MEASURING COMPETITIVENESS *,†

J. Peter Neary
University College Dublin and CEPR
November 11, 2005

Abstract

This paper reviews alternative approaches to measuring an economy’s cost competitiveness and proposes some new measures derived from the economic theory of index numbers. The indices provide a theoretical benchmark for estimated real effective exchange rates, but differ from standard measures in that they are based on marginal rather than average sectoral shares in GDP or employment. The use of the new indices is illustrated by some simple calculations which highlight the potential exposure of the Irish economy to fluctuations in the euro-sterling exchange rate.

JEL: F40, C43

Keywords: competitiveness, economic theory of index numbers, European Monetary Union (EMU); real effective exchange rates (REERs).

---

*Address for Correspondence: UCD School of Economics, University College Dublin, Belfield, Dublin 4, Ireland; tel.: (+353) 1-716 8334; fax: (+353) 1-283 0068; e-mail: peter.neary@ucd.ie.

†I am very grateful to participants at the conference Macroeconomic Perspectives in honour of Brendan M. Walsh held at University College Dublin, 7 October 2005, at a meeting of the Dublin Economics Workshop in UCD and at an IMF conference on Trade and Developing Countries for comments and to Jim Anderson, John FitzGerald, Patrick Honohan and Brendan Walsh for helpful discussions. This research is part of the International Trade and Investment Programme of the Geary Institute at UCD.
1 Introduction

Competitiveness is an elusive concept, much studied by business theorists and much invoked by politicians and commentators, but frequently dismissed as irrelevant or unimportant by economists. Krugman (1994) famously called it a “dangerous obsession” in his critique of the first Clinton administration’s flirtation with industrial policy. By contrast, Michael Porter of Harvard Business School has highlighted “competitive advantage” as the key to superior performance by firms, industries and economies as a whole. (See Porter (1990).) In part through his influence, many agencies now monitor national competitiveness, ranging from the World Economic Forum, which publishes an annual Global Competitiveness Report, to national bodies such as the U.S. Council on Competitiveness (www.compete.org) and the Irish National Competitiveness Council (www.forfas.ie/ncc). These have produced much useful data and a great deal of helpful commentary, but mainstream economic theorists have for the most part paid little attention.\footnote{For an attempt to rationalise competitive advantage in a model of oligopolistic competition in general equilibrium, see Neary (2003).}

Applied economists on the other hand have been only too aware of the importance of competitiveness as a determinant of macroeconomic performance, though they have tended to focus on the narrower concept of relative cost competitiveness. In Brendan Walsh’s work on the Irish economy this has been a recurring theme, extending from his membership of the 1981 Committee on Costs and Competitiveness, popularly known as the “Three Wise Men” (Baker, McAleese and Walsh, 1981), to his dissections of the spectacular growth performance of the “Celtic Tiger” or “Irish Hare” era (Walsh 2000, Honohan and Walsh 2002). Elsewhere the problems of measuring cost competitiveness have been extensively considered in central banks and especially in the International Monetary Fund, from the seminal work of Hirsch and Higgins (1970), Artus and Rhomberg (1973) and Rhomberg (1976), to more recent applications by Lipschitz and McDonald (1992) and Marsh and Tokarick (1996). A paper in this tradition of particular interest in the Irish context is that by Cerra, Soikkeli and Saxena (2003), who study the competitiveness of Irish manufacturing in the 1990s, and interpret periods when the employment-weighted real effective exchange rate (REER) depreciated by less than the output-weighted REER as implying that employment-intensive firms and sectors faced greater competitive pressure than the economy as a whole.

Despite the amount of work that has been done on the topic, there is no clear consensus on how to measure cost competitiveness. The theme of the present paper is that the ambiguities and controversies surrounding this issue can be illuminated, if not resolved, by applying some of the principles of the economic theory of index numbers.\footnote{See Diewert (1981) for an introduction.} In Section 2 I review some of these principles in a relatively well-known context, that of measuring the “true” cost of living and the “true” rate of inflation. I then apply them in Section 3 ...
to derive two true real effective exchange rate (REER) indices, using the same approach adopted in earlier work on true indices of trade policy restrictiveness and on international comparisons of real income.\footnote{See Anderson and Neary (1996 and 2005) and Neary (2004) respectively.} Section 4 presents an application of these indices in the context of a highly stylized model of the Irish economy, and uses it to throw light on an old policy debate concerning the wisdom of Ireland’s joining the European Monetary System (EMU) while the U.K. remained outside. Section 5 concludes.

2 Measuring the Cost of Living

The economic theory of index numbers proceeds first by requiring that an economically meaningful index number should provide an answer to some question of interest. Merely statistical indices, by contrast, provide only “answers without questions” in the words of Afriat (1977). Second, most interesting questions require taking account of behavioural responses, and this frequently implies allowing for substitution by economic agents in response to relative price changes. In this section I introduce these concepts in the context of measuring the cost of living, in order to set the scene for the later discussion of measuring competitiveness.

The starting point for measuring the price level is the consumer price index, usually taken as the ratio of the cost of purchasing a given basket of commodities at two sets of prices, the “old” or period-0 prices \( p^0 \) and the “new” or period-1 prices \( p^1 \):

\[
I(p^0, p^1, x) = \frac{\sum_i p^1_i x_i}{\sum_i p^0_i x_i} \tag{1}
\]

In practice the basket in question is often taken to be that in period 0, \( x = x^0 \), which yields the Laspeyres price index (as opposed to the Paasche index when the basket is that of period 1). This index is intended to measure the cost of living in period 1 relative to that in period 0. But whose life is it anyway? And would we not expect rational consumers to change their consumption patterns in response to the change in prices?

These objections are overcome by the “true” cost of living index due to Konüs. Unlike the consumer price index, which measures the relative cost of consuming the same commodity bundle in period 1 as in period 0, the Konüs index measures the relative cost of attaining the same level of utility.\footnote{As with the consumer price index in (1), the reference utility level is often taken to be that in period 0, \( u = u^0 \), which yields the Laspeyres-Konüs index (as opposed to the Paasche-Konüs index when the utility level is that of period 1). These distinctions are unnecessary in the case of homothetic tastes, when \( e(p, u) = u e(p) \), and so the Konüs index is independent of the choice of reference utility level.} The cost of utility, facing given prices, is measured in turn by the expenditure function \( e(p, u) \):

\[
\kappa(p^0, p^1, u) = \frac{e(p^1, u)}{e(p^0, u)} \tag{2}
\]
Like the numerator and denominator in (1), the expenditure function is also a sum of expenditures on individual goods, but now the quantities are those that would be chosen optimally by a consumer facing the appropriate prices and wishing to attain the given level of utility: \( e(p, u) = \sum_i p_i x_i(p, u) \). Thus the Konüs index allows for intercommodity substitution by the consumer in response to price changes. It also answers a well-defined and economically interesting question: “How much would it cost the reference consumer to be as well-off at the new prices as at the old?”, where the reference consumer is the one whose tastes correspond to the particular expenditure function \( e(p, u) \).

The superiority in principle of the Konüs index over the consumer price index is now widely accepted by economists. Nevertheless it is useful to rehearse some of the possible objections to it, since similar objections may be made to the measures of true competitiveness which I propose in the next section.

First is the argument that the Konüs index requires specifying an explicit behavioural model, whereas the consumer price index is apparently model-free. I say “apparently” advisedly, however, for it is well-known that the empirical index can only be rationalized in terms of an implicit model with a Leontief or fixed coefficients utility function, implying that the reference consumer does not adjust their consumption bundle at all in response to relative price changes.

A more subtle objection is that the Konüs index is conceptually redundant, because, if the true model is known, then it could be used to calculate directly the change in utility or welfare between the two periods, which is the only variable of intrinsic economic interest. This is strictly correct, but it ignores the enormous informational economy which a price index confers, and the fact that the true model can never be known with certainty. Even if the true model were known exactly, the Konüs index would still be a useful summary measure. More generally it provides a benchmark against which empirical indices can be evaluated.

A third objection is that in practice distinctions between index numbers are empirically unimportant, since all reasonable empirical indices (the values of which bound the true index) behave very similarly, often with correlation coefficients close to unity. It is true that, to a first order approximation, the change in the Konüs index is the same as that in the consumer price index. Differentiating (2) yields:

\[
\hat{\kappa} = \sum_i \omega_i \hat{p}_i \quad \text{where:} \quad \omega_i = \frac{p_i x_i}{\sum_h p_h x_h}
\]

(A circumflex over a variable denotes a proportional change.) This is a weighted average of price changes, where for small changes the weights \( \omega_i \) are the shares of each good in total expenditure in period 0. However, for large price changes the change in the consumer price index is not a good approximation to the change in the Konüs index. When the effects of small differences between the two are cumulated over time, they can make a large difference for many policy questions. The most famous demonstration of this is the report
of the U.S. Boskin Commission (1996). It showed that the U.S. consumer price index overestimated the true change in the cost of living by 1.1% per year, of which 0.4% was due to the kind of substitution bias discussed here, and that this in turn implied increased spending of billions of dollars on index-linked social security payments.

A final objection is that the true cost-of-living index is unobservable, and no estimate of it commanding universal support is likely to be available. However, direct estimation of the Konüs index is only one empirical strategy available. Alternatives are to try and place bounds on the true index, or to seek empirical indices other than the Laspeyres index which approximate it better under specific assumptions. All these methods are sensitive to model specification, and in practice scarcity of data and/or research time may leave no alternative to using the consumer price index. Nevertheless, the Konüs index provides a theoretical ideal against which the performance of empirical indices such as the consumer price index should be evaluated.

3 True Real Effective Exchange Rates

Consider next empirical measures of competitiveness or the real effective exchange rate. The standard approach to calculating them is to take a weighted average of bilateral competitiveness indices between the home country (indexed “H”) and its main trading partners, where the weights are export shares. Bilateral competitiveness in turn can be measured either by relative consumer prices, relative export unit values, or relative unit labour costs (wages corrected for labour productivity). Cerra, Soikkeli and Saxena (2003) survey the literature and conclude that the latter is the best single indicator, since it covers a greater share of trade than consumer prices and avoids the measurement and endogeneity problems associated with export unit values. This yields the following expression:

\[ \varepsilon = \sum_j \theta^j e^j \frac{ULC_H}{ULC_j} \]  \hspace{1cm} (4)

Here \( \theta^j \) is the share of country \( j \) in country H’s exports, and \( e^j \) is the bilateral nominal exchange rate (the home price of a unit of foreign currency). In the context of Ireland in the 1990s, Honohan and Walsh (2003) argue that economy-wide measures of unit labour costs are seriously misleading, because the average is increased by the dramatic shift in sectoral composition towards less labour-intensive sectors, which would have raised measured competitiveness even if marginal or average labor productivity had not changed in any sector. They propose instead using the differential change in wage rates detrended by a constant, assuming a constant rate of differential marginal productivity growth between Ireland and its trading partners. Cerra,

---

5 For examples of these approaches see Blundell, Browning and Crawford (2003) and Diewert (1976) respectively. The latter approach was used by the Boskin Commission.
Soikkeli and Saxena adopt a different approach to the same problem. They point out that industries may differ in their exposure to competition from foreign countries, so they propose instead of (4) the following:

\[
\varepsilon = \sum_i \omega_i \sum_j \theta_i^j e^{ULC_i^H \over ULC_i^j} \tag{5}
\]

This version of the REER index allows both export shares and unit labour costs to vary across industries. As for the industry weights \( \omega_i \), they may equal the share of industry \( i \) in either total output or employment. Cerra, Soikkeli and Saxena show that the growth in competitiveness of Irish industry in the second half of the 1990s was much less impressive when employment shares are used instead of output shares.

While these indices have a clear intuitive appeal, they lack a firm conceptual foundation. I propose instead starting with a specific question, or rather two specific questions, which seem to be implicit in many discussions of the REER: “Given a set of arbitrary changes in external prices or domestic costs, what change in the nominal exchange rate would restore the initial level of output or employment?” This is clearly an index number problem: it seeks a single index, corresponding to a uniform change in all traded goods prices, which would compensate for a given set of typically non-uniform changes in prices or wages. To operationalise it we need to specify a model of the production sector, and in this paper I take a very simple reduced form approach to doing so, drawing on the duality approach to international trade. (See for example Dixit and Norman (1980).) Hence, assume that the equilibrium level of nominal GDP can be written as a function of an exogenous vector of external prices \( p \) and an exogenous nominal wage rate \( w \): \( G(p, w) \). (Both \( p \) and \( w \) are measured in domestic units.) Implicit in this formulation is the institutional structure of the economy, the degree of intersectoral factor mobility and the level of technology.

We can now define a true GDP-neutral REER index as follows. Suppose that prices and wages are initially equal to \( (p^0, w^0) \) and then change to \( (p^1, w^1) \). The true GDP-neutral REER index is the scalar deflator \( \varepsilon^G \) which when applied to all prices would restore the initial level of GDP:

\[
\varepsilon^G: \quad G\left(p^1/\varepsilon^G, w^1\right) = G\left(p^0, w^0\right) \tag{6}
\]

To see how this index can be interpreted, consider two particular types of change between periods 0 and 1. If the wage does not change but external prices rise, then \( G \) increases. A value of \( \varepsilon^G \) greater than one would be needed to restore the initial level of GDP, indicating that competitiveness has increased. Alternatively, if external prices do not change but wages increase, then \( G \) falls. Now \( \varepsilon^G \) is less than one indicating a fall in competitiveness.

Alternatively, if we want to focus on total employment instead of GDP, we can write its equilibrium
level as a function of the same exogenous variables as $G$, $l(p, w)$, and, by analogy with (6), define a true employment-neutral REER index as the scalar deflator $\varepsilon^L$ which when applied to all prices would restore the initial level of employment:

$$\varepsilon^L : \quad l\left(\frac{p^1}{\varepsilon^L}, w^0\right) = l\left(p^0, w^0\right)$$

(7)

The interpretation of $\varepsilon^L$ is similar to that of $\varepsilon^G$. In addition, we can use the fact that $l$ is homogeneous of degree zero in all nominal variables (i.e., total employment is unaffected by a uniform change in all prices and wages) to rewrite (7) as:

$$\varepsilon^L : \quad l\left(p^1, \varepsilon^L w^1\right) = l\left(p^0, w^0\right)$$

(8)

From an ex ante perspective (so $w^1 = w^0$), $\varepsilon^L$ in the second expression gives the solution to the “Three Wise Men” problem: what is the permissible change in wages which would compensate for an anticipated change in external prices from $p^0$ to $p^1$? (In practical applications, allowance might also be made for labour-augmenting technical progress between the two periods, but for simplicity I ignore this.) In the remainder of this section I discuss the properties of these indices, paralleling the discussion of the Konüs true cost-of-living index in the previous section.

Just as the Konüs index requires assuming explicit tastes for the reference consumer, so the true competitiveness indices require that an explicit model of the economy be specified. The particular forms specified above, $G(p, w)$ and $l(p, w)$, can be rationalized in a number of ways. One which is explored further in the Appendix, is a perfectly competitive price-taking economy. This is consistent with the empirical REER in equation (4), provided we can match sectors with specific trading partners and hence with specific bilateral exchange rates. The index could also be rationalised in terms of a monopolistically competitive production sector as in Obstfeld and Rogoff (1995) and Lane (2001). This would be consistent with the empirical REER in equation (5). But as we shall see, the true REER indices differ in all cases from the empirical indices in (4) and (5).

One clear advantage of making explicit the underlying model of the economy is that it underlines the importance of taking account of developments in import-competing sectors as well as in exporting sectors in evaluating competitiveness. Both (6) and (7) imply that the true competitiveness indices depend on the prices of all traded goods, not just on those of exports. This makes sense: if we are concerned with the effects of changes in competitiveness because of their implications for output or employment, then we should pay equal attention to all sectors exposed to foreign competition, whether they export or not. Of course, if we want to explain export performance specifically, as in Carlin, Glyn and van Reenan (2001) for example, then a different measure of competitiveness would be appropriate, and the prices of import-competing final
goods would not be directly relevant. This underlines the general point that the appropriate measure of competitiveness depends on the question we wish to answer.

The GDP and employment functions used in defining the true REER indices above have a distinctly Keynesian, even “old Keynesian,” flavour, since the wage rate is assumed to be determined outside the production sector. However, they could be embedded in a more comprehensive model which endogenises nominal wage determination in a variety of frameworks: wage indexation, staggered wage setting, firm-union bargaining or efficiency wages. An exception is a competitive labour market, since if wages are perfectly flexible then cost competitiveness is endogenously determined and there is no reason for policy makers to take an interest in it.

In practice, analysts and commentators are likely to be more interested in changes in competitiveness rather than in its level. For small changes in prices, the change in the true GDP-neutral REER index is given by:

\[ \varepsilon^G = \sum_i \omega_i \hat{p}_i \quad \text{where:} \quad \omega_i = \frac{\frac{\partial G}{\partial p_i} p_i}{\sum_h \frac{\partial G}{\partial p_h} p_h} \]  

(9)

Once again, this is a weighted average of price changes. However, unlike the Konüs index, the weights depend on marginal rather than average output shares. This highlights a key deficiency of empirical indices such as (4) and (5). To the extent that they purport to capture the implications of external price changes for output or employment, they fail to allow for differential supply responses across sectors. At best they can be interpreted as implying that all sectors have the same output supply elasticity, in which case the weights in (9) reduce to average shares in GDP. The same point applies to the true employment-neutral REER index. Similar derivations show that the change in \( \varepsilon^L \) is:

\[ \varepsilon^L = \sum_i \omega_i \hat{p}_i \quad \text{where:} \quad \omega_i = \frac{\frac{\partial l}{\partial p_i} p_i}{\sum_h \frac{\partial l}{\partial p_h} p_h} \]  

(10)

which in the same way depends on marginal rather than average employment shares.

4 An Application: The Irish Economy’s Exposure to Sterling

To illustrate how the true indices introduced in the last section can be applied, consider their relevance to the debate which took place in Ireland in 1996 concerning the advisability of Ireland’s entering European Monetary Union (EMU) given that its main trading partner, the U.K., had chosen to remain outside. A variety of arguments was brought to bear on both sides. (See, for example, Baker, FitzGerald and Honohan

6It might be thought that the derivatives \( \frac{\partial G}{\partial p_i} \) should equal sectoral outputs by Hotelling’s Lemma. However, as shown in the Appendix, the \( G \) function is not a maximal value function, and so the Lemma does not apply.
(1996), Barry (1997) and Neary and Thom (1997).) For present purposes, the most relevant aspect of the
debate concerned the likely consequences for Irish output and employment of a sharp depreciation of sterling
against the euro. On the one hand it was pointed out that the UK share of Ireland’s trade had fallen steadily
in previous decades, and was much less than 50%. On the other hand, fears were expressed that Irish firms
exposed to competition from UK firms were disproportionally smaller, more labour-intensive and competing
primarily on price rather than on product quality or as part of the integrated manufacturing processes of
multinational corporations based in Ireland. (See Barry (1999) and Cerra, Soikkeli and Saxena (2003) for
background on the evolution of Irish manufacturing structure in the last decades of the twentieth century.)

In the event the debate between economists was redundant both ex ante and ex post, since the decision to
join EMU in any case was taken largely on political and strategic grounds, and since the feared depreciation of
sterling against the euro never materialized. (Though commentators such as Cassidy and O’Brien (2005) and
Cerra, Soikkeli and Saxena (2003) continue to express concern about its likely consequences.) Nevertheless,
it provides a context in which one potential use for the indices introduced earlier can be illustrated.

In order to operationalise the true indices introduced in the previous section, I assume that the production
sector has a relatively simple structure, a multi-sector version of the specific-factors model of Jones (1971).
In this case, the GDP function takes a particularly simple form: the output of each sector depends only on
the price of that sector’s output and on the wage:

$$ G(p, w) = \sum_i p_i y_i(p_i, w) $$

(11)

As shown in the Appendix, the weights in the expression for the change in the true GNP-neutral REER,
equation (9), can now be written as follows:

$$ \omega_i = \frac{\beta_i \phi_i}{\sum_h \beta_h \phi_h} \quad \text{where} \quad \beta_i = \frac{p_i y_i}{y} \quad \text{and} \quad \phi_i = \frac{p_i}{y_i} \frac{\partial y_i}{\partial p_i} $$

(12)

Here $\beta_i$ is the share of sector $i$ in GDP and $\phi_i$ is its price elasticity of supply. Hence, to the extent that
sectors differ in their supply responses, the use of average GDP weights will give a misleading picture of the
vulnerability of output to external price changes.

Analogous arguments apply to the employment-neutral REER index, which is probably preferable in the
Irish case, because Ireland’s GDP figures are highly inflated by transfer pricing. (See, for example, Honohan
and Walsh (2002).) For this index, we can make use of the fact that the level of employment in each sector
depends on the price of that sector’s output and on the wage:
\[ l(p, w) = \sum_i l_i(p_i, w) \]  

Similar derivations to the GDP-neutral case show that the weights in the expression for \( \varepsilon^L \), equation (10), are given by:

\[ \omega_i = \frac{\lambda_i\eta_i}{\sum_h \lambda_h\eta_h} \quad \text{where} \quad \lambda_i = \frac{l_i}{\bar{l}} \quad \text{and} \quad \eta_i = \frac{p_i}{l_i} \frac{\partial l_i}{\partial p_i} = -\frac{w}{l_i} \frac{\partial l_i}{\partial w} \]  

Here \( \lambda_i \) is the share of sector \( i \) in total employment and \( \eta_i \) is its price elasticity of labour demand (equal by homogeneity to the absolute value of its wage elasticity of labour demand). Once again therefore, if sectors differ in their labour demand elasticities, weighting by average employment shares gives a misleading picture of the vulnerability of total employment to external price changes.

To see how this effect can matter with reasonable parameter values, I give some numerical calculations for a two-sector special case, which can be thought of as a back-of-the-envelope model of the production side of the Irish economy.\(^7\) Let sector 1 be the “sterling-exposed” sector, assumed to be labour-intensive both on average and at the margin. Firms in sector 1 are assumed to compete primarily on price against foreign (i.e., U.K.) firms, mainly on the home and U.K. markets. By contrast, firms in sector 2 compete primarily on product quality with non-U.K. firms in third markets, and so their output supply is less responsive to price and their labour demand is less responsive to wages. We can parameterize the difference between the two sectors in terms of the ratio of their supply elasticities \( \alpha \), where \( \phi_1 = \alpha \phi_2 \), with \( \alpha \geq 1 \).\(^8\) This implies that the true marginal weight of sector 1 in the GDP-neutral REER index is related to its output share as follows:

\[ \omega_1 = \frac{\alpha \beta_1}{\alpha \beta_1 + 1 - \beta_1} \]  

This shows that \( \omega_1 \) equals \( \beta_1 \) when the two sectors have the same price elasticity of supply (\( \alpha = 1 \)). However, when sector 1 is more price-elastic, its true marginal weight exceeds its average weight. The true weight is strictly increasing in \( \alpha \), even when \( \alpha \) equals one.\(^9\) The relationship between \( \omega_1 \) and \( \beta_1 \) is illustrated in Figure 1 for different values of \( \alpha \). It is clear that the true weight can easily be twice as much as the average share, implying that standard measures of competitiveness greatly underestimate the vulnerability of national output and employment to a change in the bilateral nominal exchange rate with sterling.

\(^7\)Barry (1997) calibrates a similar model, augmented by a third sector producing a non-traded good.

\(^8\)Because of the formal similarity between (12) and (14), all the discussion in the remainder of this section, including Figure 1, can be reinterpreted in terms of the true weights in the employment-neutral REER index, with \( \alpha \) measuring relative wage-labour demand elasticities \( \eta_1/\eta_2 \) instead of relative price-output elasticities \( \phi_1/\phi_2 \), and with \( \lambda_1 \) replacing \( \beta_1 \).

\(^9\)\( \partial \omega_1/\partial \alpha \) equals \( \beta_1 (1 - \beta_1)/D^2 \), where \( D \) denotes the denominator of \( \omega_1 \), which equals one when \( \alpha \) equals one. Evaluated at \( \alpha = 1 \), this derivative attains its maximum value of 0.25 when \( \beta_1 \) equals one half.
5 Conclusion

This paper has reviewed alternative approaches to measuring an economy’s cost competitiveness and proposed some new measures derived from the theory of index numbers. It was shown that the indices provide a theoretical benchmark for estimated real effective exchange rates, but differ from standard measures in that they are based on marginal rather than on average sectoral shares in GDP or employment. Finally, the use of the new indices was illustrated by some simple calculations which highlight the potential exposure of the Irish economy to fluctuations in the euro-sterling exchange rate.

The desirability of basing index numbers of competitiveness on an explicit model of the economy can be traced back at least to Artus and Rhomberg (1973), who calculated real effective exchange rates at the margin using the IMF’s “MERM” (“Multilateral Exchange Rate Model”) econometric model. However, the explicit relationship between this approach and the economic theory of index numbers does not appear to have been highlighted before. Moreover, as large-scale econometric models have gone out of fashion, most users of such indices have fallen back on atheoretic methods. As noted at the end of Section 2, there are many ways in which the approach of this paper could be applied, some involving explicit modelling and others not. There are also many practical issues of implementation which have not been considered here.\footnote{See Chinn (2005) for an overview.}

At the very least the paper has hopefully laid the foundations for bridge-building between the economic theory of index numbers and the careful insightful applied work which Brendan Walsh has exemplified.
6 Appendix

6.1 Perfectly Competitive Foundations for the True Indices

The particular form of the general production model $G(p, w)$ used here is that of a price-taking small open economy as in Dixit and Norman (1980), augmented to allow for a wage rate determined outside the production sector as in Neary (1985). The model can also be interpreted as one with non-traded goods in the background, as in Neary (1988), in which case the real exchange rate can alternatively be measured by the relative price of non-traded and traded goods rather than the relative price of home and domestic output as here.

With flexible wages, the equilibrium level of GDP is the maximum value of output conditional only on exogenous prices, technology and factor endowments:

$$g(p, l) = \max_y [p'y : F(y, l) \leq 0]$$

(16)

where $F(y, l)$ is the aggregate production constraint linking the vector of outputs $y$ to the labour endowment $l$. If the wage is exogenous, the economy maximises instead the return to fixed factors, giving rise to a constrained GDP function:

$$\tilde{g}(p, w) = \max_{y,l} [p'y - wl : F(y, l) \leq 0]$$

(17)

Minus the wage derivative of this function gives the equilibrium level of employment:

$$l(p, w) = -\tilde{g}_w(p, w)$$

(18)

Finally, in the constrained economy, the actual level of GDP equals the returns to fixed factors and to labour:

$$G(p, w) = g[p, l(p, w)] = \tilde{g}(p, w) + wl(p, w)$$

(19)

Whereas $\tilde{g}$ is a maximal value function, $G$ is not. Hence as noted in the text its derivatives with respect to $p$ (unlike those of $g$ and $\tilde{g}$) do not equal the levels of outputs but reflect instead marginal responses:

$$G_p = \tilde{g}_p + wl_p$$

(20)

For this reason, the weights in the expression for the change in the true GDP-neutral REER index in (9) depend on marginal rather than on average output shares.
6.2 Derivation of Equations (9) and (12)

Finally we sketch the derivation of the expressions for changes in the true GDP-neutral REER quoted in the text. Similar steps yield the corresponding properties of the true employment-neutral REER index. To derive equation (9), totally differentiate (6) with $\rho^0$ and $w^0$ constant, which yields:

$$\sum_i p_i \frac{\partial G}{\partial p_i} \left[ \frac{1}{\varepsilon^G} dp_i - \left( \frac{1}{\varepsilon^G} \right)^2 p_i d\varepsilon^G \right] = 0 \quad (21)$$

Rearranging yields equation (9). In the specific-factors case, differentiating the GDP function (11) yields:

$$\frac{\partial G}{\partial p_i} = p_i \frac{\partial y_i}{\partial p_i} = \frac{y_i}{p_i} \frac{\partial y_i}{\partial y_i} \frac{\partial y_i}{\partial p_i} = \frac{y_i}{p_i} \beta_i \phi_i \quad (22)$$

Substituting into (9) yields the expression for the weights given by (12).

12


Fig. 1: Weight of Sterling in True Competitiveness Index

($\beta_1$ is the Share of Sterling-Exposed Sectors in GDP)