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THE ROLES OF EXPECTED PROFITABILITY, TOBIN’S Q AND CASH FLOW IN ECONOMETRIC MODELS OF COMPANY INVESTMENT

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Rain Newton-Smith
Murtaza Syed
Gertjan Vlieghe
The roles of expected profitability, Tobin’s Q and cash flow in econometric models of company investment

Stephen Bond∗  Alexander Klemm†  Rain Newton-Smith‡  Murtaza Syed§  Gertjan Vlieghe¶

June 29, 2004

Abstract

Evidence that cash flow has a significant effect on company investment spending, after controlling for Tobin’s average Q, has often been interpreted as suggesting the importance of financing constraints. Recent work on measurement error in the Q model casts doubt on this interpretation. It is possible that the Q model may not be identified if there are ‘bubbles’ in stock market valuations that are both persistent over time and that are correlated with fundamental values. Cash flow may then provide additional information about expected profitability that is not captured by a poorly measured Tobin’s average Q variable. We explore this hypothesis empirically using UK panel data on companies for which analysts’ earnings forecasts are available from the IBES database. The results point to a severe measurement error in average Q. The paper finds that, controlling for expected profitability using analysts’ earnings forecasts, cash flow becomes insignificant.

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Both sales growth and cash-stock variables do provide additional information, which could either be capturing expectations of profitability at longer horizons, or reflect misspecification of the basic $Q$ model. Results for subsamples do not suggest financing constraints as a likely explanation for these findings.

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Summary

Econometric models of company investment face the problem that current investment decisions depend on expectations of future conditions, but these expectations are generally not observed. This makes it difficult to know whether significant coefficients on financial variables, such as cash flow, in empirical investment equations indicate the importance of financing constraints, or whether these variables simply provide additional relevant information about current expectations of future profitability. In this paper we construct explicit measures of expectations of future profitability for UK firms to address this question.

The Q model of investment relates investment to the firm’s stock market valuation, which is meant to reflect the present discounted value of expected future profits. Under certain assumptions about the firm’s technology and competitive environment, the ratio of the stock market value of the firm to its replacement cost (Tobin’s Q) should be a sufficient statistic for investment. Significant coefficients on cash-flow variables after controlling for Tobin’s Q can then not be attributed to additional information about current expectations. However, if the above conditions are not satisfied, or if stock market valuations are influenced by ‘bubbles’ or any factors other than the present discounted value of expected future profits, then Tobin’s Q would not capture all relevant information about the expected future profitability of current investment. In this case additional explanatory variables, like current or lagged sales or cash-flow terms, could proxy for the missing information about expected future conditions.

This problem is particularly important in the literature that tests for an impact of financing constraints or capital market imperfections on corporate investment. Many empirical studies have added cash-flow variables to empir-
ical models that relate investment rates to Tobin’s Q, and interpreted significant coefficients on these cash-flow terms as evidence of ‘excess sensitivity’ of investment to the availability of internal funds. Although these findings are consistent with the presence of a cost premium for external sources of investment finance, they may also be explained, in the absence of financing constraints, by observed cash-flow or profits variables containing additional relevant information about expected future profitability not captured by Tobin’s Q.

Recent findings for US data suggest that much, if not all, of the significance of cash-flow variables in conventional estimates of Tobin’s Q investment equations can be attributed to the failure of Tobin’s Q to capture all relevant information about the expected profitability of current investment. Previous studies using UK company data have reported significant coefficients on cash-flow variables, both in the context of models that relate investment to Tobin’s Q, and in the context of reduced-form empirical models without explicitly forward-looking controls for expected profitability. The aim of the present study is to consider the robustness of these findings to alternative controls for expected future profitability. We obtain data on earnings forecasts from IBES International for around 700 publicly traded UK companies between 1987 and 2000. We match this information with stock market valuations and company accounts data on investment, cash flow and other financial variables obtained from Datastream International. Our main finding is that, whereas lagged cash flow is highly significant conditional on a standard measure of Tobin’s Q, the coefficient on this cash-flow variable becomes insignificantly different from zero when we include our direct measures of expected future profitability. This parallels the results found for US data by other researchers. We also examine subsamples of firms, and find that the results are robust
across subsamples of smaller firms and low-dividend firms.

Although cash-flow variables become insignificant when we control for expected profitability in this way, we find positive coefficients on both sales growth and cash-stock variables that remain statistically significant after conditioning on our measures of expected profits. These additional variables could be capturing expectations of profitability in the longer term that are not captured by our explicit measure of expectations. These longer-term expectations would be relevant for explaining investment rates under the maintained structure of the Q model. Alternatively, our findings could reflect misspecifications of the basic Q model, such as market power, decreasing returns to scale, or non-convex components of adjustment costs. In principle, the significance of these additional variables could also be due to the presence of financing constraints, although our results for subsamples do not suggest that this is a likely explanation. The coefficients on the additional sales growth and cash-stock terms appear to be broadly similar between subsamples of firms that have been considered elsewhere to be more or less likely to be subject to significant financing constraints. So the additional information these variables provide appears more likely to be explained by more general features of the investment behaviour of UK firms.
1 Introduction

Econometric models of company investment face the problem that current investment decisions depend on expectations of future conditions, and these expectations are generally not observed. This makes it difficult to know whether significant coefficients on financial variables such as cash flow in empirical investment equations indicate the importance of financing constraints, or whether these variables simply provide additional relevant information about current expectations of future profitability. In this paper we construct explicit measures of expectations of future profitability for UK firms to address this question.

The well-known $Q$ model of investment relates investment to the firm’s stock market valuation, which is meant to reflect the present discounted value of expected future profits.\footnote{See Brainard and Tobin (1968) and Hayashi (1982).} For the special case of perfectly competitive markets and constant returns to scale technology, Hayashi (1982) showed that average $Q$ - the ratio of the maximised value of the firm to the replacement cost of its existing capital stock - would be a sufficient statistic for investment rates. The usual empirical measure, which we call Tobin’s $Q$, further assumes that the maximised value of the firm can be measured by its stock market valuation. Under these assumptions, the stock market valuation would capture all relevant information about expected future profitability, and significant coefficients on cash-flow variables after controlling for Tobin’s $Q$ could not be attributed to additional information about current expectations. However if the Hayashi conditions are not satisfied, or if stock market valuations are influenced by ‘bubbles’ or any factors other than the present discounted value of expected future profits; then Tobin’s $Q$ would not capture all relevant information about the expected future profitability of current investment.
In this case additional explanatory variables like current or lagged sales or cash-flow terms could proxy for the missing information about expected future conditions. Cooper and Ejarque (2001) provide a recent illustration of this mechanism, using simulated data from a model in which firms have market power and average $Q$ is not a sufficient statistic for investment rates.

This problem is particularly important in the literature which tests for an impact of financing constraints or capital market imperfections on corporate investment. Following Fazzari, Hubbard and Petersen (1988), many empirical studies have added cash-flow variables to empirical models that relate investment rates to Tobin’s $Q$, and interpreted significant coefficients on these cash-flow terms as evidence of ‘excess sensitivity’ of investment to the availability of internal funds. While these findings are consistent with the presence of a cost premium for external sources of investment finance, they may also be explained in the absence of financing constraints by observed cash flow or profits variables containing additional relevant information about expected future profitability that is not captured by Tobin’s $Q$. Again following Fazzari, Hubbard and Petersen (1988), the literature has sought to address this concern by focusing on differential cash flow effects for subsamples of firms that are considered more or less likely to face a significant cost premium for external finance. However there are several problems with this ‘sample splitting’ approach, particularly when - as is commonly the case - the coefficients on additional financial variables are found to be significantly different from zero for all subsamples considered. Kaplan and Zingales (1997) have argued that firms facing a higher cost premium for external funds need

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2See Schiantarelli (1996), Hubbard (1998) and Bond and Van Reenen (2003) for surveys of this literature.

3The latter explanation for significant cash-flow effects is still more likely to be relevant in the context of reduced-form investment models, with no explicitly forward-looking controls for the influence of expected future profitability.
not display greater sensitivity of investment to fluctuations in cash flow.\textsuperscript{4} More straightforwardly, we cannot be confident that the additional information about expected future profitability not contained in Tobin’s $Q$ would be similar across subsamples of firms. For example, ‘bubbles’ in share prices may be more pervasive for the kinds of smaller firms, zero-dividend firms, or firms without commercial bond ratings where larger coefficients on cash-flow variables have often been reported.\textsuperscript{5}

Recent research using US company data has shown that significant coefficients on cash-flow variables may not be robust to alternative ways of dealing with measurement error in Tobin’s $Q$ or alternative controls for expected future profitability. Erickson and Whited (2000) develop a Generalised Method of Moments (GMM) estimator using higher order moment conditions that can correct for the presence of persistent ‘bubbles’, provided these ‘bubbles’ are themselves independent of the firm’s fundamental value or present discounted value of expected future profits. They find that the coefficient on an additional cash-flow variable becomes insignificant when they use this approach to correct for measurement error in Tobin’s $Q$. Bond and Cummins (2001) note that the $Q$ model of investment may not be identified using the usual measure of Tobin’s $Q$ if there are ‘bubbles’ in stock market valuations that are both persistent and themselves correlated with new information about the firm’s fundamental value. The basic idea is that this would introduce a measurement error component into the error term of the empirical investment equation which is likely to be correlated with past values of the firm’s fundamental value, and hence with past observations on all variables that

\textsuperscript{4}See also the discussion in Fazzari, Hubbard and Petersen (2000) and Kaplan and Zingales (2000).

\textsuperscript{5}This problem and other difficulties with the ‘sample splitting’ tests were noted by Alan Blinder and James Poterba in their Brookings Panel discussions of Fazzari, Hubbard and Petersen (1988).
influence this fundamental value. In this case there would be no valid instrumental variables available for the usual measure of Tobin’s $Q$ constructed using stock market valuations. Bond and Cummins (2001) consider using a direct estimate of the present discounted value of expected future profits, constructed using earnings forecasts for individual companies made by professional securities analysts. They too find that additional cash-flow variables become insignificant when this estimate is used in place of the firm’s stock market valuation to construct an alternative measure of the average $Q$ ratio.\footnote{See also Cummins, Hassett and Oliner (1999), who show that cash flow becomes insignificant in this case for all the sub-samples of firms that have commonly been used in the empirical literature on investment and financing constraints.}

These findings suggest that much if not all of the significance of cash-flow variables in conventional estimates of Tobin’s $Q$ investment equations can be attributed to the failure of Tobin’s $Q$ to capture all relevant information about the expected profitability of current investment. Previous studies using UK company data have reported significant coefficients on cash-flow variables, both in the context of models that relate investment to Tobin’s $Q$,\footnote{See, for example, Devereux and Schiantarelli (1990) and Blundell, Bond, Devereux and Schiantarelli (1992).} and in the context of reduced-form empirical models with no explicitly forward-looking controls for expected profitability.\footnote{See, for example, Bond, Harhoff and Van Reenen (1999) and Bond, Elston, Mairesse and Mulkey (2003). Nickell and Nicolitsas (1999) find a significant negative coefficient on an interest coverage measure of ‘financial pressure’, which is inversely related to cash flow.} The aim of the present study is to consider the robustness of these findings to alternative controls for expected future profitability. We follow the Bond and Cummins (2001) method and apply it to UK data. We obtain data on earnings forecasts from IBES International for a sample of around 700 publicly traded UK companies between 1987 and 2000. We match this information with stock market valuations and company accounts data on investment, cash flow and other financial...
variables obtained from Datastream International. Our main finding is that while lagged cash-flow is highly significant conditional on a standard measure of Tobin’s Q, the coefficient on this cash flow variable becomes insignificantly different from zero when we include our direct measures of expected future profitability. This parallels the results found for US data by Bond and Cummins (2001). We also examine subsamples of firms, and find that the results are robust across subsamples of smaller firms and low-dividend firms.

A potentially important difference between the earnings forecasts available for US companies and those available for UK companies is that IBES reports forecasts for ‘long-term’ earnings growth for almost all firms in their US sample, but for less than one third of the firms in their UK sample. Bond and Cummins (2001) use this growth rate to construct forecasts of earnings over a five-year horizon, and combine this with simple assumptions about discount rates and a terminal value correction to obtain estimates of the present discounted value of expected future profits. Syed (2003) shows that the long-term growth forecasts in this US data provide information which helps to explain the behaviour of corporate investment. As for the majority of UK firms in our sample we only have earnings forecasts for the current year and the following year, we do not attempt to construct an infinite horizon present discounted value measure from this information. Instead we simply use these short-term earnings forecasts as indicators of expected profitability in our empirical investment equations. Consequently we would not expect these measures of expected short-run profitability to provide sufficient statistics for company investment, and empirically we do not find this to be the case. Although cash-flow variables become insignificant when we control for expected profitability in this way, we find positive coefficients on both sales growth and cash stock variables that remain statistically significant after
conditioning on our measures of expected profits. These additional variables could either be capturing expectations of profitability in the longer term, that would be relevant for explaining investment rates under the maintained structure of the $Q$ model; or they could reflect misspecifications of the basic $Q$ model such as market power, decreasing returns to scale, or non-convex components of adjustment costs. In principle the significance of these additional variables could also be due to the presence of financing constraints, although our results for subsamples do not suggest that this is a likely explanation of our findings. We find that the coefficients on the additional sales growth and cash stock terms are broadly similar between subsamples of firms that have elsewhere been considered to be more or less likely to be subject to significant financing constraints, so that the additional information these variables provide appears more likely to be explained by some more general feature of the investment behaviour of UK firms.

The remainder of the paper is organised as follows. Section 2 outlines the basic $Q$ model and discusses the role of expected future profits in investment equations. Section 3 briefly discusses some previous empirical work on $Q$ and the financing constraint interpretation of cash-flow terms. Section 4 describes how measurement error may affect the estimation of investment equations involving $Q$. Section 5 describes the construction of our data set, Section 6 discusses the results and Section 7 concludes.

2 Expected profits and investment: the $Q$ model

The standard $Q$ model describes the investment behaviour of a competitive firm subject to constant returns to scale and strictly convex costs of adjusting its capital stock. From the first-order conditions for optimal investment,
we can write the firm’s investment rate in each period as a function of marginal $Q$, defined as the marginal value obtained from an additional unit of investment divided by the price of this unit of investment. The theoretical investment equation, which is common in the investment literature, is usually written as follows:\(^9\)

$$\frac{I_t}{K_t} = a + \frac{1}{b} Q_t + \epsilon_t$$

(1)

where $I_t$ is gross investment, $K_t$ is the net capital stock, $Q_t$ is $(q_t - 1)$ where $q_t$ is marginal $Q$ and $\epsilon_t$ is an additive shock to marginal adjustment costs. The parameters $a$ and $b$ are structural parameters of the adjustment cost function.

Hayashi (1982) showed that under certain restrictions on the profit function,\(^10\) marginal $Q$, which is unobserved, equals average $Q$, defined as follows:

$$q_t \equiv \frac{V_t}{p_t^I (1 - \delta) K_{t-1}}$$

(2)

where $V_t$ is the net present value of the firm’s expected future profits (possibly adjusted for debt and taxes, see Appendix B) and the denominator is the replacement cost at time $t$ of the capital stock inherited from the previous period. Here $p_t^I$ denotes the price of investment goods and $\delta$ is the rate of depreciation.

The influence of expected future profits on current investment behaviour reflects the forward-looking nature of the investment decision in the presence of adjustment costs. If the fundamental value $V_t$ can be measured using the firm’s stock market valuation, then under these particular assumptions there

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\(^9\)The derivation of the investment equation can be found in Appendix B. The linear functional form further requires an adjustment cost function that is quadratic.

\(^{10}\)The necessary condition is linear homogeneity of the profit function in $(K_t, I_t)$. Sufficient conditions for this to hold in the model presented here are perfect competition in output and input markets, and constant returns to scale in both production and adjustment cost technologies.
exists a single sufficient observable statistic for the firm’s investment rates. We refer to the average $Q$ ratio measured using the firm’s stock market valuation as Tobin’s average $Q$. More generally, this specification indicates that expectations of future profits should be an important explanatory variable for company investment.

3 Financing constraints and investment

There is a large body of empirical work concerned with estimating variations of equation 1 from firm-level or aggregate data. Schiantarelli (1996), Hubbard (1998) and Bond and Van Reenen (2003) review this empirical literature. But the findings have generally been disappointing. The coefficient on $Q$ is often found to be insignificant, or, if it is significant, implies implausibly slow adjustment. Moreover, although theoretically $Q$ should be a sufficient statistic for investment, other variables have commonly been found to have important additional explanatory power. These findings led to a re-evaluation of the assumptions underlying the $Q$ model. One candidate explanation for the failure of the model, although by no means the only one, is that firms face financing constraints. Other possibilities include the presence of fixed costs or irreversibilities in adjusting the capital stock, imperfect competition, decreasing returns to scale, measurement error and managerial behaviour that deviates significantly from profit maximisation.

The basic $Q$ model assumes that firms can finance as much investment as they choose at an exogenously given cost of finance. If instead there is

\footnote{Two interesting exceptions to this finding are Cummins et al (1994), who report higher coefficients on stock market $Q$ around the time of major tax reforms, and Gilchrist and Himmelberg (1995), who construct a measure of $Q$ that is not based on stock market valuations, and find that this measure performs better in an investment equation. Both of these findings are consistent with the hypothesis of measurement error in stock market $Q$.}
a cost premium for external funds from debt or new equity, compared to the required rate of return on internally generated funds, the basic $Q$ model is misspecified.\textsuperscript{12} For a given level of (marginal) $Q$, the level of investment additionally depends on the availability of internal funds. Depending on the particular type of external finance premium that is assumed, this misspecification may also lead to ‘excess sensitivity’ of investment to variables that influence the external finance premium. Candidate variables include indicators of the ‘financial health’ of the firm, such as cash flow (or internal funds), debt liabilities, and the stock of liquid assets. Fazzari, Hubbard and Petersen (1988) and many subsequent authors have found highly significant coefficients on cash-flow variables in investment equations in a number of datasets for different countries. Moreover, these cash-flow coefficients have generally been found to be larger for firms that have characteristics that make them more likely to be financially constrained, for example firms that lack bond ratings, have low dividend payout ratios, or are small. Such findings are consistent with the view that the presence of cash-flow terms reflects the impact of financing constraints, but do not exclude the possibility that other misspecifications of the $Q$ model are driving these results. One possible misspecification is mismeasurement of $Q$, which we analyse in the following section.

4 Measurement error

Underlying the result that Tobin’s average $Q$ can be used reliably in investment equations is the hypothesis that the firm’s stock market value, denoted $V^E$, reflects at all times the net present value of its discounted expected profits, denoted $V$. If this is not the case, the regressor in the investment equation

\textsuperscript{12}Such a cost premium could reflect taxes, transaction costs, or asymmetric information.
is potentially measured with error, which could have important implications for the empirical results.\textsuperscript{13}

Let us define the ‘bubble’ component \((m)\) in stock market valuations as follows:

\[
V_t^E = V_t + m_t
\]  

(3)

A measure of average \(Q\) that is based on stock market values will therefore be equal to

\[
Q_t^E = \frac{V_t + m_t}{p_t^I(1 - \delta)K_{t-1}} - 1
\]  

(4)

\[
= Q_t + \frac{m_t}{p_t^I(1 - \delta)K_{t-1}}
\]

\[
= Q_t + \mu_t
\]

The investment equation using observed average \(Q\) then becomes

\[
\frac{I_t}{K_t} = a + \frac{1}{b}Q_t^E + \left(\epsilon_t - \frac{\mu_t}{b}\right)
\]  

(5)

We distinguish three different types of measurement error, and discuss their implications for estimation. The formal derivation of the results can be found in Bond and Cummins (2001). For ease of exposition, we introduce the notation \(\kappa_t \equiv p_t^I(1 - \delta)K_{t-1}\), the current replacement cost value of the capital stock.

If \(m_t\) is a mean zero error, serially uncorrelated and independent of \(\kappa_t\), then \(\mu_t\) is serially uncorrelated and uncorrelated with \(Q_{s}^E\) for \(s \neq t\). In this case lags of \(Q_t^E\) are admissible as instrumental variables for \(Q_t^E\). If \(m_t\) follows an \(MA(k)\) process but continues to be independent of \(\kappa_t\), \(Q_{t-k-1}^E\) and longer lags are admissible instruments.

\textsuperscript{13}Our analysis in this section closely follows that in Bond and Cummins (2001).
If \( m_t \) follows a more general serially correlated process,\(^{14} \) then lagged values of \( Q^F_t \) are ruled out as admissible instruments, because they will be correlated with the \( \mu_t \) component of the error term in the empirical investment equation 5. But as long as \( m_t \) is independent of \( \kappa_s \) and other ‘fundamental’ variables such as profits, sales or investment itself, these fundamental variables will be admissible instruments. We usually rule out current values of these variables, as they are likely to be correlated with the adjustment cost shock \( \epsilon_t \), and therefore consider lagged values of these fundamental variables as potential instruments.

If however \( m_t \) follows a serially correlated process that is not independent of \( \kappa_s \) and other fundamental variables, then there may be no admissible instruments that would allow consistent estimation of the parameters of the model. This form of measurement error, where stock market values deviate persistently from fundamentals, and where the deviation is itself correlated with information that affects the fundamental value of the firm, is consistent with both rational bubbles and noisy-trader models, as described for example in Blanchard and Watson (1982), Froot and Obstfeld (1991) and Campbell and Kyle (1993). In this case the standard measure of the average \( Q \) ratio would not appear to be a sufficient statistic, even if the average \( Q \) model defined by 1 and 2 were correctly specified. Additional financial variables could then appear to be significant in the absence of financing constraints, if they simply contain relevant information about expected future profitability that is not captured by the poorly measured Tobin’s average \( Q \) variable.

To estimate the investment model consistently under this third hypothesis, one possibility is to avoid the use of stock market valuation data altogether, and to use an alternative estimate of the present discounted value

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\(^{14}\)We have in mind either a process that contains an autoregressive component, or an \( MA(k) \) process where \( k \) exceeds the time dimension of the panel.
of expected future profits. Cummins, Hassett and Oliner (1999) and Bond and Cummins (2000, 2001) have implemented this approach using data on securities analysts’ earnings forecasts to construct estimates of $V$ for samples of US companies. This approach requires long-term forecasts of earnings growth, which although reported in the IBES database for most US firms, are not available in the same data source for the majority of UK firms. In this paper we therefore adopt a less ambitious approach, and simply include the available forecasts of future profits at short horizons as additional indicators of expected profitability in empirical investment equations. To the extent that financial variables like cash flow have been found to be significant simply because they provide information about expected future profitability that is missing from the standard Tobin’s average $Q$ measures, then we would expect these financial variables to become less significant when we condition on these direct measures of expected future profitability. If on the other hand the significance of cash flow is really due to the presence of important financing constraints, then we would expect cash flow to remain significant when we include these alternative measures of expected future profitability. These issues can be further explored by considering estimates of the investment models for certain subsamples, such as smaller firms and firms with low dividend payout ratios, which have elsewhere been proposed as being more likely to be subject to significant financing constraints.

## 5 Data

We use firm-level accounting and share price data from the Datastream database, which covers UK-quoted companies from 1968 onwards. We obtain analysts’ earnings forecasts from the IBES database, which covers a subsample of UK-quoted companies from 1987 onwards.
Similarly to Blundell et al (1992), we construct a capital stock measure using the perpetual inventory method, which is based on the book value of the capital stock in the first year, and then calculates the subsequent values of the capital stock using the recursive formula:

\[ p_{t+1}^K K_{t,t+1} = p_{t+1}^K (1 - \delta) K_{t,t} + p_{t+1}^K I_{t,t+1} \]  

(6)

The theoretical timing conventions have to be adapted to fit the annual frequency of accounting data. We call \( I_{t,t} \) the investment during a particular year \( t \), and \( K_{t,t} \) is then the capital stock at the end of that year. Investment spending is measured as purchases minus sales of fixed assets, adjusted for the net book value of fixed assets in subsidiaries newly acquired or disposed of. For the depreciation rate, we take a constant 0.08, as in Bond et al (1999).

To calculate the market value of the company’s equity, we multiply the share price by the number of shares outstanding. The share price is taken near the beginning of year \( t \). To ensure that the stock market valuation is based on the same information set as the analysts’ earnings forecasts, we use the share price on the earliest day for which we have forecasts of earnings for both year \( t \) and year \( t + 1 \). Similar results were obtained using the share price on the first day of the accounting period, or using averages of the firm’s stock market valuation on these and earlier days.

Empirical results are reported using a measure of Tobin’s \( Q \) that adjusts for the firm’s use of debt financing, so that a measure of the stock of debt outstanding is added to the market value of equity to obtain the numerator of the average \( Q \) ratio. The denominator is the replacement cost value of the stock of capital that the firm inherits from the previous period, as in equation 2 above. As described further in the data appendix, we obtained very similar results using a measure of Tobin’s \( Q \) that also adjusts for corporate taxation.

Approved securities analysts are asked by IBES to provide forecasts of
earnings per share for the current year \( t \), and the years \( t + 1 \) and \( t + 2 \). They are also asked to provide a forecast of ‘long-term’ growth in ‘trend earnings’.

For the UK firms in the IBES database there is only limited availability of the forecasts for year \( t + 2 \) and for the long-term growth forecasts. We therefore focus on a sample of UK companies for which timely forecasts of earnings per share in years \( t \) and \( t + 1 \) are available. In cases where several analysts provide forecasts for the same firm, we abstract from heterogeneity across analysts by using the unweighted means of the individual forecasts, which IBES term the consensus forecasts. To get from the forecasts of earnings per share to forecasts of total profits for firm \( i \) in year \( s \) (\( \hat{\Pi}_{is} \)), we multiply the earnings per share forecast by the number of shares outstanding at the time the forecast was made. We then use the available data to construct two indicators of expected profitability as follows:

\[
E\Pi_{it} = \frac{\hat{\Pi}_{it} + \beta_{t+1}\hat{\Pi}_{i,t+1}}{p_t^q (1-\delta)K_{i,t-1}} \quad (7)
\]
\[
E\Pi1_{it} = \frac{\hat{\Pi}_{i,t+1}}{p_t^q (1-\delta)K_{i,t-1}} \quad (8)
\]

The discount factor \( \beta_t \) is constructed simply as the inverse of \( 1 + r_t + \zeta \), where \( r_t \) is the nominal yield on 20-year UK government bonds and \( \zeta \) is a constant risk premium, which we set at 0.08. \( E\Pi_{it} \) thus provides an ex-ante measure of discounted expected profitability of the firm in the current year and the following year. \( E\Pi1_{it} \) focuses on expected profitability for the following year, to reduce the degree of collinearity between these expected profitability variables and current or lagged cash-flow measures. The denominator in both cases is the replacement cost value of capital at the beginning of period \( t \), which is the same denominator that we used to construct Tobin’s \( Q \).

Using this approach, we are able to obtain a data set of 703 firms, for
Table A: Descriptive statistics of full sample

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>std.dev</th>
<th>first quartile</th>
<th>median</th>
<th>third quartile</th>
</tr>
</thead>
<tbody>
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<td>$Q$</td>
<td>2.66</td>
<td>2.16</td>
<td>1.31</td>
<td>2.00</td>
<td>3.23</td>
</tr>
<tr>
<td>$\frac{I}{K}$</td>
<td>0.15</td>
<td>0.13</td>
<td>0.08</td>
<td>0.14</td>
<td>0.20</td>
</tr>
<tr>
<td>$E\Pi$</td>
<td>0.45</td>
<td>0.33</td>
<td>0.22</td>
<td>0.36</td>
<td>0.58</td>
</tr>
<tr>
<td>$\frac{CF}{K}$</td>
<td>0.25</td>
<td>0.16</td>
<td>0.16</td>
<td>0.23</td>
<td>0.33</td>
</tr>
<tr>
<td>$\frac{CS}{K}$</td>
<td>0.26</td>
<td>0.35</td>
<td>0.05</td>
<td>0.14</td>
<td>0.34</td>
</tr>
<tr>
<td>$\Delta y$</td>
<td>0.07</td>
<td>0.20</td>
<td>-0.03</td>
<td>0.04</td>
<td>0.13</td>
</tr>
<tr>
<td>$Y$</td>
<td>1.17</td>
<td>3.67</td>
<td>0.08</td>
<td>0.22</td>
<td>0.84</td>
</tr>
</tbody>
</table>

Note: $Q$ is Tobin’s average $Q$, $\frac{I}{K}$ is the investment rate, $E\Pi$ is expected profitability as defined in equation 7, $\frac{CF}{K}$ is the ratio of cash flow (post-tax profits plus depreciation) to capital, $\frac{CS}{K}$ is the ratio of the cash stock (cash and marketable securities) to capital, $\Delta y$ is the real growth rate of sales, and $Y$ is the level of real sales in billions of 1995 pounds. Further details about the construction of variables are reported in Appendix C.

which we have at least four consecutive annual observations between 1987 and 2000. More detail on the sample and the construction of the variables is provided in Appendix C.

The descriptive statistics of the sample are reported in Table A.\(^\text{15}\)

Our sample means for the investment rate, cash flow and real sales growth are directly comparable to those reported for listed UK companies by Bond \textit{et al} (2003), who use a similar methodology to construct the variables, but do not select firms on the basis of analyst coverage. Our sample means are similar for sales growth and the investment rate, but higher for cash flow (0.13 in Bond \textit{et al}). However, the sample period in Bond \textit{et al} (2003) was different from ours, and given that these variables are highly cyclical, differing short sample periods may explain the difference.

\(^\text{15}\)Descriptive statistics of the sub-samples used in estimation can be found in Appendix C.
6 Empirical results

We used the data set described in the previous section to estimate a range of econometric investment equations for this sample of publicly traded UK companies. In all the results reported below, estimation uses the first-differenced GMM approach outlined in Arellano and Bond (1991) to control for the presence of unobserved firm-specific effects in the error term. The instrumental variables used are reported in the tables. In most cases these are lagged values of the explanatory variables or additional instruments, to allow for the endogeneity and possibly persistent measurement error in measured Tobin’s $Q$ that was discussed in Section 4.16.

Table 1 begins by reporting our estimates of the basic Tobin’s $Q$ model for this sample of UK firms. Column (i) ignores any sources of endogeneity for measured Tobin’s $Q$, and includes current as well as lagged values of this variable in the set of instrumental variables for the first-differenced equations. Lagged values of the dependent variable $(I_{it}/K_{it})$ and our cash-flow variable $(CF_{it}/K_{it})$ dated $t-2$ and $t-3$ are also included in the instrument set. Column (ii) more appropriately treats $Q_{it}$ as an endogenous variable, and excludes both $Q_{it}$ and $Q_{i,t-1}$ from the instrument set. The longer lags would be valid instruments if the average $Q$ model is correctly specified, and any deviation between stock market and fundamental values is ‘pure random noise’, serially uncorrelated and independent of the true value of the firm. Column (iii) excludes all lagged values of measured Tobin’s $Q$ from the set of instruments. The lagged values of investment and cash flow used as instruments in this case would remain valid if there is a persistent deviation between stock market and fundamental values, provided this ‘bubble’ evolves independently of these.

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16 All the reported estimates were computed using DPD08 for Gauss; see Arellano and Bond (1998).
variables.

Our results indicate that the basic Tobin’s $Q$ model is misspecified for this sample of UK companies. In particular the Sargan test of overidentifying restrictions rejects the hypothesis that the error term in the first-differenced equations is orthogonal to these instruments, regardless of which instrument set we use. This was also found to be the case for a wide range of alternative instrument sets we considered. Similar findings for a large sample of publicly traded US companies were reported by Bond and Cummins (2001). This could either be because the average $Q$ model is itself misspecified, or because stock market valuations contain a ‘bubble’ component that is both persistent and correlated with new information about the fundamental value of the firm.

The rejection of the orthogonality conditions in column (iii) of Table 1 suggests that either cash flow or the lagged dependent variable or both may be significant explanatory variables, in addition to measured Tobin’s $Q$. Column (i) of Table 2 confirms that both lagged cash flow and the lagged investment rate are highly significant when added to this empirical model. Their inclusion is also sufficient for this model not to be rejected by the Sargan test of overidentifying restrictions.

We are particularly interested in whether the significance of cash flow here can be explained by weakness of Tobin’s $Q$ as a measure of the relevant expectations of future profitability. As stressed by Erickson and Whited (2000) and Bond and Cummins (2001), this could be important if ‘bubbles’ cause stock market valuations to deviate persistently from the present discounted value of expected future profits. Alternatively, as noted by Cooper and Ejarque (2001) and Gomes (2001), this could also occur if there is a persistent wedge between average $Q$ and marginal $Q$, perhaps as a result of market power.
To investigate this issue, column (ii) of Table 2 includes a direct measure of expected profitability as an additional explanatory variable in the empirical investment model. Specifically we use the consensus forecasts for earnings in the current period and for earnings in the following period, issued by securities analysts who provide this information about a particular firm to IBES International, to construct the forward-looking measure of expected profitability, $E\Pi_{it}$, as described in Section 5.

Column (ii) of Table 2 shows that this measure of expected profitability is highly significant in our investment equation. Notice that we also treat $E\Pi_{it}$ as being endogenous and subject to persistent measurement error, and hence continue to use only lagged values of the investment rate and the cash-flow variable as instruments here. Again the validity of these moment conditions is not rejected by the Sargan statistic. However we find that the lagged cash-flow variable that was highly significant in column (i) becomes statistically insignificant when we include this direct measure of expected profitability. This is consistent with findings for US companies reported by Bond and Cummins (2001), and with their interpretation that the statistical significance of cash-flow terms in simpler specifications can be attributed to the failure of these models to control sufficiently for the influence of expected profitability on company investment decisions. Tobin’s $Q$ remains marginally significant in our empirical model, which is to be expected if stock market valuations provide some additional information about expected profitability in the longer term that is not captured by our analysts’ forecasts variable.\footnote{It should be emphasised that we measure the firm’s stock market valuation at the end of the trading day on which the analysts’ earnings forecasts are reported by I/B/E/S, so that any private information used to construct the earnings forecasts should have been incorporated in the stock market valuation.}

Column (iii) of Table 2 confirms that expectations of future profitability
are highly informative in explaining investment behaviour, by omitting the forecast of earnings in the current period and constructing the alternative expected profitability measure $\Pi_{1,i}$. Again we find that the lagged cash-flow term is statistically insignificant in the presence of this forward-looking expected profitability variable. Columns (iv) and (v) confirm that similar results are obtained when we omit either cash flow or Tobin’s $Q$ from the empirical specification.

Table 3a is included for comparison with previous papers in this literature that use sample splitting to identify credit-constrained firms. We report results for subsamples of smaller and larger firms (in terms of total real sales), and for subsamples of firms with relatively low and relatively high dividend payout ratios during our sample period. The sample splits were achieved as follows: each firm was assigned to a high (resp. low) category according to its position in the first year it enters the sample relative to the median across all firms in the first year they enter the sample. For example, firm XYZ was categorised as a high dividend payout firm if its ratio of dividends to cash flow in 1992, the first year firm XYZ entered the sample, was above the median dividend payout ratio across all firms in the first year they entered the sample. We experimented with several different methods for splitting the sample, which produced similar results.\(^\text{18}\) The cash-flow coefficient for low-dividend paying and small firms is found to be significant, whereas the cash-flow coefficient for high-dividend paying and large firms is not, consistent with the hypothesis that the low-dividend paying firms and smaller firms may face financing constraints. The more general finding that

\(^{18}\)In addition to the reported results, we also compared the firm’s median characteristics (measured over time) with the median characteristics of all firms. A third method involved comparing a firm in its first year in the sample with the characteristics of all firms in that same year.
one group of firms, assumed to be credit-constrained on a priori grounds, has a higher cash-flow coefficient is consistent with other split-sample findings in the literature.\footnote{For example, Fazzari, Hubbard and Peterson (1988), Devereux and Schiantarelli (1990), Hoshi, Kashyap and Scharfstein (1991), Whited (1992), Elston (1993) and Gilchrist and Himmelberg (1995).}

Table 3b reports estimates of the same specification used in column (ii) of Table 2, but by subsample. The main finding of interest here is that the coefficient on the cash-flow variable is found to be insignificantly different from zero in each of these subsamples, after controlling for the influence of expected profitability on investment by including our analysts’ forecasts variable.\footnote{Similar findings for sub-samples of US companies are reported by Cummins, Hassett and Oliner (1999) and Syed (2003). The finding that the cash-flow coefficient becomes insignificant after controlling for expected profitability is robust to various choices of instrument sets.} The relationship between investment rates and this measure of expected profitability is found to be broadly similar across these subsamples, while Tobin’s $Q$ is found to be marginally significant in each of the subsamples.

We experimented with a wide range of additional financial variables, such as stock and flow measures of gearing, and with additional sales terms in our empirical investment equations. Since we only have data on analysts’ forecasts of profits in the short term, it is not surprising to discover that some of these variables contain additional information that helps to explain company investment. Two variables that were found to be particularly informative were the current growth rate of real sales ($\Delta y_{it}$) and the lagged ratio of the stock of cash and short-term financial assets to the capital stock ($CS_{it}/K_{it}$).

Table 4 reports some specifications where these variables are included. The instrumental variables used here are lagged values of Tobin’s $Q$ and these sales growth and cash stock terms, although similar findings were ob-
tained using a range of different instrument sets. Columns (i) and (ii) show that the inclusion of either of these terms is sufficient to make Tobin’s $Q$ insignificant.\textsuperscript{21} Thus any relevant information in measured Tobin’s $Q$ about expected profitability in the longer term seems to be proxied better by these sales growth or cash stock variables. Cash flow continued to be insignificant when added to either of these specifications.\textsuperscript{22} Columns (iii) and (iv) show that cash stock and sales growth each provide independent information that helps to explain company investment rates, after controlling for expected short-term profitability using analysts’ earnings forecasts.

The significance of these additional terms could indicate that they provide additional information about expected profitability in the longer term. Such information would be relevant for explaining company investment if the average $Q$ model of investment was correct. Alternatively they could reflect one of several possible sources of misspecification of the average $Q$ model. Market power or decreasing returns to scale would introduce a wedge between expectations of average profitability and the expectations of the future marginal profitability of additional investment that are relevant for explaining investment behaviour under strictly convex adjustment costs (see Hayashi (1982)). Non-convex components of adjustment costs would imply a non-linear relation between investment rates and expectations of average profitability (see Abel and Eberly (1996)). The combination of a concave net revenue function and non-convex adjustment costs would lead to a more fundamental misspecification of the $Q$ model, since in this case investment would be influenced by the value of the ‘real option’ to delay investing until

\textsuperscript{21}The coefficient on Tobin’s $Q$ remains insignificantly different from zero when the lags of Tobin’s $Q$ are omitted from the instrument set.

\textsuperscript{22}This was the case whether or not lags of cash flow were included in the instrument set.
more information has accumulated (see, for example, Caballero (1991)).

In principle the significance of these additional variables could also be explained by the presence of financing constraints, or a wedge between the costs of internal and external sources of investment finance. This could be suspected particularly in the case of the cash stock term. To explore this possibility further, Table 5 reports estimates of our preferred empirical specification from column (iv) of Table 4 for the subsamples of firms considered previously in Table 3. Following Fazzari, Hubbard and Petersen (1988), numerous authors have argued that if there is a cost premium for external finance, it is more likely to be significant for smaller firms or for firms with relatively low dividend payout ratios. If that were the main explanation for the significance of the cash-stock variable in our empirical model, we would therefore expect the significance of this term to be concentrated among our subsamples of smaller or low-dividend firms. In contrast the results in Table 5 show that the coefficient on the cash-stock variable is significantly different from zero, and broadly similar, in each of these four subsamples. If anything, there is more heterogeneity in the relationship between investment rates and sales growth, although even here the hypothesis of common coefficients is not rejected at conventional significance levels. The additional information provided by these variables, after controlling for our direct measure of expected short-term profitability, seems likely to reflect some more general feature of the investment behaviour of UK companies.

It is important to stress at this point that our subsample splits consider firms that are small relative to our sample. Since our sample is based on stockmarket listed firms only, it contains firms that are large compared to the whole population of firms. However, the empirical work on UK and US data that we compare our work to has largely been carried out on stockmarket
listed firms as well. All of this work still leaves open the possibility that cash flow has a quite different impact on the investment behaviour of truly small firms.23

One interpretation of these findings is that financing constraints do not matter much for the sample of firms we have considered. An alternative interpretation of our findings is that financing constraints are similar across the subsamples we have considered, which would explain why the cash stock coefficient does not differ significantly across subsamples. A third interpretation is that financing constraints enter the investment process in a way not captured by the variables we have experimented with.

An important policy implication of our findings is that cash-flow coefficients reported in standard Q models of investment are unlikely to be informative about the importance of financing constraints for investment. The significance of cash-flow variables in this context seems to reflect the weakness of stock-market based Tobin’s Q measures as controls for the influence of expected future profitability on current investment decisions. An avenue for further research is to check to what extent the inclusion of more informative expected profitability measures weakens the role of cash-flow terms in alternative investment model specifications, such as error-correction, accelerator and Euler equation models. In addition, it would be useful to try to identify financing constraints empirically in a structural model that specifies how financing constraints enter the firm’s decision problem.24

23 According to the Bank of England Quarterly Report on Small Business Statistics, small firms in the UK (with fewer than 250 employees) accounted for 45% of turnover in 1999. In our sample, in 1999, firms with fewer than 250 employees accounted for 0.1% of turnover, confirming that our sample is not representative of the overall size distribution of UK firms. On the other hand, according to the Office for National Statistics, listed companies accounted for 74% of the market value of the total UK corporate capital stock in 1999, suggesting that in terms of capital (and therefore investment), focusing on listed firms may be quite informative about aggregate investment.

24 Gilchrist and Himmelberg (1998) provide one example of a structural approach to
7 Conclusions

Our principal conclusion is that, in line with standard economic theory, direct measures of expected future profits are very informative explanatory variables for the behaviour of company investment. In contrast, Tobin’s $Q$ measures based on stock market valuations are much less informative, providing only marginally significant additional information after controlling for short-term earnings forecasts. Moreover, cash-flow variables, which appear to be highly significant in reduced-form models or in models which control for Tobin’s $Q$, become insignificant once we control for expected future profitability using analysts’ earnings forecasts. These empirical results for UK companies are consistent with recent evidence reported for US firms. They indicate that the apparent significance of cash-flow terms in many econometric investment equations can be explained by the absence of sufficiently informative controls for the influence of expected future profitability on company investment decisions.

The stock market valuations contained in Tobin’s $Q$ become completely uninformative in our empirical investment equations when additional variables like sales growth or cash stock are included together with expectations of short-term profitability. The limited information in this measure of the average $Q$ ratio is consistent with the presence of pervasive and persistent ‘bubbles’, or deviations between stock market values and the present discounted value of expected future profits. Alternatively this could indicate a failure of the Hayashi conditions - perfect competition, constant returns to scale and strictly convex adjustment costs - under which average $Q$ is a sufficient statistic for investment rates. Our results do not discriminate between these possibilities, as additional variables like sales growth or cash modelling financial frictions.
stock could be expected to provide relevant information about expected profitability in the longer term, not contained in analysts’ short-term earnings forecasts, even if the average Q model was correctly specified. However our results for subsamples of smaller and low-dividend firms do not suggest that the presence of financing constraints is a likely explanation for these empirical findings.
## Results

### Table 1 - Basic Tobin’s $Q$ models

First-differenced GMM

<table>
<thead>
<tr>
<th></th>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_t$</td>
<td>0.035</td>
<td>0.039</td>
<td>0.047</td>
</tr>
<tr>
<td></td>
<td>(.003)</td>
<td>(.005)</td>
<td>(.008)</td>
</tr>
<tr>
<td>$m1$</td>
<td>-9.77</td>
<td>-9.73</td>
<td>-9.67</td>
</tr>
<tr>
<td>$m2$</td>
<td>-1.33</td>
<td>-1.27</td>
<td>-1.11</td>
</tr>
<tr>
<td>Sargan</td>
<td>0.006</td>
<td>0.033</td>
<td>0.006</td>
</tr>
<tr>
<td>IVs</td>
<td>$Q$: t, t-1, t-2</td>
<td>$Q$: t-2, t-3</td>
<td>$Q$: t-2, t-3</td>
</tr>
<tr>
<td></td>
<td>$\left(\frac{CF}{K}\right)$: t-2, t-3</td>
<td>$\left(\frac{CF}{K}\right)$: t-2, t-3</td>
<td>$\left(\frac{CF}{K}\right)$: t-2, t-3</td>
</tr>
<tr>
<td></td>
<td>$\left(\frac{CF}{L}\right)$: t-2, t-3</td>
<td>$\left(\frac{CF}{L}\right)$: t-2, t-3</td>
<td>$\left(\frac{CF}{L}\right)$: t-2, t-3</td>
</tr>
</tbody>
</table>


Notes to all tables:

Time dummies are included as regressors and instruments. Asymptotic standard errors are reported in parentheses. Standard errors are robust to general time-series and cross-section heteroskedasticity. $m1$ and $m2$ are tests for first and second-order serial correlation, asymptotically distributed as $N(0,1)$ under the null of no serial correlation. The Sargan test statistic is a test of the overidentifying restrictions, asymptotically distributed as $\chi^2(k)$ under the null that they are valid. The p-value of this test is reported.
Table 2 - Tobin’s Q, cash flow and expected profits

First-differenced GMM

Dependent variable \( \left( \frac{I}{K} \right)_t \)

<table>
<thead>
<tr>
<th></th>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
<th>(iv)</th>
<th>(v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_t )</td>
<td>0.027</td>
<td>0.017</td>
<td>0.016</td>
<td>0.017</td>
<td></td>
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<tr>
<td></td>
<td>(0.009)</td>
<td>(0.008)</td>
<td>(0.009)</td>
<td>(0.009)</td>
<td></td>
</tr>
<tr>
<td>( \left( \frac{CF}{K} \right)_{t-1} )</td>
<td>0.152</td>
<td>-0.078</td>
<td>-0.085</td>
<td>-0.071</td>
<td>-0.071</td>
</tr>
<tr>
<td></td>
<td>(0.058)</td>
<td>(0.095)</td>
<td>(0.095)</td>
<td>(0.095)</td>
<td></td>
</tr>
<tr>
<td>( \Pi_t )</td>
<td>0.248</td>
<td>0.248</td>
<td></td>
<td>0.194</td>
<td>0.305</td>
</tr>
<tr>
<td></td>
<td>(0.089)</td>
<td>(0.089)</td>
<td></td>
<td>(0.049)</td>
<td>(0.083)</td>
</tr>
<tr>
<td>( \Pi_{t+1} )</td>
<td></td>
<td></td>
<td>0.445</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.157)</td>
<td></td>
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</tr>
<tr>
<td>( \left( \frac{I}{K} \right)_{t-1} )</td>
<td>0.088</td>
<td>0.068</td>
<td>0.071</td>
<td>0.075</td>
<td>0.081</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.028)</td>
<td>(0.027)</td>
<td>(0.027)</td>
<td>(0.028)</td>
</tr>
<tr>
<td>m1</td>
<td>-10.72</td>
<td>-10.41</td>
<td>-10.52</td>
<td>-10.47</td>
<td>-10.41</td>
</tr>
<tr>
<td>m2</td>
<td>-0.06</td>
<td>-0.31</td>
<td>-0.27</td>
<td>-0.23</td>
<td>-0.36</td>
</tr>
<tr>
<td>Sargan</td>
<td>0.52</td>
<td>0.41</td>
<td>0.42</td>
<td>0.42</td>
<td>0.35</td>
</tr>
</tbody>
</table>


IVs: \( \left( \frac{CF}{K} \right): \) t-2, t-3; \( \left( \frac{I}{K} \right): \) t-2, t-3
### Table 3a - Subsample results

First-differenced GMM

Dependent variable $\left( \frac{I}{R} \right)_t$

<table>
<thead>
<tr>
<th></th>
<th>Small firms</th>
<th>Large firms</th>
<th>Low dividends</th>
<th>High dividends</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_t$</td>
<td>0.023</td>
<td>0.050</td>
<td>0.025</td>
<td>0.036</td>
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<tr>
<td></td>
<td>(0.008)</td>
<td>(0.015)</td>
<td>(0.008)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>$(\frac{CF}{K})_{t-1}$</td>
<td>0.189</td>
<td>0.122</td>
<td>0.211</td>
<td>0.074</td>
</tr>
<tr>
<td></td>
<td>(0.066)</td>
<td>(0.089)</td>
<td>(0.080)</td>
<td>(0.071)</td>
</tr>
<tr>
<td>$(\frac{I}{K})_{t-1}$</td>
<td>0.099</td>
<td>0.051</td>
<td>0.111</td>
<td>0.062</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.040)</td>
<td>(0.039)</td>
<td>(0.042)</td>
</tr>
<tr>
<td>$m1$</td>
<td>-8.32</td>
<td>-7.13</td>
<td>-7.31</td>
<td>-7.63</td>
</tr>
<tr>
<td>$m2$</td>
<td>0.85</td>
<td>-0.89</td>
<td>0.30</td>
<td>-0.54</td>
</tr>
<tr>
<td>Sargan</td>
<td>0.78</td>
<td>0.48</td>
<td>0.38</td>
<td>0.52</td>
</tr>
<tr>
<td>Firms</td>
<td>351</td>
<td>352</td>
<td>351</td>
<td>352</td>
</tr>
<tr>
<td>Observations</td>
<td>1,824</td>
<td>2,439</td>
<td>2,200</td>
<td>2,063</td>
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</tbody>
</table>

IVs: $(\frac{CF}{K})$: t-2, t-3; $(\frac{I}{K})$: t-2, t-3
Table 3b - Subsample results

First-differenced GMM
Dependent variable \( \left( \frac{I}{K} \right)_t \)

<table>
<thead>
<tr>
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<th>Small firms</th>
<th>Large firms</th>
<th>Low dividends</th>
<th>High dividends</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_t )</td>
<td>0.015</td>
<td>0.027</td>
<td>0.016</td>
<td>0.029</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.014)</td>
<td>(0.009)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>( \left( \frac{CF}{K} \right)_{t-1} )</td>
<td>0.030</td>
<td>-0.077</td>
<td>-0.159</td>
<td>-0.050</td>
</tr>
<tr>
<td></td>
<td>(0.068)</td>
<td>(0.155)</td>
<td>(0.136)</td>
<td>(0.095)</td>
</tr>
<tr>
<td>( E\Pi_t )</td>
<td>0.193</td>
<td>0.241</td>
<td>0.292</td>
<td>0.214</td>
</tr>
<tr>
<td></td>
<td>(0.064)</td>
<td>(0.149)</td>
<td>(0.106)</td>
<td>(0.121)</td>
</tr>
<tr>
<td>( \left( \frac{I}{K} \right)_{t-1} )</td>
<td>0.082</td>
<td>0.046</td>
<td>0.070</td>
<td>0.053</td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
<td>(0.040)</td>
<td>(0.036)</td>
<td>(0.043)</td>
</tr>
<tr>
<td>( m1 )</td>
<td>-8.03</td>
<td>-6.66</td>
<td>-7.29</td>
<td>-7.13</td>
</tr>
<tr>
<td>( m2 )</td>
<td>0.78</td>
<td>-1.19</td>
<td>0.33</td>
<td>-0.77</td>
</tr>
<tr>
<td>Sargan</td>
<td>0.88</td>
<td>0.26</td>
<td>0.59</td>
<td>0.32</td>
</tr>
<tr>
<td>Firms</td>
<td>351</td>
<td>352</td>
<td>351</td>
<td>352</td>
</tr>
<tr>
<td>Observations</td>
<td>1,824</td>
<td>2,439</td>
<td>2,200</td>
<td>2,063</td>
</tr>
</tbody>
</table>

IVs: \( \left( \frac{CF}{K} \right): \) t-2, t-3; \( \left( \frac{I}{K} \right): \) t-2, t-3
Table 4 - Expected profits and additional variables

First-differenced GMM

Dependent variable \( \left( \frac{I}{K} \right)_t \)

<table>
<thead>
<tr>
<th></th>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
<th>(iv)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E\Pi_t )</td>
<td>0.169</td>
<td>0.240</td>
<td>0.168</td>
<td>0.200</td>
</tr>
<tr>
<td></td>
<td>(0.057)</td>
<td>(0.050)</td>
<td>(0.054)</td>
<td>(0.039)</td>
</tr>
<tr>
<td>( \left( \frac{CS}{K} \right)_{t-1} )</td>
<td>0.078</td>
<td>0.062</td>
<td>0.060</td>
<td>0.019</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td></td>
<td>(0.022)</td>
<td></td>
</tr>
<tr>
<td>( \Delta y_t )</td>
<td>0.165</td>
<td>0.137</td>
<td>0.132</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.045)</td>
<td>(0.045)</td>
<td></td>
<td>(0.040)</td>
</tr>
<tr>
<td>( Q_t )</td>
<td>0.009</td>
<td>0.001</td>
<td>0.007</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.007)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \left( \frac{I}{K} \right)_{t-1} )</td>
<td>0.083</td>
<td>-0.110</td>
<td>-0.022</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.056)</td>
<td>(0.048)</td>
<td>(0.056)</td>
<td></td>
</tr>
<tr>
<td>m1</td>
<td>-7.62</td>
<td>-7.19</td>
<td>-7.19</td>
<td>-10.00</td>
</tr>
<tr>
<td>m2</td>
<td>-0.43</td>
<td>-2.11</td>
<td>-1.14</td>
<td>-1.11</td>
</tr>
<tr>
<td>Sargan</td>
<td>0.48</td>
<td>0.37</td>
<td>0.49</td>
<td>0.63</td>
</tr>
</tbody>
</table>


IVs: \( Q \): t-2, t-3; \( \left( \frac{CS}{K} \right) \): t-2, t-3; \( \Delta y \): t-2, t-3
### Table 5 - Subsample results

**First-differenced GMM**

Dependent variable \( \left( \frac{L}{K} \right) \),

<table>
<thead>
<tr>
<th></th>
<th>Small</th>
<th>Large</th>
<th>Low dividends</th>
<th>High dividends</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E\Pi_t )</td>
<td>0.216 (0.042)</td>
<td>0.257 (0.078)</td>
<td>0.200 (0.046)</td>
<td>0.218 (0.058)</td>
</tr>
<tr>
<td>( \left( \frac{CS}{K} \right)_{t-1} )</td>
<td>0.039 (0.026)</td>
<td>0.082 (0.026)</td>
<td>0.057 (0.026)</td>
<td>0.061 (0.025)</td>
</tr>
<tr>
<td>( \Delta y_t )</td>
<td>0.158 (0.045)</td>
<td>0.069 (0.067)</td>
<td>0.156 (0.051)</td>
<td>0.115 (0.044)</td>
</tr>
<tr>
<td>m1</td>
<td>-7.58</td>
<td>-6.95</td>
<td>-7.38</td>
<td>-7.19</td>
</tr>
<tr>
<td>m2</td>
<td>0.15</td>
<td>-1.74</td>
<td>0.32</td>
<td>-1.15</td>
</tr>
<tr>
<td>Sargan</td>
<td>0.88</td>
<td>0.10</td>
<td>0.47</td>
<td>0.70</td>
</tr>
<tr>
<td>Firms</td>
<td>351</td>
<td>352</td>
<td>351</td>
<td>352</td>
</tr>
<tr>
<td>Observations</td>
<td>1,824</td>
<td>2,439</td>
<td>2,200</td>
<td>2,063</td>
</tr>
</tbody>
</table>

IVs: \( Q: t-2, t-3; \) \( \left( \frac{CS}{K} \right): t-2, t-3; \) \( \Delta y: t-2, t-3 \)
Derivation of the standard \( Q \) model

We consider a profit-maximising firm operating in a perfectly competitive environment. The profit function is assumed to be of the form

\[
\Pi(K_t, I_t, \epsilon_t) = p_t F(K_t) - p_t^I [I_t + G(I_t, K_t, \epsilon_t)]
\]  

where \( F(K_t) \) is output, \( I_t \) is investment, \( p_t \) is the price of output, \( G(I_t, K_t, \epsilon_t) \) is an adjustment cost function, \( p_t^I \) is the price of investment goods, and \( \epsilon_t \) is a stochastic shock to the adjustment cost function.

We assume that adjustment costs are quadratic, and of the form

\[
G(I_t, K_t, \epsilon_t) = \frac{b}{2} \left[ \left( \frac{I_t}{K_t} \right) - a - \epsilon_t \right]^2 K_t
\]  

The firm maximises the present value of future discounted profits, given by

\[
V_t = E_t \left[ \sum_{i=0}^{\infty} \beta^i \Pi(K_{t+i}, I_{t+i}, \epsilon_{t+i}) \right]
\]  

subject to

\[
K_{t+i} = (1 - \delta)K_{t+i-1} + I_{t+i}
\]  

Here \( \beta \) is the one-period discount factor, assumed constant for simplicity, and \( E_t [\cdot] \) denotes an expected value given information at time \( t \).

The two first-order conditions of this maximisation problem are

\[
\frac{\partial \Pi_t}{\partial I_t} = -\lambda_t
\]

\[
\frac{\partial \Pi_t}{\partial K_t} = \lambda_t - (1 - \delta) \beta E_t \lambda_{t+1}
\]
where \( \lambda_t \) is the shadow value associated with the constraint 12 in period \( t \).

If we assume linear homogeneity of the profit function, then we can write

\[
\Pi_t = K_t \frac{\partial \Pi_t}{\partial K_t} + I_t \frac{\partial \Pi_t}{\partial I_t}
\]  
(15)

By substituting equations 13 and 15 into 14, we obtain

\[
\lambda_t = \left( \frac{\Pi_t}{K_t} + \frac{I_t \lambda_t}{K_t} \right) + \beta (1 - \delta) E_t \lambda_{t+1}
\]  
(16)

Using equation 12, we can rearrange this as

\[
\lambda_t (1 - \delta) K_{t-1} = \Pi_t + \beta E_t \left[ \lambda_{t+1} (1 - \delta) K_t \right]
\]  
(17)

Solving this forward, we recover the maximised value of the firm,

\[
\lambda_t (1 - \delta) K_{t-1} = E_t \left[ \sum_{i=0}^{\infty} \beta^i \Pi_{t+i} \right] = V_t
\]  
(18)

We now define marginal \( q_t \) as the ratio of the shadow value of an additional unit of capital, \( \lambda_t \), to its replacement cost, \( p_t' \). Expressing \( q_t \) in terms of observable variables, we get

\[
q_t \equiv \frac{\lambda_t}{p_t'} = \frac{V_t}{p_t' (1 - \delta) K_{t-1}}
\]  
(19)

This is Hayashi’s (1982) result that under linear homogeneity of the profit function, marginal \( q \) equals average \( \bar{q} \). To obtain an investment equation, we rewrite the first-order condition 13 making use of the functional form for \( \Pi_t \) that we assumed. This gives the familiar investment equation

\[
\frac{I_t}{K_t} = a + \frac{1}{b} Q_t + \epsilon_t
\]  
(20)

where \( Q_t \equiv (q_t - 1) \).
Allowing for debt finance and taxes as in Blundell et al (1992), the basic structure of the investment equation remains unchanged, but the definition of observable $Q$ changes to

$$Q_t = \left[ \frac{V_t - A_t + H_t}{p_t'(1 - \delta)K_{t-1}(1 - n_t)} - 1 \right] \frac{(1 - n_t)}{(1 - \tau_t)} \quad (21)$$

where $H$ is a measure of the stock of debt, $A$ is the present value of expected future depreciation allowances related to past investment, and $n$ is the present value of expected future depreciation allowances on a unit (expressed in money) of current investment. The corporate tax rate is denoted $\tau$. 

34
C Data

This section describes in some detail how the data was constructed. We provide the Datastream item code, indicated by a number preceded by ‘ds’. Further details, as well as the necessary programs to download the raw data from Datastream and construct the variables are available in two technical appendices that can be downloaded from www.bankofengland.co.uk/wp/index.html.

C.1 Investment and capital stock

We define investment as follows:

\[ I = ds1026 + ds479 \]

where ds1026 is net payments for fixed assets (where net means less sales of fixed assets) and ds479 is net fixed assets of subsidiaries acquired or sold. If ds1026 is not available, we define investment as:

\[ I = ds431 - ds423 + ds479 \]

where ds431 is purchases of fixed assets and ds423 is sales of fixed assets. We calculate investment in two ways since ds1026 replaces ds423 and ds431 after an accounting change in 1990. Since companies report investment in nominal terms we then deflate investment using the quarterly business investment deflator implied by the UK National Accounts to create an investment series in constant (1995) prices (NS codes: NPEK/NPEL).

Next, we need an estimate of the initial capital stock \((K_0)\) for each firm. As a general rule, we use the book value of fixed capital (ds339) in the first year of data for each firm. This can be modified to allow for some inflation in previous years, by revaluing the first available book value to reflect investment goods price inflation over the preceding three years.
We can now estimate the evolution of the real capital stock as

\[ K_{t+i} = (1 - \delta)K_{t+i-1} + I_{t+i} \]

For the baseline estimate of the capital stock we use an annual depreciation rate \((\delta)\) of 8 % for all capital goods, in line with Bond et al (2003). We drop observations if the estimated capital stock is negative, or if our estimate is out of line with book value by more than a factor of four. Most variables in our estimation are defined as a ratio of a nominal value to the nominal capital stock. We define the nominal capital stock and nominal investment simply as \(p_t^iK_t\) and \(p_t^iI_t\), ie we inflate the real capital stock by the business investment deflator.

C.2 Tobin’s average \(Q\)

As described in Appendix B, a measure of Tobin’s \(Q\) that allows for debt financing is

\[ Q_t \equiv \left[ \frac{V_t + H_t}{p_t^i(1 - \delta)K_{t-1}} - 1 \right] \]

For the estimate of \(V\), we use the firm’s share price multiplied by the number of shares outstanding. These are both taken at the end of the trading day on which the analysts’ earnings forecasts that we use were issued. This is to ensure that no more information is included in Tobin’s \(Q\) than in the analysts’ forecasts. Following Blundell et al (1992) and others we approximate \(H_t\) by the current stock of debt. This is calculated as total long-term debt (ds321) less net current assets (ds390). Note that this implicitly includes short-term debt, because short-term debt enters net current assets with a negative sign (net current assets = current assets – current liabilities).

We also considered a tax-adjusted measure of Tobin’s \(Q\), for which the tax parameters defined in equation 21 were constructed following the proce-
dures described in Blundell et al (1992). Very similar empirical results were obtained using this tax-adjusted measure.

C.3 Construction of other variables

\( (\frac{CF}{K})_t \): cash flow in period \( t \) divided by capital stock at replacement cost at the end of period \( t \). Cash flow is constructed as after tax profits (ds182) plus depreciation (ds136).

\( (\frac{CS}{K})_t \): cash stock at the end of period \( t \) divided by capital stock at replacement cost at the end of period \( t \). Cash stock is total cash and cash equivalent (ds375).

\( \Delta y_t \): real sales growth in period \( t \) relative to period \( t - 1 \). We first construct real sales as nominal sales (ds104) deflated by the GDP deflator, and then calculate the growth rate of real sales.

To achieve the sample split according to dividend payout ratios, we calculated the dividend payout ratio as dividends (ds187) as a ratio to cash flow. Negative payout ratios (due to negative profits) are recoded so that they are considered as extremely high payout ratios.

C.4 Descriptive statistics of subsamples

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>std.dev</th>
<th>first quartile</th>
<th>median</th>
<th>third quartile</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q )</td>
<td>2.86</td>
<td>2.40</td>
<td>1.27</td>
<td>2.06</td>
<td>3.69</td>
</tr>
<tr>
<td>( \frac{L}{K} )</td>
<td>0.16</td>
<td>0.13</td>
<td>0.08</td>
<td>0.14</td>
<td>0.22</td>
</tr>
<tr>
<td>( \frac{\Pi}{K} )</td>
<td>0.48</td>
<td>0.36</td>
<td>0.23</td>
<td>0.39</td>
<td>0.64</td>
</tr>
<tr>
<td>( \frac{CF}{K} )</td>
<td>0.27</td>
<td>0.17</td>
<td>0.16</td>
<td>0.24</td>
<td>0.35</td>
</tr>
<tr>
<td>( \frac{CS}{K} )</td>
<td>0.25</td>
<td>0.33</td>
<td>0.03</td>
<td>0.12</td>
<td>0.34</td>
</tr>
<tr>
<td>( \Delta y )</td>
<td>0.10</td>
<td>0.22</td>
<td>-0.02</td>
<td>0.06</td>
<td>0.17</td>
</tr>
<tr>
<td>( Y )</td>
<td>0.09</td>
<td>0.08</td>
<td>0.04</td>
<td>0.07</td>
<td>0.11</td>
</tr>
</tbody>
</table>
### Table C2: Large (352 firms)

<table>
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<th>first quartile</th>
<th>median</th>
<th>third quartile</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$</td>
<td>2.50</td>
<td>1.92</td>
<td>1.34</td>
<td>1.96</td>
<td>2.95</td>
</tr>
<tr>
<td>$\frac{I}{K}$</td>
<td>0.14</td>
<td>0.13</td>
<td>0.08</td>
<td>0.13</td>
<td>0.19</td>
</tr>
<tr>
<td>$E\Pi$</td>
<td>0.41</td>
<td>0.30</td>
<td>0.21</td>
<td>0.34</td>
<td>0.52</td>
</tr>
<tr>
<td>$\frac{CF}{K}$</td>
<td>0.24</td>
<td>0.14</td>
<td>0.16</td>
<td>0.22</td>
<td>0.31</td>
</tr>
<tr>
<td>$\frac{CS}{K}$</td>
<td>0.27</td>
<td>0.36</td>
<td>0.07</td>
<td>0.15</td>
<td>0.34</td>
</tr>
<tr>
<td>$\Delta y$</td>
<td>0.04</td>
<td>0.17</td>
<td>-0.04</td>
<td>0.03</td>
<td>0.11</td>
</tr>
<tr>
<td>$Y$</td>
<td>2.04</td>
<td>4.76</td>
<td>0.30</td>
<td>0.71</td>
<td>1.93</td>
</tr>
</tbody>
</table>

### Table C3: Low dividend (351 firms)

<table>
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<th>first quartile</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$Q$</td>
<td>2.83</td>
<td>2.34</td>
<td>1.37</td>
<td>2.07</td>
<td>3.43</td>
</tr>
<tr>
<td>$\frac{I}{K}$</td>
<td>0.16</td>
<td>0.13</td>
<td>0.09</td>
<td>0.14</td>
<td>0.22</td>
</tr>
<tr>
<td>$E\Pi$</td>
<td>0.46</td>
<td>0.36</td>
<td>0.22</td>
<td>0.36</td>
<td>0.61</td>
</tr>
<tr>
<td>$\frac{CF}{K}$</td>
<td>0.26</td>
<td>0.16</td>
<td>0.16</td>
<td>0.23</td>
<td>0.34</td>
</tr>
<tr>
<td>$\frac{CS}{K}$</td>
<td>0.26</td>
<td>0.37</td>
<td>0.05</td>
<td>0.14</td>
<td>0.33</td>
</tr>
<tr>
<td>$\Delta y$</td>
<td>0.08</td>
<td>0.19</td>
<td>-0.03</td>
<td>0.05</td>
<td>0.15</td>
</tr>
<tr>
<td>$Y$</td>
<td>1.01</td>
<td>2.57</td>
<td>0.08</td>
<td>0.21</td>
<td>0.78</td>
</tr>
</tbody>
</table>

### Table C4: High dividend (352 firms)

<table>
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<th>first quartile</th>
<th>median</th>
<th>third quartile</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$</td>
<td>2.49</td>
<td>1.92</td>
<td>1.25</td>
<td>1.93</td>
<td>3.04</td>
</tr>
<tr>
<td>$\frac{I}{K}$</td>
<td>0.13</td>
<td>0.13</td>
<td>0.07</td>
<td>0.13</td>
<td>0.19</td>
</tr>
<tr>
<td>$E\Pi$</td>
<td>0.42</td>
<td>0.30</td>
<td>0.22</td>
<td>0.35</td>
<td>0.54</td>
</tr>
<tr>
<td>$\frac{CF}{K}$</td>
<td>0.25</td>
<td>0.15</td>
<td>0.16</td>
<td>0.23</td>
<td>0.32</td>
</tr>
<tr>
<td>$\frac{CS}{K}$</td>
<td>0.26</td>
<td>0.33</td>
<td>0.05</td>
<td>0.14</td>
<td>0.34</td>
</tr>
<tr>
<td>$\Delta y$</td>
<td>0.05</td>
<td>0.20</td>
<td>-0.04</td>
<td>0.03</td>
<td>0.12</td>
</tr>
<tr>
<td>$Y$</td>
<td>1.33</td>
<td>4.55</td>
<td>0.08</td>
<td>0.23</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Note: $Q$ is Tobin’s average $Q$, $\frac{I}{K}$ is the investment rate, $E\Pi$ is expected profitability as defined in equation 7, $\frac{CF}{K}$ is the ratio of cash flow (post-tax profits plus depreciation) to capital, $\frac{CS}{K}$ is the ratio of the cash stock (cash and marketable securities) to capital, $\Delta y$ is the real growth rate of sales, and $Y$ is the level of real sales in billions of 1995 pounds.

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We dropped observations that failed to meet a set of criteria for data quality. We dropped values of Tobin’s $Q$ that were negative or in the top decile of the empirical distribution. We also dropped observations where the first difference of Tobin’s $Q$ was in the top or bottom 5% of the empirical distribution. We applied similar rules to our direct measures of expected profitability based on analysts’ earnings forecasts. Finally, we also dropped extreme outliers - never amounting to more than a percentile - in the level of $I/K$ and $CF/K$. 
References


