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Monetary Policy and the Dangers of Deflation: Lessons from Japan*

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This paper investigates how monetary policy can help to avoid the liquidity trap by studying the experience of Japan. First, I analyze how the Bank of Japan conducted interest rate policy over the 1990s as the economy entered a deflationary slump. I use a new method of estimating the policy rule with a time-varying inflation target and a time-varying natural rate of interest. The estimation strategy reveals that the Bank’s implicit inflation target declined to about 1% in the 1990s from about 2.5% in the 1980s. I also find that the policy rule respects the Taylor principle and is forward looking. Such a Taylor rule does not depart from what was perceived as current best practice. It thus seems that the problem arose because of a series of adverse shocks and not because of an extraordinary monetary policy mistake. Next, I investigate whether an alternative monetary policy rule could have avoided the liquidity trap despite these shocks. I find that targeting a higher rate of inflation of 2-3% would not have provided much protection against hitting the zero bound on nominal interest rates. Similarly, a policy of responding more aggressively to the inflation gap while keeping the low inflation target would have provided little improvement in economic performance. The economy also still enters the trap under a nonlinear policy rule that commits the central bank to keeping interest rates at zero even after the economy begins to recover. However, I find that a rule that combined both (i) a higher inflation target of about 3%, and (ii) a more aggressive response to the inflation gap would have improved the economy’s performance and avoided the zero bound.

Key Words: Deflation, Kalman filter, liquidity trap, Taylor rule.

JEL Classification: C22, E31, E52

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1. INTRODUCTION

The fear of some economies falling into a liquidity trap has stimulated research into how to avoid the zero bound on nominal interest rates and the dangers of deflation. This paper seeks to shed light on this topic by studying the experience of Japan, a country that has entered a liquidity trap. The floundering of the Japanese economy over the last twelve years with slow growth, rising unemployment, and deflation has stimulated a lively debate about what went wrong. Many authors blame the Bank of Japan for being “behind the curve and then intentionally deflationary” (Posen, 2003) and for making “wrongheaded monetary policy” (Friedman, 2003) that allowed Japan to slip into a liquidity trap. What policy was the Bank of Japan following and what, if anything, could it have done to avoid this “economic quagmire” (Krugman, 2003)?

This paper addresses these questions by first investigating how the Bank of Japan conducted monetary policy as conditions deteriorated in the 1990s. For this, I use a new method of estimating the Taylor rule that allows the implicit inflation target and the natural rate of interest to vary over time. The inflation target and the natural rate are modelled as random walks and are estimated using maximum likelihood and the Kalman filter. Secondly, the paper conducts counter-factual simulations to see whether an alternative

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For the purposes of this paper, an economy is said to be in a liquidity trap when nominal interest rates on short-term assets have been driven to zero. Some authors claim that a liquidity trap only occurs once all nominal rates, including long-term rates, have fallen to zero and that this has not happened in Japan. However, even the 10-year Japanese Government Bond yield has very little room to fall. The average yield was less than 1% over the first half of 2003.
monetary policy approach would have helped to avoid the liquidity trap. To this end, I estimate a standard rational expectations model and identify the shocks that were hitting the economy.

My first finding is that the implicit inflation target seems to have declined to about 1% in the 1990s from near 2.5% in the 1980s. Also, as in Clarida et al. (1998) and Ahearne et al. (2002), Japanese interest rates fit a forward-looking policy rule that respects the Taylor principle. Such a policy rule fits the orthodoxy of the era and does not depart from what was perceived as current best practice. This finding also suggests that monetary policy may not have been directly responsible for the economy’s disappointing performance.

Secondly, I identify a number of adverse shocks over the 1990s. I find that with more benign shocks, such as those that occurred over the 1980s, the economy would have avoided the deflationary slump. This finding supports the notion that monetary policy was not the primary cause of the economy’s poor performance. However, it does not follow that the Bank could have done nothing more to stabilize the economy.

In the counterfactual analyses, I try the following alternative monetary policy rules to see whether they would have helped Japan avoid the deflationary slump and the zero bound: (i) a policy rule with a higher inflation target, as suggested by Hunt and Laxton (2001) and by Reifschneider and Williams (2000); (ii) a policy rule with a very strong response to deviations of inflation forecasts from the target, along the lines of Orphanides and Williams (1998) and Kato and Nishiyama (2001); (iii) a policy rule that combines a higher inflation target with a stronger response to the inflation
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gap; and (iv) a non-linear Taylor rule that commits the central bank to holding nominal rates at zero even after the economy begins to recover as in Reifschneider and Williams (2000) and Eggertsson and Woodford (2003).

Several authors have found that such alternative policy rules can reduce the likelihood of hitting the zero bound using artificial data sets and stochastic simulations. For example, Hunt and Laxton (2001) generate 100 artificial data sets of 100 years each and find that targeting inflation rates greater than 2 percent reduces the probability of interest rates reaching zero. In contrast to such existing research, I investigate the effectiveness of these policy proposals in the context of an estimated model of the Japanese economy and the actual historical series of shocks that occurred in Japan over the 1990s.

I find that simply raising the inflation target to 3% but keeping everything else unchanged would not have provided much protection against hitting the zero bound. Similarly, a policy of simply responding more aggressively to inflation while keeping the inflation target at the low original level would have provided little insurance against falling into the deflationary slump. Importantly, however, I find that a policy that both (i) raised the inflation target to about 3% and (ii) responded more energetically to the inflation gap would have helped Japan avoid the liquidity trap. Finally, I find that the nonlinear policy rule would have provided only a minor improvement in performance.

Part 2 of the paper explains my methodology and reports my estimates of the Bank of Japan policy rule. Part 3 explains the specification and estimation of the model used for the counterfactual experiments. Part 4 reports the results of the simulations. Part 5 concludes the paper.
FIG. 1. The Japanese Economy Since 1980

2. TAYLOR RULE ANALYSIS

In this section, I investigate how the Bank of Japan conducted interest rate policy as economic conditions deteriorated. The first step is to define the variables that I use, i.e. inflation, the output gap and the policy interest rate. Japan’s experience in the 1990s is sufficiently well known so that a detailed description of the data seems unnecessary. Figure 1 displays the data. My estimation sample is 1979:Q4-1995:Q4. The sample ends in 1995:Q4 as interest rates were at or near the zero bound after that date.

Inflation

I use two measures of inflation based on the CPI excluding perishables (CPIX) and on the GDP deflator (GDPD). There were two jumps in the
price level in 1989:Q2 and in 1997:Q2 due to the introduction of a consumption tax, and its increase from 3% to 5%, respectively. Since monetary policymakers probably do not respond to purely tax induced inflation changes, I use the consumption-tax-adjusted inflation series provided by the Bank of Japan.².

**Output Gap**

In the Taylor rule literature, the output gap is typically measured by detrending the log of real GDP using the Hodrick-Prescott (1997) filter. For example, a negative output gap of 4% implies that output is 4% below “potential” GDP.

However, as McCallum (2000) explains, using the HP filter to measure the Japanese output gap over the 1990s is problematic. When GDP declines, the HP filter “potential GDP” declines along with it. Thus, if the economy is in a long recession towards the end of the sample, as in the case of Japan, the HP filter output gap tends to understate the degree of slack in the economy. In effect, the HP filter eliminates recessions that occur towards the end of a sample period. For example, over 2000:Q1-2001:Q4, the average value of the HP-filtered output gap is positive at 0.2% which suggests that the economy was operating at just above potential. However, there is a consensus that Japanese output was actually well below potential over that period.

Ideally, I would have the official output gap data that BoJ policymakers consider when deciding on interest rate changes. However, the BoJ keeps its official real-time output gap estimates confidential.

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²Yasuo Hirose kindly provided me with the tax-adjusted CPI data
I thus use an alternative measure of the output gap provided by the International Finance Department of the Federal Reserve Board of Governors. The measure is judgemental in that it represents the Fed economists’ best estimates of the Japanese output gap based on various models. Haltmaier (2001) discusses some of the methods used at the Fed to construct output gaps including an approach based on the Cobb-Douglas production function. Also, the Fed output gaps are retrospective, i.e. they are computed for the entire sample based on information available as of 2002. As explained in Ahearne et al. (2002), the Fed does not have real time output gap data for Japan.

The Fed output gap data are available for the 1970:Q4 to 1996:Q4 sample but post-1996:Q4 observations are confidential. For dates after 1996:Q4, I construct the output gap using real GDP data and the assumption that potential GDP grows at the rate estimates of the IMF staff provided in Bayoumi (2000). The IMF staff estimates of potential output growth obtained using a Cobb-Douglas production function and data on adjusted inputs of labour, capital, and an assumed level of technological progress. The estimates are 1.87% per annum over 1995-98 and 1.09% per annum over 1999-2002. For subsequent periods (2003:Q1 and 2003:Q2), I assume potential continues to grow at 1.09% per annum.\footnote{The post-1995:Q4 data are not used in the Taylor rule estimation. However, in the counterfactual simulations in Section 3, the simulation sample ends in 2003.}

**Nominal Interest Rate**

The uncollateralized interbank overnight call money interest rate is considered to be the main operating instrument of monetary policy. For the
analysis, however, I also tried using an alternative interest rate, the discount rate, and the main findings described below were unchanged.\(^4\)

### 2.1. The Taylor Rule

The Taylor rule model of monetary policy assumes that central banks respond in a systematic fashion to deviations of expected inflation from the desired level. For instance, when the inflation forecast rises above the target, the Taylor rule prescribes raising nominal interest rates enough to raise real interest rates. The rule also allows for some output stabilization by prescribing lower interest rates when output falls below potential.

The target nominal interest rate is

\[
i_t^* = r^n + E_{t-1} \hat{\pi}_{t+3} + (\mu_\pi - 1)E_{t-1}(\hat{\pi}_{t+3} - \pi^*) + \mu_y E_{t-1} y_t
\]

where \(r^n\) is the natural rate of interest, \(\pi^*\) is the implicit inflation target, and \(\hat{\pi}_t\) is four-quarter inflation, i.e. \(\hat{\pi}_t = 100(P_t - P_{t-4})/P_{t-4}\) where \(P_t\) is the aggregate price level in quarter \(t\). The term \(E_{t-1} \hat{\pi}_{t+3}\) denotes the inflation forecast. The forecast horizon is four quarters (from quarter \(t - 1\) to \(t + 3\)).\(^5\)

Previous work, such as Clarida et al. (1998), finds that the forward-looking version of the Taylor rule fits Japanese interest rates well. In addition, statements by Bank of Japan officials about the importance of taking “preemptive” action and the need to “promptly alter interest rates once ... the risk of damaging sustainable price stability has become higher” (Hayami, 2000) support the notion that policy is forward-looking.

\(^4\)Miyao (2002) analyses the institutional details of the operating procedures of the BoJ and concludes that the best measure of monetary policy is the call market rate (i.e. the interbank overnight rate).

\(^5\)For the purposes of this paper, the natural rate of interest is defined as the interest rate consistent with output equal to potential, i.e. with a zero output gap.
I assume that expectations are based on information available at the end of period $t - 1$. This assumption is consistent with the notion of information lags as discussed in Taylor (1998), i.e. policymakers decide on interest rates in quarter $t$ before they observe the current state of the economy. Collecting the inflation forecast terms, the Taylor rule can be rewritten as

$$i_t^* = \mu_0 + \mu_\pi E_{t-1} \hat{\pi}_{t+3} + \mu_y E_{t-1} y_t$$

(2)

where $\mu_0 = r^n - \pi^*(\mu_\pi - 1)$. 

Expectations are assumed to be rational so that

$$E_{t-1} \hat{\pi}_{t+3} = \hat{\pi}_{t+3} + u_{t+3}^\pi$$

$$E_{t-1} y_t = y_t + u_t^y$$

(3)

where the forecast errors, $u_{t+3}^\pi$ and $u_t^y$ are mean zero and are uncorrelated with any information available as of date $t - 1$.

An important condition for the Taylor rule to stabilize inflation is $\mu_\pi > 1$, i.e. when the inflation forecast rises above target, the policymaker raises nominal interest rates enough to raise real interest rates. This condition is the so-called “Taylor principle.” Output stabilization, or “leaning against the wind” implies a positive value for $\mu_y$.

### 2.2. Empirical specification

One problem with estimating Taylor rule equation (2) is that it does not allow for the widespread tendency of central bankers to smooth interest rates. I thus consider the partial adjustment model

$$i_t = (1 - \rho_1 - \rho_2) i_t^* + \rho_1 i_{t-1} + \rho_2 i_{t-2} + \varepsilon_t$$

(4)
where the parameters $\rho_1$ and $\rho_2$ capture the degree of interest rate smoothing and $\varepsilon_t$ is a mean zero exogenous i.i.d. shock to the interest rate. Substituting the expression for the target rate, equation (2), into equation (4) yields the following regression equation:

$$i_t = (1 - \rho_1 - \rho_2)(\mu_0 + \mu_\pi \pi_{t+3} + \mu_y y_t) + \rho_1 i_{t-1} + \rho_2 i_{t-2} + \nu_t$$

(5)

where $\nu_t$ is a composite error term defined as

$$\nu_t = \varepsilon_t - (1 - \rho_1 - \rho_2)(\mu_\pi \pi_{t+3} + \mu_y y_t)$$

(6)

that is uncorrelated with any variables in the information set at time $t-1$.

I therefore instrument for the right-hand-side variables of equation (5) using four lags of the call rate, of inflation, and of the output gap, a constant and a time trend.

2.3. Estimation of the Implicit Inflation Target

This subsection develops a method for estimating the inflation objective, $\pi^*$. This approach might prove useful for future studies of monetary policy as well as for the analysis below.

Although regression equation (5) does not separately identify $\pi^*$ and $r^n$, it implies the following relation between them:

$$\pi^* = \frac{r^n - \mu_0}{\mu_\pi - 1}$$

(7)

The canonical method of estimating $\pi^*$ is based on this relation and on the assumption that $r^n$ and $\pi^*$ are unchanged over the estimation sample.

---

6This specification is used in Clarida et al. (1998).

period. For example, Clarida et al. (1998) first compute a constant value for $r^*_{n}$ by taking the sample mean of the ex post short real interest rate and then use this value along with the estimates of $\mu_0$ and $\mu_\pi$ to obtain a value for $\pi^*$. Over the sample period that I consider, 1979:Q4-1995:Q4, using the canonical approach yields an estimate of $\pi^* = 2.4\%$. The estimate is based on a value of $r^*_{n} = 3.5\%$ found by taking the sample mean of the short real rate, $i_t - \bar{\pi}_{t+3}$.

However, over this sample period, 1979:Q4-1995:Q4, the assumption of constant $r^*_{n}$ and $\pi^*$ values is unduly restrictive. It seems that the natural rate of interest, $r^*_{n}$, declined in the 1990s relative to the 1980s. The mean of the short real rate over the 1979-1990 period is 4% but only 1.4% over 1991-2002. Explanations for the apparent fall in the natural rate include the reduction in investment following the bursting of the asset price bubble in 1990-1991. A decline in $r^*_{n}$ could also reflect slowing productivity growth. Posen (2003b) finds that labour productivity declined steadily in Japan after 1989.

In addition, inflation has declined significantly over the sample period. The 1980s started with inflation near 6% but, by the mid-1990s, inflation had been brought down to near zero. Watanabe (2002) and other authors claim that the Bank was aiming for a very low inflation level in the 1990s. Kuttner and Posen (2001) study the statements of Bank of Japan policy board members and conclude that the BoJ was targeting zero inflation. Eiko Shinotsuka, a member of the policy board of the Bank of Japan has said that the “Bank should aim at zero inflation” (Shinotsuka, 2000) except to the extent that there are biases in the measurement of inflation. Shiratsuka
(1999) estimates that the measurement bias in CPI inflation in Japan is about 0.9% per annum. This evidence suggests that the Bank’s inflation objective in the 1990s was low and near to 1%. Thus although the value of $\pi^* = 2.4\%$ obtained using the canonical method may seem reasonable for the 1980s, it is questionable for the 1990s.

I now propose a new method for estimating the implicit inflation target using a time-varying parameter model. I first estimate the natural rate of interest, $r^n_t$, by specifying a simple reduced-form “IS” equation similar to that in Laubach and Williams (2002). The output gap is determined by its own lags and the lagged short real interest rate gap (the difference between the short real interest rate, $r_t$, and the natural rate of interest $(r^n_t)$):

$$y_t = a_{y,1}y_{t-1} + a_{y,2}y_{t-2} - a_r(r_{t-1} - r^n_{t-1}) + \varepsilon_{1,t} \quad (8)$$

where $\varepsilon_{1,t}$ is a mean zero i.i.d. normal disturbance. The short real interest rate is $r_t = i_t - E_t\pi_{t+3}$ and I instrument for the future inflation rate using the same $t-1$ dated variables as above, i.e. four lags of $y_t$, $\pi_t$, and $i_t$. I also assume that the law of motion of the natural rate is a random walk,

$$r^n_t = r^n_{t-1} + \eta_{1,t} \quad (9)$$

where $\eta_{1,t}$ is a mean zero i.i.d. normal disturbance. I estimate this model using maximum likelihood and the Kalman filter following Hamilton (1994).\(^8\)

Secondly, I use the estimates of the natural rate, $r^n_t$, along with the Taylor rule equation (5) to identify the inflation target. The $\pi_t^*$ and $r^n_t$ are now

\(^8\)Orphanides and Williams (2002) estimate the natural rate using this random walk specification and U.S. data.
TABLE 1.
Taylor Rule Estimation Results

<table>
<thead>
<tr>
<th>Parameters Based on CPI</th>
<th>Based on GDP Deflator</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_\pi$</td>
<td>1.457 (0.634)</td>
</tr>
<tr>
<td>$\mu_y$</td>
<td>0.0746 (0.374)</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>1.173 (0.090)</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>−0.353 (0.082)</td>
</tr>
</tbody>
</table>

...time-varying, i.e.

\[
i_t = (1 - \rho_1 - \rho_2)(\pi_t^* + E_{t-1}\tilde{\pi}_{t+3} + (\mu_\pi - 1)E_{t-1}(\tilde{\pi}_{t+3} - \pi_t^*) + \mu_yE_{t-1}y_t + \rho_1i_{t-1} + \rho_2i_{t-2} + \varepsilon_t)
\]

(10)

where $\varepsilon_t$ is a mean zero i.i.d. normal disturbance.

I assume that the inflation target, $\pi_t^*$, follows a random walk,

\[
\pi_t^* = \pi_{t-1}^* + \eta_{2,t}
\]

(11)

where $\eta_{2,t}$ is another mean zero i.i.d. normal disturbance that is uncorrelated with $\varepsilon_t$.

I use maximum likelihood and the Kalman filter to obtain estimates of the policy rule parameters and of the implicit inflation target using the 1979:Q4 to 1995:Q4 sample. Standard errors are derived using the delta method.

2.4. Taylor Rule Estimation Results

Having discussed how I estimate the Taylor rule, I turn to the results. Table 1 shows the maximum likelihood parameter estimates based on CPI and GDPD inflation, respectively.
The inflation response coefficient is 1.5 in the CPIX version and in the GDPD version. This value conforms with the “Taylor principle,” i.e. it is greater than unity. The inflation response coefficients are similar to other estimates in the existing literature as in Ahearne et al. (2002) and in Clarida et al. (1998).

Also, the output gap response coefficient is small, positive and statistically indistinguishable from zero. The BoJ’s response to output (beyond output’s role as a predictor of future inflation) is thus nil. Again, this finding is consistent with the relevant literature. As in Clarida et al. (1998), the sum of the interest-rate smoothing coefficients, $\rho_1 + \rho_2$, is about 0.8, indicating significant interest rate inertia.

Figure 2 displays the estimates of $\pi_t^*$ based on CPIX (along with the 95% confidence interval). The graph suggests that the Bank’s desired inflation level declined from about 2.5% in the early 1980s to about 1% by 1995. The graph of $\pi^*$ estimates based on the GDP deflator was very similar and I do not show it here.

Figure 3 shows the estimates of $r_t^*$ and suggests that the natural rate of interest declined to about 1.5% by 1995 from a peak of about 4.5% in the late 1980s. The decline in $r_t^*$ can be interpreted as reflecting low productivity growth in the 1990s, as discussed in Iwamura et al. (2004). For comparison, the graph also shows the trend of the real interest rate obtained using the Hodrick-Prescott filter with a smoothing parameter of 6400.

*Actual Interest Rate Versus Target Rate*
I next compare the actual nominal interest rate, $i_t$, with the target interest rate, $i_t^*$, implied by the estimated Taylor rule, a standard practice in the policy rules literature. The actual interest rate ($i_t$) follows its estimated target ($i_t^*$) with a delay that reflects the high degree of interest-rate inertia.
FIG. 4. Actual Call Rate, \(i_t\), Target Rate, \(i^*_t\), and Fitted Call Rate

The graph also shows the fitted actual policy rate (fine dashes) that is always very close to the actual rate.

Recall that the policymaker gradually adjusts the nominal interest rate, \(i_t\), towards \(i^*_t\) using the second-order partial adjustment model. Figure 4 illustrates how the actual policy interest rate follows its target rate.\(^9\)

Overall, this section suggests that the Bank of Japan was following what was considered best practice. A new finding is that the implicit inflation target declined to near 1% in the early 1990s. However, the consensus amongst academics and policymakers was that central banks ought to follow a Taylor rule with a low inflation target. There was thus nothing unorthodox about Japanese interest rate policy.

\(^9\)This graph is based on CPI inflation. Using GDP deflator inflation yields similar results.
3. COUNTERFACTUAL SIMULATIONS: A STRUCTURAL MODEL

An important aim of this paper is to conduct counterfactual policy experiments for Japan in the 1990s. This requires an empirical model of the Japanese economy that fits the output, inflation and interest rate dynamics in Japan. In this section, I first develop and then estimate a rational expectations model comprising a Phillips curve, an IS curve and a Taylor rule.

3.1. Model Equations

Inflation evolves according to Equation (12), the hybrid variant of the New Keynesian Phillips curve.

\[
\pi_t = \beta E_{t-1}\pi_{t+1} + (1 - \beta)\pi_{t-1} + \kappa E_{t-1}y_t + \varepsilon_T^\pi
\] (12)

Current quarter inflation, \(\pi_t\), is the percent change in prices from quarter \(t-1\) to quarter \(t\), i.e. \(400(P_t/P_{t-1} - 1)\). A fraction \((1 - \beta)\) of prices are indexed to the previous quarter’s inflation rate. This specification allows for a share of backward-looking as well as forward-looking behaviour. Again, expectations of future inflation, \(E_{t-1}\pi_{t+1}\) are formed on the basis of information available at the end of the previous period, as in Svensson (2001). Gali and Gertler (1999) estimates a similar specification but has expectations dated in period \(t\), i.e. \(E_t\pi_{t+1}\) and \(y_t\) appear on the right-hand-side. The term \(\varepsilon_T^\pi\) denotes an exogenous inflation (supply) shock.

Equation (13) governs the dynamics of the output gap using the standard Euler equation augmented by a lag of output,

\[
y_t = \alpha_1 E_{t-1}y_{t+1} + \alpha_2 y_{t-1} - \sigma(i_{t-1} - E_{t-2}\pi_{t+2} - r^m) + \varepsilon_T^y
\] (13)
where $\pi_{t+2}$ is four-quarter inflation between quarters $t-2$ and $t+2$. The inclusion of the output lag can be rationalized using the concept of habit formation as in Fuhrer (1998) and allows the model to capture the serial correlation of output. $\epsilon_t^y$ denotes an exogenous output (demand) shock.\(^{10}\)

Interest rates move according to the Taylor rule that is subject to the zero bound, equation (14).

\[
\begin{align*}
i_t &= \max\{0, (1 - \rho_1 - \rho_2)(r^* + E_{t-1}\tilde{\pi}_{t+3} + (\mu_\pi - 1)(E_{t-1}\tilde{\pi}_{t+3} - \pi^*) + \\
&\quad \mu_y E_{t-1}y_t + \rho_1 i_{t-1} + \rho_2 i_{t-2} + \epsilon_t^i) \}\end{align*}
\]

The term $\epsilon_t^i$ denotes an exogenous monetary disturbance. I enforce the zero lower bound by setting the nominal interest rate equal to the maximum of zero and the unconstrained Taylor rule. Details of the procedure for enforcing the zero bound are described in Appendix A.1.

I make the stability assumption of Reifschneider and Williams (2000) which is designed to rule out self-fulfilling deflationary spirals. The assumption is that there will be an emergency fiscal policy that will stimulate the economy if the zero bound is binding for a very long period. Agents expect that an emergency fiscal package, capable of offsetting the zero bound constraint, will be used if interest rates are projected to be at zero for seven years in the future. This condition rules out a deflationary spiral.

\(^{10}\)The parameters in this model are functions of underlying structural parameters, such as labor supply and demand elasticities. Since I do not use any additional information from the underlying model, I treat them as structural.
3.2. Minimum Distance Estimation of the Model

I now turn to the estimation of the model’s parameters. I use the minimum distance approach found in Rotemberg and Woodford (1998) and in Boivin and Giannoni (2002). The parameters are estimated so as to make the model’s impulse responses following a monetary shock, $\varepsilon^i$, fit those estimated by an unrestricted VAR as closely as possible. The advantage of estimating the model on the basis of impulse response functions (IRFs) is that I do not need to assume anything about the processes that drive the three shocks $\varepsilon^y$, $\varepsilon^\pi$, and $\varepsilon^i$.

The first step in the approach is to estimate the unrestricted VAR model for the output gap, $y_t$, the inflation rate, $\pi_t$, and the nominal call rate, $i_t$ using quarterly data for the 1979:Q4-1995:Q4 period, i.e., the same sample as in the Taylor rule analysis. In the VAR model, $y_t$, $\pi_t$, and $i_t$ (in that order) are regressed on four lags of each of these variables, along with a constant (additional lags were insignificant and were omitted). The VAR ordering is standard and the Cholesky decomposition allows me to identify the structural shocks, $\varepsilon^y$, $\varepsilon^\pi$, and $\varepsilon^i$.

I then use the just-identified VAR to estimate impulse responses of all three variables to a monetary policy shock. Figure 5 shows the impulse responses of all three variables to a 100-basis-point shock to the policy interest rate, along with the associated 95% confidence intervals. I computed the confidence bands using Runkle’s (1987) bootstrap procedure with 1000

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11 Jean Boivin and Marc Giannoni kindly provided me with the Matlab code for implementing the minimum distance estimation approach.
iterations. The shapes of the IRFs are broadly consistent with the results in the relevant literature.  

The next step is to estimate the model equations. To reduce the dimension of the estimation, I calibrate the parameters of the Taylor rule using the values reported in Table 1. The remaining parameters that need to be estimated are five: \( \{\alpha_1, \alpha_2, \sigma, \beta, \kappa\} \). I estimate the parameters by matching the model-based impulse responses, \( IRF_M \), with those of the empirical VAR, \( IRF_{VAR} \). The model-based impulses are obtained using the VAR representation of the model’s rational expectation solution described by equation (A.2) in Appendix A.1. Following Boivin and Giannoni (2002), I impose  

\[ \text{See for example Kim (1999) that conducts an analysis of G-7 countries including Japan.} \]
constraints on the signs and magnitudes of the parameters. For instance, \( \beta \) must be non-negative and smaller than unity. I consider the responses of the variables over the first twelve periods (i.e. three years) following the monetary shock.

The estimation procedure is thus to minimize the value of the loss function

\[
L = D'WD
\]  

(15)

where \( D = IRF_M - IRF_{VAR} \) and \( W \) is an asymptotically non-stochastic positive definite weighting matrix. Following Boivin and Giannoni (2002), I determine the weighting matrix to be a diagonal matrix that involves the inverse of each impulse response’s variance on the main diagonal.

### 3.3. Estimation Results

Table 3 gives the estimates of the model parameters. These are in line with those reported for Japan in the literature. In the Phillips curve, the proportion of forward-looking price setters, \( \beta \), is estimated to be 0.50. This result is similar to that of Coenen and Wieland (2002) that provides estimates of a very similar, though not identical, Phillips curve specification as that described above using Japanese data over 1979:Q2-1995:Q2.\footnote{The specification estimated by Coenen and Wieland does not contain information lags, i.e. it is \( \pi_t = \beta \pi_t + (1 - \beta) \pi_{t-1} + \kappa y_t + \varepsilon_t \).} Their estimate of the fraction of forward-looking agents in the economy, \( \beta \), is 0.48. The slope of the Phillips curve, \( \kappa \), is estimated at 0.01. This compares with the Coenen and Wieland (2002) estimate of 0.007.

The Phillips curve coefficients play an important role in determining the sacrifice ratio, i.e. the percentage loss in output relative to potential that is
TABLE 2.

Structural Parameter Estimates

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>0.381 (0.349)</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.602 (0.284)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.038 (0.015)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.496 (0.072)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.010 (0.0009)</td>
</tr>
</tbody>
</table>

required to reduce the inflation rate permanently by one percentage point.

To derive the sacrifice ratio implied by the model parameters, I conducted a disinflation experiment by lowering the inflation target (i.e. the long-run inflation level) from 2% to 1%. The resulting sacrifice ratio was 1.5 at an annual rate, i.e. the economy had to experience the equivalent of one year with output at 1.5% below potential to lower inflation by one percentage point. This estimate of the Japanese sacrifice ratio is similar to the Zhang (2001) estimate of 1.85 found using 1960-1999 data.

For the IS curve, my estimates of the lag and lead coefficients on output are 0.6 and 0.4, respectively. The estimate of the interest rate sensitivity of output, $\sigma$, is 0.04. For comparison, Coenen and Wieland (2002) estimates an IS equation that is similar to mine except that it omits the lead of output term, i.e. it imposes $\alpha_1 = 0$ in equation (13). The Coenen and Wieland estimate of the coefficient on lagged output, $\alpha_2$, is 0.9 and the interest sensitivity estimate, $\sigma$, is 0.08.

Figure 5 plots the optimized model-based impulse responses (solid line) along with the empirical VAR responses (circles). The fit is very good.
and the model captures the decline in inflation and output following the monetary shock, $\varepsilon^1$. The model also closely tracks the path of the interest rate.

4. COUNTERFACTUAL SIMULATIONS

Having estimated the parameters of the model, I now investigate whether an alternative monetary policy rule could have helped Japan to avoid the deflationary slump. In order to perform counterfactual simulations of the model, I first extract a time series for each of the exogenous disturbances $\{\varepsilon^y, \varepsilon^\pi, \varepsilon^1\}$. I identify the shocks using an adaptation of the procedure in Coenen, Orphanides and Wieland (2003) that I discuss in Appendix A.2. Using the derived shocks together with the structural equations of the model reproduces exactly the historical time series of all variables.

It seems interesting to compare the size of the shocks occurring in the 1990s with those that arrived during the preceding two decades. Figures 6 and 7 show the exogenous output (demand) and inflation (supply) shocks hitting the economy over the three decades.\textsuperscript{14} The shocks can be compared according to mean and volatility.

Regarding the mean, Figure 6 suggests that the output shocks were worse in the 1990s than in the other two decades. Specifically, the demand shocks had a mean of -0.30%. The 90s shocks were also relatively volatile with a standard deviation of 0.98%. Overall, the behaviour of the demand shocks over this period is intuitive and is in line with the findings of other stud-

ies. For example, the negative shocks in the early 1990s are explained by the decline in investment demand that followed the bursting of the asset price bubble. The large negative shocks in 1997-98 are associated with the consumption tax hike and the effects of the Asian financial crisis.

In contrast to the negative shocks that occurred in the 1990s, the demand shocks over the 1980s were benign and had a positive mean of 0.15%. One factor that contributed to the positive disturbances in the late 1980s was the asset price bubble with higher investment demand. The shocks were also less volatile over this period and had a standard deviation of only 0.61%.

In the 70s, the mean of the demand shocks was slightly negative at -0.05%. The volatility was higher than in the 80s with a standard deviation of 0.74%. The worst shock, -3.3% in 1974:Q1 is associated with the first OPEC oil price shock and is comparable in size to the worst shock in the 1990s, i.e.-3.8% in 1997:Q1.

\footnote{See Ahearne et al. (2002) for more narrative details of this period}
As Figure 7 suggests, the exogenous inflation disturbances have a near zero mean over all three decades. Regarding volatility, the inflation shocks have a higher standard deviation in the 80s than in the 90s (0.84% versus 0.61%). However, the volatility is highest in the 70s with a standard deviation of 3.76%. The OPEC oil price hikes in 1973 and 79 explain the largest inflation shocks.

The inflation shocks over the 1998-2003 period need to be interpreted carefully. As several authors find, there is a puzzle regarding inflation in Japan in the late 1990s. A standard Phillips curve, such as the one in my model, seems to break down. With the large output gaps, the standard Phillips curve predicts rapidly falling inflation. However, actual (CPI) inflation has remained near to -1%. The failure of inflation to fall translates into positive inflation shocks over the 1998-2003 period. However, there is little reason to interpret these disturbances as actual supply shocks. Rather, these shocks are a symptom of how the Phillips curve changes in the presence of defla-
tion. For example, DeVeirman (2003, forthcoming) attributes the apparent breakdown of the Phillips curve to downward nominal wage rigidity. In the absence of deflation, the model would not imply such positive inflation disturbances. Therefore, I set the $\varepsilon^{\pi}$ shocks to zero over the 1998-2003 period in simulations that avoid deflation.

Finally, the exogenous interest rate shocks also have a zero mean and a low standard deviation of less than 0.5% over the three decades. Overall, the three graphs suggest that, although the 90s demand shocks had a low mean, they were not unprecedented in terms of volatility. In addition, the supply shocks of the 70s were much worse than those of the 1990s.

4.1. Alternative Shocks

The discussion of the shocks suggests that there was some “bad luck” in the 1990s. To confirm the role of bad luck in explaining the performance of the economy, consider a simulation of the 1990s under the estimated policy rule but with a different distribution of shocks. First, I simulate how the economy would have evolved under the shocks that were identified for the 1980s assuming as initial conditions the state of the economy in 1991:Q1. Secondly, I run a simulation of the 1990s under the shocks of the 1970s. As the Taylor rule analysis suggested that the Bank of Japan was aiming for a low inflation rate near 1% over this decade, I set the implicit inflation target equal to $\pi^* = 1\%$.

**Counterfactual: 1990s Policy Rule and 80s Shocks**
Figure 8 shows the results of applying the shocks of the 1980s starting from the initial values of 1991:Q1. The solid line shows the actual historical path of the variables, while the dashed line shows the simulated path.

Under the relatively benign shocks of the 1980s, the economy avoids deflation and the large output slump, and interest rates stay clear of the zero bound. For example, by the end of the decade, the economy is in a boom, inflation is near 2% (instead of at -1%) and nominal interest rates are at 3.8% (instead of at zero).

Counterfactual: 1990s Policy Rule and 70s Shocks

Now I investigate what happens to the economy with the shocks of the 1970s starting from the initial values of 1991:Q1. It seems interesting to see whether the large shocks of the 70s would have pushed the economy into a liquidity trap, had they occurred in the 90s under the low inflation Taylor
rule. In Figure 9, the solid line again shows the actual historical path of the variables in the 90s, while the dashed line shows the simulated path.

Interestingly, the results suggest that the shocks of the 70s were large enough to push the economy into a liquidity trap. One reason why the economy avoided the zero bound in the 70s is that interest rate policy had a higher implicit inflation target. Consequently, agents’ inflation expectations were higher in the 70s than in the 90s. The higher inflation expectations kept inflation and nominal interest rates far above zero.

In addition, the inflation response coefficient in the Taylor rule estimated over the 1970s sample is lower than one. The implication of this response coefficient is that policy accommodated the increases in inflation instead of actively countering them. By contrast, in the simulation with the 90s rule, the central bank actively counters the higher inflationary pressures by raising real interest rates and pushing the economy into a recession.
4.2. Higher Inflation Target

Having considered what would have happened with the estimated policy rule but with different shocks, I now consider different policy rules. In the following counterfactual experiments, the simulation sample is 1991:Q1 to 2003:Q2, i.e. up to the latest available observation.

In the next experiment, I raise the inflation target to $\pi^* = 3\%$. Everything else is left unchanged. Several studies, such as Reifschneider and Williams (2000) and Hunt and Laxton (2001) find that an inflation target of greater than 2\% as opposed to less than 2\% can reduce the probability of hitting the zero bound. Watanabe (2002) claims that the BoJ’s pursuit of very low inflation in the 1990s actually precipitated Japan’s descent into the deflationary slump. The higher inflation target, i.e. a higher inflation “buffer,” makes it easier to lower real interest rates below $r^*_n$ when the nominal interest rate is at the zero bound. My finding of a decline in the natural rate of interest, $r^*_n$, further suggests the need for a higher inflation buffer in the 90s.

It thus seems appropriate to see if a higher inflation target would have helped Japan avoid the liquidity trap. As Figure 10 suggests, the higher inflation target translates into higher inflation expectations and thus lower real rates over 1991-1994. Consequently, simulated output is slightly higher than actual output and inflation is near to the $\pi^* = 3\%$ target over the early to mid 1990s.

However, when the large deflationary shocks arrive in 1997 and beyond, the central bank cuts interest rates slowly until 1999. Two factors explain this delay in cutting rates. First, the deflationary shocks are unexpected and
interest rates are kept higher than they would be if the forecasts correctly capture the rapid decline in inflation. Secondly, the policy rates are lowered in proportion to the inflation response parameter, $\mu_\pi = 1.5$. With a stronger response, the interest rates would have fallen faster.

Given the slow policy easing, simulated output falls to as low as actual output does and remains there until 2001. Also, by the time rates start falling faster and are cut to zero at the end of 1999, the economy is already experiencing deflation and a large output gap. With no more room for interest rate easing, the economy remains in a liquidity trap until the sample end.

This first counter-factual policy experiment thus yields an important finding: simply raising the inflation target to 3% would not have provided much insurance against hitting the zero bound. It also seems interesting to in-
investigate whether a more aggressive rule that responded faster to inflation declines would have provided a sufficient stimulus to avoid the liquidity trap.

### 4.3. A Stronger Response to the Inflation Gap

I now consider the performance of a Taylor rule that responds more aggressively to deviations of the inflation forecast from target. The inflation target is kept at the original 1% level over the 1990s but I raise the $\mu_\pi$ parameter from the original 1.5 to an “aggressive” 3.0. In the theory literature on Taylor rules in a low inflation environment, several authors prescribe a strong response to inflation.  

I am interested in seeing if such a policy rule would have helped Japan avoid the liquidity trap.

As Figure 11 illustrates, the central bank starts by lowering inflation towards the 1% target over 1991-92 by raising nominal and real interest rates.

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Note that under the stronger response coefficient, the policy rule is more contractionary whenever the inflation forecast is above target, i.e. above 1%. On the other hand, the policy is relatively more expansionary whenever the inflation forecast is below target. The real rate hikes in 1991-92 lower output towards potential in 1992. Simulated output then declines further as the bad shocks arrive in the early to mid 1990s.

In response to the growing output gap and to declining inflation, the aggressive Taylor rule cuts interest rates to almost zero by 1994. Real rates are below the actual historical level over 1993-96. This monetary easing raises output over 1994-96 relative to the actual historical level and inflation hovers near to the 1% target. This inflation rate is slightly above actual inflation over 1995-98.

However, when the severe deflationary contractions arrive in and after 1997, there is not much of an inflation buffer to absorb the shocks. The nominal interest rate is quickly cut to zero but, given the low inflation expectations, real rates do not decline by much. As a result, the output gap grows and remains in the -3 to -5% range until the decade’s end. Deflation sets in and the economy stagnates in a liquidity trap.

This counter-factual experiment thus suggests another important result: a Taylor rule with a rapid response to the inflation gap \( \mu_\pi = 3.0 \) but the original low inflation target would not have warded off the deflationary episode. A natural next step thus seems to evaluate a rule that combines the two proposals and has both a higher inflation target and a stronger response to the inflation forecast.
4.4. A Higher Inflation Target and a Stronger Response to the Inflation Gap

Figure 12 reports the results of the simulation using a Taylor rule that has both the higher inflation target, $\pi^* = 3.0\%$, and the stronger inflation response, $\mu_\pi = 3.0$.

As the negative demand shocks arrive over 1992-1995, the central bank lowers nominal and real interest rates. Consequently, the simulated output path stays closer to potential than does the historical path. Inflation also stays near to the 3% target.

Next, as the contractionary shocks arrive in and after 1997, the central bank quickly lowers interest rates. This rapid response to the deflationary shocks thus contrasts with the Taylor rule that had a high inflation target but the original response coefficients. Combined with the larger inflation buffer, the interest rate cuts help to mitigate the output slump. Following
the adverse demand shocks in 1997-98 and in 2001, the simulated path of output comes back to potential faster than does the actual historical path. By 2003, the simulated output gap has closed, whereas actual output is about 4% below potential. Note that there are bad shocks in 2001 but that the economy recovers relatively quickly with the alternative rule. Over the whole sample, the average simulated output gap is 1.3 percentage points smaller than the actual output gap.

Given the higher output and, importantly, higher inflation expectations, the simulated inflation level never reaches zero. In fact, inflation remains above 1% throughout the 1991-2003 period. Consequently, nominal interest rates do not get stuck at zero, and only hit the constraint in one quarter, 2000:Q1. In terms of reducing the amount of time spent at the zero bound, the rule produces a greater improvement than the sum of the improvements under each of the two previous rules. This counter-factual experiment thus yields a novel finding: a rule that had both (i) a higher inflation target and (ii) a stronger response to the inflation forecast would have outperformed a rule that had only one of either (i) or (ii). The intuition for this finding is that the higher inflation target provides the inflation buffer, i.e. the “ability” to cut interest rates in response to the deflationary shocks, while the stronger inflation response coefficient provides the “will” to respond aggressively.

4.5. Nonlinear Taylor Rules

In addition to the proposals of a higher inflation target and a stronger response to inflation, some authors have proposed a nonlinear Taylor rule. Orphanides and Wieland (1999) consider a rule in which the inflation re-
response increases as the average level of inflation declines. For example, this rule has an inflation response of 3.0 when average inflation is near 1% but a response of 1.5 when average inflation is near 4%. In a counter-factual simulation, the Orphanides and Wieland rule produces the same result as the strong response rule does in section 4.3. Over the 1991-2003 period that I am considering, inflation is low and the strong response rule in section 4.3 is thus very similar to a rule that is aggressive in a low inflation environment.

Another proposal in the literature is for the central bank to commit to holding interest rates at zero even after the economy begins to recover. The rational is that agents expect the longer zero interest rate period to bring about a boom with higher inflation in the future. These expectations of higher inflation and activity in the future translate into higher inflation and output in the current quarter. 17

In this final counterfactual experiment I consider such a rule, as proposed by Reifschneider and Williams (2000). The proposal is similar in spirit to that in Eggertsson and Woodford (2003). Reifschneider and Williams (2000) let\( d_t = i_t - i_{Taylor}^t \) denote the difference at time \( t \) between the actual nominal rate and the nominal rate prescribed by the unconstrained Taylor rule. The unconstrained Taylor rule is just the estimated rule reported in Table 1. Thus \( d_t \) is only positive when the zero bound binds and the unconstrained Taylor rule setting falls below zero. Next, let \( Z_t \) denote the

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17 A recent Federal Open Market Committee statement suggests that the Federal Reserve may be incorporating such a policy proposal into its interest rate decisions. Despite the apparent strong recovery in US growth in 2003:Q2 and Q3, the FOMC decided to keep the fed funds rate at 1%. The October 28, 2003 FOMC statement says that “[t]he Committee judges that, on balance, the risk of inflation becoming undesirably low remains the predominant concern for the foreseeable future. In these circumstances, the Committee believes that policy accommodation can be maintained for a considerable period.”
cumulative sum of all past deviations \( Z_t = \sum_{j=0}^{\infty} \{ i_{t-j} - i_{t-j}^{\text{Taylor}} \} \). The Reifschneider and Williams (2000) policy rule is thus written as

\[
i_t = \max\{i_t^{\text{Taylor}} - \alpha Z_t, 0\}
\]

where \( \alpha \in (0, 1] \).

The augmented policy rule deviates from the Taylor rule whenever \( Z_t \) is positive. As long as interest rates are constrained at zero, \( d_t \) is positive and \( Z_t \) increases. Once the economy recovers and the unconstrained Taylor rule rises above zero, \( d_t \) is negative and \( Z_t \) declines. Interest rates stay at zero for as long as it takes to reduce \( Z_t \) back to zero.

I ran the simulation using this Taylor rule (with various values of \( \alpha \)) but the results were very similar to the actual historical evolution of the economy. Figure 13 shows the results for the simulation with \( \alpha = 1.0 \).
As the graph suggests, the simulation does not improve much over the actual historical paths. Interest rates decline and get stuck near to zero in 1995. The policy can only mitigate the deflationary trap to the extent that there are expectations of higher inflation in the future. However, because the economy is still far below potential and there is deflation in 2002, the recovery is still distant in agents’ expectations. In addition, because only a fraction $\beta = 0.5$ of agents set prices in a forward-looking manner, any expectations of eventual future inflation are discounted heavily and thus have a muted impact on inflation in the current quarter.

5. CONCLUSION

A combination of high unemployment, deflation and low growth have made the 1990s “A Lost Decade” for Japan (Hiyashi and Prescott, 2002). Many economists blame the BoJ for policy errors that precipitated the economy into a liquidity trap. Using a new method of estimating the policy rule that allows the implicit inflation target and the natural rate of interest to vary, I find that Bank of Japan’s inflation target declined to about 1% in the 1990s from about 2.5% in the 1980s. I also find that the policy rule respects the Taylor principle and is forward looking. Such a rule was regarded as best practice at the time and was also followed by other central banks.

Next, using an estimated rational expectations model, I identify a number of adverse shocks occurring over the 1990s. These contrast with the relatively benign shocks of the 1980s. Counterfactual simulations suggest that with the same policy rule but with the more benign shocks of the 1980s, the economy would have avoided a deflationary slump. It thus follows that
monetary policy was not solely responsible for the economy’s poor performance.

However, the Bank’s orthodox policy rule had an Achilles’ heel. Aiming for a low inflation level and responding to the economy according to the standard Taylor rule provided insufficient insurance against the contractionary shocks that occurred over the 1990s. Moreover, when I run a counterfactual simulation with the 1990s estimated policy rule but with the large shocks that occurred over the 1970s, the economy also goes into a liquidity trap. This finding suggests that the Bank’s policy rule did not provide sufficient insurance against the type of shocks that occurred in two out of three decades.

In my counterfactual policy experiments, I find that a rule that combines (i) a higher inflation target of about 3% and (ii) a more energetic response to the inflation gap would have improved the economy’s performance and would have avoided the zero bound on nominal interest rates. Importantly, rules that had only (i) or (ii) would have provided little protection against the large deflationary shocks.

So what lessons does Japan’s experience offer for the rest of the world? Many central banks still regard an interest rate policy that aims for very low inflation, such as 0-2% as best practice. Some of these central banks operate in economies that have similar structures to the Japanese economy and are prone to similar shocks. The results in this paper suggest that such economies may need to take out greater insurance against adverse shocks by (i) raising their inflation objectives and (ii) responding aggressively to the inflation gap.
To impose the non-negativity constraint on the model, I use the procedure of Reifschneider and Williams (2000). The basic idea is that the zero lower bound implies non-negative shocks to the nominal interest rate. Here is how the procedure works. First, the linear model described by equations (12), (13) and (14) is augmented with $k$ additional variables, corresponding to anticipated shocks to the policy interest rate in periods $t$ (the current period) and $t + 1, ..., t + k$, for some value of $k$.

The model can then be written as

$$
\Gamma_0 y_t = \Gamma_1 y_{t-1} + C + \Psi_\varepsilon_t + \Pi_\eta_t
$$

(A.1)

where $y_t$ is the vector of all the variables in the model dated (i.e. known in period $t$), $C$ is a vector of constants, $z_t$ is the vector of exogenous shocks, and $\eta_t$ is an expectational error, satisfying $E_t \eta_{t+1} = 0$ for all $t$. The expectations errors are treated as determined as part of the model solution. Using the Sims (2000) solution algorithm, I can solve for the VAR representation of the model

$$
y_t = F y_{t-1} + G + H_\varepsilon_t
$$

(A.2)

Using the VAR representation, I simulate the model for periods $t, t+1, ..., t + k$, without imposing the zero bound. If the interest rate remains above zero in every point from $t$ through $t + k$, then period $t$ has been solved and I move on to period $t + 1$. 
If, however, the interest rate falls below zero at some point over the stipulated horizon, I run a special subroutine. This subroutine finds a time series of non-negative shocks to the interest rate corresponding to the periods $t$, $t+1$, $..., t+k$ that satisfy the zero lower bound for forecasts of the interest rate from $t$ through $t+k$. I wrote an algorithm that can do this in Matlab.

Expectations of future interest rates will include these notional shocks, so that the zero bound is respected in the current period and in expectation over the subsequent $k$ periods. Expectations of the interest rate in periods $t+k+1$ and thereafter do not respect the zero bound restriction. I set $k$ at 28 quarters, i.e. 7 years. This specification is justified on the grounds that, in my model, fiscal policy can eventually be expected to offset the effects of the zero bound on interest rates. Throughout all my simulations, this fiscal package is never actually expected to occur and the results are not sensitive to the precise length of the horizon $k$.

The results in the paper are robust to alternative solution algorithms. For instance, I obtained similar results using the Fair-Taylor solution algorithm, an adaptation of which is used in Coenen, Orphanides and Wieland (2003). I also tried the stacked-time, forward-solution-path algorithm for nonlinear models used in Hunt and Laxton (2001).

A.2. IDENTIFICATION OF THE SHOCKS

The shocks are identified using an adaptation of the procedure in Coenen, Orphanides and Wieland (2003). In a purely backward-looking model, the shocks would be equal to the residuals from the single equations. In a rational expectations model, the procedure is less straightforward. I obtain
the structural shocks by simulating the full model and computing the time series of rational expectations with respect to the actual historical data. The structural shocks then differ from the simple residuals to the extent that there are rational expectations forecast errors. Using the derived shocks together with the structural equations of the model reproduces exactly the historical time series of all variables.

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