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Media as Watchdogs:
The Role of News Media in Electoral Competition*

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Abstract. We develop an equilibrium model to analyze the role of the media in electoral competition. Policy payoffs in the model are state dependent. We show that voters cannot infer the state directly from off-equilibrium party policies. As a result, party policies do not converge to the median voter’s ideal policy when the media report only party strategies. News analyses that convey information about the state allow voters to identify the party that serves their interests. Competition for positive news coverage causes political parties to adopt more centrist policies. In equilibrium voters interpret the news strategically. Mass media that are likely to have the same policy preference as the median voter are more effective in promoting electoral competition, resulting in greater policy convergence. However, since voters are rational, media that are more biased need not be less effective.

JEL Classification. D720, D820

Keywords. media bias, cheap talk, policy convergence

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1 Introduction

In elections voters have little incentive to gather information about complex social issues. Instead they rely on information provided by various sources to decide which party to support (Downs 1957; Popkin 1994). Chief among these information sources are mass media of communications: television and radio news, magazines, and newspapers. Surveys have found that a majority of American voters regularly watch television news and read newspapers, and many of them consider the mass media their principal source of campaign information (Popkin 1994; Stromberg 2004a).

In order to cast an informed vote, voters need information about the policies of the political parties as well as the desirability of these policies. Since the media can be held accountable for demonstrable falsehoods, they have little leeway in the reporting of party policies. News analyses, however, are often based on opinions of policy experts. By choosing which experts to interview and which comments to quote, the media may influence voters’ perception of the desirability of a policy. Conservative commentators in the United States have long maintained that mainstream media slant news in favor of the liberal cause. More recently, the rising popularity of conservative cable-television and radio news programs and the increasing commercialization of mainstream media have led liberal commentators and journalists to question whether the media are becoming dominated by business and conservative interests. The view that the media are biased is shared by a large fraction of the public. A recent Gallup poll finds that the media are considered “too liberal” by 45 percent of the respondents and “too conservative” by 14 percent. In a study of the relationship between information in the news pages and opinions in the editorial pages, Kahn and Kenney (2002) find that newspapers’ coverage of senatorial campaigns in the United States is often slanted in favor of the candidates they endorse.\footnote{Page (1996) presents several examples of biased news coverage by the American media.}

How does media bias affect policy outcomes? Though widely believed, there is a serious flaw with the conventional wisdom that the media push policy in the direction they favor. Since the political stances of major news outlets are known, voters may not be easily swayed by media outlets they consider biased. Voters, for example, may interpret a liberal newspaper’s tepid endorsement of the Democratic candidate as a sign that the Republican candidate is superior. It is often claimed that The Sun’s endorsement of the Labour Party in the 1992 general election in Britain was influential because The Sun was supposed to be pro-Conservative.
In game-theoretic terms, news reports are cheap talk, whose actual meaning is determined by the strategic interactions between political parties, voters, and news media. In this paper we explicitly model information transmission between news media and voters in a Downsian framework. Using this model, we derive the equilibrium meaning of news reports and analyze the media’s effects on the policy choices of political parties.

In the model, there are two political parties, each of which has single-peaked preferences over a one-dimensional policy space. Ideal policies of the parties and the median voter are increasing functions of a one-dimensional state variable that summarizes information pertaining to the desirability of a policy. The parties have opposite policy preferences, and each prefers a more extreme policy than the median voter. The electoral process involves three steps. First, each party chooses a policy based on the state. Then voters cast their ballots; the party that receives a majority of the votes is elected. Finally the elected party implements its policy. Before they vote, voters acquire information about party policies and the state through the media. While the media must report the parties’ policies truthfully, they may lie about the state in order to promote its preferred policy. Voters and parties observe the policy preferences of the media imperfectly. In equilibrium they respond to the news rationally. A media outlet is biased if the median voter expects its policy preference to be systematically different from its own.

One school of thought in journalism maintains that “objective” political news should only report actions of political actors and refrain from comments and analyses that are inevitably subjective.\(^2\) Implicit in this view is the assumption that voters can decide for themselves which candidate serves their best interests based on the candidates’ actions and policies. The first part of the paper investigates the validity of this assumption.

Previous research in cheap-talk games (Krishna and Morgan 2001a, 2001b) has showed that in the two-sender, one-dimensional-uncertainty case, the sender’s first-best outcome is implementable. In our context, the result means that there are equilibria in which the median voter infers the state and implements his ideal policy solely on the basis of the information conveyed by the parties’ policies. In such equilibria, the only role of the media is to report party policies; news analyses that convey information the state are redundant. This seems to suggest that the problem of media bias can be solved by getting into the “right” equilibrium.

Our analysis, however, shows that these equilibria must involve unrea-

\(^2\)See Fallows (1996) for a debate between proponents of this views and those who believe that news reporting should actively promote a policy agenda in public interest.
sonable off-equilibrium beliefs of the voters. Under any reasonable beliefs, voters cannot infer the state directly from off-equilibrium party policies. As a result, party policies do not converge to the median voter’s ideal policy when they are the voters’ only source of information. The intuition is as follows. For any pair of off-equilibrium party policies, there are two possibilities: either the party that prefers higher policies has deviated to a higher policy in a lower state, or the party that prefers lower policies has deviated to a lower policy in a higher state. Since parties have opposite policy preferences, each will claim that the state is such that its policy is better for the median voter. Without an independent source of information, voters cannot tell which party has lied. This captures a common problem of real-life politics. For example, a party that supports free trade would claim that free trade creates new jobs while a party that opposes free trade would argue that it leads to higher unemployment. Voters may find it difficult to judge which theory is correct.

The second part of the paper shows that news analyses, even when potentially biased, allow voters to probabilistically identify the state off the equilibrium path. In equilibrium voters are more likely to vote for a party when it receives positive media coverage. As a result, parties adopt more centrist policies in order to compete for positive media coverage. The effectiveness of the media in promoting policy convergence depends on two factors. The first is the perceived level of media responsiveness to policy changes—how likely a small policy change may cause the media to change the tone of their coverage—which increases with the likelihood that the media have the same preference as the median voter. The second factor is the influence of the media on voting behavior—how likely a change in coverage may change the outcome of the election—which depends on media bias. When party preferences are symmetric, biased media are less influential. However, in general we find that the media are the most influential when their policy preferences match the equilibrium election probabilities of the parties. Media that are liberal, for example, may be more influential than media that are neutral if the more liberal party is more likely to be elected than the conservative one. Furthermore, media bias, when harmful, results in polarized, rather than biased, policies. In sum, while our results confirm the importance of media bias, they also depart from the conventional wisdom in significant ways.

One way to increase the effectiveness of electoral competition is to ensure that the media contain multiple independent voices. We find that parties

\[\text{A formal definition of “reasonable” beliefs is introduced in Section 4.}\]
adopt more centrist policies when media outlets have diverse policy preferences. Since voters have access to the news reports from multiple sources, the bias of one particular media outlet becomes less important. As the number of independent media outlets becomes arbitrarily large, there is full policy convergence.

The rest of the paper is organized as follows. Section 2 summarizes the related literature. Section 3 introduces the formal model. Section 4 proves that policy does not converge when voters ignore news analyses. Section 5 introduces several restrictions on voters’ beliefs that rule out unreasonable equilibria. Section 6 proves that news reports are informative despite media bias. Section 7 derives the unique linear equilibrium when there is a single media outlet. Section 8 extends the results to the case where there are multiple media outlets. Section 9 concludes.

2 Related Literature

Downs (1957) is the first to give an economic analysis of the political role of the media. Recently, there is a growing interest on this issue. Stromberg (2004a, 2004b) argues that public policies tend to favor groups that receive more news coverage from the media, and he shows that U.S. counties with a higher ratio of radio ownership received more funds from a New Deal relief program. Mullainathan and Shleifer (2002) construct a behavioral model of “spin,” in which media competition may lead to inferior outcomes. Djankov et al. (2001), examining the patterns of media ownership in 97 countries, find that state ownership of the media tends to be associated with poor social and economic outcomes, particularly when the state-owned media outlet is a monopoly. Besley and Prat (2001) construct a model of media capture. They find evidence that state-ownership and high concentration enhance the likelihood of media capture, and that foreign ownership is correlated with more efficient media.

In our model, competition for media endorsement compels the two parties to adopt more centrist policies. Grossman and Helpman (1999) consider another channel through which an information provider may affect political outcomes. In their model, voters are divided into two groups with distinct policy preferences. One of the groups is represented by an interest group, which endorses one of the parties after observing their policies. Grossman and Helpman show that parties tend to adopt policies that favor the interest group because members of the interest group are more responsive to the interest-group endorsement than are non-members. Since parties have no
policy preference in their model, the logic behind their results is fundamentally different from ours.

Our theoretic model is related to the literature on two-sender information transmission games (Gilligan and Krehbiel 1987, 1989; Krishna and Morgan 2001a, 2001b; Battaglini 2002). While these papers focus on the amount of information that can be revealed when there are two senders, the main goal of this paper is to analyze how the incentives of the two senders (the parties) are affected by the presence of a third sender (the media). We also show that some of the equilibria discussed in this literature do not satisfy some reasonable restrictions on the receiver’s off-equilibrium beliefs.

Battaglini (2002) shows that full-information revelation is generically possible in two-sender cheap talk games when the state variable is multi-dimensional. Our state variable is one-dimensional, so his result does not apply. More importantly, our model of electoral competition is different from his cheap-talk model in that our median voter is not free to implement any feasible policy. Although the median voter can deduce the state in equilibrium, he can only choose between policies proposed by the two parties. Hence, policy convergence may not be possible even when the state variable is multi-dimensional.

3 The Model

Two political parties, referred to as parties 1 and 2, compete in a one-dimensional policy space $Y \equiv \mathbb{R}$. The outcome of a policy $y$ is $y - \theta$, where $\theta$ is a state variable in the space $\Theta \equiv \mathbb{R}$. There is an odd number of voters. Each of them has a utility function

$$U(y, \theta, b_v) = -(y - \theta - b_v)^2,$$

where $b_v$ is an individual-specific parameter that denotes the “taste” of the voter.

Intuitively, $\theta$ represents contextual information that is costly for individual voters to learn. For example, if the policy is about social expenditures, then $\theta$ may include information about the marginal productivity of social

\footnote{Gilligan and Krehbiel (1989) and Krishna and Morgan (2001b) examine the performance of legislative committees under different procedural rules. Without the media, our model of two-party competition resembles the case of modified rule in their models, whereby a legislature is restricted to choose among the status quo and two competing bills submitted by the majority and minority members of a committee. Our result is different from theirs in part because in our model voters do not have a third choice (the status quo in their models).}
spending and the deadweight loss of taxation. Voters know their own tastes but not the state variable \( \theta \). The parties, on the other hand, know both \( \theta \) and the preference of every voter.

Policy preferences of the two parties are also in the form of (1). Let \( b_i \) be the taste parameter of party \( i \). We normalize the median voter’s taste parameter to zero and assume \( b_1 = -b_2 = b > 0 \). Thus, for any \( \theta \), the median voter’s ideal policy is lower than that of party 1 and higher than that of party 2. For convenience, we refer to high policies as “liberal” and low policies as “conservative.” Thus, party 1 is more liberal than party 2. In addition to the utility it derives from the policy that is being implemented, a party receives a rent of \( r \) if it is in power. Thus, the total utility of party \( i \) is

\[
U(y, \theta; b_i) + I_i r,
\]

where \( I_i = 1 \) if party \( i \) is elected, and \( I_i = 0 \) otherwise. The magnitude of \( r \) measures the importance of the office-holding motive. When \( r \) is zero, the parties care only about policy.

In the beginning of the game, nature chooses \( \theta \) according to some atomless distribution with strictly positive density everywhere. The state \( \theta \) is revealed to the parties. Each party \( i \) then independently proposes a policy. A strategy of party \( i \) is a function that assigns a policy to each \( \theta \). To understand how media bias affect policy outcomes, we focus on linear strategies of the form

\[
\gamma_i(\theta) = \theta + \gamma_i,
\]

where \( \gamma_i(\theta) \) is the policy chosen in state \( \theta \). Let \( \gamma_i \) denote a linear strategy as well as the constant gap between party \( i \)’s policy \( \gamma_i(\theta) \) and the median voter’s ideal point \( \theta \). A strategy profile of the parties is denoted by \( \gamma \equiv (\gamma_1, \gamma_2) \). Focusing on linear strategies greatly simplifies the exposition that follows. Since a linear strategy is characterized by a single parameter, linear equilibria are easy to analyze. The magnitude of the constant policy gap between the parties in a linear equilibrium provides a convenient measure of the efficiency of the electoral process. Moreover, there is a unique linear equilibrium that meets several restrictions on voters’ off-equilibrium beliefs. This enables us to apply standard comparative statics techniques to analyze the equilibrium relationship between policy preferences of the media and the policy gap.

We allow for multiple media outlets in Section 8. In the meantime we assume that there is a single media outlet, which we refer to as a “newspaper.” The policy preference of the newspaper is also in the form of (1), with a taste parameter \( b_m \). The newspaper prefers a more liberal policy than the
median voter when \( b_m > 0 \), and it prefers a more conservative policy than the median voter when \( b_m < 0 \). In reality, while parties and voters may know that a media outlet tends to favor a certain type of policies, they may not know its exact policy position on any particular issue. For example, they may know that *The New York Times* is liberal without knowing its precise stance on school vouchers. Hence, from the perspectives of parties and voters, the newspaper’s taste parameter is a random variable \( b_m \), which is uniformly distributed in \( B \equiv [-c(1 - \tau)/2, c(1 + \tau)/2] \) with \( c > 0 \) and \( \tau \in (-1, 1) \). (In the following, \( b_m \) refers to the random variable and \( b_m \) the value of the random variable.) It is crucial that \( B \) contains 0, so that there is always some positive probability that the newspaper is arbitrarily close to being unbiased. Note that voters’ beliefs about the newspaper’s policy preference have two dimensions. If \( c \) is small, it is more likely that the newspaper’s policy preference is close to that of the median voter.\(^5\) The parameter \( \tau \) measures the extent of the newspaper’s bias. The newspaper tends to favor liberal policies when \( \tau > 0 \).

Throughout, we treat the newspaper’s policy preference as exogenous. In general, there is no reason why the policy preference of the media must coincide with that of the median voter. Media outlets controlled by individuals or families may have viewpoints that reflect those of their owners, while those owned by profit-maximizing corporations may adopt the viewpoints of demographic groups targeted by advertisers (Stromberg 2004a). We do not address the question of how competition among media outlets affect their policy preferences.

The role of the newspaper is to inform voters of the parties’ policies and the state. We assume that the newspaper can lie only about the state but not about party policies, as lies about the latter are (relatively) easy to disprove. In general the newspaper can make any statements about the state. Let \( \mathcal{M} \) denote the set of such statements. We say the newspaper is influential when the policies are \( y \) if given \( y \) there exist \( x, x' \in \mathcal{M} \) such that the probability that party 1 is elected when the newspaper makes statement \( x \) is different from that when the newspaper makes statement \( x' \). In other words, a newspaper is influential when its statements affect political outcomes. The newspaper chooses a statement in \( \mathcal{M} \) to maximize the probability of election of the party it prefers. Conditional on the newspaper being influential in \( y \), statements in \( \mathcal{M} \) that are chosen with positive probability can be partitioned into two subsets such that every liberal news-

\(^5\)All results apply to the case where the distribution of \( b_m \) is not uniform. In general \( c \) corresponds to the density at \( b_m = 0 \).
paper chooses from one subset and every conservative newspaper chooses from another. Furthermore, while statements within each subset are nominally different, they are strategically equivalent in the sense that they all lead to the same election probabilities. Thus, news reports about the state can be modeled without loss of generality as a binary decision, which, for convenience, we refer to as an endorsement. Intuitively, a report that says $\theta > (y_1 + y_2)/2$ is tantamount to an endorsement of the party with the more conservative policy. Formally, denote the newspaper’s endorsement strategy by a function $\rho : Y^2 \times \Theta \times B \rightarrow \{1, 2\}$, where $\rho(y, \theta, b_m) \in \{1, 2\}$ is the party endorsed by the newspaper with taste parameter $b_m$ in state $\theta$ when the party policies are $y \equiv (y_1, y_2)$.

The election takes place after the parties chose their policies and the newspaper made its endorsement. Voters update their beliefs about $\theta$ on the basis of party policies and media endorsement. They then cast their ballots for the party that maximizes their expected utilities. The party that receives a majority of votes wins and implements its policy.

A voting strategy of the median voter is a function $\pi : Y^2 \rightarrow [0, 1]^2$, where $\pi^1(y, e)$ and $\pi^2(y, e)$ are the probabilities (summing to one) of voting for parties 1 and 2, respectively. The median voter’s posterior belief over $\Theta$ is denoted by $\mu : Y^2 \times \{1, 2\} \rightarrow \mathcal{P}$, where $\mathcal{P}$ is the space of probability distributions over $\Theta$; $\mu(y, e)$ is the median voter’s posterior belief conditional on policies $y$ and endorsement $e$.

Since there are an odd number of voters, whose preferences satisfy the single-crossing condition in $(y, b_v)$, the party chosen by the median voter always receives a majority of votes (Gans and Smart 1996). The outcome of the game is therefore fully characterized by the strategies of the parties, the newspaper, and the median voter.

A voting strategy $\pi$ is optimal if for all $y \in Y^2$ and $e \in \{1, 2\}$,

$$\pi^1(y, e) \in \arg \max_{x \in [0, 1]} \int_{\theta \in \Theta} (xU(y_1, \theta, 0) + (1 - x)U(y_2, \theta, 0)) \, d\mu(y, e). \quad (4)$$

Let $\theta_i(y, \gamma)$ denote the state where $\gamma_i$ prescribes $y_i$, and let $\varepsilon(y, \gamma) \equiv (y_1 - y_2) - (\gamma_1 - \gamma_2)$ denote the difference between the actual policy gap and the policy gap under $\gamma$. A policy profile $y$ is consistent with $\gamma$ if and only if $\varepsilon(y, \gamma) = 0$. A posterior belief $\mu$ is consistent with $\gamma$ if for all $y$ such that $\varepsilon(y, \gamma) = 0$ and $e = 1, 2$,

$$\mu(\theta_1(y, \gamma) \mid y, e) = 1. \quad (5)$$

Since both $\gamma_1$ and $\gamma_2$ are one-to-one from $\Theta$ to $Y$, the median voter can infer $\theta$ from any $y$ consistent with $\gamma$. 


Given \( \pi \), the newspaper makes its endorsement decision to maximize the election probability of the party it prefers. It chooses \( \rho \) such that for all \( y \in Y^2 \), \( \theta \in \Theta \), and \( b_m \in B \),

\[
\rho(y, \theta, b_m) \in \arg \max_{e \in \{1,2\}} \sum_{i=1}^2 \pi^i(y, e)U(y_i, \theta, b_m). \tag{6}
\]

We let \( \beta^i(y, \theta) \) denote the ex ante probability that party \( i \) is endorsed by the newspaper, given its endorsement strategy \( \rho \).

A party’s policy choice affects both its probability of being endorsed and its probability of being elected conditional on the endorsement decision. We assume that a party is committed to carrying out its policy platform after winning the election.\(^6\) A strategy profile of the parties \( \gamma \) is optimal if for \( i, j \in \{1,2\}, j \neq i, \) and \( \theta \in \Theta \),

\[
\gamma_i(\theta) \in \arg \max_{y_i \in Y} p^i((y_i, \gamma_j(\theta)), \theta) (U(y_i, \theta, b_i) + r) + p^j((y_i, \gamma_j(\theta)), \theta)U(\gamma_j(\theta), \theta, b_i), \tag{7}
\]

where

\[
p^i(y, \theta) \equiv \sum_{e=1}^2 \beta^c(y, \theta)\pi^i(y, e)
\]

is party \( i \)'s ex ante probability of winning the election. Note that (7) requires that \( \gamma_i \) be a best response among all feasible strategies, including those that are non-linear.

**Definition 1** A quadruple \((\gamma, \rho, \pi, \mu)\) is a perfect Bayesian equilibrium if it satisfies (4), (5), (6), and (7).

A quadruple \((\gamma, \rho, \pi, \mu)\) is a perfect Bayesian equilibrium if each of the players acts optimally given other players’ strategies, and the median voter’s posterior belief is derived from the players’ strategies via Bayes’ rule on the equilibrium path.

### 4 Ignoring the News

If news about the state are biased, why don’t voters ignore them? In this section we consider what happen when voters vote solely on the basis of

\(^6\)See Alesina (1988) for an analysis of a reputation mechanism for credible commitment.
party policies $y$. Since the model is symmetric, we begin with symmetric equilibrium of the form: $\gamma_1 = -\gamma_2 = k$.

It is easy to see that there is an equilibrium in which each party chooses its own favorite policy. Consider the following equilibrium strategy profile:

- Party strategies: $\gamma_1 = \gamma_2 = -b$;
- Median voter strategy: $\pi^1(y) = 0.5$ for all $y \in Y$;
- Median voter beliefs: if $\varepsilon(y, \gamma) = 0$, then $\mu(\theta_1(y, \gamma) \mid y) = 1$;
  otherwise, $\mu(\theta_1(y, \gamma) \mid y) = \mu(\theta_2(y, \gamma) \mid y) = 0.5$.

In this equilibrium, each party is elected with probability 0.5 under all circumstances. Since a party’s policy choice does not affect its electoral prospects, its best response is to choose its favorite policy. The equilibrium policy gap is $2b$. If the median voter observes such a gap, he concludes that neither party has deviated and, hence, is indifferent between the two parties. If the median voter observes a policy gap not equal to $2b$, he knows at least one party has deviated. Since off-equilibrium beliefs are not pinned down by equilibrium conditions, the median voter in principle can entertain any beliefs over $\Theta$. Here, he believes that the state is equally likely to be $\theta_1(y, \gamma)$, where party 1 has not deviated, or $\theta_2(y, \gamma)$, where party 2 has not deviated. Given these beliefs, he is again indifferent between the two parties. Thus, the median voter’s strategy is optimal.

[Figure 1 about here.]

The above example shows that electoral competition may be completely ineffective when news about the state are ignored. Are there any other symmetric equilibria? Consider the possibility that there is an equilibrium with $\gamma_1 = -\gamma_2 = k \neq b$. Since the parties are not choosing their own favorite policies, there is some $\varepsilon$ such that for each $\theta$ party 1 strictly prefers $y_1' = \theta + k + \varepsilon$ to $y_1 = \theta + k$ in state $\theta$, and party 2 strictly prefers $y_2' = \theta + k$ to $y_2 = \theta + \varepsilon - k$ in state $\theta + \varepsilon$. Figure 1 describes a situation where $k < b$. In Figure 1, the median voter cannot tell whether the state is $\theta_2 = \theta$ or $\theta_1 = \theta + \varepsilon$ when he observes $y' \equiv (y_1', y_2')$. In order to deter party 1 from deviating from $y_1$ to $y_1'$ in state $\theta_2$, the median voter must elect party 1 with a higher probability when he observes $(y_1, y_2')$ than when he observes $(y_1, y_2)$. Similarly, in order to deter party 2 from deviating from $y_2$ to $y_2'$ in $\theta_1$, he must elect party 2 with a higher probability when he observes $(y_1', y_2)$ than when he observes $(y_1', y_2')$. Hence,

$$\pi^1(y_1, y_2') > \pi^1(y_1', y_2') > \pi^1(y_1', y_2).$$
The argument applies to any $\theta$ and $\theta'$ that are $\varepsilon$ apart. If $(\gamma_1, \gamma_2)$ is part of an equilibrium, then we can construct a sequence $\theta^1, \theta^2, \theta^3, \ldots$ such that for $i \geq 1$, $\pi^1(\gamma(\theta^i)) - \pi^1(\gamma(\theta^{i+1})) = \delta > 0$, violating the restriction that $\pi^1$ be bounded between 0 and 1. There is thus no equilibrium with $k < b$. Using a similar argument, we can rule out equilibria with $k > b$. We have established the following proposition.

**Proposition 1** If voters vote only on the basis on party policies, then in any symmetric linear equilibrium, the parties adopt policies $\gamma_1 = -\gamma_2 = b$.

Proposition 1 explains why voters cannot simply ignore biased news and why newspapers cannot stay politically “neutral” by reporting only party policies. Hotelling (1929) shows that party policies converge to the median voter’s ideal point when political parties are motivated solely by the office-holding motive. Calvert (1985) shows that this result is robust in the sense that departures of policies from the median are small as long as the policy motive is weak relative to the office-holding motive. But without information about the state, the equilibrium may be complete policy divergence even when the policy motive is small. All voters are made worse off when that happens, as the utility function $U$ is concave in the policy outcome.

Although voters can infer the state from the party policies on the equilibrium path, that information alone is not sufficient to compel the parties to adopt policies closer to the median voter’s ideal policy. Since the parties have incentives to pursue partisan policies in opposite directions, off equilibrium the median voter cannot distinguish whether it is the conservative policy that is “too conservative” or the liberal policy “too liberal.” Letting parties make statements about the state will not remedy the situation, as their statements are not credible. Due to the incentive problem, information about the state cannot be obtained from political parties. News analyses provide an independent source of information, which creates incentives for the parties to adopt centrist policies. Before we proceed with our analysis of this mechanism, we first show that reasonable restrictions on off-equilibrium beliefs justify our focus on symmetric equilibria.

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7In Wittman (1983) and Calvert (1985), complete policy divergence cannot be an equilibrium, because a marginal deviation toward the median leads to a second-order loss in utility but reaps a first-order increase in the probability of winning.

8In a spatial model of product competition, locating away from the center can help consumers in the periphery because they do not have to make purchases with the producer that is located on the opposite end. In a electoral competition model, however, the elected party implements its policy to all voters. Risk aversion then implies that divergence to extreme policies are bad for voters.
5 Voter Beliefs

Upon observing an off-equilibrium policy profile $y$ and the newspaper endorsement $e$, the median voter must form new beliefs over $\Theta$. We can think of the way these new beliefs are formed as a two-stage process. First, the median voter comes up with interim beliefs about which party in what state may have deviated. Then, he updates these beliefs on the basis of $e$ via Bayes’ rule. Suppose $\beta^e(y, \theta)$ is the probability that the newspaper endorses party $e \in \{1, 2\}$ in state $\theta$ when the policies are $y$. The median voter’s posterior belief that $\theta \in A$ is

$$
\mu(A \mid y, e) = \frac{\int_A \beta^e(y, \theta)dq(\theta; y, e)}{\int_{\Theta} \beta^e(y, \theta)dq(\theta; y, e)},
$$

(8)

where $q(\cdot; y, e)$, a probability measure over $\Theta$, denotes the median voter’s interim beliefs. Writing $\mu$ in this way only pushes the indeterminacy of beliefs one step back. Since $q$ is not pinned down by equilibrium, for any $\mu$ and $\beta$ we can find $q$ such that (8) holds. To say anything more specific about the median voter’s off-equilibrium beliefs, we need to impose additional restrictions on the interim beliefs.

The first restriction requires that off equilibrium the median voter believe that only one party has deviated.

**Condition 1** Given $\gamma$, for $y$ such that $\varepsilon(y, \gamma) \neq 0$ and for $e = 1, 2$,

$$
\mu(\theta_1(y, \gamma) \mid y, e) + \mu(\theta_2(y, \gamma) \mid y, e) = 1.
$$

The median voter expects a constant policy gap $\gamma_1 - \gamma_2$ in equilibrium. When the actual policy gap $y_1 - y_2$ is different from $\gamma_1 - \gamma_2$, the median voter realizes that some party has deviated, but he does not know which and in what state. In Figure 2, $(y_1, y_2)$ may be caused by, among other possibilities, (i) party 2 deviating downward at $\theta_1$; (ii) party 1 deviating upward at $\theta_2$; or (iii) both parties deviating at $\theta_3$. Condition 1 requires that the posterior beliefs should put positive weight only on (i) and (ii). Deviations by definition are rare events. If the median voter believes that each party deviates independently with a small probability, then he should believe that it is much more likely for one party to deviate than for both to deviate simultaneously.\(^9\)

\(^9\)Condition 1 is related to the notion of consistency in Kreps and Wilson (1982). If the median voter believes that each party $i$ independently deviates to some alternative mixed strategy $\sigma_i$ with a small probability $\xi$, then his off-equilibrium beliefs satisfy Condition 1 as $\xi$ tends to zero (holding $\sigma_i$ constant).
Condition 1 implies that posterior beliefs belong to the class of two-point distributions over $\theta_1(y, \gamma)$ and $\theta_2(y, \gamma)$. It rules out beliefs that are “discontinuous” (when the parties’ policies are strictly increasing and continuous in $\theta$). If a party $i$ deviates to $y_i = \gamma_i(\theta) + \varepsilon$ for some $\varepsilon$ in state $\theta$, then the median voter believes that the state is at most $\varepsilon$ away from $\theta$. A small discrepancy between the parties’ policies will not cause the median voter to drastically revise his beliefs.

The second restriction is similar to the Intuitive Criterion of Cho and Kreps (1987).

**Condition 2** Let $u^*_i(\theta)$, $i = 1, 2$, denote the equilibrium payoff for party $i$ at state $\theta$, and let $\theta_i$ be the state which is consistent with policy $y_i$ under $\gamma_i$. If $u^*_i(\theta_j) > u(y_i, \theta_j, b_j) + r$ and $u^*_j(\theta_i) \leq u(y_j, \theta_i, b_j) + r$, then for any $\varepsilon \in \{1, 2\}$,

$$
\mu(\theta_i | y, \varepsilon) = 1.
$$

Suppose the observed policies $y$ imply that the states are either $\theta_1(y, \gamma)$ or $\theta_2(y, \gamma)$. If party 1 is worse off surely by deviating from the equilibrium policy under state $\theta_2$ to the observed policy $y_1$, while party 2 may potentially gain by deviating from the equilibrium policy under state $\theta_1$ to $y_2$, then the median voter should believe that it is party 2 which has deviated.

The last restriction requires that the median voter react to the newspaper endorsement in a sensible way.

**Condition 3** For any off-equilibrium $y$, let $\theta'$ and $\theta''$ be the states consistent with $y$ according to Condition 1. Then for $i, j \in \{1, 2\}$, $j \neq i$, if $\beta^i(y, \theta') > \beta^j(y, \theta'')$, then $\mu(\theta' | y, i) \geq \mu(\theta' | y, j)$, with the strict inequality relation holds when $\mu(\theta' | y, j) < 1$ or $\mu(\theta' | y, i) > 0$.

Condition 3 says that if the newspaper is more likely to endorse party $i$ in state $\theta'$ than in state $\theta''$, then the median voter should assign a higher posterior belief to state $\theta'$ when party $i$ is endorsed than when party $i$ is not endorsed.

By Bayes’ rule, the posterior probability ratio between $\theta'$ and $\theta''$ can be written as

$$
\frac{\mu(\theta' | y, \varepsilon)}{\mu(\theta'' | y, \varepsilon)} = \frac{q(\theta'; y, \varepsilon) \beta^e(y, \theta')}{q(\theta''; y, \varepsilon) \beta^e(y, \theta'')},
$$

10Thus is another reason why restricting the message space of the newspaper to $\varepsilon \in \{1, 2\}$ is without loss of generality.
Condition 3 is satisfied if the newspaper endorsement affects the posterior beliefs only through the ratio $\beta^e(y, \theta') / \beta^e(y, \theta'')$; that is, if the interim beliefs are independent of the newspaper endorsement.

The conditions imply that party strategies are symmetric in equilibrium.

**Proposition 2** If a perfect Bayesian equilibrium $(\gamma, \rho, \pi, \mu)$ satisfies Conditions 1 and 2, then $\gamma_1 = -\gamma_2$.

Suppose there is an equilibrium in which $|\gamma_1| < |\gamma_2|$. By Condition 1, upon observing $y$ that is inconsistent with $\gamma$, the median voter believes that the true state must be either $\theta_2(y, \gamma)$ or $\theta_1(y, \gamma)$. Since $|\gamma_1| < |\gamma_2|$, the median voter should strictly prefer $y_1$ to $y_2$ in either state $\theta_1$ or $\theta_2$ when these two states are sufficiently close to one another. See Figure 2. Thus, party 1 will be elected so long as $\varepsilon(y, \gamma) = \theta_2 - \theta_1$ is small. If $\gamma_1 \neq b_1$, party 1 can gain by deviating slightly toward its ideal point. It follows that $\gamma_1$ must be equal to $b_1$. The situation is depicted in Figure 3. Consider the policy profile $(y_1, y_2)$. Whereas party 2 has incentives to choose $y_2$ in state $\theta_1$, party 1 strictly prefers the equilibrium outcome to $y_1$ in state $\theta_2$. By Condition 2, the median voter should believe that the state is $\theta_1$ and choose $y_2$, the policy proposed by party 2. Hence, $(\gamma_1, \gamma_2)$ cannot be part of an equilibrium. Intuitively, the median voter recognizes that the losing party has a stronger incentive to deviate and adopt a policy favorable to him than the winning party does. This shows that in equilibrium $|\gamma_1| = |\gamma_2|$. Finally, note that $\gamma_1 = \gamma_2 \neq 0$ cannot be part of an equilibrium, because in that case a party not winning certainly would gain by deviating slightly toward the median voter’s ideal point.

![Figure 3 about here.](image)

The above argument does not rely on the fact that party strategies are linear. Using a similar argument we can show that even without the restriction to linear strategies it is impossible to achieve the first-best outcome for the median voter using only information about party policies.

**Corollary 1** Even without the restriction to linear strategies, there is no perfect Bayesian equilibrium such that voters’ off-equilibrium beliefs satisfy Conditions 1 and 2 and depend only on party policies, and the equilibrium policy outcome is $\theta$ for all $\theta$.\(^\text{11}\)

\(^{11}\)We proved in the previous section that there is no linear equilibrium in which $\gamma_1 = -\gamma_2 = 0$. If there is a perfect Bayesian equilibrium in which the equilibrium policy outcome is $\theta$ for all $\theta$, then there is some interval $I$ such that for all $\theta \in I$, either $\gamma_1(\theta) \neq \gamma_2(\theta) = \theta$ or $\gamma_2(\theta) \neq \gamma_1(\theta) = \theta$. The interval $I$ exists because the proof in Section 4 is valid as long
Conditions 1 and 2 ensure that voters’ off-equilibrium beliefs are reasonable. If these conditions are ignored, it is no longer clear why it is insufficient for voters to know only party policies. For example, the argument of Krishna and Morgan (2001a, 2001b) can be used to establish that, in our setting with $r = 0$, the following strategy profile constitutes a perfect Bayesian equilibrium:

Party strategies: $\gamma_1 = 0$, $\gamma_2 = -2b$;

Median voter strategy: for $e \in \{1, 2\}$, if $|y_1 - y_2| \leq 2b$, then $\pi^1(y, e) = 1$; otherwise, $\pi^1(y, e) = 0$.

Party 1 chooses the median voter’s ideal policy and always wins the election. The equilibrium policy gap between the two parties is $2b$. Party 1 is prevented from deviating to a more liberal position because the median voter chooses party 2 whenever the observed policy gap is greater than $2b$. Since the median voter obtains its favorite outcome without relying on the newspaper’s endorsement, it may seem that the endorsement is redundant. But the voting strategy in the above equilibrium can only be justified by an unreasonable belief that $\theta$ is substantially lower than $\theta_1(y, \gamma)$ or $\theta_2(y, \gamma)$ when the policy gap is arbitrarily close to $2b$. Such a belief is ruled out by Condition 1.

6 Media Credibility

In this section, we analyze the effect of newspaper endorsement on the median voter’s voting decision. Consider any symmetric perfect Bayesian equilibrium $(\gamma, \rho, \pi, \mu)$ with $\gamma_1 > 0 > \gamma_2$. In state $\theta$ if party $i$ strictly prefers the equilibrium outcome in $\theta$ to winning the election with policy $y_i$, then party $i$ will never gain from a deviation to $y_i$. In that case we say that the off-equilibrium policy profile $(y_1, \gamma_2(\theta_2))$ is “equilibrium-dominated” for party $i$. Write $Q$ for the set of off-equilibrium policy profiles that are not equilibrium-dominated for either party. That is,

$$Q \equiv \{y \in Y^2 : U(y_i, \theta_j(y, \gamma), b_i) + r > u_i^*(\theta_j(y, \gamma)), i \in \{1, 2\}, j \neq i \},$$

as the set of $\theta$ where $\gamma_1(\theta) = \gamma_2(\theta) = \theta$ is dense. Within $I$, there must exist some $\theta \in I$, $i \in \{1, 2\}$, and $j \neq i$ such that $\gamma_i(\theta) = 0$, $|\theta - \gamma_j(\theta)| = \varepsilon > 0$, $\gamma_i(\theta + \varepsilon) = \theta + \varepsilon$, and $\gamma_i(\theta - \varepsilon) = \theta - \varepsilon$. Under Condition 1, party $i$ will be better off deviating to either $\theta + \varepsilon$ or $\theta - \varepsilon$ in state $\theta$.

12 Battaglini (2002) makes a similar point.
where $u_i^*(\theta)$ denotes party $i$’s equilibrium payoff in state $\theta$. The optimality of $\gamma$ is affected by the median voter’s reaction to the newspaper endorsement only when the policy profile is contained in $Q$.13

The newspaper is influential (off the equilibrium path) if at any $y \in Q$ endorsing a party enhances the party’s probability of election; that is, if for $i, j \in \{1, 2\}$, $i \neq j$

$$\pi^i(y, i) - \pi^i(y, j) > 0.$$ 

Whether the newspaper is influential obviously depends on its endorsement strategy. Call the simple strategy of endorsing the party it prefers $\rho^*$. For all $\theta \in \Theta$, $y \in Y^2$, and $b_m \in B$,

$$\rho^*(y, \theta, b_m) = 1 \iff |y_1 - \theta - b_m| \leq |y_2 - \theta - b_m|. \quad (9)$$

Choosing $\rho^*$ is always optimal for the newspaper. If the newspaper is influential at $y$, then $\rho^*$ is the unique best response at $y$.14 We shall prove that the converse is also true: a newspaper is influential if it adopts $\rho^*$ at all $y \in Q$.

Suppose the newspaper’s strategy is $\rho^*$. For $y_1 \geq y_2$, the probability that party 1 is endorsed in state $\theta$ when the policies are $y$ is

$$\beta^1(y, \theta) = \begin{cases} 0 & \text{if } \hat{\beta}(y, \theta) < 0, \\ \hat{\beta}(y, \theta) & \text{if } \hat{\beta}(y, \theta) \in [0, 1], \\ 1 & \text{if } \hat{\beta}(y, \theta) > 1; \end{cases}$$

where

$$\hat{\beta}(y, \theta) = \frac{c(1 + \tau) + 2\theta - (y_1 + y_2)}{2c}.$$ 

By Condition 1, upon observing $y$ the median voter believes that only $\theta_1(y, \gamma)$ and $\theta_2(y, \gamma)$ are possible. The relative likelihood for the newspaper to endorse party 1 in these two state is

$$\begin{align*}
\frac{\beta^1(y, \theta_1(y, \gamma))}{\beta^1(y, \theta_2(y, \gamma))} &< 1 & \text{if } \varepsilon(y, \gamma) < 0, \\
= 1 & \text{if } \varepsilon(y, \gamma) = 0, \\
> 1 & \text{if } \varepsilon(y, \gamma) > 0;
\end{align*}$$

13If an off-equilibrium policy profile $y$ is equilibrium-dominated for only one party, then by Condition 2 the median voter should conclude that the deviating party is the one for which $y$ is undominated. In this case, the newspaper endorsement has no impact on the median voter’s posterior beliefs. If an off-equilibrium policy profile $y$ is equilibrium-dominated for both parties, then the median voter’s reaction to $y$ by definition has no bearing on the optimality of the parties’ equilibrium strategies.

14If the newspaper is not influential at $y$, then by definition any strategy is optimal.
where \( \varepsilon(y, \gamma) = \theta_1(y, \gamma) - \theta_2(y, \gamma) \).

Recall that party 1 deviates in \( \theta_2 \) and party 2 deviates in \( \theta_1 \). When \( \varepsilon(y, \gamma) < 0 \) (i.e., a party has deviated toward the median voter’s ideal point) the newspaper is more likely to endorse party 1 if party 1 deviates, whereas for \( \varepsilon(y, \gamma) < 0 \) (i.e., a party has deviated away from the median voter’s ideal point) it is less likely to endorse party 1 when party 1 deviates. Hence, the newspaper’s endorsement is informative—a party is more likely to be endorsed when it is the one better for the median voter. This result is due to the assumption that 0 belongs to \( B \). Intuitively, since party strategies are symmetric in equilibrium, the newspaper switches its endorsement when a small deviation occurs only if its policy preference is almost exactly the same as that of the median voter.

By Condition 3 the median voter infers that a party is more likely to be the one he prefers when it is endorsed by the news media. Hence, if he weakly prefers party \( i \) when it is not endorsed, he must strictly prefer party \( i \) when it is. Upon observing any off-equilibrium \( y \), if the median voter elects a party with strictly positive probability when the party is not endorsed, then he must elect the party with probability one when it is. That is, for \( i, j \in \{1, 2\}, i \neq j, \)
\[
\pi^i(y, j) > 0 \Rightarrow \pi^i(y, i) = 1. \tag{10}
\]

Since \( y \in Q \), the median voter cannot elect a party with probability one regardless of the newspaper endorsement; for if it did, that party would gain from the deviation by definition of \( Q \). It then follows from (10) that the newspaper endorsement must strictly increase a party’s probability of election. We have thus established the following proposition.

**Proposition 3** Suppose the median voter’s posterior beliefs \( \mu \) satisfy Conditions 1 and 3. Then in any symmetric perfect Bayesian equilibrium, the newspaper is influential if and only if it chooses \( \rho^* \) at all \( y \in Q \). For any off-equilibrium \( y \), let \( \theta' \) and \( \theta'' \) denote the two states that are consistent with \( y \) according to Condition 1. When the newspaper is influential, for any off-equilibrium \( y \) and for any \( i \in \{1, 2\}, j \neq i, \)
\[
U(y_i, \theta', 0) > U(y_i, \theta'', 0) \Rightarrow \mu(\theta'|y, i) > \mu(\theta''|y, i),
\]
and
\[
\pi^i(y, j) > 0 \Rightarrow \pi^i(y, i) = 1.
\]
Furthermore, if \( y \in Q \), then \( \pi^i(y, j) < \pi^i(y, i) \).

18
Proposition 3 establishes that communication between the newspaper and the median voter is incentive incompatible despite their different policy objectives. Off equilibrium the newspaper endorsement increases a party’s probability of election.

Note that not all equilibria entails an influential newspaper. For example, there are equilibria in which the newspaper randomly makes endorsements, which are ignored by the median voter. This type of babbling equilibria is not a good description of the behavior of the media, whose primary function is to communicate.\(^\text{15}\)

7 Determination of Equilibrium Policies

We now examine how the median voter’s off-equilibrium voting behavior determines the parties’ policy choices. We illustrate our results in two steps. Recall that when the newspaper’s endorsement is ignored, each party chooses its own ideal point as policy in equilibrium. We first show that this is no longer an equilibrium when voters pay attention to the newspaper’s endorsement.

**Proposition 4** There is no perfect Bayesian equilibrium \((\gamma, \rho, \pi, \mu)\) where (1) \(\mu\) satisfies Conditions 1–3; (2) \(\rho = \rho^*\); (3) \(\gamma_1 = -\gamma_2 = b\); and (4) each party is elected with probability \(p^* \in (0,1)\) in every state \(\theta \in \Theta\).

Suppose, by way of contradiction, that there is such an equilibrium. For any \(\varepsilon < 0\), find some off-equilibrium policy profile \(y\) with \(\varepsilon(y, \gamma) = \varepsilon\). Write \(\theta_1\) for \(\theta_1(y, \gamma)\), the state where party 2 deviates, and \(\theta_2\) for \(\theta_2(y, \gamma)\), the state where party 1 does. Note that \(\theta_1 - \theta_2 = \varepsilon < 0\). In state \(\theta_2\), party 1 prefers \(\gamma_1(\theta_2)\) to \(y_1\); hence,

\[
p^*r - (1 - p^*)(2b)^2 \geq p^1(y, \theta_2)(-\varepsilon^2 + r) - (1 - p^1(y, \theta_2)) (2b)^2.
\]

This is equivalent to

\[
p^*(4b^2 + r) \geq p^1(y, \theta_2)(4b^2 + r - \varepsilon^2). \tag{11}
\]

Similarly, in state \(\theta_1\) party 2 prefers \(\gamma_2(\theta_1)\) to \(y_2\), which implies

\[
(1 - p^*)(4b^2 + r) \geq (1 - p^1(y, \theta_1))(4b^2 + r - \varepsilon^2). \tag{12}
\]

\(^{15}\text{Babbling equilibria in this model can be ruled out by more sophisticated arguments. See Farrell (1993), Farrell and Rabin (1996), and Rabin (1990).}\)
Recall that by definition
\[ p^1(y, \theta) = \beta^1(y, \theta)\pi^1(y, 1) + \beta^2(y, \theta)\pi^1(y, 2). \]

Combining (11) and (12) and substituting in the definition of \( p^1 \) gives

\[
\frac{p^*(4b^2 + r)}{4b^2 + r - \varepsilon^2} \geq \beta^1(y, \theta)\pi^1(y, 1) + \beta^2(y, \theta)\pi^1(y, 2)
\geq 1 - \frac{(1 - p^*)(4b^2 + r)}{4b^2 + r - \varepsilon^2} + \frac{\varepsilon}{c} (\pi^1(y, 1) - \pi^1(y, 2)).
\]

(13)

As \( \varepsilon \) tends to zero, both the first and the last term in (13) converge to \( p^* \), and hence the middle term converges to \( p^* \) as well. By Proposition 3, for \( i \in \{1, 2\}, j \neq i, \pi^i(y, j) > 0 \) implies \( \pi^i(y, i) = 1 \). Hence, \( \lim_{\varepsilon \to 0} \pi^1(y(\varepsilon), 2) = 0 \)
if \( (1 + \tau)/2 \geq p^* \), and \( \lim_{\varepsilon \to 0} \pi^1(y(\varepsilon), 1) = 1 \) if \( (1 + \tau)/2 < p^* \). Thus,

\[
\lim_{\varepsilon \to 0} \pi^1(y(\varepsilon), 1) = \begin{cases} 
\frac{2p^*}{1+\tau} & \text{and } \lim_{\varepsilon \to 0} \pi^1(y(\varepsilon), 2) = \begin{cases} 
0 & \text{when } \tau \geq 2p^* - 1; \\
\frac{2p^*-1-\tau}{1-\tau} & \text{when } \tau < 2p^* - 1.
\end{cases}
\end{cases}
\]

(14)

Since the newspaper is more likely to endorse a party when the party’s policy is the better one for the median voter (Proposition 3), the median voter elects the party with a higher probability when it is endorsed by the newspaper. As the size of the deviation tends to zero, the influence of the newspaper’s endorsement on the median voter’s voting strategy converges to

\[
\lim_{\varepsilon \to 0} (\pi^1(y(\varepsilon), 1) - \pi^1(y(\varepsilon), 2)) = \begin{cases} 
\frac{2p^*}{1+\tau} & \text{if } \tau \geq 2p^* - 1; \\
\frac{2(1-p^*)}{p^*1-\tau} & \text{if } \tau < 2p^* - 1.
\end{cases}
\]

(14)

For small deviations the equilibrium influence of the newspaper is determined by the newspaper’s bias and the parties’ equilibrium election probabilities. The newspaper’s influence is the greatest when \( \tau = 2p^* - 1 \); that is, when the newspaper’s bias matches the voting tendency of the median voter. When \( p^* = 0.5 \), a newspaper with \( \tau = 0 \) is the most influential. However, if \( p^* \neq 0.5 \), a neutral newspaper can be less influential than a biased one. Equation (13) implies that the probability that a deviating party get elected must converge to the its equilibrium probability of election. Suppose in equilibrium party 1 is elected with probability \( p^* = 0.6 \). A newspaper with bias \( \tau = 0.2 \) would endorse party 1 with probability 0.6. Therefore, the median voter elects party 1 if and only if the newspaper endorses party 1. On the other hand, a neutral newspaper (\( \tau = 0 \)) would endorse party 1.
with probability 0.5. Then, the median voter elects party 1 with probability 1 when the newspaper endorses party 1. But even when the newspaper endorses party 2, the median voter elects party 1 with probability 0.2. Hence, the neutral newspaper is less influential in the latter case.

Return to the parties’ incentive compatibility constraints. Add (11) to (12), we obtain

\[ \frac{\epsilon^2}{4b^2 + r - \epsilon^2} \geq p^1(y(\epsilon), \theta_2) - p^1(y(\epsilon), \theta_1) \]

\[ = \frac{-\epsilon}{c} \left( \pi^1(y(\epsilon), 1) - \pi^1(y(\epsilon), 2) \right) > 0. \] (15)

Equation (15) implies that a party’s probability of election is higher when it is the one that has deviated toward the median voter’s ideal position. Since a small deviation causes a party only a second-order loss when elected but leads to a first-order increase in the probability of election, a party will gain from deviating toward the political center. Thus, it cannot be an equilibrium for the parties to choose their respective ideal policies. Formally, since the left-hand-side of (15) converges to zero at a faster rate than the right-hand-side of (15), the inequality is violated when \( \epsilon \) is sufficiently small. Note that this result, like Proposition 1, does not depend on assigning specific off-equilibrium beliefs to the median voter.

We are now in a position to state our main result.

**Proposition 5** Suppose the support of \( b_m \) contains \([-b, b] \). Then, there exists perfect Bayesian equilibria in which (1) the median voter’s beliefs satisfy Conditions 1–3; and (2) the newspaper endorses the party that it prefers. Furthermore, in any such equilibrium each party is elected with probability 0.5 on the equilibrium path, and the parties’ strategies are given by \( \gamma_1 = -\gamma_2 = k \geq 0 \), where

\[ \frac{b - k}{4bk + r} \leq \frac{1}{2c(1 + |\tau|)}, \] (16)

with the equality sign holds when \( k > 0 \).

Proposition 5 extends Proposition 4 in several ways. In Proposition 4 the parties’ equilibrium election probabilities are assumed to be constant. Proposition 5 proves that this must be the case. The exact election probabilities are pinned down by Condition 2, and they equal 0.5 when the parties’

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16 Compare this with the equilibrium we constructed in Section 4. In the latter case, each party is elected with probability 0.5 off the equilibrium path.
policy preferences are symmetric—if in equilibrium the parties were elected with different probabilities, the one elected with a lower probability would have a stronger incentive to deviate toward the median voter’s position.\footnote{See Lemma 1 in Appendix A.} Proposition 4 is based on the necessary condition that parties cannot gain from marginal deviations in equilibrium. Proposition 5 shows that when the support of $b_m$ contains $[-b, b]$, this condition is sufficient as well. A formal proof of the proposition is provided in the Appendix.

The numerator of the fraction on the left-hand-side of (16) is the product of the equilibrium probability of winning the election and the marginal gain from adopting a more partisan policy conditional on winning, and the denominator is the difference in utility between winning and losing. This expression is therefore the benefit-cost ratio of a marginal deviation toward a more partisan policy. Since this ratio is decreasing in $k$, the parties have weaker incentives to deviate when their policies are farther apart.

The right-hand-side of (16), whose value depends on the characteristics of the newspaper, measures the extent to which choosing a marginally more partisan policy lowers a party’s probability of winning the election. When the party adopts a policy that is $\varepsilon$ more partisan than its equilibrium policy, there is a probability of $\varepsilon/2c$ that the newspaper may switch and endorse its opponent. The loss of endorsement in turn reduces the party’s election probability by $1/(1 + |\tau|)$ (equation (14) with $p^* = 0.5$).

When $1/(2c(1 + |\tau|)) > (b - k)/(4bk + r)$, the loss from a more centrist policy is more than compensated by an increase in election probability, and the parties are better off from deviating toward the median voter’s ideal point. Hence, if $1/(2c(1 + |\tau|)) > b/r$, then $k = 0$ in equilibrium; that is, policies converge to the median voter’s ideal point. Otherwise, the parties choose policies such that the gain from a marginal deviation is balanced by the loss in election probability.

The comparative statics for the equilibrium policy gap are easy to derive. When $k > 0$, $k$ increases in $b$ and decreases in $r$. The more extreme are the parties’ ideological positions, the farther are their policy platforms from the median voter’s ideal position. The higher is the rent from holding office, the more eager each party is to win the election by engaging in a Hotelling-type competition for newspaper endorsement. When the rent $r$ is sufficiently large, equilibrium entails full policy convergence (i.e., $k = 0$).

Since the parties adopt symmetric policies in equilibrium, the newspaper changes its endorsement for small deviations only when its taste parameter, $b_m$, is close to 0. The density of $b_m$ at 0 is $1/c$. Hence, a newspaper with a
smaller $c$ is more “responsive” to policy changes. Since parties have stronger incentives to compete when the newspaper is responsive, the equilibrium policy gap increases with $c$.

In equilibrium a newspaper with a larger policy bias, $|\tau|$, has a weaker influence on the median voter’s voting behavior. Since the parties’ incentives to compete for the newspaper endorsement is proportional to the newspaper’s influence, the policy gap between the parties grows with the bias of the newspaper. It is important to note that this result is in part due to the fact each party is elected with probability 0.5 in equilibrium. In general, although the parties’ equilibrium policies are still symmetric (i.e., $\gamma_1 = -\gamma_2$) when their policy preferences are asymmetric (i.e., $b_1 \neq -b_2$), their election probabilities are not the same.\footnote{Condition 2 implies that the party whose policy preferences are closer to the median voter’s is elected with a higher probability in equilibrium.} In that case, an unbiased newspaper may not be the most effective. If, for example, the median voter elects party 1 with probability 0.6, then the most effective newspaper is one that endorses party 1 with probability 0.6. Thus, our results do not support the simple notion that biased media are necessarily bad. Since on the equilibrium path voters can directly infer the state from party policies, the election probabilities of the parties are not affected by the newspaper’s bias; rather a newspaper is more effective when its bias matches the equilibrium election probabilities of the parties.

Several aspects of our results require further comments. The conditions on voters’ off-equilibrium beliefs play an important role in our analysis. Conditions 1 and 2 rule out equilibria relying on implausible off-equilibrium beliefs. They imply that linear equilibria are symmetric. This implication in turn leads to one of the key results, namely, that voters cannot determine from the parties’ policies alone which party has deviated (Proposition 1). Condition 2 also helps pin down the parties’ equilibrium election probabilities, which, together with the policy bias of the newspaper, determine the equilibrium policy gap in Proposition 5. Condition 3 ensures that voters cannot ignore the media endorsement when it is informative. If Condition 3 is not required, we can construct linear equilibria with policy gap anywhere between $b$ and the equilibrium policy gap in Proposition 5. The linear equilibrium in Proposition 5 fully utilizes the information conveyed the endorsement. There is no linear equilibrium with a narrower policy gap even when Condition 3 is not required.

Throughout we have focused on linear equilibria because of their tractability and because they nicely capture the incentive problem that prevents vot-
ers from learning the state from the parties. A limitation of this approach is that it ignores the role of the media on the equilibrium path. A natural way to allow the media to play an active role on the equilibrium path is to assume there are fewer policies than states. Alternatively, one can try to characterize the set of bundling equilibria where parties choose the same policy in different states. The main question then becomes when and how do parties choose bundling strategies. Answering these questions goes beyond the scope of this paper.

The basic idea behind Proposition 5 can be generalized to the case where parties use non-linear but strictly increasing strategies. In that case, the policy gap is of course no longer constant, but the equilibrium is still symmetric, and we can establish a lower bound on the maximum policy gap. But solving the equilibrium is cumbersome when the parties’ strategies are non-linear. As a result, it would be difficult to compare the effectiveness of newspapers with different biases.

8 Multiple Newspapers

Thus far we have assumed there is a single newspaper. In this section, we consider the case when there are multiple newspapers. Suppose there are two newspapers, $A$ and $B$, with taste parameters $b_A$ and $b_B$, respectively. Let $b_A$ be uniformly distributed on $[-c(1 - \tau_A), c(1 + \tau_A)]$ and $b_B$ be uniformly distributed on $[-c(1 - \tau_B), c(1 + \tau_B)]$, with $\tau_A, \tau_B \in (-1, +1)$. We assume that voters know the endorsements of both newspapers. Let $e \equiv (e_A, e_B) \in \{1, 2\}^2$ denote an endorsement profile, where $e_i, i \in \{A, B\}$, is the party endorsed by newspaper $i$. A voting strategy of the median voter is denoted by $\pi(y, e) \equiv (\pi^1(y, e), \pi^2(y, e))$. Since voters have the same information, the electoral outcome is determined solely by the vote of the median voter. The probability that party 1 is elected in state $\theta$ when the policy profile $y$ is

$$ p^1(y, \theta) = \sum_{i=1,2} \sum_{j=1,2} \beta^{ij}(y, \theta) \pi^1(y, (i, j)), $$

where $\beta^{ij}(y, \theta)$ is the probability that newspaper $A$ endorses party $i$ and newspaper $B$ endorses party $j$.

Many results developed in previous sections can be readily applied to the current situation. Proposition 2 implies that the equilibrium must be symmetric. Proposition 5 applies in a more general form with equation (16)
replaced by

\[
\frac{b - k}{4bk + r} \leq \lim_{\varepsilon \to 0} \frac{p^1(y(\varepsilon), \theta_2(y(\varepsilon), \gamma)) - p^1(y(\varepsilon), \theta_1(y(\varepsilon), \gamma))}{\varepsilon},
\]

which is independent of the number of newspapers. In equilibrium each party is elected with probability 0.5 in every state \(\theta\), and the cost-benefit ratio of a marginal deviation should be equal to (if \(k > 0\)), or less than (if \(k = 0\)), the marginal change in election probability caused by the deviation. To determine \(k\), we simply have to express the right-hand-side of equation (17) as a function of the characteristics of the two newspapers.

The equilibrium policy outcomes depend on the correlation between \(b_A\) and \(b_B\). At one extreme, we can imagine that they are perfectly correlated; that is, \(b_B = b_A - c(\tau_A - \tau_B)\). When the biases are perfectly correlated, there is a constant gap of \(|c(\tau_A - \tau_B)|\) between the ideal policies of the two newspapers. If the newspaper that is more conservative endorses party 1, the liberal party, then the one that is more liberal must do the same. At the other extreme, we can imagine that \(b_A\) and \(b_B\) are independent, in which case it is possible that the more conservative newspaper endorses party 1 while the liberal newspaper endorses party 2. In the following, we briefly describe the equilibrium under each assumption. Formal propositions and proofs are provided in the Appendix.

**Perfectly Correlated Biases**

When the newspapers have perfectly correlated biases, equilibrium party policies depend on whether the biases have the same sign \((\tau_A\tau_B \geq 0)\) or opposite signs \((\tau_A\tau_B < 0)\). Assume without loss of generality that newspaper A is more liberal than newspaper B (i.e., \(\tau_A > \tau_B\)). Since A endorses party 1 whenever B does, only three endorsement profiles are possible, namely \((1,1)\), \((1,2)\), and \((2,2)\). The probability of \((1,1)\) is the probability that B (the more conservative party) endorses party 1, and the probability of \((2,2)\) is the probability that A endorses party 2.

Suppose newspapers A and B are both liberal. Since newspaper B is liberal, on the equilibrium path the probability of \((1,1)\) is greater than 0.5, while the combined probability of \((1,2)\) and \((2,2)\) is less than 0.5. For small deviations, the median voter treats \((1,1)\) as a neutral signal (meaning that he votes for both parties with positive probability) and either \((1,2)\) or \((2,2)\) as one in favor of party 2 (meaning that he votes for party 2 with probability one). Equilibrium party policies are identical to those when there is a single newspaper with a bias equal to \(\min\{|\tau_A|, |\tau_B|\}\) (which equals \(|\tau_B|\) in this case). The case where both newspapers are conservative is similar.
Suppose newspaper A is liberal and newspaper B is conservative. Now on the equilibrium path the probability of (1, 1) and that of (2, 2) are both less than 0.5. The endorsements are therefore more evenly distributed than that when the newspapers have like biases. For small deviations the median voter treats (1, 1) as a signal in favor of party 1 (i.e., $\pi^1(y, (1, 1)) = 1$), (2, 2) as one in favor of party 2 (i.e., $\pi^1(y, (2, 2)) = 0$), and (1, 2) as one that is neutral. A marginal change in policy toward the median would raise the probability of election by

$$\frac{1}{2c} (\pi^1(y, (1, 1)) - \pi^1(y, (2, 2))) = \frac{1}{2c}.$$

Equilibrium party policies are therefore identical to those when there is a single unbiased newspaper (with the same c). Hence, two newspapers with opposite biases always lead to more centrist policies than two with like biases.

Independent Biases

The median voter has two sources of information when the newspapers’ biases are independent. When a party deviates, each of the two newspapers may switch to endorse its opponent. Parties therefore have stronger incentives to compete for newspaper endorsements. Equilibrium policies are generally closer to the median voter’s ideal point than when there is a single unbiased newspaper.

It is noteworthy that, contrary to the case of perfectly correlated biases, parties do not always adopt more centrist policies when newspapers have opposite biases rather than like biases. A full characterization of the equilibrium policies is provided in the Appendix. An illustrative example is as follows. Suppose in the case of like biases, $\tau_A = 0.6$ and $\tau_B = 0.5$. On the equilibrium path newspaper A endorses party 1 with probability $(1 + \tau_A)/2 = 0.8$, and newspaper B endorses party 2 with probability 0.75. In the case of opposite biases, let $\tau_A = 0.6$ and $\tau_B = -0.5$. The distribution of endorsements is shown in the following table.
At first glance, the endorsements may appear to be more heavily biased in favor of party 1 when the media have like biases than when they have opposite biases. But the median voter do not take endorsements at face value. For small deviations the median voter treats \((1, 1)\) as a neutral signal in the case of like biases, and he treats \((1, 2)\) as a neutral signal in the case of opposite biases. The voting strategy \(\pi^1(y, e)\) shown in the table is derived using the condition that each party wins the election with probability 0.5 in equilibrium. The lower panel of this table shows the effect of a marginal policy deviation on the probability of winning (omitting the multiplicative constant \(1/(2c)\)). For example, the first row of this panel indicates that, conditional on an endorsement of party 1 by newspaper \(A\) (which occurs with probability 0.8), a switch in the endorsement of newspaper \(B\) has a greater influence on the median voter in the case of like biases than in the case of opposite biases.\(^{19}\) In this example, the calculation shows that two independent newspapers with like biases are more effective in promoting policy convergence than are two independent newspapers with opposite biases.\(^{20}\)

Both independent and perfectly correlated biases are obviously extreme cases. Nevertheless, two lessons can be drawn from the above analysis. First, the presence of multiple media outlets encourages political parties to adopt policies that reflect the policy preference of the median voter. Second, the extent of media bias cannot be assessed by examining the bias of each individual media outlet in isolation. The influence of one newspaper depends on the biases of other newspapers and on whether or not these biases are

\(^{19}\)In the case of like bias, the voter’s presumption is that the signal is \((1, 1)\). So a switch in the endorsement of newspaper \(B\) lowers his probability of electing party 1 from \(5/6\) to 0. In the case of opposite biases, the presumption is that the signal is \((1, 2)\). A switch in the endorsement of the newspaper \(B\) only raises his probability of electing party 1 from \(1/2\) to 1.

\(^{20}\)In the Appendix, we show that this conclusion holds if \((1 + |\tau_A|)(1 + |\tau_B|) > 2.\n
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<table>
<thead>
<tr>
<th>(e)</th>
<th>((1,1))</th>
<th>((1,2))</th>
<th>((2,1))</th>
<th>((2,2))</th>
<th>((1,1))</th>
<th>((1,2))</th>
<th>((2,1))</th>
<th>((2,2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta^2(y, \theta))</td>
<td>0.60</td>
<td>0.20</td>
<td>0.15</td>
<td>0.05</td>
<td>0.20</td>
<td>0.60</td>
<td>0.05</td>
<td>0.15</td>
</tr>
<tr>
<td>(\pi^1(y, e))</td>
<td>5/6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1/2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>((1,2) \rightarrow (1,1))</td>
<td>0.8(5/6 - 0) = 0.667</td>
<td>0.8(1 - 1/2) = 0.4</td>
<td></td>
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<tr>
<td>((2,2) \rightarrow (2,1))</td>
<td>0.2(0 - 0) = 0</td>
<td>0.2(0 - 0) = 0</td>
<td></td>
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<tr>
<td>((2,1) \rightarrow (1,1))</td>
<td>0.75(5/6 - 0) = 0.625</td>
<td>0.25(1 - 0) = 0.25</td>
<td></td>
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<tr>
<td>((2,2) \rightarrow (1,2))</td>
<td>0.25(0 - 0) = 0</td>
<td>0.75(1/2 - 0) = 0.375</td>
<td></td>
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<tr>
<td>total influence</td>
<td>1.292</td>
<td>1.025</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
correlated. It is the ecology of the media rather than the biases of individual outlets that determine their role in the political process.

These lessons are further illustrated by the case when the mass media is made up of $n$ independent newspapers with identical bias $\tau$. As all newspapers are identical, the posterior beliefs of the median voter depends only on the number of endorsements each party receives. Let $x$ be the number of newspapers that endorse party 1. In equilibrium $x$ follows a binomial distribution,

$$f(n, x) \equiv \binom{n}{x} \rho^x (1 - \rho)^{n-x},$$

(18)

where $\rho \equiv (1 + \tau)/2$.

Let $x^*$ be the median of $x$. Then $x = x^*$ is the neutral signal. The median voter votes for party 1 with probability 1 if $x \geq x^* + 1$ and he votes for party 1 with probability 0 if $x \leq x^* - 1$. Thus any individual newspaper is influential if and only if there are exactly $x^* - 1$ or $x^*$ other newspapers that endorses party 1. The influence of the media as a group is just $n$ times of the influence of an individual newspaper. In the Appendix, we show that the influence of an individual newspaper goes to 0 at the rate $1/\sqrt{n}$ as $n$ approaches infinity. Hence, regardless of the magnitude of the bias $\tau$, the influence of the media as a group grows at the rate $\sqrt{n}$. We have the following result.

**Proposition 6** Suppose all news media outlets are identical and independent, and that the support of $b_t$ contains $[-b, b]$. Then as the number of media outlets tends to infinity, the equilibrium policy gap tends to zero.

## 9 Conclusion

Empirical research has shown that political news coverage is often slanted in favor of the party favored by the media. Since the political stance of all major news outlets are known, voters respond to news strategically. Surveys have found that voters tend to choose news outlets whose policy preferences are close to their own. There is also anecdotal evidence that endorsements by media outlets holding opposite policy preferences carry more weight among voters.

Thus, one must take into account of voters’ strategic response to biased news in order to understand the effects of the media. The main contribution of this paper is to provide such a framework. Our results confirm the common belief that news media play an important role in electoral politics.
Since voters cannot deduce the state from party policies alone, the problem of media bias cannot be solved by requiring the media to report only party policies, and voters cannot avoid the influence of the media by ignoring biased news analyses.

Since in reality voters are unlikely to be fully rational as we have assumed, some of our results must be interpreted carefully. We show that media bias is potentially harmful even when voters are fully rational and aware of the media bias. In addition, we show that in this case media bias may lead to policy polarization. If some voters are naive, then the media may affect the general direction of policies, as well as the policy gap between the parties.

Throughout we have assumed that every voter consumes all the news provided by the media. In reality, voters consume different amount of news and from different news sources. Voters’ media consumption decisions are important in two ways. First, they affect the information voters receive. Second, commercial media outlets may tailor news to appeal to particular segments of the electorate. We plan to address these issues in future work.
Appendix

A. Proof of Proposition 5

We first establish an auxiliary result.

**Lemma 1** A symmetric perfect Bayesian equilibrium with $\gamma_1 = -\gamma_2 = k$ must violate Conditions 1–2 if there exists an interval $[\theta_l, \theta_r]$, $\theta_r - \theta_l > 2k$, and $\delta < 0.5$ such that the probability of election of one of the parties is less than $\delta$ for all $\theta \in [\theta_l, \theta_r]$. Condition 2 is satisfied if each party is elected with probability 0.5 on the equilibrium path.

**Proof.** Define for $p \in [0, 1],$

$$t(p) \equiv b - \sqrt{p(b - k)^2 + (1 - p)((b + k)^2 + r)}.$$

By definition, party $i \in \{1, 2\}$ is indifferent between winning with the election with probability $p$ (given the parties’ equilibrium strategies) and winning the election with probability 1 with a policy $\gamma_i = (-1)^{i+1}t(p)$. Hence a policy $\theta + (-1)^{i+1}t$ in state $\theta$ is equilibrium-dominated for party $i$ if $(-1)^{i+1}t < (-1)^{i+1}t(p)$. Note that $t(p) \leq k$.

Suppose party 1 is elected with a probability $p(\theta) < \delta < 0.5$ for $\theta \in [\theta_l, \theta_r]$, where $\theta_r - \theta_l > 2k$. Since $t(p)$ increases in $p$, $t(1 - \delta) > t(\delta)$. There exists $\bar{t} \in (t(\delta), t(1 - \delta))$. Consider the off-equilibrium policy profile $\gamma \equiv (\overline{\theta} + \bar{t}, \theta - k)$. Upon observing $\gamma$, the median voter believes that the state is either $\theta_l$ or $\theta_l + (\bar{t} - k) \in [\theta_l, \theta_r]$. By construction, in state $\theta_l$ the policy $\overline{\theta} + \bar{t}$ is not equilibrium-dominated for party 1, while in state $\theta_l - (\bar{t} - k)$ the policy $\theta - k$ is equilibrium-dominated for party 2. Condition 2 therefore implies that the median voter should elect party 1 with probability one when the policy profile is $\gamma$. Hence, choosing $\overline{\theta} + k$ is not optimal for party 1 in state $\theta_l$.

Suppose each party is elected with probability 0.5 in every state. Then for any off-equilibrium $\gamma \equiv (y_1, y_2)$, if $y_i$ is equilibrium-dominated for party $i$ in $\theta_j(\gamma)$, $j \neq i$, then $y_j$ must also be equilibrium-dominated in $\theta_i(\gamma)$. Condition 2 is therefore satisfied.

Suppose $(\gamma, \rho, \pi, \mu)$ is a perfect Bayesian equilibrium where (1) $\mu$ satisfies Conditions 1–3; (2) $\rho = \rho^*$; (3) $\gamma_1 = -\gamma_2 = k$. We adopt the following notation. Let $p(\theta)$ denote party 1’s equilibrium probability of election in state $\theta$, $q(\theta', \theta, i)$ denote party 1’s probability of election when party $i$ deviates by choosing $\gamma_i(\theta')$ in state $\theta$, $\beta_i(\theta', \theta'', \theta)$ denote the probability that the newspaper endorses party $i$ when party 1 chooses $\gamma_1(\theta')$ and party 2 chooses $\gamma_2(\theta'')$ in state $\theta$, and $\pi_i(\theta', \theta'', e)$ denote party $i$’s probability of election.
when party 1 chooses $\gamma_1(\theta')$, party 2 chooses $\gamma_2(\theta'')$ and the newspaper endorses party $e$. Consider any pair of states $\theta, \theta' \in \Theta$. Incentive compatibility requires that party 1 prefers $\gamma_1(\theta)$ to $\gamma_1(\theta')$ in state $\theta$, and that party 2 prefers $\gamma_2(\theta')$ to $\gamma_2(\theta)$ in state $\theta'$. That is,

$$p(\theta) (U(\gamma_1(\theta), \theta, b_1) + r) + (1 - p(\theta)) U(\gamma_2(\theta), \theta, b_1) \geq q(\theta', \theta, 1) (U(\gamma_1(\theta'), \theta, b_1) + r) + (1 - q(\theta', \theta, 1)) U(\gamma_2(\theta), \theta, b_1);$$

(19) and

$$1 - p(\theta') (U(\gamma_2(\theta'), \theta', b_2) + r) + p(\theta') U(\gamma_1(\theta'), \theta', b_2) \geq (1 - q(\theta', \theta', 2)) (U(\gamma_2(\theta), \theta', b_2) + r) + q(\theta, \theta', 2) U(\gamma_1(\theta'), \theta', b_2).$$

(20)

Using the definition of $U$, these incentive compatibility constraints can be written as

$$p(\theta) \geq q(\theta', \theta, 1) \left(1 + \frac{2(b - k)\varepsilon - \varepsilon^2}{4bk + r}\right)$$

(21) and

$$1 - p(\theta') \geq (1 - q(\theta, \theta', 2)) \left(1 + \frac{2(b - k)\varepsilon - \varepsilon^2}{4bk + r}\right),$$

(22)

where $\varepsilon \equiv \theta' - \theta$ is the difference between $\theta'$ and $\theta$.

The analysis proceeds in several steps.

**Step 1: Show that $p$ is continuous.**

Write $H(\varepsilon)$ for $(2(b - k)\varepsilon - \varepsilon^2)/(4bk + r)$. Equations (21) and (22) can be combined to give

$$p(\theta) - p(\theta') + 1 \geq (1 + H(\varepsilon)) (q(\theta', \theta, 1) - q(\theta', \theta', 2) + 1).$$

(23)

By definition,

$$q(\theta', \theta, 1) - q(\theta', \theta', 2) = \sum_{i \in \{1, 2\}} (\beta^i(\theta', \theta) - \beta^i(\theta', \theta', i)) \pi^1(\theta', \theta, i)$$

$$= \frac{\varepsilon}{c} (-\pi^1(\theta', \theta, 1) + \pi^1(\theta', \theta, 2)).$$

(24)

Substituting equation (24) into (23) gives

$$p(\theta) - p(\theta') \geq H(\varepsilon) + \frac{\varepsilon}{c} (1 + H(\varepsilon)) (-\pi^1(\theta', \theta, 1) + \pi^1(\theta', \theta, 2)).$$

(25)
Equation (25) holds for all $\theta$ and $\theta'$. Interchanging the roles of $\theta$ and $\theta'$ in equation (25), we get
\[ p(\theta) - p(\theta') \leq -H(-\varepsilon) + \frac{\varepsilon}{C} (1 + H(-\varepsilon)) \left( -\pi^1(\theta, \theta', 1) + \pi^1(\theta, \theta', 2) \right). \quad (26) \]
Since $H(0) = 0$, equations (25) and (26) imply that $p$ is continuous in $\theta$.

**Step 2:** Derive the limits of $\pi^1(\theta, \theta', 1)$ and $\pi^1(\theta, \theta', 2)$ as $\theta'$ tends to $\theta$.

Equations (21) and (22) can be written as
\[ p(\theta) \geq q(\theta', \theta, 1) (1 + H(\varepsilon)) \]
\[ \geq p(\theta') + H(\varepsilon) + \frac{\varepsilon}{C} (1 + H(\varepsilon)) \left( -\pi^1(\theta', \theta, 1) + \pi^1(\theta', \theta, 2) \right). \quad (27) \]
Since the right-hand-side of equation (27) converges to $p(\theta)$ as $\theta'$ tends to $\theta$, both $q(\theta', \theta, 1)$ and $q(\theta', \theta, 2)$ converge to $p(\theta)$ as $\theta'$ tend to $\theta$. Since $\pi^1(y, j) > 0$ implies $\pi^1(y, i) = 1$ (Proposition 3), it follows that
\[ \lim_{\theta' \to \theta} \pi^1(\theta, \theta', 1) = \lim_{\theta' \to \theta} \pi^1(\theta', \theta, 1) = \begin{cases} \frac{1}{2\rho(\theta)} & \text{if } p(\theta) \geq (1 + \tau)/2, \\ \frac{2\rho(\theta) - (1 + \tau)}{1 - \tau} & \text{if } p(\theta) < (1 + \tau)/2; \end{cases} \]
and
\[ \lim_{\theta' \to \theta} \pi^1(\theta, \theta', 2) = \lim_{\theta' \to \theta} \pi^1(\theta', \theta, 2) = \begin{cases} \frac{2\rho(\theta) - (1 + \tau)}{1 - \tau} & \text{if } p(\theta) \geq (1 + \tau)/2, \\ 0 & \text{if } p(\theta) < (1 + \tau)/2. \end{cases} \]

**Step 3:** Show that $p$ is differentiable.

Combining equations (25) and (26) and dividing every term by $-\varepsilon$, we have
\[ \frac{H(-\varepsilon)}{\varepsilon} - \frac{1}{C} \left( -\pi^1(\theta, \theta', 1) + \pi^1(\theta, \theta', 2) \right) + O(\varepsilon) \geq \frac{p(\theta) - p(\theta')}{\theta - \theta'} \]
\[ \geq - \frac{H(\varepsilon)}{\varepsilon} - \frac{1}{C} \left( -\pi^1(\theta', \theta, 1) + \pi^1(\theta', \theta, 2) \right) + O'(\varepsilon) \quad (28) \]
where $\lim_{\varepsilon \to 0} O(\varepsilon) = \lim_{\varepsilon \to 0} O'(\varepsilon) = 0$. As $\varepsilon$ tends to 0, both the right-hand-side and the left-hand-side of equation (28) converge to the same limit. Hence, $p$ is differentiable with
\[ \frac{dp}{d\theta} = \begin{cases} \frac{2\rho(\theta)}{4\rho(\theta) + r} + \frac{2(1 - \rho(\theta))}{c(1 - \tau)} & \text{if } p(\theta) \geq (1 + \tau)/2, \\ \frac{2\rho(\theta)}{4\rho(\theta) + r} + \frac{2\rho(\theta)}{c(1 + \tau)} & \text{if } p(\theta) < (1 + \tau)/2. \end{cases} \quad (29) \]
Step 4: Solve for $p(\theta)$.

When $c$ tends to infinity, $dp/d\theta$ is strictly negative for all $\theta$ when $k < b$ and strictly positive for all $\theta$ when $k > b$, violating the condition that $p$ be bounded between zero and one. Proposition 1 is thus a special case of Proposition 5. More generally, define $\hat{k}$ such that

$$\frac{2(b - \hat{k})}{4\hat{k} + r} = \frac{1}{c}.$$  

Note that when $p(\theta) \geq (1 + \tau)/2$, $2(1 - p(\theta))/(1 - \tau) \leq 1$; and when $p(\theta) < (1 + \tau)/2$, $2p(\theta)/(1 + \tau) < 1$. Since $2(b - k)/(4bk + r)$ decreases in $k$ when $k < \hat{k}$. On the other hand, $dp/d\theta$ is strictly positive for all $\theta$ when $k > b$. In either case $p$ will violate the boundary condition. There is thus no equilibrium outside the range $[\hat{k}, b]$. For any $k \in [\hat{k}, b]$, the differential equation (29) has three solutions. There are two stationary solutions obtained by setting $dp/d\theta$ to zero:

$$p(\theta) = p^* = \frac{(b - k)c(1 + \tau)}{4bk + r},$$

and

$$p(\theta) = p^{**} = 1 - \frac{(b - k)c(1 - \tau)}{4bk + r}.$$  

Note that when $k > \hat{k}$, $p^* < (1 + \tau)/2 < p^{**}$. Finally, there is a class of non-stationary solutions of the form

$$p(\theta) = p^{***}(\theta) = \begin{cases} p^{**} + \left(\frac{1 + \tau}{2} - p^{**}\right) \exp\left\{\frac{2}{c(1 - \tau)}(\theta - \theta^*)\right\} & \text{for } \theta \geq \theta^*, \\ p^* + \left(\frac{1 + \tau}{2} - p^*\right) \exp\left\{\frac{2}{c(1 - \tau)}(\theta - \theta^*)\right\} & \text{for } \theta < \theta^*, \end{cases}$$

where $\theta^*$ is an arbitrary constant. This third solution converges to $p^{**}$ as $\theta$ tends to infinity and to $p^*$ as $\theta$ tends to negative infinity.

Define $k^*$ such that equation (16) of Proposition 5 holds as an equality; that is,

$$\frac{(b - k^*)}{4k^* + r} = \frac{1}{2c(1 + |\tau|)}.$$  

For $\tau \geq 0$, $p^* = 0.5$ at $k = k^*$. For $\tau < 0$, $p^{**} = 0.5$ at $k = k^*$. For all $k \neq k^*$, neither of the stationary solutions equal 0.5. By Lemma 1, they must violate Conditions 1 and 2. Since the non-stationary solution converges to the two stationary solutions, it violates Conditions 1 and 2 as well. Hence, the only solution consistent with Conditions 1–3 is $\gamma_1 = \gamma_2 = k^*$ and $p(\theta) = 0.5$ for all $\theta$. This establishes the necessity of equation (16) for equilibrium.
Step 5: Show that the incentive compatibility constraints are satisfied.

To demonstrate the sufficiency of equation (16) for equilibrium, we need to show that the incentive compatibility constraints (19) and (20) are satisfied at $k = k^*$. We defer the case $k = 0$ to the next step.

Let the median voter's strategy be given by

$$\pi^1(\theta', \theta, 1) = \begin{cases} \frac{1}{1+\tau} & \text{and } \pi^1(\theta', \theta, 2) = \begin{cases} 0 & \text{when } \tau \geq 0; \\ \frac{1}{1+|\tau|} & \text{when } \tau < 0. \end{cases} \end{cases}$$

(30)

This voting strategy is consistent with Proposition 3 and satisfies $p(\theta) = 0.5$ for all $\theta$.

Note that both $q(\theta', \theta, 1) < p(\theta)$ and $U(\gamma_1(\theta'), \theta, b_1) < U(\gamma_1(\theta), \theta, b_1)$ if $\theta' - \theta > b - k$ or $\theta' - \theta < -2k$. In these two cases, the incentive compatibility constraint (19) is always satisfied. We therefore need to consider only the case where $\varepsilon = \theta' - \theta$ lies in the range $[-2k, b - k]$. If the support of $b_m$ contains $[-b, b]$, we have

$$c(1 - |\tau|) > 2b \geq |\varepsilon|$$

for all $\varepsilon \in [-2k, b - k]$. Therefore, $\beta^1(\theta', \theta, \theta')$ and $\beta^1(\theta', \theta, \theta')$ are strictly between 0 and 1. Substituting (30) into the definition of $q$, we have

$$q(\theta', \theta, 1) = 0.5 - \frac{\varepsilon}{2c(1+|\tau|)}.$$  

(31)

The incentive compatibility constraint (21) then reduces to

$$0.5 \geq \left(0.5 - \frac{\varepsilon}{2c(1+|\tau|)}\right) \left(1 + \frac{2(b - k)\varepsilon - \varepsilon^2}{4bk + r}\right).$$

(32)

Let $G(\varepsilon)$ represent the right-hand-side of (32). One can verify that $G(0) = 0.5$, $dG(0)/d\varepsilon = 0$ at $k = k^*$, and

$$\frac{d^2G(\varepsilon)}{d\varepsilon^2} = -\frac{4(b - k - \varepsilon) + c(1 + |\tau|) - \varepsilon}{2c(1 + |\tau|)(4bk + r)} < 0.$$  

Hence, when $k = k^*$, the maximum value of $G(\varepsilon)$ is 0.5 for all $\varepsilon \in [-2k, b - k]$. This establishes that the constraint (19) for party 1 is satisfied for all $\varepsilon$. The argument for party 2 is parallel.

Step 6: The case of $k = 0$.  

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Finally, consider the case where the parameter values are such that \( r/b > 2c(1 + |\tau|) \), so that \( k = 0 \) by equation (16). As in the previous case, the incentive constraint (19) is satisfied for all \( \theta' - \theta > b \) or \( \theta' - \theta < 0 \). Therefore, we need to consider only the case where \( \varepsilon \in [0, b] \). Using the \( G \) function defined earlier, we have
\[
\frac{dG(0)}{d\varepsilon} = -\frac{1}{2c(1 + |\tau|)} + \frac{b}{r} < 0
\]
at \( k = 0 \). Furthermore, since
\[
\frac{d^2G(\varepsilon)}{d\varepsilon^2} = -\frac{4(b - \varepsilon) + (c(1 + |\tau|) - \varepsilon)}{2rc(1 + |\tau|)} < 0,
\]
we have \( G(\varepsilon) \leq G(0) = 0.5 \) for all \( \varepsilon \in [0, b] \). Therefore the incentive constraint (19) is satisfied at \( k = 0 \). A parallel argument establishes that (20) is satisfied for party 2.

**B. Characterization of Equilibria with Two Newspapers**

**Proposition 7** Suppose the support of \( b_i, i = A, B \), contains \([-b, b]\). Let \( b_A \) and \( b_B \) be perfectly correlated. Then, there exists perfect Bayesian equilibria in which (1) \( \mu \) satisfies Conditions 1–3; and (2) \( \rho = \rho^* \). Furthermore, in any such equilibrium the parties’ strategies are given by \( \gamma_1 = -\gamma_2 = k \geq 0 \), where
\[
\frac{b - k}{4bk + r} \leq \frac{1}{2c(1 + \min\{|\tau_A|, |\tau_B|\})}
\]
if \( \tau_A \tau_B \geq 0 \), and
\[
\frac{b - k}{4bk + r} \leq \frac{1}{2c}
\]
if \( \tau_A \tau_B < 0 \); with the equality signs hold when \( k > 0 \).

**Proof.** We just provide a sketch of the proof here. The notation and many of the arguments are similar to those used in the proof of Proposition 5.

Consider first the case of like bias. Assume specifically that \( \tau_A > \tau_B \geq 0 \). (The other cases are similar.) Since newspaper \( A \) is more biased for party 1 than is newspaper \( B \), Proposition 3 implies that
\[
\pi^1(\theta', \theta, (1, 1)) \geq \pi^1(\theta', \theta, (1, 2)) \geq \pi^1(\theta', \theta, (2, 2)).
\]
Furthermore, at most one of these probabilities is strictly between 0 and 1. Let the median voter’s strategy be

\[
\begin{align*}
\pi^1(\theta', \theta, (1, 1)) &= \frac{1}{1 + \tau_B}, \\
\pi^1(\theta', \theta, (1, 2)) &= 0, \\
\pi^1(\theta', \theta, (2, 2)) &= 0.
\end{align*}
\] (35)

Since \(b_A\) and \(b_B\) are perfectly correlated, for small \(\theta\) and \(\theta'\) such that \(\varepsilon = \theta' - \theta\) is small, we have

\[
\begin{align*}
\beta^{11}(\theta', \theta, \theta) &= \frac{(1 + \tau_B)}{2} - \frac{\varepsilon}{2c}, \\
\beta^{12}(\theta', \theta, \theta) &= \frac{\tau_A - \tau_B}{2}, \\
\beta^{22}(\theta', \theta, \theta) &= \frac{1 - \tau_A}{2} + \frac{\varepsilon}{2c}.
\end{align*}
\]

Therefore, the voting strategy (35) satisfies \(p(\theta) = 0.5\) for all \(\theta\), and it is consistent with Proposition 3 and Lemma 1. If the support of \(b_i\), \(i = A, B\), contains \([-b, b]\), we have

\[
q(\theta', \theta, 1) = 0.5 - \frac{\varepsilon}{2c} \frac{1}{1 + \tau_B}
\]

for \(\varepsilon \in [-2k, b - k]\). This is the same as equation (31) in the proof of Proposition 5, with \(|\tau|\) replaced by \(\tau_B\). The rest of the proof follows.

Next, consider the case of like bias. Assume without loss of generality that \(\tau_A \geq 0 \geq \tau_B\). Let the median voter’s strategy be

\[
\begin{align*}
\pi^1(\theta', \theta, (1, 1)) &= 1, \\
\pi^1(\theta', \theta, (1, 2)) &= \frac{-\tau_B}{\tau_A - \tau_B}, \\
\pi^1(\theta', \theta, (2, 2)) &= 0.
\end{align*}
\]

This strategy implies \(p(\theta) = 0.5\). For \(\varepsilon \in [-2k, b - k]\), we have

\[
q(\theta', \theta, 1) = 0.5 - \frac{\varepsilon}{2c}.
\]

This is the same as the expression in equation (31), with \(|\tau|\) replaced by 0. The rest of the proof follows.

Proposition 8 Suppose the support of \(b_i\), \(i = A, B\), contains \([-b, b]\). Let \(b_A\) and \(b_B\) be independent. Then, there exists perfect Bayesian equilibria in which (1) \(\mu\) satisfies Conditions 1–3; and (2) \(\rho = \rho^*\). Furthermore, in any such equilibrium the parties’ strategies are given by \(\gamma_1 = -\gamma_2 = k \geq 0\), where

\[
\frac{b - k}{4bk + r} \leq \frac{1}{2c} \left( \frac{1}{1 + |\tau_A|} + \frac{1}{1 + |\tau_B|} \right) \tag{36}
\]
if $(1 + \tau_A)(1 + \tau_B) \geq 2$ and $\tau_A \tau_B \geq 0$;

$$\frac{b - k}{4bk + r} \leq \frac{2c}{1 + \min\{|\tau_A|, |\tau_B|\}}$$

(37)

if $(1 + |\tau_A|)(1 + |\tau_B|) < 2$ and $\tau_A \tau_B \geq 0$; and

$$\frac{b - k}{4bk + r} \leq \frac{2c}{1 + \max\{\tau_A, \tau_B\}} + \frac{1}{1 - \min\{|\tau_A|, |\tau_B|\}}$$

(38)

if $\tau_A \tau_B < 0$; with the equality signs hold when $k > 0$.

**Proof.** We only sketch a proof of this proposition for the case $\tau_A \geq 0$, $\tau_B \geq 0$, and $(1 + \tau_A)(1 + \tau_B) \geq 2$. The proof for the other cases are similar.

Let the median voter’s strategy be given by

$$\pi^1(\theta', \theta, (1, 1)) = \frac{2}{(1 + \tau_A)(1 + \tau_B)},$$

and $\pi^1(\theta', \theta, e) = 0$ for $e \in \{(1, 2), (2, 1), (2, 2)\}$. For small $\varepsilon$, the probability of observing the endorsement $(1, 1)$ is

$$\beta^{11}(\theta', \theta, \varepsilon) = \left(1 + \frac{\tau_A}{2} - \frac{\varepsilon}{2c}\right) \left(1 + \frac{\tau_B}{2} - \frac{\varepsilon}{2c}\right).$$

Therefore $p(\theta) = 0.5$ for all $\theta$.

If the support of $b_i$ contains $[-b, b]$, we have

$$q(\theta', \theta, 1) = 0.5 - \frac{\varepsilon}{2c} \left(\frac{1 + \tau_A}{2} + \frac{1 + \tau_B}{2} - \frac{\varepsilon}{2c}\right) \frac{2}{(1 + \tau_A)(1 + \tau_B)},$$

for $\varepsilon \in [-2k, b - k]$. As in the case for the proof of Proposition 5, the incentive compatibility constraint requires that

$$0.5 \geq q(\theta', \theta, 1) \left(1 + \frac{2(b - k)\varepsilon - \varepsilon^2}{4bk + r}\right).$$

Let $G(\varepsilon)$ represent the right-hand-side of the above equation, and let

$$m = \frac{1}{2c} \left(\frac{1}{1 + \tau_A} + \frac{1}{1 + \tau_B}\right).$$

We have

$$G(\varepsilon) = \left(0.5 - m\varepsilon + \frac{\varepsilon^2}{(2c)^2} \frac{2}{(1 + \tau_A)(1 + \tau_B)}\right) \left(1 + 2m\varepsilon - \frac{\varepsilon^2}{4bk + r}\right)$$

$$\leq (0.5 - m\varepsilon + m^2\varepsilon^2) (1 + 2m\varepsilon)$$

$$= 0.5 - m^2\varepsilon^2 (1 - 2m\varepsilon).$$

(39)
Note that for \( \varepsilon \in [-2k, b-k] \), \( \varepsilon \leq 2b \leq \min\{c(1-\tau_A), c(1-\tau_B)\} \). Therefore
\[
2m\varepsilon \leq \min\{1 - \tau_A, 1 - \tau_B\} \left( \frac{1}{1 + \tau_A} + \frac{1}{1 + \tau_B} \right) \\
\leq \min\{1 - \tau_A, 1 - \tau_B\} \frac{1 + \tau_A + 1 + \tau_B}{2} \\
\leq \min\{1 - \tau_A, 1 - \tau_B\} \max\{1 + \tau_A, 1 + \tau_B\} \\
\leq 1.
\]
Substituting this into equation (39), we obtain \( G(\varepsilon) \leq 0.5 \), as required. ■

**Corollary 2** Equilibrium policies with two newspapers \( A \) and \( B \) are closer to the median point than are equilibrium policies with newspaper \( A \) or newspaper \( B \) alone, regardless of the value of \( \tau_A \) and \( \tau_B \) and regardless of whether \( b_A \) and \( b_B \) are independent or perfectly correlated.

**Proof.** The expressions on the right-hand-side of (33), (34), (36), (37), (38) are all greater than \( 1/(2c\min\{\tau_A, \tau_B\}) \). ■

**Corollary 3** If \( b_A \) and \( b_B \) are independent, and if \( (1 + |\tau_A|)(1 + |\tau_B|) \geq 2 \), then equilibrium policies are closer to the median in the case of like biases \( (\tau_A \tau_B \geq 0) \) than in the case of opposite biases \( (\tau_A \tau_B < 0) \).

**Proof.** The right-hand-side expression of equation (36) is greater than that of equation (38). ■

**C. Proof of Proposition 6**

Suppose there are \( n \) independent newspapers, each with bias \( \tau \). Let \( e = (e_1, \ldots, e_n) \in \{1, 2\}^n \) be the endorsements of the newspapers, and let \( x \) denote the number of endorsements for party 1. In equilibrium the distribution of \( x \) is given by equation (18), and we let \( x^* \) denote the median of this distribution.

Consider the voting strategy given by
\[
\pi^1(\theta', \theta, e) = \pi^1(x) = \begin{cases} 
1 & \text{if } x \geq x^* + 1 \\
d & \text{if } x = x^* \\
0 & \text{if } x \leq x^* - 1,
\end{cases}
\]
where the constant \( d \) is chosen such that the equilibrium probability of winning \( p(\theta) \) is equal to 0.5.
Let $\beta^{e_t}(\theta', \theta, \theta)$ be the probability that newspaper $t$ endorses party $e_t$ when party 1 deviates to $\gamma_1(\theta')$ in state $\theta$. Write

$$\beta^{e_t}(\theta', \theta, \theta) = \beta^{e_1}(\theta', \theta, \theta) \ldots \beta^{e_t}(\theta', \theta, \theta).$$

We have

$$q(\theta', \theta, 1) = \sum_e \beta^e(\theta', \theta, \theta)\pi^1(\theta', \theta, e).$$

Let $e_{-t}$ be the vector of endorsements with the element $e_t$ removed, and let $(e_{-t}, e_t) = e$. Define

$$s_t(i) \equiv \sum_{e_{-t}} \beta^{e_{-t}}(\theta', \theta, \theta)\pi^1(\theta', \theta, (e_{-t}, i)).$$

That is, $s_t(i)$ is the conditional probability that the median voter votes for party 1 when newspaper $t$ endorses party $i$. Then, we can write the marginal effect of deviation on winning probability as

$$\left. \frac{\partial q(\theta', \theta, \theta)}{\partial \theta'} \right|_{\theta' = \theta} = -\sum_{i=1}^n \frac{1}{2c}(s_t(1) - s_t(2)).$$

Equation (41) expresses the marginal effect as a sum of $n$ terms, each of which represents the effect of an individual newspaper.

Given the voting strategy (40), the influence of a newspaper $t$ can be written as

$$s_t(1) - s_t(2) = \frac{f(n-1, x^* - 1)d + f(n-1, x^*)(1-d)}{n\rho} + (1-d) \left( \frac{f(n, x^*)(n\rho - x^*)}{n\rho(1-\rho)} \right).$$

(42)

Since the distribution of the standardized binomial variable

$$x' = (x' - n\rho)/\sqrt{n\rho(1-\rho)}$$

tends to the unit normal distribution as $n$ tends to infinity, $(x^* - n\rho)/\sqrt{n}$ tends to 0 and $f(n, x^*)$ tends $1/\sqrt{2\pi n\rho(1-\rho)}$. Hence, the second term in (42) tends to zero, and the first tends to $1/\sqrt{2\pi n\rho(1-\rho)}$ (Feller 1950, Theorems 7.1 and 7.2). Thus the expression in (41) approaches infinity at the rate $\sqrt{n}$. For $n$ large, we must have

$$\frac{b - k}{4bk + r} < \lim_{\varepsilon \to 0} \frac{p^1(y(\varepsilon), \theta_2(y(\varepsilon), \gamma)) - p^1(y(\varepsilon), \theta_1(y(\varepsilon), \gamma))}{\varepsilon}.$$ 

Hence, $k = 0$. 

39
References


Figure 1
Figure 2
Figure 3