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## Article

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# Public Debt, Child Allowances and Pension Benefits with Endogenous Fertility

*Masaya Yasuoka and Atsushi Miyake*

## Abstract

The public debt stock in some economically developed countries continues to increase because of a lack of tax revenues and the concomitant burdens of social security. Many of those countries suffer from lower birth rates and consequently, have fewer children. Child allowances might be an effective way to increase fertility, leading to higher future tax revenues through an increase in the number of younger people. In this paper, the authors examine whether or not child allowances reduce the public debt stock as a share of GDP in an economy with a pension system. As long as a non-negative debt policy is adopted in the long run, child allowances financed by bonds always increase the public debt stock as a share of GDP.

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**Keywords** Public debt; endogenous fertility; child allowances; pension

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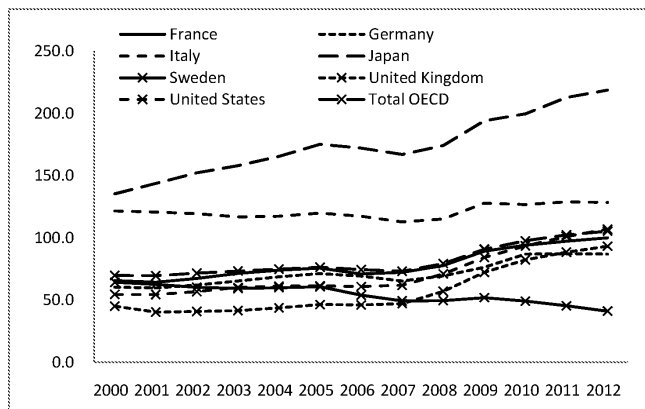
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## 1 Introduction

Public debt accumulation is a social problem in many economically developed countries. After the financial shocks that occurred in 2008, the pace of public debt accumulation has accelerated. General government gross financial liabilities (public debt stock) compared to the Gross Domestic Product (GDP) of OECD countries reached almost 100% in 2012 (Fig. 1). Perhaps more importantly, the accumulation of the public debt stock in Japan became greater than 200% of GDP in 2012.

Some people have expressed anxiety that a country that owes so much debt might declare bankruptcy. To avoid sovereign bankruptcy, some countries must undertake fiscal reform measures. Two basic means exist to decrease the public debt stock per unit of GDP: raising the tax burden and increasing the population. Even if the tax burden per capita remains constant, population growth can bring higher tax revenues. Population growth contributes to economic growth, which increases tax revenues. Therefore, population growth might be more effective at decreasing the public debt stock compared to GDP than an increased tax burden would be.



**Figure 1:** General government gross financial liabilities as a percentage of nominal GDP (Data: OECD Statistics).

Child allowances are commonly regarded as instruments used by governments for family support policies. Some earlier papers have presented examination of the effects of child allowances in the economy with endogenous fertility. Zhang (1997), Oshio (2001), van Groezen et al. (2003), and van Groezen and Meijdam (2008) reported that the fertility level is raised by child allowances in the economy with a pay-as-you-go pension system.<sup>1</sup>

Some earlier research papers have described related aspects of public debt. Diamond (1965) considered public debt in an overlapping-generations model. Samuelson (1958) and Azariadis (1993) examined whether a fiscal policy that brings about fiscal deficit is sustainable or not in terms of fiscal management. Sustainability depends on the primary fiscal deficit and on the gap separating interest rates and the population growth rate. Government expenditures in these models are regarded as public consumption. Similarly, Chalk (2000) and Bräuninger (2005) investigated whether public bond financing is sustainable or not in a model incorporating government consumption. Ono (2003) examined the dynamics of the public debt stock and capital stock and found that they depend on parametric conditions and the initial level of the public debt stock. That paper describes a system under which the government issues public debt to finance the wedge between contributions for public pension and benefits of that in a closed economy with a fixed contribution rate and benefits. Meijdam et al. (1996) examined the dynamics of the public debt stock in a small open economy and derived the manner in which taxation affects the dynamics of public debt. Yakita (2008) investigated public capital formation financed by public debt and examined the sustainability of fiscal management.

Our paper presents development of a model with endogenous fertility and analyzes whether child allowances financed by public debt can reduce the public debt stock per unit of GDP or not in the long run. Child allowances raise fertility and then bring about a population effect that decreases the public debt stock per unit of GDP because an increase in the population of younger people raises income tax revenues. In Japan, total fertility was about 1.4 at 2010, which is a low level in

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<sup>1</sup> However, child allowances can not always raise fertility. Fanti and Gori (2009) found that taxation for children raises fertility because of the income effect in a closed economy. In other words, they showed that a child allowance lowers fertility because of decreased capital per capita, i.e. personal income.

OECD countries.<sup>2</sup> The Japanese government is seeking to raise fertility through provision of child allowances to sustain an aging society. Results of our analyses show that child allowances financed by public debt increase the public debt stock per unit of GDP in the long run if the government targets zero public debt stock in the long run. Indeed, child allowances financed by public debt can raise fertility, and there exists a population effect to decrease the public debt stock per unit of GDP through an increase in tax revenue. However, increased public debt prevents capital accumulation. Consequently, GDP decreases. In short, child allowances financed by public debt can not decrease the public debt stock per unit of GDP because this effect is greater than the population effect. Even if the government targets positive public debt stock per unit of GDP in the long run, child allowances can not decrease the public debt stock per unit of GDP. Therefore, given a non-negative public debt stock policy, a population effect by which revenue rises because of an increase in younger people is small. It is insufficient to reduce the public debt stock per unit of GDP.

The remainder of this paper presents the following. Section 2 establishes the model. Section 3 presents derivation of the equilibrium in a closed economy. Section 4 examines the effect of child allowances on the public debt stock per unit of GDP. The final section presents results.

## **2 The Model**

This model economy consists of a two-period (young and old) overlapping generations model. Three agents exist in this model: households, firms, and a government. In the following subsection, we explain each agent.

### **2.1 Households**

Each household lives in three periods—childhood, young, and old—and supplies labor to earn an income during the young period. Young people supply labor inelastically for consumption during the young period and use savings to consume during the old period, in addition to caring for children. A government provides

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<sup>2</sup> Data: OECD Statistics.

not only a pension system that gives older people a fixed benefit, but also a child allowance for younger people. The budget constraint is given as

$$c_{1t} + \frac{c_{2t+1}}{1+r_{t+1}} + (z_t - q_t)n_t = (1-\tau)w_t + \frac{p_{t+1}}{1+r_{t+1}}. \quad (1)$$

Therein,  $q_t$  denotes the child allowance. Furthermore,  $n_t$  represents the number of children. Necessary goods to bring up a child are represented as  $z_t$ . In addition,  $c_{1t}$  and  $c_{2t+1}$  respectively denote consumption during young and old periods. Here,  $w_t$  shows the wage rate. Interest rate  $1+r_{t+1}$  is returned to savings. Younger people face income taxation (tax rate or contribution rate  $\tau$ ). Older people receive a pension benefit  $p_{t+1}$ . Furthermore,  $t$  signifies the period. We assume that the child-care cost  $z_t$  depends on wage income such as  $z_t = \hat{z}w_t$  ( $\hat{z} > 0$ ).<sup>3</sup> Moreover, the government provides a child allowance as  $q_t = \hat{q}w_t$  ( $\hat{z} > \hat{q} > 0$ ) and pension benefit as  $p_{t+1} = \hat{x}w_t$  ( $\hat{x} > 0$ ).<sup>4</sup> A household's utility function is assumed as

$$u_t = \alpha \ln c_{1t} + \beta \ln c_{2t+1} + (1-\alpha-\beta) \ln n_t, \quad 0 < \alpha, \beta < 1, \quad \alpha + \beta < 1. \quad (2)$$

Under the budget constraint (1), the allocation of  $c_{1t}$ ,  $c_{2t+1}$ , and  $n_t$  to maximize their utility is

$$c_{1t} = \alpha \left( 1 - \tau + \frac{\hat{x}}{1+r_{t+1}} \right) w_t, \quad (3)$$

$$c_{2t+1} = (1+r_{t+1})\beta \left( 1 - \tau + \frac{\hat{x}}{1+r_{t+1}} \right) w_t, \quad (4)$$

$$n_t = \frac{(1-\alpha-\beta) \left( 1 - \tau + \frac{\hat{x}}{1+r_{t+1}} \right)}{\hat{z} - \hat{q}}. \quad (5)$$

<sup>3</sup> van Groezen et al. (2003), Fanti and Gori (2009), and Oshio (2001) also assume the same fixed child-care cost. van Groezen and Meijdam (2008) describe an economy with child-care cost  $z$  as a wage increasing function.

<sup>4</sup> Zhang and Zhang (2007) explain that the assumed pension benefit is practiced by many economically developed countries such as France, Germany, and Japan.  $\hat{x}$  denotes the replacement rate.

## 2.2 Firms

A representative firm produces final good  $Y_t$  with constant returns to scale or a neoclassical product function as

$$Y_t = K_t^\theta (A_t N_t)^{1-\theta}, \quad 0 < \theta < 1, \quad A_t \equiv a \frac{K_t}{N_t}, \quad 0 < a. \quad (6)$$

The firm inputs capital stock  $K_t$  and labor (population size of younger people)  $N_t$ . The productivity  $A_t$  is given as a Romer-type externality, as described by Romer (1986) and Grossman and Yanagawa (1993).  $\theta$  and  $a$  are given exogenously. With a perfectly competitive market, the wage rate  $w_t$  and the interest rate  $r_t$  are

$$w_t = (1 - \theta)a^{1-\theta}k_t \text{ and} \quad (7)$$

$$1 + r_t = \theta a^{1-\theta}, \quad (8)$$

where  $k_t \equiv \frac{K_t}{N_t}$ , and where capital stock depreciates fully in a single period. The interest rate is constant over time. Fertility  $n_t$  is constant ( $n_t = n$ ) also.

## 2.3 Government

The government executes two policies: one for the pension and one for child allowances. A payroll tax rate  $\tau$ , which we can regard as the contribution rate, is levied on younger people. Older people receive pension benefit  $p_t$ . Assuming a balanced budget in each period, the government must change the tax rate to balance the budget. However, allowing a fiscal deficit, the government need not change the tax rate in each period. Therefore, the government budget is shown as

$$b_{t+1} = \frac{1+r}{n}b_t + \frac{\hat{x}w_{t-1}}{n^2} + \left(\hat{q} - \frac{\tau}{n}\right)w_t. \quad (9)$$

$b_t$  denotes the public debt stock per young individual, i.e.,  $b_t \equiv \frac{B_t}{N_t}$ , which  $B_t$  denotes the aggregate public debt stock.<sup>5</sup>  $\frac{\hat{x}w_{t-1}}{n^2} + \left(\hat{q} - \frac{\tau}{n}\right)w_t$  shows the primary balance. Even if this primary balance equals zero, the public debt per capita  $b_t$

<sup>5</sup> Considering  $B_{t+1} = (1+r)B_t + \hat{x}w_{t-1}N_{t-1} + \hat{q}w_t n_t N_t - \tau w_t N_t$ , we obtain (9).

increases when  $1 + r > n$ . In economically developed countries, no population growth exists. Then  $1 + r > n$  holds. Eq. (9) becomes the following equation.

$$b_t = \frac{b_{t+j}}{\left(\frac{1+r}{n}\right)^j} - \sum_{s=1}^j \frac{\frac{\hat{x}w_{t-2+s}}{n^2} + \left(\hat{q} - \frac{\tau}{n}\right)w_{t-1+s}}{\left(\frac{1+r}{n}\right)^s}$$

If a non-Ponzi condition prevails, then we obtain  $\lim_{j \rightarrow \infty} \frac{b_{t+j}}{\left(\frac{1+r}{n}\right)^j}$ . Therefore, a fiscal surplus is necessary to sustain positive  $b_t$ .

### 3 Equilibrium

Having examined the agents' behavior, we proceed to an analysis of the equilibrium. The equilibrium of this economy depends on the amount of capital per capita  $k_t \left( \equiv \frac{K_t}{N_t} \right)$ . Representing the savings per household as  $s_t$ , the capital market clearing condition is given as  $K_{t+1} + B_{t+1} = N_t s_t$  or  $k_{t+1} + b_{t+1} = \frac{s_t}{n_t}$ . Consequently, we obtain the following equation.

$$k_{t+1} + b_{t+1} = \left( \frac{1-\tau}{n} - \frac{(1-\beta)(\hat{z}-\hat{q})}{1-\alpha-\beta} \right) (1-\theta)a^{1-\theta}k_t \quad (10)$$

An increase in  $b_{t+1}$  prevents capital accumulation such that income per capita  $y_t$  and wage rate  $w_t$  decrease. This is a crowding-out effect. Then, the equilibrium is determined by the following two equations,<sup>6</sup>

$$b_{t+1} = Hb_t + Ik_t, \quad (11)$$

$$k_{t+1} = -Hb_t + Jk_t, \quad (12)$$

where

$$H \equiv \frac{\beta(\hat{z}-\hat{q})((1+r)(1-\tau)+\hat{x})}{(1-\alpha-\beta)\left(1-\tau+\frac{\hat{x}}{1+r}\right)\left(\beta(1-\tau)-\frac{(1-\beta)\hat{x}}{1+r}\right)} > 0,$$

<sup>6</sup> See Appendix for a detailed proof.



$$I \equiv \frac{(\hat{z} - \hat{q}) \left( \frac{\hat{x}}{\beta(1-\tau) - \frac{(1-\beta)\hat{x}}{1+r}} - \frac{(1-\theta)(1+r)\tau}{\theta} \right)}{(1-\alpha-\beta) \left( 1 - \tau + \frac{\hat{x}}{1+r} \right)} + \frac{(1-\theta)(1+r)\hat{q}}{\theta},$$

$$J \equiv \frac{(\hat{z} - \hat{q}) \left( \frac{(1-\theta)(1+r) \left( \beta + (1-\beta)\tau - \frac{(1-\beta)\hat{x}}{1+r} \right)}{\theta} - \frac{\hat{x}}{\beta(1-\tau) - \frac{(1-\beta)\hat{x}}{1+r}} \right)}{(1-\alpha-\beta) \left( 1 - \tau + \frac{\hat{x}}{1+r} \right)} - \frac{(1-\theta)(1+r)\hat{q}}{\theta}.$$

$H$  corresponds to  $\frac{1+r}{n}$  in (9) if  $\hat{x} = 0$ . An increase in the child allowance  $\hat{q}$  decreases  $H$  because fertility  $n$  increases. Then  $\frac{1+r}{n}$  decreases.  $I$  corresponds partially to  $\frac{\hat{x}w_{t-1}}{n^2} + \left( \hat{q} - \frac{\tau}{n} \right) w_t$ . Without a pension and no child allowance, this sign becomes negative, which indicates a surplus of the primary balance. An increase in pension  $\hat{x}$  can change this sign from negative to positive. Given  $\tau$ , an increase in  $\hat{x}$  crowds out capital accumulation. Because  $H > 0$  and because  $k_{t+1} \geq 0$ ,  $J > 0$  must be assumed. Defining  $v_t \equiv \frac{b_t}{k_t}$  and considering (11) and (12), we obtain

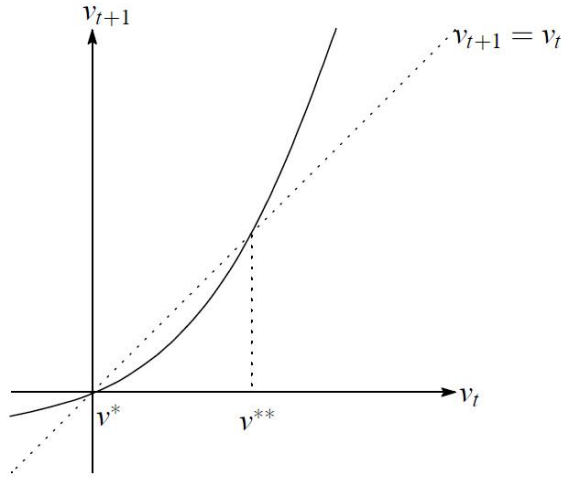
$$v_{t+1} = \frac{Hv_t + I}{-Hv_t + J}. \quad (13)$$

Because of  $k_t = \frac{y_t}{a^{1-\theta}}$  and  $v_t = a^{1-\theta} \frac{B_t}{Y_t}$ , we consider  $v_t$  as the public debt stock/GDP ratio. We consider the equilibrium without child allowances  $\hat{q} = 0$ . Additionally, we consider the economy with  $I = 0$ , which is the primary balanced fiscal policy.<sup>7</sup> Then, we obtain  $\frac{\partial v_{t+1}}{\partial v_t} > 0$  and  $\frac{\partial^2 v_{t+1}}{\partial v_t^2} > 0$ . Assuming  $J > H$ , the dynamics of  $v_t$  is shown in Fig. 2.<sup>8</sup>

Defining  $v^*$  and  $v^{**}$  as the public debt stock per unit of GDP in a stable steady state and that in an unstable steady state respectively, then  $v^* = 0$  and  $v^{**} = \frac{J-H}{H}$ . Considering a stable steady state, this fiscal policy  $I = 0$  gives no public debt stock in the long run. In the following section, we examine whether child allowances can decrease the public debt stock per unit of GDP or not.

<sup>7</sup> The pension benefit  $\hat{x}$  must be held by  $\hat{x} = \frac{\beta\tau(1-\tau)(1+r)}{(1-\beta)\tau + \frac{\theta}{1-\theta}}$  to obtain  $I = 0$ .

<sup>8</sup> The condition to be  $J > H$  is  $\frac{(1-\theta)(1+r) \left( \beta + (1-\beta)\tau - \frac{(1-\beta)\hat{x}}{1+r} \right)}{\theta} > \frac{\beta(1+r)(1-\tau) + (1+\beta)\hat{x}}{\beta(1-\tau) - \frac{(1-\beta)\hat{x}}{1+r}}$ .



**Figure 2:** Dynamics of the public debt/ capital stock ratio.

#### 4 Policy Effects

First, we examine the effect of an increase in child allowances  $\hat{q}$  on  $v^* = 0$  the public debt stock per unit of GDP in the steady state. Calculating  $\frac{dv_{t+1}}{d\hat{q}}$  at an approximation of  $\hat{q} = 0$  for any  $v_t$ , we obtain

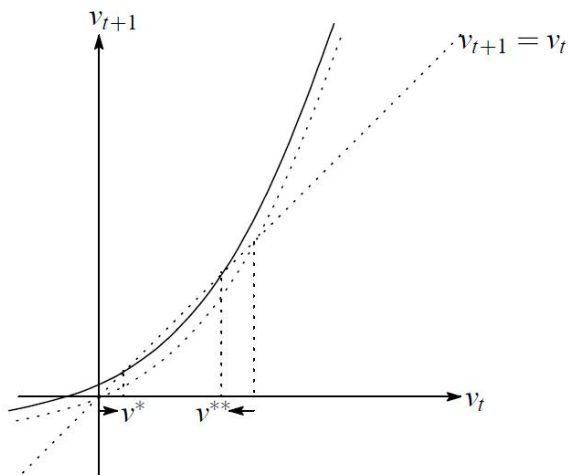
$$\frac{dv_{t+1}}{d\hat{q}} = \frac{HJ}{(-Hv_t + J)^2} > 0. \quad (14)$$

As depicted in Fig. 3, an increase in  $\hat{q}$  increases  $v^*$ .

Then, the following proposition is established.

**Proposition 1** If the public debt stock per unit of GDP is zero in a stable steady state, then an increase in child allowances raises the public debt stock per unit of GDP in a stable steady state.

An increase in child allowances financed by public debt raises the fertility given in an earlier equation (5). Then, population growth increases. The share of older



**Figure 3:** Increase in the public debt/ capital stock ratio.

people to total population shrinks. Tax revenues are growing for pension benefits for older people. Therefore, we can infer that child allowances financed by public debt reduce the public debt stock per unit of GDP in the long run because of an increase in tax revenue given by an increase in the younger population. However, this effect is weak. Moreover, it is dominated by a direct increase in public debt.

Second, we examine the effect of child allowances on the public debt stock per unit of GDP in the economy with  $I > 0$ . If  $I > 0$ , then  $v^*$  is given as a positive value. The government in each country targets some positive public debt per unit of GDP such as the budget rule in the European Union (EU). In the EU, each country must obey the budget rule that the public debt stock per unit of GDP be less than 60% and that the annual fiscal deficit be less than 3%. Therefore, in addition to the policy that the government targets no public debt stock in the long run, it is

important to examine the fiscal budget rule  $v^* > 0$  as a realistic target. Calculating  $\frac{dv^*}{d\hat{q}}$  at an approximation of  $\hat{q} = 0$  in  $I > 0$ , we obtain the following<sup>9</sup>

$$\frac{dv^*}{d\hat{q}} = \left( \frac{(1-\theta)(1+r)}{\theta} \right)^2 \frac{\hat{z} \left( \beta(1-\tau) - \frac{(1-\beta)\hat{x}}{1+r} \right)}{(1-\alpha-\beta) \left( 1-\tau + \frac{\hat{x}}{1+r} \right) \left( 1 - \frac{H(J+I)}{(-Hv^*+J)^2} \right)} > 0. \quad (15)$$

Then, the following proposition is established.

**Proposition 2** We assume an economy with positive  $v^*$  in a steady state. An increase in child allowances raises the public debt stock per unit of GDP.

Regarding non-negative public debt stock per unit of GDP, an increase in child allowances raises the public debt stock per unit of GDP in the steady state  $v^*$ . As shown by  $I$ , the first term in  $I$  includes  $\hat{q}$ . With  $I > 0$ , this first term is positive unless child allowances are not provided. This positive term shows that payments for older people as pension benefits are greater than tax revenue. An increase in  $\hat{q}$  shrinks this term. This effect reduces  $b_{t+1}$ , i.e.,  $v^*$ . An increase in  $\hat{q}$  increases fertility. An increase in population growth raises tax revenues from younger people, compared with pension benefits for older people. However, in the endogenous growth model given by production function (6), the population effect that reduces the public debt stock is small.

## 5 Concluding Remarks

This paper describes an endogenous fertility model with a pay-as-you-go pension model including public debt. It examines the effects of child allowances. Child allowances are used to raise fertility. A government in an economically developed country considers that an increase in fertility brings about an increase in tax revenue in the future because of an increase in younger people. This policy therefore copes with an aging society. This paper presents analysis of whether child allowances financed by public debt can decrease the public debt stock per unit of GDP by

<sup>9</sup> The stable condition holds  $1 - \frac{H(J+I)}{(-Hv^*+J)^2} > 0$ .

virtue of an increase in younger people or not. However, this paper presents one derivation that child allowances financed by public debt raise the public debt stock per unit of GDP in the model with no public debt policy in the long run. Even if the government adopts a positive public debt stock policy in the long run, child allowances can not reduce the public debt stock per unit of GDP. Therefore, an increase in the younger population given by child allowances does not make sense for reduction of the public debt stock per unit of GDP. Child allowances increase the public debt stock directly, which constitutes a direct and immediate effect. However, child allowances raise fertility. Therefore, the ratio of younger people to older people increases. Consequently, tax revenue increases for pension benefit for older people. Child allowances have the effect of decreasing the public debt stock, which is an indirect effect. Our results show that the indirect effect is less than the direct effect. For that reason, child allowances can not reduce the public debt stock per unit of GDP.

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## Appendix

Considering (10), the following equation is obtained.

$$k_{t-1} = \frac{k_t + b_t}{(1 - \theta)a^{1-\theta} \left( \frac{1-\tau}{n} - \frac{(1-\beta)(\hat{z}-\hat{q})}{1-\alpha-\beta} \right)}. \quad (16)$$

Substituting (5), (7), and (16) into (9), the following equation is obtained.

$$\begin{aligned} b_{t+1} &= \frac{1+r}{n}b_t + \frac{\hat{x}}{n^2} \frac{k_t + b_t}{\frac{1-\tau}{n} - \frac{(1-\beta)(\hat{z}-\hat{q})}{1-\alpha-\beta}} + \left( \hat{q} - \frac{\tau}{n} \right) (1 - \theta)a^{1-\theta}k_t \\ &= \frac{\beta(\hat{z} - \hat{q})((1 - \tau)(1 + r) + \hat{x})}{(1 - \alpha - \beta) \left( 1 - \tau + \frac{\hat{x}}{1+r} \right) \left( \beta(1 - \tau) - \frac{(1-\beta)\hat{x}}{1+r} \right)} b_t \\ &\quad + \left[ \frac{(\hat{z} - \hat{q}) \left( \frac{\hat{x}}{\beta(1-\tau) - \frac{(1-\beta)\hat{x}}{1+r}} - \frac{(1-\theta)(1+r)\tau}{\theta} \right)}{(1 - \alpha - \beta) \left( 1 - \tau + \frac{\hat{x}}{1+r} \right)} + \frac{(1 - \theta)(1 + r)\hat{q}}{\theta} \right] k_t. \end{aligned}$$

Substituting (5) and (11) into (10), the following equation is obtained.

$$\begin{aligned} k_{t+1} &= -Hb_t + \frac{1}{n} \frac{(1 - \theta)(1 + r)}{\theta} \left[ 1 - \tau - (1 - \beta) \left( 1 - \tau + \frac{\hat{x}}{1+r} \right) \right] k_t - Ik_t, \\ &= -Hb_t + \left[ \frac{(\hat{z} - \hat{q}) \left( \frac{(\beta + (1-\beta)\tau - \frac{(1-\beta)\hat{x}}{1+r})(1-\theta)(1+r)}{\theta} - \frac{\hat{x}}{\beta(1-\tau) - \frac{(1-\beta)\hat{x}}{1+r}} \right)}{(1 - \alpha - \beta) \left( 1 - \tau + \frac{\hat{x}}{1+r} \right)} \right. \\ &\quad \left. - \frac{(1 - \theta)(1 + r)\hat{q}}{\theta} \right] k_t. \end{aligned}$$

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