisure time and ETWH declines.

Compared with the historical evolution the model predicts a somewhat too steep increase of life expectancy, a somewhat too flat increase of years of schooling, and a somewhat too late decline of labor supply. But overall, it is hard to argue that the model's predictions are contradicted by the data. Interestingly, the model predicts also that labor supply stops declining roughly at the end of Hazan's period of observation, that is for cohorts born in the second half of the 20th century. This prediction seems in line with the evidence in Ramey and Francis (2009) for hours worked per employed person (see below). The turning point for labor supply, however, theoretically identified in Proposition 4, is outside the relevant numerical range of Figure 3.

Figure 3: Long Run Evolution of Longevity, Education, and Labor Supply

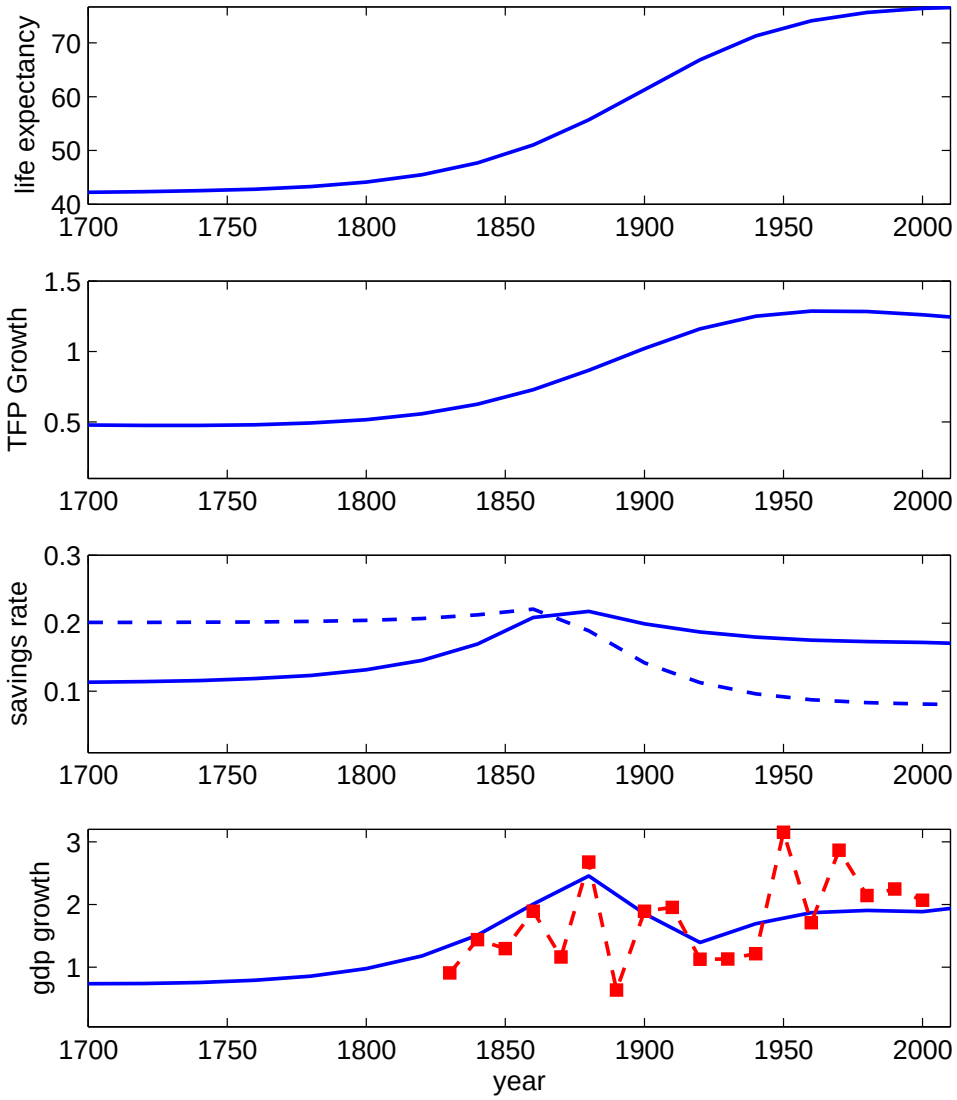


Solid lines: model prediction, dashed lines: historical data (Hazan, 2009). Labor supply is normalized such that the historical peak is at unity. A model period is translated into 20 years.

Ceteris paribus, it would be found where $\tau_1 = 3.89$, that is for an active life expectancy of 97.8 years.

Figure 4 shows the implications for economic growth, aggregate savings, and TFP. For comparison, the first panel reiterates the longevity trajectory from Figure 3. Note that the abscissa is now indexed by year (no longer by birth year of cohort as in Figure 3). Increasing life expectancy, with some delay, causes a gradually increasing growth rate of TFP, which reaches a maximum in the 1970s. Since returns are only mildly decreasing (with ϕ set to 0.85), the decline

Figure 4: Long Run Evolution of TFP Growth, Aggregate Savings, and Growth of GDP per Capita



Solid lines: model prediction, dashed (red) lines: historical data for (Maddison, 2003). GDP growth is the annual growth rate of GDP per capita. Savings rate panel: solid line: aggregate savings rate during middle age $\tau_1 s$, dashed line: savings rate per time increment s.

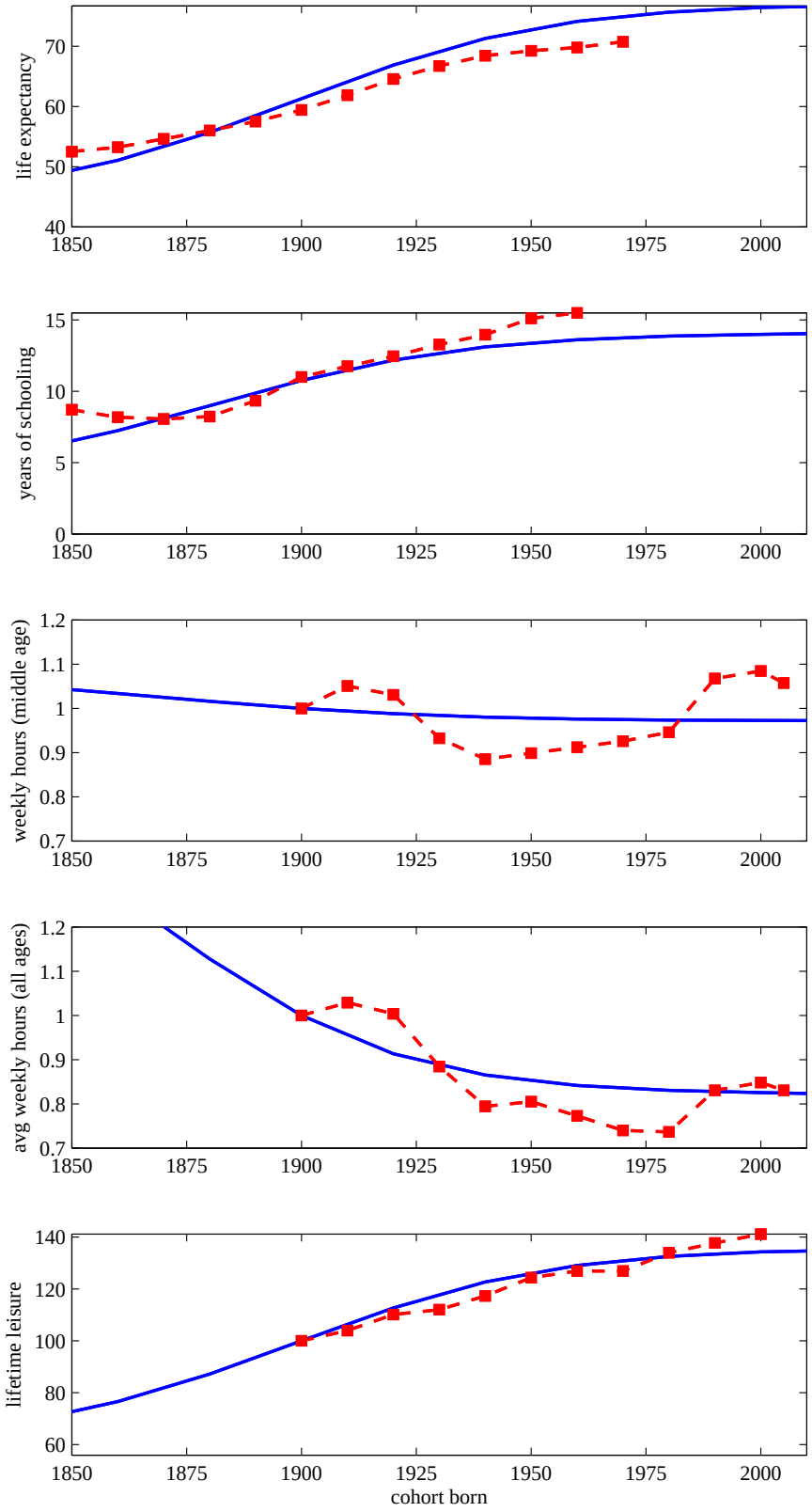
of TFP growth is not yet markedly visible in the trajectory, it looks more like a plateau for the second half of the 20th century.

For the savings rate per time increment (dashed line) the model predicts an almost constant profile for pre-industrial times and a decline with the onset of the Second Industrial Revolution (around 1860). The aggregate savings rate of middle-age aged adults (solid line) is predicted to increase before and during the industrialization period, in line with the historical observation for Britain (Crafts, 1985). This outcome simply reflects the fact that people have increasingly more years of active life at their disposal to accumulate wealth. For the 20th century the aggregate savings rate is predicted to stabilize at a plateau of about 17 percent, in line with the historical observation for the US (Maddison, 1992).

During the second Industrial Revolution the growth rate of GDP is predicted to largely surpass TFP growth, in line with the historical evidence for Britain (Crafts and Harley, 1992) and the US (Gordon, 1999). Dashed lines in the GDP panel show annual GDP growth rates per decade for the US computed from the Maddison (2003) data. During industrialization, according to the model, GDP growth is quite high relative to TFP growth because it is fueled by increasing labor supply and higher aggregate savings, which in turn are driven by a longer duration of active life. For the first half of the 20th century the model predicts declining GDP per worker (yet increasing GDP per working hour) due to the declining labor supply. For the second half of the 20th century labor supply has stabilized at a low level and GDP growth is first increasing mildly, fueled by increasing TFP growth, after which it stays (observationally) constant for the rest of the century.

4.4. **Re-interpretation: A Century of Work and Leisure.** So far we imagined the individual of our model as a male US American in order to match Hazan's (2009) data. The picture changes, however, if we imagine the individual as a representative (unisex) member of a household consisting actually of husband and wife. Francis and Ramey (2009) have demonstrated that the declining labor supply of US males over the last century has been accompanied by a rising labor supply of females. As a consequence, average labor supply of prime age adults displayed no discernable time trend. Average labor supply declined basically as a result of less labor supplied in youth and old age. While labor supply in youth was largely substituted by education, increasing leisure in old age was largely a result of increasing length of life.

Figure 5: A Century of Work and Leisure



Solid lines: model prediction, dashed lines: data from Hazan (2009) for life expectancy and education and from Ramey and Francis (2009) for hours worked and leisure. See text for details.

In order to take the alternative notion of "the individual" into account we recalibrate the model such that it approximates Ramey and Francis' (2009) data. For that purpose we keep all parameters from the benchmark run but set $\eta = 0.4$. Given the much lower labor supply elasticity, the response of labor supply to increasing active life expectancy turns out to be much smaller. Results are shown in Figure 5. For better comparison the upper two panels re-iterate results for life expectancy and education, which are the same as for the benchmark model. Only results on labor supply are affected. Dashed lines show the data from Francis and Ramey.

The third and forth panel show average labor supply per time increment of middle aged adults, that is l_{t+1} for the generation born in t, and average labor supply per time increment over the life cycle, $\hat{l}_t \equiv (1 - \epsilon_t + \tau_{1,t} l_{t+1})/(1 + \tau_{1,t} + \tau_{2,t})$ for the generation born at t. The variable l_{t+1} corresponds with average weekly hours of 25-54 year old persons in Ramey and Francis (2009) and the variable $\hat{l}$ corresponds with average life-long weekly hours of persons above age 14. For better comparison we have chosen the same scale for both panels and normalized the values obtained for 1900 to unity for both time series. The re-calibrated model matches the Ramey and Francis data reasonably well. In particular, there is little change of weekly labor supply of middle-aged adults while average weekly hours drop by about 20 percent during the course of the century because of less labor supply of the young and a longer life of the old.

The final panel shows total life–time leisure, computed as $\ell \equiv [\tau_{1,t}(1-l_{t+1})+\tau_{2,t}]/(1+\tau_{1,t}+\tau_{2,t})$ for the generation born at t. The prediction of the model is confronted with the Ramey and Francis data, both series are normalized to 100 in the year 1900. The model matches the 40 percent increase of total leisure during the last century reasonably well. In theory as well as in the data, this tremendous increase is largely explained by a longer life in old age.

5. Conclusion

In this paper we proposed a simple life cycle model that reconciles theory with the evidence on the historical evolution of life expectancy, education, and labor supply. In particular, our theory predicts that increasing life expectancy causes more education and, if the labor supply elasticity is sufficiently high, less life-time labor supply. The key mechanism is that increasing life expectancy is associated with increasing active life expectancy and that the active and healthy part of life increases (mildly) relative to the inactive and frail part. This entails an income and substitution effect. If labor supply is sufficiently elastic, the substitution effect dominates

and aggregate labor supply in middle age declines. The mechanism re-establishes increasing life expectancy as a driver of education and long-run growth. We have demonstrated the quantitative importance of this fact by a calibration with US Data. Increasing life expectancy explains the historical evolution of education, labor supply, and economic growth in the US since the 1830s reasonably well.

The notion of the "representative" individual in the model determines how strongly labor supply reacts to increasing life expectancy. A calibration with respect to males requires a relatively high elasticity of labor supply to match the historical data (as in Hazan, 2009). A calibration with respect to a unisex average member of a two sex household, requires a much lower supply elasticity to match the data (as in Ramey and Francis, 2009). The main point of the paper is thus of theoretical nature: no matter how the representative individual is conceptualized, observing simultaneously increasing education and declining labor supply does not contradict the life-cycle model and its prediction of increasing life expectancy as a powerful driver of education and economic development.

Appendix A. Derivation of the optimal values ϵ, l, s

These values are derived from the FOCs (5), (6), (7) as follows. Inserting both $u(x) = \log(x)$ and $v(x) = Bx^{\frac{1}{\eta}}$ yields

$$\frac{1}{1-\epsilon_t} = \frac{\beta\tau_1}{l_{t+1}h(\epsilon_t)w_{t+1} - s_{t+1}} h'(\epsilon_t) l_{t+1} w_{t+1} \tag{A.1}$$

$$\frac{h(\epsilon_t)w_{t+1}}{l_{t+1}h(\epsilon_t)w_{t+1} - s_{t+1}} = \frac{1}{\eta} \cdot B \cdot (l_{t+1})^{\frac{1-\eta}{\eta}} \tag{A.2}$$

$$\frac{\tau_1\beta}{l_{t+1}h(\epsilon_t)w_{t+1} - s_{t+1}} = \frac{\gamma\tau_2}{s_{t+1}}. \tag{A.3}$$

Rearranging equation (A.3) to obtain

$$\frac{1}{h(\epsilon_t)l_{t+1}w_{t+1} - s_{t+1}} = \frac{\gamma\tau_2}{s_{t+1}\tau_1\beta} \quad \Leftrightarrow \quad s_{t+1} = \frac{\gamma\tau_2 l_{t+1}h(\epsilon_t)w_{t+1}}{\beta\tau_1 + \gamma\tau_2} \tag{A.4}$$

and insert this into equation (A.1) yields

$$\begin{aligned}
&(1-\epsilon_t)\beta\tau_1 h'(\epsilon_t) l_{t+1} w_{t+1} = l_{t+1}h(\epsilon_t)w_{t+1} - s_{t+1} \\
\Leftrightarrow \quad &(1-\epsilon_t)\beta\tau_1 h'(\epsilon_t) l_{t+1} w_{t+1} = l_{t+1}h(\epsilon_t)w_{t+1} - \frac{\gamma\tau_2 l_{t+1}h(\epsilon_t)w_{t+1}}{\tau_1\beta + \gamma\tau_2} \\
\Leftrightarrow \quad &\epsilon_t = \frac{h(\epsilon_t)}{h'(\epsilon_t)} \frac{1}{\beta\tau_1 + \gamma\tau_2}.
\end{aligned}$$

Let $h(\epsilon_t) = \omega \cdot \exp(\theta\epsilon_t)$, then $\frac{h(\epsilon_t)}{h'(\epsilon_t)} = \theta$. Hence, the optimal schooling time is

$$\epsilon = \max\left\{0,\ 1 - \frac{1}{\gamma\theta\tau_2 + \beta\theta\tau_1}\right\}. \tag{A.5}$$

Rearranging equation (A.2) and using (A.4) yields

$$\frac{1}{l_{t+1}h(\epsilon_t)w_{t+1} - s_{t+1}} = \frac{\frac{1}{\eta}B\left(l_{t+1}\right)^{\frac{1-\eta}{\eta}}}{h(\epsilon_t)w_{t+1}} \tag{A.6}$$

$$\Leftrightarrow \quad h(\epsilon_t)w_{t+1} = \frac{1}{\eta}B\left(l_{t+1}\right)^{\frac{1-\eta}{\eta}} \left(l_{t+1}h(\epsilon_t)w_{t+1} - \frac{\gamma\tau_2 l_{t+1}h(\epsilon_t)w_{t+1}}{\tau_1 + \gamma\tau_2}\right) \tag{A.7}$$

$$\Leftrightarrow \quad l = \left(\frac{\beta\tau_1 + \gamma\tau_2}{\beta\tau_1 B^{\frac{1}{\eta}}}\right)^{\eta}. \tag{A.8}$$

The optimal savings rate follows by inserting the optimal schooling time ϵ from (A.5) and the optimal labor supply l from (A.8) into equation (A.4).

$$\tilde{s} = \frac{\gamma\tau_2 h(\epsilon) l}{\tau_1 + \gamma\tau_2} \tag{A.9}$$

Appendix B. Proof of the Propositions

Proof of Proposition 1. The partial derivatives of the optimal value ϵ, c.f. (A.5), are

$$\begin{aligned}\frac{\partial \epsilon}{\partial \tau_1} &= \frac{\beta}{\theta\left(\beta\tau_1+\gamma\tau_2\right)^2} &> 0\\ \frac{\partial \epsilon}{\partial \tau_2} &= \frac{\gamma}{\theta\left(\beta\tau_1+\gamma\tau_2\right)} &> 0\\ \frac{\partial \epsilon}{\partial \theta} &= \frac{1}{\theta^2\left(\beta\tau_1+\gamma\tau_2\right)} &> 0.\end{aligned}$$

□

Proof of Proposition 2. The partial derivatives of the optimal value l, c.f. (A.8), with respect to $\tau_1,\ \tau_2$ are

$$\begin{aligned}\frac{\partial l}{\partial \tau_1} &= -\frac{B\beta\gamma\tau_2\left(\frac{\eta(\beta\tau_1+\gamma\tau_2)}{B\beta\tau_1}\right)^{1+\eta}}{\left(\beta\tau_1+\gamma\tau_2\right)^2} &< 0\\ \frac{\partial l}{\partial \tau_2} &= \frac{\eta\gamma\left(\frac{\beta\tau_1+\gamma\tau_2}{B\beta\tau_1}\right)^{\eta}}{\beta\tau_1+\gamma\tau_2} &> 0\end{aligned}$$

□

Proof of Proposition 3. The partial derivative of the aggregate labor supply L with respect to τ_1 is

$$\frac{\partial L}{\partial \tau_1} = \frac{\left(\frac{\eta(\beta\tau_1+\gamma\tau_2)}{B\beta\tau_1}\right)^{\eta}\left(\beta\tau_1-\gamma(-1+\eta)\tau_2\right)}{\left(\beta\tau_1+\gamma\tau_2\right)}.$$

This leads to

$$\frac{\partial L}{\partial \tau_1} < 0 \ \Leftrightarrow\ \beta\tau_1-\gamma(-1+\eta)\tau_2 < 0 \ \Leftrightarrow\ \eta > \frac{\beta\tau_1+\gamma\tau_2}{\gamma\tau_2}.$$

□

Proof of Proposition 4. The first order condition for a minimum is

$$\begin{aligned}\frac{\partial L}{\partial \tau_1} = \frac{\left(\frac{\eta(\beta\tau_1+\gamma\tau_2)}{B\beta\tau_1}\right)^{\eta}\left(\beta\tau_1-\gamma(-1+\eta)\tau_2\right)}{\left(\beta\tau_1+\gamma\tau_2\right)} &\stackrel{!}{=} 0\\ \Leftrightarrow\quad \beta\tau_1-\gamma(-1+\eta)\tau_2 &= 0\\ \Leftrightarrow \tau_1^{min} &= \frac{(\eta-1)\gamma\tau_2}{\beta}.\end{aligned}$$

Since

$$\frac{\partial^2 L}{\partial \tau_1^2}|\tau_1^{min} = \frac{\beta\left(\frac{\eta^2}{B(\eta-1)}\right)^{\eta}}{\gamma\eta\tau_2} > 0 \quad \text{for} \quad \eta > 1,$$

τ_1^{min} is a minimum. □

Proof of Proposition 5. The partial derivatives of $\tilde{s}$, c.f. (A.9), with resprect to π_2, θ are

$$\begin{aligned}
\frac{\partial \tilde{s}}{\partial \tau_2} &= \frac{h(\epsilon_t) \cdot l \cdot \gamma \left(\beta\tau_1(-1+\beta\tau_1) + \beta\gamma(1+\eta)\tau_1\tau_2 + \gamma^2\eta\tau_2^2\right)}{\tau_1(\beta\tau_1+\gamma\eta\tau_2)^3} && > 0 \\
\frac{\partial \tilde{s}}{\partial \theta} &= \frac{h(\epsilon_t) \cdot l \cdot \gamma\tau_2}{\beta\tau_1+\gamma\tau_2} && > 0 \\
\frac{\partial \tilde{s}}{\partial \tau_1} &= -\frac{h(\epsilon_t) \cdot l \cdot \gamma\tau_2 \left(\beta\tau_1(-1+\beta\tau_1) + \beta\gamma(1+\eta)\tau_1\tau_2 + \gamma^2\eta\tau_2^2\right)}{\tau_1(\beta\tau_1+\gamma\tau_2)^3}.
\end{aligned}$$

The sign of the derivative $\frac{\partial \tilde{s}}{\partial \tau_1}$ is negative if and only if

$$\begin{aligned}
&\beta\tau_1(-1+\beta\tau_1) + \beta\gamma(1+\eta)\tau_1\tau_2 + \gamma^2\eta\tau_2^2 > 0 \\
\Leftrightarrow\ &\eta > \frac{\beta\tau_1\left(1-\beta\tau_1-\gamma\tau_2\right)}{\gamma\tau_2(\beta\tau_1+\gamma\tau_2)}.
\end{aligned}$$

The partial derivative of the aggregate savings rate $\widetilde{S}$ with respect to the period length τ_1 is

$$\frac{\partial \widetilde{S}}{\partial \tau_1} = -\frac{h(\epsilon_t) \cdot l \cdot \gamma\tau_2 \left(\gamma^2(-1+\eta)\tau_2^2 + \beta\tau_1(-1+\gamma(-1+\eta)\tau_2)\right)}{\left(\beta\tau_1+\gamma\tau_2\right)^3}.$$

The sign of this derivative is positive if and only if

$$\begin{aligned}
&\gamma^2(-1+\eta)\tau_2^2 + \beta\tau_1(-1+\gamma(-1+\eta)\tau_2) < 0 \\
\Leftrightarrow\ &\eta < \frac{\beta\tau_1 + \beta\gamma\tau_1\tau_2 + \gamma^2\tau_2^2}{\beta\gamma\tau_1\tau_2 + \gamma^2\tau_2^2}.
\end{aligned}$$

□

3. Proof that optimization yields a maximum

If the Hessian is negative definite in the critical point $(\epsilon,\ s,\ l)$, then this point is a maximum. The Hessian of U, c.f. (4), with $u(x) = \log(x)$ and $v(x) = B \cdot x^{\frac{1}{\eta}}$ is

$$H_U(\epsilon_t,\ l_{t+1},\ s_{t+1}) := \begin{pmatrix} \frac{\partial^2 U}{\partial s_{t+1}^2} & \frac{\partial^2 U}{\partial s_{t+1}\partial \epsilon_t} & \frac{\partial^2 U}{\partial s_{t+1}\partial l_{t+1}} \\ \frac{\partial^2 U}{\partial \epsilon_t \partial s_{t+1}} & \frac{\partial^2 U}{\partial \epsilon_t^2} & \frac{\partial^2 U}{\partial \epsilon_t \partial l_{t+1}} \\ \frac{\partial^2 U}{\partial l_{t+1}\partial s_{t+1}} & \frac{\partial^2 U}{\partial l_{t+1}\partial e_t} & \frac{\partial^2 U}{\partial l_{t+1}^2} \end{pmatrix} := \begin{pmatrix} H_{1,1} & H_{1,2} & H_{1,3} \\ H_{2,1} & H_{2,2} & H_{2,3} \\ H_{3,1} & H_{3,2} & H_{3,3} \end{pmatrix} \tag{A.10}$$

with

$$\begin{aligned}
H_{1,1} &= -\frac{\beta\tau_1}{(s_{t+1} - h(\epsilon_t) l_{t+1} w_{t+1})^2} - \frac{\gamma\tau_2}{s_{t+1}^2}, \\
H_{1,2} &= \frac{h(\epsilon_t) l_{t+1} w_{t+1} \beta\theta\tau_1}{(s_{t+1} - h(\epsilon_t) l_{t+1} w_{t+1})^2}, \\
H_{1,3} &= \frac{h(\epsilon_t) w_{t+1} \beta\tau_1}{(s_{t+1} - h(\epsilon_t) l_{t+1} w_{t+1})^2}, \\
H_{2,1} &= H_{1,2}, \\
H_{2,2} &= -\frac{1}{(1-\epsilon_t)^2} - \frac{h(\epsilon_t) l_{t+1} s_{t+1} w_{t+1} \beta\theta^2\tau_1}{(s_{t+1} - h(\epsilon_t) l_{t+1} w_{t+1})^2}, \\
H_{2,3} &= -\frac{h(\epsilon_t) s_{t+1} w_{t+1} \beta\theta\tau_1}{(s_{t+1} - h(\epsilon_t) l_{t+1} w_{t+1})^2}, \\
H_{3,1} &= H_{1,3}, \\
H_{3,2} &= H_{2,3}, \\
H_{3,3} &= \beta\tau_1 \left(-\frac{1}{\left(l_{t+1} - \frac{s_{t+1}}{h(\epsilon_{t+1}) w_{t+1}} \right)^2} + \frac{B(-1+\eta) l_{t+1}^{-2+\frac{1}{\eta}}}{\eta^2} \right).
\end{aligned}$$

The three principal minors of H_U evaluated at the critical point $(\epsilon,\ s,\ l)$ are

$$\begin{aligned}
H_{U,1} &= -\frac{\beta\tau_1}{(s_{t+1} - h(\epsilon_t) l_{t+1} w_{t+1})^2} - \frac{\gamma\tau_2}{s_{t+1}^2}, \\
H_{U,2} &= \frac{B \exp\left[-2\theta + \frac{2}{\beta\tau_1 + \gamma\tau_2} \right] \theta^2 (\beta\tau_1 + \gamma\tau_2)^4 \left(\frac{\eta(\beta\tau_1 + \gamma\tau_2)}{B\beta\tau_1} \right)^{1-2\eta}}{w_{t+1}^2 \gamma\eta\tau_2}, \\
H_{U,3} &= -\frac{B \exp\left[-2\theta + \frac{2}{\beta\tau_1 + \gamma\tau_2} \right] \theta^2 (\beta\tau_1 + \gamma\tau_2)^4 \left(\frac{\eta(\beta\tau_1 + \gamma\tau_2)}{B\beta\tau_1} \right)^{1-4\eta} \left(\gamma\eta^2\tau_2 + \beta\tau_1 \left(\eta^2 - (\eta - 1) \left(\frac{\eta(\beta\tau_1 + \gamma\tau_2)}{\beta\tau_1} \right) \right) \right)}{w_{t+1}^2 \gamma\eta^3\tau_2}.
\end{aligned}$$

It is well known that H_U is negative definite if $H_{U,1} < 0,\ H_{U,2} > 0$ and $H_{U,3} < 0$. Obviously $H_{U,1} < 0,\ H_{U,2} > 0$ and

$$H_{U,3} < 0 \ \Leftrightarrow\ \gamma\eta^2\tau_2 + \beta\tau_1 \left(\eta^2 - (\eta - 1) \left(\frac{\eta(\beta\tau_1 + \gamma\tau_2)}{\beta\tau_1} \right) \right) > 0 \ \Leftrightarrow \frac{1}{\eta} > 0.$$

Hence, H_U is negative definite and U is maximized in the critical point $(\epsilon,\ s,\ l)$.

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