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MOTHERS' EMPLOYMENT AND THE USE OF CHILDCARE IN THE UNITED KINGDOM

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THE INSTITUTE FOR FISCAL STUDIES
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Mothers' Employment and Use of Childcare in the United Kingdom

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1 Introduction

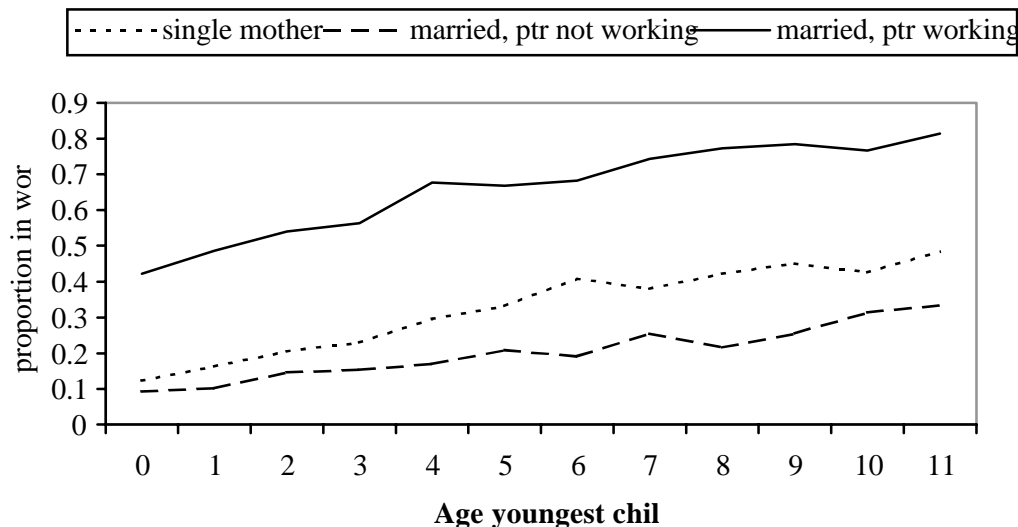
The availability and affordability of good quality childcare is a central issue in the public policy debate around the world on the role of mothers of pre-school children in the labour market. The UK Government's stated policy in this regard has emphasised the importance of work as the means to escape poverty for families with children, particularly for single parent families. There have also been a series of initiatives to assist parents financially in meeting the costs of childcare, the most recent of which takes the form of a targeted tax credit designed specifically to compensate for the costs of registered childcare.¹

The focus of the debate on women with pre-school age children is not unwarranted. Figure 1 shows the labour force participation rate of mothers in the United Kingdom, differentiated by the age of their youngest child. Some 62 percent of all working-age women are in the labour force. For mothers with an employed partner, the proportion working rises from 42% for those with a youngest child under 1, to 82% for those with the oldest children. For women with non-working partners and for single mothers, participation rates are much lower. For example, only just over 10% of single mothers with a child under 1 year are in work, and this figure rises to just 48% for those with older children. For mothers without a partner's earnings, means-tested benefits generate substantial disincentives to work.

But why should such employment choices be a source for public concern? After all, it could be argued that mothers are merely choosing the best option in the light of the real costs of working and the constraints confronting them. One response is that these individual or household choices do not take into account the potential externalities inherent in the labour supply choice of parents. Society may value labour force participation over and above the wage offered to women, since working mothers acquire human capital through employment. The state will have invested in the human capital of women through training and education and these sunk costs will fail to bring benefits if women do not return to the labour force after childbirth. For single parents especially, society would benefit from reduced social security payments if women could be encouraged back into work, although such gains from lower benefit expenditure can often be exaggerated, as shown in Duncan, Giles, and Webb (1994). Similarly, positive externalities may exist in the use of childcare for pre-school children. Research cited by Currie and Thomas (1995) and Holtermann (1992) suggests that good quality childcare is beneficial to children's development and is therefore desirable for parents as a consumption good in addition to being a means to facilitate

¹The UK Working Families' Tax Credit (WFTC) contains an element specifically to pay for registered childcare costs.

Figure 1: Employment rates and age of youngest child, by family type



employment. However, the quantity of care demanded may fall below the efficient level either because parents fail to perceive the benefits or are unable to take advantage of them due to credit constraints or because the social benefit of pre-school children receiving childcare is greater than the individual private benefits to a child’s development.

Given the externalities present in the separate labour participation decision and childcare demand decision, a theoretical case exists for government intervention to subsidise the cost of that care. If the two decisions are joint, the case for government assistance becomes stronger because the under-supply of labour will compound the lack of childcare used and visa versa. There also exists a fundamental equity consideration for government. Care for pre-school children has traditionally been the responsibility of the female in a couple, and as such women taking time out of the labour market tend to come back into the labour market at wages not only far below men of the same cohort but also below women in similar circumstances who did not have an interrupted career. If the interruption was caused by the constraint of childcare costs on female labour force participation, women face unequal opportunities and the lack of affordable care can perpetuate the economic subordination of women in society. Equally, the increased likelihood of women returning rapidly to the labour market after childbirth reduces the incentives for employers to discriminate against women when investing in human capital.

Data from the UK Family Resources Survey are used to estimate jointly the choices of whether to participate in employment and whether to use formal paid childcare.

In analysing the determinants of these choices, particular focus is given to the role of the price of childcare and so the results shed some light on the potential impact of childcare subsidies on employment rates. The models we use are developments of earlier studies (for example, Duncan and Giles (1996)) and are innovative to the literature in two main respects.²

First, we develop a range of empirical specifications which focus carefully both on the joint nature of the childcare take-up and labour supply decisions, and also on the presence of quantity constraints which force the number of childcare hours consumed to at least equal the number of hours in paid employment for many mothers with children. We show theoretically and in our empirical work that childcare price effects tend to be *overestimated* when constraints are not taken into account. This in turn has important implications for the potential effect of welfare reform on patterns of employment and use of childcare.

Second, the price of childcare used in the empirical analysis is an *hedonic* price. That is to say, it is adjusted to a standard quality choice. Childcare is not an homogeneous good, as has been highlighted by the emphasis on good quality childcare in recent policy discussions. The absence of appropriate controls for quality differences in childcare price can lead to the seemingly counter-intuitive result that higher childcare prices are correlated, an empirical result which can be rationalised if higher quality childcare is being chosen at higher levels of childcare use. Some studies have utilised direct information on the quality of care to analyse the relationship with cost and with labour market choices.³ Although quality choice is not the focus of this paper, we take advantage of the hedonic childcare price indices estimated by Duncan *et al* (2000) to adjust the childcare price to a standardised level of quality. We also allow for non-linearities in the relationship between price and quantity of childcare. The quality-adjusted price is shown to be negatively and significantly related to both labour market participation and childcare take-up, suggesting a potentially important role for higher childcare subsidies in encouraging mothers with young children into work.

²Duncan and Giles (1996) developed the first innovation of the joint modelling of labour supply and childcare take-up, but did not use hedonic prices or adjust for quality.

³See Blau and Hagy (1998) or Duncan, Paull, and Taylor (2001).

2 An Economic Model of Employment and Child-care

The model used in the empirical analysis is a one-period model of the choice of formal childcare among women with a least one dependent child below school age. The general economic structure in this present study is similar in many respects to earlier studies⁴. We make two substantive assumptions about behaviour and the nature of the childcare market. First, it is assumed that the price paid for formal childcare reflects heterogeneity in childcare quality. Mothers are confronted with a market-determined price for some base quality of childcare, but can select differing quality levels over a range of prices. Hence, the observed expenditure on childcare reflects the two endogenous choices of quantity and quality, in addition to the exogenous parameter of the price for base quality care. Second, we relax the constraint used in some models⁵ that hours of work must strictly equal non-maternal childcare time. In order to recognise the possibility of positive demand for formal childcare as a good in its own right, rather than simply to provide necessary cover for the time the mother is in paid employment, we assume only that hours of work are less than or equal to non-maternal childcare time.

For ease of analysis, we also make the following simplifying assumptions. First, the labour supply of the male partner in the household is taken as given.⁶ Second, the level of informal childcare (that is, free care provided by close relatives or friends) available to the mother is also taken as given. If informal care is available, it is assumed to be used to its limit. The model is therefore a conditional model of the choice of formal childcare take-up and labour force participation rather than a more general model including the use of informal care as a choice variable. Thirdly, a single aggregate of formal childcare is modelled, although this could easily be extended to multiple types with preferences and choices defined over different types of formal care as well as quality of care.

The mother in each household makes decisions about her labour force participation and the care of her child so as to maximise the value of a preference function defined over private non-childcare consumption C , time not in employment (which we term "leisure" L for shorthand), the quality of childcare (Q) and a vector of exogenous

⁴See *inter alia* Blau and Hagy (1998), Blau and Robins (1988), Connelly (1992), Hotz and Kilburn (1991) and Ribar (1991).

⁵See for example Connelly (1992) or Jenkins and Symons (1995).

⁶This is of course a simplification relative to a more complex model of household labour supply. There is nevertheless some empirical evidence to suggest that the labour supply of the male partner in a two-adult household is relatively inflexible.

characteristics (Z_1). The leisure term may equally be expressed as the difference between the mother's full time endowment T and her labour supply H , so that $T = H + L$. The preference function may be written as

$$U = U(C, L, Q | Z_1). \quad (1)$$

It is expected that the preference function will rise with increases in C , L and Q . Childcare quality is described by some "production process" of the form

$$Q = F(K_M, K_I, qK_F | Z_2), \quad (2)$$

where quality is a function of the mother's time devoted to the care of her children (K_M), the time spent in formal (K_F) and informal (K_I) childcare, and the quality of formal childcare (q). The quality of care is normalised at 1 for the "base quality", with $q > 1$ indicating better than base quality and $0 < q < 1$ indicating poorer than base quality.

The model includes a budget constraint:

$$C + pqK_F = wH - T(w, H, V, pqK_F) + V \quad (3)$$

where p is the hourly price of the basic quality level of childcare, w is the hourly wage of the mother and V represents exogenous household income, including the income of the partner. Hence, total expenditure on formal childcare (pqK_M) depends upon the exogenous base price and the chosen quality and quantity levels. In addition to the budget constraint, there are two time constraints. First, the total amount of childcare must sum to the total time that the children require care:

$$T = K_M + K_F + K_I. \quad (4)$$

Second, there is a time constraint for the mother which requires that the sum of time spent caring for children, work hours and non-childcare leisure time is equal to the total time available:

$$T = H + K_M + (L - K_M). \quad (5)$$

Combining these two time constraints generates

$$(L - K_M) = (K_F + K_I) - H, \quad (6)$$

which simply states that the mother's childfree leisure time (denoted by $L - K_M$) equals the amount by which non-maternal childcare $K_F + K_I$ exceeds her hours of work H .

Models that assume that hours of work strictly equal the sum of informal and formal childcare characterise a very particular use of childcare. They implicitly assume that $L = K_M$, such that any time a mother spends away from employment is devoted to the care of her child and that there is no demand for childcare independent of its role in facilitating employment for the mother. However, there is evidence to support the view that childcare is a good which confers benefits upon both mother and child, to the extent that women may well choose to consume more childcare than is required simply to cover for the time that they are working. We therefore relax this assumption to permit hours of employment to be less than the sum of non-maternal childcare, introducing the possibility of positive childfree leisure time for the mother. That is:

$$K_F + K_I \geq H. \tag{7}$$

As we will go on to demonstrate, this can have important implications for a mother's response to changes in the hourly price of childcare p and the hourly wage rate w .

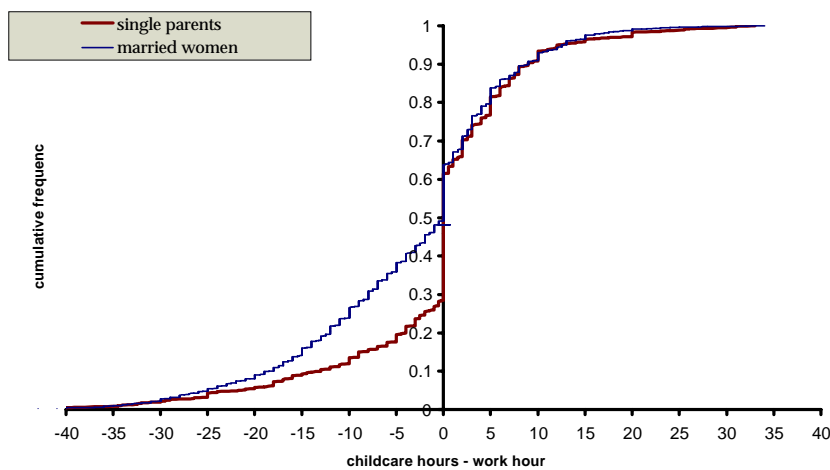
2.1 labour supply, childcare hours and quantity constraints

There is some evidence to support the view that childcare is a good which confers benefits upon both mother and child, to the extent that women may well choose to consume more childcare than is required simply to cover for the time that they are working.

Yet it is incumbent upon parents to organise care for their children at least for the period during the day over which all parents are in work. Indeed, the requirement that non-maternal childcare must, at least, cover the mothers' hours of work is implicit in the model's time constraint. Under two types of conditions (or even a combination of the two), the strict equality constraint between work hours and childcare hours will bind. First, childcare hours may be constrained upwards by work hours, that is, desired work hours may be in excess of the optimal level of childcare hours that would be chosen if the mother were not working. Second, work hours may be constrained by the use of childcare, in that, the labour supply of families with children, and particularly lone parent households, may be conditioned heavily on the availability, cost and quality of childcare.

To demonstrate the potential prevalence of a strict inequality constraint between mothers' work hours and childcare hours, Figure 2 looks at the distribution of the difference between childcare hours and work hours for a sample of working single parents and two-adult households with pre-school children from the UK Family Resource

Figure 2: cumulative frequency of excess childcare hours

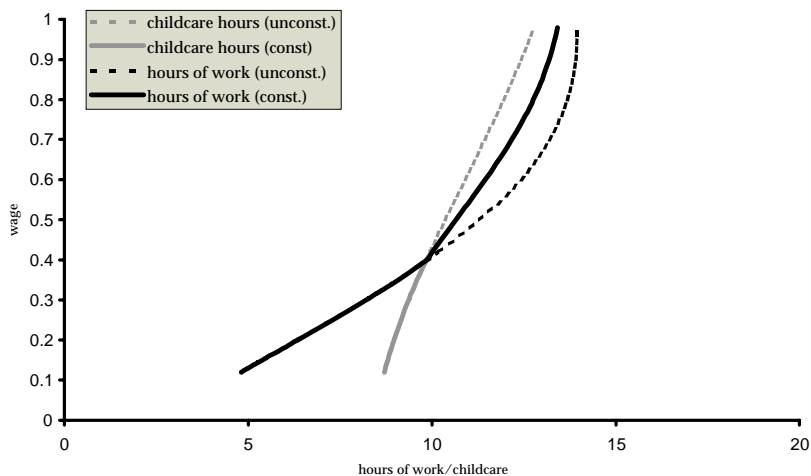


Survey data.⁷ Taking a difference of zero as a reference point (where hours of childcare and hours of work are equal) allows a picture to be drawn of the extent to which hours constraints bind for the two types of household. Allowing equality to include differences of less than five hours, we see that around 20 per cent of single parent households consume fewer hours of childcare than work, some 55 per cent consume roughly equal amounts, and around 25 per cent of households use more childcare than they strictly require to cover their time in work. For single parents, therefore, there is strong evidence of a binding constraint on the choice of childcare hours. Among working women in couples where the male partner is in full-time employment, the pattern is somewhat different. Many more women (perhaps 40 per cent) have fewer hours of childcare than hours of work, although the pattern of 'excess' childcare use is similar to that of single parent households.

The constraint between work and childcare is rarely recognized in the empirical literature in this area. More often, one finds models which treat all observations as the outcome of unconstrained choices. This is of course not consistent, and could po-

⁷This sample excludes those who report using no childcare. The analysis in earlier chapters has shown that a significant proportion of mothers seem able to work without using informal or formal childcare, suggesting the possibility that childcare by the mother and work can be combined. In terms of the model, this would violate the mutually exclusive nature of time use implicit in the mother's time constraint. However, the use of maternal childcare while the mother is working can be thought of as an alternative form of informal care, available in limited quantities to some mothers and not to others. Although it differs from other types in that it cannot be used to facilitate leisure, inclusion of such time as informal childcare is a parsimonious method of allowing the model to be consistent with the observed data.

Figure 3: simulated wage responses in the presence of quantity constraints



tentially lead to biases in the prediction parameters of an empirical model of childcare demand and labour supply. It is a particular concern that the failure to accommodate quantity constraints might alter predicted price and wage effects.

To see how this might be so, we include an illustrative simulation of the economic model laid out in the earlier section. For a hypothetical set of parameters for work and childcare preferences, we compare the relationships between labour supply, childcare hours and wage rates for two scenarios. The first allows a free choice of childcare and work. The second enforces a quantity constraint which requires that childcare hours must at least equal hours of work.

Figure 3 plots a series of functions, each of which show the optimized labour supply or childcare choice at different levels of the wage rate either in the presence or absence of quantity constraints. For the particular parameterisation chosen here, we see that for low wage rates, childcare demand exceeds labour supply. As wage rates increase, so too does the supply of labour, to a point where labour supply equals childcare demand. In the absence of quantity constraints, further increases in the wage rate would lead to a labour supply choice in excess of childcare demand (compare the hashed black and grey schedules respectively). However, imposing the quantity constraint in this model does two things; firstly, it reduces labour supply relative to its unconstrained level. And second, childcare hours increase beyond that level which would be chosen in the absence of the constraint. It is interesting to note from Figure 3 the effect of the quantity constraint on wage elasticities. For the pattern of behaviour implied by our illustrative model, labour supply is less elastic with respect to wages in the presence of quantity constraints. On the other hand,

the effective wage elasticity of childcare hours is forced higher by the presence of the constraint. A similar distortion is evident in the response to changes in childcare price. For a low price of childcare, the demand for childcare is higher than the supply of work hours. As the price of childcare increases, demand for childcare falls to a point of equality with hours of work. In the absence of any quantity constraint, childcare demand would continue to fall as prices rise further. However, this might reduce childcare below the level required to cover for desired hours of work. Instead, one sees a higher consumption of childcare than would have been the case in the absence of the quantity constraint.

To fully quantify the relationships between childcare price and childcare hours, and between wage rates and labour supply, we need to develop an econometric model which deals explicitly with quantity constraints.

3 A Econometric Model of Employment and Childcare Use with Quantity Constraints

One of the principal aims in the estimation of a joint model of labour supply and childcare demand is to isolate childcare price and wage effects in the two related decisions. By doing so, the hope is to cast some light on the potential effects of childcare subsidies on work incentives among families with children. To do so requires some care in model specification, for two principle reasons. First, childcare choices are affected by the quantity constraint that the number childcare hours are at least sufficient to cover the time that all parents are in paid employment.⁸ Second, our FRS data has information on childcare choices only among working mothers.

3.1 an unconstrained approach

The econometric model developed here builds on an approach adopted by Duncan, Giles, and Webb (1995). Their data source was a one-off supplement to the 1991-92 UK General Household Survey which collected data on childcare choices for a sample of households with dependent children, regardless of their employment status. Following the empirical observation that a good proportion of non-working households purchase childcare to some degree, Duncan, Giles, and Webb (1995) choose to model opt for a *bivariate* or a *simultaneous probit* model of labour force participation and

⁸As the earlier theoretical analysis shows, the presence of quantity constraints has an important influence on apparent childcare price effects, to an extent which can magnify price elasticities in a labour supply relationship among those for whom the quantity constraint binds.

Table 1: Four-state work and childcare decision matrix

	Use formal childcare?	
Choose to work?	Yes	No
Yes	work + childcare: $h_w^* > 0, h_c^* > 0$ $\varepsilon_w > -x\beta_w$ $\varepsilon_c > -z\beta_c$	work + no childcare: $h_w^* > 0, h_c^* \leq 0$ $\varepsilon_w > -x\beta_w$ $\varepsilon_c \leq -z\beta_c$
No	no work + childcare: $h_w^* \leq 0, h_c^* > 0$ $\varepsilon_w \leq -x\beta_w$ $\varepsilon_c > -z\beta_c$	no work + no childcare: $h_w^* \leq 0, h_c^* \leq 0$ $\varepsilon_w \leq -x\beta_w$ $\varepsilon_c \leq -z\beta_c$

childcare take-up.⁹ This model builds on the premise that observed choices of work (denoted h_w) and childcare (h_c) are driven by a person's propensity to work (denoted h_w^*) and to consume formal childcare (h_c^*). These propensities are assumed to depend on a series of factors x and z respectively (including individual socio-demographic characteristics, price, wage and income variables, and factors associated with the level of supply of childcare, or the level of demand for labour), and take the following general form:

$$h_w^* = x\beta_w + \varepsilon_w; \quad (8)$$

$$h_c^* = z\beta_c + \varepsilon_c, \quad (9)$$

where β_w and β_c are parameters, and ε_w and ε_c represent the intrinsic propensities to work and to consume childcare which are not related to observed characteristics x and z .¹⁰ This gives rise to the four possible combinations of choices described in Table 1.

The simultaneous probability model applied in Duncan, Giles, and Webb (1995) allows for non-working households to consume formal childcare ($h_w = 0, h_c > 0$) which is both an advantage, and a development on earlier work. However, three

⁹The latter was proposed initially by Mallar (1977). The former is covered in standard texts, for example Amemiya (1981) or Maddala (1983).

¹⁰One interpretation of h_w^* is that it represents the difference in the utility from work ($U_{h>0}^*$) and the utility from not working ($U_{h=0}^*$), so that $h_w^* = U_{h>0}^* - U_{h=0}^*$. Alternatively, one can look on h_w^* as a term directly proportional to desired hours. Either way, individuals are assumed to work if $h_w^* > 0$, and to purchase childcare if $h_c^* > 0$.

Table 2: Three-state work and childcare decision matrix

	Use formal childcare?	
Choose to work?	Yes	No
Yes	work + childcare: $h_w^* > 0, h_c^* > 0$ $\varepsilon_w > -x\beta_w$ $\varepsilon_c > -z\beta_c$	work + no childcare: $h_w^* > 0, h_c^* \leq 0$ $\varepsilon_w > -x\beta_w$ $\varepsilon_c \leq -z\beta_c$
No	no work: $h_w^* \leq 0$ $\varepsilon_w \leq -x\beta_w$	

modifications to this model are required for the current analysis.

The first modification is driven by limitations in the FRS data collection. The FRS data, providing no consistent information on childcare use among non-working mothers, does not permit non-working households to be separated into those who consume childcare and those who don't. Hence, the two states in the bottom row of Table 1 must be merged into one "No work, unspecified childcare" state, leaving a three state model of the form described in Table 2:

3.2 including quantity constraints

The second modification reflects the fact that the option to work but consume no formal childcare ($h_w > 0, h_c = 0$) fails to respect the quantity constraint which requires that childcare hours must at least equal work hours. Moreover, in the presence of quantity constraints, the interpretation of the choice both to work and consume childcare ($h_w > 0, h_c > 0$) needs to be treated with some caution. To observe a positive consumption of childcare (that is, $h_c > 0$) does not necessarily correspond to a positive desire for childcare (for which $h_c^* > 0$), but rather the need for someone with strong latent desire to work to provide necessary childcare cover. The three-state characterisation of labour supply and childcare demand is one way of dealing with missing data on childcare hours for non-workers, but it still fails to deal appropriately with the quantity constraints on childcare hours and work hours. The consequence of this misspecification will be to overstate the childcare price effect on employment incentives, and understate the price effect on childcare demands.

We deal explicitly with quantity constraints on employment and childcare use,

Table 3: Quantity-constrained three-state decision matrix for work and childcare

	Use more than required hours of formal childcare?	
Choose to work?	<i>Yes</i>	<i>No</i>
<i>Yes</i>	work + excess childcare: $h_w^* > 0, h_c^* > h_w^*$ $\varepsilon_w > -x\beta_w$ $\varepsilon_c > (x\beta_w - z\beta_c) + \varepsilon_w$	work + minimum childcare: $h_w^* > 0, h_c^* \leq h_w^*$ $\varepsilon_w > -x\beta_w$ $\varepsilon_c \leq (x\beta_w - z\beta_c) + \varepsilon_w$
<i>No</i>	no work, indeterminate level of childcare: $h_w^* \leq 0$ $\varepsilon_w \leq -x\beta_w$	

starting with the same propensities (8) and (9) that were used in the unconstrained four-state model described earlier. For working households with observed childcare hours strictly in excess of work hours, it is assumed that their desired level of childcare exceeds the (positive) desire for work, so that $h_w^* > 0$ and $h_c^* > h_w^*$. In this case, the quantity constraint on childcare hours does not bind. For the remainder of working households, we assume that they prefer to work (so that $h_w^* > 0$), but that their desire for childcare falls short of this hours level ($h_c^* \leq h_w^*$). This latter condition is very different to the previous three-state model¹¹, and recognises that observed childcare hours h_c (which are predominantly positive for working households) do not necessarily reflect the latent desire for childcare (h_c^* , which may be negative). Rather, the binding quantity constraint forces a level of use of childcare that exceeds the underlying desire for childcare. Table 3 describes these three choices.

Our third and final modification extends the three-state model of employment and childcare using the same underlying representation of tastes given by (8) and (9). The extension further distinguishes between part-time and full-time employment, according to the strength of the latent propensity to work.¹² It is assumed that an individual chooses not to work if $h_w^* \leq \gamma_1$. For part-time workers, the propensity is bounded according to the rule $h_w^* \in (\gamma_1, \gamma_2)$, whereas full-time workers have an employment propensity h_w^* which exceeds γ_2 . For those women who are in paid employment, we further distinguish between those who choose to purchase more than the minimum

¹¹where $h_w^* > 0$ and $h_c^* > 0$ for working households who use any childcare.

¹²We categorise an individual to be in part-time employment if she works less than 30 hours per week, and full-time if she works for at least 30 hours.

Table 4: Quantity-constrained five-state decision matrix for work and childcare

	Use more than required hours of formal childcare?	
Which labor market state?	Yes	No
<i>Full-time</i> ($h_w \geq 30$)	$h_w^* \geq \gamma_2, h_c^* > h_w^*$ $\varepsilon_w > \gamma_2 - x\beta_w$ $\varepsilon_c > (x\beta_w - z\beta_c) + \varepsilon_w$	$h_w^* \geq \gamma_2, h_c^* \leq h_w^*$ $\varepsilon_w > -x\beta_w$ $\varepsilon_c \leq (x\beta_w - z\beta_c) + \varepsilon_w$
<i>Part-time</i> ($0 < h_w < 30$)	$h_w^* \in (\gamma_1, \gamma_2), h_c^* > h_w^*$ $\varepsilon_w \in (\gamma_1 - x\beta_w, \gamma_2 - x\beta_w)$ $\varepsilon_c > (x\beta_w - z\beta_c) + \varepsilon_w$	$h_w^* \in (\gamma_1, \gamma_2), h_c^* \leq h_w^*$ $\varepsilon_w \in (\gamma_1 - x\beta_w, \gamma_2 - x\beta_w)$ $\varepsilon_c \leq (x\beta_w - z\beta_c) + \varepsilon_w$
<i>No paid work</i>	$h_w^* \leq \gamma_1$ $\varepsilon_w \leq \gamma_1 - x\beta_w$	

level of childcare (for whom $h_c^* > h_w^*$) and those who use only the minimum level of childcare necessary to cover for their time in paid employment ($h_c^* \leq h_w^*$). Taken together, these criteria allow us to distinguish five states of employment and childcare use, and at the same time respect the constraints which act as a restriction on hours choices.¹³ Table 4 shows the relationship between the five observed states of (1) no paid employment; (2) part-time work at minimum childcare; (3) part-time work with more than minimum childcare; (4) full-time work at minimum childcare; and (5) full-time work with more than minimum childcare.

As a test of the importance of allowing for the quantity constraint between the hours of work and hours of childcare, a similar five-state model ("without quantity constraints") can be estimated with the childcare division being made on the use of formal childcare rather than on the excessive use of formal care.

¹³Ideally, it might be desirable to model the entire range of employment hours, but, as is typical of similar models, the discrete choice is most parsimonious in the light of non-linear budget constraints and the degree of computational burden inherent in continuous models. There are, of course, more general economic models of discrete employment choices than the one presented in this paper (see Blau and Hagy (1998), Blundell, Duncan, McCrae, and Meghir (2000), Keane and Moffitt (1998) or Ribar (1995) *inter alia*). Our focus is more on the development of appropriate methods to correct for quantity constraints in a bivariate model of employment and childcare.

3.3 An econometric specification for the three-state model with quantity constraints

For the basic specification, a linear relationship of the form

$$\begin{aligned} h_w^* &= x\beta_w + \varepsilon_w; \\ h_c^* &= z\beta_c + \varepsilon_c \end{aligned}$$

is chosen, where the two disturbances ε_w and ε_c are assumed to be joint-normally distributed with unit variances and correlation ρ .¹⁴ Defining $\varepsilon = (\varepsilon_w, \varepsilon_c)'$, we have that $\varepsilon \sim N(0, \Sigma_{wc})$, where

$$\Sigma_{wc} = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}. \quad (10)$$

Given observed hours of work h_w and childcare h_c , the observability criteria for the three-state model with quantity constraints on childcare are the following:

$$\begin{aligned} h_w &= 0 && \text{if } h_w^* \leq 0; \\ h_w > 0, h_c &= h_w && \text{if } h_w^* > 0 \text{ and } h_c^* \leq h_w^*; \\ h_w > 0, h_c &> h_w && \text{if } h_w^* > 0 \text{ and } h_c^* > h_w^*. \end{aligned}$$

Together with the assumption of joint normality of $\varepsilon = (\varepsilon_w, \varepsilon_c)'$, these observability criteria may be used to derive the following probabilities:

$$\begin{aligned} \Pr(h_w = 0 | x) &= \Pr(h_w^* \leq 0 | x) \\ &= \Pr(\varepsilon_w \leq -x\beta_w); \end{aligned} \quad (11)$$

$$\begin{aligned} \Pr(h_w > 0, h_c = h_w | x, z) &= \Pr(h_w^* > 0, h_c^* \leq h_w^* | x, z) \\ &= \Pr(\varepsilon_w > -x\beta_w, z\beta_c + \varepsilon_c < x\beta_w + \varepsilon_w) \\ &= \Pr(\varepsilon_w > -x\beta_w, \varepsilon_c - \varepsilon_w < x\beta_w - z\beta_c); \end{aligned} \quad (12)$$

¹⁴The unit variance condition is a required normalisation when modelling discrete specifications of this form, and controls for the fact that variances of ε_w and ε_c cannot be identified separately from the parameters in the two linear propensity equations.

$$\begin{aligned}
\Pr(h_w > 0, h_c > h_w | x, z) &= \Pr(h_w^* > 0, h_c^* > h_w^* | x, z) \\
&= \Pr(\varepsilon_w > -x\beta_w, z\beta_c + \varepsilon_c > x\beta_w + \varepsilon_w) \\
&= \Pr(\varepsilon_w > -x\beta_w, \varepsilon_c - \varepsilon_w > x\beta_w - z\beta_c). \quad (13)
\end{aligned}$$

In Figure 4 we describe a typical contour of the bivariate density associated with the two stochastic terms ε_w and ε_c in (8) and (9). The shaded regions in the figure give a graphical representation of probabilities (11),(12) and (13). Given data on characteristics x which are thought to influence employment, we can derive quite straightforwardly the first probability (11) of observing a non-working household as

$$\begin{aligned}
P^{nw} &= \Pr(h_w = 0 | x) \\
&= \Pr(\varepsilon_w \leq -x\beta_w) \\
&= \Phi(-x\beta_w) \\
&= 1 - \Phi(x\beta_w) \quad (14)
\end{aligned}$$

where $\Phi(\cdot)$ represents the (univariate) standard normal distribution function. This probability corresponds to the lower shaded region in Figure 4.

The remaining two probabilities for working households need a little more care. For working households who consume more than the minimum required level of child-care, the required probability $P^{wc} = \Pr(h_w > 0, h_c > h_w | x, z)$ corresponds to the vertical shaded section of Figure 4. Given a bivariate normal distribution for ε_w and ε_c , we have that

$$\begin{aligned}
P^{wc} &= \Pr(h_w > 0, h_c > h_w | x, z) \\
&= \Pr(\varepsilon_w > -x\beta_w, \varepsilon_c - \varepsilon_w > x\beta_w - z\beta_c) \quad (15)
\end{aligned}$$

$$\begin{aligned}
&= \Pr[\varepsilon_w > -x\beta_w, \varepsilon_c > (x\beta_w - z\beta_c) + \varepsilon_w] \\
&= \int_{-x\beta_w}^{\infty} \int_{(x\beta_w - z\beta_c) + \varepsilon_w}^{\infty} \phi_2(\varepsilon_w, \varepsilon_c; \rho) \partial\varepsilon_c \partial\varepsilon_w, \quad (16)
\end{aligned}$$

where $\phi_2(\varepsilon_w, \varepsilon_c; \rho)$ represents the bivariate normal density. The evaluation of (16) is complicated by the presence of ε_w in the lower limit of the inner integral. However, we can get further with a change in variable, by re-expressing P^{wc} in terms of the differenced stochastic term $v = \varepsilon_c - \varepsilon_w$ using the formulation (15) above. Now, v is clearly normal, and correlated with ε_w . More specifically, $var(v) = 2(1 - \rho)$ and

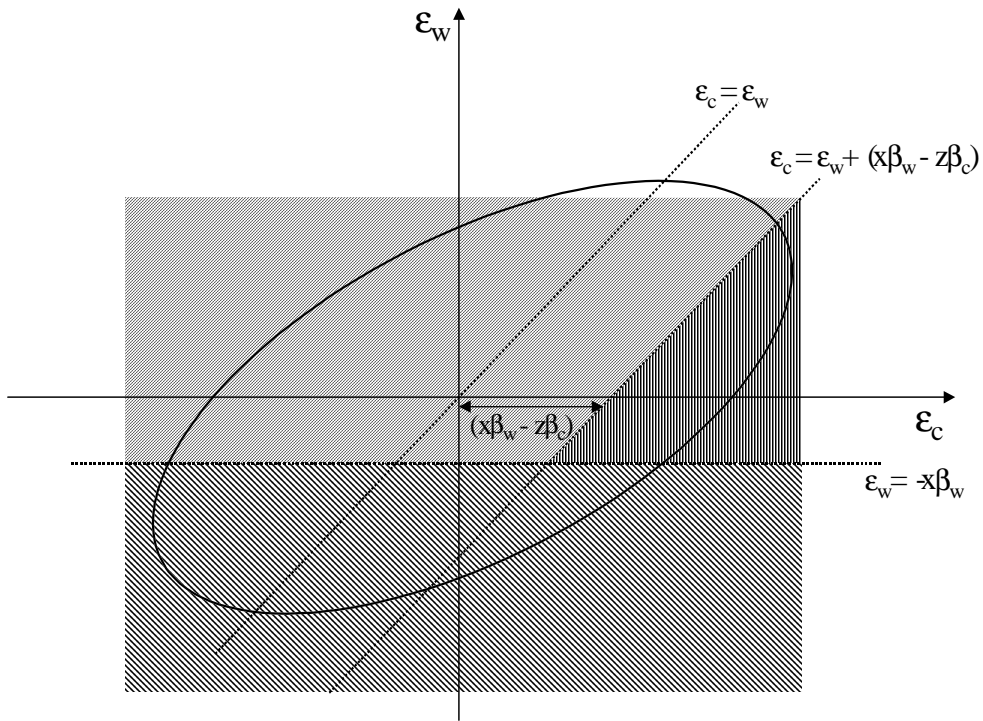


Figure 4: Contours and probabilities for the three-state model with quantity constraints

$cov(\varepsilon_w, v) = \rho - 1$,¹⁵ so that the distribution of ε_w and v is jointly normal $N(0, \Sigma_{\varepsilon_w v})$ with covariance matrix

$$\Sigma_{\varepsilon_w v} = \begin{pmatrix} 1 & \rho - 1 \\ \rho - 1 & 2(1 - \rho) \end{pmatrix}.$$

The original probability P^{wc} may be re-stated using the change in variable approach, to give

$$\begin{aligned} P^{wc} &= \Pr(\varepsilon_w > -x\beta_w, v > x\beta_w - z\beta_c) \\ &= \Pr\left[\varepsilon_w > -x\beta_w, \frac{v}{\sqrt{2(1-\rho)}} > \frac{x\beta_w - z\beta_c}{\sqrt{2(1-\rho)}}\right] \\ &= \Phi_2\left[x\beta_w, \frac{z\beta_c - x\beta_w}{\sqrt{2(1-\rho)}}; \frac{\rho-1}{\sqrt{2(1-\rho)}}\right]. \end{aligned} \quad (17)$$

The remaining probability P^{wmc} of working but using no more than the minimum necessary childcare follows directly;

$$\begin{aligned} P^{wmc} &= \Pr(\varepsilon_w > -x\beta_w, v \leq x\beta_w - z\beta_c) \\ &= \Pr\left[\varepsilon_w > -x\beta_w, \frac{v}{\sqrt{2(1-\rho)}} \leq \frac{x\beta_w - z\beta_c}{\sqrt{2(1-\rho)}}\right] \\ &= \Phi(x\beta_w) - \Phi_2\left[x\beta_w, \frac{z\beta_c - x\beta_w}{\sqrt{2(1-\rho)}}; \frac{\rho-1}{\sqrt{2(1-\rho)}}\right]. \end{aligned} \quad (18)$$

3.3.1 likelihood specification: three-state model with quantity constraints

Consider a sample of data $\{h_{iw}, h_{ic}, x_i, z_i, i = 1, \dots, n\}$ on employment and childcare choices among a sample of n households, and define the following indicators I_i^{nw} , I_i^{wmc} and I_i^{wc} for each household to represent the three states of non-work (nw), work with minimum childcare (wmc), and work with more than minimum childcare (wc):

$$I_i^{nw} = \mathbf{1}(h_{iw} \leq 0);$$

¹⁵since $var(v) = E[(\varepsilon_c - \varepsilon_w)^2] = E[\varepsilon_c^2 + \varepsilon_w^2 - 2\varepsilon_w\varepsilon_c] = 1 + 1 - 2E(\varepsilon_w\varepsilon_c) = 2(1 - \rho)$, and $cov(\varepsilon_w, v) = E[\varepsilon_w(\varepsilon_c - \varepsilon_w)] = E[\varepsilon_w\varepsilon_c - \varepsilon_w^2] = \rho - 1$. Taken Together, these imply a correlation $corr(\varepsilon_w, v) = \frac{\rho-1}{\sqrt{2(1-\rho)}}$ between $\{\varepsilon_w, v\}$.

$$\begin{aligned}
I_i^{wmc} &= \mathbf{1}(h_{iw} > 0, h_{ic} \leq h_{iw}); \\
I_i^{wc} &= \mathbf{1}(h_{iw} > 0, h_{ic} > h_{iw}),
\end{aligned}$$

where $\mathbf{1}(\cdot)$ represents the indicator function. The log likelihood for the quantity-constrained three-state model may then be written as

$$\begin{aligned}
\ln L(\beta_w, \beta_c, \rho) &= \sum_{i=1}^n [I_i^{nw} \ln(P_i^{nw}) + I_i^{wmc} \ln(P_i^{wmc}) + I_i^{wc} \ln(P_i^{wc})] \\
&= \sum_{i=1}^n I_i^{nw} \ln(1 - \Phi(x_i \beta_w)) \\
&\quad + \sum_{i=1}^n I_i^{wmc} \ln \left(\Phi(x_i \beta_w) - \Phi_2 \left[x_i \beta_w, \frac{z_i \beta_c - x_i \beta_w}{\sqrt{2(1-\rho)}}; \frac{\rho-1}{\sqrt{2(1-\rho)}} \right] \right) \\
&\quad + \sum_{i=1}^n I_i^{wc} \ln \Phi_2 \left(x_i \beta_w, \frac{z_i \beta_c - x_i \beta_w}{\sqrt{2(1-\rho)}}; \frac{\rho-1}{\sqrt{2(1-\rho)}} \right) \tag{19}
\end{aligned}$$

This specification combines appropriate controls for the absence of childcare data among non-working households, and second, it factors quantity constraints explicitly into the likelihood used for estimation of parameters β_w, β_c and ρ in the two latent propensity equations (8) and (9).

We described earlier how this model might further distinguish between part-time and full-time employment with either minimum or more than minimum childcare. Appendix B model B3 describes the econometric specification and likelihood derivation for this model. For illustration, and for empirical comparison, we also include likelihood details in Appendix B for the alternative (unconstrained) specifications covered in the previous discussion.

4 Empirical Results

4.1 data

The data source used in this paper is a pooled series of cross-sections from the UK Family Resources Surveys (FRS) over the period 1994/5 to 1998/9 inclusive. The FRS is an annual cross-section survey of around 25,000 UK households. It collects a wide range of information on the incomes and circumstances of all family members, and includes detailed questions about childcare use for each child in the family. The information collected on childcare includes a multiple response question on who looks

after the child (eg. relatives, friends/neighbours, childminder, nursery/playgroup, creche, 'other'). Questions are also asked about total weekly hours of care and the cost of this care, in term time and school holidays separately. As noted earlier, the childcare choice data in the FRS are only reliable for working households. Our econometric models take explicit account of this sample defect in the construction of the likelihood functions used in estimation.

4.2 conditioning variables

The empirical models include the following set of conditioning variables that are thought to influence the both decisions to work and consume childcare:

- i. the number of pre-school children by age
- ii. the number of school children and the number of older children
- iii. the age of the mother
- iv. the age at which the mother left full-time education

A number of exclusion restrictions are necessary to identify the full set of parameters in our propensity equations. From the employment equation the following variables were excluded which are deemed not to have a direct impact on the probability of employment:

- i. receipt of maintenance income
- ii. number of older children
- iii. number of families in the household
- iv. years lived at current address

The receipt of income maintenance may indicate a source of income that is earmarked to provide for the care of the children, while the remaining three factors may all be related to the provision of alternative informal care. From the childcare take-up equation we exclude one factor:

- i. the level of unearned family income

Although the analysis presented above shows a positive relationship between other family income and the use of formal childcare, it is presumed to operate through its influence on labour supply rather than directly on the demand for childcare.

4.2.1 predicting wages, childcare prices and childcare costs

In addition to these variables in the basic model, two further extensions are made. First, a wage variable is added to the employment equation and a childcare price variable to the childcare equations. Wage rates are typically not observed for non-employed women and a wage for non-workers is estimated using a standard selection-corrected wage equation. The childcare price measure is derived from an estimated price equation that controls for quality and potential non-linearities in the pricing structure, in the manner of Duncan, Paull, and Taylor (2001). These prices vary by Local Authority and the number and ages of children.¹⁶

Childcare costs are clearly of prime concern to a mother who is seeking paid employment. These gross costs depend on a number of factors, including the local hourly cost of care for pre-school and school-aged children, and the number of children in each age group. Of course, a child requires no purchased care for the time that she or he is in school. Our data source does not record the time-of-day patterns of work for mothers of school-aged children, which presents something of a problem. In calculating the minimum hourly cost of H_c hours childcare for a mother who works for H hours, we adopt the following convention: let \overline{H}_{sch} represent the (fixed) length of the school period (in hours per week). Then the marginal hourly cost of childcare $P_c(H_c, \overline{H}_{sch})$ for a mother who currently uses H_c hours of care is assumed to be given by

$$P_c(H_c, \overline{H}_{sch}) = N_{ps} \cdot P_{ps} + \mathbf{1}(H_c > \overline{H}_{sch}) \cdot N_{sc} \cdot P_{sc}, \quad (20)$$

where P_{ps} and P_{sc} represent the (locally varying) cost of an hour of standard quality childcare for a pre-school and school-aged child respectively, N_{ps} and N_{sc} denote the number of pre-school and school-aged children in the family respectively, and $\mathbf{1}(\cdot)$ denotes the indicator function.¹⁷

Given (20), we can derive the total cost to a mother of H_c hours of care, $TC(\cdot)$ as

$$TC(H_c, \overline{H}_{sch}) = \sum_{h=1}^{H_c} P_c(h, \overline{H}_{sch})$$

¹⁶Because Duncan, Paull, and Taylor (2001) applies grouping methods to predict the quality and hours adjusted prices, there are missing price data when grouping cells contain no observations on price. The sample is therefore rebalanced so that the proportions of working and non-working households remain representative of the full population.

¹⁷It is assumed that the mother arranges her employment and childcare patterns initially during school hours. So, if a woman requires H_c hours of childcare cover, then the actual level of childcare for a school child will be limited to excess out-of-school hours $H_c - \overline{H}_{sch}$. This of course may not be universally true. Nevertheless, it is an assumption which reflects many mothers' experiences. Furthermore, in the absence of time-of-day data on womens' work patterns, we feel that the approach represents an adequate solution to a data problem. In our empirical work, \overline{H}_{sch} is set at 30.

$$= N_{ps} \cdot P_{ps} \cdot H_c + \mathbf{1}(H_c > \bar{H}_{sch}) \cdot N_{sc} \cdot P_{sc} \cdot (H_c - \bar{H}_{sch}) \quad (21)$$

Thus, the *minimum total cost* of childcare $MTC(H)$ for a woman who is in paid employment for H hours is $MTC(H) = TC(H, \bar{H}_{sch})$.

4.2.2 modelling the effects of the tax and transfer system on employment and childcare

The second extension additionally controls for the effects of taxes and transfers on employment and childcare decisions. The relative incomes in and out of work and the average hourly pay-off from increasing hours of work are important determinants of a mother's decision to seek or increase her level of paid employment. For families with children, the decision to work will also be affected by the cost of childcare, and the extent to which welfare programs will offset those costs. As is the case in many countries, the modelling of taxes and transfers in the United Kingdom is not straightforward and these complexities are abstracted from in the formal specification of the empirical models, in much the same way, it could be argued, that families do when they make their own labour supply decisions.

This is achieved by including variables in the employment equations that measure the average hourly income gain from employment. Full account is taken both of minimum childcare costs and any government subsidies for those costs. In doing so, we introduce a relationship between childcare price and the propensity to work, since each income variable depends directly on childcare price. In the absence of any subsidisation for childcare costs, an increase in childcare price will reduce the income gain from additional employment. If the tax or transfer system delivers any compensation for childcare costs, the income gain will be adjusted.

4.2.3 income and childcare subsidy variables in the employment and childcare equations

All income terms are based on net household income Y_H when the mother works H hours per week. The first is simply the level of household income Y_0 when the mother chooses not to seek paid employment. We also include a series of income-related variables which capture the average hourly income gain $Y_{H_i \rightarrow H_j}$ when the mother increases her employment from H_i to H_j hours per week, defined as

$$Y_{H_i \rightarrow H_j} = \frac{1}{(H_j - H_i)} [Y_{H_j} - Y_{H_i}] - \sum_{h=H_i+1}^{H_j} P_c(h, \bar{H}_{sch}) \quad (22)$$

The first term in (22) captures the effect of increasing maternal labour supply from H_i to H_j on net household income, and the second (summation) term represents the

difference in minimum childcare costs at H_i and H_j . In our empirical work we include two realisations of (22); the relative income gain from increasing labour supply beyond part-time ($Y_{20 \rightarrow 30}$), and the rewards from moving to full-time employment ($Y_{30 \rightarrow 40}$).

We also extend the set of factors which are used to model childcare demand. In addition to the gross cost of childcare (either the unit cost of childcare, or the total cost to the mother of an increase in childcare coverage), we include a measure of the *average hourly childcare subsidy* $S(H_c)$ for H_c hours of childcare.¹⁸ This measure is included in order to capture the direct impact of such subsidies to use formal childcare independent of the labour supply response.

If the tax and transfer system were fixed over the period of our estimation sample, the income-related variables would not be identified separately (beyond the non-linearity of the tax and transfer system). All variation would be captured by other explanatory variables, principally gross wages and household demographics. However, the 1994-1999 period covered by the data included a number of important policy changes in the UK, particularly in relation to support for low-wage working households, which ensures that the income-related variables exhibit independent variation and so can be used to identify separately the effects of the tax and transfer system on employment and childcare.

The inclusion of family net income variables among the control set is also advantageous from the point of view of microsimulation. Once estimated, the econometric model can be used to predict changes in employment and childcare status in response to changes in the tax or transfer system. Of particular interest here is the potential to simulate employment and childcare responses to the introduction of the new Working Families' Tax Credit (WFTC) in the UK.

4.3 model estimates

Empirical results for the five-state specifications of employment and childcare are presented in Tables 11 and 12 of Appendix A. Models are estimated separately for single parents and married women in our sample, using specifications which explicitly account for quantity constraints in the use of childcare.¹⁹

¹⁸To calculate the average hourly childcare subsidy $S(H_c)$, we first generate two tax unit net income measures. The first net income $Y_c(H_c)$ is calculated under the assumption that H_c hours of formal childcare is purchased by a woman who works for H_c hours. The second income measure, $Y_0(H_c)$, presumes that the same woman works for H_c hours, but purchases no formal childcare. The average hourly childcare subsidy is then calculated as $S(H_c) = H_c^{-1} \cdot [Y_c(H_c) - Y_0(H_c)]$.

¹⁹Results for the corresponding three-state models, which do not differ markedly from the more general models, are available from the authors on request.

Most of the results are consistent with expectations. The level of family non-work income (a term which including savings, maintenance payments and any partner’s earnings) is negatively related to the employment propensity, indicating that higher out-of-work incomes act as an employment disincentive. Families with very young children are less likely to work, but more likely to demand formal childcare than are families with older children. Women with higher formal education qualifications are more likely both to be employed, and to use formal childcare. These results hold qualitatively for both single parent and married households, although some differences do exist in the magnitudes of these effects. In particular, the propensity for single parents with young children to use formal childcare systematically exceeds that of married women, a result which is consistent with the earlier analysis of FRS data on excess childcare for these two demographic groups.

A comparison of our quantity-constrained models (Tables 11 and 12) with unconstrained model estimates reveal two significant results. First, we find the parameters of the unconstrained employment (childcare) propensities generally to be biased downwards (upwards). The second important result concerns the sign of the estimated correlation coefficient ρ . For both single parents and married women, the correlation between the unobserved propensities to work and use childcare is (somewhat unusually) negative and highly significant in the unconstrained models, suggesting that women who are intrinsically more likely to seek employment are less likely to choose formal childcare. In contrast, the correlation coefficients for the quantity-constrained models are at least insignificant, and for the married women’s sample, significant at 1 per cent. We conclude from this comparison that a failure to control appropriately for quantity-constrained data in the estimation method can lead to general parameter bias, and a spurious negative correlation between childcare and employment decisions. Both potential pitfalls may be avoided by a careful specification of the model likelihood along the lines suggested in (19).

4.4 wage and price effects

In the most general five-state specifications, we include predicted wage rates for the mother into the employment propensity equation, and a prediction of the standard-quality hourly price of pre-school childcare into the childcare propensity, using the approach developed in Duncan, Paull, and Taylor (2001). Concentrating on our preferred quantity-constrained model, we find there to be evidence of a positive relationship between wage rates and employment for married women (Table 12). For single parents (Table 11), the relationship is less strong, for reasons which may have something to do with the relative lack of variation in wage rates among the working

sample.

The estimated childcare price effects on the probability of choosing formal childcare are very strongly negative for both single parents and married women. Our estimated price effects are stronger than previous UK estimates, and more in line with results found in US studies (eg. Hotz and Kilburn (1991)). Recall that these prices have been adjusted to control for possible quality differences in the set of observed prices across the working sample, and may therefore be indicative of pure price elasticities. The strength and consistency of these empirical results lends strong support to the claim that childcare price subsidies will have strong positive incentive effects in the take-up of formal childcare among families with children. A joint test reveals that these price and wage effects are highly significant.

4.5 introducing tax and transfer effects

To control for the effects of taxes and transfers on employment and childcare decisions, we include in the employment propensity equations a series of variables which measure the average hourly income gain from employment (from 0 to 20 hours, from 20 to 30 hours, and from 30 to 40 hours).²⁰ This introduces a relationship between childcare price and employment propensity, since each income variable depends directly on childcare price. In the absence of any subsidisation for childcare costs, an increase in childcare price will reduce the income gain from additional employment. If the tax or transfer system delivers any compensation for childcare costs, the income gain will of course be adjusted. For each childcare equation, we add the average hourly subsidy to compensate for the childcare costs incurred when 20 hours of standard-quality childcare is purchased.

The results for single parents (Table 11) and married women (Table 12) show that the greater the income gain from working, the more likely is the mother to seek employment. These results also condition on gross wage rates, and so are identified through variation in taxes and transfers over time and across household. We may conclude from these results that a tax reform which increases the reward to, say, full-time employment will improve work incentives to a significant degree. Of particular interest in this regard will be the likely effects of the new Working Families' Tax Credit, since for many lower earning households this is precisely the effect that is anticipated (see also Tables 1 to 4 above). We also find a positive association between the level of childcare subsidy and the take-up of formal childcare among working

²⁰We covered the construction of these additional variables in some detail in previous sections. It should again be noted, however, that each income-related variable is fully consistent with the detail of the UK tax and transfer system.

households. Again, this has implications in the current policy debate on childcare cost subsidisation, particularly given the relative generosity of the childcare credit element of the WFTC.

5 Simulating the Working Families' Tax Credit Reform

It has long been recognised in the UK that the necessity for households to provide for the care of their children is a significant barrier to their seeking employment. It may be that a mother chooses to remain away from paid employment in the early years of her child's life. But for those mothers with young children who would in principle be willing to return to work, the costs associated with formal childcare can actively discourage re-entry into the labour market. There have been a number of initiatives within the United Kingdom's welfare system to provide financial assistance with childcare costs. In large part, this financial help has been contingent on the mother working. Thus, from the governments perspective, the most important reason for intervening in the childcare market is to alleviate the barriers to employment associated with childcare costs. Chronologically, the first significant assistance the state provided with childcare costs came with the 1995 reform to Family Credit.

5.1 childcare expenditure disregards in Family Credit (1995)

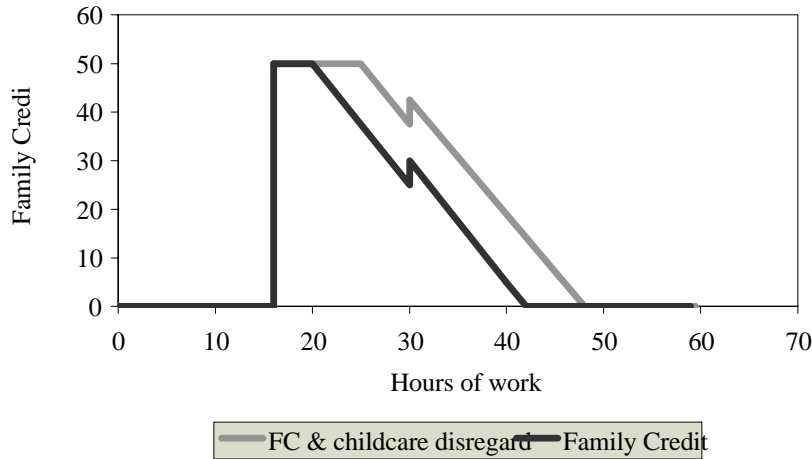
In 1995, the government provide some additional financial help through the Family Credit system for those low-income families who purchased formal (registered) childcare. This help was in the form of a childcare expenditure disregard, whereby costs of up to £60 per week on childcare could be disregarded from the Family Credit means test. The effect of the childcare disregard was to extend rather than increase entitlement to Family Credit, as shown in Figure 5.

5.2 childcare tax credits in the Working Families' Tax Credit (1999)

The Working Families' Tax Credit (WFTC) replaced FC in 1999. Its structure is similar to FC, but it is substantially more generous. The government expects a near doubling of the number of recipients compared with FC: around 1,500,000. The main differences between WFTC and the latest FC (at April 1999) are:

- i. increases in the adult credit: from £49.80 under FC to £53.15 under WFTC

Figure 5: Childcare expenditure disregards in Family Credit



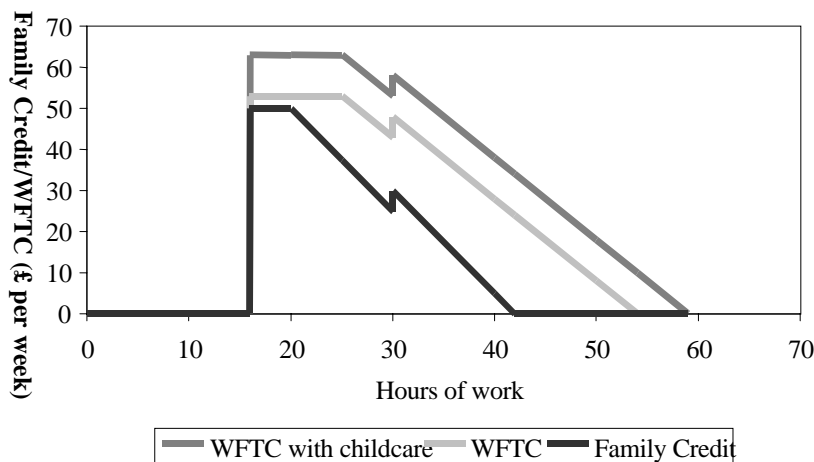
- ii. increases in the child credit: from £15.15 to £25.60 under WFTC²¹;
- iii. an increase in the threshold before payment is withdrawn: from £80.65 to £91.45 per week;
- iv. a reduction in the withdrawal rate from 70 per cent to 55 per cent;
- v. a new childcare credit of 70 per cent of actual childcare costs, up to a maximum of £150 per week, to replace the childcare disregard;

The childcare credit under WFTC increases entitlement by 70 per cent of childcare costs up to 100 per week for people with one child (£150 per week for people with two or more children). With such generous support available, the WFTC childcare credit is likely to have a considerable (expansive) impact on the childcare market in the UK. There have been some early simulations of the possible work incentive consequences of the WFTC including the childcare credit, but as yet there has been little, if any, work on the likely impact of the childcare credit on the extent of use of formal childcare services. By way of illustration, we report later in the paper a comparative simulation of the effect of the childcare credit on net incomes among single parent households under a range of scenarios regarding hours of work, hours of childcare used and the hourly cost of care.

A stylised comparison of WFTC and FC is shown in Figure 6. The figure shows the value of the two credits at various hours of work. It is interesting to note that the largest cash gains from WFTC go to people who are currently just at the end of

²¹for children under 11

Figure 6: Childcare tax credits in WFTC



the taper under FC. For them, the introduction of WFTC will create an entitlement to in- work support whereas before they were ineligible for FC. This brings more families into the WFTC caseload. The effect of the childcare credit is also illustrated, as the highest line in Figure 6. In comparison with the childcare disregard in Family Credit (Figure 5), the WFTC childcare credit increases the total financial support available to mothers who work and pay for childcare. Indeed, the potential generosity of the childcare credit is one of the more interesting features of the new WFTC; at its maximum, it will be worth up to £105 per week.²²

5.3 financial support for childcare costs under Family Credit and WFTC

The childcare credit element of the WFTC is by far the most generous subsidy for childcare costs in the history of the UK tax system. With a childcare credit of up to £105 in value, the costs of employment for families who purchase formal childcare have been significantly reduced. It is very likely that the pattern of childcare use among WFTC-eligible households will change considerably over the short-term, to an extent which might force the UK government to review the generosity of the childcare credit. It is therefore important than any analysis of labour supply among families with children takes adequate account of the potential support for costs incurred in purchasing childcare.

One can get some feel for the generosity of the childcare credit component of the

²²see Blundell, Duncan, McCrae, and Meghir (2000).

WFTC over different patterns of childcare use and labour market choices, and the extent to which this generosity might encourage an expansion both in hours of work and childcare demand. Tables 1 and 2 show the average net incomes among a sample of single parent households drawn from the FRS for different combinations of work hours and childcare hours choices. Because we don't observe childcare expenditures for the full sample of single parents, we base our simulations on a fixed hourly price for childcare services. The resulting pattern of incomes therefore represents the opportunity set of hours and childcare choices open to a single parent, in the sense that it gives some indication of the net incomes that she might enjoy were she to choose a particular combination of work and childcare.

The first panel of Table 1 simply reports an illustrative series of childcare expenditures, at various levels from 0 to 40 hours of childcare per week, given a fixed hourly price of £3.38 per hour of childcare. The second panel of Table 1 shows the average net income pattern at different combinations of work hours and childcare hours for the sample of single parents under the Family Credit regime, including the childcare expenditure disregard. The third panel shows the value of the childcare disregard component of Family Credit in monetary units (£ per week), and the last panel expresses this figure as a proportion of the childcare expenditure outlay. Given that single women with children can only qualify for Family Credit if they work at or beyond 16 hours, there is no financial gain from the childcare disregard when the mother works either 0 and 10 hours per week. A single women who purchases as many hours of childcare as hours of work, the absolute values of the childcare disregard component of Family Credit are relatively small. On average, taking account of heterogeneity of wages and demographics, a single parent working at 40 hours and purchasing 40 hours of childcare would see only a small increase of £17.96 per week in the value of Family Credit, compared with a similar women who purchases no care. In proportionate terms, this increase would cover only 13 per cent of the cost of childcare, assuming an hourly price of £3.38.

In comparison, the value of the childcare credit component of the WFTC is very large. On average, the value of the childcare credit among single parents on 40 hours of work, and consuming 40 hours of childcare at £3.38 per hour, more than doubles to £41.05 (see Tables 3 and 4). This represents 30 per cent of total childcare expenditures at the assumed price level. For part-time women, the average childcare credit covers more than half of the total cost of childcare.

There have been a number of empirical studies both in the UK and the US on the relationships between employment and childcare demand²³. In the UK, the Blundell

²³See, for example, Duncan, Giles, and Webb (1995), Blundell, Duncan, McCrae, and Meghir

Table 5: Lone parents' net income under Family Credit - low cost childcare

I. Net household income under FAMILY CREDIT (£ per week)					
	<i>Hours of childcare</i>				
	0	10	20	30	40
<i>childcare expenditure</i>	0	9.61	19.21	28.82	38.42
<i>hours of work:</i>					
0	144.99	144.99	144.99	144.99	144.99
10	155.19	155.19	155.19	155.19	155.19
20	189.23	190.87	191.86	192.47	192.84
30	209.95	214.09	216.96	218.60	219.64
40	223.56	228.14	232.54	235.94	238.15
II. Value of childcare credit component of FAMILY CREDIT					
	0	10	20	30	40
<i>a). in £ per week</i>					
0	-	0.00	0.00	0.00	0.00
10	-	0.00	0.00	0.00	0.00
20	-	1.64	2.63	3.24	3.61
30	-	4.13	7.00	8.65	9.68
40	-	4.58	8.98	12.38	14.60
<i>b). as a percentage of childcare expenditure</i>					
0		0%	0%	0%	0%
10		0%	0%	0%	0%
20		17%	14%	11%	9%
30		43%	36%	30%	25%
40		48%	47%	43%	38%

Table 6: Lone parents' net income under Family Credit - high cost childcare

I. Net household income under FAMILY CREDIT (£ per week)					
	<i>Hours of childcare</i>				
	0	10	20	30	40
<i>childcare expenditure</i>	0	33.75	67.50	101.25	135.00
<i>hours of work:</i>					
0	144.99	144.99	144.99	144.99	144.99
10	155.19	155.19	155.19	155.19	155.19
20	189.23	192.67	193.34	193.38	193.38
30	209.95	219.20	220.94	221.17	221.17
40	223.56	237.21	241.03	241.52	241.52
II. Value of childcare credit component of FAMILY CREDIT					
	0	10	20	30	40
<i>a). in £ per week</i>					
0	-	0.00	0.00	0.00	0.00
10	-	0.00	0.00	0.00	0.00
20	-	3.44	4.11	4.15	4.15
30	-	9.25	10.98	11.22	11.22
40	-	13.65	17.48	17.96	17.96
<i>b).as a percentage of childcare expenditure</i>					
0		0%	0%	0%	0%
10		0%	0%	0%	0%
20		10%	6%	4%	3%
30		27%	16%	11%	8%
40		40%	26%	18%	13%

Table 7: Lone parents' net income under WFTC - low cost childcare

I. Net household income under WFTC (£ per week)					
	<i>Hours of childcare</i>				
	0	10	20	30	40
<i>childcare expenditure</i>	0	9.61	19.21	28.82	38.42
<i>hours of work:</i>					
0	144.99	144.99	144.99	144.99	144.99
10	155.19	155.19	155.19	155.19	155.19
20	203.06	208.33	214.03	220.00	225.80
30	229.73	235.53	241.52	247.64	253.54
40	245.19	251.05	257.07	263.22	269.15
II. Value of childcare credit component of WFTC					
	0	10	20	30	40
<i>a). in £ per week</i>					
0	-	0.00	0.00	0.00	0.00
10	-	0.00	0.00	0.00	0.00
20	-	5.27	10.98	16.94	22.74
30	-	5.80	11.79	17.90	23.81
40	-	5.86	11.88	18.03	23.96
<i>b). as a percentage of childcare expenditure</i>					
0		0%	0%	0%	0%
10		0%	0%	0%	0%
20		55%	57%	59%	59%
30		60%	61%	62%	62%
40		61%	62%	63%	62%

Table 8: Lone parents' net income under WFTC - high cost childcare

I. Net household income under WFTC (£ per week)					
	<i>Hours of childcare</i>				
	0	10	20	30	40
<i>childcare expenditure</i>	0	33.75	67.50	101.25	135.00
<i>hours of work:</i>					
0	144.99	144.99	144.99	144.99	144.99
10	155.19	155.19	155.19	155.19	155.19
20	203.06	223.05	238.19	242.71	242.71
30	229.73	250.75	266.16	270.67	270.67
40	245.19	266.35	281.73	286.24	286.24
II. Value of childcare credit component of WFTC					
	0	10	20	30	40
<i>a). in £ per week</i>					
0	-	0.00	0.00	0.00	0.00
10	-	0.00	0.00	0.00	0.00
20	-	19.99	35.14	39.65	39.65
30	-	21.02	36.42	40.93	40.93
40	-	21.16	36.54	41.05	41.05
<i>b). as a percentage of childcare expenditure</i>					
0		0%	0%	0%	0%
10		0%	0%	0%	0%
20		59%	52%	39%	29%
30		62%	54%	40%	30%
40		63%	54%	41%	30%

et al (2000) study looks at the likely behavioural effects following the introduction of the WFTC. Their study controls for observed patterns of childcare use among single parent and two-adult households, and can therefore simulate the full impact of the WFTC including the childcare credit. It does not seek explicitly to model independently the demand for childcare, but nevertheless represents an advance on previous labour supply simulation methods. The work of Duncan, Giles, and Webb (1995) seeks to model employment and childcare choices as simultaneous decisions, in which the propensity to use childcare directly influences the probability of employment, and *vice versa*. Nevertheless, there has been relatively little attention paid in this literature to the range of practical and theoretical difficulties highlighted in our earlier discussions. The two main issues of concern to us are, first, the importance of controlling for quality differences in observed childcare costs, and second, the constraints on the level of childcare use among working households. We seek to address some of these modelling issues in this paper. First, we set out an economic framework with which to highlight the specific issues in the modeling of labour supply and childcare choices. We look at the specific issue of quantity constraints which prevent households from choosing certain combinations of work and childcare, and how those quantity constraints can affect the relationship between childcare price, childcare hours and labour supply. We develop a range of empirical specifications with which to quantify the effects of childcare price on labour supply and childcare demand. We introduce appropriate controls for quality differences when modelling the relationship between childcare price, childcare use and employment. Moreover, our specification can be extended to simulate the potential impact of the childcare credit element of the Working Families' Tax Credit on employment and use of childcare.

5.4 simulating the WFTC reform

The empirical models of employment and childcare presented in the previous section have been estimated using a pooled sample of FRS data from 1994/5 to 1998/9, a period during which Family Credit comprised the main system of financial support to low-wage working families with children. Although Family Credit included some compensation for childcare costs through an earnings disregard, the level of compensation was generally quite small (as evidenced in Tables 1 and 2). Following the introduction in 1999 of the Working Families' Tax Credit, the levels of generosity today are quite different. The basic credit under WFTC is significantly more generous than Family Credit. Moreover, the childcare credit component of WFTC offers a much greater

(2000), Ribar (1995), Blau and Hagy (1998); Michalopoulos, Robins, and Garfinkel (1992), and also the useful survey piece by Anderson and Levine (1999).

level of compensation for childcare costs than have hitherto been the case.²⁴ The degree to which these reforms might improve employment incentives among families with children is a question of some policy interest.

Table 9: Simulated transitions from WFTC among single parent families

	<i>under WFTC</i>					
	NP	PT(min)	PT(extra)	FT(min)	FT(extra)	proportion under FC
<i>under Family Credit</i>						
non-participation (NP)	55.0	2.4	0.7	0	0	58.1
part-time, minimum childcare - PT(min)	0	16.5	0.3	2.8	0	19.6
part-time, extra childcare - PT(extra)	0	0	7.0	0	0.3	7.3
full-time, minimum childcare - FT(min)	0	0.6	0.1	11.2	0.3	12.2
full-time, extra childcare - FT(extra)	0	0	0.2	0	2.6	2.8
proportion under WFTC	55.0	19.5	8.4	14.0	3.2	100
change relative to FC	-3.1	-0.1	+1.1	+1.8	+0.4	

Table 10: Simulated transitions from WFTC among married womens

	<i>under WFTC</i>					
	NP	PT(min)	PT(extra)	FT(min)	FT(extra)	proportion under FC
<i>under Family Credit</i>						
non-participation (NP)	29.3	0.6	0	0	0	29.9
part-time, minimum childcare - PT(min)	0.4	35.5	0.1	0.5	0	36.5
part-time, extra childcare - PT(extra)	0	0.2	5.0	0	0	5.2
full-time, minimum childcare - FT(min)	0	0.3	0	25.0	0	25.3
full-time, extra childcare - FT(extra)	0	0	0	0.1	2.8	2.9
proportion under WFTC	29.7	36.6	5.2	25.6	2.8	100
change relative to FC	-0.2	+0.1	.	+0.3	-0.1	

We have been careful in this paper to formulate economic models which can give some indication of the likely impact of WFTC on employment and childcare choices. The most general series of models (Tables ?? and ??, model 3) include terms which capture the effects of taxes and transfers on net income gains and average hourly childcare subsidies. These in turn have been found to influence employment and childcare choices. This opens up the possibility of simulating the effects on employment incentive of replacing the incumbent Family Credit system with the recent WFTC regime. To do so, we generate a new series of income gains under WFTC using the IFS tax-benefit model TAXBEN2. Using the estimated models reported in the previous section, we predict how employment and childcare choices might change under this new tax credit regime. Tables 5 and 6 report, for single parents and

²⁴Tables 3 and 4 indicate that up to 60 per cent of the childcare costs for working single parents may be covered through the childcare credit element of the WFTC alone.

married women respectively, the probabilities of non-participation (NP), part-time employment with minimum childcare (PTmin) and more than minimum childcare (PTextra), and full-time work with minimum (FTmin) and more than minimum (FTextra) childcare under the pre-existing Family Credit system, and under the new WFTC with childcare credits. For single parents, we simulate an increase of 3.1 per cent in employment following the introduction of WFTC, mainly into part-time employment with the minimum level of childcare (2.4 per cent). There are some offsetting employment responses among women who move from full-time to part-time employment to take advantage of the increased generosity of tax credits at lower hours levels. Nevertheless, the net positive incentive effect in our study exceeds by some margin earlier simulations of the incentive impact of WFTC on single parents' employment rates. For married women, the results are more ambiguous. We predict around 0.6 per cent of married women to respond to the added generosity of WFTC by moving into employment. Conversely, around 0.4 per cent of women are predicted to move out of paid employment, principally due to their partners' increased WFTC entitlement. Overall, the net employment effect for married women is negligible.

6 Conclusions

In this paper we consider how best to formulate economic models of employment and childcare among families with children, taking account of a range of factors which could potentially influence those decisions. Among the factors we deal with explicitly in theoretical discussion and empirical application, the following feature most prominently:

- i. We account for non-observability of childcare choices among non-working families in the FRS through a sequential structuring of our modelling of employment and childcare choices.
- ii. We confront directly the issue of quantity constraints in employment and childcare choices which require that, for many households, the number of hours of childcare used must at least equal the number of hours of paid employment. Our solution is innovative in the literature in this area, and provides evidence that more common modelling methods in earlier work were unable to isolate authentic childcare price effects.
- iii. Most applied work in this area fails to control appropriately for quality differences in the observed price of childcare among working families with children.

The consequences of such failure are severe; childcare price effects on employment and childcare choices will typically be biased, and can in some case be perverse (in the sense that demand for childcare may rise with apparent childcare price if quality is not accounted for in estimation). We avoid these difficulties by including predictions of standard-quality childcare prices using the methods described in our earlier paper (Duncan, Paull and Taylor, 2001a). Our results suggest that price remains an important factor in the decision to use childcare, and, by implication, the decision of a mother to seek employment.

- iv. Taxes and transfers (particularly childcare cost subsidies) are included in our empirical models in a manner which makes it possible to simulate employment responses to a change in government policy. A prime example of this is the introduction of the new Working Families' Tax Credit, including the childcare tax credit. Using predictions of childcare costs among pre-school and school-aged children, controlling for local differences in quality, we simulate net incomes among all families in our FRS sample at different combinations of employment and childcare hours. These incomes (which control fully both for potential childcare costs and government compensation for those costs) are included among the range of factors which explain employment and childcare choices. We go on to simulate some significant positive employment responses to the introduction of potentially generous childcare credits through the Working Families' Tax Credit.

The empirical results in this study suggest a strong and significant childcare price effect on the demand for childcare, and, through that, on the employment choices of mothers with children. When quantity constraints are correctly accounted for in estimation, we find price effects to fall, suggesting that some of the apparently high price elasticities in previous models of employment and childcare were in fact due to quantity constraints which require that childcare hours must at least equal hours of work for many working mothers. We also find the level of government compensation for childcare costs has a significantly positive impact on childcare use, which again suggests that the subsidisation of childcare costs through programmes such as the WFTC will have an independent impact on the take-up of childcare.

We use these empirical models to examine the effects on employment and childcare choices of the recent move to WFTC. When childcare choices and quantity constraints are fully accounted for in the modelling approach, we predict that up to 3.1 per cent of lone mothers will move to (mainly part-time) employment with minimum childcare following the introduction of the WFTC. Among married women, the results are more ambiguous. Some women increase their labour supply (moving from non-employment

to part-time work, or from part-time to full-time employment) to take advantage of the increases subsidisation of childcare that the WFTC provides. Others move out of employment, taking advantage of their partners' increased entitlement to the basic WFTC. Nevertheless, we find a net positive effect on employment rates among married women of around 0.6 per cent. We also find a marginal increase in the use of childcare beyond that which is necessary simply to cover for mothers' hours of employment, suggesting that childcare subsidies can have an independent effect on the demand for childcare. Both results have important implications in the context of the current policy debate on the effects of subsidisation of childcare costs on incentives and family welfare.

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Appendix A

Table 11: Five-state work and childcare model - lone parents, constrained

Variable	Employment propensity		Childcare propensity			
	coeff.	std.error	coeff.	std.error		
Number of children aged:						
<1	-0.723	0.093	**	0.460	0.298	
1	-0.409	0.081	**	0.930	0.263	**
2	-0.395	0.073	**	0.663	0.269	**
3	-0.411	0.071	**	0.494	0.276	*
4	-0.407	0.073	**	-0.366	0.262	
5-11	-0.103	0.034	**	-2.970	6.202	
(age - 35)/10	-0.142	0.035	**	-0.181	0.161	
(age - 35) ² /100	-0.233	0.032	**	-0.340	0.136	**
left education aged 17-18	0.306	0.054	**	0.581	0.151	**
left education aged 19-21	0.302	0.078	**	1.135	0.231	**
left education aged > 21	0.250	0.102	**	1.503	0.293	**
non-white	-0.149	0.062	**			
log(other income)	-0.023	0.010	**	excluded		
mother's hourly wage	0.024	0.014	*	excluded		
income gain, 0 to 20 hours	-0.120	0.009	**	excluded		
income gain, 20 to 30 hours	0.147	0.020	**	excluded		
income gain, 30 to 40 hours	0.119	0.018	**	excluded		
mother works at home	excluded			-0.099	0.429	
number of families in household	excluded			0.167	0.124	
number of older children	excluded			-1.480	0.395	**
years at current address	excluded			-0.002	0.023	
receives maintenance payments	excluded			-0.178	0.161	
hourly childcare price	excluded			-0.194	0.079	**
hourly childcare subsidy (20 hours)	excluded			2.704	0.913	*
Constant	-	-		-0.944	0.348	**
first threshold (γ_1)	0.027	0.066	**			
second threshold (γ_2)	0.882	0.069	**			
correlation	0.227	0.236				
log likelihood		-3,342.1				
number of observations		3810				
R_1 : no wage and price effects (χ^2_2 , p -value)	16.3	0.0003				
R_2 : no welfare or subsidy effects (χ^2_4 , p -value)	489.6	0.0000				
$R_1 + R_2$: (χ^2_6 , p -value)	437.3	0.0000				

Notes: *Income Gain* represents the average hourly gain from increasing labour supply. *Childcare*

Appendix B

B1: A sequential probit with no quantity constraints

A three-state model of employment and childcare without quantity constraints may be formulated using the same relationships for h_w^* and h_c^* and the same stochastic assumptions for ε_w and ε_c which underpin the equivalent quantity-constrained model.

Given observed hours of work h_w and childcare h_c , our observability criteria for the three-state model with quantity constraints on childcare are the following:

$$\begin{aligned} h_w = 0 & && \text{if } h_w^* \leq 0; \\ h_w > 0, h_c = 0 & && \text{if } h_w^* > 0 \text{ and } h_c^* \leq 0; \\ h_w > 0, h_c > 0 & && \text{if } h_w^* > 0 \text{ and } h_c^* > 0. \end{aligned}$$

We may use these observability criteria, together with the assumption of joint normality of $\{\varepsilon_w, \varepsilon_c\}$, to derive the following probabilities:

$$\begin{aligned} \Pr(h_w = 0 | x) &= \Pr(h_w^* \leq 0 | x) \\ &= \Pr(\varepsilon_w \leq -x\beta_w); \end{aligned} \tag{23}$$

$$\begin{aligned} \Pr(h_w > 0, h_c = 0 | x, z) &= \Pr(h_w^* > 0, h_c^* \leq 0 | x, z) \\ &= \Pr(\varepsilon_w > -x\beta_w, \varepsilon_c < -z\beta_c); \end{aligned} \tag{24}$$

$$\begin{aligned} \Pr(h_w > 0, h_c > 0 | x, z) &= \Pr(h_w^* > 0, h_c^* > 0 | x, z) \\ &= \Pr(\varepsilon_w > -x\beta_w, \varepsilon_c > -z\beta_c) \end{aligned} \tag{25}$$

The first probability $P^{nw} = 1 - \Phi(x_i\beta_w)$ is identical to the three-state model with quantity constraints. The probability P^{wc} of working and using childcare is

$$\begin{aligned} P^{wc} &= \Pr(\varepsilon_w > -x\beta_w, \varepsilon_c > -z\beta_c) \\ &= \Phi_2 [x\beta_w, z\beta_c; \rho]. \end{aligned} \tag{26}$$

Table 12: Five-state work and childcare model - married women, constrained

Variable	Employment propensity		Childcare propensity			
	coeff.	std.error	coeff.	std.error		
Number of children aged:						
<1	-0.633	<i>0.037</i>	**	0.348	<i>0.099</i>	
1	-0.387	<i>0.034</i>	**	0.337	<i>0.084</i>	**
2	-0.363	<i>0.033</i>	**	0.292	<i>0.083</i>	**
3	-0.301	<i>0.033</i>	**	0.311	<i>0.084</i>	*
4	-0.310	<i>0.032</i>	**	-0.464	<i>0.081</i>	
5-11	-0.185	<i>0.016</i>	**	-2.717	<i>0.358</i>	
(age - 35)/10	-0.213	<i>0.020</i>	**	-0.182	<i>0.071</i>	**
(age - 35) ² /100	-0.264	<i>0.019</i>	**	-0.661	<i>0.070</i>	**
left education aged 17-18	0.201	<i>0.026</i>	**	0.136	<i>0.056</i>	**
left education aged 19-21	0.263	<i>0.036</i>	**	0.296	<i>0.073</i>	**
left education aged > 21	0.175	<i>0.041</i>	**	0.467	<i>0.082</i>	**
non-white	-0.185	<i>0.036</i>	**	-	-	
partner at home	-0.274	<i>0.056</i>		-0.464	<i>0.103</i>	**
log(other income)	-0.033	<i>0.005</i>	**	excluded		
partner's earnings	-0.016	<i>0.005</i>	**	excluded		
mother's hourly wage	0.119	<i>0.014</i>	**	excluded		
income gain, 0 to 20 hours	-0.012	<i>0.009</i>		excluded		
income gain, 20 to 30 hours	0.092	<i>0.025</i>	**	excluded		
income gain, 30 to 40 hours	0.181	<i>0.023</i>	**	excluded		
mother works at home	excluded			-1.046	<i>0.191</i>	**
number of families in household	excluded			-0.252	<i>0.098</i>	**
number of older children	excluded			-2.465	<i>0.567</i>	**
years at current address	excluded			-0.016	<i>0.008</i>	*
receives maintenance payments	excluded			-0.244	<i>0.398</i>	
hourly childcare price	excluded			-0.342	<i>0.027</i>	**
hourly childcare subsidy (20 hours)	excluded			1.251	<i>0.126</i>	**
Constant	-	-		0.368	<i>0.138</i>	**
first threshold (γ_1)	-0.605	<i>0.043</i>	**			
second threshold (γ_2)	0.575	<i>0.043</i>	**			
correlation	0.490	<i>0.236</i>				
log likelihood	-13,426.3					
number of observations	12,011					
R_1 : no wage and price effects ($\chi^2_2, p\text{-value}$)	49.7	<i>0.0000</i>				
R_2 : no welfare or subsidy effects ($\chi^2_4, p\text{-value}$)	4370.1	<i>0.0000</i>				

For the remaining probability P^{wnc} of working but using no childcare, we simply subtract P^{wc} from the (marginal) probability of working;

$$\begin{aligned} P^{wmc} &= \Pr(\varepsilon_w > -x\beta_w, \varepsilon_c < -z\beta_c) \\ &= \Phi(x\beta_w) - \Phi_2[x\beta_w, z\beta_c; \rho]. \end{aligned} \quad (27)$$

To formulate the likelihood for this model, define indicators I_i^{nw} , I_i^{wnc} and I_i^{wc} for each household to represent the three states of non-work (nw), work with no childcare (wnc), and work with positive childcare (wc) as follows:

$$\begin{aligned} I_i^{nw} &= \mathbf{1}(h_{iw} = 0) ; \\ I_i^{wmc} &= \mathbf{1}(h_{iw} > 0, h_{ic} = 0); \\ I_i^{wc} &= \mathbf{1}(h_{iw} > 0, h_{ic} > 0), \end{aligned}$$

where $\mathbf{1}(\cdot)$ represents the indicator function. The log likelihood for the three-state model with no quantity constraints may be written as

$$\begin{aligned} \ln L(\beta_w, \beta_c, \rho) &= \sum_{i=1}^n [I_i^{nw} \ln(P_i^{nw}) + I_i^{wnc} \ln(P_i^{wnc}) + I_i^{wc} \ln(P_i^{wc})] \\ &= \sum_{i=1}^n I_i^{nw} \ln(1 - \Phi(x_i\beta_w)) \\ &\quad + \sum_{i=1}^n I_i^{wnc} \ln(\Phi(x\beta_w) - \Phi_2[x\beta_w, z\beta_c; \rho]) \\ &\quad + \sum_{i=1}^n I_i^{wc} \ln \Phi_2[x\beta_w, z\beta_c; \rho] \end{aligned} \quad (28)$$

B2: a sequential ordered probit with no quantity constraints

Given observed hours of work h_w and childcare h_c , our observability criteria for the five-state model with no quantity constraints on childcare are the following:

$$\begin{aligned}
h_w &= 0 && \text{if } h_w^* \leq \gamma_1; \\
h_w \in PT, h_c &= 0 && \text{if } h_w^* > \gamma_1 \text{ and } h_w^* \leq \gamma_2 \text{ and } h_c^* \leq 0; \\
h_w \in PT, h_c &> 0 && \text{if } h_w^* > \gamma_1 \text{ and } h_w^* \leq \gamma_2 \text{ and } h_c^* > 0; \\
h_w \in FT, h_c &= 0 && \text{if } h_w^* > \gamma_2 \text{ and } h_c^* \leq 0. \\
h_w \in FT, h_c &> 0 && \text{if } h_w^* > \gamma_2 \text{ and } h_c^* > 0
\end{aligned}$$

We may use these observability criteria, together with the assumption of joint normality of $\{\varepsilon_w, \varepsilon_c\}$, to derive the following probabilities:

$$\begin{aligned}
\Pr(h_w = 0 | x) &= \Pr(h_w^* \leq \gamma_1 | x) \\
&= \Pr(\varepsilon_w \leq \gamma_1 - x\beta_w); \\
&= \Phi(\gamma_1 - x\beta_w)
\end{aligned}$$

$$\begin{aligned}
\Pr(h_w \in PT, h_c = 0 | x, z) &= \Pr(\gamma_1 < h_w^* \leq \gamma_2, h_c^* \leq 0 | x, z) \\
&= \Pr(\gamma_1 - x\beta_w < \varepsilon_w \leq \gamma_2 - x\beta_w, \varepsilon_c \leq -z\beta_c) \\
&= \Pr(\varepsilon_w \leq \gamma_2 - x\beta_w, \varepsilon_c \leq -z\beta_c) - \Pr(\varepsilon_w \leq \gamma_1 - x\beta_w, \varepsilon_c \leq -z\beta_c) \\
&= \Phi_2[\gamma_2 - x\beta_w, -z\beta_c; \rho] - \Phi_2[\gamma_1 - x\beta_w, -z\beta_c; \rho] \tag{29}
\end{aligned}$$

$$\begin{aligned}
\Pr(h_w \in PT, h_c > 0 | x, z) &= \Pr(h_w \in PT | x) - \Pr(h_w \in PT, h_c = 0 | x, z) \\
&= \Pr(\gamma_1 - x\beta_w < \varepsilon_w \leq \gamma_2 - x\beta_w) - \Pr(h_w \in PT, h_c = 0 | x, z) \\
&= (\Phi[\gamma_2 - x\beta_w] - \Phi[\gamma_1 - x\beta_w]) - \Pr(h_w \in PT, h_c = 0 | x, z) \\
&= (\Phi[\gamma_2 - x\beta_w] - \Phi[\gamma_1 - x\beta_w]) \\
&\quad - (\Phi_2[\gamma_2 - x\beta_w, -z\beta_c; \rho] - \Phi_2[\gamma_1 - x\beta_w, -z\beta_c; \rho]) \tag{30}
\end{aligned}$$

$$\begin{aligned}
\Pr(h_w \in FT, h_c = 0 | x, z) &= \Pr(h_w \in FT | x) - \Pr(h_w \in FT, h_c > 0 | x, z) \\
&= \Pr(\varepsilon_w > \gamma_2 - x\beta_w) - \Pr(\varepsilon_w > \gamma_2 - x\beta_w, \varepsilon_c > -z\beta_c) \\
&= (1 - \Phi[\gamma_2 - x\beta_w]) - \Phi_2[-(\gamma_2 - x\beta_w), z\beta_c; \rho] \tag{31}
\end{aligned}$$

$$\begin{aligned}
\Pr(h_w \in FT, h_c > 0 | x, z) &= \Pr(\varepsilon_w > \gamma_2 - x\beta_w, \varepsilon_c > -z\beta_c) \\
&= \Pr(\varepsilon_w > \gamma_2 - x\beta_w, \varepsilon_c > -z\beta_c) \\
&= \Phi_2[-(\gamma_2 - x\beta_w), z\beta_c; \rho]
\end{aligned} \tag{32}$$

B3: a five-state model with quantity constraints

The same basic propensity equations h_w^* and h_c^* are used for the five-state model of employment and childcare. Given observed hours of work h_w , we distinguish between part-time workers ($h_w \in PT$, for hours of work greater than zero but less than 30) and full-time workers ($h_w \in FT$ when $h_w \geq 30$). The observability criteria for the five-state model with quantity constraints on childcare are the following:

$$\begin{aligned}
h_w = 0 & \quad \text{if } h_w^* \leq \gamma_1; \\
h_w \in PT, h_c = h_w & \quad \text{if } h_w^* > \gamma_1 \text{ and } h_w^* \leq \gamma_2 \text{ and } h_c^* \leq h_w^*; \\
h_w \in PT, h_c > h_w & \quad \text{if } h_w^* > \gamma_1 \text{ and } h_w^* \leq \gamma_2 \text{ and } h_c^* > h_w^*; \\
h_w \in FT, h_c = h_w & \quad \text{if } h_w^* > \gamma_2 \text{ and } h_c^* \leq h_w^*. \\
h_w \in FT, h_c > h_w & \quad \text{if } h_w^* > \gamma_2 \text{ and } h_c^* > h_w^*
\end{aligned} \tag{33}$$

Using these observability criteria, together with the assumption of joint normality of $\{\varepsilon_w, \varepsilon_c\}$, we may derive the following probabilities:

$$\begin{aligned}
\Pr(h_w = 0 | x) &= \Pr(h_w^* \leq \gamma_1 | x) \\
&= \Pr(\varepsilon_w \leq \gamma_1 - x\beta_w); \\
&= \Phi(\gamma_1 - x\beta_w)
\end{aligned} \tag{34}$$

$$\begin{aligned}
\Pr(h_w \in PT, h_c = h_w | x, z) &= \Pr(\gamma_1 < h_w^* \leq \gamma_2, h_c^* \leq h_w^* | x, z) \\
&= \Pr(\gamma_1 - x\beta_w < \varepsilon_w \leq \gamma_2 - x\beta_w, z\beta_c + \varepsilon_c \leq x\beta_w + \varepsilon_w) \\
&= \Pr(\gamma_1 - x\beta_w < \varepsilon_w \leq \gamma_2 - x\beta_w, \varepsilon_c - \varepsilon_w \leq x\beta_w - z\beta_c) \\
&= \Pr(\gamma_1 - x\beta_w < \varepsilon_w \leq \gamma_2 - x\beta_w, \frac{v}{\sqrt{2(1-\rho)}} \leq \frac{x\beta_w - z\beta_c}{\sqrt{2(1-\rho)}}) \\
&= \Pr(\varepsilon_w \leq \gamma_2 - x\beta_w, \frac{v}{\sqrt{2(1-\rho)}} \leq \frac{x\beta_w - z\beta_c}{\sqrt{2(1-\rho)}}) - \\
&\quad \Pr(\varepsilon_w \leq \gamma_1 - x\beta_w, \frac{v}{\sqrt{2(1-\rho)}} \leq \frac{x\beta_w - z\beta_c}{\sqrt{2(1-\rho)}}) \\
&= \Phi_2 \left[\gamma_2 - x\beta_w, \frac{x\beta_w - z\beta_c}{\sqrt{2(1-\rho)}}; \frac{\rho-1}{\sqrt{2(1-\rho)}} \right]
\end{aligned}$$

$$-\Phi_2 \left[\gamma_1 - x\beta_w, \frac{x\beta_w - z\beta_c}{\sqrt{2(1-\rho)}}; \frac{\rho-1}{\sqrt{2(1-\rho)}} \right] \quad (35)$$

$$\begin{aligned} \Pr(h_w \in PT, h_c > h_w | x, z) &= \Pr(h_w \in PT | x) - \Pr(h_w \in PT, h_c = h_w | x, z) \\ &= \Pr(\gamma_1 - x\beta_w < \varepsilon_w \leq \gamma_2 - x\beta_w) - \Pr(h_w \in PT, h_c = h_w | x, z) \\ &= (\Phi[\gamma_2 - x\beta_w] - \Phi[\gamma_1 - x\beta_w]) - \Pr(h_w \in PT, h_c = h_w | x, z) \\ &= \Phi[\gamma_2 - x\beta_w] - \Phi[\gamma_1 - x\beta_w] \\ &\quad - \Phi_2 \left[\gamma_2 - x\beta_w, \frac{x\beta_w - z\beta_c}{\sqrt{2(1-\rho)}}; \frac{\rho-1}{\sqrt{2(1-\rho)}} \right] \\ &\quad + \Phi_2 \left[\gamma_1 - x\beta_w, \frac{x\beta_w - z\beta_c}{\sqrt{2(1-\rho)}}; \frac{\rho-1}{\sqrt{2(1-\rho)}} \right] \end{aligned} \quad (36)$$

$$\begin{aligned} \Pr(h_w \in FT, h_c = h_w | x, z) &= \Pr(h_w \in FT | x) - \Pr(h_w \in FT, h_c > h_w | x, z) \\ &= \Pr(\varepsilon_w > \gamma_2 - x\beta_w) - \Pr(\varepsilon_w > \gamma_2 - x\beta_w, z\beta_c + \varepsilon_c > x\beta_w + \varepsilon_w) \\ &= \Pr(\varepsilon_w > \gamma_2 - x\beta_w) - \Pr(\varepsilon_w > \gamma_2 - x\beta_w, \varepsilon_c - \varepsilon_w > x\beta_w - z\beta_c) \\ &= \Pr(\varepsilon_w > \gamma_2 - x\beta_w) - \Pr(\varepsilon_w > \gamma_2 - x\beta_w, \frac{v}{\sqrt{2(1-\rho)}} > \frac{x\beta_w - z\beta_c}{\sqrt{2(1-\rho)}}) \\ &= (1 - \Phi[\gamma_2 - x\beta_w]) - \Phi_2 \left[-(\gamma_2 - x\beta_w), \frac{z\beta_c - x\beta_w}{\sqrt{2(1-\rho)}}; \frac{\rho-1}{\sqrt{2(1-\rho)}} \right] \end{aligned} \quad (37)$$

$$\begin{aligned} \Pr(h_w \in FT, h_c > h_w | x, z) &= \Pr(\varepsilon_w > \gamma_2 - x\beta_w, \varepsilon_c - \varepsilon_w > x\beta_w - z\beta_c) \\ &= \Pr(\varepsilon_w > \gamma_2 - x\beta_w, \frac{v}{\sqrt{2(1-\rho)}} > \frac{x\beta_w - z\beta_c}{\sqrt{2(1-\rho)}}) \\ &= \Phi_2 \left[-(\gamma_2 - x\beta_w), \frac{z\beta_c - x\beta_w}{\sqrt{2(1-\rho)}}; \frac{\rho-1}{\sqrt{2(1-\rho)}} \right] \end{aligned} \quad (38)$$

For a sample of data $\{h_{iw}, h_{ic}, x_i, z_i, i = 1, \dots, n\}$, define indicators I_i^{nw} , I_i^{ptmc} , I_i^{ptc} , I_i^{ftmc} and I_i^{ftc} to represent the five states of non-work (*nw*), part-time work with minimum childcare (*ptmc*), part-time work with more than minimum childcare (*ptc*), full-time work with minimum childcare (*ftmc*), and full-time work with more than minimum childcare (*ftc*) as follows:

$$I_i^{nw} = \mathbf{1}(h_{iw} \leq 0); \quad (39)$$

$$I_i^{ptmc} = \mathbf{1}(h_{iw} \in PT, h_{ic} \leq h_{iw}); \quad (40)$$

$$I_i^{ptc} = \mathbf{1}(h_{iw} \in PT, h_{ic} > h_{iw}); \quad (41)$$

$$I_i^{ftmc} = \mathbf{1}(h_{iw} \in FT, h_{ic} \leq h_{iw}); \quad (42)$$

$$I_i^{ftc} = \mathbf{1}(h_{iw} \in FT, h_{ic} > h_{iw}); \quad (43)$$

where $\mathbf{1}(\cdot)$ represents the indicator function. The log likelihood for the quantity-constrained five-state model may then be written as

$$\ln L(\beta_w, \beta_c, \rho) = \sum_{i=1}^n \left[I_i^{nw} \ln(P_i^{nw}) + I_i^{ptmc} \ln(P_i^{ptmc}) + I_i^{ptc} \ln(P_i^{ptc}) + I_i^{ftmc} \ln(P_i^{ftmc}) + I_i^{ftc} \ln(P_i^{ftc}) \right] \quad (44)$$

where each of the probabilities are as derived in equations (34) to (38).