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**A Dynamic Model of Cross Licensing**

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# A dynamic model of cross licensing

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## Abstract

In sectors with cumulative and complementary technologies, some firms build patent portfolios in order to block their competitors' access to the technology and/or to negotiate cross licensing agreements. We propose a dynamic model that captures this behaviour in an integrated duopoly where the firms invest successively in upstream patentable technologies and downstream marketable products. We study the impact of legal patent strength on competition and investment. We then consider two alternative settings. One where the firms cross license or pool their patents and another where the patent strength is restricted. We verify whether and when such alternatives are socially efficient.

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# 1 Introduction

In industries where innovations are cumulative and complementary, a patent on an early innovation grants its owner more than a mere protection against copies or illegal use of its innovation. A patent owner may be in a position to block the access of other innovations to market, and thereby to gather a revenue from licensing agreements. Indeed, a patent allows its owner to legally claim a revenue from all innovations deemed as infringing. The legal strength of patents has led firms in many industries to build up large patent portfolios. They do so to protect their innovations but can also use these to free ride on future discoveries.

In this paper we focus at blocking patents and consider their implications for upstream and downstream investments. We define blocking patents as instruments that not only protects an innovation against copies but also gives its owner the right to sue other innovators for infringement so as to claim part of the surplus they create. We contrast our findings with two different settings. First, we examine the possibility for the firms to use cross licensing agreements. By doing so they also commit not to sue each other later on. Second, we consider the possibility for a regulatory agency to introduce restricted patents. These do not allow firms holding a valid infringement claim to escape competition downstream. Moreover, the regulator may also restrict the royalty charged to an infringing firm. Such patents come close, yet as a milder version, to the concept of compulsory licensing which is enforced in some industries.

The next section gives a summary of the literature on blocking patents and cross licensing. It allows us to situate this paper within the literature. We especially show how the way we capture the legal concept of essentiality articulates with the economic concept of complementarity. Section 3 presents the model which outlines a very simple setting where 2 firms may invest in sequential innovations. The upstream innovation is patentable. It has no stand alone value but is necessary to develop subsequent innovations. Each patent is characterized by its legal strength which reflects its ability to validate an infringement claim. If both firms succeed upstream they can either cross licence their technologies and pay each other a royalty that maximizes joint profits, or pursue investment downstream without signing any agreements.

In section 4 we focus at the downstream investments considering unrestricted, restricted patents and cross licensing. We show that these investments are sub-optimal for any positive level of legal patent strength and for any positive agreed upon royalty. The unique possibility to restore efficient investments is to impose *free* compulsory licensing. In section 5 we then consider the possibility for the firms to cross licence their technologies and show that they do so for two very distinct motives. One only is pro-competitive as it aims at pooling highly blocking patents. The other is a way for the firms to save on otherwise high investments and to escape competition.

In section 6 we consider upstream investments and total welfare. We show that cross licensing cannot restore efficiency as firms will always agree on a positive royalty which then distorts investments. Findings in this section also

corroborate the fact that cross licensing agreements are welfare improving when and only when patents are legally strong and thus most likely to be blocking. This result goes along the lines of Shapiro (2001) and Lerner and Tirole (2004) who state that it is optimal to pool patents that exhibit a strong complementarity.

## 2 Blocking patents and cross licensing.

Blocking patents have been mostly analyzed in terms of sequential innovations, where the owner of one upstream patent may hold up a subsequent innovator (Scotchmer (1991); Green & Scotchmer (1995) and Denicolò (2000)). This setting has also been generalized to a chain of cumulative innovators in several articles (Hunt (1995); O'Donoghue (1998); O'Donoghue, Scotchmer and Thisse (1998); Bessen & Maskin (2000)). An important result of this literature is that ex post licensing contracts between upstream patent holders and downstream innovators may not provide incentives to innovate because of the hold up problem. Thus ex ante licensing agreements are necessary to allocate the incentives to innovate when upstream patents are blocking. In this chapter, we generalize this approach by considering that several upstream patents can block downstream innovations. However we do not study the relationship between upstream and downstream innovators, but rather between the different upstream patent holders whose patents may be complementary.

Such a pattern where one final product is blocked by several complementary but non-sequential patents has been focused on more recently (Shapiro (2000), Gilbert (2002), Lerner and Tirole (2004)). This approach has especially been privileged to analyze cross licensing and patent pool agreements. One hardship in this approach is to define the complementarity between patents. In a seminal paper, Shapiro (2001) considers innovations that are perfect complements. This enables him to match the antitrust definition of "essential patents" that "have no substitutes; one needs licenses to each of them to comply with [a] standard"<sup>1</sup>. Neither essentiality nor pure complementarity do however capture all possibilities of combining innovations with each other. Lerner and Tirole (2004) especially emphasizes that complementary patents at time  $t$  may become substitutes at time  $t + 1$  if both enable the development of competing subsequent innovations. Lerner and Tirole (2004) thus goes back to the more general definition of substitutability and complementarity, namely that goods  $A$  and  $B$  are substitutes (respectively complements) if increasing the price of  $A$  increases (respectively decreases) the demand for  $B$ . They propose a model where patents are complements for low prices - because increasing the price of one patent increases the price of the whole bundle - and substitutes for high prices - because beyond a price threshold the technology user will only buy one patent (and, for instance, invent around the second one).

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<sup>1</sup>Letter of Joel I. Klein to R. Carey Ramos, Esq., June 10, 1999, <http://www.usdoj.gov:80/atr/public/busreview/2485.wpd>.

In this paper, we propose another interpretation of complementarity which is closer to the legal definition of a patent. By focusing on one product, we indeed consider that the courts ultimately decide whether patents are essential inputs of this product. This requires for each patent that the court rules *(i)* that the patent scope includes the product, so that an infringement claim is valid, and *(ii)* that the patent itself is not invalid, e.g. that it satisfies all patentability requirements. An important consequence of this definition is that complementarity does no more depend on the technology underlying the patents, nor on the prices and demands for other patents as in the Lerner and Tirole (2004) model, but on the probability that a patent is held essential by a court. The definition thus builds upon Shapiro (2003) and Shapiro and Lemley (2004) who emphasize the probabilistic nature of patents but did not explicitly derive the probabilistic nature of patent complementarity.

The problems raised by complementary patents has been mostly studied in a static environment. Shapiro (2001) and Lerner and Tirole (2004) focus on the same issue, namely that decentralized pricing of complementary patents yields the Cournot (1838) multiple marginalization. They conclude that cross-licensing or pooling the patents raises welfare because they are a way to coordinate the pricing of complementary patents. By contrast, cross licensing or pooling substitute patents harm welfare, which upholds the antitrust requirement that only essential patents should be pooled. Lerner and Tirole (2004) also demonstrates that requiring that the members of a pool have the possibility to license their patents independently is sufficient to screen out inefficient pools. Lerner and alii (2002) validates empirically these results regarding the efficiency (respectively, inefficiency) of pooling complementary (respectively, substitute) patents, as well as the independent licensing requirement. By contrast with these papers, our model of patent settlements ignores the multiple marginalization issue to focus on dynamic R&D competition within an integrated industry.

Both Shapiro (2001) and Lerner and Tirole (2004) indeed consider a basic framework where patent owners and patent users are perfectly separated. Lerner and Tirole however propose extensions of their static model. In one of them, each patent owner is also a patent user on one different downstream market. More importantly for our analysis, they also consider the case of two patent owners competing on the same downstream market. Interpreting patents as differentiation factors, they show especially that making cross royalties illegal per se would impeach the creation of welfare increasing patent pools. In our model, we similarly consider two patents owners that compete on the same market. Thanks to our probabilistic definition of complementary patents, we can however suppose that the downstream products are perfect substitutes, and thereby have a simpler specification of the product market.

Shapiro (2003) analyses several forms of patent settlements in a framework that captures all forms of static competition between owners of probabilistic patents. He demonstrates that in this static pattern, and independently of any litigation cost, there always exists pro-competitive settlements that *(i)* are acceptable for the firms and *(ii)* "leave the competitors as well off as they would have been from ongoing patent litigation". He concludes that the latter

condition should be used as an antitrust rule. This rule however only focuses on static competition and double-marginalization issues, without taking into account how the strategic use of probabilistic patents affects R&D investments.

As far as we know, few theoretic papers study the dynamic impact of blocking patents and cross-licensing agreements on innovation. Fershtman and Kamien (1992) develop a model in which two firms engage in a patent race for two complementary patents. They use it especially to evaluate the impact of cross-licensing agreements that may take place if each firm has patented one different complementary innovation. They show first that cross-licensing agreements do not allow a perfect coordination of the firms' R&D efforts. Although it takes more time to achieve both innovations if cross-licensing is forbidden, such agreements indeed do not match the R&D efficiency that a centralized coordination would achieve. This is due to inefficient strategic behaviors by the firms, who tend for example to retard the development of the technology in which they have a cost advantage, and seek to patent first the other technology in order to deter their competitor. Fershtman and Kamien (1992) also shed light on the social trade-off underlying cross-licensing agreements. On the one hand cross-licensing improves the efficiency of the R&D investments by eliminating the duplication of efforts. But on the other hand, it favors price collusion between the firms. Our model also describes a patent race, but it differs from Fershtman and Kamien (1992) in two main aspects. First the complementary between patents becomes probabilistic. In that respect pure complementarity becomes a particular case, and we can explore further how probabilistic complementarity may determine the firms' cooperation and investment strategies. Second, our R&D race setting includes not only a stage of research investment for the patents, but also a stage of product development upon the patents. We thereby introduce in our analysis a dimension of R&D investments which has often been neglected in the literature, namely the follow on investments that need to be done until a new technology is commercialized. As we show in our model, this is indeed necessary to provide a comprehensive picture of cross-licensing in a dynamic environment.

The setting we propose permits to get closer to the findings of empirical studies. Hall and Ziedonis (2001) show for instance that in the US semi-conductor industry, the need to cross-license complementary patents has led the firms to engage into strategic patenting, which may finally be detrimental to competition and raise the cost of innovation. Testing a theoretical model developed by Bessen (2003), Bessen and Hunt (2004) show that a weak enforcement of patent requirements in the software industry leads to similar strategies of patent portfolio building and cross-licensing. They find that strategic patenting has a negative effect on innovation, because it does not correspond to real innovation, and it raises the cost of innovation for other firms. The monograph of Beckers and alii (2001) on the GSM (global system for mobile communications) standard also provides interesting insights on how blocking patents affect the firms' behaviors. Beckers and alii show first how the firms of the industry, following a first aggressive move by Motorola, have engaged in a costly patent race to preempt the essential patents that would be included in the standard. They also

find that the market shares in the market for GSM equipment reflect both, the weights of the firms in the GSM patent pool and their alliance networks influence. This suggests that patent agreements offer an opportunity for collusion, while blocking patents provide bargaining power in such arrangements.

Our model provides theoretic explanations for these empirical findings. We can especially characterize the cases in which the firms invest all the more so in upstream patentable research as patents are likely to be blocking downstream. The model furthermore permits us to identify when the firms will sign ex ante agreements, and in whether such agreements are procompetitive or not. Interestingly, our conclusions converge with those of the static literature on patent pools, although for different reasons. In our dynamic setting, agreements may indeed be procompetitive when blocking patents induce underincentives to invest downstream, and anticompetitive otherwise.

### 3 The model

We consider a situation where two symmetric firms (referred to as firm A and firm B) sequentially invest in R&D. Initially, both firms invest in R&D to create a basic innovation that is necessary to develop a new product at the second stage. Each firm can achieve a basic innovation with a probability  $x$  at a cost  $c(x)$ . When achieved, the first stage innovations are granted a patent.

Besides the technical information disclosed in patents, the basic innovations also consist in knowledge that is protected by trade secret. We consider that both this secret knowledge and the information disclosed in the patents are necessary to build on the basic innovation. This assumption is consistent with the reality of some industries, such as computer hardware and semiconductors, where "the disclosure of information through patents is seldom sufficient for a rival to replicate the innovation" (FTC, 2003). Therefore, we consider that a firm can be successful at stage 2 only if it was successful at stage 1. We consider such a setting because we want to focus explicitly on the implications following the strategic use of legally blocking patents.

In the second stage, each patent owner invests  $c(y)$  and develops a product with a probability  $y$ . We assume that there are no additional production cost. The demand for a final product comes from a mass of consumer that is set equal to 1, with a willingness to pay equal to 1 for either product. Thus a monopoly price on the product market grants a profit equal to 1. If both firms are successful, we assume that they compete a la Bertrand on the product market. This very simple setting does not take into account any deadweight loss effect, so that the dynamic effects that the model exhibits are independent from the multiple margins issue.

For a better exposition and in order to reach explicit results we assume that the cost function is such that  $c(t) = \frac{\delta}{2}t^2$ , with  $t = x, y$  and  $\delta > 1$ . (Results hold for different cost parameters  $\delta_t$  at stages 1 and 2 or under more general convex cost functions.)

We now come to the concept of blocking patents. Before competing on



the product market, a firm can use its basic patent to sue its competitor for infringement. If the plaintiff's infringement claim is held valid by the Court, then its patent becomes an essential input of the defendant's product. Let  $\phi_i$  ( $i = A, B$ ) denote the probability with which firm  $i$ 's claim is held valid by a court. Let

$$\phi_i = \phi + \tilde{\varepsilon}_i, i = A, B.$$

where  $\phi$  reflects a patent's legal strength and  $\tilde{\varepsilon}_i$  is a realization of a random variable  $\tilde{\varepsilon}$  with mean zero.

According to this formalization the firms are symmetric as each patent has the same probability of being held essential by a court on average. We introduce  $\tilde{\varepsilon}_i$  indexed by  $i$ , to disentangle the effect of a firm's own patent strength on its investment with the one triggered by its opponent's patent strength.

Note here that the parameter  $\phi$  provides a proxy of the complementarity between the firms' basic patents. Indeed, as a Court holds one firm's patent essential to its competitor's product, which occurs with probability  $\phi$ , it acknowledges a complementarity between this patent, as a legal input, and the infringer's basic patent, as a technical input. Moreover, if each patent is held essential to the competitor's product, which happens with probability  $\phi^2$ , then the patents become perfect legal complements for all the industry.

## 4 Three scenarios

In this section, we focus at the downstream investments. We consider successively unrestricted, restricted patents and cross licensing in case the two firms have innovated at stage 1. The two first scenarios describe the cooperation strategies available to the firms. The third scenario is a variant of the first one, in which the exclusion power of the patents is exogeneously restricted by a system of cross licensing. As a first step, we quickly describe what happens if only one firm has innovated at stage 1.

### 4.1 One firm has succeeded at stage 1

When a single firm owns a patent after stage 1, it is the only one who can innovate downstream and will be a monopolistic seller if it succeeds. Therefore whether an agreement has been signed or not is irrelevant in this case and the firm always invests  $y^m$  such that

$$y^m \in \arg \max_y y - c(y).$$

This leads to  $y^m = \frac{1}{\delta}$  and generates a profit  $\Pi^m = \frac{1}{2\delta}$ . Second period investments will differ as we consider situations where both firms were initially successful. For each possible scenario we now evaluate second period investments when both firms are patent owners.

## 4.2 No ex-ante agreement, unrestricted patent

Let us focus now on what happens at stage 2 when the two firms have innovated at stage 1. We consider first patent protection in its most general form, that is when it is not restricted to compulsory licenses. When the two firms have innovated at stage 1, both own a potentially blocking patent. Assume, for notation purpose, that  $\phi_A$  and  $\phi_B$  are common knowledge before the firms invest. Let  $\pi_i^j$  with  $i, j = A, B$  denote firm  $j$ 's profit when firm  $i$  only succeeded downstream. Let  $\pi_{AB}^i$  denote firm  $i$ 's profit when both firms succeeded downstream.

When firm  $i$  is the only successful firm downstream, we have:

$$\begin{aligned}\pi_i^i &= (1 - \phi_j) + \frac{\phi_j}{2}, \\ \pi_i^j &= \frac{\phi_j}{2}.\end{aligned}$$

When only firm  $i$  ( $i = A, B$ ) succeeds at stage 2, firm  $j$ ,  $j \neq i$ , can use its patent to sue  $i$  for infringement. If the Court rejects the infringement claim, firm  $i$  is a monopoly. If the Court upholds the claim firm  $j$ 's patent is essential and firm  $i$  cannot sell its product without  $j$ 's agreement. In that case, firms  $i$  and  $j$  share equally the monopoly profits which corresponds to the Nash bargaining solution.

Assume now that both firms succeed at stage 2. We then have:

$$\pi_{AB}^i = \phi_i (1 - \phi_j) + \phi_i \phi_j \frac{1}{2}, \quad i = A, B.$$

Both firms can use their initial patent to sue their competitor for infringement. If both claims are rejected, the firms have no choice but compete a la Bertrand and get no profit in equilibrium. If both patents are held essential then the firms are entitled to extract and share equally the monopoly profit. If only one patent is held essential, the firm with the essential patent can exclude its opponent from the market and extract the monopoly profit.

Given the above, we can express firm  $i$ 's expected profit in the second stage as

$$\Pi_2^U = y_i (1 - y_j) \left(1 - \frac{\phi_j}{2}\right) + y_i y_j \phi_i \left(1 - \phi_j \frac{1}{2}\right) + (1 - y_i) y_j \frac{\phi_i}{2} - \frac{\delta}{2} y_i^2, \quad i = A, B, i \neq j.$$

Since  $\phi_A$  and  $\phi_B$  are equal on expectation, we have a symmetric equilibrium investment:

$$y^U = \frac{1 - \frac{1}{2}\phi}{1 + \delta - \phi + \frac{1}{2}\phi^2}.$$

(We use the upper-script  $U$  to refer to unrestricted patent.)

**Lemma 1** *The second period investment decreases with the expected patents' legal strength. (The proof is obvious and thus omitted.)*

**Corollary 1:** *Firm  $i$ 's downstream investment increases with its own expected patent strength  $E(\phi_i)$  but it decreases with its opponent's expected patent strength  $E(\phi_j)$ .*

**Proof.** See Appendix 1. ■

The above corollary permits to better understand Lemma 1. As the opponent's blocking prospect increases a firm's expected benefits from investing decrease and thus it has less incentives to invest. The reason why this is not systematically compensated by an increase of the firm's own patent strength is due to a free rider effect triggered by the fact that a firm can extract part of the monopoly profit even if it fails. The term  $(1 - y_i) y_j \frac{\phi_i}{2}$  in the expression of  $\Pi_2^U$  characterizes this free rider benefit. As a firm's legal patent strength increases the potential free riding revenue increases and this softens the incentive to increase investment that would result from possessing a stronger patent. Thus, the free riding effect leads to an investment that is overall decreasing in  $\phi$ .

**Lemma 2** *The expected payoff is inverse U shaped with respect to  $\phi$ .*

**Proof.** See Appendix 2. ■

Lower levels of patent strength are associated with higher investments. Firms are then more likely to succeed and compete away their profits since infringement claims will most likely be rejected. Thus expected profits are low for weak patents. Higher levels of patent strength are associated with low investments. Firms are more unlikely to succeed as each counts on the free riding revenue. We face a situation comparable to under-provision of a public good in which both firms hope that the other will invest to generate some value. As a result both invest too little and the expected revenue is small.

### 4.3 Ex ante agreement

We investigate as a second step how ex ante agreements affect the firms' incentives at stage 2. Assume that the firms have the possibility to sign an arrangement before investing in the second period. This agreement consists in fixing a royalty  $\psi$  per unit of output conditional on developing the downstream product. The firms then commit not to sue each other for infringement afterwards.

If a single firm succeeds in developing the product we have:

$$\begin{aligned}\pi_i^i &= 1 - \psi, \\ \pi_i^j &= \psi.\end{aligned}$$

The successful firm gets the monopoly profit minus the royalty which is paid to the non successful firm.

If both firms succeed, they compete à la Bertrand and each firm's marginal cost is equal to  $\psi$ . Thus we have:

$$\pi_{AB}^i = \frac{\psi}{2}, i = A, B.$$

Expected profits for the second period are then given by:

$$\Pi_2^A = y_i (1 - y_j) (1 - \psi) + y_i y_j \frac{\psi}{2} + (1 - y_i) y_j \psi - \frac{\delta}{2} y_i^2, i = A, B, i \neq j.$$

In equilibrium each firm invests

$$y^A = \frac{1 - \psi}{1 + \delta - \frac{\psi}{2}}.$$

**Lemma 3** *When an ex ante agreement is settled, the second period investment is decreasing in royalty  $\psi$ . (The proof is obvious and thus omitted.)*

A higher royalty only encourages investment when both firms succeed and each gathers rents through the royalty. In any other case a higher royalty either means lower rents or greater free riding revenue. Thus overall the investment decreases with the royalty.

**Lemma 4** *If an ex ante agreement is settled, the expected payoff is inverse U shaped with respect to  $\psi$  and there exists a unique  $\hat{\psi} = \underset{\psi}{\text{ArgMax}} \Pi_2^A(\psi)$ .*

**Proof.** See Appendix 3. ■

Though the structure of the payoffs differs from the case without ex ante agreement, these results thus establish that the main features of the competition that were valid without ex ante agreement remain valid. The patent royalties induce a free rider behavior by the firms, which reduces their investment efforts. This has first a positive effect on the firms' expected payoffs when the royalties are low, but the effect on the expected payoffs becomes negative when the royalties are too high. We can deduce from that the firms will settle on the royalty rate  $\hat{\psi}$  that maximizes their expected payoffs at stage 2.

#### 4.4 No ex-ante agreement, restricted patent

We consider now a variant of the first scenario in which the firms, if they do not agree ex ante, have their patents restricted by a system of compulsory licenses. Under this regime, a restricted patent does not permit its owner to escape competition when his opponent's product infringes his own. Instead the patent owner has to grant a license to his competitor who pays him a royalty. Hence, even when both products infringe each other, the patent holders must compete a la Bertrand and cannot use their patent rights to share the monopoly profit.

Let  $r$  denote the royalty that a firm must pay if it held infringing by a court. If firms decide on the royalty to be paid, we will assume that it corresponds to the Nash bargaining outcome and we have  $r = \frac{1}{2}$ . Alternatively, the royalty

could be set by some regulatory agency so as to maximize total welfare. Assume that only firm  $i$  ( $i = A, B$ ) succeeds at stage 2. We have:

$$\begin{aligned}\pi_i^i &= (1 - \phi_j) + \phi_j(1 - r), \\ \pi_i^j &= \phi_j r.\end{aligned}$$

If a single firm succeeds, competition does not come into play and payoffs are the same as those achieved under unrestricted patent for  $r = 1/2$ . Assume that both firms succeed at stage 2. There are now 2 products on the market and firms compete a la Bertrand. Whether a firm's patent is essential determines its opponent's marginal cost. We have

$$\pi_{AB}^i = \phi_i(1 - \phi_j)r + \phi_i\phi_j\frac{r}{2} \text{ with } i = A, B \text{ and } j \neq i.$$

If no patent is essential, then marginal cost equals zero for firms compete away their profits. If both patents are essential then both firms must pay each other a royalty and marginal cost equals  $r$  for both and in equilibrium  $p = r$ , each firm sells to half of the market. All the firms earn is the royalty revenue. Finally, if one patent only is essential firms become asymmetric with one firm with zero marginal cost (the one with the valid patent) and one firm with a marginal cost equal to  $r$ . Bertrand predicts that  $p = r$  and both sell  $q = 1/2$ . The firm without essential patent makes no profit, while the firm with the essential patent gets  $r$ .

Firm  $i$  ( $i = A, B$ ) expects:

$$\Pi_2^R = y_i(1 - y_j)(1 - \phi r) + (1 - y_i)y_j\phi r + y_i y_j \phi r \left(1 - \phi \frac{1}{2}\right) - \frac{\delta}{2} y_i^2, i = A, B, i \neq j.$$

In equilibrium we have:

$$y^R = \frac{1 - r\phi}{1 + \delta - r\phi + \frac{r}{2}\phi^2}.$$

**Lemma 5** *The downstream investment is strictly decreasing (and concave) with the expected patent breadth  $\phi$ . It also decreases with the royalty  $r$ . (The proof is obvious and thus omitted.)*

We could also prove that investment increases with a firm's own patent strength but decreases with its opponent's patent strength. The free riding effect is still present but the return it triggers depends on  $r$ . The main difference with the case of unrestricted patent is that now, in the event of both succeeding, firms cannot escape competition. This diminishes the expected profit of success and deters investment.

**Lemma 6** *For any  $\delta > 0$ , there exists a range of royalties  $[0, \hat{r}]$  with  $\hat{r} < \frac{1}{2}$  such that for any  $r \in [0, \hat{r}]$ ,  $\Pi_2^W$  reaches a maximum at  $\phi = 1$ . As the cost of investment increases, this interval shrinks as  $\hat{r}$  decreases (but it never disappears). For any  $\delta > 0$ , and any  $r > \hat{r}$ , the second period expected revenue is inverse U shaped with respect to  $\phi$ .*

**Proof.** See Appendix 4. ■

Once again low levels of patent breadth are associated with high investments and firms often compete away their profits. Higher level of patent breadth are associated with low investments and raises the problem of under-provision of a public good unless  $r$  is sufficiently small. By implementing small enough royalties a regulatory agency has the possibility to counteract the free riding incentive. Notice in particular that setting  $r = 0$  wipes out any free riding revenue and transforms the game in a patent race in which revenue accrues to a successful firm provided it is the only winner.

## 5 Comparing the scenarios

In the previous sections, we have characterized the firms' behaviors and expected payoffs at stage 2 with normal or restricted patents and with ex ante agreement. In this section, we compare the different scenarios. After some comments on the firms' investments at the second stage, we study when the firms will decide or not to make an ex ante agreement.

### 5.1 Investment in R&D

Let us first characterize the socially optimal level of downstream investment as

$$y^S = \underset{y}{\text{ArgMax}} \left[ 1 - (1 - y)^2 \right] - 2c(y)$$

where we maximize the expected generated surplus minus the cost of obtaining this surplus. We have  $y^S = \frac{1}{1 + \delta}$ .

The table below summarizes our findings in terms of downstream investments.

No ex-ante agreement Unrestricted patent	Ex ante agreement	No ex-ante agreement restricted patent
$y^U = \frac{1 - \frac{1}{2}\phi}{1 + \delta - \phi + \frac{1}{2}\phi^2}$	$y^A = \frac{1 - \psi}{1 + \delta - \frac{\psi}{2}}$	$y^R = \frac{1 - r\phi}{1 + \delta - r\phi + \frac{r}{2}\phi^2}$

**Proposition 1:** *In absence of an ex-ante agreement, downstream investments are suboptimal (provided  $r > 0$ ) for any positive level of patent strength ( $\phi > 0$ ). An ex-ante agreement will always fail to reach the socially optimal investments.*

**Proof.** Recall that  $y^U$  and  $y^R$ , and  $y^A$  are decreasing in  $\phi$  and  $\psi$  respectively (provided  $r > 0$ ). We have  $y^T = y^S$  with  $T = U, R, A$  if and only if  $\phi = 0$  for  $T = U, R$  and  $\psi = 0$  for  $T = A$ . Thus for any  $\phi, \psi > 0$ , we have  $y^T < y^S$ , for  $T = U, R, A$ . The fact that ex-ante agreement will always fail to lead to socially optimal investment stems from the fact that  $\left. \frac{d\Pi^A}{d\psi} \right|_{\psi=0} > 0$ . Thus firms will always set a strictly positive royalty. ■

That patent protection lowers R&D investments may seem surprising. Recall however that the parameter  $\phi$  only capture the likelihood that a product infringes while it has been developed upon a different basic innovation. By contrast, we have assumed that patent protection is perfect vis-a-vis imitations. Thus the result above does not imply that no protection of innovation is optimal. It just captures the effect of patent protection as a legal mean for the firms to hold up products that have been developed independently by their competitor. And it states that the ability given by patents to obtain rights of such independent products is detrimental to innovation.

**Corollary:** *A regulatory agency can implement efficient investments by imposing compulsory free licensing (set  $r = 0$ ).*

When patents owners are in a position to legally block infringing innovations, they are tempted to free ride on their competitor's investments. By imposing free licensing a regulatory body can inhibit free riding and restore efficient investments. A similar remark can be made as regards the ex ante agreement. An ex ante agreement with  $\psi = 0$  is equivalent to a case where the firms cannot use their patents to block their rival and simply compete without being threatened by imitators. This case too appears to trigger optimal downstream investments. Introducing a positive royalty would indeed reduce the profit of a successful innovator, and thereby lower the incentives to innovate.

## 5.2 Agreement versus non-agreement

For any given cost of investment, the firms decide to settle ex-ante if and only if  $\Pi_2^A(\hat{\psi}) \geq \Pi_2^T(\phi)$  with  $T = U, R$ . We can rewrite the second period profits as

$$\Pi_2^U = \frac{\phi}{2}y^U + \frac{\delta}{2}(y^U)^2,$$

and

$$\Pi_2^R = r\phi y^R + \frac{\delta}{2}(y^R)^2,$$

and

$$\Pi_2^A = \psi y^A + \frac{\delta}{2}(y^A)^2, \tag{1}$$

We have, for any  $r \geq 0$  and  $\delta > 1$ ,

$$\Pi_2^U|_{\phi=0} = \Pi_2^R|_{\phi=0} = \Pi_2^A|_{\psi=0}.$$

**Proposition 2:** *For any cost parameter  $\delta > 1$ , there exists  $\underline{\phi}^U < 1$  and  $\overline{\phi}^U > \underline{\phi}^U$  such that firms settle ex-ante in either cases:*

*-when patents are weak (that is  $\phi \in [0, \underline{\phi}^U]$ ) so as to save on otherwise high investments and escape competition.*

*-when patents have a strong blocking capacity (that is  $\phi \geq \min\{\overline{\phi}^U, 1\}$ ) to overcome the free riding issue and settle on low royalties.*

**Proof.** See appendix 5. ■

Figure 1 gives a visual representation of the above proposition.

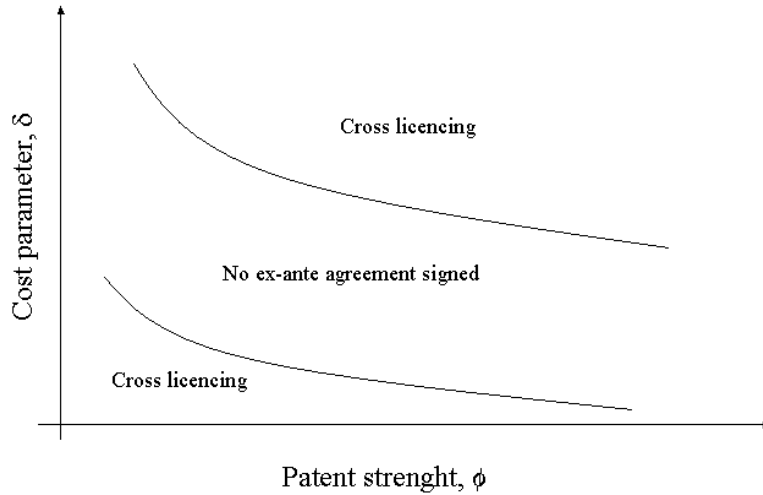


Figure 1: Decision to cross licence under unrestricted patents.

Given any  $\delta$ , the firms will *not* sign any ex ante agreement when the parameter  $\phi$  is in between the two decreasing lines. From the above proposition we learn that firms may settle for two very distinct reasons. The first is to pool really blocking patents (characterized by a high patent strength parameter  $\phi$ ) when downstream investment is costly. This corresponds to the northeast region of the graph. This motive is pro-competitive. Indeed, firms prefer to settle on (lower) royalties and overcome the free riding issue that deters investment. The second motive is to seize an opportunity to collude when patents are unlikely to be held essential and investment in R&D is not expensive. Without an ex ante agreement investment in R&D would be high and firms would compete away their profits. The ex ante agreement leads to lower investments and limits the probability of competing away their profits. This motive is not pro-competitive.

**Proposition 3:** *If compulsory licensing is implemented and firms settle on a royalty  $r = 1/2$  (corresponding to the Nash bargaining solution) then the set of parameters  $\delta$  and  $\phi$  for which firms settle shrinks as it appears in figure 2.*

*If compulsory free licensing is implemented then firms will always settle ex-ante.*



**Proof.** The second statement is obvious since  $\Pi_2^R(0) = \Pi_2^A(0) < \Pi_2^A(\widehat{\psi})$ . The first statement relates to the previous proposition and stems from the fact that for  $r = 1/2$ , we have

$$\Pi_2^R(\phi) \leq \Pi_2^U(\phi) \text{ for all } \phi, \text{ with equality at } \phi = 0 \text{ only.}$$

■

To reach efficient investments, a regulatory authority should implement compulsory free licensing. Setting  $r = 0$  is optimal for any level of patent strength. Unfortunately, such a policy would result in firms systematically agreeing ex-ante, and thus would be ineffective. Basically, for any  $r < \widehat{r}$  will always choose to sign an ex-ante agreement to save on otherwise high investments and to escape the dramatic consequences of the Bertrand competition leading to a revenue at most equal to  $r$ . Thus, the royalty must be sufficiently high to prevent firms resorting to systematic ex-ante agreement.

## 6 Upstream investment and welfare

We can now move to the first stage. A first question consists in analyzing the determinants of the upstream investment. Let the variable  $\gamma$  refer to either  $\phi$  or  $\psi$ . The expected profit from the first stage investment is given by:

$$\Pi_1(\gamma) = x_i(1 - x_j) \frac{1}{2\delta} + x_i x_j \Pi_2^T(\gamma) - \frac{\delta}{2} (x_i)^2, i = A, B, j \neq i, T = U, R, A.$$

If a firm succeeds while its opponent fails it has no competitor in the second period. In that case it invests  $y^m$  and gathers  $\Pi_2^M = \frac{1}{2\delta}$  in the second period. If both firms succeed, the expected profit is given by  $\Pi_2^T(\gamma)$  with  $T = U, R, A$  depending on what regime prevails in the second period.

A firm selects the investment level non-cooperatively, the Nash solution is symmetric and is given by:

$$x(\gamma) = \frac{\Pi_2^M}{\delta + \Pi_2^M - \Pi_2^T(\gamma)}$$

Given that  $\Pi_2^T(\gamma) < \Pi_2^M$  for any  $T = R, U, A$ , we have  $x(\gamma) \in [0, 1]$ .

**Lemma 7** *The upstream investment is such that  $\text{sign} \frac{dx}{d\gamma} = \text{sign} \frac{d\Pi_2^T}{d\gamma}$ . (The proof is obvious and thus omitted.)*

As one could expect the impact of either a royalty or a patent strength increase on upstream investments is contingent on the impact it has on future expected profit.

We have  $x^U(1) > x^*$  for  $\delta < \delta^U$  (with  $\delta^U > 1$ ). Thus, there is over-investment in the first period for any  $\phi$  only when the cost parameter is sufficiently small. However, as investment becomes dear, firms will under-invest in both periods for high values of  $\phi$ .

**Proposition 4:** *Total welfare is decreasing in both  $\phi$  and  $\psi$  (for any  $r > 0$ ).*

**Proof.** See Appendix 6. ■

The above result states that granting firms a legal possibility to block innovations deters welfare. Besides, offering firms the possibility to pool potentially blocking patents will not restore efficiency. This result suggests that the only form of protection that would lead to efficient investment is one against copies. The narrower the patent breadth, the better.

**Lemma 8** *From the above results we can deduce that*

1) *Any strictly positive level of cooperative royalties will always fail to maximize total welfare.*

2) *Compulsory licensing will also fail to maximize total welfare. At best, if  $r = 0$  and if no ex-ante settlements are permitted it will restore second period efficiency but lead to over investment in the first period.*

3) *There exists  $\phi^U$  with such that for all  $\phi < \phi^U$  welfare is higher without ex-ante settlements and for all  $\phi > \phi^U$  welfare is higher with ex-ante settlements. This corroborates the finding according to which it is best for firm to settle only when patents are very likely to be blocking.*

**Proof.** Point 1 and point 3 are obvious as welfare decreases with  $\psi$  and with  $\phi$  and we have  $W^A(0) = W^U(0)$ . Thus, there exists a unique  $\phi^U$  such that  $W^U(\phi) > W^A(\hat{\psi})$  if and only if  $\phi < \phi^U$ .

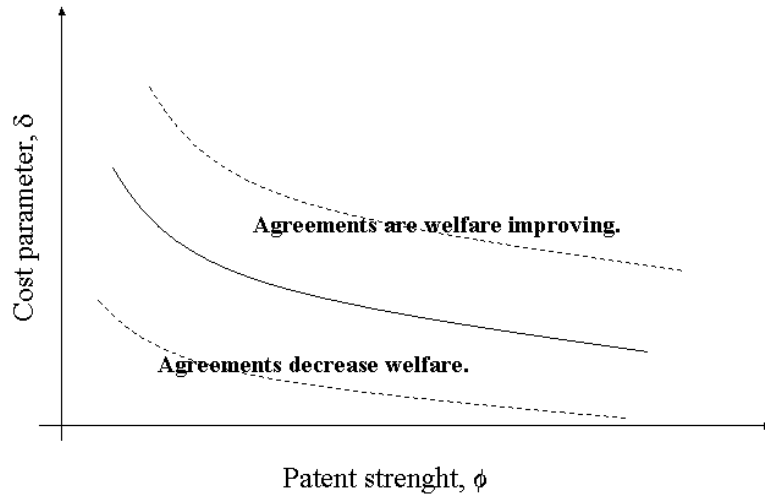
Point 2: Let  $(x^*, y^*)$  denote the socially optimal level of investments. The maximization of welfare leads to

$$y^* = \frac{1}{1 + \delta},$$

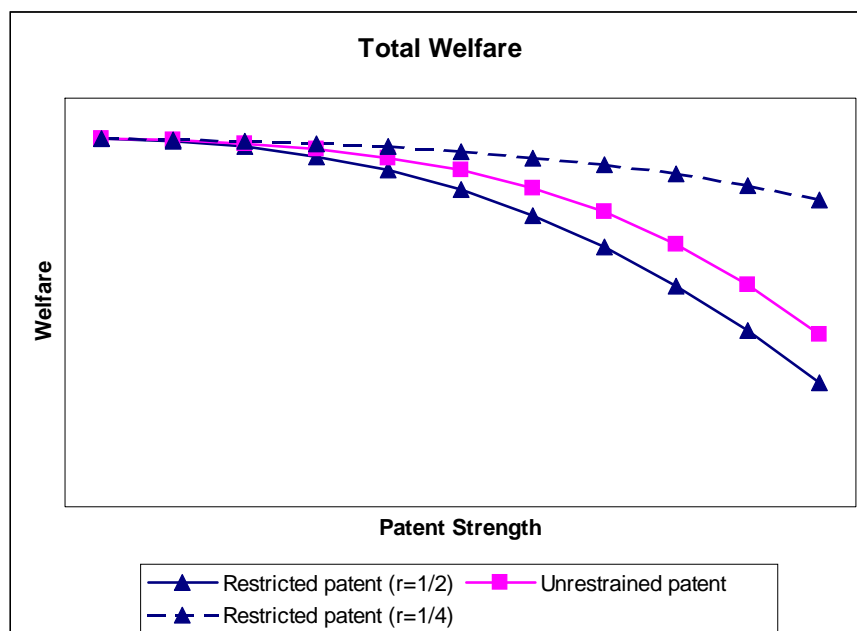
$$x^* = \frac{1 + \delta}{2(1 + \delta^2(1 + \delta))}.$$

One can easily verify that  $x^R(r = 0) = \frac{1 + \delta}{(2\delta^2 + 1)(1 + \delta)^2 - \delta^2} > x^*$ , while  $y^U(r = 0) = y^*$ .

Figure 2 illustrates the last point in the above proposition. It shows the values of  $\phi$  above which setting an agreement is welfare improving and contrasts it with the regions for which the firm would set an ex-ante agreement. ■



Finally, figure 3 represents total welfare under unrestricted patents and compares it with the one reached under restricted patents  $r = 1/2$  and  $r = 1/4$ .



As one can see in the graph above, restricted patents can be welfare improving if the regulator can impose a low royalty to balance free riding.

## 7 Conclusion

This paper focuses on an integrated industry where two competitors have to achieve basic patentable innovations in order to be able to invest in new products as a second step. In this setting, it appears that the firms' ability to block their competitors by using a patent on the basic innovation yields a free riding behavior as regards the development of new products. Indeed, the broader the basic patents, the lower the firms' investments in new products. As a result, the firms' expected payoffs if both have patented basic innovations is an inverse U shaped function of the patent breadth. Put differently, too strong patents affect negatively the innovators' downstream profits.

The firms can however decide to settle an ex ante agreement before investing in new products, by setting cooperatively a royalty on their patents. In this case, the level of the royalty has a similar effect to that of the patent breadth without ex ante agreement. Indeed, important royalties induce a free rider behavior by the firms, and may even reduce their expected profits if they are too high. The firms will thus choose to settle ex ante on the royalty that maximizes their expected onwards profit.

This happens in two different cases. First the firms will settle if the patents are broad enough to be probably essential, while the creation of new products

requires important downstream investments. In this case setting a relatively low ex ante royalty is a way for the firms to cope with the free riding issue and to foster downstream investments. But the firms will also settle ex ante in the opposite case, that is if the patents are narrow while creating new products is cheap. In this case, the firms would indeed invest in excess and compete away the profit from innovation. Thus they use relative high ex ante royalties in order to collude on lower investments in product creation. By this way they also reduce the risk of competing away the profit from innovation on the product market.

The proposed model also shows that the firms will not sign some agreements that would be welfare improving if the R&D cost and patent breadth are not important enough. In any ex ante agreement that is signed, the firms will furthermore fix excessive royalties with regards to social welfare. This is because they do not take into account the impact of their decision on the upstream R&D costs. This bias results in insufficient investments in the development of new products at the second stage, and in most cases, in a costly patent race at the R&D stage. We show then that introducing a legal restriction on patents, namely the duty to grant compulsory licensing on essential patents, is a way to foster the signing of ex ante agreement. Indeed patent restriction lower the firms' profits and the social welfare if no agreement is signed, so that the firms have additional incentives to organize the development of new products through ex ante agreements. We warn however that a policy of patent restriction through compulsory licences would favour both pro- and anti-competitive agreements.

## 8 Appendix

Appendix 1: proof of corollary 1.

We have

$$y_i^U = \frac{2\delta(2 - E(\phi_j)) - (2 - E(\phi_i))(2 - E(\phi_i) - E(\phi_j) + E(\phi_i)E(\phi_j))}{4\delta^2 - (2 - E(\phi_i) - E(\phi_j) + E(\phi_i)E(\phi_j))^2}.$$

Differentiating with respect to  $E(\phi_i)$ , leads to a numerator which can be simplified as:

$$E(\phi_j) \left[ 4\delta^2 - (2 - E(\phi_j) - E(\phi_i) + E(\phi_j)E(\phi_i))^2 \right] + 4\delta(1 - E(\phi_j)) \left[ (2 - E(\phi_i))(2\delta - 2 + E(\phi_j)) + E(\phi_j)(2 - E(\phi_j))(1 - E(\phi_i)) \right]$$

which is positive given that  $\delta > 1$  and  $E(\phi) \in [0, 1]$ .

Taking derivative with respect to  $E(\phi_j)$  leads to an obviously negative expression since the the numerator is positive but decreases with  $E(\phi_j)$ , while the denominator is also positive but increases with  $E(\phi_j)$ .

Appendix 2: Proof of lemma 2.

By combining the expressions of  $y^U(\phi)$  and  $\Pi_2^U(y^U(\phi))$  we obtain:

$$\Pi_2^U = \frac{(2-\phi)(2\delta+2\phi+\delta\phi-2\phi^2+\phi^3)}{2(2\delta-2\phi+\phi^2+2)^2}$$

It can be checked that:

$$\frac{d\Pi_2^U}{d\phi} = \frac{12\delta - 8\phi - 22\delta\phi + 6\phi^2 - 2\phi^3 + 12\delta\phi^2 - 2\delta^2\phi - 3\delta\phi^3 + 4}{(2\delta - 2\phi + \phi^2 + 2)^3}$$

$$\frac{d\Pi_2^U}{d\phi}(0) = \frac{12\delta + 4}{(2\delta + 2)^3} > 0$$

and

$$\frac{d\Pi_2^U}{d\phi}(1) = -\frac{\delta}{4\delta + 4\delta^2 + 1} < 0$$

Let us assume now that there exists at least one  $\hat{\phi}$  such that  $\frac{d\Pi_2^U}{d\phi}(\hat{\phi}) = 0$ .

The problem is unicity. Since  $\frac{d\Pi_2^U}{d\phi}$  is continuous,  $\hat{\phi}$  is unique if and only if  $\frac{d^2\Pi_2^U}{d\phi^2}(\hat{\phi}) < 0$ .

The sign of  $\frac{d^2\Pi_2^U}{d\phi^2}(\hat{\phi}) < 0$  is the same as the sign of

$$-8 - 22\delta - 2\delta^2 + 12\phi(1+2\delta) - 3\phi^2(3\delta+2),$$

which is the derivative of the numerator of  $\frac{d\Pi_2^U}{d\phi}$ . Since we care about the derivative at  $\hat{\phi}$ , we need not worry about the rest which is 0 at  $\hat{\phi}$ . The function

$$H(x) = -8 - 22\delta - 2\delta^2 + 12x(1+2\delta) - 3x^2(3\delta+2),$$

is concave in  $x$ . We have  $H(0) < 0$  and  $H(1) < 0$ . Moreover it maximizes at

$$x^* = \frac{2+4\delta}{2+3\delta} > 1$$

Thus for any  $x < 1$ , we have  $H(x) < 0$ . Thus we have  $\frac{d^2\Pi_2^U}{d\phi^2} < 0$  at any  $\hat{\phi}$  such that  $V_2'(\hat{\phi}) = 0$ . Thus  $\hat{\phi}$  is unique.

It follows that for all  $\phi < \hat{\phi}$  we have  $\frac{d\Pi_2^U}{d\phi} > 0$  and for all  $\phi > \hat{\phi}$  we have  $\frac{d\Pi_2^U}{d\phi} < 0$ . Thus there is only one value of  $\hat{\phi}$  maximizing  $\Pi_2^U$ , and  $\Pi_2^U(\phi)$  is inverse-U-shaped on  $[0; 1]$ .

Appendix 3: Proof of lemma 4.

By combining the expressions of  $y^A(\psi)$  and  $\Pi_2^A(y^A(\psi))$  we obtain:

$$\Pi_2^A = \frac{2(\psi^2 - 2\psi - \delta\psi - \delta)(\psi - 1)}{(\psi - 2\delta - 2)^2}$$

It can be checked that:

$$\begin{aligned} \frac{d\Pi_2^A}{d\psi} &= \frac{2(10\psi - 6\delta + 16\delta\psi - 6\psi^2 + \psi^3 - 6\delta\psi^2 + 4\delta^2\psi - 4)}{(\psi - 2\delta - 2)^3} \\ \frac{d^2\Pi_2^A}{d\psi^2} &= \frac{4(2\psi - \delta - 4)(2\delta + 1)^2}{(\psi - 2\delta - 2)^4} < 0 \\ \frac{d\Pi_2^A}{d\psi}(0) &= \frac{(3\delta + 2)}{2(\delta + 1)^3} > 0 \end{aligned}$$

and

$$\frac{d\Pi_2^A}{d\psi}(1) = -\frac{2}{2\delta + 1} < 0$$

Thus  $\Pi_2^A(\psi)$  is inverse-U-shaped on  $[0, 1]$ , and there is only one value of  $\hat{\psi}$  maximizing  $\Pi_2^A$ .

Appendix 4: proof of lemma 6.

Concavity of  $y^R(\phi)$ .

We have

$$\frac{dy^R}{d\phi} = r \left[ \left( \frac{1 - \phi}{1 + \delta - r\phi + \frac{r}{2}\phi^2} \right) y^R(\phi) - \frac{1}{1 + \delta - r\phi + \frac{r}{2}\phi^2} \right]$$

Using the fact that  $\frac{dy^R}{d\phi} < 0$ , and given the above expression we have  $\frac{d^2y^R}{d\phi^2} < 0$ .

We can rewrite the expected profit as

$$\Pi_2^R = y^R - (y^R)^2 \left[ 1 + \frac{\delta}{2} - r\phi + \frac{r}{2}\phi^2 \right].$$

We have

$$\left. \frac{d\Pi_2^R}{d\phi} \right|_{\phi=0} = \frac{r(1 + 2\delta)}{(1 + \delta)^3} > 0,$$

and

$$\frac{d\Pi_2^R}{d\phi}\Big|_{\phi=1} = \frac{dy^R}{d\phi}\Big|_{\phi=1} \left[ r\delta + \frac{r}{2}\phi - (1-r)^2 \right].$$

The terms in brackets it negative for some  $r < \hat{r}$  with  $\hat{r} > 0$  and decreasing in  $\delta$ . Given that the function  $\Pi_2(\phi)$  is continuous, and continuously differentiable there exists at least one  $\hat{\phi} \in (0, 1)$  such that  $\frac{d\Pi_2^W}{d\phi}\Big|_{\phi=\hat{\phi}} = 0$  for  $r > \hat{r}$ . We will prove that it is unique by showing that the second derivative at such a point is always negative. The first order condition leads to:

$$2y(\hat{\phi}) \left( 1 + \frac{\delta}{2} - r\hat{\phi} + \frac{r}{2}\hat{\phi}^2 \right) = 1 + \frac{r(1-\hat{\phi})y^2(\hat{\phi})}{y'(\hat{\phi})}.$$

The second derivative at  $\hat{\phi}$  can be expressed as

$$\frac{d^2\Pi_2^R}{d\phi^2}\Big|_{\phi=\hat{\phi}} = y''(\hat{\phi}) - 2y''(\hat{\phi})y(\hat{\phi}) \left( 1 + \frac{\delta}{2} - r\hat{\phi} + \frac{r}{2}\hat{\phi}^2 \right) + \Delta(\hat{\phi}).$$

with  $\Delta(\hat{\phi}) < 0$ . Using the first order condition we get

$$\frac{d^2\Pi_2^R}{d\phi^2}\Big|_{\phi=\hat{\phi}} = -\frac{r(1-\hat{\phi})y^2(\hat{\phi})}{y'(\hat{\phi})}y''(\hat{\phi}) + \Delta(\hat{\phi}) < 0.$$

Thus  $\hat{\phi}$  is unique and we have  $\frac{d\Pi_2}{d\phi} > 0$  for  $\phi < \hat{\phi}$  and  $\frac{d\Pi_2}{d\phi} < 0$  for  $\phi > \hat{\phi}$ .

#### Appendix 5: Proof proposition 2.

The profit maximizing royalty  $\hat{\psi}$  solves

$$2(1+\delta) - \psi^3 + \psi^2(4+3\delta) - 2\psi(1+\delta)(3+\delta) = 0.$$

This expression is decreasing in  $\psi$  and strictly negative for  $\psi = \frac{1}{2}$ . Thus we have  $\hat{\psi} < \frac{1}{2}$ .

Consider any given  $\psi \in [0, 1/2]$ , let  $\phi = 2\psi$ , we have

$$\Pi_2^U(2\psi) = \psi y^U(2\psi) + \frac{\delta}{2} [y^U(2\psi)]^2,$$

with

$$y^U(2\psi) = \frac{1-\psi}{1+\delta-2\psi(1-\psi)} > y^A(\psi).$$



Given (1), we have  $\Pi_2^U(2\psi) > \Pi_2^A(\psi)$ . Thus we have proved that for any  $\psi \in [0, 1/2]$ , there exists at least one value for  $\phi \in [0, 1]$  such that  $\Pi_2^U(\phi) > \Pi_2^A(\psi)$ . Since  $\widehat{\psi} < \frac{1}{2}$ , there exists at least one  $\phi \in [0, 1]$  such that  $\Pi_2^U(\phi) > \Pi_2^A(\widehat{\psi})$ . Since expected profits are all inverse U shaped, it means that there exists  $\underline{\phi}^U < 1$ , such that  $\Pi_2^U(\phi) < \Pi_2^A(\widehat{\psi})$  for all  $\phi < \underline{\phi}^U$ . Finally  $\Pi_2^U(1)$  decreases with  $\delta$ , as we have

$$\Pi_2^U(1) = \frac{3\delta + 1}{2(2\delta + 1)^2}.$$

Moreover we have:

$$\Pi_2^T(0) = \frac{\delta}{2(1 + \delta)^2} \text{ for } T = U, R, A.$$

For sufficiently large  $\delta$ , we have  $\Pi_2^U(1) < \Pi_2^A(0)$  thus, for sufficiently large  $\delta$ , there exists  $\overline{\phi}^U \in ]\underline{\phi}^U, 1]$  such that  $\Pi_2^U(\phi) < \Pi_2^A(\widehat{\psi})$  for all  $\phi > \overline{\phi}^U$ .

#### Appendix 6: Proof of proposition 4.

Let  $W(\gamma)$  denote the total welfare. We have

$$W(\gamma) = 2x(\gamma)(1 - x(\gamma))\frac{1}{2\delta} + (x(\gamma))^2 \left[ 1 - (1 - y(\gamma))^2 - \delta y(\gamma)^2 \right].$$

After simplifications, we have

$$\begin{aligned} \frac{dW}{d\gamma} &= \frac{dx}{d\gamma} \left[ \frac{1}{\delta} + 2x(\gamma) \left[ 2y(\gamma) - y^2(\gamma)(1 + \delta) - \frac{1 + \delta^2}{\delta} \right] \right] \\ &\quad + 2(x(\gamma))^2 \frac{dy}{d\gamma} [1 - y(\gamma)(1 + \delta)]. \end{aligned}$$

Since

$$\frac{dx}{d\gamma} = 2\delta \frac{d\Pi_2^T}{d\gamma} [x(\gamma)]^2,$$

we can rewrite the derivative of total welfare as

$$\frac{dW}{d\gamma} = 2(x(\gamma))^2 \left\{ \frac{d\Pi_2^T}{d\gamma} (1 + 2\delta x(\gamma) F^T(\gamma)) + \frac{dy^T}{dy} [1 - y^T(\gamma)(1 + \delta)] \right\}.$$

with

$$F^T(\gamma) = 2y^T(\gamma) - (y^T(\gamma))^2(1 + \delta) - \frac{1 + \delta^2}{\delta}.$$

Since  $\frac{dy^T}{dy} < 0$  and since  $y^T(\gamma) < \frac{1}{1 + \delta}$ , the second term in the brackets

is always negative. We know that  $\frac{d\Pi_2^T}{d\gamma}$  can be positive or negative. We will consider both cases separately.

- Consider first all  $\gamma$  such that  $\frac{d\Pi_2^T}{d\gamma} > 0$ . It is trivial to show that  $\frac{dF^T}{d\gamma} < 0$ , and since  $F^T(0) < 0$ ,  $F^T(\gamma) < 0$ . Thus, since the sign of the derivative of  $x(\gamma)$  is same as the sign of the derivative of the second period profit, we have

$$\frac{d}{d\gamma} [1 + 2\delta x(\gamma) F^T(\gamma)] < 0 \text{ for all } \gamma \text{ such that } \frac{d\Pi_2^T}{d\gamma} > 0.$$

Finally since  $[1 + 2\delta x(0) F^T(0)] < 0$ , we have  $[1 + 2\delta x(\gamma) F^T(\gamma)] < 0$  when  $\frac{d\Pi_2^T}{d\gamma} > 0$  and we can conclude that the derivative of welfare is negative.

- Consider now all  $\gamma$  such that  $\frac{d\Pi_2^T}{d\gamma} < 0$ . Over that range, it is not clear whether  $\frac{d}{d\gamma} [1 + 2\delta x(\gamma) F^T(\gamma)] < 0$ . This derivative might be positive for some  $\gamma$  and thus it may be that for large  $\gamma$ , we have  $[1 + 2\delta x(\gamma) F^T(\gamma)] > 0$ . However if there exists any such  $\gamma$ , then it is obvious that welfare is decreasing for such values. Whether welfare decreases is ambiguous when we have both,  $[1 + 2\delta x(\gamma) F^T(\gamma)] < 0$  and  $\frac{d\Pi_2^T}{d\gamma} < 0$ . Let us then focus at this particular case.

The proof relies on several elements. Let

$$\Sigma = \left\{ \frac{d\Pi_2^T}{d\gamma} [1 - 2\delta x(\gamma) F^T(\gamma)] + \frac{dy^T}{d\gamma} [1 - y^T(\gamma)(1 + \delta)] \right\}. \quad (2)$$

We will prove that  $\Sigma$  is bounded above by a negative term and is therefore negative.

First, note that for any  $T$ , we may write the second period profit as

$$\Pi_2^T = y^T(\gamma) - (y^T(\gamma))^2 K^T(\gamma),$$

with

$$K^A(\psi) = 1 + \frac{\delta}{2} - \frac{\psi}{2},$$

$$K^U(\phi) = 1 - \phi + \frac{\phi^2}{2} + \frac{\delta}{2},$$

$$K^R(\phi) = 1 - \phi r \left( 1 - \frac{\phi}{2} \right) + \frac{\delta}{2}.$$

Notice that in all cases,  $K^T > 0$  and  $\frac{dK^T}{d\gamma} < 0$ . Differentiating the profit function leads to

$$\frac{d\Pi_2^T}{d\gamma} = \frac{dy^T}{d\gamma} [1 - 2K^T(\gamma)y^T(\gamma)] - (y^T(\gamma))^2 \frac{dK^T}{d\gamma}.$$

To have  $\frac{d\Pi_2^T}{d\gamma} < 0$ , it must be that  $[1 - 2K^T(\gamma)y^T(\gamma)] > 0$  and we can conclude that

$$\frac{dy^T}{d\gamma} [1 - y^T(\gamma)(1 + \delta)] < \frac{d\Pi_2^T}{d\gamma} \frac{[1 - y^T(\gamma)(1 + \delta)]}{[1 - 2K^T(\gamma)y^T(\gamma)]}. \quad (3)$$

Second, it is trivial to show that for any  $\gamma$ , we have

$$F^T(\gamma) > -\frac{\delta^2 + 1}{\delta}. \quad (4)$$

Substituting in (2) both (3) and (4) we have for all  $\gamma$  such that  $\frac{d\Pi_2^T}{d\gamma} < 0$ ,

$$\Sigma < \frac{d\Pi_2^T}{d\gamma} \left[ 1 - 2x^T(\gamma)(1 + \delta^2) + \frac{1 - y^T(\gamma)(1 + \delta)}{1 - 2K^T(\gamma)y^T(\gamma)} \right].$$

We shall then prove that the expression on brackets is always positive for all  $\gamma$  such that  $\frac{d\Pi_2^T}{d\gamma} < 0$ . Since  $x(\gamma) < \frac{1}{2\delta^2}$ , we have

$$2x^T(\gamma)(1 + \delta^2) - 1 < \frac{1}{\delta^2}.$$

Furthermore one can show that for any  $T$ , the function

$$G(\gamma) = \frac{1 - y^T(\gamma)(1 + \delta)}{1 - 2K^T(\gamma)y^T(\gamma)}$$

is decreasing in  $\gamma$  since

$$\text{sign} \frac{dG}{d\gamma} = \text{sign} \left[ \frac{dy^T}{d\gamma} (2K^T - (1 + \delta)) + 2y^T(1 - y^T(1 + \delta)) \frac{dK^T}{d\gamma} \right] < 0.$$

For any  $T$ , we have  $G^T(1) > \frac{1}{\delta^2}$ . Thus we can conclude that for all  $\gamma$  we have

$$G^T(\gamma) > \frac{1}{\delta^2} > 2x^T(\gamma)(1 + \delta^2) - 1,$$

thus the term in brackets is positive and welfare decreases for all  $\gamma$ .

## References

- [1] Arora, A., Fosturi, A., Ganbardella, A, 2001, *Markets for Technology*, Cambridge, Mass., MIT Press.
- [2] Arrow, K., 1962, "Economic Welfare and the Allocation of Resources for Inventions", in Nelson, R. (Ed.), *The Rate and Direction of Inventive Activity*. Princeton, Princeton University Press.
- [3] Beckers, R.,
- [4] Bessen, J., 2003, "Patent Thickets: Strategic Patenting of Complex Technologies", Research on Innovation Working Paper.
- [5] Bessen, J., Hunt, R., 2004, "An Empirical Look at Software Patents", Research on Innovation Working Paper no. 03-16/R
- [6] Bessen, J., Maskin, E., 2000, "Sequential Innovations, Patents and Imitation", MIT, mimeo.
- [7] Cournot, A., 1838, *Recherches sur les Principes Mathématiques de la Théorie des Richesses*, Calmann-Lévy, 1974, Paris.
- [8] Federal Trade Commission, 2003, *To Promote Innovation: A Proper Balance of Competition and Patent Law and Policy*, available at <http://www.ftc.gov/opa/2003/10/cpreport.htm>.
- [9] Fershtman, C., Kamien, M.I., 1992, "Cross Licensing of Complementary Technologies", *International Journal of Industrial Organization*, 10, 329-348.
- [10] Gilbert, R.J., 2002, "Antitrust for Patent Pools: A Century of Policy Evolution", University of California at Berkeley Working Paper.
- [11] Green, J.R., Scotchmer, S., 1995, "On the Division of Profit in Sequential Innovation", *RAND Journal of Economics*, 26, 20-33.
- [12] Hall, B.H., Ziedonis, R.H., 2001, "The Patent Paradox Revisited: an Empirical Study of Patenting in the US Semiconductor Industry, 1979-1995", *RAND Journal of Economics*, 32, 101-128.
- [13] Hunt, R., 1995, "Nonobviousness and the Incentive to Innovate: An Economic Analysis of Intellectual Property Reform", Mimeo, Congressional Budget Office and Department of Economics, University of Pennsylvania.
- [14] Lemley, M.A., Shapiro, C., 2004, "Probabilistic Patents", University of California at Berkeley Working Paper CPC 04-046.
- [15] Lerner, J., Tirole, J, 2004, "Efficient Patent Pools", *American Economic Review*

- [16] Mazzoleni, R., Nelson, R.R., 1998, "The Benefits and Costs of Strong Patent Protection: A Contribution to the Current Debate", *Research Policy*, 27, 273-284.
- [17] O'Donoghue, T., 1998, "A Patentability Requirement for Sequential Innovations", *RAND Journal of Economics*, 29, 654-679.
- [18] O'Donoghue, T., Scotchmer, S., Thisse, J.-F., 1998, "Patent Breadth, Patent Life, and the Pace of Technological Progress", *Journal of Economics and Management Strategy*, 7, 1-32.
- [19] Scotchmer, S., 1991, "Standing on the Shoulders of Giants: Cumulative Research and the Patent Law", *Journal of Economic Perspective*, 5, 29-41.
- [20] Shapiro, C., 2001, "Navigating the Patent Thicket: ...
- [21] Shapiro, C., 2003, "Antitrust Limits to Patent Settlements", *RAND Journal of Economics*, 34, 391-411.
- [22] Shapiro, C., Lemley, M.,