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## Growth effects of age-related productivity differentials in an ageing society: A simulation study for Austria

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Economics Series

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Productivity Differentials  
in an Ageing Society:  
A Simulation Study for Austria**

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November 2005

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Founded in 1963 by two prominent Austrians living in exile – the sociologist Paul F. Lazarsfeld and the economist Oskar Morgenstern – with the financial support from the Ford Foundation, the Austrian Federal Ministry of Education and the City of Vienna, the Institute for Advanced Studies (IHS) is the first institution for postgraduate education and research in economics and the social sciences in Austria. The **Economics Series** presents research done at the Department of Economics and Finance and aims to share “work in progress” in a timely way before formal publication. As usual, authors bear full responsibility for the content of their contributions.

Das Institut für Höhere Studien (IHS) wurde im Jahr 1963 von zwei prominenten Exilösterreichern – dem Soziologen Paul F. Lazarsfeld und dem Ökonomen Oskar Morgenstern – mit Hilfe der Ford-Stiftung, des Österreichischen Bundesministeriums für Unterricht und der Stadt Wien gegründet und ist somit die erste nachuniversitäre Lehr- und Forschungsstätte für die Sozial- und Wirtschaftswissenschaften in Österreich. Die **Reihe Ökonomie** bietet Einblick in die Forschungsarbeit der Abteilung für Ökonomie und Finanzwirtschaft und verfolgt das Ziel, abteilungsinterne Diskussionsbeiträge einer breiteren fachinternen Öffentlichkeit zugänglich zu machen. Die inhaltliche Verantwortung für die veröffentlichten Beiträge liegt bei den Autoren und Autorinnen.

## **Abstract**

We integrate age specific productivity differentials into a long-run neoclassical growth model for the Austrian economy with a highly disaggregated labor supply structure. We assume two life time productivity profiles reflecting either small or large hump-shaped productivity differentials and compute an average labor productivity index using three different aggregation functions: linear, Cobb-Douglas, and a nested Constant Elasticity of Substitution (CES). Model simulations with age specific productivity differentials are compared to a base scenario with uniform productivity over age groups. Depending on the aggregation function, the simulation results show only negligible or small negative effects on output and other macroeconomic key variables.

## **Keywords**

Age specific productivity, demographic change, model simulation

## **JEL Classification**

O41, J11, E17

**Comments**

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## 1. Introduction and overview

Demographic projections throughout Europe show similar patterns of rapidly ageing societies. Although long-run projections of the population depend crucially on the assumptions about migration, the outcomes are reliable enough to cause a steady flow of studies and policy recommendations. All major international organizations devote resources to long-term analysis and concentrate on the impact of ageing on public finances (*Leibfritz et al.*, 1996; *Koch – Thiemann*, 1997; *World Bank*, 1994, *Holzmann – Stiglitz*, 2001; *Economic Policy Committee*, 2001, 2002; *European Commission*, 2001, 2002).

Another strand of literature emphasizes the consequences of demographic change on long run growth perspectives. The channel of transmission is usually the direct influence on the size of the labor force and indirect effects of a changing population structure on the capital intensity of production. *Bloom – Williamson* (1998), for example, add demographic variables to Barro-type cross country growth regressions and plug in time variable dependency ratios, i. e. the ratio of the non-working age population to the working age population, into the equation to assess the role of demographic change in the Asian growth experience between 1965 and 1990. Bloom and Williamson suggest that a baby boom generation would create a wave like pattern for real GDP per capita over time. As the baby boomers increase the head count immediately after birth, they reduce per capita income. At the same time they draw on available resources for caring and educating and consequently reduce savings. When baby boomers enter the working age population their additional labor supply contributes to higher output and life cycle consumption smoothing will generate more saving. When the baby bust period follows, output tends to fall below the baseline scenario, because the labor force shrinks and the elderly run down their assets. *Cutler et al.* (1990) use a Ramsey type growth model and show that the baby bust phase requires lower capital accumulation to equip new workers and to house new families, while permitting more consumption for a given capital-output ratio.

Both approaches are aware of the potential effects from demographic change on the long run growth performance but the transmission channel of demographic change is limited to its direct effect on the size of labor supply and its indirect effect on capital accumulation. In this paper we want to extend the prior analysis by introducing age specific productivity profiles into a calibrated neoclassical growth model for the Austrian economy, i. e. we extend the direct effect of the baby bust on the head count by a change in labor input as measured by efficiency units. This extension is comparable to the approach in *Bryant – McKibbin* (2004) where the aggregate wage bill depends on age specific earnings profiles. By applying non-linear age-earning profiles to simplified versions of the IMF Multimod and the McKibbin-Wilcoxon G-cubed model, Bryant and McKibbin show that baby boom shocks reduce output during the baby bust phase by some 20 percentage points as compared to a scenario without hump-shaped age-earning profiles.

*Baumgartner et al.* (2005) calibrate a long-run neoclassical growth model for the Austrian economy (A-LMM), which is based on the population projections by Statistics Austria and is used to forecast the effect of ageing on the development of potential output, employment, public finances, and other macroeconomic variables. Additionally, the social security system takes a central role in the model. The development of productivity over time depends on an exogenous rate of Harrod neutral technological progress. In *Baumgartner et al.* (2005) ageing results mainly in a shift of the relative size of age cohorts with differential participation rates. Due to the shrinking size of the working age population and age-specific activity rates for each cohort the labor force declines over time.

In this paper we will go a step further and implement age-specific productivity differentials into a calibrated neoclassical growth model based on actual population forecasts. We take the projection of a declining working age population as given and analyse what happens if, on top of the changing head count of individual cohorts, age-related productivity profiles affect aggregated labor supply expressed in efficiency units. There are mainly two arguments why productivity should vary with age. First, training on the job and experience add to human capital and second, physical and cognitive abilities diminish with increasing age. The upward slope of the age productivity profile over the early years of the working career results from the accumulation of the human capital acquired by education and from training on the job. *Becker* (1975) provides a comprehensive analysis about the incentives to human capital accumulation. Becker's optimality constraint for investing into human capital equates the present value of investment costs with the present value of additional income from this investment. Assuming constant investment costs, the accumulated gains from investment automatically get smaller as workers grow older and the number of years over which gains can be appropriated shrinks. Consequently, the incentive to invest into human capital decreases with age which motivates at least concavity of the age-productivity profile.

Physical and cognitive abilities, competency of inductive reasoning, and retentiveness start to decline from a peak around the age of 50. At the same time persons need more time for the reception of signals, aggravating the loss in productivity due to age (*Verhaegen – Salthouse*, 1997; *Skirbekk*, 2002, 2003A). Interestingly, interactive skills do not depreciate with age, but as structural change accelerates demand for experience based skills abates. This is a universal phenomenon across countries, population subgroups, sexes, and persons with high or with low ability level. Training programs can soften or even halt the decline but are not yet widespread across industrialized countries. Summarizing *Skirbekk's* (2003A) overview, the empirical evidence suggests an inverted U-shape for productivity with a peak around the age of 50 and with different estimates for the discount in productivity for younger and older workers. The size of the discount regularly depends on the measurement method. Supervisory ratings usually support no clear link between age and productivity, whereas employer-employee matched data series show more pronounced differentials.

The long-run simulation model calibrated on Austrian data is a highly aggregated macroeconomic model with an aggregate production function. Capital and labor measured in efficiency units are combined to produce aggregate output. We will relate aggregate labor efficiency units to age specific cohorts with different labor productivity following the approach developed by *Fürnkranz-Prskawetz –Fent* (2004). We will apply three aggregation functions (linear, Cobb-Douglas, CES) with deviating assumptions on the substitutability of age groups to compute an average productivity index that reflects cohort specific productivity differentials.

The advantage of integrating age specific productivity in a long-run simulation model is a complete feedback of the ageing labor force into the production possibility frontier. *Andersson* (2001), for example, estimates the effect of ageing on the average growth rate and concludes that within Scandinavian countries, the distribution of age groups contributes significantly to the growth rate of output. Andersson's approach, however, does not allow for a decomposition into direct effects resulting from productivity differentials and effects from changing savings behavior. Besides the direct impact of ageing on the amount of labor efficiency units available we also have indirect effects as life expectancy increases over time. In response to a longer life-span, individual households lower their propensity to consume out of permanent income. But as the number of households in higher age brackets rises, the average expected remaining life-span declines and causes a reverse effect on the propensity to consume. The overall response of household saving depends on which of both effects dominates. In the simulation model we can integrate the adaption of household savings by allowing for a time dependent probability of death, which directly results in a change of the propensity to consume.

The paper is structured as follows; in the next section we present the relevant subset of equations determining the labor market and the production of goods and services. Subsequently, we present estimates for productivity differentials and various aggregation functions for the computation of labor efficiency units. Section 4 presents the necessary details on the consumption function and shows how ageing will change the accumulation of capital. After presenting the simulation results in section 5 we finally conclude.

## 2. The simulation model

A-LMM is a long-run macroeconomic model for the Austrian economy and has been designed to analyze the macroeconomic impact of long-term issues on the Austrian economy, to develop long-term scenarios, and to perform simulation studies. The current version of the model foresees a time horizon until the year 2075 and is documented in *Baumgartner et al.* (2005). The model is firmly based in neoclassical growth theory with exogenous Harrod-neutral technological progress. In combination with a Cobb-Douglas production function this implies constant factor income shares over the simulation horizon and is consistent with stylized facts about

growing market economies. The model attains a steady state growth path determined by exogenous growth rates of the working age population and technical progress.

The model is populated by two agents whose actions are the result of forward looking optimizing behavior. The agents are private firms and private households. Private agents' behavioral equations are derived from dynamic optimization principles under constraints and based on perfect foresight. As the third major actor we consider the general government. We assume a constant legal and institutional framework for the whole projection period. The government is constrained by the balanced budget requirement according to the Stability and Growth Pact. The structure of A-LMM is shown in Figure 1.

The long-run growth path is determined by supply side factors. Thus, the modeling of firm behavior becomes decisive for the properties of our model.<sup>1</sup> Firms are assumed to produce goods and services using capital and labor as inputs. The production technology is a Cobb-Douglas production function with exogenous Harrod-neutral technical progress. Factor demand is derived under the assumption of profit maximization subject to resource constraints and the production technology. Capital accumulation is based on a modified neoclassical investment function with forward-looking properties. In particular, the rate of investment depends on the ratio of the market value of new additional investment goods to their replacement costs. This ratio (Tobin's Q) is influenced by expected future profits net of business taxes. Domestic savings creates no limit for investment activities since Austria is modeled as a small open economy with perfect capital markets. Labor demand is derived from the first order condition of the firms' profit maximization problem.

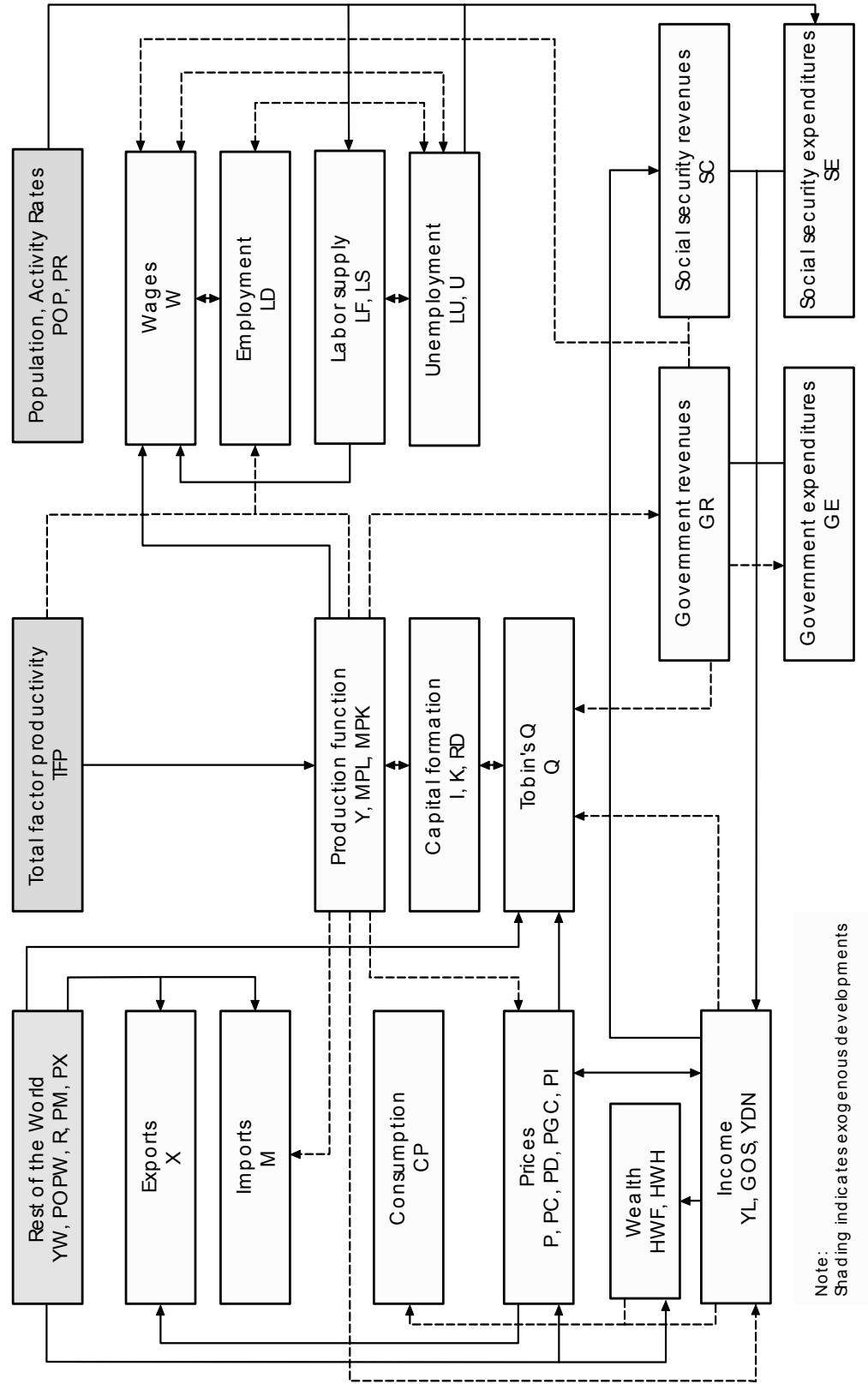
Private households' behavior is derived from intertemporal utility maximization according to an intertemporal budget constraint. Within this set-up, decisions about consumption and savings (financial wealth accumulation) are formed in a forward-looking manner. Consumption depends on the present value of expected future disposable income (human wealth) and financial wealth but also on current disposable income since liquidity constraints are binding for some households.

Households offer their labor service to firms and receive income in return. A special characteristic of A-LMM is the focus on disaggregated labor supply. In general, the labor force can be represented as a product of the size of population and the labor market participation rate. In the model we implement highly disaggregated (by sex and by six age groups) participation rates. This gives us the opportunity to account for the different behavior of males and females (where part-time work is a major difference) and young and elderly employees (here early retirement comes into consideration).

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<sup>1</sup> See, for example, *Allen - Hall (1997)*.

Figure 1: A-LMM Structure



Another special characteristic of A-LMM is a disaggregated model of the social security system as part of the public sector. We explicitly model the expenditure and revenue side for the pension, health and accident, and unemployment insurance, respectively. Additionally, expenditures on long-term care are modeled. Demographic developments are important explanatory variables in the social security part. Although, individual branches of the public sector may run permanent deficits, for the public sector as a whole, a short-run balanced-budget condition is forced to hold.

A-LMM as a long-run model is supply side driven. The demand side adjusts in each period to secure equilibrium in the goods market. The adjustment mechanism runs via disequilibria in the trade balance. The labor market equilibrium is characterized by a time varying natural rate of unemployment. Prices and financial markets are not modeled explicitly; rather we view Austria as a small open economy. Consequently, the real interest and inflation rates coincide with their foreign counterparts. Real output growth in the rest of the world is set above the Austrian trend level to allow for the convergence of per capita income to Austria's level. Thus we impose a golden rule value for real interest rate in Austria. Furthermore, we assume that the European Central Bank continues to follow an inflation target of smaller than but close to two percent. To achieve equilibrium in the capital market and to close the model, we impose that domestic excess savings corresponds to the transfer balance in the current account. This closure rule allows for a significant build up of the net foreign assets position during the transition phase towards a smaller population with a higher share of elderly and a reduction of foreign assets during the retirement phase of the baby boom generation. In the long run steady state, the model converges to a zero net foreign asset position.

Because of the long projection horizon and a comparatively short record of sensible economic data for Austria, the parameterization of the model draws extensively on economic theory.<sup>2</sup> This shifts the focus towards theoretical foundations, economic plausibility, and long-run stability conditions and away from statistical inference. As a consequence, many model parameters are either calibrated or estimated under theory-based constraints.<sup>3</sup>

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<sup>2</sup> For consistency A-LMM relies on the system of national accounts. On the basis of the current European System of National Accounts framework (ESA 1995), official data are available from 1976, in part only from 1995, onwards. The projection outreaches the estimation period by a factor of three.

<sup>3</sup> "[S]o called 'calibrated' models [...] are best described as numerical models without a complete and consistent econometric formulation [...]" *Dawkins et al.* (2001, p. 3655). Parameters are usually calibrated so as to reproduce the benchmark data as equilibrium. A typical source for calibrated parameters is empirical studies, which are not directly related to the model at hand, for example cross section analysis or estimates for other countries, or simple rules of thumb that guarantee model stability. For a broader introduction and discussion of the variety of approaches subsumed under the term 'calibrated models' see *Hansen - Heckman* (1996), *Watson* (1993) and *Dawkins et al.* (2001).

## 2.1 The neoclassical production function

Output is produced with a Cobb-Douglas technology by combining labor and physical capital under constant returns to scale. After taking the natural logarithm, the Cobb-Douglas production function is given by:

$$\log(Y_t) = CONY + (1 - ALPHA)TFP \cdot t + ALPHA \log(K_t) + (1 - ALPHA) \log(LD_t) \quad (2.0)$$

where  $Y_t$  denotes gross domestic product at constant 1995 prices.  $CONY$  denotes a shift parameter in the production function,  $TFP$  is the constant growth rate of labor enhancing technical progress,  $t$  is a time trend,  $LD_t$  the number of full-time equivalent employees,<sup>4</sup> and  $K_t$  the stock of capital. The parameter  $ALPHA = 0.491$  is the output elasticity of capital. The value of  $(1 - ALPHA)$  corresponds to share of labor income in nominal GDP in 2002. The labor income share in Austria is lower than in most other developed countries. This can be partially explained by Austria's practice of including incomes of self-employed into the gross operating surplus, i.e., profits. This makes our specification closer in spirit to the augmented neoclassical growth model along the lines of *Mankiw - Romer - Weil* (1992). By augmenting the production function with human capital, these authors obtain an estimate the labor coefficient of 0.39.

The Cobb-Douglas production function implies a unit elasticity of substitution between factor inputs. The elasticity of substitution is a local measure of technological flexibility. It characterizes alternative combinations of capital and labor, which generate the same level of output. In addition, under the assumption of profit maximization (or cost minimization) on the part of the representative firm, the elasticity of substitution measures the percentage change in the relative factor input as a consequence of a change in relative factor prices. In our case, factor prices are the real wage per full-time equivalent and the user costs of capital. Thus, other things being equal, an increase of the ratio of the real wage to user costs will lower the ratio of the number of employees to capital by the same magnitude.

A Cobb-Douglas production function implies constancy of the income shares of factor inputs in the total value added. These are given by the ratios of the gross operating surplus and of wages to GDP at constant prices. Another feature of Cobb-Douglas technology is that the marginal and the average products of input factors grow at identical rates, their levels differing by the respective factor shares. In the baseline, we assume a constant annual rate of change of labor productivity of  $TFP=1.7$  percent. The corresponding annual rate of change of total factor productivity is  $TFP(1-ALPHA) = 0.85$  percent.

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<sup>4</sup> Following the convention of the National Accounts, the compensation of self-employed are included in the gross operating surplus and therefore are not part of the compensation of employees. We therefore exclude labor input by the self-employed from the production function.



## 2.2 The labor market

The labor market block of the model consists of four parts (labor supply; labor demand; wage setting, and unemployment). In the first part aggregate labor supply is projected until 2075. Total labor supply is determined by activity rates of disaggregated sex-age cohorts and the respective population shares. Labor demand is derived from the first order conditions of the cost minimization problem. Real wages are assumed to be determined in a bargaining framework and depend on the level of (marginal) labor productivity, the unemployment rate, and a vector of so-called wage push factors (tax burden on wages and the income replacement rate from unemployment benefits).

The development of the Austrian labor force depends on the future activity rates and the population scenario. In our model population dynamics is exogenous. We use only the main variant of the most recent population projections 2000 to 2075 by Statistics Austria<sup>5</sup> (*Statistics Austria*, 2003, *Hanika et al.*, 2004).

We use the following data with respect to labor. Total labor supply,  $LF_t$ , comprises the dependent employed,  $LE_t$ ,<sup>6</sup> the self-employed,  $LSS_t$ , and the unemployed,  $LU_t$ . We take our data from administrative sources (Federation of Austrian Social Security Institutions<sup>7</sup> for  $LE_t$ , the Public Employment Service for  $LU_t$ , and the Austrian Institute for Economic Research for  $LSS_t$ ).<sup>8</sup> Only this database provides consistent long-run time series for the calculation of labor force participation rates. Dependent labor supply (employees and unemployed),  $LS_t$ , and the unemployed are calculated as:

$$LS_t = QLS_t LF_t . \quad (2.1)$$

$$LU_t = LS_t - LE_t . \quad (2.2)$$

In the projections we set  $QLS = 0.9$ , the value for the year 2002. Therefore  $LSS_t$  amounts to 10 percent of  $LF_t$ .

We use the number of dependent employed in full-time equivalents,  $LD_t$ , as labor input in the production function. The data source for employment in full-time equivalents is Statistics Austria. Employment (in persons) is converted into employment in full-time equivalents through the

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<sup>5</sup> We received extended population projections from Statistics Austria until the year 2150. Given the forward looking behavior of households and firms this mitigates end point problems for our simulation period from 2003 through 2075.

<sup>6</sup> In  $LE_t$  persons on maternity leave and persons in military service (Karenzgeld- bzw. Kindergeldbezieher und Kindergeldbezieherinnen und Präsenzdienstler mit aufrechtem Beschäftigungsverhältnis -  $LENA_t$ ) are included due to administrative reasons. In the projection of  $LENA_t$  we assume a constant relationship,  $QLENA_t$ , between  $LENA_t$  and the population aged 0 to 4 years,  $POPC_t$ , which serves as proxy for maternity leave.

<sup>7</sup> Hauptverband der österreichischen Sozialversicherungsträger.

<sup>8</sup> For a description of the respective data series see *Biffi* (1988).

factor  $QLD_t$ . For the past,  $QLD_t$  is calculated as  $LD_t/(LE_t-LENA_t)$ .  $QLD_t$  is kept constant over the whole forecasting period at 0.98, the value for 2002).

In our model the production technology is expressed in terms of a two-factor (labor and capital) constant returns-to-scale Cobb-Douglas production function. Labor input,  $LD_t$ , is measured as the number of dependent employed persons in full-time equivalents. Consistent with the production technology, optimal labor demand,  $LD_t^*$ , can be derived from the first order conditions of the cost minimization problem as follows

$$\log(LD_t^*) = \log(1 - ALPHA) - \log(W_t) + \log(Y_t) . \quad (2.3)$$

Labor demand rises with output,  $Y_t$ , and is negatively related to real wages,  $W_t$ . As it takes time for firms to adjust to their optimal workforce (*Hamermesh*, 1993), we assume the following partial adjustment process for employment. The partial adjustment parameter  $ALD$  denotes the speed of adjustment:

$$\left( \frac{LD_t}{LD_{t-1}} \right) = \left( \frac{LD_t^*}{LD_{t-1}} \right)^{ALD} , \quad (2.4)$$

with  $0 < ALD < 1$ . Actual labor demand is then given by

$$\log(LD_t) = ALD(\log(1 - ALPHA) - \log(W_t) + \log(Y_t)) + (1 - ALD)\log(LD_{t-1}) . \quad (2.5)$$

The speed of adjustment parameter  $ALD$  is set to 0.5.

$QWT_t$  denotes an average working time-index, which takes the development of future working hours into account.  $QWT_t$  is calculated in the following way: the share of females in the total labor force times females average working hours plus the share of males in the labor force times the average working hours of males. The average working time for males and females is 38.7 hours per week and 32.8 hours per week, respectively. These values are taken from the Microcensus 2002.  $QWT_t$  is standardized to 1 in 2002. In our scenarios we assume constant working hours for males and females, respectively, over time. An increasing share of females in the labor force implies that total average working time will fall. The relationship between  $LE_t$  and  $LD_t$  is as follows:

$$LE_t = \frac{LD_t}{QLD_t QWT_t} + LENA_t . \quad (2.6)$$

### 2.2.1 Implementing age related productivity

The implementation of age related productivity into the model mimics the approach we already use to account for part time work and the calculation of full time equivalents of labor input. Additionally to the working time index  $QWT_t$ , we introduce a labor productivity index  $IAP_t$ , that reflects the changing composition of the labor force and the shift in average labor productivity associated with the ageing process.

$$LE_t = \frac{LD_t}{QLD_t QWT_t IAP_t} + LENA_t, \quad (2.6')$$

where  $IAP_t$  is normalized such that its value in 2002 is equal to one. This normalization guarantees that the values in the starting year fully reflect the data from labor market statistics and the national accounts.

There are several ways to compute the labor productivity index each associated with different assumptions on the substitutability of labor across age cohorts. We use six age cohorts in our model because participation rates do not vary much within each of these cohorts.

We project the activity rates for 6 male ( $PRM_{1t}$  to  $PRM_{6t}$ ) and 6 female ( $PRF_{1t}$  to  $PRF_{6t}$ ) age cohorts separately. The following age groups are used ( $PRM_{it}$  and  $PRF_{it}$ : 15 to 24 years; 25 to 49 years; 50 to 54 years; 55 to 59 years; 60 to 65 years, 65 years and older).  $POPM_{1t}$  to  $POPM_{6t}$  and  $POPF_{1t}$  to  $POPF_{6t}$  denote the corresponding population groups. Total labor supply,  $LF_t$ , is given by

$$LF_t = \sum_{i=1}^6 (PRM_{it} POPM_{it} + PRF_{it} POPF_{it}). \quad (2.7)$$

In order to consider economic repercussions on future labor supply we model future activity rates as trend activity rates,  $PRT_t$ , which are exogenous in A-LMM, and a second part, depending on the development of wages and unemployment:

$$PRM_{it} = PRTM_{it} + ELS \cdot WA_t; \quad (2.8a)$$

$$PRF_{it} = PRTF_{it} + ELS \cdot WA_t. \quad (2.8b)$$

$ELS$  denotes the uniform participation elasticity with respect to  $WA_t$ , where  $WA_t$  is given by

$$WA_t = \log \left( \frac{w_t(1-u_t)}{w_{2002}(1+g_{wa})^t(1-u_{wa})} \right). \quad (2.9)$$

$WA_t$  is a proxy for the development of the ratio of the actual wage to the reservation wage. It measures the (log) percentage difference between the actual wage at time  $t$ , weighted by the employment probability  $(1 - u_t)$ , and an alternative wage.<sup>9</sup> For the path of the alternative wage (see the denominator in 2.9) we assume for the future a constant employment probability  $(1 - u_{wa})$  and that wages grow at a constant rate  $g_{wa}$ . In our simulations we set  $g_{wa}$  to 1.8 percent and  $u_{wa}$  to 5.4 percent.

Since no actual estimate for the Austrian participation elasticity is available we use an estimate for Germany with respect to gross wages and set  $ELS = 0.066$  (Steiner, 2000). This estimate implies that a 10 percent increase in the (weighted) wage leads to a 0.66 percentage point increase in the participation rate. For detailed motivation of the activity rate scenarios cf. Baumgartner et al. (2005).

### 3. The aggregation of age specific productivity

The base version of the model assumes homogenous labor and thus ignores possible age related productivity differentials between cohorts. Skirbekk (2002, 2003A, 2003B) provides an overview of sources for age related productivity differentials and estimates of their magnitude. Productivity reductions at older ages are small for tasks with a need for experience and communication skills, whereas older workers show a declining productivity for tasks requiring problem solving, learning, and speed. Skirbekk (2003B) presents an estimate of age related productivity differentials. We have to adjust our age cohort structure to model age specific productivities. We calculate labor supply by sex for the following six age groups: 15-19, 20-24, 25-34, 35-44, 45-49, and 50 years and older. We denote the activity rates in this age classification as  $PRAM_{1t}$  to  $PRAM_{6t}$  (males) and  $PRAF_{1t}$  to  $PRAF_{6t}$  (females), and the corresponding population groups as  $POPAM_{1t}$  to  $POPAM_{6t}$  and  $POPAF_{1t}$  to  $POPAF_{6t}$ .

The first column of Table 1 replicates Skirbekk's estimates standardized such that the productivity of 25 through 34 year olds equals one. The productivity of the youngest age group is eight percent smaller as compared to the reference group. A similar picture emerges for the oldest age group of 50 years and older. This group shows a productivity gap of 6 percent. Since other studies provide evidence for productivity gaps in the size of some 25 percent, we multiply Skirbekk's values by a factor of three and use those values for a sensitivity analysis. For the implementation in the model we will standardize the productivity differentials such that they sum up to one. This guarantees homogeneity of degree one in the aggregation function, i. e. a proportional increase of labor supply in all age groups will result in an equivalent proportional increase in efficiency labor units. The age specific productivity weights are given in columns three and four of Table 1.

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<sup>9</sup> We use lagged  $WA_t$  instead of current  $WA_t$  to avoid convergence problems in EViews®.

*Table 1: Productivity Differentials, Weighting Factors, and Per-Capita Wage Distribution Across Age Groups*

Age group	Productivity differentials relative to group 3		Weighting factors $AP_i$		Distribution of per capita wages
	Small	Large	Small	Large	
Up to 19	0.92	0.76	0.16	0.14	0.03
20 to 24	0.93	0.80	0.16	0.15	0.10
25 to 34	1.00	1.00	0.17	0.19	0.16
35 to 44	1.01	1.03	0.18	0.19	0.21
45 to 49	0.97	0.92	0.17	0.17	0.22
50 to 65	0.94	0.81	0.16	0.15	0.27
			1.00	1.00	1.00

Source: Productivity differentials from *Skirbekk* (2003B), distribution of per capita wages across cohorts from Statistics Austria "Wage and Income Tax Statistics 2001", note: Productivity differentials are defined with respect to age group 25 to 34. Weighting factors and income distribution represent shares adding up to 1.

For a comparison we also tabulate the distribution of per capita wages across age groups in Table 1. There is a markable difference between the distribution of per capita wages across age brackets from the distribution of productivity levels. Very likely this is the result of a deviation of age specific wages from the age specific marginal product, i. e. a consequence of seniority wages.

In the base version of the simulation model we do not distinguish between different age related productivity levels. The equation relating the head count of labor input to full time equivalents assumes that the age related productivity index  $IAP_t=1$  for all periods. We now follow *Fürnkranz-Prskawetz – Fent* (2004) and use three possible ways to construct this index. First we allow for perfect substitutability between different age groups by using a linear specification of the aggregation function:

$$IAP_t = \frac{\left( \sum_{i=1}^6 AP_i (PRAM_{it} POPAM_{it} + PRAF_{it} POPAF_{it}) \right)}{LF_t} \cdot \frac{LF_{2002}}{IAP_{2002}}, \quad (3.0a)$$

where  $AP_i$  is the weight of the  $i$ -th age group from Table 1 and  $IAP_{2002}$  is the value of the productivity index for the last sample year 2002. In order to isolate the effect from the shift in the population structure from the change in the population size we divide the weighted sum of labor force components by the unweighted sum used in the base scenario. Division by the initial value of the age related productivity index guarantees that we replicate the data for the starting year

2002 exactly. The productivity weights  $AP_i$  can also be interpreted as the elasticities of the efficiency of labor with respect to a cet. par. change in the size of one age group. An alternative specification that restricts the substitutability of age groups is a Cobb-Douglas function:

$$IAP_t = \frac{\prod_{i=1}^6 (PRAM_{it} POPAM_{it} + PRAF_{it} POPAF_{it})^{AP_i}}{LF_t}, \quad (3.0b)$$

$$IAP_{2002}$$

The elasticity of substitution between different age groups for this aggregation function is restricted to one. A further alternative is to allow for elasticities of substitution that differ between age cohorts. For this purpose we use a nested Constant Elasticity of Substitution (CES) function with three different age groups, where we assume perfect substitutability of labor within each age group but imperfect substitutability between those three age groups. We distinguish between middle-aged workers (25 through 49), old workers (50 and older), and young workers (up to 24). Middle-aged workers can be easiest substituted by someone from the old age group with the slightly lower relative productivity (see Table 1). The substitutability of older workers with younger ones is again smaller. This sequence of nesting is also suggested by the magnitude of the productivity differentials in Table 1. The set of two nested CES-aggregation functions is:

$$IAP_t = \frac{\left( (1 - AP_a)(P_1)^{-\rho_1} + (AP_a) \left( \sum_{i=3}^5 AP_i (PRAM_{it} POPAM_{it} + PRAF_{it} POPAF_{it}) \right)^{-\rho_1} \right)^{-\frac{1}{\rho_1}}}{LF_t}, \quad (3.0c.a)$$

$$IAP_{2002}$$

$$P_1 = \left[ (1 - AP_b) \left( \sum_{i=1}^2 AP_i (PRAM_{it} POPAM_{it} + PRAF_{it} POPAF_{it}) \right)^{-\rho_2} + (AP_b) (PRAM_{6t} POPAM_{6t} + PRAF_{6t} POPAF_{6t})^{-\rho_2} \right]^{-\frac{1}{\rho_2}}, \quad (3.0c.b)$$

where  $\rho_1$ , and  $\rho_2$  are the parameters of substitution at each level of nesting,  $P_1$  represents the nested labor aggregate of older and younger workers, and  $AP_a$  and  $AP_b$  are the corresponding productivity weights of the respective age groups. The parameters  $AP_i$  correspond to the values in the linear case in equation 3.0a given in Table 1 but normalized such that their respective

sums equal one. We assume that the substitution parameters fulfill the condition  $\rho_1 < \rho_2$ . This implies that the elasticity of substitution between workers of different age groups declines from middle aged persons to elderly workers and again from the elderly to the younger age group (Layard – Walters, 1978). Kratena (2004) calibrates a static CGE model for Austria, where he draws a distinction between skilled and unskilled labor. His Allen elasticity of substitution between skilled and unskilled labor in manufacturing is 1.88, implying a parameter of substitution,  $\rho$ , in the magnitude of 1.53. For the rest of the economy Kratena assumes a substitution parameter of 2.06. A weighted average for the total economy would be 1.96. We use the two extreme values for our simulation and set  $\rho_1 = 1.53$  and  $\rho_2 = 2.06$ . For all aggregation functions of age groups into efficiency labor units we use high and low productivity differentials (cf. Table 1).

## 4. The relation between ageing and savings behavior

The consumption function in A-LMM is a hybrid between perfect foresight and liquidity constrained households. This mixed consumer behavior fits actual data better than random walk models, which would be the consequence of pure perfect foresight behavior. We follow Campbell – Mankiw (1989) and introduce two groups of consumers. The first group follows the optimal consumption rule resulting from the solution of the intertemporal utility maximization problem. A fraction  $\lambda$  of the population belongs to the second group, which follows a different rule. The second group is called rule-of-thumb consumers, because they consume their real disposable income  $YDN_t/P_t$ .

The rule of thumb behavior can be motivated by liquidity constraints that prevent households from borrowing the amount necessary to finance the optimal consumption level (Deaton, 1991). Quest II, the multi country business cycle model of the European Commission also uses this approach (Roeger – In't Veld, 1997).

By assuming two groups of consumers we arrive at the following aggregate consumption function:

$$CP_t = CONCP + (1 - \lambda)(RTP + PRD)(HWH_t + HWF_t) \frac{P_t}{PC_t} + \lambda \frac{YDN_t}{PC_t}, \quad (4.0)$$

where  $CONCP$  is a constant. The fraction of liquidity constrained households  $\lambda = 0.3$ , the rate of time preference  $RTP = 0.0084$ , and the probability of death  $PRD = 0.02$  are set in accordance with Roeger – In't Veld (1997).  $HWH_t$  is the value of human wealth accumulated by private households and corresponds to the present value of future net wage income and public monetary transfer payments e.g. pension payments.  $HWF_t$  represents financial wealth of private households and corresponds to the present value of future capital earnings as well as changes

in the net foreign asset position. In this formulation the amount of private wealth does not depend on accumulated savings, rather it depends on the present value of future earnings associated with the investment of current and future savings. The sum of both present values represents lifetime wealth, out of which rational consumers with a preference for a smooth consumption path consume a constant fraction (*Blanchard – Fischer, 1989*)

The value of 0.02 for *PRD* implies a fifty year forward looking horizon for the average household. Due to the increase in life expectancy, the forward looking horizon of an individual household will grow. On the other hand, ageing will shift the population structure towards more elderly households with shorter remaining life-span. The overall effect on the average expected life-span is ambiguous but both forces are unlikely to cancel. For this reason we integrate a time varying probability of death into the model.

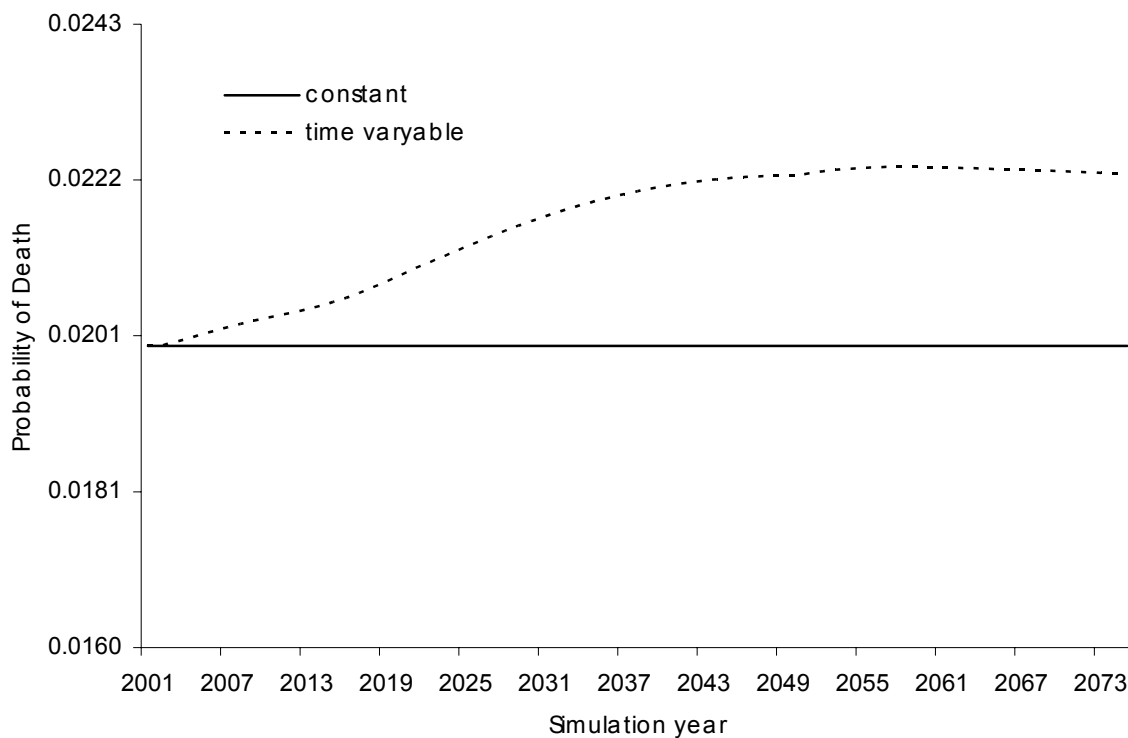
The current population projection for Austria shows an increase in the life expectancy of females from 81.7 years in 2002 to 88.0 years in 2050. The corresponding values for males are 75.8 years (2002) and 83.0 years (2050), respectively. After 2050 mortality remains constant (*Hanika et al., 2004*). Starting from the life table 2000/2002 we use cohort, *i*, and sex specific life expectancies,  $\varepsilon_{i,2002}^f$  and  $\varepsilon_{i,2002}^m$ , for females, *f*, and males, *m*, respectively, with *t* = 2002 as the starting point. In accordance with the assumptions in the population projection, life expectancy increases linearly until 2050 by sex specific constant rates:  $g^f$  and  $g^m$ . Furthermore, we weight the life expectancies of each cohort by its share in the population  $\omega_{i,t}^f$  and  $\omega_{i,t}^m$ , respectively. Because participation rates are very low in the age groups up to 19 years we restrict the computation to age groups from 20 years and older:

$$PRD_t = \sum_{i=20}^{99} \varepsilon_{i,2002}^f (1 + g^f)^{t-2002} \omega_{i,t}^f + \sum_{i=20}^{99} \varepsilon_{i,2002}^m (1 + g^m)^{t-2002} \omega_{i,t}^m, \quad t=2003, \dots, T \quad (4.1)$$

The combination of increasing individual longevity with ageing results in a hump-shaped pattern for the aggregate  $PRD_t$  with a peak around 2060. Figure 2 shows that the aggregate ageing effect dominates over the individual effect of higher life expectancy. Due to ageing the fraction of the present value of human and financial wealth consumed every year will go up from 0.02 in the year 2002 towards 0.0223 in 2075.



Figure 2: Constant versus Time Varying Probability of Death



Note: Time varying probability of death according to increasing life expectancy reported by *Hanika et al.* (2004)

## 5. Simulation results

Demographers expect an articulate change in the age structure of the population. The effect of the decline in the work force on the Austrian economy has already been presented in *Baumgartner et al.* (2005). The base scenario uses the main variant of the population forecast for Austria (*Hanika et al.*, 2004). In this variant the working age population (15-64) increases until 2012 reaching a peak value of 5.61 million persons. Afterwards, the working age population diminishes quickly until 2030 and continues to shrink at a smaller rate until 2050. Despite the starting decline in the size of the working age population in 2012, the labor force keeps rising until 2015 and shows a weak downward trend until 2070. This pattern is due to the increase in the overall participation rate throughout the simulation period by 8 percentage points. With only a modest degree of capital deepening and lower employment due to the decelerating size of the working age population, the model predicts an average annual growth rate of real GDP of 1.6 percent. We will not use this base scenario as the reference point but already use a version of the model that incorporates a time variable probability of death. The main effect of introducing a time variable probability of death is a shift in consumption over time and thus the net foreign asset position. There is no effect on investment, labor demand and production because inflation

and interest rates are fixed to their foreign counterparts by the small country assumption. The following presentation of our simulation results therefore uses deviations from this modified base line.

Given the hump shaped profile of individual productivity over the lifetime, we expect an additional effect on output growth resulting from the ageing labor force. Figure 3 shows how ageing translates into a slightly shrinking share of juveniles. The size of the youngest age group up to 19 years with the lowest productivity declines between 2002 and 2075 by 1.3 percentage points. The cohort aged 20 through 24 declines as well by 1.8 percentage points. On the other hand, the share of the 50 through 65 year olds increases by 12.8 percentage points and shows by far the strongest consequence of ageing. The size of the cohort 45 through 49 is unaffected by the structural shift, whereas the size of the most productive age groups shrinks by 6.3 percentage points (35 through 44) and 3.5 percentage points (25 through 34), respectively. Overall, about a quarter of the age related decrease in average productivity is compensated by an equivalent reduction in the size of younger cohorts. The remainder fully drags on the extension of production possibilities. Demographic change mainly occurs in the period up to 2030 and more or less subdued from 2031 through 2050. After 2050 the demographic structure remains stable.

Figure 3: *Distribution of Labor Supply by Age Group over Time*

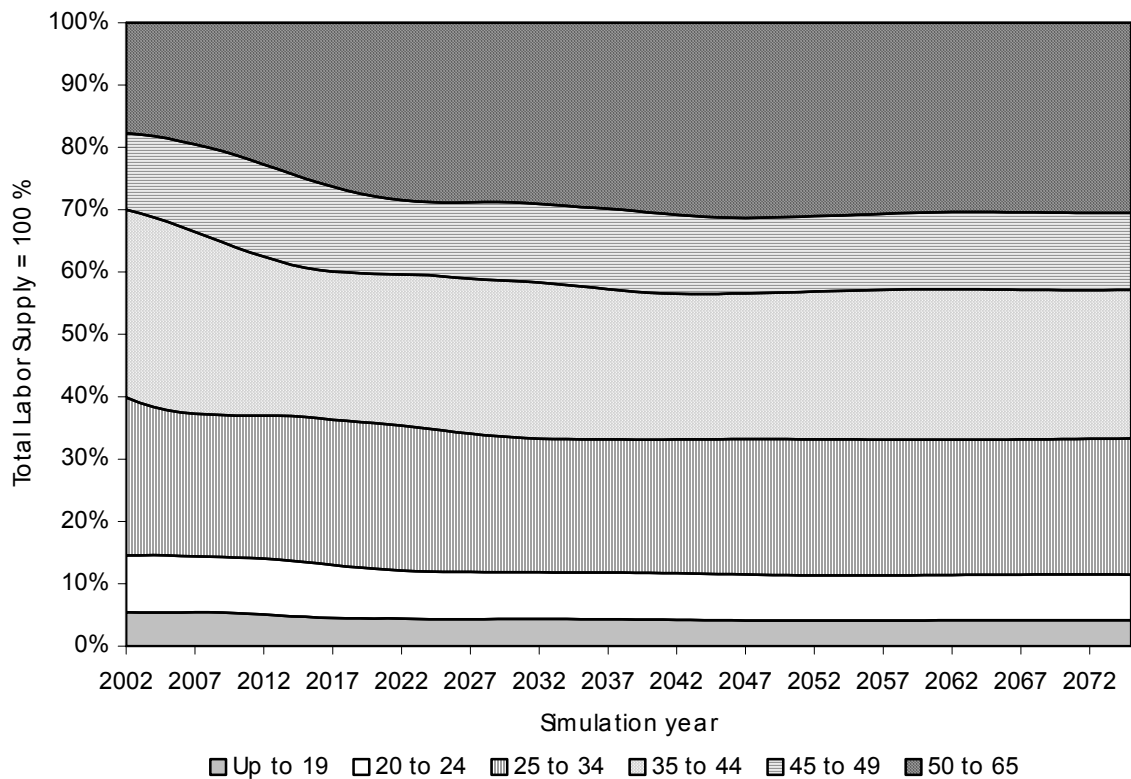
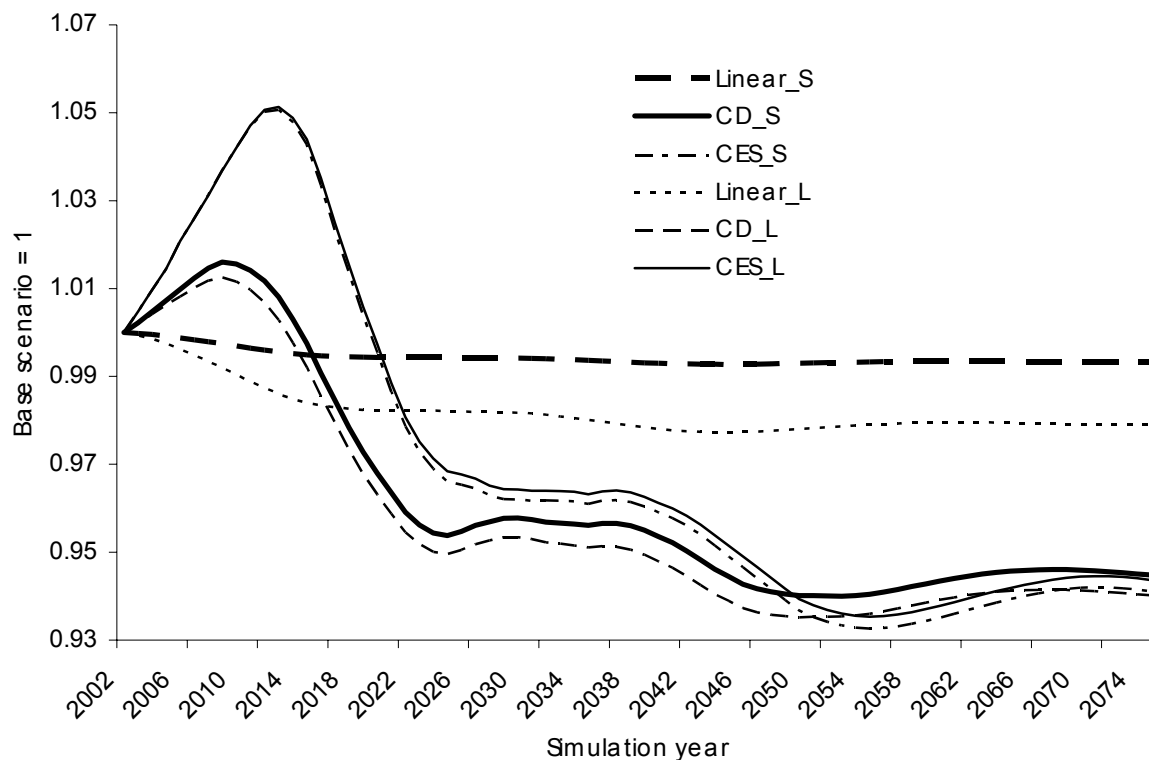


Figure 4: Index of Age Specific Productivity for Linear, Cobb-Douglas, and Nested CES aggregation



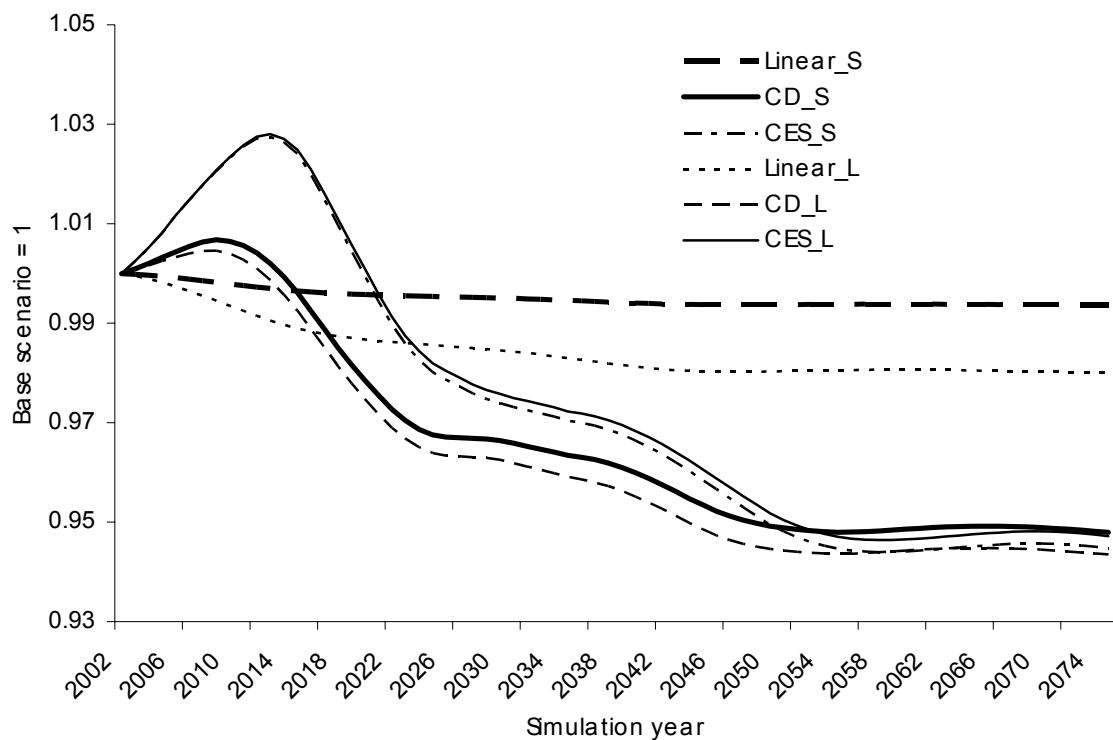
Note: Linear represents linear aggregation, CD represents Cobb-Douglas aggregation, Nested CES represents a nested Constant Elasticity of Substitution aggregation. Suffix L indicates large productivity differential and suffix S indicates small productivity differential.

Correspondingly, the age related productivity index declines in the long run. The size of the productivity differential depends on the choice of the aggregation function, i. e. the size of the elasticity of substitution, and the assumption whether the productivity differential is small or large. Under the assumption of perfect substitution we find negligible effects of ageing on the average productivity for the case of small productivity differentials (Figure 4). Even for large differentials we can only identify a decline in the average productivity by 2.3 percentage points. Relaxing the assumption on labor elasticity and allowing for unit elasticity causes a long run drop in the productivity index by some six percentage points, regardless of the assumption on the productivity differential. The nested CES-aggregation shows almost the same picture, although both substitution elasticities are lower as compared to the unit elasticity case. The response to the temporary expansion of high productivity groups peaks around 2015 and is more pronounced under the nested CES aggregation. The similarity between Cobb-Douglas and CES aggregation is also due to a smaller number of age groups in the nested CES-function. Allowing for six age groups in a CES-function but using uniform elasticities of substitution between age groups exposes a considerable weakening of productivity by 13 and 18 percentage points, for  $\rho = 1$  and  $\rho = 2$  respectively (not shown in Figure 4). The trough in the productiv-

ity index occurs between 2045 and 2055, when the share of the oldest age group reaches a maximum.

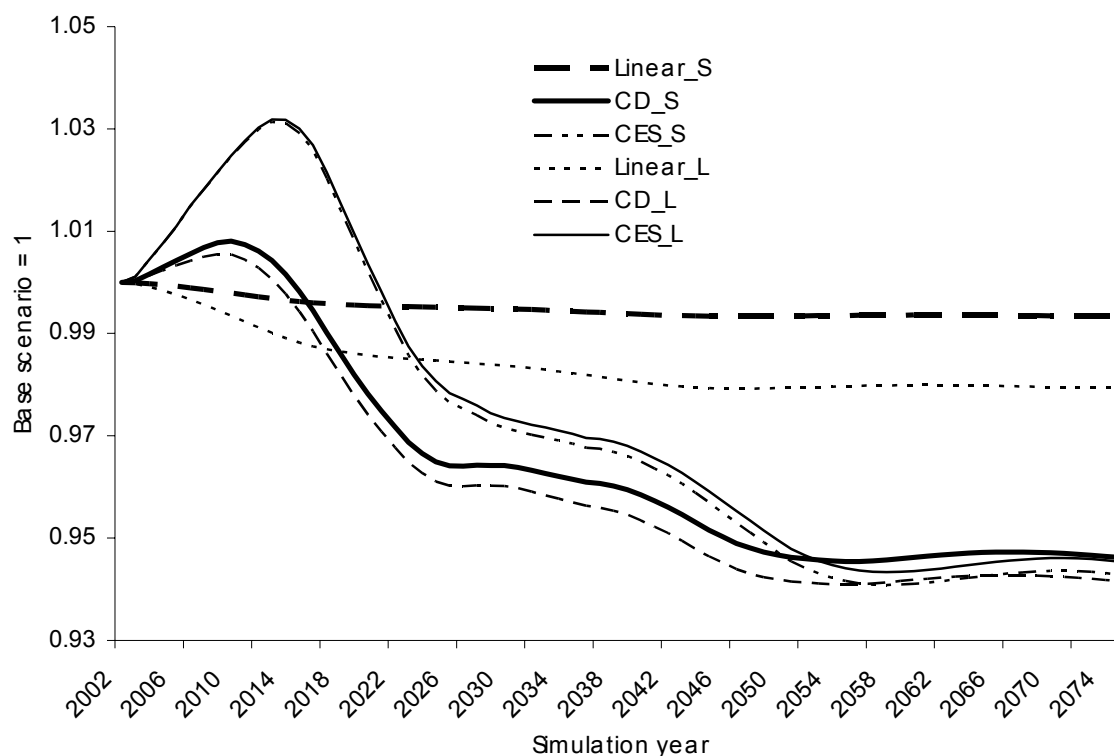
Figures 5 and 6 present results for real GDP and the wage per employee, to which we will refer as per-worker wage in the following. Both figures show the ratio of the result from alternative simulations to the base scenario with a time varying probability of death. The overall impression is that the response of these two variables matches the productivity index in shape, but the size is somewhat smaller. The stability properties of the model guarantee that by the time continual perturbations from the changing working age population die away, the model solution will return to a steady state growth path. Consequently, the deviation from the base scenario becomes constant. The wave like pattern supports the suggestion by Bloom – Williamson (1998): an increase in per-worker wages occurs when baby boomers enter the age brackets with high-productivity and a decline sets in after their movement into lower productivity age brackets and into retirement.

Figure 5: Simulation Results for real GDP, 1995=100



Note: Linear represents linear aggregation, CD represents Cobb-Douglas aggregation, Nested CES represents a nested Constant Elasticity of Substitution aggregation. Suffix L indicates large productivity differential and suffix S indicates small productivity differential.

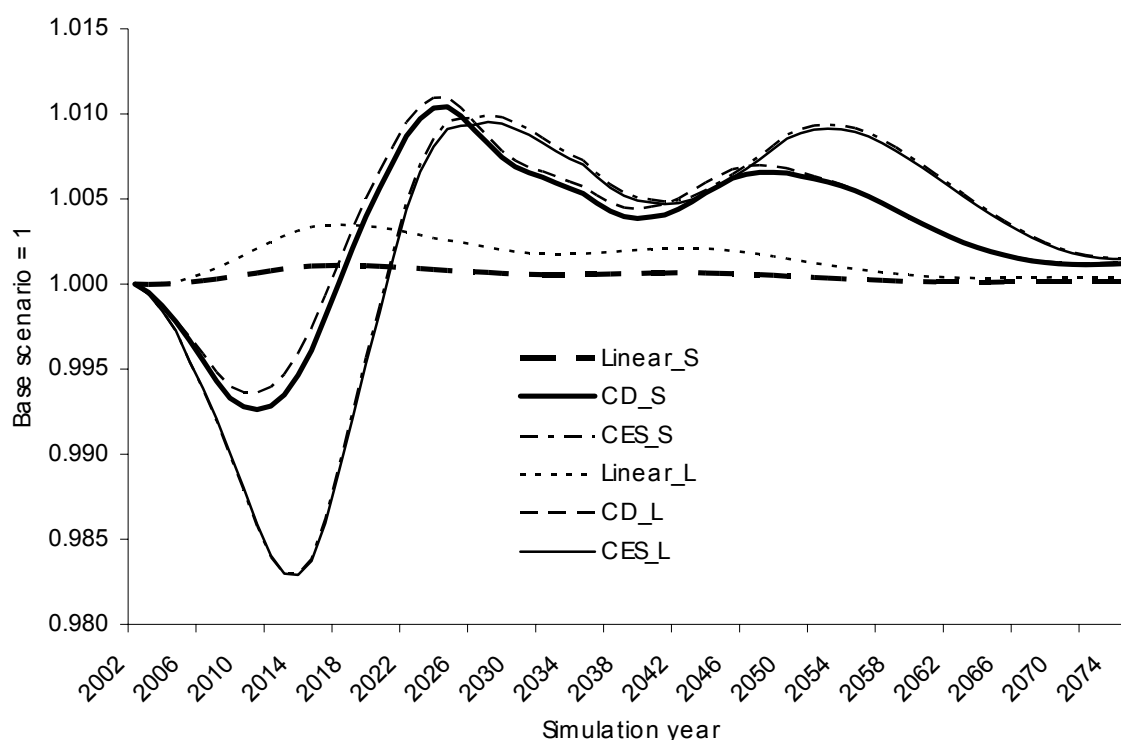
Figure 6: Simulation Results for real Wage per employee, 1995=100



Note: Linear represents linear aggregation, CD represents Cobb-Douglas aggregation, Nested CES represents a nested Constant Elasticity of Substitution aggregation. Suffix L indicates large productivity differential and suffix S indicates small productivity differential.

So, how come first that productivity has only a comparatively small effect, and second that swings in productivity do not translate fully into ups and downs in output and per-worker wages? The main answer to these questions is that the simulation model uses first order optimality conditions for factor demand, which respond to the development of the marginal productivity of labor and capital. Figure 7 shows, how the marginal productivity of labor evolves over time. Essentially the productivity related increase in efficiency labor is mirrored by a temporary decline in the marginal productivity of labor, which in the long run is reversed into a small cycling gain. The marginal productivity of capital, on the other hand, has a pattern more or less in line with the development of the productivity index. This is reasonable, since *cet. par.* capital gets more productive if the efficiency labor input goes up. Figures 8 and 9 show the response of the investment to output ratio and the capital labor ratio, respectively, to ageing. Starting from a capital labor ratio of 2.4, only small adjustments are to be expected. The investment output ratio drops only by some 0.1 units with the maximum occurring between 2015 and 2020. Accumulated over time, the reduction in investment lowers the capital labor ratio of the economy by roughly two percent (linear case) and six percent (all other cases), respectively. The adjustment in the capital labor ratio mirrors the projected response of output to ageing in size and direction (cf. Figure 5).

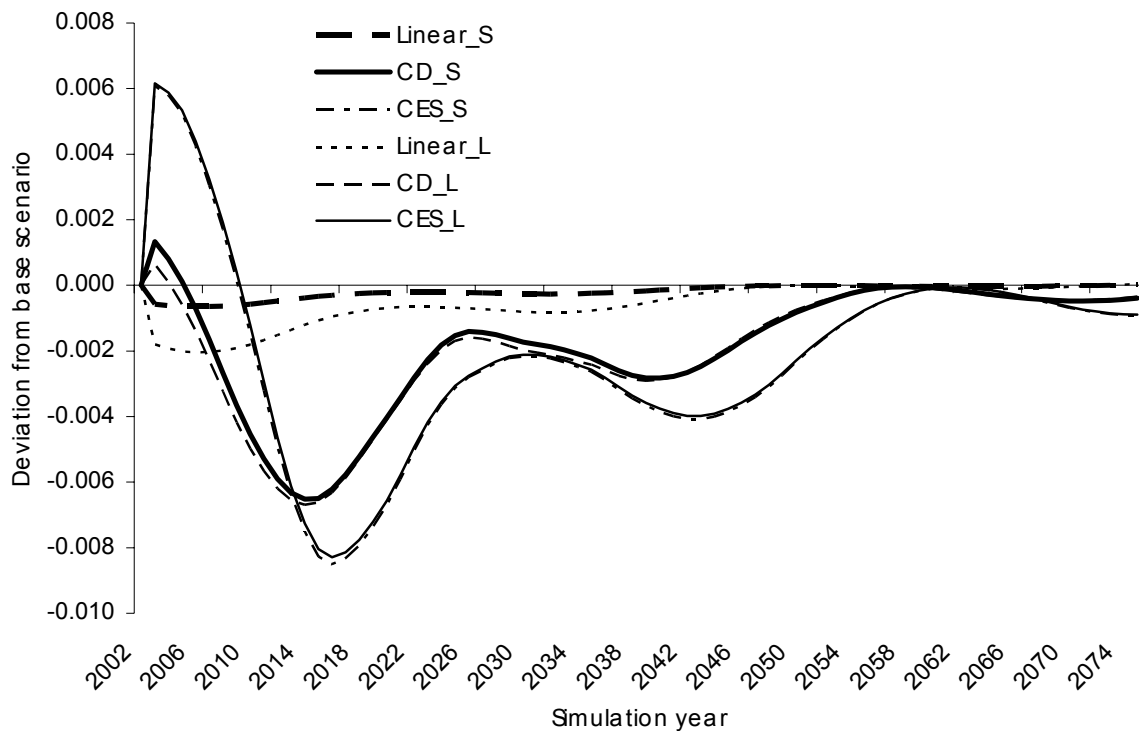
Figure 7: Simulation Results for Marginal Productivity of Labor



Note: Linear represents linear aggregation, CD represents Cobb-Douglas aggregation, Nested CES represents a nested Constant Elasticity of Substitution aggregation. Suffix L indicates large productivity differential and suffix S indicates small productivity differential.

An equally important factor for the similarity between the development of productivity and output or per-worker wages is that our experiment did not affect the exogenous rate of technical progress,  $TFP$ , in equation 2.0. There is no feedback mechanism in the model that links ageing with the rate of productivity growth. With the exogenously set value of 0.85 for total factor productivity growth, the rate of technical progress is obviously the driving force of the model. Reductions in labor supply or in labor efficiency units have only a temporary effect on output growth. Nevertheless, the growth rate of real output drops from around two percent per year in the first 15 simulation years to a meager 1.4 percent per year at the end of the simulation period. More important, the wave like pattern of the productivity index directly translates into a similar cycle for output growth rates. Thus, the higher labor supply in efficiency units in the beginning creates a higher base for all alternative simulations which is only modestly compensated by the negative growth differential starting after 2015.

Figure 8: Simulation Results for investment to GDP Ratio



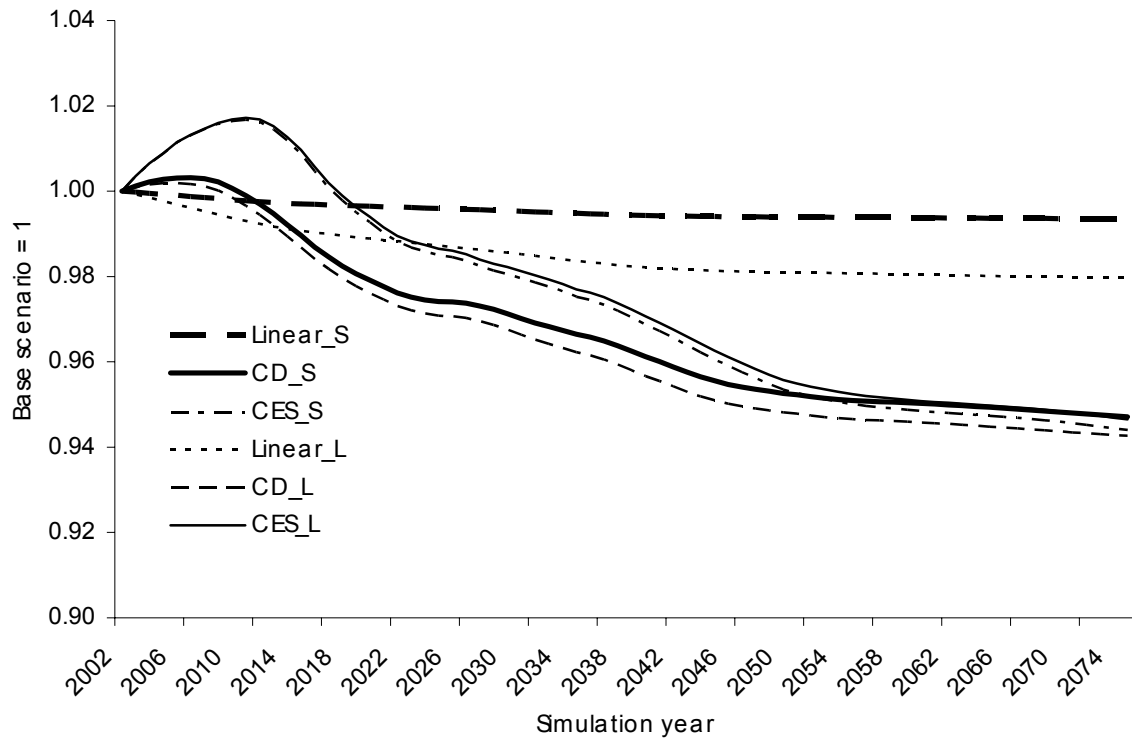
Note: Linear represents linear aggregation, CD represents Cobb-Douglas aggregation, Nested CES represents a nested Constant Elasticity of Substitution aggregation. Suffix L indicates large productivity differential and suffix S indicates small productivity differential.

Finally, we want to mention the effect of ageing on the net foreign asset position of the Austrian economy. This variable provides an insight into the closure of the model. Austria as a small open economy takes international output growth rates, interest rates, and prices as given. Furthermore, imbalances in national savings will be compensated by corresponding current account imbalances, which accumulate over time into the stock of net foreign assets. Interest payments on net foreign assets either increase (net creditor) or decrease (net debtor) the consumption possibility set of domestic households and act as a, albeit slow, correction mechanism on the domestic savings imbalance. Basically due to the assumed negative growth differential between Austria and the rest of world, the net foreign asset position gets positive after ten years and grows continuously thereafter.<sup>10</sup> In Figure 10 we relate all alternative solutions of the model to the original base scenario with a constant probability of death taken from *Baumgartner et al.* (2005) and we add the scenario with the time varying probability of death. Increases in the probability of death result in a lower but still positive net foreign asset position. This confirms theoretical expectations, because a shorter average life-span should result in lower aggregate

<sup>10</sup> We assume convergence of per capita income in the rest of the world at the rate suggested by conventional growth regressions (e.g. Barro – Sala-i-Martin, 1995).

savings. Interestingly, the case of CES-aggregation also leads to lower asset accumulation as compared to a constant probability of death scenario. On the other hand, linear and Cobb-Douglas aggregation further drive up the net foreign asset position. The trajectories for the ratio of net foreign assets to GDP are unstable, because we continue to shock the model with an increasing life expectancy until 2055 and the stock adjustment process works only slowly.

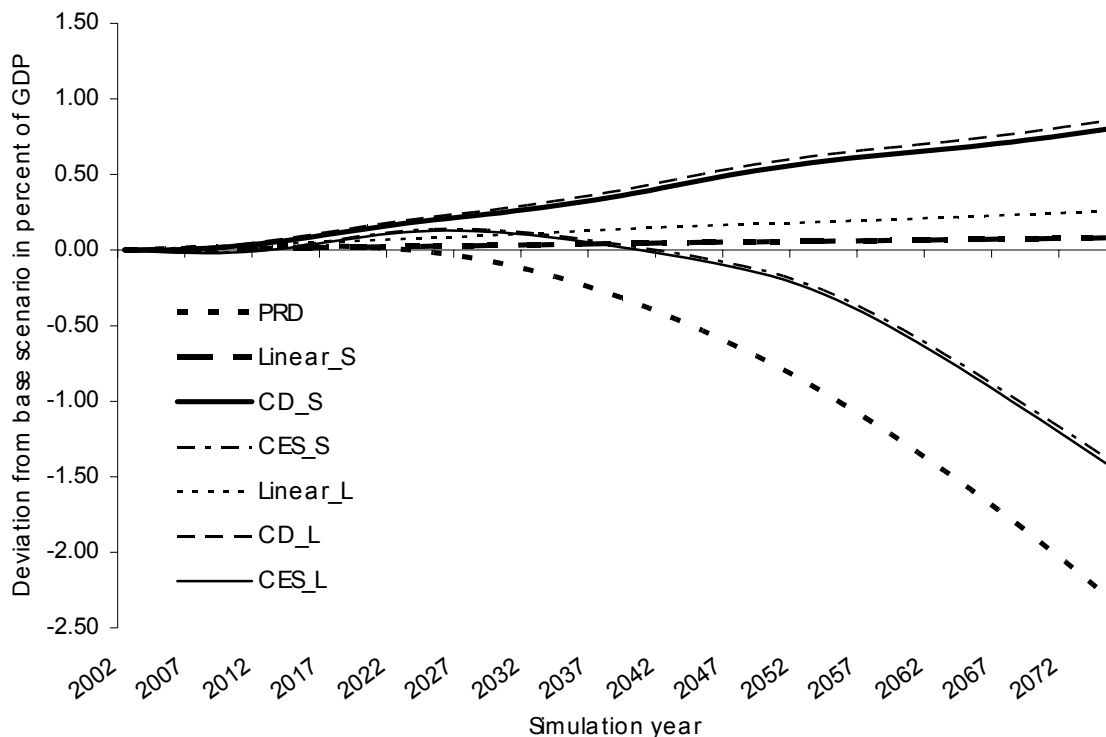
Figure 9: Simulation Results for Capital Labor Ratio



Note: Linear represents linear aggregation, CD represents Cobb-Douglas aggregation, Nested CES represents a nested Constant Elasticity of Substitution aggregation. Suffix L indicates large productivity differential and suffix S indicates small productivity differential.



Figure 10: Simulation Results for Net Foreign Asset Position relative to GDP



Note: PRD represents time varying probability of death, Linear represents linear aggregation, CD represents Cobb-Douglas aggregation, Nested CES represents a nested Constant Elasticity of Substitution aggregation. Suffix L indicates large productivity differential and suffix S indicates small productivity differential. Comparison to base scenario with constant probability of death.

Relaxing another assumption of the model mitigates the already small negative effect from age-specific productivity differentials. We assume an exogenous rate of total factor productivity growth of 1.67 percent per year. Due to the dwindling working age population and the shift between age groups the growth rate of real output decelerates to 1.4 percent annually. *Cutler et al.* (1990) suggest a possible negative relationship between the growth of the labor force and the rate of technical progress: If labor gets scarce, innovation will be focused on labor saving technical progress and ageing feeds positively back on the overall rate of technical progress. In a cross section of industrialized countries they can prove a significant negative relationship between total factor productivity growth and labor force growth with a coefficient between -0.26 and -0.97, depending on the specification of the model. Accordingly, a decline in the long run growth rate of the labor force by one percentage point would improve the annual rate of total factor productivity growth by roughly 0.3 to 1.0 percentage points per year.

In our base scenario the average growth rate of the labor force over the simulation period from 2003 through 2075 will be 0.7 percentage points below its value during the three decades preceding the year 2003. The estimates by *Cutler et al.* (1990) would thus imply an upsurge in the total factor productivity growth rate in the range between 0.2 to 0.7 percentage points. This

is a rather challenging amount, given the expected lower receptiveness of an ageing society for new technologies. To get an idea about the necessary amount of the additional total factor productivity growth, which is needed to fully compensate for the reduction in output per worker suggested by our simulations, we introduce an endogenous feedback mechanism from changes in the labor force to total factor productivity growth into A-LMM. We calibrate the adjustment parameter of total factor productivity such that the level of output per worker in 2075 corresponds to the base scenario without age specific productivity differentials. The rate of total factor productivity has to increase from 1.67 percent per year to 1.674 (linear aggregation with small productivity differentials) and 1.722 (Cobb-Douglas aggregation with large productivity differentials), respectively.

## 6. Summary and conclusions

We integrate age specific productivity differentials into a long-run calibrated neoclassical growth model for the Austrian economy, which fully integrates the actual population forecast. The model is based on an aggregate production function of Cobb-Douglas type with exogenous technical progress. The labor market is highly disaggregated with six age groups for males and females in the working age and accounts for cohort specific participation rates. The literature on age specific productivity differentials usually stresses that training on the job builds up human capital and that starting around the age of 50 physical and cognitive abilities, competency of inductive reasoning, and retentiveness start to decline. This results in a hump shaped profile for age specific productivity.

We take two profiles reflecting either small or large differentials in productivity relative to the cohort of 25 through 34 year olds. This assumption results in a more or less hump shaped pattern for the age specific productivity. Then we compute an average productivity index using three different aggregation functions: linear, Cobb-Douglas, and a nested Constant Elasticity of Substitution (CES). The productivity index allows us to rescale labor in the simulation model from full time equivalents into efficiency units.

If we restrict the aggregation function to the linear case, the simulation results show only a small variation in output and other macroeconomic key variables. If one is prepared to assume perfect substitutability of labor, the consequences of ageing to the long-run growth potential are negligible. Introducing imperfect substitutability by using Cobb-Douglas or nested CES aggregation functions creates relatively small long run effects on average productivity, real output, and per-worker wages in the magnitude of minus six percent as compared to the base scenario with uniform labor productivity. The expected change in the demographic structure is not evenly distributed over time.

The most important reason for the small size of age related productivity differentials on the levels of output and wages is that the development of the working age population is not unidirectional over the whole simulation horizon. In the first fifteen years of the simulation demographers expect an increase of the age bracket with the highest productivity. Starting from approximately 2015, the age group of 50 to 65 year olds gains in size, mainly at the expense of high productivity middle aged cohorts. Age related productivity and less than perfect substitutability imply comparatively high growth over the first years of the simulation. Lower growth rates start to set in around 2015 but the higher base reached in 2015, as compared to the base scenario, suffices to generate only small differences at the end of the simulation period.

The marginal productivity condition for factor demand is a stabilizing force in the model. The marginal product of labor and capital, respectively, react with opposite sign to ageing. The lower amount of labor in efficiency units, due to the lower productivity of the youngest and oldest workers, is accompanied by a lower capital stock such that the marginal productivity of capital converges to the same long run value as in a model with uniform labor productivity.

We also show that the negative growth effect from age-specific productivity profiles can be easily compensated, if one is willing to believe that the abating dynamism in an ageing society is compensated by a slightly bigger increase in ingenuity caused by the advancing scarcity of labor. The rate of total factor productivity growth has to be increased from 1.67 percentage per year to 1.674. This fully compensating increase is far below estimates for the feed back between productivity growth and changes in the labor force from a cross section of countries, and it remains clearly within the confidence interval for estimates of the rate of total factor productivity growth. All in all, we conclude that age-specific productivity profiles appear likely to have small consequences for the future economic performance.

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