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**Working Paper**

## Modeling national accounts sub-aggregates: An application of non-linear error correction

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149

Reihe Ökonomie  
Economics Series

**Modeling National Accounts  
Sub-Aggregates**  
An Application of Non-Linear  
Error Correction

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March 2004

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Founded in 1963 by two prominent Austrians living in exile – the sociologist Paul F. Lazarsfeld and the economist Oskar Morgenstern – with the financial support from the Ford Foundation, the Austrian Federal Ministry of Education and the City of Vienna, the Institute for Advanced Studies (IHS) is the first institution for postgraduate education and research in economics and the social sciences in Austria. The **Economics Series** presents research done at the Department of Economics and Finance and aims to share “work in progress” in a timely way before formal publication. As usual, authors bear full responsibility for the content of their contributions.

Das Institut für Höhere Studien (IHS) wurde im Jahr 1963 von zwei prominenten Exilösterreichern – dem Soziologen Paul F. Lazarsfeld und dem Ökonomen Oskar Morgenstern – mit Hilfe der Ford-Stiftung, des Österreichischen Bundesministeriums für Unterricht und der Stadt Wien gegründet und ist somit die erste nachuniversitäre Lehr- und Forschungsstätte für die Sozial- und Wirtschaftswissenschaften in Österreich. Die **Reihe Ökonomie** bietet Einblick in die Forschungsarbeit der Abteilung für Ökonomie und Finanzwirtschaft und verfolgt das Ziel, abteilungsinterne Diskussionsbeiträge einer breiteren fachinternen Öffentlichkeit zugänglich zu machen. Die inhaltliche Verantwortung für die veröffentlichten Beiträge liegt bei den Autoren und Autorinnen.

## **Abstract**

Many macroeconometric models depict situations where the shares of the major demand aggregates in output are stable over time. The joint dynamic behavior of the considered demand aggregate and output may thus be approximated by a cointegrated vector autoregression. However, the shares of many demand sub-aggregates in output are rather mobile and changing over time. In order to simultaneously capture the flexibility of the shares of the sub-aggregates and the long-run constancy of the share of the total aggregate, we consider trivariate systems of two macroeconomic sub-aggregates and output with error-correction terms that are non-linear functions of the original variables. The merits of the models are evaluated by means of several forecasting experiments.

## **Keywords**

Macroeconomic accounts, great ratios, non-linear error correction, forecasting

## **JEL Classification**

C32, C53, E27

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# 1 Introduction

Econometric forecasting necessarily has to strike a balance between statistical evidence and plausibility. It is well known that excellent short-run macroeconomic forecasts can be obtained from models that generate quite infeasible economies in the longer run. For example, ENGLE AND YOO (1987) have shown that cointegration is able to improve prediction only at larger forecast horizons, even in simulated structures, where the cointegrated model is known to be true (see also CHRISTOFFERSEN AND DIEBOLD, 1998). However, cointegration expresses plausible and well accepted economic equilibrium conditions. This means that, for example, good one-quarter forecasts for private consumption can be derived from models that imply enormous or negative household saving in the longer run. Similarly, good forecasts for the unemployment rate sometimes can be derived from models implying a longer-run rate below zero or above 50%.

If a forecaster aims at longer horizons—developing scenarios for several decades—pure extrapolation of statistically identified coefficient structures often fails. In a period of slow growth, predicting the unemployment rate to rise by 0.5 percentage points a year may work for some more years. To predict that rate for values of 20% after 30 years may appear implausible. In order to attain plausibility, long-run equilibrium conditions have to be accounted for, even if they are rejected on statistical grounds for a sample of limited time range. On the basis of this reasoning, the current paper focuses on the longer-run implications of certain error-correction models for macroeconomic scenarios rather than on statistical tests.

The problem at hand is one that concerns longer-run scenarios, even though it has been mostly ignored in the forecasting literature. Demand aggregates such as private consumer expenditure, in short consumption, and gross fixed capital formation, in short investment, are known to keep to a rather stable share of total output as measured by gross domestic product (GDP). In some theoretical models, the properties of long-run constant ratios of consumption to output and of investment to output can also be derived from the assumptions of collective utility maximization and relatively general forms of aggregate production functions (see, e.g., ROMER, 1996). In macroeconomic works, this long-run constancy of ‘great ratios’ is usually captured by the econometric condition of cointegration that ties demand aggregates to roughly constant shares of output in dynamic equilibrium (see also STOCK AND WATSON, 1988, KUNST AND NEUSSER, 1990).

Usually, economic forecasters are also required to compile predictions for subaggregates of the main demand aggregates, such as consumption of durable goods and investment in machinery. For these subaggregates, no long-run relationships are known or have been found to hold. For example, wealthier economies are found to spend more on durable goods than poorer economies, thus we would expect the share of durables in output to increase as the economy develops. Imposing cointegration or error correction with regard to the subaggregates may result in implausible longer-run scenarios. Unfortunately, imposing long-run constancy of the main demand shares in equations for subaggregates implies non-linear error correction models. These will be outlined in Section 2.

For longer-run scenarios, asymptotic properties of such error-correction models are crucial. Simulating the basic non-linear error-correction models usually leads to a high probability of one of the subaggregates disappearing from the market, while the other subaggregates take over the whole time-constant share. For example, construction investment may disappear altogether, while the total investment share remains at 20%. Although a universally accepted economic theory for the long-run demand for investment components is missing, such a perspective seems implausible. We show how restricting the deterministic drift part of the cointegrating models may serve in a considerable delay of such unwanted long-run features.

As empirical examples for the techniques, we use data for investment components in Austria, France, and the United Kingdom. Minor components are aggregated such that total capital formation is split among the two parts of construction investment, including residential structures, and the remainder. The scenarios serve to highlight the main longer-run features of the models. In real applications, the same techniques can be used for consumption components and for a finer disaggregation of investment. The data are described in Section 3, while Section 4 presents the main empirical results. Section 5 presents a small prediction evaluation experiment for the British series and points to some of the problems of merits of the non-linear error-correction model in empirical applications. Section 6 concludes.



## 2 Methodology

### 2.1 Traditional error-correction models

According to economic theory and also to observation, the share of a major demand aggregate—say, consumption or investment—in output is roughly constant in the long run. If a modeler just wished to fulfil the task of developing a joint model for the considered demand aggregate and output, she may approximate dynamic behavior by a cointegrated vector autoregression, otherwise known as vector error-correction model (VECM)

$$\begin{pmatrix} \Delta x_t \\ \Delta y_t \end{pmatrix} = \mu + \alpha(x_{t-1} - y_{t-1}) + \Gamma \begin{pmatrix} \Delta x_{t-1} \\ \Delta y_{t-1} \end{pmatrix} + \varepsilon_t \quad . \quad (1)$$

In (1),  $x$  denotes the demand aggregate in logarithms and  $y$  denotes output in logarithms. Depending on the loading vector  $\alpha$ , which typically has a negative first and a positive second entry, and on  $\Gamma$ , the model is marginally stable in the sense that first differences  $(\Delta x, \Delta y)$  form a stationary and ergodic process, while  $(x, y)$  do not. For some applications, particularly for the consumption-output ratio, the second entry of  $\alpha$  is close to zero and  $y$  becomes weakly exogenous for  $\alpha$ . The model is called error-correcting, as the variable  $x - y$  tends to move back to its long-run equilibrium. Nonlinear functions of  $x - y$ , such as the original ‘great’ or characteristic ratio  $\exp(x - y)$ , will also remain close to their equilibrium. For a good presentation of the linear VECM and its statistical features, see JOHANSEN (1995).

In large macroeconometric models, many demand aggregates are decomposed into subaggregates. Gross fixed investment is disaggregated into investment on construction and on equipment. Private consumer spending may be disaggregated into spending on services, durable goods, and non-durables. Therefore, there is a theoretical exact adding-up condition

$$\exp(x) = \exp(z_1) + \dots + \exp(z_q)$$

with  $z_j$ ,  $j = 1, \dots, q$  denoting the subaggregates. In some cases, the share of some subaggregate in output will itself be constant in the long run. Then, bivariate models of  $(z_1, y)$  can be built in the vein of (1), albeit with some loss of information relative to larger models. Alternatively, one may consider  $(q + 1)$ -variate VECMs for  $(z_1, \dots, z_q, y)$  with a matrix  $\alpha$  of dimension  $q \times (q - 1)$  and the  $q - 1$  error-correction terms  $z_1 - y, \dots, z_q - y$ .

In many other cases, individual ratios  $\exp(z_j)/\exp(y)$  are rather mobile and changing through time, while  $\exp(x)/\exp(y)$  remains stable. For example, after World War II, the share of construction in total investment and hence in total output was decreasing for several decades, in many main European economies. During that phase, perhaps excepting the immediate aftermath of the war, the investment-output ratio was roughly constant. Expenditures on machinery simply replaced expenditures on construction. Similarly, the consumption-output ratio has shown a remarkable constancy over the last few decades, while the components of household expenditure were subject to trends that reflected the increasing wealth and also shifts in taste. A relative decrease in expenditures on non-durables reflects the lesser importance of basic goods. Simultaneously, durable goods showed a relative expansion. Later on, an increased demand for luxury services implied a rising share of services in consumer expenditures. In summary, sizeable shifts occur among the subaggregates, while the total aggregate grows in parallel with the general economy.

## 2.2 A nonlinear error-correction model

In the following, the focus will be on the case  $q = 2$  in order to keep notation simple. Generalizations to larger  $q$  are straightforward. A trivariate variable  $X = (z_1, z_2, y)'$  consists of two parts of a demand aggregate and gross output. We consider the model

$$\begin{aligned} \begin{pmatrix} \Delta z_{1t} \\ \Delta z_{2t} \\ \Delta y_t \end{pmatrix} &= \mu + \alpha [\ln \{ \exp(z_{1,t-1} - y_{t-1}) + \exp(z_{2,t-1} - y_{t-1}) \} - \delta] \\ &+ \Gamma \begin{pmatrix} \Delta z_{1,t-1} \\ \Delta z_{2,t-1} \\ \Delta y_{t-1} \end{pmatrix} + \varepsilon_t, \end{aligned} \quad (2)$$

which contains a non-linear error-correction term. It is convenient to include an explicit target for the logarithmic ‘great’ ratio  $\delta$  and to separate it from economic growth represented by  $\mu$ . As in (1), the third element of  $\alpha$  may be close to zero, expressing the fact that there is no tendency in overall output to adjust to disequilibrium. In that case,  $y$  can be regarded as exogenous for the longer-run characteristics  $\alpha$  and  $\delta$ .

The model (2) is a member of a class of nonlinear dynamic models that was analyzed by ESCRIBANO AND MIRA (2001, EM). These models are char-

acterized by two main features. Firstly, the equilibrium term is a nonlinear function of a linear transform of the original variables, in EM's notation  $J(\beta'X)$  and

$$J(w) = J(w_1, w_2) = \alpha [\ln \{\exp(w_1) + \exp(w_2)\} - \delta] \quad .$$

This corresponds to (2) for  $X = (z_1, z_2, y)'$  and

$$\beta = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{pmatrix} \quad .$$

Secondly, the function  $\partial J(w)/\partial w$  follows a Lipschitz condition. This derivative can be represented as

$$\frac{\partial J(w)}{\partial w'} = \alpha \left[ \{1 + \exp(w_2 - w_1)\}^{-1}, \{1 + \exp(w_1 - w_2)\}^{-1} \right] = J_1(w) \quad .$$

The function  $J(w)$  has bounded derivatives and therefore obeys the required Lipschitz condition. According to EM's Theorem 3.7, the model (2) is stable in the sense that  $\Delta X$  has a stationary solution, if the spectral radius of the matrix

$$\begin{pmatrix} \Gamma & J_1(w) \\ \beta'\Gamma & I_2 + \beta'J_1(w) \end{pmatrix} \quad (3)$$

is less than  $1 - \epsilon$ .

For the model (2), estimation constitutes no problems, as the error-correction vector is given and thus OLS can be applied. Linear regression yields estimates  $\hat{\mu}^*, \hat{\alpha}, \hat{\Gamma}$  of parameters  $\mu^*, \alpha, \Gamma$ , where  $\mu^*$  denotes the total intercept  $\mu - \alpha\delta$ . Then,  $\hat{\delta}$  is obtained as the sample mean of the error-correction variable  $\ln \{\exp(z_1 - y) + \exp(z_2 - y)\}$ . In a second step,  $\hat{\mu}$  is obtained as  $\hat{\mu}^* + \hat{\alpha}\hat{\delta}$ .

Unfortunately, this approach may yield estimated model structures that do not fulfil important features of the observed data. Firstly, unrestricted  $\hat{\alpha}$  may contain elements that violate stability conditions. A simple remedy is to replace such elements by zero. For example,  $\hat{\alpha}_3$  may be positive, thus driving away the output variable  $y$  from the equilibrium. A valid model is obtained by replacing  $\hat{\alpha} = (\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3)'$  by  $\hat{\alpha}^* = (\hat{\alpha}_1, \hat{\alpha}_2, 0)'$ . This is not the maximum-likelihood estimate under the restriction  $\alpha_3 = 0$ , which would require a GLS-type correction.

Another critical feature of the OLS estimates may be inhomogeneous growth in  $X$ , whenever  $\mathbf{E} \Delta X$  is not scalar. This does not impair non-linear integratedness and stationarity of  $\Delta X$  or of the error-correction variable. However, inhomogeneity tends to drive the ratios  $\exp(z_j - y)$ ,  $j = 1, 2$ , to marginal values in the sense that, for the slower-growing  $z_j$ ,  $\exp(z_j - y)$  approaches 0 and the other  $\exp(z_k - y)$  with  $k \neq j$  approaches  $\exp(\delta)$ .

Growth homogeneity can be imposed in the following steps. If the mean error-correction variable is  $\delta$ , one obtains

$$\mathbf{E} \Delta X = \mu + \Gamma \mathbf{E} \Delta X$$

and therefore

$$\mathbf{E} \Delta X = (\mathbf{I} - \Gamma)^{-1} \mu \quad ,$$

where  $\mathbf{I}$  denotes the identity matrix. For example, the estimate  $\hat{\mu}$  will lead to an estimate  $m_x = (\mathbf{I} - \hat{\Gamma})^{-1} \hat{\mu}$  of  $\mathbf{E} \Delta X$ . The averaged version

$$\tilde{m}_x = \mathbf{3}^{-1} (\mathbf{1}' m_x) \mathbf{1},$$

with  $\mathbf{1} = (1, \dots, 1)'$ , then yields a modified estimate of  $\mu$  as

$$\tilde{\mu} = (\mathbf{I} - \Gamma) \tilde{m}_x \quad . \quad (4)$$

The estimate  $\tilde{\mu}$  enforces homogeneous growth among components and gives more realistic trajectories. Again, an alternative solution would be to use restricted maximum-likelihood estimation. If the generating model is simulated using the model (2) with an intercept  $\mu^* = \mu - \alpha\delta$ , then an estimate for this intercept is formed according to  $\tilde{\mu}^* = \tilde{\mu} - \hat{\alpha}\hat{\delta}$ .

### 3 The data

We use three sets of parallel country data from Austria, France, and the United Kingdom. For each country, (quarterly) total fixed investment (or gross fixed capital formation, GFCF) is the sum of investment in equipment and machinery, investment in residential construction, investment in non-residential construction, and some minor positions. As a general rule, we simplify this breakdown by dividing total GFCF into two categories, construction investment and non-construction investment, though we will refer to the latter position in a slight abuse of wording as ‘equipment investment’.

Additionally to the two investment subaggregates, we use a quarterly time series of gross domestic product (GDP) for each country. All variables are at constant prices.

*Austrian* series cover a relatively short time range from 1988 to 2002. This time range has been dictated by the availability of the new national accounts (according to the ESA 95 standard) on a quarterly basis. GFCF is decomposed into construction investment and the remainder, which is equated to equipment investment. Figure 1 shows that all variables, particularly construction investment and to a lesser degree all other series including GDP, are affected by strong seasonal variation. Unreported tests on seasonal unit roots following HYLLEBERG *et al.* (1990) find seasonal unit roots at least in the construction series, which confirms the time-varying nature of the seasonal cycles. In order to safely remove potential seasonal unit roots and simultaneously permit a uniform treatment of all variables in a multivariate framework, all series were subjected to de-seasonalizing filters of the form  $S(B) = 1 + B + B^2 + B^3$ , where  $B$  denotes the lag operator. This filter is a main ingredient of the mentioned test procedure by HYLLEBERG *et al.* Because of the uncertain effects of officially used seasonal adjustment filters on subsequent analysis, such filters were avoided. Henceforth, all variable names such as ‘construction investment’ and ‘GDP’ will refer to the filtered series.

Table 1 summarizes some statistical unit-root tests. Unit roots cannot be rejected for the ratios of the subaggregates to output, while the evidence for the ratio of total GFCF to GDP is ambiguous. The Dickey-Fuller test rejects a unit root at the significance level of 10%. According to Figure 1, there is some longer-run increase of the share of equipment investment and some decrease of the share of construction investment. A characteristic of the Austrian series is the large slump in total investment in 2002, which may exert a strong influence on all results. Because of the relatively short time range of the Austrian data, we did not experiment with reducing the sample.

For *France*, ‘new’ national accounts are available for a rather short time range only, hence we used the ‘old’ national accounts data for the time range 1970 to 1998. Figure 2 displays the data set as ratios of investment components over GDP. Four subaggregates of GFCF are available for the ‘old’ French national accounts: livestock and plants, machinery and equipment, residential construction, and other construction. We added the relatively small first category to the second one to obtain ‘equipment investment’, while the latter two categories represent ‘construction investment’. Figure

2 demonstrates that a longer-run relative decrease of construction investment is reflected in a shrinking investment/GDP ratio, while the share of equipment investment remains approximately constant. These features are slightly different from the other two countries, while the relative decrease of construction investment is a common feature. According to formal unit-root tests, all three variables are classified as first-order integrated. In line with visual evidence, test statistics for the equipment investment share come closest to a rejection and to stationarity yet fail to surpass the 10% significance boundary. These statistical results are summarized in Table 2.

*British* series were taken from the UK quarterly national accounts for the time range 1965:1 to 2002:3. Figure 3 shows the evolution over time of ratios of total GFCF and of some raw investment components to GDP. The investment components were aggregated to the two main components later. It is seen that the ratio of total GFCF over GDP has remained fairly stable over the whole time range, at around 17–18%. By contrast, the share of equipment investment has increased from less than 5% to around 8%, while the share of residential construction has fallen from 5–6% to less than 3% over the same time range. These three subaggregates do not sum to total investment. Besides some smaller components and discrepancies, a fourth major position of ‘transport equipment’ adds to the overall increase in equipment investment.

In order to keep the historical distinction of the two major components of investment (see, e.g., BERNDT, 1996), we form the two subaggregates ‘construction investment’ from the residential and non-residential series and summarize the remainder, i.e., the difference of total GFCF and construction investment, in a variable ‘non-construction investment’, which we will identify with ‘equipment investment’ in the following. The share of these two components in GDP output is shown in Figure 4. In the notation of the previous section, the logarithms of construction and of non-construction investment correspond to  $z_1$  and  $z_2$ , while  $y$  corresponds to the logarithm of GDP.

Some descriptive unit-root test statistics are summarized in Table 3. The variables are logarithms of the share of construction investment in GDP, of equipment investment in GDP, and of total GFCF in GDP. While for the Dickey-Fuller tests the lag order was determined by the AIC information criterion, a window length of 4 was generally used for the Phillips-Perron version of the test. In summary, unit roots are never formally rejected for any variable, although the share of total GFCF comes closer to a rejection

than the shares of the subaggregates. This result is slightly at odds with visual evidence and is likely due to the long swings and high volatility of the total share series. The result is confirmed by a multivariate cointegration test on the three variables. A search for the cointegrating rank according to the JOHANSEN method (see JOHANSEN, 1995, for a detailed description) yields a rank of zero and hence no cointegrating vector. This excludes the possibility of self-cointegration and stationarity of any of the individual variables.

In summary, stationarity of the investment quota is not supported statistically, although it should be imposed for longer-run prediction, for reasons of plausibility. By contrast, stationarity of subaggregate quotas is unsupported by statistics as well as by plausibility.

## 4 Estimation

### 4.1 Austrian data

Table 4 gives the results of a preliminary unrestricted VAR estimation in differences, with the non-linear error-correction term and a constant included as additional regressors. One lag of the differences of the three variables  $c$  (logarithm of construction investment),  $e$  (logarithm of equipment investment), and  $y$  (logarithm of GDP) was used in the VAR, according to the recommendation by information criteria. As outlined in the previous section, Austrian series have been seasonally filtered. Consequently,  $c$  now corresponds to the logarithm of the seasonally filtered construction investment series, as  $e$  and  $y$  denote the logarithms of seasonally filtered equipment (non-construction) investment and output. In order to predict a seasonal series that corresponds to raw data, seasonality can be easily generated from the forecasts of the non-seasonal variables and actual starting values. For brevity we restrict attention to the non-seasonal variables in the following. Note that first differences of the filtered variables, for example  $\Delta y_t$ , equal seasonal differences of the original seasonal variables. Seasonal filtering was applied to the original non-logarithmic series, therefore  $\Delta y_t$  does not equal  $y_t^* - y_{t-4}^*$ , when  $y_t^*$  denotes the logarithm of quarterly output.

The influence of the error correction terms for the investment aggregates is rather small, while the coefficient in the  $\Delta y$  equation is significant. That coefficient must be set to zero as it has the wrong sign and would cause unstable behavior in prediction. Kruskal's Theorem (see, e.g., DAVIDSON

AND MCKINNON, 1993) implies that an unrestricted VAR with additional regressors is estimated efficiently by least squares. This does not hold for a system with imposed restrictions, where efficiency gains can be achieved by a transformation based on the estimated error variance structure. In small samples, however, the gains from using this procedure are unclear. Therefore, we estimate all restricted systems by least squares. The restricted estimates for the  $y$  equation are reported in the last column of Table 4, while coefficient estimates for the other two equations are identical to those shown in the first two columns. Excepting the coefficient of the error correction term, which was set at zero, the coefficient estimates are close to those for the unrestricted VAR.

The restricted VAR permits the calculation of forecasts with zero residuals. These form a benchmark scenario that is shown in Figure 5. The share of construction investment tends to re-increase to its sample maximum, while equipment investment tends to lag behind. Seen from a judgmental viewpoint, this scenario is not so plausible, as the downturn of the construction share in the late 1990s is mainly due to a budget consolidation. The scenario of Figure 5 may require either a budget expansion, with large construction projects financed by the government, or crowding-in effects, with an emphasis on construction in the private sector. Assuming zero residuals for prediction does not yield the conditional expectation in nonlinear models, hence the graphs of deterministic forecasts are only meant as rough sketches.

The results of a stochastic forecast of the non-linear Austrian system are displayed in Figures 6 and 7. These figures are based on 999 random draws from a normal distribution, with standard deviations conforming to those estimated from the sample. This procedure is sometimes called the parametric bootstrap. At each forecast horizon  $\tau = 1, \dots, 100$ , the 999 values were sorted. Suppose they are sorted upward. Then, the value at position 50 represents the lower 5% fractile, the value at position 500 is the median and the value at position 950 is the upper 5% fractile. Note that these values stem from different trajectories of the time-series model at different  $\tau$ . Specific trajectories typically show large variation within the bounds and thus their behavior corresponds neatly to the observed series in the sample range. Comparing the scenario for the total investment quota in Figure 6 to the baseline prediction of Figure 5, the considerable downward risk in 2003 becomes visible. However, even if the pessimistic lower bound is assumed, the investment quota is predicted to recover fast. The parallel scenarios for the component quotas show that the distribution of total investment to its



components is far less certain than the baseline of Figure 5 may insinuate. There is even a sizeable risk of equipment investment overtaking construction.

If growth homogeneity is imposed according to (4), forecasting performance changes from the one depicted in Figure 7 to the one of Figure 8. This version assumes identical growth rates across variables, i.e.,  $\mathbb{E} \Delta c = \mathbb{E} \Delta e = \mathbb{E} \Delta y$ . In short, it is assumed that the deterministic part of the trend is the same, while there is no restriction on the stochastic trend except for the non-linear cointegration term, which imposes long-run constancy in the total GFCF to GDP ratio but not in its components. This model takes an intermediate position between the hitherto used model and a traditional cointegrating model with long-run constancy of component quotas. Figure 8 shows that the median prediction is constant, according to assumptions, but also that strong deviations from this median are not uncommon. Specifically, the probability of equipment investment overtaking construction is slightly higher for the model with growth homogeneity than for the model without that restriction. Note that, even if growth homogeneity is imposed, the component ratios are not assumed as stationary. The widening cones reflect this assumption. For the (less plausible) assumption of linear cointegration for both subaggregates, the median forecast is similar to Figure 8, while the confidence bands remain much tighter.

## 4.2 French data

Table 5 gives the results of a preliminary VAR estimation. One lag of the differences of the three variables  $c$ ,  $e$ ,  $y$  was used in the VAR, according to the recommendation by information criteria. The  $t$ -values show that many entries in  $\Gamma$  are statistically insignificant. Aggregate output growth shows some reaction to the investment variables, and construction reacts to lagged equipment investment. The error correction term has the correct sign in the  $e$  and in the  $y$  equation, though it is hardly significant. The sign in the  $c$  equation is wrong, therefore it has to be dropped or otherwise the system becomes unstable, which must be ruled out for forecasting. Again, we estimate the restricted system by least squares. The coefficient estimates for the  $c$  equation are shown in the last column of Table 5.

A baseline forecast for the restricted nonlinear error-correction system is given as Figure 9. In the long run, the investment quota converges to its in-sample mean, while equipment investment overtakes construction investment, extrapolating the in-sample evolution. Stochastic simulation allows a more

adequate evaluation of the properties of the forecasting model. Figure 10 summarizes the distribution of the predictions by their upper and lower 5% fractiles and their median.

Figure 10 shows that the distribution is asymmetric. Positive deviations from the median position are more probable than large negative deviations, while the median comes close to the baseline of Figure 9. Note that the confidence intervals are comfortably strict. This is even true for the predicted component quotas in Figure 11. It appears that the model forecasts with high probability that equipment investment will overtake construction investment in the near future.

If growth homogeneity is imposed, one gets the scenario shown in Figure 12. The median forecast for the construction to GDP ratio remains at its current value, while the equipment to GDP ratio continues to increase for a while. There is a sizeable probability that both components remain in the same range for some years, although the model supports a higher share for equipment in the longer run.

An instructive experiment is depicted in Figure 13. Instead of the non-linear error-correction model used otherwise in this paper, we assumed a linear error-correction model with constant long-run component ratios. The variables of logged construction investment to output and equipment investment to output were inserted as error-correction terms instead of the total ratio. The effect of the error-correction terms in the  $\Delta c$  equation resulted unstable and explosive, therefore this equation was formed in pure first differences. Similarly, the effect of the equipment quota had the wrong sign in the  $\Delta y$  equation. As a consequence, the construction quota continues its decline in the prediction scenario, while the equipment quota stabilizes. The share of total investment in output also declines, which may not be very plausible. Even more disturbing are the narrow confidence bands that reflect the cointegration assumptions and add an undue precision to the implausible scenario. The experiment confirms that the non-linear error-correction model used here may be a necessary requirement to obtain plausible longer-run scenarios. There is no better linear alternative.

If linear modeling is applied to the Austrian and British data sets, linear models generate scenarios that are similar to the French one in Figure 13. If the influence of destabilizing error-correction terms is ignored and these coefficients are estimated freely, the resulting scenario is equally unlikely and displays extremely wide confidence bands.

### 4.3 British data

Table 6 gives the results of the preliminary unrestricted VAR estimation. Again, only one lag of differences was included according to information criteria. The non-linear error-correction term has the correct sign for the investment subaggregates, though it has a wrong sign for output and must therefore be excluded from the  $\Delta y$  equation. For this restricted VAR system with non-linear error correction, mean forecasts with zero residuals are shown in Figure 14, while Figures 15 and 16 show stochastic forecasts. The low values of  $R^2$  in Table 6 imply that the model has less explanatory power for the British data than for the other two countries, hence the confidence bands are relatively wide. The coefficient that is most severely affected by the exclusion of the error-correction term is the intercept, which reflects the fact that the error-correction variable is more ‘constant’ than the other regressors.

The forecast for the total investment quota in Figure 15 reflects the fact that the British investment quota is right at its historical mean close to the end of the sample. Therefore, the median forecast is almost constant over the whole forecast time range. Figure 16 shows that construction investment is in a long-run decline that is assumed to continue into the future. Nevertheless, the lower confidence boundary with values around 2% for this quota is difficult to accept. On the other hand, the upper confidence bound overlaps with the confidence band for equipment investment. This means that there is a small but non-zero possibility that construction investment again overtakes equipment investment.

If growth homogeneity is imposed on the British system (see Figure 17), median forecasts for both investment components become flat at the end-of-sample value, thus reflecting the low degree of dynamic dependence in the model. This variant avoids the probably too low construction shares of the model with inhomogeneous growth, at the price of reducing the scenario to an uninformative random-walk behavior. Contrary to a usual cointegration model, there is no *stochastic* restriction on the difference between investment components, excepting the error-correction term for the sum, which keeps the components from undue expansion.

## 5 Forecasting evaluation

### 5.1 Different concepts of evaluating forecasts

Even though we suggest to mainly rely on visual evaluation of the plausibility of longer-range prediction rather than on a numerical prediction evaluation, because of the prominence of sample-specific features in any selected finite time range, we also report here the results of an out-of-sample prediction evaluation for completeness. The British series offer the best opportunities for such an experiment, due to the relatively long time range of observations.

Regarding the stochastic assumption about the prediction model and the true model, one can distinguish four types of forecast evaluations. The simplest way of evaluating forecasts is by comparing a mean forecast  $x_{s+h}^*$  for an observation  $x_{s+h}$ , which is a function of the observations  $x_1, \dots, x_s$  and the observation  $x_{s+h}$ , typically by a distance function  $g(x_{s+h}^*, x_{s+h})$ . Typically, such evaluations are summarized by averaging over a range of values for  $s$ , tacitly assuming that  $m^{-1} \sum_{s=n-m+1}^n g(x_{s+h}^*, x_{s+h})$  converges to some constant, which we denote symbolically by  $g(x^*, x)$ . The forecasting model with the lowest value for  $g(x^*, x)$  is then interpreted as the best one. This approach is not appropriate for non-linear forecasting models and for stochastic prediction.

Most prediction experiments rely on a variant of approximating the integral

$$\int g(x^*, x) f(x) dx,$$

where  $x$  denotes the data-generating process,  $x^*$  is the forecast, and  $g(x, y)$  is a distance function, for example the squared distance  $g_2(x, y) = (x - y)^2$ . It is obvious that this approach is used as a backdrop for the prediction accuracy tests, as suggested by DIEBOLD AND MARIANO (1995), for example. Again, for a stochastic forecast  $x^*$ , this interpretation of measuring accuracy is inconvenient, and the assumption of a true probability model for the data also appears to be restrictive.

As an alternative, one may consider integrals of the type

$$\int g(x^*, x) f(x^*) dx^*,$$

where the expectation of  $g(x^*, x)$  is conditioned on the observed data  $x$  and the forecast  $x^*$  is random. Such an interpretation is in line with the current

popularity of fan charts and appears to be more appropriate for our purposes. For empirical applications, the integral is to be approximated by

$$\sum_{t=1}^m \sum_{\omega=1}^{\Omega} g(x_t^*(\omega), x_t),$$

where  $\omega$  is drawn according to the probability distribution of the stochastic forecast. For our experiment, we use  $\Omega = 200$ , i.e. there are 200 replications of the stochastic forecast, and  $m = 50$ , i.e. we evaluate predictive accuracy over the last 12.5 years of the sample. We repeat the experiment for a whole range of horizons, ranging from single-step to forty-step forecasts, and remember to focus on a prediction horizon of five to ten years. Note that for a horizon of  $h$  and a time range of  $m$ , only the first  $n - h - m + 1$  observations can be used for estimating the model parameters, if the prediction is supposed to be truly out-of-sample.

A fourth variant assumes stochastic processes for both the forecast  $x^*$  and the true model  $x$ , i.e.

$$\int \int g(x^*, x) f(x^*, x) dx^* dx.$$

Here, the difficulty is that the generating law for  $x$  is unknown. A workable solution would be to act as if the estimated parameters from the observations  $x_1, \dots, x_s$  determined the true structure and to draw from the assumed statistical distribution. One obtains an approximation by sums of the form

$$\sum_{t=1}^m \sum_{\omega_1=1}^{\Omega_1} \sum_{\omega_2=1}^{\Omega_2} g(x_t^*(\omega_1), x_t(\omega_2)),$$

where the notation simplifies the fact that both the generating laws for  $x^*$  and for  $x$  are time-changing in the sense that an increasing ‘training sample’ is used to determine the parametric structures. Such evaluations answer the question whether the suggested prediction *method* performs satisfactorily, *if the true model class is given*. For example, the assumed true model class may be non-linear cointegration models and the methods may be linear or non-linear error-correction models. Because of sampling variation in parameter estimation, a match between method and true class does not necessarily define the best method. We shall first consider the above mentioned alternative with fixed  $x$  and then return to the double-stochastic version.

## 5.2 Evaluations conditional on observed data

As candidates for stochastic forecasts, we use model-based predictions for five models: an unrestricted VAR in differences, the non-linear error-correction model without and with imposing growth homogeneity, a linear VECM with stationary subcomponent ratios, and a VAR in differences with growth homogeneity. For all error-correction models, instability was excluded by changing unstable influences of the error-correction terms to zero. This criterion was used separately at each time point, such that the experiment is out-of-sample in all regards. While various other models could be used for a comparison, note that it is not necessary to impose growth homogeneity on the linear cointegration model, as it is fulfilled automatically.

For the function  $g_2(x, y) = (x - y)^2$ , i.e. mean squared errors, results are displayed in Figures 18 and 19. For  $g_1(x, y) = |x - y|$ , i.e. mean absolute errors, the ranking of forecasts is very similar. The benchmark model in differences without any further restriction clearly yields inferior forecasts. Contrary to the simulations of ENGLE AND YOO (1987), cointegrating models dominate at almost all horizons for all series, not only at larger horizons. Note, however, that we do not use the VAR in levels as a benchmark that was used by ENGLE AND YOO but in differences, and that we evaluate predictive accuracy for the (stationary or at least bounded) ratios and not for the assumedly integrated variables, such as  $y$ .

For the total investment quota and for the construction subaggregate, the nonlinear cointegration models dominate convincingly. Growth homogeneity appears to achieve a further reduction in forecast errors. The linear cointegration model ranks third, well ahead of the VAR variants in differences. For the equipment investment quota, the evidence is less clear. Linear cointegration of subcomponents and therefore also of the equipment quota yields the best forecasts at shorter horizons, while nonlinear models take over at longer ones. Interestingly, imposing growth homogeneity implies a deterioration for this subaggregate, probably reflecting the marked longer-run shift in the contribution from this subcomponent. Presumably, growth homogeneity would prove more beneficial at even longer horizons, while different growth rates for subaggregates are acceptable within the limits of the experiment. The general impression from all three variables is anyway that the nonlinear error-correction model with growth homogeneity yields the most satisfactory results.

Similar results were obtained for the French data set, which is much

shorter than the British one. Therefore,  $m$  was reduced to 40, which corresponds to 10 years. Thus, the last 10 out of 29 years were predicted using the first nine to 19 years for estimating the forecast model for the longest horizon  $h = 40$ , while longer samples were available for the shorter horizons.

The results are shown in Figures 20 and 21, again for the distance function  $g_2$ . It is evident that the nonlinear cointegration models are optimal for the total investment quota and the equipment investment quota. For construction investment, imposing growth homogeneity fails at large forecast horizons. For the French data, the linear error-correction model is not really competitive, as error correction of subcomponent ratios is not found at nearly all considered subsamples. Rather, usage of these ratios would exert a destabilizing influence due to reaction parameters with wrong signs. These were automatically set at zero, thus there is virtually no difference between the linear cointegration model and the VAR in differences. In summary, the nonlinear error-correction model with growth homogeneity shows the best performance, excepting one of the subaggregate quotas. This outcome matches the evidence from the British data. Results for the Austrian series were also qualitatively similar. Due to the even shorter time range, they are not so reliable and are therefore not shown.

### 5.3 Evaluations conditional on simulated data

These evaluations assume that a specified model class is the correct one and determines the free parameters by estimation from the full available sample. From this estimated ‘pseudo-true’ model, artificial samples are generated (‘parametric bootstrap’), which are then ‘predicted’ using all of the previously specified methods. One of the methods corresponds to the class used for generating the data. These evaluations are helpful, as they provide information on the relative merits with regard to the accuracy of forecasts from correctly specifying the model class. Because of sampling variation in parameter estimation, the true model class is not necessarily the best one at all forecast horizons.

Figures 22 and 23 rely on experiments for the British data set and 100 replications both for the stochastic predictors and for the pseudo-true model. The assumed true model class is the nonlinear cointegration model with the growth homogeneity restriction. Figures are drawn for mean absolute errors rather than mean squared errors to allow a more instructive visual separation of curves. For both criteria, the ranking is the same. Predictions based

on the true model dominate at all horizons, while the ranking of the other predictors varies. For the equipment investment quota, the nonlinear model without the homogeneity restriction falls behind the linear error-correction model, while for the construction and total quotas, linear cointegration performs worse than the unrestricted nonlinear model. The primitive models without any error-correction restriction are worst for the construction and total quotas, whereas the differences VAR with growth homogeneity achieves a similar performance as the unrestricted nonlinear model for the equipment investment quota.

For the French data, mean absolute errors are summarized in Figures 24 and 25. The linear error-correction model is not a good choice for this data. Its performance is even worse for the mean-squared errors evaluation, which corresponds to the empirical results reported above. While setting error-correction adjustment at zero is a satisfactory solution for the given data, it implies locally unstable behavior for the bootstrapped version.

These simulations also offer an informal test of whether the assumed model is a likely data-generating mechanism for the British data, even though such tests are not in the focus of our investigation. If a nonlinear error-correction model actually had generated the British investment data, Figures 22 and 23 should roughly match the features seen in the observational counterparts, Figures 18 and 19. While that correspondence is acceptable in general, there are some noteworthy differences. For the British data, the empirical plots support the linear cointegration model as a forecasting tool at some prediction horizons, while this model is not among the preferred ones for the simulation graphs. This mismatch may indicate that true data behavior is ‘in between’ the linear and the nonlinear model, in the sense that the persistence of subcomponent quotas is stronger than would be implied by the nonlinear error-correction model, though not as strong as would be implied by the linear error-correction model. For both data sets, prediction errors increase monotonously for the bootstrap version, while they deviate from monotonicity for the empirical version. This may indicate that longer-run cycles play a larger role in empirical data than in all suggested model classes. These longer-run cycles may reflect cycles in political attitudes, as particularly construction investment is severely influenced by policy decisions. Finally, the numerical values of mean absolute and mean squared errors show noteworthy differences, which however is to be expected due to sampling variation, if the data is viewed as a single observation of a trajectory from a time-series process.



## 6 Summary and conclusion

Non-linear error correction modeling was applied to three-variable systems for two main investment components and aggregate output in three industrialized economies. While macroeconomic theory is informative with respect to the share of total investment in output, which is supposed to be constant in the steady state of an economy, there is little information on the development of its components. Attempting to forecast the subcomponents under the restriction of long-run constancy of the total investment share yields a relatively simple example for non-linear cointegration. We suggest the elimination of unstable features from longer-run scenarios in an iterative dialog with the data. Particularly in longer-run forecasting, statistical test decisions obtained from samples of limited length should be overridden in favor of admissibility restrictions imposed by plausibility and economic theory. We also demonstrate that automatically generated forecasts from statistical analysis of linear structures may not fulfil such plausibility requirements. In scenarios with a very long time horizon, restricting the deterministic part of the system can also become crucial in order to avoid that one subcomponent disappears in the longer run.

The presented partial models on investment and output can be used as building blocks in larger macroeconometric models, where the relationships between the investment sector and other sectors of the economy can be fully captured. In order to permit a focus on the main issues, we exclude such extensions from the present paper. The study could, however, form a basis for future work on this subject.

Without substantial modification, similar techniques to the one outlined in this paper can be used for other cases where the total share is known to be ‘more constant’ than the shares of components, such as in modeling components of consumer demand. We feel, however, that the basic modeling idea may have even wider applicability in other areas of economic modeling.

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## Figures and Tables

Table 1: Unit root tests on Austrian series

	log(IFC/GDP)	log(IFE/GDP)	log(IF/GDP)
Dickey-Fuller tests			
augmenting lags	2	1	3
$\mu$ -statistics	-0.077	-1.681	-2.762
Phillips-Perron tests			
window length	3	3	3
statistics	0.326	-1.519	-1.239

IFC is construction investment, IFE is equipment investment, IF is total fixed investment. All series, including GDP, were subjected to a seasonal moving average filter  $S(B) = 0.25 * (1 + B + B^2 + B^3)$ .

Table 2: Unit root tests on French series

	log(IFC/GDP)	log(IFE/GDP)	log(IF/GDP)
Dickey-Fuller tests			
augmenting lags	5	8	8
$\mu$ -statistics	-1.033	-2.269	-2.084
Phillips-Perron tests			
window length	4	4	4
statistics	0.109	-2.074	-0.902

IFC is construction investment, IFE is equipment investment, IF is total fixed investment.

Table 3: Unit root tests on British series

	log(IFC/GDP)	log(IFE/GDP)	log(IF/GDP)
Dickey-Fuller tests			
augmenting lags	2	1	1
$\mu$ -statistics	-0.752	-1.234	-2.003
Phillips-Perron tests			
window length	4	4	4
statistics	-1.121	-1.406	-2.310

IFC is construction investment, IFE is equipment investment, IF is total fixed investment.

Table 4: Coefficient estimates for the Austrian data. Estimation time range is 1989:1-2002:4.

	$\Delta c_t$	$\Delta e_t$	$\Delta y_t$	$\Delta y_t$
$\Delta c_{t-1}$	0.421 [ 2.671]]	-0.218 [-0.780]	-0.022 [-0.506]	-0.041 [-0.908]
$\Delta e_{t-1}$	0.124 [ 1.291]	0.503 [ 2.945]	0.060 [ 2.295]	0.027 [1.082]
$\Delta y_{t-1}$	-0.207 [-0.407]	0.985 [ 1.090]	0.593 [ 4.253]	0.722 [5.144]
$\mu^*$	-0.218 [-1.838]	-0.140 [-0.666]	-0.089 [-2.741]	0.002 [2.169]
$\alpha$	-0.151 [-1.853]	-0.094 [-0.651]	-0.062 [-2.790]	
$R^2$	0.229	0.370	0.644	0.589

The first three columns give the results for the unrestricted non-linear error-correction model. The last column gives least-squares estimates for the  $\Delta y_t$  equation without the error-correction term.

Table 5: Coefficient estimates for the French data. Estimation time range is 1970:3-1998:4.

	$\Delta c_t$	$\Delta e_t$	$\Delta y_t$	$\Delta c_t$
$\Delta c_{t-1}$	-0.014 [-0.140]]	0.461 [ 2.637]	0.119 [ 3.194]	0.001 [0.008]
$\Delta e_{t-1}$	0.097 [ 1.651]	0.047 [ 0.468]	0.080 [ 3.744]	0.093 [1.591]
$\Delta y_{t-1}$	0.343 [ 1.211]	0.570 [ 1.181]	-0.052 [-0.503]	0.386 [1.383]
$\mu^*$	0.021 [ 0.827]	-0.074 [-1.715]	0.019 [ 2.026]	-0.002 [-0.746]
$\alpha$	0.015 [ 0.893]	-0.051 [-1.808]	0.009 [ 1.432]	
$R^2$	0.081	0.128	0.259	0.075

The first three columns give the results for the unrestricted non-linear error-correction model. The last column gives least-squares estimates for the  $\Delta c_t$  equation without the error-correction term.

Table 6: Coefficient estimates for the UK data. Estimation time range is 1965:3-2002:3.

	$\Delta c_t$	$\Delta e_t$	$\Delta y_t$	$\Delta y_t$
$\Delta c_{t-1}$	-0.156 [-1.839]	0.007 [ 0.069]	-0.016 [-0.665]	-0.017 [-0.701]
$\Delta e_{t-1}$	0.069 [ 0.969]	-0.213 [-2.482]	0.036 [ 1.779]	0.036 [1.763]
$\Delta y_{t-1}$	0.619 [ 1.971]	0.418 [ 1.110]	-0.028 [-0.310]	-0.026 [-0.293]
$\mu^*$	-0.136 [-1.808]	-0.083 [-0.922]	-0.000 [-0.017]	0.006 [6.041]
$\alpha$	-0.076 [-1.806]	-0.051 [-1.014]	-0.003 [-0.280]	
$R^2$	0.074	0.055	0.026	0.025

The first three columns give the results for the unrestricted non-linear error-correction model. The last column gives least-squares estimates for the  $\Delta y_t$  equation without the error-correction term.

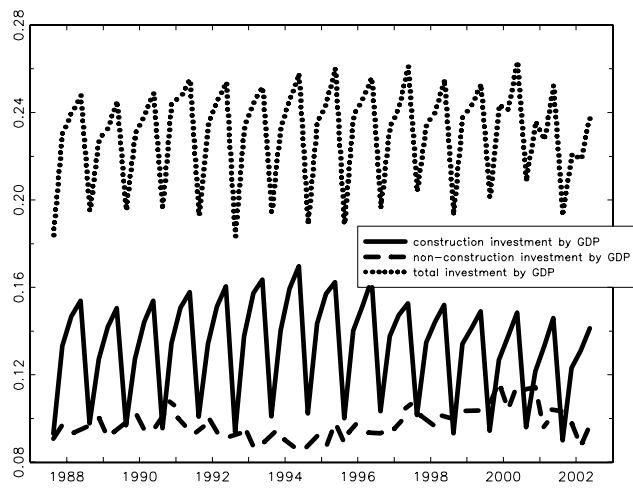


Figure 1: Shares of investment components in Austrian GDP. Time range is 1988–2002.



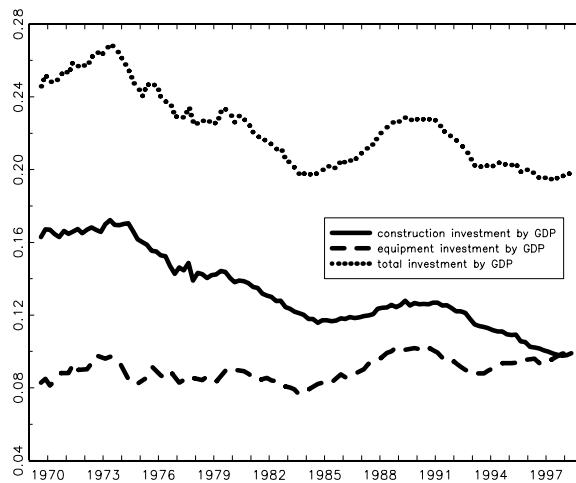


Figure 2: Ratios of construction investment to GDP, equipment investment to GDP, and total investment to GDP for France. Sample period is 1970:1–1998:4.

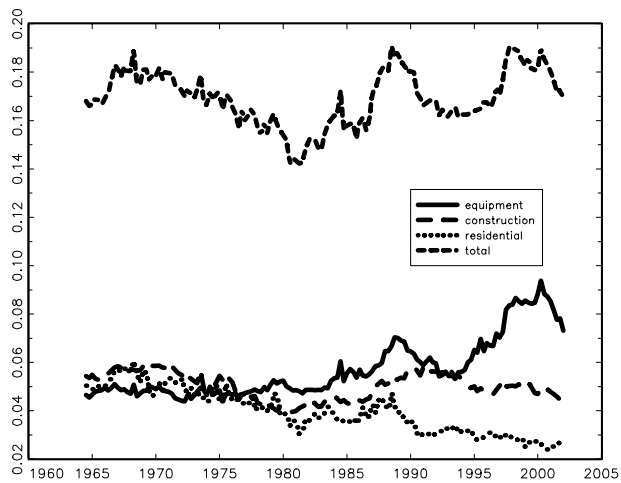


Figure 3: Shares of investment components in British GDP. Quarterly data 1965:1–2002:3.

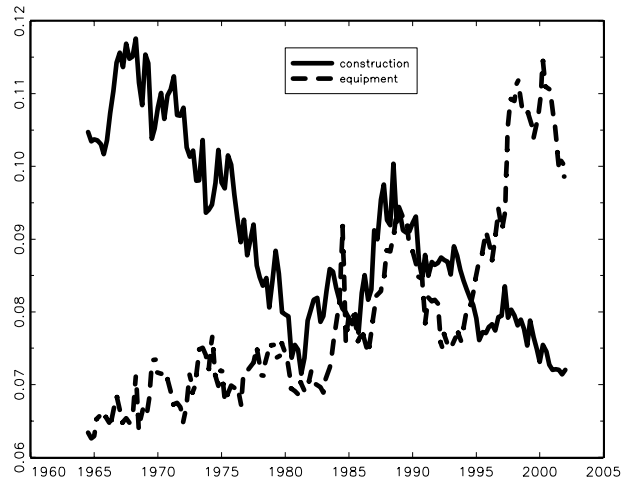


Figure 4: Shares of constructed investment components in British GDP. “construction” comprises residential and non-residential construction, while “equipment” refers to all categories of total fixed investment excluding construction.

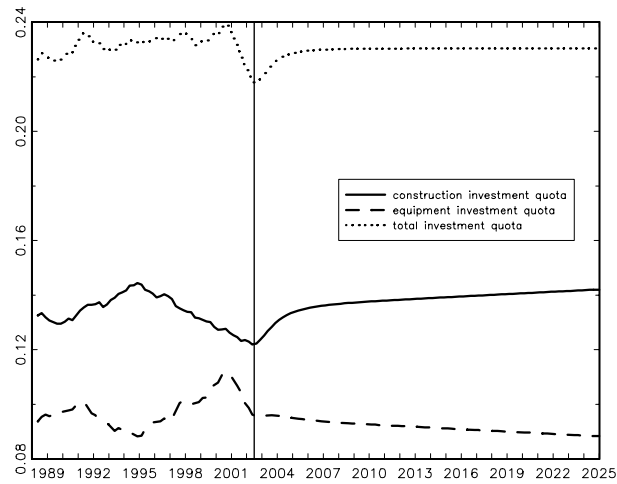


Figure 5: 25 years of forecasting for the Austrian nonlinear system with zero residuals. A vertical bar separates the sample from the prediction interval.

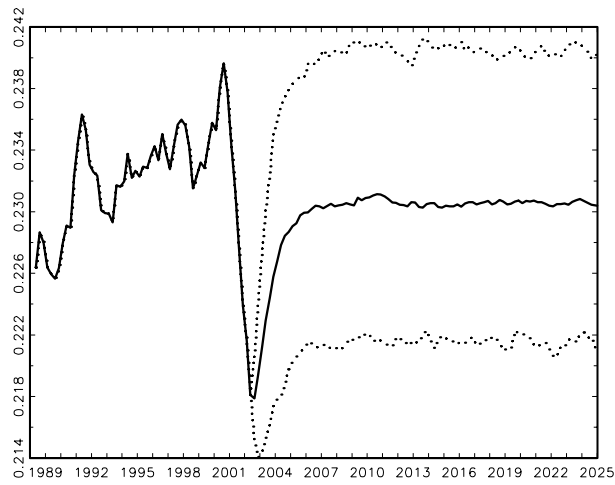


Figure 6: Median and upper and lower 5% fractiles for stochastic forecasts from the Austrian nonlinear system. Predicted variable is the total investment quota.

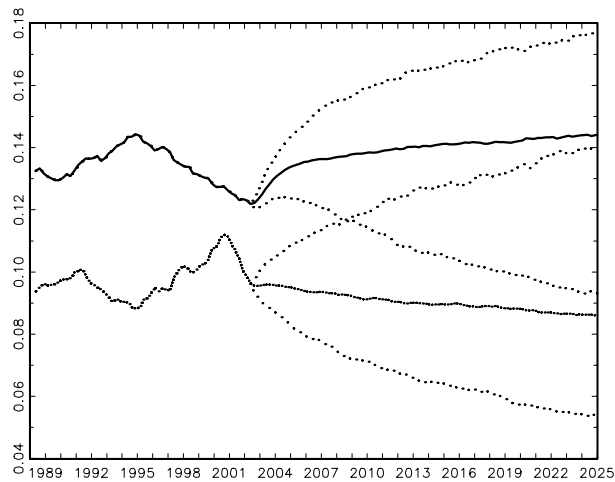


Figure 7: Stochastic prediction from the nonlinear model for the Austrian data. Median forecasts for construction investment quota (solid curve) and for equipment investment quota (dashed curve). Dotted curves represent lower and upper 5% fractiles of the forecast distribution.

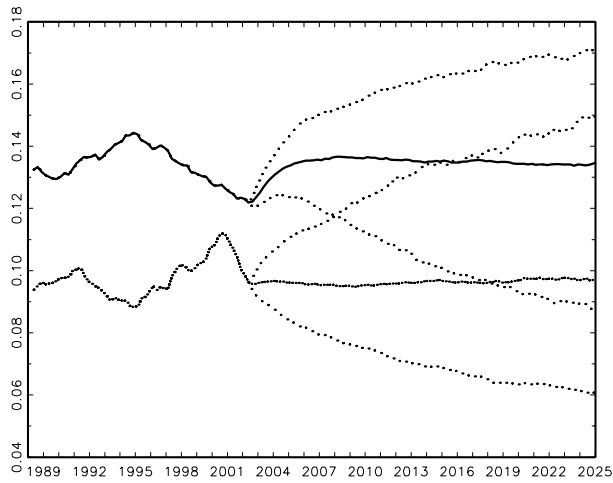


Figure 8: Stochastic prediction from the nonlinear model for the Austrian data under the restriction of homogeneous deterministic growth. Median forecasts for construction investment quota (solid curve) and for equipment investment quota (dashed curve). Dotted curves represent lower and upper 5% fractiles of the forecast distribution.

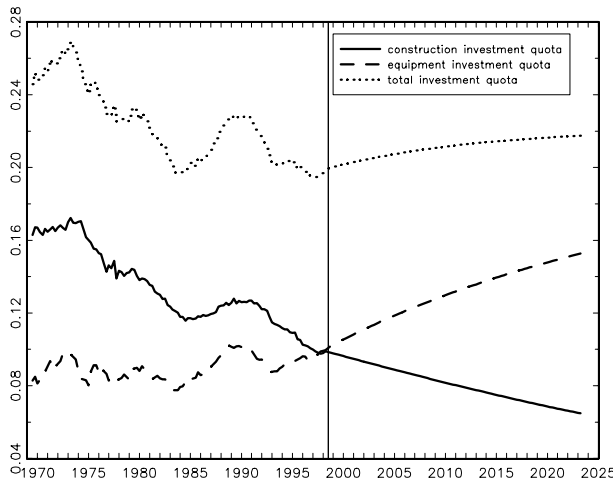


Figure 9: 25 years of forecasting for the French nonlinear system with zero residuals. A vertical bar separates the sample from the prediction interval.

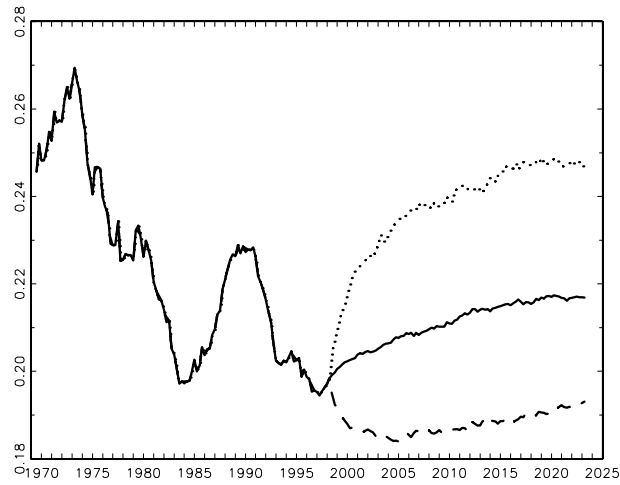


Figure 10: Median and upper and lower 5% fractiles for stochastic forecasts from the French nonlinear system. Predicted variable is the total investment quota.

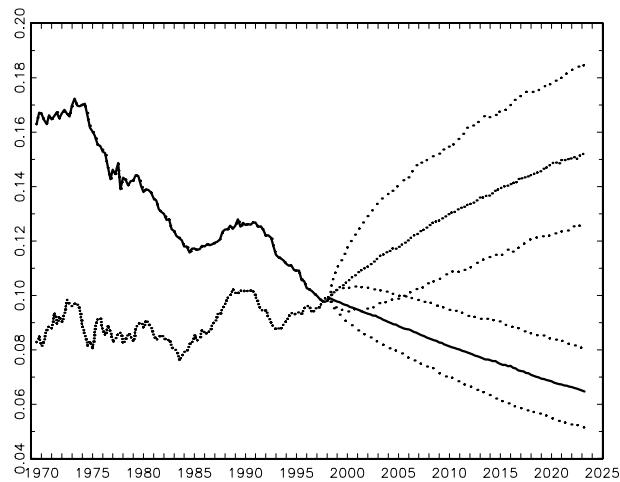


Figure 11: Stochastic prediction from the nonlinear model for the French data. Median forecasts for construction investment quota (solid curve) and for equipment investment quota (dashed curve). Dotted curves represent lower and upper 5% fractiles of the forecast distribution.

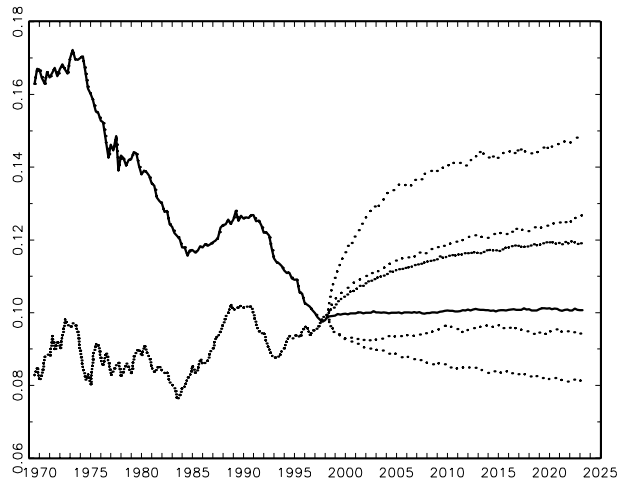


Figure 12: Stochastic prediction from the nonlinear model for the French data using the restriction of growth homogeneity. Median forecasts for construction investment quota (solid curve) and for equipment investment quota (dashed curve). Dotted curves represent lower and upper 5% fractiles of the forecast distribution.

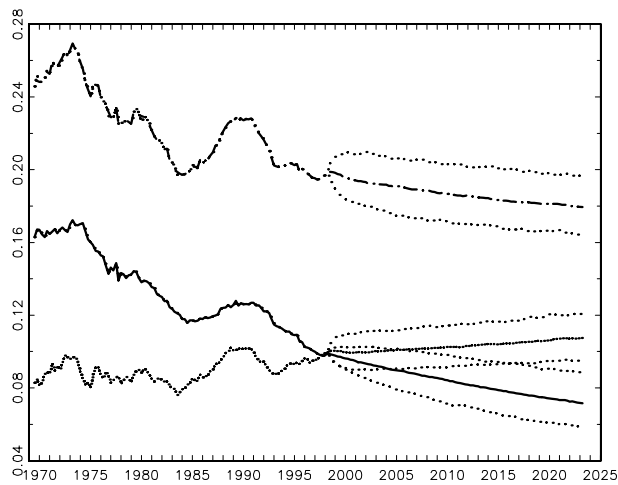


Figure 13: Stochastic prediction for a linear error-correction model based on French data. Solid curve corresponds to construction investment, dashed curve to equipment investment, and dashes and dots to total investment.

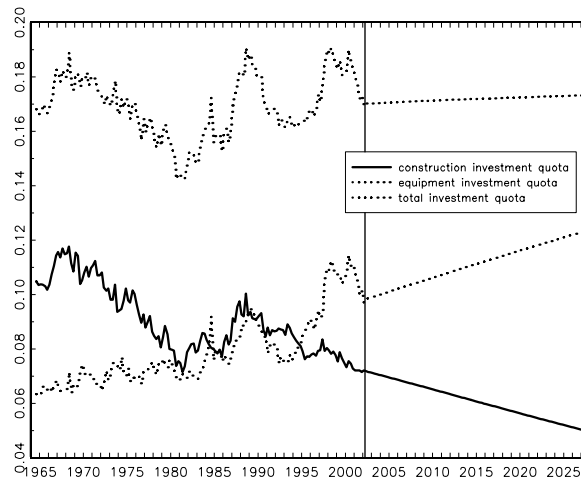


Figure 14: 25 years of forecasting for the UK nonlinear system with zero residuals. A vertical bar separates the sample from the prediction interval.

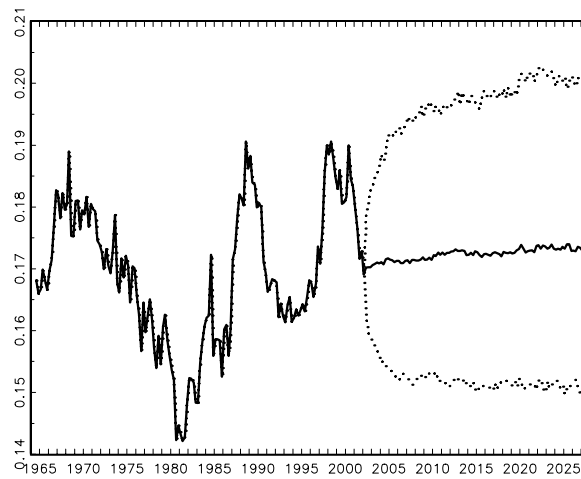


Figure 15: Median and upper and lower 5% fractiles for stochastic forecasts from the UK nonlinear system. Predicted variable is the total investment quota.

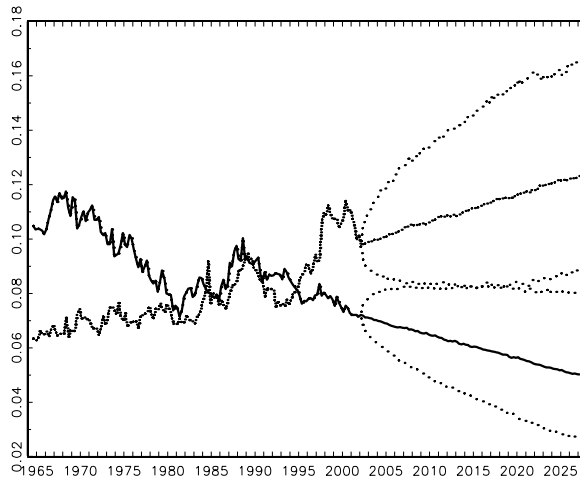


Figure 16: Stochastic prediction from the nonlinear model for the UK data. Median forecasts for construction investment quota (solid curve) and for equipment investment quota (dashed curve). Dotted curves represent lower and upper 5% fractiles of the forecast distribution.

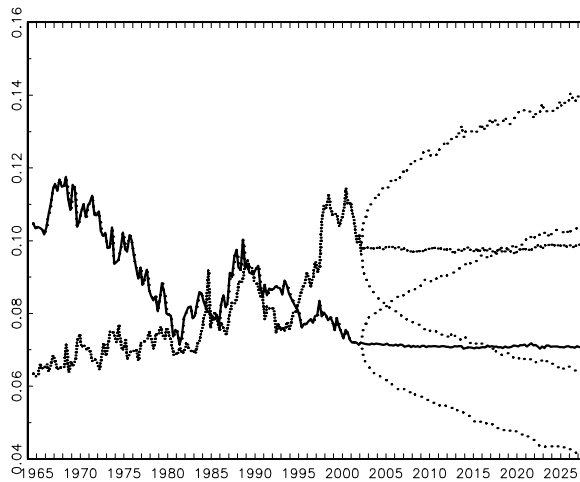


Figure 17: Stochastic prediction from the nonlinear model for the UK data under the restriction of growth homogeneity. Median forecasts for construction investment quota (solid curve) and for equipment investment quota (dashed curve). Dotted curves represent lower and upper 5% fractiles of the forecast distribution.



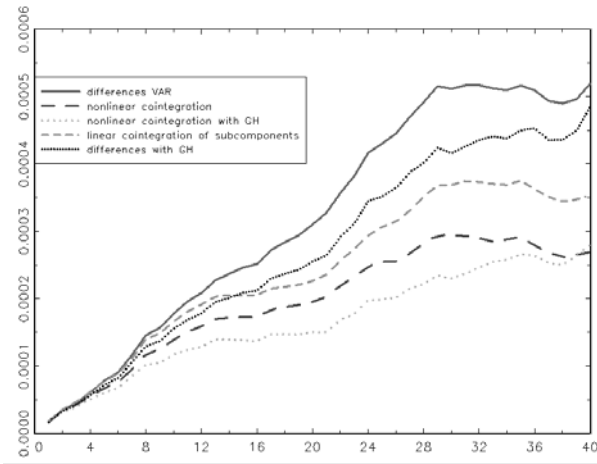


Figure 18: Mean squared errors for prediction horizons  $h = 1$  to  $h = 40$ . Predicted variable is the UK construction investment quota.

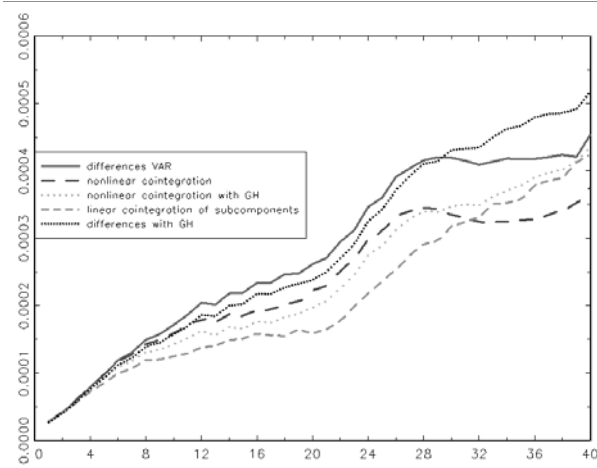


Figure 19: Mean squared errors for prediction horizons  $h = 1$  to  $h = 40$ . Predicted variable is the UK equipment investment quota.

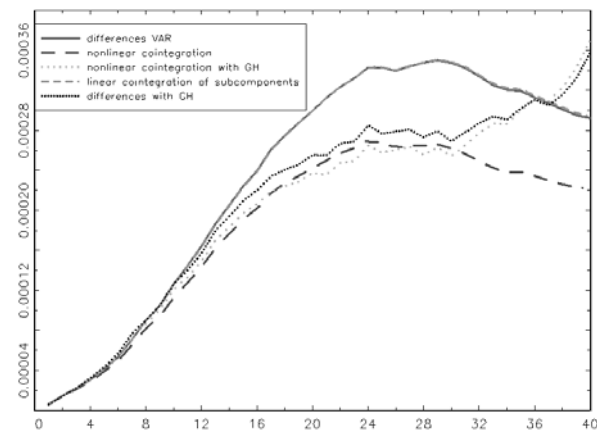


Figure 20: Mean squared errors for prediction horizons  $h = 1$  to  $h = 40$ . Predicted variable is the French construction investment quota.

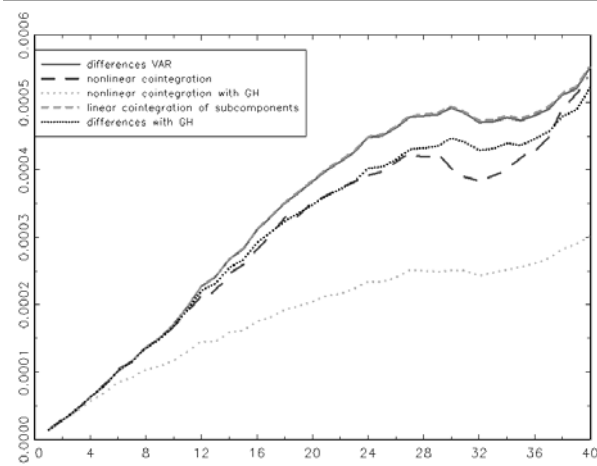


Figure 21: Mean squared errors for prediction horizons  $h = 1$  to  $h = 40$ . Predicted variable is the French equipment investment quota.

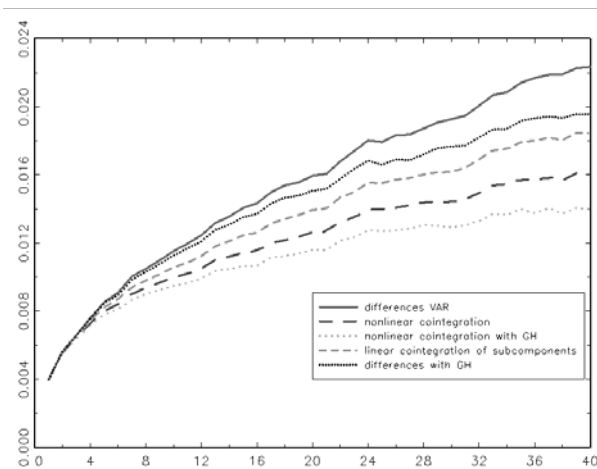


Figure 22: Mean absolute errors for prediction horizons  $h = 1$  to  $h = 40$ . Predicted variable is a parametric bootstrap version of the British construction investment quota.

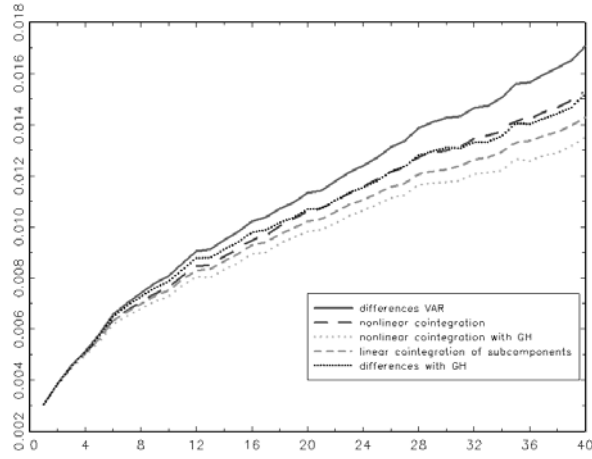


Figure 23: Mean absolute errors for prediction horizons  $h = 1$  to  $h = 40$ . Predicted variable is a parametric bootstrap version of the British equipment investment quota.

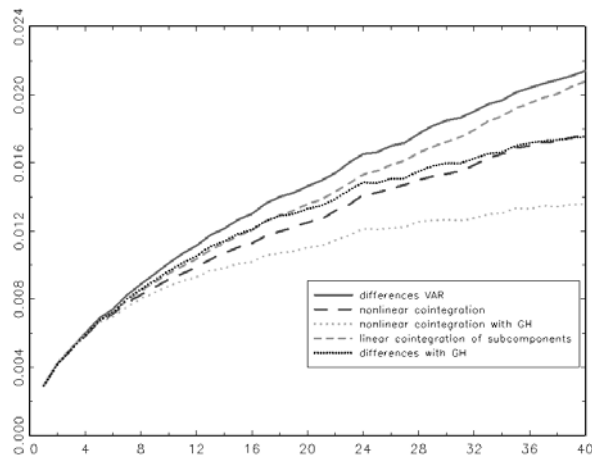


Figure 24: Mean absolute errors for prediction horizons  $h = 1$  to  $h = 40$ . Predicted variable is a parametric bootstrap version of the French construction investment quota.

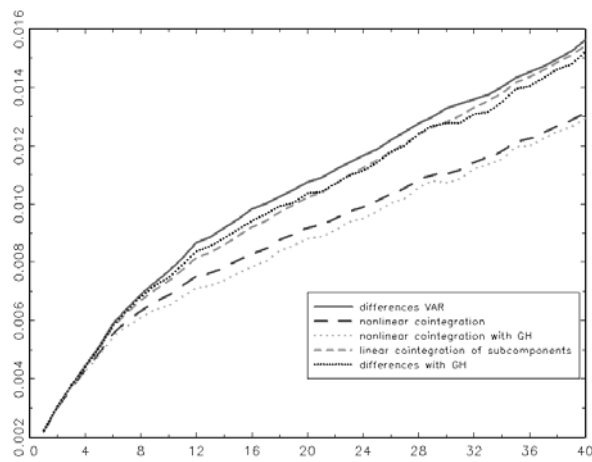


Figure 25: Mean absolute errors for prediction horizons  $h = 1$  to  $h = 40$ . Predicted variable is a parametric bootstrap version of the French equipment investment quota.

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