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150

**Reihe Ökonomie
Economics Series**

A Sunspot Paradox

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March 2004

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Founded in 1963 by two prominent Austrians living in exile – the sociologist Paul F. Lazarsfeld and the economist Oskar Morgenstern – with the financial support from the Ford Foundation, the Austrian Federal Ministry of Education and the City of Vienna, the Institute for Advanced Studies (IHS) is the first institution for postgraduate education and research in economics and the social sciences in Austria. The **Economics Series** presents research done at the Department of Economics and Finance and aims to share “work in progress” in a timely way before formal publication. As usual, authors bear full responsibility for the content of their contributions.

Das Institut für Höhere Studien (IHS) wurde im Jahr 1963 von zwei prominenten Exilösterreichern – dem Soziologen Paul F. Lazarsfeld und dem Ökonomen Oskar Morgenstern – mit Hilfe der Ford-Stiftung, des Österreichischen Bundesministeriums für Unterricht und der Stadt Wien gegründet und ist somit die erste nachuniversitäre Lehr- und Forschungsstätte für die Sozial- und Wirtschaftswissenschaften in Österreich. Die **Reihe Ökonomie** bietet Einblick in die Forschungsarbeit der Abteilung für Ökonomie und Finanzwirtschaft und verfolgt das Ziel, abteilungsinterne Diskussionsbeiträge einer breiteren fachinternen Öffentlichkeit zugänglich zu machen. Die inhaltliche Verantwortung für die veröffentlichten Beiträge liegt bei den Autoren und Autorinnen.

Abstract

Calibrated models of the business cycle typically assume a certain frequency at which economic agents take decisions. In this paper I show that the local stability properties of dynamic stochastic general equilibrium macro models may depend on the length of a period in the model economy. This leads to the following paradoxical situation: For given parameters, and in particular those assigning values of imperfections in the economy, the economy may be driven by sunspots at some frequencies while sunspots can have no impact at other frequencies.

Keywords

Sunspots, indeterminacy, high frequency, temporal aggregation

JEL Classification

C60, E30

Comments

I gratefully acknowledge that comments of Roger Farmer have significantly improved this paper.

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1 Introduction

Recently there has been a growing literature¹ pointing to the possibility of business cycles driven by self-fulfilling beliefs or “animal spirits”. It has turned out that standard dynamic general equilibrium models of the business cycle can give rise to those phenomena, if they are augmented by some kind of imperfection, like externalities or imperfect competition. Typically, the literature studying the theoretical possibility of sunspots in those models uses a continuous time setup, because the local stability properties corresponding to the existence of stationary sunspot equilibria are more easily formulated in such a setting. On the other hand, calibrated models of the business cycle typically assume a certain frequency at which economic agents take decisions, which usually corresponds to the frequency at which data is collected, say quarterly, in order to make comparison with real time series easier. The purpose of this paper is to study the impact of the modelling frequency on the existence of sunspots. It turns out that moving along the spectrum between the usually considered cases of continuous time and quarterly or annual frequency may have interesting implications for the determinacy properties of an economy.

The model is defined in section 2, and solved in section 3. Section 4 explains the strategies adopted for calibrating the model economy at different frequencies. Some analytic results on the dynamics in discrete time are collected in section 5. In section 6 I formulate the main result as a paradox. Section 7 is devoted to highlighting the frequency dependence of determinacy and its geometric interpretation. Section 8 concludes and resolves the paradox.

2 The Model

The analysis is framed in a discrete time version of the model considered by Benhabib and Farmer [2].

There is a representative agent maximizing her utility over an infinite lifetime, choosing sequences of consumption, C_t , and labor supplied to the market, L_t .

$$Max \sum_{t=0}^{\infty} \psi^t (\log C_t - L_t), \quad (1)$$

Output is produced using the inputs capital and labor. There are externalities in production resulting from the fact that individual output also depends on economy-wide averages of capital, \bar{K}_t , and labor, \bar{L}_t , employed. This is the only non-standard feature of the model and may be the cause of indeterminacy or sunspot equilibria. Each individual firm takes input decisions by the rest of the firms as given. The restriction $a + b = 1$ implies constant returns to scale at the firm level.

¹See the recent survey by Benhabib and Farmer [3] for an account of the literature.

$$Y_t = AK_t^a (z_t L_t)^b \left[\bar{K}_t^a (z_t \bar{L}_t)^b \right]^\theta, \quad \theta > 0 \quad (2)$$

Hence, aggregating individual production possibilities to a social production function in a symmetric equilibrium implies increasing returns to scale at the social level, with an elasticity of scale equal to $1 + \theta$.

Output can be used for either consumption or investment purposes.

$$Y_t = C_t + I_t \quad (3)$$

Capital is accumulated according to

$$K_{t+1} = K_t (1 - \delta) + I_t, \quad (4)$$

where δ is the depreciation rate.

Labor augmenting technological progress evolves over time according to,

$$\log z_t = \log z_{t-1} + \mu. \quad (5)$$

3 Solution

The (symmetric) equilibrium conditions are:

$$C_t = b \frac{Y_t}{L_t} \quad (6)$$

$$\frac{1}{C_t} = \psi \frac{1}{C_{t+1}} \left(1 - \delta + a \frac{Y_{t+1}}{K_{t+1}} \right) \quad (7)$$

$$Y_t = AK_t^{a(1+\theta)} (z_t L_t)^{b(1+\theta)} \quad (8)$$

$$K_{t+1} = K_t (1 - \delta) + Y_t - C_t \quad (9)$$

The growth rate of Y_t , C_t and K_t in this economy is

$$\gamma = \mu \frac{b(1+\theta)}{1 - a(1+\theta)}, \quad (10)$$

since the effect of labor augmenting technical progress is magnified by increasing returns.

Defining the transformed variables

$$y_t = \frac{Y_t}{e^{\gamma t}}, \quad c_t = \frac{C_t}{e^{\gamma t}}, \quad k_t = \frac{K_t}{e^{\gamma t}}, \quad (11)$$

we obtain the following system of equations:

$$c_t = b \frac{y_t}{L_t} \quad (12)$$

$$\frac{1}{c_t} = \frac{\psi}{e^\gamma} \frac{1}{c_{t+1}} \left(1 - \delta + a \frac{y_{t+1}}{k_{t+1}} \right) \quad (13)$$

$$y_t = A k_t^{a(1+\theta)} L_t^{b(1+\theta)} \quad (14)$$

$$e^\gamma k_{t+1} = k_t (1 - \delta) + y_t - c_t \quad (15)$$

Hence, the balanced growth path is defined as the solution to the equations:

$$\bar{c} = b \frac{\bar{y}}{\bar{L}} \quad (16)$$

$$1 = \frac{\psi}{e^\gamma} \left(1 - \delta + a \frac{\bar{y}}{\bar{k}} \right) \quad (17)$$

$$\bar{y} = A \bar{k}^{a(1+\theta)} \bar{L}^{b(1+\theta)} \quad (18)$$

$$e^\gamma \bar{k} = \bar{k} (1 - \delta) + \bar{y} - \bar{c} \quad (19)$$

Let hats over variables denote relative deviations from their steady state values. Then a linear approximation of the equilibrium conditions is as follows:

$$\hat{c}_t = \hat{y}_t - \hat{L}_t \quad (20)$$

$$-\hat{c}_t = -\hat{c}_{t+1} + \left[1 - \frac{\psi}{e^\gamma} (1 - \delta) \right] \hat{y}_{t+1} - \left[1 - \frac{\psi}{e^\gamma} (1 - \delta) \right] \hat{k}_{t+1} \quad (21)$$

$$\hat{y}_t = a(1 + \theta) \hat{k}_t + b(1 + \theta) \hat{L}_t \quad (22)$$

$$e^\gamma \hat{k}_{t+1} = (1 - \delta) \hat{k}_t + \left[\left(\frac{1}{a} \right) \left(\frac{e^\gamma}{\psi} - 1 + \delta \right) \right] \hat{y}_t - \left[\left(\frac{1}{a} \right) \left[e^\gamma \left(\frac{1}{\psi} - a \right) - (1 - \delta) (1 - a) \right] \right] \hat{c}_t \quad (23)$$

The linearized system can be formulated as:

$$A_1 x_t + A_2 z_t = 0, \quad (24)$$

$$A_3 x_t + A_4 z_t + A_5 x_{t+1} + A_6 z_{t+1} = 0, \quad (25)$$

where x_t denotes the vector of state variables $(\hat{k}_t, \hat{c}_t)'$, and z_t denotes the vector of subsidiary variables $(\hat{L}_t, \hat{y}_t)'$. Then the reduced form of the dynamic system can be expressed as:

$$x_{t+1} = Mx_t, \quad (26)$$

with

$$M = -(A_5 - A_6A_2^{-1}A_1)^{-1}(A_3 - A_4A_2^{-1}A_1). \quad (27)$$

The matrices A_1 through A_6 are defined as follows:

$$A_1 = \begin{bmatrix} 0 & -1 \\ a(1+\theta) & 0 \end{bmatrix} \quad (28)$$

$$A_2 = \begin{bmatrix} -1 & 1 \\ b(1+\theta) & -1 \end{bmatrix} \quad (29)$$

$$A_3 = \begin{bmatrix} 0 & 1 \\ (1-\delta) & -\left[\left(\frac{1}{a}\right)\left[e^\gamma\left(\frac{1}{\psi} - a\right) - (1-\delta)(1-a)\right]\right] \end{bmatrix} \quad (30)$$

$$A_4 = \begin{bmatrix} 0 & 0 \\ 0 & \left(\frac{1}{a}\right)\left(\frac{e^\gamma}{\psi} - 1 + \delta\right) \end{bmatrix} \quad (31)$$

$$A_5 = \begin{bmatrix} -\left[1 - \frac{\psi}{e^\gamma}(1-\delta)\right] & -1 \\ -e^\gamma & 0 \end{bmatrix} \quad (32)$$

$$A_6 = \begin{bmatrix} 0 & \left[1 - \frac{\psi}{e^\gamma}(1-\delta)\right] \\ 0 & 0 \end{bmatrix} \quad (33)$$

4 Calibration Strategy

When modelling a specific economy at different frequencies, a choice has to be made about how to adjust the parameters of the model to the modelling frequency.

I take the steady-state ratios \bar{y}_k and \bar{c}_k as the starting point. These have certain values for a the benchmark quarterly modelling frequency. In order to reparametrize the model at higher frequencies, first these steady-state ratios are adjusted for consistency across frequencies. Let n be the number of subdivisions required to change from the low (quarterly) frequency to a higher frequency. So n would be 13 for a weekly model, 91 for a daily model, 2190 for an hourly model, etc. Let

$$\bar{y}_k^n \quad \text{and} \quad \bar{c}_k^n \quad (34)$$

denote the steady-state ratios \bar{y}_k and \bar{c}_k corresponding to a model at frequency n .

Since output and consumption are flows while capital is a stock, we impose the following conditions:

$$\bar{y}_k^n = \frac{1}{n} \bar{y}_k^1, \quad (35)$$

and

$$\bar{c}_k^n = \frac{1}{n} \bar{c}_k^1. \quad (36)$$

This ensures consistency of steady-state ratios across frequencies under time aggregation. For instance, if \bar{y}_k^{13} is the steady state output/capital ratio in the weekly model then \bar{y}_k^{13} is required to be 1/13 times the output/capital ratio in the quarterly model, denoted \bar{y}_k^1 .

Second, given these values we solve equations (17) and (19) for the parameters defined per unit of time: the discount rate contained in ψ and the depreciation rate δ . The solution is

$$\delta = 1 - e^\gamma + \bar{y}_k^n - \bar{c}_k^n, \quad (37)$$

$$\psi = \frac{e^\gamma}{1 - \delta + a\bar{y}_k^n}. \quad (38)$$

It seems worth pointing out that the parameter for the imperfection (externality), θ , does not affect the calibration of δ and ψ over frequencies.

For exogenous growth the change in the modelling frequency corresponds to stretching the time line. Therefore γ is adjusted as $\gamma = \gamma^*/n$, where γ^* denotes the growth drift at the basic (quarterly) frequency. This approach will be called “consistent calibration”² or “method 1” in the rest of the paper, since I consider it the preferred method.

A number of authors have calibrated high-frequency models of the business cycle, e.g. Christiano [5], Cogley and Nason [6], and Chari, Kehoe and McGrattan [4]. They use simple transformation rules to adjust parameter values across frequencies. In our model this would correspond to the following: Let δ^* and ψ^* denote the quarterly depreciation rate and discount factor, respectively. Then this “standard” method amounts to solving, at each frequency n , the equations

$$(1 - \delta)^n = 1 - \delta^*, \quad (39)$$

$$\psi^n = \psi^*, \quad (40)$$

for the depreciation rate δ and the discount factor ψ . The simplicity of this method comes at the cost of a fundamental inconsistency: Nothing guarantees that the steady

²Aadland and Huang [1] use the same expression for their approach, which is similar to mine. However, instead of using the output-capital ratio and the consumption-capital ratio as the starting point, they impose the consistency requirement across frequencies directly on levels of all the variables, which, depending on the model, may lead to an identified system in the parameters or not. Matching the two ratios in the model considered here leads to an exactly identified system, and can cope with situations where there is steady state growth, which cancels out in the ratios.

state around which we evaluate the model is the same at two different frequencies; “the same” in the sense that steady state values would be consistently aggregated over time. So comparing properties (e.g. stability properties) of models at different frequencies may not really be “comparing like with like”, but maybe rather “comparing apples with pears”. The latter approach will be called “standard calibration” or “method 2” in what follows.

The baseline quarterly calibration is according to the values given in Schmitt-Grohé [7]: $a = 0.3$; $b = 0.7$; $\gamma = 0.004$; $\psi = 0.994$; $\delta = 0.026$.

The following tables shows the calibrated values for the depreciation rate and the discount factor at annual, quarterly, monthly, weekly, daily, and hourly frequencies.

Depreciation rate, δ		
Frequency per quarter	Method1	Method2
0.25	0.10388990	0.10000000
1	0.025996254	0.025996254
3	0.0086671707	0.0087416110
13	0.0020002717	0.0020241125
91	0.00028575881	0.00028941008
2190	1.1874034e-005	1.2027384e-005

Discount factor, ψ		
Frequency per quarter	Method1	Method2
0.25	0.97738476	0.97692308
1	0.99418016	0.99418016
3	0.99804735	0.99805628
13	0.99954825	0.99955111
91	0.99993542	0.99993586
2190	0.99999732	0.99999733

The differences between the methods appear rather small in absolute terms. Certainly the numbers are not the same if we depart from the baseline quarterly calibration, and it is not clear, a priori, what those differences imply for the stability properties of the model, which we are interested in. Hence, for all results about indeterminacy regions given below, I will perform sensitivity analysis checking the two methods against each other.

Figures 1 and 2 illustrate how the relation between parameters obtained by methods 1 and 2 changes, as the frequency increases from annual to hourly. Figure 1 gives the ratio between the depreciation factors $(1 - \delta)$ obtained by the two different methods of frequency adjustment. Figure 2 does the same for the discount factor ψ . For both parameters, the biggest relative differences occur for very low frequencies.

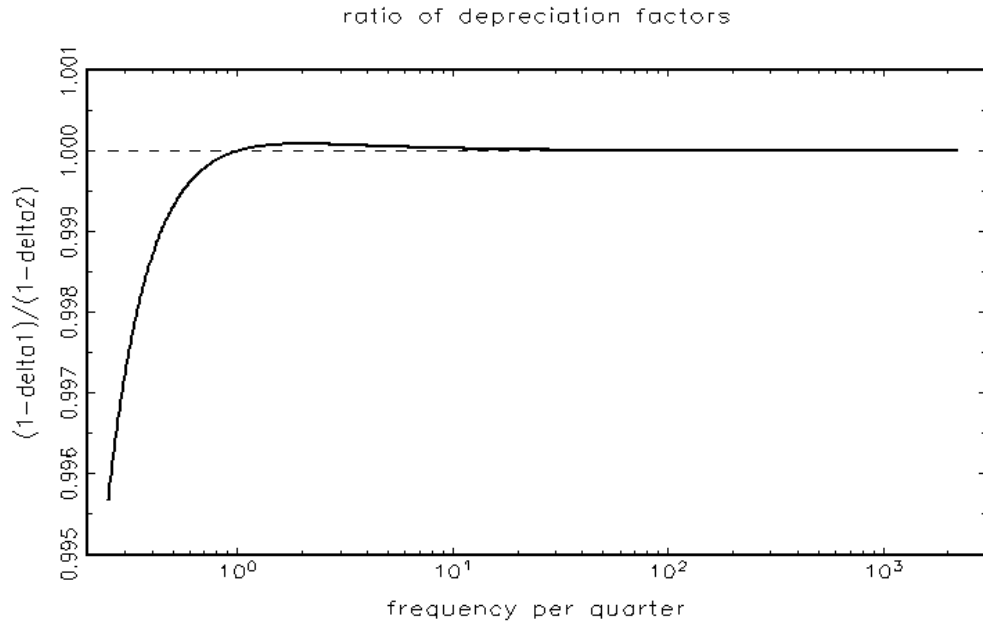


Figure 1

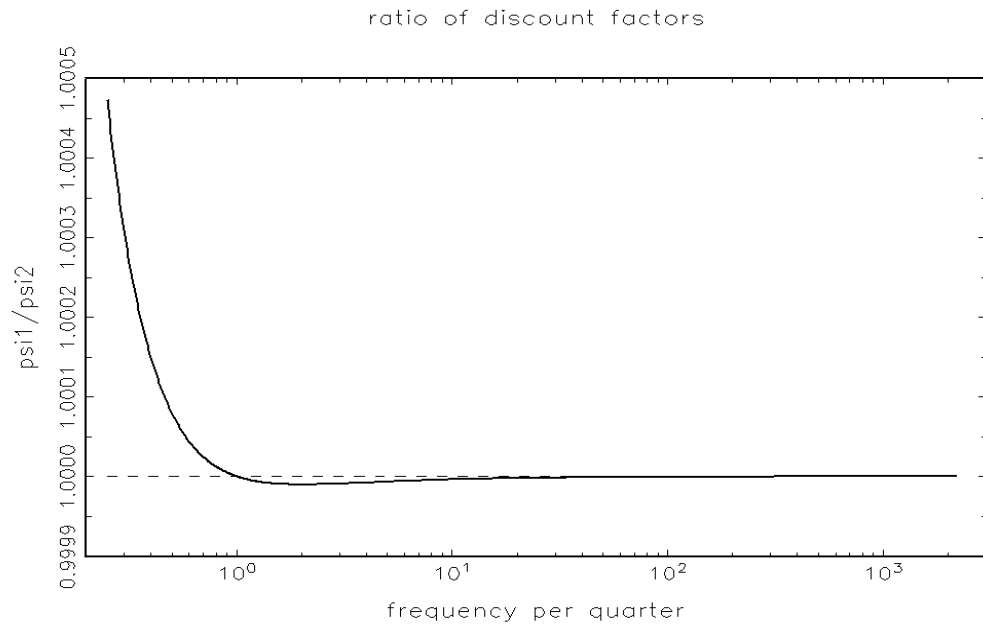


Figure 2

5 Analytic Conditions

Starting from the solution of the model, conditions under which the dynamics of the economy become indeterminate can be derived. What needs to be checked are the local stability conditions of the reduced form of the dynamic system

$$x_{t+1} = Mx_t. \quad (41)$$

The economy is indeterminate if and only if both eigenvalues are less than one in modulus. An equivalent formulation is the following:

$$-1 < Det(M) < 1 \quad (42)$$

$$-(1 + Det(M)) < Trace(M) < (1 + Det(M)) \quad (43)$$

The analytic expressions for the trace and the determinant of the discrete time system are:

$$Det(M) = \frac{1}{\psi} \left(1 + \frac{\theta(e^\gamma - \psi(1 - \delta))}{\tau} \right), \quad (44)$$

$$\text{where } \tau = e^\gamma - b\psi(1 + \theta)(1 - \delta). \quad (45)$$

$$Trace(M) = \frac{1}{e^\gamma} \left(1 - \delta + \frac{\frac{e^\gamma}{\psi} - (1 - \delta)}{1 - b(1 + \theta)} \right) + \frac{e^\gamma}{\tau} (1 - b(1 + \theta)) \dots \quad (46)$$

$$\dots + \frac{\theta(e^\gamma - \psi(1 - \delta))}{e^\gamma \tau} \frac{1}{a} \left((1 - \delta)(1 - a) - e^\gamma \left(\frac{1}{\psi} - a \right) - \left(\frac{e^\gamma}{\psi} - 1 + \delta \right) \frac{b(1 + \theta)}{1 - b(1 + \theta)} \right)$$

Considering the expression for the determinant, the following two remarks become straightforward:

Remark 1 *If $\theta = 0$, i.e. if there is no imperfection in the economy, equilibrium is determinate, since $1/\psi > 1$.*

Remark 2 *The discrete time analogue of the Benhabib-Farmer [2] necessary condition is*

$$\tau = e^\gamma - b\psi(1 + \theta)(1 - \delta) < 0. \quad (47)$$

The latter statement follows from the fact that otherwise, given that $(e^\gamma - \psi(1 - \delta)) > 0$, the determinant could not be less than unity. However, as the frequency goes to infinity, and $\gamma, \delta \rightarrow 0, \psi \rightarrow 1$, their well known slope condition (labor demand sloping up steeper than labor supply)

$$b(1 + \theta) - 1 > 0$$

is obtained for the limiting continuous time case.

As will also be emphasized below when discussing the results for different frequencies, in the discrete time case the qualification of this condition as a necessary one becomes crucial. Unlike in the continuous time case the greatest lower bound of the indeterminacy region (in the space of the imperfection, as parametrized by the externality θ) is not necessarily obtained by just solving for $\tau = 0$. On top of the necessary condition (47) it is the entire system of inequalities in (42) and (43) that matters. Especially at low frequencies the gap between values for θ that make this necessary condition hold and those which actually satisfy all inequalities, as necessary and sufficient for indeterminacy, can be considerable.

The main message is that at frequencies smaller than infinity effects from parameters defined per unit of time, such as growth, depreciation and discounting, become relevant.

6 The Paradox

Now let's take the natural step of combining the previous two sections: First, take the methods for recalibrating the model at different frequencies. Second, given this recalibration of parameters across frequencies, check for indeterminacy at all frequencies. In particular, check for the range of the indeterminacy region in the space of the parameter θ , which can be interpreted as the magnitude of externalities or, equivalently, returns to scale for which the economy becomes a sunspot economy. The paradoxical outcome can be formulated as the following

Remark 3 *Let economies A and B be defined by the preferences and technology as given in the model above. The only difference between economy A and B being that they are calibrated at different frequencies, n_A and n_B . Then there are values for the imperfection, as parametrized by θ , such that economy A is an indeterminate (sunspot) economy, while equilibrium in economy B is determinate.*

This is illustrated in the following Figure 3. The x-axis refers to the value of θ and the y-axis to the frequency of the model on a logarithmic scale. So the horizontal lines correspond to indeterminacy regions at annual, quarterly, monthly, weekly, daily, and hourly frequency of the model calibrated as explained above. We see that we could put vertical lines for θ equal to some value which would hit some of the horizontal lines but not others. Figure 3 also shows two further results, mentioned in the section on analytic conditions above:

First, there is a dashed vertical line, that shows the bifurcation point between determinate and sunspot economies for the continuous time case. We see that the indeterminacy region in discrete time converges to this value as the frequency increases.

Second, there are crosses denoting at each frequency the value for the externality parameter θ where the frequency-adjusted necessary condition in (47) starts to hold.

It turns out that there is a gap between this point and the point where the indeterminacy region actually starts. The gap gets smaller as the frequency increases and vanishes for the limiting continuous time case.

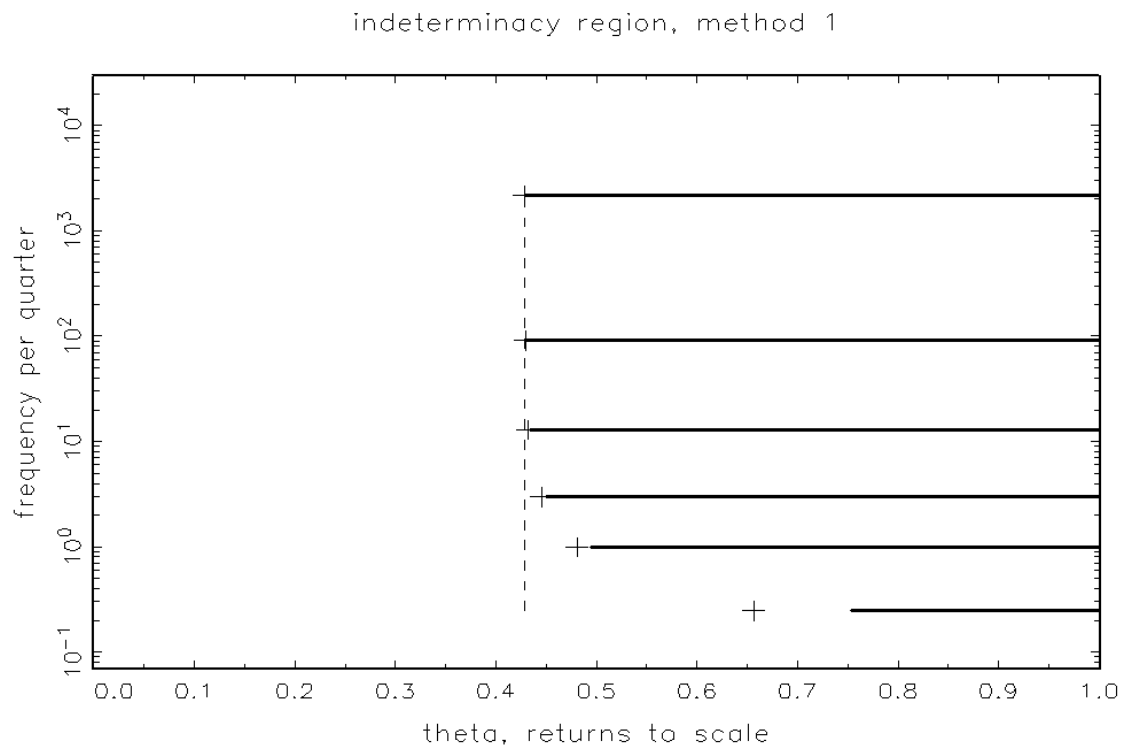


Figure 3

Figure 4 shows the same under the assumption that recalibration across frequencies is performed according to method 2 instead of method 1. The pattern looks very similar. This means that the frequency dependence of indeterminacy region is not a consequence of the specific recalibration method used. This is not surprising keeping in mind the analytic conditions above, which suggested that growth, depreciation and discounting matter as absolute magnitudes for indeterminacy in discrete time models. The magnitude of those parameters follows immediately from the frequency and was already shown in section 4 to be quite independent of the calibration strategy adopted.

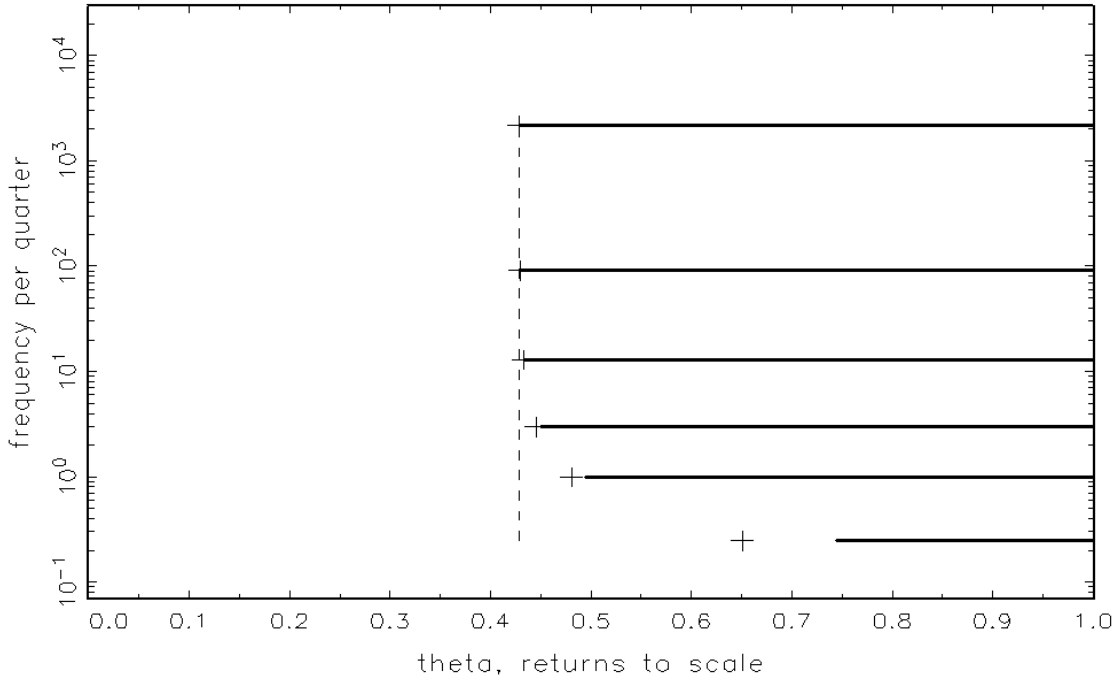


Figure 4

7 Frequency Dependence

While in the previous section the focus was on the indeterminacy region for given frequencies, we now turn to the impact of frequency change on stability properties, given a certain calibration, in particular a given value of the externality parameter θ . Let's first consider a result for a specific case, and then take a look at the more general principle of frequency change, and what it means for the geometry of stability conditions.

Figure 5 shows how the moduli of the eigenvalues of the linearized dynamic system, as represented by the matrix M , change, as the frequency changes. We use the benchmark calibration defined above and set $\theta = 0.5$. The only change to the model made is that parameters defined per unit of time, i.e. γ, δ, ψ , are adjusted to the frequency as described in the section on the calibration strategy (according to method 1). We consider a broad spectrum of frequencies, moving continuously from the annual frequency to the hourly frequency.

The striking result is that a single model can display a great variety of dynamics, depending on the frequency at which it is evaluated. For low frequencies there is one root outside the unit circle and one root inside, indicating saddle path stability, or determinacy. As the frequency increases, both roots move inside the unit circle,

meaning that there is indeterminacy. If frequency is increased even further, the both roots remain stable but become (conjugate) complex, which shows in the picture as the part of the graph, where there is just one line (since conjugate complex eigenvalues have the same modulus). In order to avoid the trivial result that eigenvalues simply converge to the unit root as frequency increases, they are rescaled by raising them to power n , where n denotes the (quarterly) frequency.

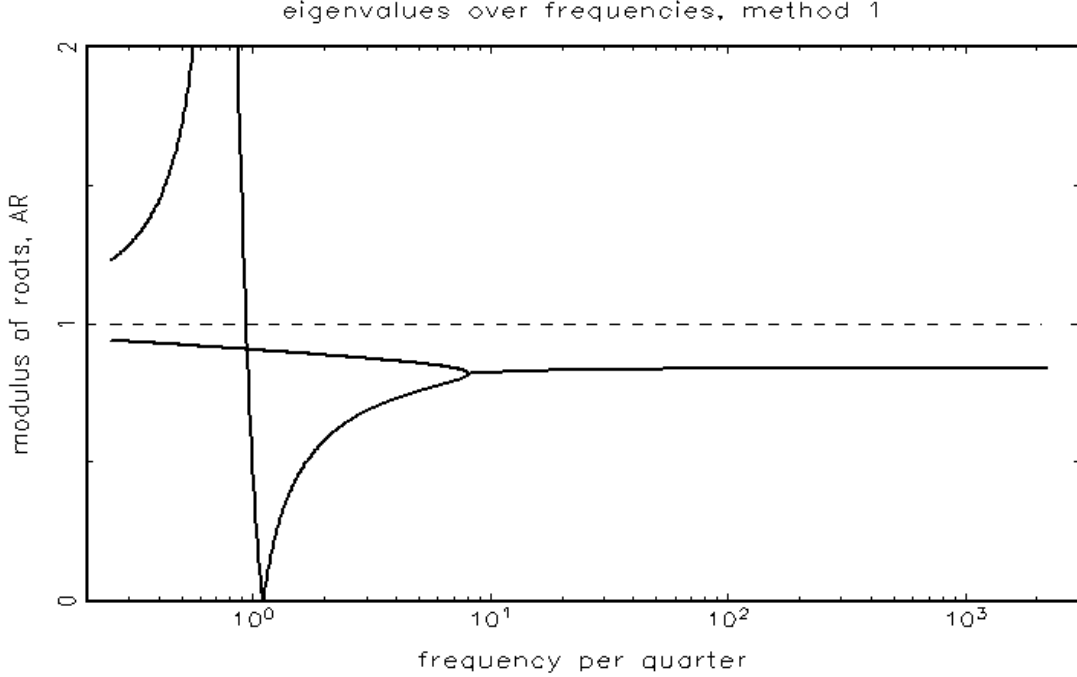


Figure 5

When changing the frequency of a linear dynamic system and studying its stability properties, the geometric interpretation of such a frequency change becomes very useful. In particular, this interpretation explains how the limiting continuous time stability conditions arise as we let frequency go to infinity. Equation (26) is transformed to difference form and rescaled by the frequency n .

$$x_{t+1} - x_t = n(M - I)x_t \quad (48)$$

The stability region, as expressed in terms of trace and determinant, for this system is not the centered triangle described by (42) and (43) anymore, but rather a distorted triangle with corners at $(0, 0)$; $(-2, 0)$; $(-4, 4)$. As the frequency changes away from 1, the trace is rescaled linearly by n , while the determinant grows by n^2 . This is illustrated in Figure 6.

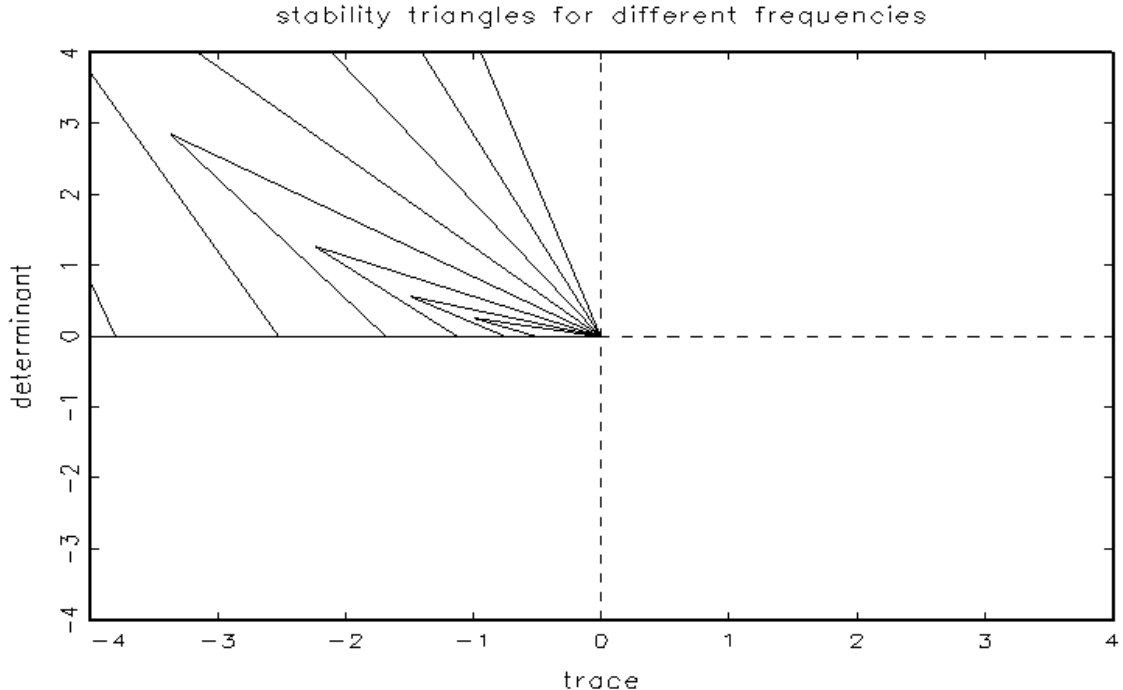


Figure 6

Hence, as the frequency goes to infinity the stability triangle explodes to the entire $(-/+)$ quadrant, nesting the limiting continuous time case where sink stability is equivalent to negative trace and positive determinant. Let's now apply this principle to the present model.

Figure 7 shows the combinations of trace and determinant of (48) for a sequence of frequencies, ranging from annual to hourly. A subsequence of elements is labeled $P1$ to $P6$ in the picture, to facilitate explanation. We use the same calibration as mentioned above for the eigenvalues. Combinations for those frequencies that do not correspond to indeterminate equilibria are indicated by a plus sign, while indeterminate frequencies are indicated by a solid circle. Combinations for very low frequencies (starting at $P1$ for the annual frequency) are closely spaced in the $(+/-)$ quadrant, and as the frequency increases points (take as an example $P2$) still remain determinate. At a certain frequency the trace/determinant combination jumps from $P3$ to $P4$ in the $(-/+)$ quadrant. The jump occurs at the frequency that makes the adjusted necessary condition in (47) hold true. But equilibrium is still not indeterminate at $P4$, as indicated by the plus sign. This illustrates graphically the gap between necessity and sufficiency for those discrete time models. As frequency increases further, combinations move (take as an example $P5$), and, importantly, at the same time the triangles corresponding to areas of sink stability increase in size. There is a frequency, in this case it is 0.94539896, at which the trace and determinant just enter their corresponding triangle. (Note that this can be checked to correspond to the frequency at which both eigenvalues become less than one in modulus in Figure

5.) The dotted lines correspond to the triangle of the frequency corresponding to $P5$. It is smaller, since the frequency is lower, and does not fit $P5$. At this point trace and determinant which are still too far out with respect to the appropriate triangle. It is only at the next (higher) frequency that the combination of trace and determinant enters the corresponding stability triangle.

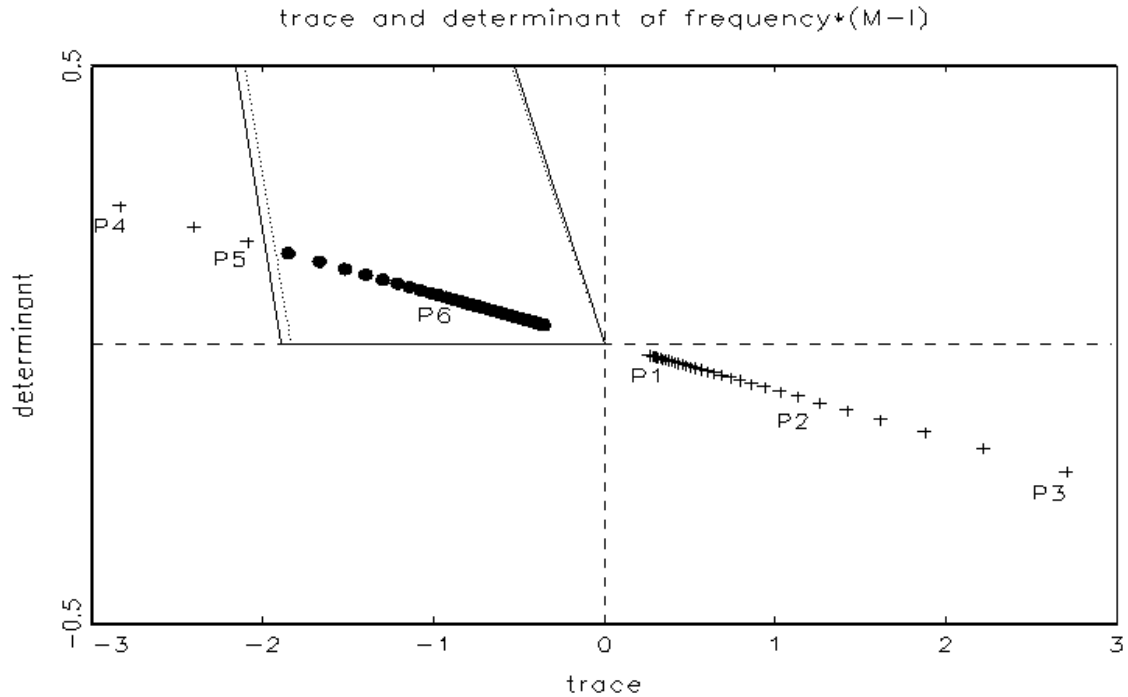


Figure 7

8 Conclusion

This exercise has shown that indeterminacy regions of economic models of the business cycle may depend on the modelling frequency. The results are robust to the method by which reparametrization across frequency is performed. Among those methods one seems preferred to the other, since it guarantees that steady state ratios are consistent, if time aggregated over frequencies. The frequency dependence of stability conditions gives rise to a paradoxical situation, where identical economies can be determinate or sunspot economies, depending on the frequencies at which decisions of agents are modelled. I conclude that the paradox is one which falls into the category of “insufficient distinction”: The paradox is resolved by recognizing that the economies are just not identical because agents face different decision frequencies. The length of a period turns out to be a fundamental parameter.

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