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How to Counter Union Power? Equilibrium Mergers in International Oligopoly

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How to Counter Union Power? Equilibrium Mergers in International Oligopoly*

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April 2013

Abstract

We re-examine the common wisdom that cross-border mergers are the most effective merger strategy for firms facing powerful unions. In contrast, we obtain a domestic merger outcome whenever firms are sufficiently heterogeneous (in terms of productive efficiency and product differentiation). A domestic merger unfolds a “wage-unifying” effect which limits the union’s ability to extract rents. When asymmetries among firms vanish, then cross-border mergers are the unique equilibrium. However, they may be either between symmetric or asymmetric firms. Social welfare is never higher under a domestic merger outcome than under a cross-border merger outcome.


Keywords: Unionization, International Oligopoly, Endogenous Mergers, Countervailing Power.

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1 Introduction

We re-examine the question whether national or international mergers should be expected in the presence of powerful unions. Lommerud et al. (2006) argue in favor of an “only cross-border merger” equilibrium. By creating an “outside option” abroad, an international firm can threaten to move production into a different country which creates downward pressure on domestic wage demands. In contrast, we show that a domestic merger exhibits a “wage-unifying” effect which may more effectively counter union power than an international merger. For the wage-unifying effect to arise it is necessary that the merging firms differ with regard to their productive efficiency. Moreover the effect is re-enforced by product differentiation. An “only domestic merger” equilibrium then exists, in which asymmetric firms producing differentiated products merge in their home country to counter union power.

The wage-unifying effect of a merger is sometimes a direct result of labor law. For instance, in Germany the tariff unity (“Tarifeinheit”) principle stipulates that only one collective agreement should apply within a firm to the same type of labor. Accordingly, a merged entity will “unify” labor contracts simply by the fact that it must reach a new collective agreement which then applies to all its employees. A recent example is the RWTÜV/ TÜVNord merger in 2011. Both firms had different collective agreements before the merger. After the merger, a new collective wage agreement was concluded with the services labor union Verdi. That collective contract defines a uniform wage profile for all workers of the merged firm (see Verdi, 2011).

Another recent example of the wage-unifying effect is the creation of Vattenfall Europe in Germany. The merger included previously independent public utility operators BEWAG, HEW and LAUBAG. Before the merger, employees at BEWAG and HEW enjoyed much better working conditions and higher wages than those employed by LAUBAG. Right after the merger, Vattenfall Europe announced in a restructuring plan that it wants to reach a new collective agreement for the entire group to reduce wage levels at HEW and BEWAG locations.

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1The adjustment towards a more uniform wage structure after a merger may take some time as workers are protected to some extent by previous collective agreements (see Haucap et al., 2007, for more details on German labor market institutions).

2HEW (Hamburg) and BEWAG (Berlin) were located in former West Germany and West Berlin, respectively, whereas LAUBAG was active in former East Germany (Senftenberg/Brandenburg).

3Immediately after its formation Vattenfall Europe announced that wages at BEWAG and HEW locations had
4th, 2010 the daily newspaper *Der Tagesspiegel* published an interview under the title “*Die BEWAG war am großzügigsten*” (“The BEWAG was most generous”) with the Head of Human Resources at Vattenfall Europe, Mr. Udo Bekker. In that interview Mr. Bekker stated that the new tariff agreement concluded in 2007 has reduced annual salaries of employees at former BEWAG locations by 7,500 Euro. He also reported salary cuts at former HEW locations in Hamburg of about 2,000 Euro.\(^4\)

Even in the absence of a legal provision as the tariff unity principle in Germany, the wage-unifying effect should be considered as a part of a (domestic) merger. First, unions have strong preferences for egalitarian wage-setting and it can be expected that this objective is most effective at the firm-level.\(^5\) Second, there is some casual evidence that a unifying effect is also present in non-labor input markets. It should be expected that right after a merger contractual relations with suppliers are compared. If a certain supplier was able to discriminate before the merger, then the merged entity should be able to renegotiate contractual terms to the better. Such a behavior was expected by most suppliers according to an investigation conducted by the German Federal Cartel Office in association with its decision on the *EDEKA/Tengelmann* merger (see Bundeskartellamt, 2008).

By considering the uniformity effect of domestic mergers, our model combines aspects from the literature on price-discrimination in input markets (e.g., Yoshida, 2000) and downstream mergers in vertically related industries. More specifically, our paper builds on a growing literature which analyzes mergers in a vertical structure where upstream firms (or unions in the case of labor) have market power vis-à-vis downstream oligopolists.\(^6\) Making the vertical structure to be reduced significantly in order to align them with the much lower wage levels at LAUBAG. See newspaper article “Vattenfall plant neuen Tarifvertrag,” *Hamburger Abendblatt*, 3 January 2006, online article (available at: http://www.abendblatt.de/wirtschaft/article372957/Vattenfall-plant-neuen-Tarifvertrag.html).

\(^4\) The interview is available online (http://www.tagesspiegel.de/wirtschaft/unternehmen/udo-bekker-die-bewag-war-am-grosszuegigsten/1712310.html).

\(^5\) The trade union principle “equal pay for equal work” summarizes this nicely. See Freeman (1982) for an early empirical study which shows that unionism reduces within-establishment wage dispersion.

\(^6\) Works which assume linear wholesale prices (or, the right-to-manage approach in the case of labor) include Horn and Wolinsky (1988a), Dobson and Waterson (1997), von Ungern-Sternberg (1997), Zhao (2001), and Symeonidis (2010). Another approach is to assume “efficient contracts” in input market relations (see, for instance, Horn and Wolinsky, 1988b) which avoids double marginalization issues.
explicit this literature has uncovered new incentives for downstream mergers resulting from improved purchasing conditions on input markets. We depart from those works by analyzing an international setting and we apply the approach of endogenous merger formation as put forward by Horn and Persson (2001a, 2001b).  

We extend Lommerud et al. (2006) by considering asymmetric firms. Lommerud et al. (2006) analyze a two-country model with four symmetric firms (two in each country) each producing an imperfect substitute. In each country a monopoly union sets wages at the firm level. Within such a symmetric setting, Lommerud et al. (2006) obtain their main result that the endogenous merger equilibrium only exhibits cross-country mergers. Under the resulting market structure wages reach their minimum as both merged firms can most effectively threaten to scale up production abroad if a union raises its wage.

By allowing for asymmetric firms in each country, we qualify the “only cross-border merger” result as follows: First, given that products are sufficiently differentiated, a domestic merger equilibrium follows whenever cost asymmetries between national firms are large enough. Second, as products become more substitutable, the cross-country merger equilibrium becomes more likely; however, both a symmetric and an asymmetric cross-border merger outcome are possible. If products are close substitutes, then a cross-border merger induces intense competition between the unions to the benefit of the international firm. If, however, products become more differentiated the “threat-point” effect of “internal” union competition becomes less effective. Considering cost asymmetries gives then rise to our main result that a domestic merger equilibrium emerges.

From the perspective of the low-cost firm, a national merger with the high-cost firm becomes attractive as this constrains the wage demand of the domestic union. It is, therefore, the wage-
unifying effect of a domestic merger that prevents the labor union from extracting rents from a low-cost plant in order to maintain employment at a high-cost plant. The merged entity can partially shift production domestically from a less towards a more efficient plant, rendering a domestic merger even more profitable.\textsuperscript{10} As a result, depending on cost asymmetries among firms and the degree of product differentiation, we find that either domestic or cross-border mergers may result in equilibrium.

There is some empirical evidence that the internationalization of firms unfolds negative effects on wages, so that it may serve as a mean to counter union power. For instance, Clougherty et al. (2011) show that international mergers unfold a threat effect which increases international firms’ bargaining power vis-à-vis unions.\textsuperscript{11} Concerning domestic merger outcomes, we note two empirical observations which are aligned with our finding. First, while cross-border mergers have become increasingly important, the major amount of mergers and acquisitions is still domestic in nature (Gugler et al., 2003; UNCTAD, 2012). Second, mergers typically occur between rather asymmetric firms which is documented in Gugler et al. (2003) who report that target firms are on average only 16 percent of the size of their acquirers.

The remainder of this paper is organized as follows. In the following Section, we present the basic model and the cooperative merger formation process. Firms’ merger incentives, in the form of wage and employment effects of different merger types, are analyzed in Section 3. Based on these findings, we determine the equilibrium industry structure and discuss the welfare implications of our results in Sections 4 and 5. Finally, Section 6 offers a short discussion and concluding remarks.

\textsuperscript{10}Breinlich (2008) has shown that the liberalization of trade between the US and Canada triggered substantial merger activity between asymmetric firms in Canada. Those mergers allowed for an optimal re-allocation of production which strengthened the merged entities’ competitiveness vis-à-vis US firms.

\textsuperscript{11}Recent empirical labor research obtains mixed results concerning the relationship between labor demand and internationalization of firms. Fabbri et al. (2003) provide an empirical study which shows that labor demand of UK and US firms for low skilled workers between 1958 and 1991 (UK data are available until 1986) has become more elastic. They argue that increased activity of multinational firms is (partially) responsible for this trend. Barba-Navaretti et al. (2003) provide a cross-country firm-level study of European countries where they find that multinationals adjust their labor demand more rapidly than domestic firms in response to shocks. However, they report a more inelastic demand curve with respect to wages for multinationals which they contribute to differences in skill structure.
2 The Model

We consider an oligopolistic industry with initially four independent firms, \( i \in N = \{1, 2, 3, 4\} \). Each firm operates one single plant and produces one variant of a differentiated good. There are two countries \( A \) and \( B \). Firms 1 and 2 are located in country \( A \), while firms 3 and 4 reside in country \( B \).

Firms compete in quantities in an internationally integrated product market. This set-up resembles a “third-market” model (see e.g. Brander, 1995). The (inverse) demand function for product \( i \) is given by

\[
P_i = 1 - q_i - \beta \sum_{k \in I \setminus \{i\}} q_k \quad \text{for all } i \in N,
\]

where \( q_i \) denotes the quantity supplied by plant \( i \), and \( \beta \in (0, 1) \) measures the degree of product differentiation. As \( \beta \) approaches 1, products become perfect substitutes, while for \( \beta \to 0 \) products are virtually independent.

Firms use labor and non-labor inputs in fixed proportions to produce the good. We consider a constant-returns-to-scale production technology, such that one unit of output of product \( i \) requires one unit of labor at wage \( w_i \) and one unit of a non-labor input at unit-price \( c_i \). Firms differ in their non-labor production costs. We assume that firms 1 and 3 are the low-cost firms with \( c_1 = c_3 = 0 \), while firms 2 and 4 are the high-cost producers, with \( c_i = c \geq 0 \) for \( i = 2, 4 \).\(^{12}\)

We can express firm \( i \)'s cost function (with \( i \in N \)) as

\[
C_i(q_i) = [w_i + D(i)c]q_i \quad \text{with } D(i) := \begin{cases} 1, & \text{if } i = 1, 3 \\ 0, & \text{if } i = 2, 4 \end{cases}.
\]

Note that \( D(i) \in \{0, 1\} \) is an indicator such that \( D(i) = 1 \) for the high-cost firms \( i = 2, 4 \) and \( D(i) = 0 \) for the low-cost firms \( i = 1, 3 \).\(^{13}\) The profit function of firm \( i \) is thus given by

\[
\pi_i = [p_i(\cdot) - w_i - D(i)c]q_i \quad \text{for all } i \in I.
\]

\(^{12}\) For \( c = 0 \), all firms are ex ante identical and we are back in the model analyzed by Lommerud et al. (2006).

\(^{13}\) We abstract from the option that mergers induce efficiency gains with respect to marginal costs. We calculated another version of this model where mergers induced marginal cost savings for the high cost plants to \( \mu c \), where \( \mu \in (0, 1) \) measures the degree of efficiency gains. Our results are not affected by the introduction of merger synergies, only the scope for domestic mergers is reduced the larger the cost savings through mergers becomes. The results are available from the authors upon request.
Workers are organized in centralized labor unions in their respective countries.\textsuperscript{14} We consider a monopoly union model and we adopt the right-to-manage approach, which stipulates that labor unions set wages for the firms residing in their countries, whereas the responsibility to determine employment remains with the firms. Unions make take-it or leave-it wage offers to firms to maximize their wage bills. The wage-setting of labor unions adjusts to the industry structure which the plant owners determine cooperatively.

Wage setting depends crucially on whether or not a domestic merger occurs. In market structures without a domestic merger (i.e., in which either cross-border mergers or no merger has taken place), each labor union \( j = A, B \) sets a firm-specific (and hence, plant-specific) wage to maximize its wage bill

\[
U_j = \sum_i w_i q_i(D(i)),
\]

where \( i = 1, 2 \) in country \( j = A \) and \( i = 3, 4 \) in country \( j = B \). We denote by \( w_i \) the wage paid by firm \( i \) and \( q_i(D(i)) \) is the labor demand of firm \( i \) which depends on its non-labor costs.\textsuperscript{15}

If a domestic merger occurs, then the labor union offers a uniform wage rate to the merged entity which now operates two asymmetric plants.\textsuperscript{16} The labor union \( j \)'s wage bill in those cases is then given by

\[
U_j = w_j \sum_i q_i(D(i)),
\]

with \( i = 1, 2 \) in country \( j = A \) and \( i = 3, 4 \) in country \( j = B \), where \( w_j \) is the uniform wage rate in country \( j \in \{A, B\} \).

We analyze the following three-stage game. In the first stage, firms merge in pairs according to the cooperative merger formation process proposed by Horn and Persson (2001a, 2001b).\textsuperscript{17}

\textsuperscript{14}A crucial assumption is that workers are unable to organize in unions across borders. Although there have been attempts towards more cooperation among labor unions at a European level, in general, labor market regimes are bound locally at the national level (Traxler and Mermet, 2003).

\textsuperscript{15}Workers’ reservation wages are normalized to zero.

\textsuperscript{16}In an industry structure with two domestic mergers, both unions set uniform wages. In contrast, when only one domestic merger has occurred (and the plants in the second country stay independent) only the union in whose country a merger has taken place sets a uniform wage rate. The second union sets two separate plant-specific wage rates.

\textsuperscript{17}That is, we only allow mergers between two firms, so that the most concentrated market is a duopoly. We are interested in highlighting the incentives for domestic versus cross-border mergers and the role asymmetries
In the second stage, labor unions simultaneously and non-cooperatively set wages after having observed the outcome of the merger process. Finally, in the third stage of the game, firms compete in quantities in the final product market ("Cournot competition").

We solve for the subgame perfect equilibrium. In the third stage of the game we obtain a unique quantity vector depending on the market structure and wages. In the second stage of the game, unions set wages depending on the market structure while foreseeing firms’ subgame perfect quantity choices. In the first stage, a merger formation process applies, in which all parties foresee perfectly unions’ wage demands and optimal Cournot quantities depending on the resulting market structure.

**Merger formation process.** We apply the method developed in Horn and Persson (2001a, 2001b) by modelling the merger formation process as a cooperative game of coalition formation. An ownership structure $M^r$ describes a partition of the set $N$ into voluntary coalitions. As in Lommerud et al. (2006), we consider only two-firm mergers. We obtain ten such partitions, two being mirror images, which leaves us with eight relevant industry structures of the merger formation process:18

1. no merger: $M^0 = \{1, 2, 3, 4\}$,

2. one domestic merger: $M^{D1} = \{12, 3, 4\}$ or $M^{D1'} = \{1, 2, 34\}$,

3. two domestic mergers: $M^{D2} = \{12, 34\}$,

4. one symmetric cross-border merger between the efficient firms: $M^{C1se} = \{13, 2, 4\}$,

5. one symmetric cross-border merger between the inefficient firms: $M^{C1si} = \{1, 3, 24\}$,

between firms play in this formation process. If firms have the opportunity to monopolize the market, an all-encompassing merger is the obvious outcome, regardless of firm asymmetries. In addition, three- or four-firm mergers are more likely to be blocked by antitrust authorities. Finally, cost of administering a merger may grow overproportionally making mergers of three or four plants unprofitable.

18We use the following abbreviations for $r$ to describe a market structure $M^r$. A merger can be domestic ($D$) or cross-border ($C$) and there can be one merger ($D1$; $C1$) or two mergers ($D2$; $C2$) in either case. If two cross-country mergers occur, then they can be symmetric ($C2s$) or asymmetric ($C2a$). If one cross-border merger occurs, then it can be symmetric ($C1s$) or asymmetric ($C1a$). Finally, in case of a single cross-border merger between symmetric firms it can be either between the efficient firms ($C1se$) or between the inefficient ($C1si$).
6. two symmetric cross-border mergers: $M^{C2s} = \{13, 24\}$,

7. one asymmetric cross-border merger: $M^{C1a} = \{14, 2, 3\}$ or $M^{C1a'} = \{1, 4, 23\}$, and

8. two asymmetric cross-border mergers: $M^{C2a} = \{14, 23\}$.

As firms are not symmetric, cross-border mergers can take place in different constellations.\(^{19}\) First, firms with the same non-labor production costs can merge, which we call symmetric cross-border mergers (cases 4., 5. and 6.). When there is only one international symmetric merger, it can either be the two efficient ($M^{C1se}$) or the two inefficient ($M^{C1si}$) firms that merge. The ownership structure with two mergers between the symmetric (low-cost and high-cost) firms is represented by structure $M^{C2s}$. Thus, in structure $M^{C2s}$ there is one firm producing brands 1 and 3 at low costs, and one firm producing brands 2 and 4 at high costs.

Second, there can be cross-border mergers between two firms of different cost types, which we call asymmetric cross-border mergers (cases 7. and 8.). If there is only one asymmetric cross-border merger, the outcome is obviously identical for structures $M^{C1a}$ and $M^{C1a'}$. Industry structure $M^{C2a}$ indicates that there have been two cross-border mergers each between one low-cost and one high-cost firm. As a result each merged firm produces one brand at low cost and the other brand at high cost.

The determination of the outcome of the cooperative merger formation process is based on dominance relations between the partitions of $N$. If an ownership structure is dominated by another structure, it cannot be the equilibrium outcome of the cooperative merger formation game. The approach involves a comparison of each structure $M^{r}$ against all other structures $M^{-r}$ separately. $M^{r}$ dominates a structure $M^{r'}$ if the combined profits of the decisive group of owners in structure $M^{r}$ exceeds those in structure $M^{r'}$.

Decisive owners can influence which coalition is formed. All firm owners which belong to identical coalitions in ownership structures $M^{r}$ and $M^{r'}$ are not decisive. By that we exclude the possibility of transfer payments among all firms.\(^{20}\) Within a coalition of firms, owners are

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\(^{19}\)When firms are symmetric, then partitions $M^{C1se}$, $M^{C1si}$, and $M^{C1a}$ are structurally equivalent. The same holds for structures $M^{C2s}$ and $M^{C2a}$.

\(^{20}\)Clearly, if we allow for transfers between all firms, then the equilibrium structure is the one which maximizes industry profits.
free to distribute the joint profit among each other. Thus, an industry structure $M^r$ dominates another structure $M^{r'}$ if the decisive group of owners prefers $M^r$ over $M^{r'}$ which is the case if the combined profit of this group is larger in $M^r$ than in $M^{r'}$.

Applying the bilateral dominance relationship, it is possible to rank different ownership structures. We then search for the equilibrium industry structure (EIS) which is undominated. Undominated structures belong to the core of a cooperative game of coalition formation where the characteristic function follows from the subgame perfect strategies unions and firms choose for a given industry structure.

**Parameter restriction.** A well-known problem associated with a uniform input price (or wage) is that the input supplier (or union) may prefer to set a price (wage) so high that the less efficient plant is shut down.\(^{21}\) In our model, this issue arises in structures when domestic firms merge and marginal non-labor cost, $c$, of the inefficient firm becomes large. The following assumption ensures that all plants $i \in I$ produce strictly positive quantities under all market structures.\(^{22}\)

**Assumption 1.** The high-cost firms’ marginal cost, $c$, fulfills $0 < c < \overline{c}(\beta)$. The critical value $\overline{c}(\beta)$ is monotonically decreasing in $\beta$, with $\lim_{\beta \to 0} \overline{c}(\beta) = 2 - \sqrt{2}$ and $\lim_{\beta \to 1} \overline{c}(\beta) = 0$.

We maintain Assumption 1 throughout the entire analysis. In Appendix B we show that the critical value $\overline{c}(\beta)$ is derived from market structure $M^{D1}$. In case of a single domestic merger, the union has the strongest incentive to raise the uniform wage rate up to a level which makes production at the high-cost plant unprofitable. By assuming $c < \overline{c}(\beta)$ we ensure that the union prefers a relatively low wage rate which keeps the inefficient plant active.

Before we analyze the equilibrium of the merger formation process, we present the following preliminary result. All proofs are relegated to the Appendix.

**Lemma 1.** The no-merger ($M^0$) and all one-merger structures ($M^{D1}, M^{C1se}, M^{C1si}$, and $M^{C1a}$) are dominated by at least one two-merger structure ($M^{D2}, M^{C2a}$, or $M^{C2s}$).

A comparison of profit levels reveals that industry structures involving two mergers ($M^{D2},

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\(^{21}\)For instance, Haucap et al. (2001) show that a union may have an incentive to raise a uniform industry-wide wage rate above a certain level to drive inefficient firms out of the market.

\(^{22}\)We provide the derivation of Assumption 1 in Appendix B.
$M^{C2a}$, and $M^{C2s}$) unambiguously provide higher total profits for the decisive group of firms than industry structures in which more than two firms prevail in the market. The equilibrium outcome of the merger formation process will therefore always result in a downstream duopoly. As a consequence, when analyzing possible candidates for equilibrium industry structures, only structures with two merged firms have to be considered. Therefore, we restrict our attention in the following analysis to the three candidate equilibrium industry structures: $M^{D2}$, $M^{C2a}$ and $M^{C2s}$, i.e., we focus on the incentives for either two domestic or two cross-border mergers, where we distinguish between coalitions of symmetric plants (two efficient and two inefficient plants merge) and coalitions between asymmetric plants (one efficient producer merges with one inefficient producer each).

3 Merger Incentives

We solve our model for all possible industry structures in Appendix A. As we focus on the driving forces behind domestic and cross-border mergers when firms are asymmetric, it will be instructive to analyze first of all the impact of different types of mergers on wages and employment.

3.1 Wage and Employment Effects

As wage rates are determined endogenously, unions may react to each market structure by adjusting their wage demands accordingly. How do different types of mergers affect wage rates? As we can restrict attention to two-merger structures, wage rates in countries $A$ and $B$ are always symmetric in equilibrium. However, there can be differences in the wage rates paid by efficient and inefficient plants if labor unions set plant-specific wages (i.e., in structures $M^{C2s}$ and $M^{C2a}$). In those cases, we use subscript $I$ to indicate wages paid by inefficient plants (plants 2 and 4) and subscript $E$ to indicate wages paid by efficient plants (1 and 3). As there is only one equilibrium uniform wage for $M^{D2}$, we do not use a subscript in this case.

When we compare the wage rates set by the labor unions in countries $A$ and $B$ for structures $M^{D2}$, $M^{C2s}$ and $M^{C2a}$, we find that the plant-specific wages in industry structures involving cross-border mergers can be ranked unambiguously. When including the uniform wage set for domestic merger participants, the ranking is not distinctly possible. The relation between
the wage rates in the different industry structures then depends on the degrees of product

differentiation ($\beta$) and cost asymmetry between firms ($c$).

Proposition 1. Consider all market structures with two mergers, i.e., $M^{D2}$, $MC^{2s}$ and $MC^{2a}$. Then, equilibrium wages can be ranked as follows:

i) The ranking of wage rates set by labor unions in structures $MC^{2s}$ and $MC^{2a}$ is unambiguously given by $w_E^{C2a} > w_E^{C2s} > w_I^{C2s} > w_I^{C2a}$.

ii) The equilibrium wage under structure $M^{D2}$ is always larger than the equilibrium wage of the inefficient firms under market structures $MC^{2s}$ and $MC^{2a}$, i.e., $w^{D2} > w_I^{C2s} > w_I^{C2a}$ holds always.

iii) The comparison of the equilibrium wage under structure $M^{D2}$ with the equilibrium wage of the efficient firms under market structures $MC^{2s}$ and $MC^{2a}$ depends on two uniquely determined critical values $c_1(\beta)$ and $c_2(\beta)$, with $c_1(\beta) > c_2(\beta) > 0$ such that $w_E^{C2s} > w^{D2}$ holds for $c > c_1(\beta)$, $w_E^{C2a} > w^{D2} > w_E^{C2s}$ holds for $c_1(\beta) > c > c_2(\beta)$, and $w^{D2} > w_E^{C2a}$ holds for $c < c_2(\beta)$.

Moreover, $c_1(\beta \to 0) = c_2(\beta \to 0) = 0$ and $c_1(\beta)$ and $c_2(\beta)$ are monotonically increasing.

Part i) of Proposition 1 compares both cross-country merger structures and says that efficient plants pay unambiguously higher plant-specific wage rates than inefficient plants. Quite obviously, as labor unions can discriminate in case of cross-country mergers, they are able to extract a higher surplus from efficient plants. Post-merger wages depend on which type of plants have formed a coalition. Recall that in structure $MC^{2a}$ each merged firm operates one efficient and one inefficient plant. To save on non-labor cost of production, each merged firm will partially reallocate production from the high- to the low-cost plant. The magnitude of this reallocation depends on the degree of substitutability between brands. Consequently, the efficient plants increase their market shares in $MC^{2a}$ giving labor unions the opportunity to raise wages $w_E^{C2a}$ while balancing wage demands and respective effects on employment.

In contrast, in structure $MC^{2s}$ firms of the same cost type merge and do not create an option to reallocate production among each other to save on non-labor cost. Unions adjust their wage demands to these different constellations of ownership. The respective production shifting opportunities in the two structures yield higher wages for efficient plants in $MC^{2a}$ than $MC^{2s}$. For inefficient plants, obviously the reverse holds true.
Comparing the uniform wage $w^{D2}$ with the plant-specific wage rates under cross-border merger structures is less easy. Part \textit{ii)} of Proposition 1 shows that the wage in case of domestic mergers is always larger than the wage which prevails at the inefficient plant in case of cross-country mergers. Hence, a domestic merger outcome is always good news for employees at inefficient firms which would otherwise suffer from wage cuts in case of cross-country mergers.

Part \textit{iii)} of Proposition 1 shows that the comparison of the wages at the efficient plants depends on both the cost asymmetries and product differentiation. Figure 1 illustrates the different rankings. The three areas in Figure 1 follow from Proposition 1, such that the following orderings hold:

- **Area A**: $w_E^{C2a} > w_E^{C2s} > w^{D2} > w_I^{C2s} > w_I^{C2a}$;
- **Area B**: $w_E^{C2a} > w^{D2} > w_E^{C2s} > w_I^{C2s} > w_I^{C2a}$;
- **Area C**: $w^{D2} > w_E^{C2a} > w_E^{C2s} > w_I^{C2s} > w_I^{C2a}$.

A domestic merger allows the merged entity to reallocate production domestically towards the more efficient plant. The union has an incentive to balance this threat of production shifting by adjusting the uniform wage rate downward. As part \textit{iii)} of Proposition 1 shows, union power is most effectively constrained through a domestic merger when products are sufficiently
differentiated and/or firms are sufficiently asymmetric. In Figure 1, area $A$ represents all parameter constellations where a domestic merger allows to operate the efficient firms at the lowest possible wage level. If the labor unions were allowed to discriminate between efficient and inefficient firms in that area (as it is the case when firms merge cross-border), then unions would optimally increase the wage rates at the efficient plant in anticipation of increased production. Thus, as can be seen from Figure 1, for higher values of $c$, $w^{D2}$ is driven below the levels of $w^{C2a}_E$ and $w^{C2s}_E$.

The reason for this result is that the non-labor cost of the inefficient firms affects wage rates differently. Note that

$$\frac{\partial w^{D2}}{\partial c} = -\frac{1}{4 + 2\beta} < 0$$

for $\beta \in (0, 1)$. When the non-labor cost of production of the inefficient plants marginally increases, the wage rate paid by the merged firm falls. As uniformity of wages restricts the labor union in exploiting the production efficiency of the low-cost producer, it limits its wage demand when firms become more asymmetric in order to maintain employment at the high-cost plant. In contrast, in cross-border merger structures, low-cost plants’ wages rise if non-labor costs of high-cost plants increase; i.e., in equilibrium it holds that

$$\frac{\partial w^{C2a}_E}{\partial c} = \frac{\beta}{4 - \beta} > 0$$

and

$$\frac{\partial w^{C2s}_E}{\partial c} = \frac{2\beta(1 - \beta)}{(4 - \beta)(4 - 3\beta)} > 0.$$ 

Next to the impact of firm asymmetry and uniformity of wages, a merger further affects the choice of wage rates through changes in the elasticities of labor demand at the merged firms. Different merger types may result in different changes in labor demand elasticities due to the relation between national labor unions and international firms. While for a domestic merger plants with relation to the same labor union merge, cross-border mergers induce rivalry between nationally organized labor unions due to the threat of moving production abroad.

To analyze the changes in labor demand elasticities, first consider structure $M^{D2}$ in relation to the no-merger case. Using the results for derived labor demands presented in Appendix A, we can write the slopes of the labor demand curves as follows,

$$\frac{\partial q^0_{I,E}}{\partial w^0_E} = \frac{\partial q^0_{I,I}}{\partial w^0_I} = \frac{2 + 2\beta^2}{(\beta - 2)(2 + 3\beta)},$$

and

$$\frac{\partial q^{D2}_{I,E}}{\partial w^{D2}} = \frac{\partial q^{D2}_{I,I}}{\partial w^{D2}} = \frac{2 - 2\beta^2}{4(\beta - 1)(1 + 2\beta)}.$$
for the pre- and post-merger cases, where $q_E^r$ and $q_I^r$, $r = \{D, 2\}$, are the derived labor demand functions of efficient and inefficient plants, respectively. Comparison of the two expressions reveals that

$$\left| \frac{\partial q_D^2}{\partial w} \right| - \left| \frac{\partial q_0}{\partial w} \right| = \frac{\beta(1 + \beta)(4 + 3\beta)}{2(-4 - 12\beta - 5\beta^2 + 6\beta^3)} < 0.$$  

Reduced product market competition after the two domestic mergers reduces the responsiveness of firms’ labor demand. Ceteris paribus, labor demand becomes less elastic in a domestic merger case and the labor unions have an incentive to raise wages. However, the previously described wage-unifying effect countervails this incentive, because the union would raise wages for all workers in both plants.

On the other hand, a cross-border merger induces union rivalry through the threat effect. The slope of labor demand in both cross-border merger structures $M^{C2s}$ and $M^{C2a}$ is given by

$$\frac{\partial q_E^{C2s}}{\partial w_E^{C2s}} = \frac{\partial q_I^{C2s}}{\partial w_I^{C2s}} = \frac{\partial q_E^{C2a}}{\partial w_E^{C2a}} = \frac{\partial q_I^{C2a}}{\partial w_I^{C2a}} = \frac{2 + 2\beta + \beta^2}{4(\beta - 1)(1 + 2\beta)}.$$  

Comparison with the slope of labor demands in the no-merger case reveals that

$$\left| \frac{\partial q_E^{C2s}}{\partial w_E^{C2s}} \right| - \left| \frac{\partial q_0}{\partial w} \right| = \frac{3\beta^2(2 + 2\beta + \beta^2)}{4(4 - 8\beta - 7\beta^2 - 11\beta^3 + 6\beta^4)} > 0.$$  

Ceteris paribus, cross-border mergers increase the responsiveness of labor demand of the firms, which would lead to a decrease in wage demands by unions. The difference in labor demand responsiveness for different merger types is in line with the results by Lommerud et al. (2006). However, a countervailing effect may arise in our model increasing firms’ incentives to merge domestically: the constraining effect of a uniform wage on a labor union’s ability to extract surplus from efficient firms.

To understand which types of mergers will be chosen in equilibrium, it is also instructive to look at the employment effects of different merger types. Total employment is given by the sum of firms’ output levels. Accordingly, define $Q := \sum_i q_i$. The following Lemma summarizes the impact of different merger types on total employment when compared with the pre-merger employment level.
Lemma 2. Total employment, $Q$, under the three two-merger structures ($M^{D2}$, $M^{C2s}$, and $M^{C2a}$) and the no merger structure $M^0$ can be ranked as follows: Employment levels are identical in the two cross-border merger structures ($Q^{C2s} = Q^{C2a}$). Employment is always lower in the domestic merger structure than in the cross-border and in the no merger structure ($Q^{C2s} = Q^{C2a} > Q^{D2}$ and $Q^0 > Q^{D2}$). Whether cross-border mergers reduce or increase total employment compared to no merger depends on the degree of product differentiation in the following way:

i) $Q^0 > Q^{C2s} = Q^{C2a}$ if $\beta \in (0, 1/2)$, and

ii) $Q^{C2s} = Q^{C2a} > Q^0$ if $\beta \in (1/2, 1)$.

Moreover, equality holds, $Q^{C2s} = Q^{C2a} = Q^0$, if $\beta = 1/2$.

Three interesting observations can be made from Lemma 2. First of all, we find that total employment is always lowest in the domestic merger structure compared to cross-border merger structures and the no-merger benchmark.\textsuperscript{23} Inspection of the plant-specific employment rates (see Appendix A) reveals that this mainly hinges upon the low employment of inefficient plants in the domestic merger structure. The increase in market concentration leads to a contraction of total employment.

Second, total employment in the two cross-border merger structures is identical, although different types of mergers are formed in the two structures. The reason for this result becomes obvious from the ranking of wage rates above. In the two cross-border merger structures, labor unions set wages as to balance total costs for the firms in the two structures. Note that, however, this does not mean that the distribution of output across plants is identical for the merger structures. This is not the case, as firms shift production towards more efficient plants in structure $M^{C2a}$ while this is not possible for structure $M^{C2s}$, where plants with identical technologies merge.

Third, for lower degrees of product differentiation, total employment is higher with cross-border mergers than in the no merger case. If products are closer substitutes ($\beta$ close to 1) the opportunity for firms to shift production, for either labor or non-labor cost savings, becomes

\textsuperscript{23} Note that uniformity of wages in the domestic merger structure $M^{D2}$ does not influence this result. Essentially, uniformity has no effect on total employment compared to plant-specific (discriminatory) wages when market demand is linear (Schmalensee, 1981; Yoshida, 2000). Assuming symmetric firms, total employment is the same as in the model analyzed by Lommerud et al. (2006). Differences in total employment are therefore only a result of firm asymmetries.
larger. Thereby, efficient firms produce a higher output compared to the no-merger case, which has an overall increasing effect on employment.

4 Equilibrium Industry Structures

The previous Section has examined how different merger types influence wage and employment levels. We now turn to the industry structures which will result in equilibrium as the outcomes of the merger formation process. Since firms will anticipate the wage-setting behavior of the unions, they will take into account the effect their merger decisions will have on union behavior. The following proposition summarizes which industry structures will arise in equilibrium.

Proposition 2. If $\beta > \bar{\beta} \approx 0.913$, then the equilibrium industry structure is $M^{C2a}$. If $\beta < \bar{\beta}$, then there exists a critical value $\bar{c}(\beta)$, such that the equilibrium industry structure is $M^{D2}$ if $c > \bar{c}(\beta)$ and $M^{C2a}$ if $c < \bar{c}(\beta)$. Moreover, $\lim_{\beta \to 0} \bar{c}(\beta) = 0$ and $\partial \bar{c}(\beta)/\partial \beta > 0$ in the relevant interval $0 < \bar{c}(\beta) \leq \bar{c}(\beta)$ and $\bar{c}(\beta) = \bar{c}(\beta)$ for $\beta \approx 0.351$.

In contrast to previous work with homogenous firms and purely plant-specific wages, the equilibrium industry structure in our model can consist of either domestic or cross-border mergers. Two domestic mergers will be the unique equilibrium if products are sufficiently differentiated and firms are sufficiently asymmetric, more specifically when $c > \bar{c}(\beta)$. Figure 2 illustrates these results. The result that either domestic or cross-border mergers can occur is in contrast to the findings by Lommerud et al. (2006), where domestic mergers never occur in equilibrium. The incentives for firms to merge domestically when plants are sufficiently asymmetric stem from the two effects described above. A domestic merger induces the labor unions to limit their wage demands from the efficient plant in order to maintain employment at the inefficient plant. The mergers decrease competition in the product market and induce a reduction in overall employment. Concerning the distribution of employment among the plants, merged firms domestically shift production from an inefficient to the efficient plant.

When these two effects dominate the gains from cross-border mergers – namely the reduction of market power of labor unions through the threat of reallocation – two domestic mergers will emerge as an equilibrium industry structure. More specifically, merging domestically becomes more attractive the more asymmetric firms become. Thus, we should expect that the threat
effect will conversely dominate the wage-unifying effect if firms are rather symmetric or products are closer substitutes.

For a wide range of parameter values, we observe that cross-border mergers between symmetric firms \((M^{C2s} = \{13, 24\})\) will occur in equilibrium. In this region, the threat effect of cross-border mergers dominates the benefits of uniformity for the firms. When products become less differentiated, the reallocation of production becomes easier, thereby strengthening the firms’ threat position vis-à-vis the labor unions.

Closer inspection of this equilibrium reveals that the driving factor is the gain in profits of the merged efficient plants compared to the other two industry structures in this parameter range. As they are not able to reshu­le production for non-labor cost savings, incentives are even stronger to threaten production reallocation to put downward pressure on wages. Nevertheless, also the inefficient plants gain through the increase in market concentration.

Finally, \(M^{C2a} = \{14, 23\}\) is the equilibrium outcome for \(\beta > \bar{\beta}\), i.e. when products and firms are almost homogeneous. In this area, the production shifting effect becomes strongest while market shares are distributed rather evenly between firms. Note that equilibrium cross-border mergers will not necessarily lead to higher employment compared to a no-merger case. Only for
the region $\beta \in (0.5, 1)$ cross-border mergers will increase total employment.

## 5 Welfare

Finally, we inspect the welfare implications of our results. At first glance, a domestic merger might have welfare improving effects because of the redistribution of production from less efficient to more efficient firms. However, the employment effect of domestic mergers gives rise to the following result:

**Proposition 3.** The ownership structure involving two domestic mergers, $M^{D2}$, is never socially optimal. The optimal industry structure from a welfare perspective can be either no merger ($M^0$), one domestic merger ($M^{D1}$), one cross-border merger between the inefficient plants ($M^{C1i}$) or two asymmetric cross-border mergers ($M^{C2a}$).

Calculating the global welfare as the sum of firms’ profits, labor union wage bills and consumer surplus, we see that industry structure $M^{D2}$ is never welfare optimal. For all parameter constellations of $\beta$ and $c$, it is welfare dominated by other structures. Although a domestic merger results in a partial reallocation of production from less towards more efficient plants, the reduction in overall quantity in the market causes this structure to be never optimal from a welfare perspective.

Establishing which industry structure is welfare optimal (from a global welfare point of view) is, however, not easy in practice. Since the production asymmetry may cause a reallocation of production from inefficient to efficient plants in some structures, total quantity sold in the market does not necessarily indicate when a structure is also most desirable from a welfare perspective. Figure 3 summarizes the industry structures, which can be welfare optimal in given parameter regions. Interestingly, there can be also welfare optimal industry structures which will never be the equilibrium outcome of the merger formation process between firms ($M^0$, $M^{D1}$, and $M^{C1i}$). Most notably, while two domestic mergers are never optimal from a welfare perspective, an industry structure with one domestic merger can be when firms are rather asymmetric and product differentiation is rather strong. This parameter constellation roughly coincides with the area where two domestic mergers are the equilibrium industry structure (see Figure 2). From a welfare perspective, too many domestic mergers occur for these parameter constellations.
We do find, following Lommerud et al. (2006), that two cross-border mergers ($M^{C2a}$) always welfare dominate structure $M^{D2}$. However, this result is only true for asymmetric cross-border mergers which result in one efficient and one inefficient firm in the industry. In contrast, we cannot establish a pattern leading to the conclusion that cross-border mergers are the welfare optimal industry structure for a wide range of parameters.

A comparison with the results of the equilibrium outcomes of the merger formation shows that firms only choose the welfare maximizing industry structure when products are close substitutes, i.e. when $\beta > \tilde{\beta}$. This result supports empirical findings of an increasing trend in cross-border mergers where target and acquiring firms may strongly differ in size (Gugler et al., 2003).

6 Concluding Remarks

We have presented an extension of the model analyzed by Lommerud et al. (2006) to uncover the role of cost asymmetries among firms in a unionized oligopoly. Our results suggest that domestic mergers may result as an equilibrium outcome of the merger formation process when firms are asymmetric in their non-labor costs of production.
The incentives for domestic mergers critically depend on the labor unions’ inability to discriminate among workers belonging to the same employer. Thereby, firms face a trade-off between domestic and cross-border mergers in the coalition formation game: cross-border mergers give rise to the threat effect — the opportunity to reallocate production from one country to another — which puts downward pressure on wages. Domestic mergers constrain the labor unions in their freedom to extract surplus from efficient plants. This uniformity effect provides incentives for firms to merge domestically. On the one hand a domestic merger may lower the wage paid by the efficient plant, on the other hand production may be reshuffled within one country from the less to the more efficient producer.

We obtain, therefore, both domestic and cross-border mergers in equilibrium, depending on the degree of product substitutability and the asymmetry between firms. If cross-border mergers occur, mergers between symmetric plants will be the prevailing industry structure for the widest range of parameter constellations. However, asymmetric international merger outcomes are also possible whenever products are sufficiently homogeneous.

A comparison with the optimal industry structures from a global welfare perspective reveals that firms do not choose the welfare optimal industry structures, unless products are close substitutes. While two domestic mergers are never welfare optimal, no unambiguous pattern in the industry structures according to welfare effects can be established. The welfare optimal structure can involve no mergers at all, one, or two mergers. For intermediate to low degrees of product differentiation, the global welfare maximizing industry structure involves two mergers between asymmetric firms, i.e. between an efficient and an inefficient plant each. Obviously, this result is enforced by the positive welfare effect of these mergers because of the reallocation of production from less to more efficient firms. A comparison to the equilibrium industry structure chosen by firms, reveals that such an industry structure is however rarely chosen by firms.

How do these results relate to the evaluation of merger proposals in the light of collective bargaining institutions? The presence of powerful labor unions and egalitarian wage-setting principles (“one firm, one wage”) affects firms’ merger decisions and gives rise to equilibrium industry structures which do not result when wages are set purely firm-specific. The wage-setting regime has a considerable impact on the optimal, welfare maximizing industry structure. In contrast to previous research on domestic and cross-border mergers, our model supports the
idea that one domestic merger can be welfare maximizing under certain parameter constellations. However, the presence of a wage-unifying effect triggers too much merger activity from a welfare perspective. A domestic merger, or a no merger outcome, maximize global welfare when firms are rather heterogeneous in terms of productive efficiency and product differentiation. In our model firms choose two domestic mergers in this area. In reality we observe an increasing amount of international mergers in this region (when firms are rather asymmetric) as put forward in the introduction to this paper. A relevant question which arises in this context is, therefore, whether merger policies should take into account the prevailing wage-setting institutions and thereby generated wage effects of mergers when evaluating merger proposals.
Appendix A

In this Appendix we explicitly solve all possible industry structures. All structures are solved by backward induction.

No merger \((M^0)\): \(\{1,2,3,4\}\) Given the demands (1), firms’ profit functions are given by

\[
\begin{align*}
\pi_1(\cdot) &= (1 - q_1 - \beta(q_2 + q_3 + q_4) - w_1) q_1, \\
\pi_2(\cdot) &= (1 - q_2 - \beta(q_1 + q_3 + q_4) - w_2 - c) q_2, \\
\pi_3(\cdot) &= (1 - q_3 - \beta(q_1 + q_2 + q_4) - w_3) q_3, \\
\pi_4(\cdot) &= (1 - q_4 - \beta(q_1 + q_2 + q_3) - w_4 - c) q_4,
\end{align*}
\]

Solving the four first-order conditions yields the following optimal quantities which are also the derived labor demands:

\[
\begin{align*}
\hat{q}_1(\cdot) &= \frac{-2c\beta + 2w_1 + \beta + 2\beta w_2 - \beta w_3 - \beta w_4 - 2}{(\beta - 2)(3\beta + 2)}, \\
\hat{q}_2(\cdot) &= \frac{2c + \beta + 2w_2 + \beta w_1 + 2\beta w_3 - \beta w_4 - 2}{(\beta - 2)(3\beta + 2)}, \\
\hat{q}_3(\cdot) &= \frac{-2c\beta + 2w_3 + \beta - \beta w_1 + \beta w_2 + 2\beta w_3 - \beta w_4 - 2}{(\beta - 2)(3\beta + 2)}, \text{ and} \\
\hat{q}_4(\cdot) &= \frac{2c + \beta + 2w_4 + \beta w_1 + \beta w_2 - \beta w_3 + 2\beta w_4 - 2}{(\beta - 2)(3\beta + 2)}.
\end{align*}
\]

The labor unions’ wage bills are given by

\[
\begin{align*}
U_A(\cdot) &= w_1 \hat{q}_1(\cdot) + w_2 \hat{q}_2(\cdot), \text{ and} \\
U_B(\cdot) &= w_3 \hat{q}_3(\cdot) + w_4 \hat{q}_4(\cdot).
\end{align*}
\]

The unions set wages to maximize their wage bills. Solving the four first-order conditions yields the optimal wage rates

\[
w_1^0 = w_3^0 = \frac{(4 + \beta(-2 + c))}{8} \quad \text{and} \quad w_2^0 = w_4^0 = \frac{(4 + \beta(-2 + c) - 4c)}{8}.
\]

Using the expressions for \(w_1^0, w_2^0, w_3^0\) and \(w_4^0\), we obtain the union wage bills

\[
U_A^0 = U_B^0 = \frac{4(-2 + \beta)^2(2 + \beta) - 4(-2 + \beta)^2(2 + \beta)c + (16 + \beta(8 + (-2 + \beta)\beta))c^2}{32(2 - \beta)(2 + 3\beta)},
\]

and production quantities

\[
\begin{align*}
q_1^0 &= q_3^0 = \frac{8 + \beta^2(-2 + c) + 63c}{8(2 - \beta)(2 + 3\beta)}, \text{ and} \\
q_2^0 &= q_4^0 = \frac{8 + \beta^2(-2 + c) - 8c - 63c}{8(4 + (4 - 3\beta)\beta)}.
\end{align*}
\]
It follows immediately that \( \pi_1^0 = (q_1^0)^2, \pi_2^0 = (q_2^0)^2, \pi_3^0 = (q_3^0)^2 \) and \( \pi_4^0 = (q_4^0)^2 \).

**One Domestic Merger** \((M^{D1})\) \(\{12,3,4\}\) Here, we only consider the interior solution in which all four plants produce a positive output. In Appendix B, we will derive a sufficient condition to ensure an interior solution in all market structures.

When all plants produce positive outputs in the last stage of the game, the profit functions are given by

\[
\pi_{12}(\cdot) = (1 - q_1 - \beta(q_2 + q_3 + q_4) - w_A)q_1 + (1 - q_2 - \beta(q_1 + q_3 + q_4) - w_A - c)q_2,
\]

\[
\pi_3(\cdot) = (1 - q_3 - \beta(q_1 + q_2 + q_4) - w_3)q_3, \text{ and}
\]

\[
\pi_4(\cdot) = (1 - q_4 - \beta(q_1 + q_2 + q_3) - w_4 - c)q_4.
\]

Solving the four first-order conditions yields the following optimal quantities which are also the derived labor demands:

\[
\tilde{q}_1(\cdot) = \frac{2+(\beta^2-3\beta)(1-c) + (\beta-\beta^2)(w_3+w_4) + (\beta+\beta^2-2)w_A}{2(\beta-1)(-2-3\beta+\beta^2)},
\]

\[
\tilde{q}_2(\cdot) = \frac{2(1-c)\beta(3-\beta) + (\beta-\beta^2)(w_3+w_4) + (\beta+\beta^2-2)w_A}{2(\beta-1)(-2-3\beta+\beta^2)},
\]

\[
\tilde{q}_3(\cdot) = \frac{4-4\beta+4\beta c-2+4\beta^2 c-4(4+4\beta-2\beta^2)w_3+2\beta w_4+(4\beta-2\beta^2)w_A}{2(4+4\beta-5\beta^2+\beta^3)}, \text{ and}
\]

\[
\tilde{q}_4(\cdot) = \frac{(4-2\beta)(1-c)+2\beta c+4\beta c+4(2\beta-2\beta^2)w_3+4\beta(4\beta-2\beta^2)w_A}{2(4+4\beta-5\beta^2+\beta^3)}.
\]

In the second stage, unions maximize their wage bills by simultaneously setting their wage rates \(w_A, w_3\) and \(w_4\). The wage bills are given by

\[
U_A(\cdot) = w_A (\tilde{q}_1(\cdot) + \tilde{q}_2(\cdot)), \text{ and}
\]

\[
U_B(\cdot) = w_3\tilde{q}_3(\cdot) + w_4\tilde{q}_4(\cdot).
\]

Solving the first-order conditions yields the optimal wage rates

\[
w_A^{D1} = \frac{(2-c)(2\beta+2-\beta^2)}{2(4+\beta)(6+\beta)},
\]

\[
w_3^{D1} = \frac{4+\beta(4+\beta(-1+c)+c)}{2(4+\beta)(6+\beta)},
\]

\[
w_4^{D1} = \frac{4-4c-\beta(-4+\beta+5c)}{2(4+\beta)(6+\beta)}.
\]

Using the expressions for \(w_A^{D1}, w_3^{D1}\) and \(w_4^{D1}\), we obtain the union wage bills

\[
U_A^{D1} = \frac{(\beta+2)(c-2)^2(4\beta^2+2\beta+2)}{4(-\beta^2+3\beta+2)(\beta^2+6\beta+4)},
\]

\[
U_B^{D1} = \frac{(34\beta^3-10\beta^4-\beta^6+124\beta^5+112\beta^3+32\beta^2+1)(-2\beta^6+18\beta^5+28\beta^4-80\beta^3+64\beta^2+160\beta+64)}{4(4+(-4+\beta)(1+\beta)(4+\beta)(6+\beta))},
\]

(4)
and optimal quantities

\[ q_{1D} = \frac{2c + 2\beta + 11c\beta - 6\beta^2 - \beta^3 + 11c\beta^2 - c\beta^4 + 4}{2\beta^3 + 4\beta^4 - 38\beta^3 - 16\beta^2 + 32\beta + 16}, \]
\[ q_{2D} = \frac{-6c + 2\beta - 13c\beta - 6\beta^2 - \beta^3 + 5c\beta^2 + 2c\beta^4 + 4}{2\beta^3 + 4\beta^4 - 38\beta^3 + 16\beta^2 + 32\beta + 16}, \]
\[ q_{3D} = \frac{12\beta + 6c\beta - 2\beta^2 - 5\beta^3 + 4\beta^4 + 9c\beta^2 + c\beta^4 + 8}{2\beta^3 + 2\beta^4 - 44\beta^3 + 16\beta^2 + 80\beta + 32}, \]
\[ q_{4D} = \frac{-8c + 12\beta - 18c\beta - 2\beta^2 - 5\beta^3 + 4\beta^4 - 7c\beta^2 + 4c\beta^4 + 8}{2\beta^3 + 2\beta^4 - 44\beta^3 + 16\beta^2 + 80\beta + 32}. \]

The final profits of the unmerged firms in country B are given by \( \pi_{3D}^{D1} = (q_{3D}^{D1})^2 \) and \( \pi_{4D}^{D1} = (q_{4D}^{D1})^2 \). The profit of the merged firm in country A is given by

\[ \pi_{12}^{D1} = \frac{2(1-c)(1-\beta^2)(-4-6\beta+\beta^3)^2+(4\beta(216+\beta(426+\beta(332+\beta(24+\beta(4+\beta((-17+3\beta))))))))\beta^2}{4(1-\beta)(-2+(-3+3\beta)\beta^2)(4+\beta(6+\beta))^2}. \]

**Two domestic mergers \( M^{D2} \): \( \{12,34\} \)** As in the previous industry structure, we solve for an interior solution with all four firms producing a positive output. We will show below that whenever the sufficient condition \( c < \overline{c}(\beta) \) is fulfilled, also in the two domestic mergers case all plants produce a positive output. The firms’ profit functions are consequently given by

\[ \pi_{12}(\cdot) = (1 - q_1 - \beta(q_2 + q_3 + q_4) - w_A)q_1 + (1 - q_2 - \beta(q_1 + q_3 + q_4) - w_A - c)q_2, \]
\[ \pi_{34}(\cdot) = (1 - q_3 - \beta(q_1 + q_2 + q_4) - w_B)q_3 + (1 - q_4 - \beta(q_1 + q_2 + q_3) - w_B - c)q_4. \]

Solving the first-order conditions of the firms’ profit maximization problems, the optimal quantities (derived labor demands) are given by:

\[ \hat{q}_1(\cdot) = \frac{2 - 2\beta - 2w_A + 3c\beta + 2\beta w_B + 2\beta^2 w_A - 2\beta^2 w_B}{4\beta - 8\beta^2 + 4}, \]
\[ \hat{q}_2(\cdot) = \frac{2 - 2c - 2\beta - 2w_A - c\beta + 2\beta w_B + 2\beta^2 w_A - 2\beta^2 w_B}{4\beta - 8\beta^2 + 4}, \]
\[ \hat{q}_3(\cdot) = \frac{2 - 2\beta - 2w_B + 3c\beta + 2\beta w_A - 2\beta^2 w_A + 2\beta^2 w_B}{4\beta - 8\beta^2 + 4}, \]
\[ \hat{q}_4(\cdot) = \frac{2 - 2c - 2\beta - 2w_B - c\beta + 2\beta w_A - 2\beta^2 w_A + 2\beta^2 w_B}{4\beta - 8\beta^2 + 4}. \]

In the second stage, unions maximize their wage bills by simultaneously setting their wage rates \( w_A \) and \( w_B \). The wage bills are given by

\[ U_A(\cdot) = w_A(\hat{q}_1(\cdot) + \hat{q}_2(\cdot)), \]
\[ U_B(\cdot) = w_B(\hat{q}_3(\cdot) + \hat{q}_4(\cdot)). \]
Solving the first-order conditions yields the optimal wage rates

\[ w_A^{D2} = w_B^{D2} = \frac{2-c}{1+\beta}. \]

Using the expressions \( w_A^{D2} \) and \( w_B^{D2} \), wage bills are given by

\[ U_A^{D2} = U_B^{D2} = \frac{(\beta+1)(c-2)^2}{(\beta+2)(2\beta+4)(4\beta+2)}. \] (5)

Finally, we obtain optimal quantities

\[ q_1^{D2} = q_3^{D2} = \frac{2+3c+5\beta c+\beta^2(-2+3c)}{8+12\beta-12\beta^2-8\beta^3}, \]
\[ q_2^{D2} = q_4^{D2} = \frac{2+3c+5\beta c+\beta^2(2+c)}{4(-2-3\beta+3\beta^2+2\beta^3)}, \]

and firm profits

\[ \pi_A^{D2} = \pi_B^{D2} = \frac{-4(-1+\beta)(1+\beta)^3+4(-1+\beta)(1+\beta)c+(5+\beta(22+3\beta(11+\beta(6+\beta))))c^2}{8(1-\beta)(2+\beta)^2(1+2\beta)^2}. \]

**One efficient symmetric international merger \( \{M^{1Cse}\}: \{13,2,4\} \)** Firms’ profit functions are given by

\[ \pi_{13}(\cdot) = (1-q_1 - \beta(q_2 + q_3 + q_4) - w_1) q_1 + (1-q_3 - \beta(q_1 + q_2 + q_4) - w_3) q_3, \]
\[ \pi_2(\cdot) = (1-q_2 - \beta(q_1 + q_3 + q_4) - w_2 - c) q_2, \] and
\[ \pi_4(\cdot) = (1-q_4 - \beta(q_1 + q_2 + q_3) - w_4 - c) q_4. \]

Solving the four first-order conditions yields the following quantities:

\[ \tilde{q_1}(\cdot) = \frac{(2-2w_1+\beta(-3+2c+2w_3+w_2-w_1+w_4)-\beta(-1+2c+w_2-w_1+w_4))}{2(2+\beta-4\beta^2+\beta^3)}, \]
\[ \tilde{q_2}(\cdot) = \frac{(4(-1+c+w_2)+\beta(2(-1+\beta)c+\beta(w_1-2w_2+w_3)+2(1+w_1-2w_2+w_3+w_4)))}{2(4+(-4+\beta)(-1+\beta)\beta)}, \]
\[ \tilde{q_3}(\cdot) = \frac{(2-2w_3+\beta(-3+2c+2w_1+w_3-w_2+w_4)-\beta(-1+2c+w_2-w_3+w_4))}{2(2+\beta-4\beta^2+\beta^3)}, \] and
\[ \tilde{q_4}(\cdot) = \frac{(4+(-4+\beta)(-1+\beta)\beta)}{2(4+(-4+\beta)(-1+\beta)\beta)}. \]

The labor unions’ wage bills are given by

\[ U_A(\cdot) = w_1\tilde{q_1}(\cdot) + w_2\tilde{q_2}(\cdot), \] and
\[ U_B(\cdot) = w_3\tilde{q_3}(\cdot) + w_4\tilde{q_4}(\cdot). \]
The unions set wages to maximize their wage bills. Solving the four first-order conditions yields the optimal wage rates:

\[
\begin{align*}
  w_1^{C_{1se}} &= w_3^{C_{1se}} = \frac{(2-\beta)(4+\beta(-2+c))}{16+(-12+\beta)\beta}, \\
  w_2^{C_{1se}} &= w_4^{C_{1se}} = \frac{(2-\beta)(4-3\beta+2(-2+c))}{16+(-12+\beta)\beta},
\end{align*}
\]

and union wage bills

\[
U_A^{C_{1se}} = U_B^{C_{1se}} = \frac{(2\beta^5-16\beta^4+54\beta^3-60\beta^2-32\beta+64)c^2+(-6\beta^5+38\beta^4-60\beta^3-48\beta^2+192\beta-128)c+(5\beta^6-32\beta^5+42\beta^3+84\beta^2-224\beta+128)}{(16-\beta(12-\beta))2(2+)3-\beta)2}. 
\]

Finally, quantities and firm profits are given by

\[
\begin{align*}
  q_1^{C_{1se}} &= q_3^{C_{1se}} = \frac{(2-\beta)(-8+\beta^2-6\beta c)}{2(16+(-12+\beta)\beta)(-2+(-3+\beta)\beta)}, \\
  q_2^{C_{1se}} &= q_4^{C_{1se}} = \frac{8(-1+c)+\beta(2+\beta(4+\beta(-1+c)-7c)+4c)}{(16+(-12+\beta)\beta)(-2+(-3+\beta)\beta)}, \\
  \pi_1^{C_{1se}} &= \frac{(2-\beta)^2(1+\beta)(-8+\beta^2-6\beta c)^2}{2(16+(-12+\beta)\beta)^2(-2+(-3+\beta)\beta)^2}, \\
  \pi_2^{C_{1se}} &= \frac{(8(-1+c)+\beta(2+\beta(4+\beta(-1+c)-7c)+4c))^2}{(16+(-12+\beta)\beta)^2(-2+(-3+\beta)\beta)^2}.
\end{align*}
\]

One inefficient symmetric international merger \(M^{C_{1se}}\): \{1,3,24\} Firms’ profit functions are given by

\[
\begin{align*}
  \pi_2(\cdot) &= (1-q_2-\beta(q_1+q_3+q_4)-w_2-c)q_2 + (1-q_4-\beta(q_1+q_2+q_3)-w_4-c)q_4, \\
  \pi_1(\cdot) &= (1-q_1-\beta(q_2+q_3+q_4)-w_1)q_1, \text{ and} \\
  \pi_3(\cdot) &= (1-q_3-\beta(q_1+q_2+q_4)-w_3)q_3.
\end{align*}
\]

Solving the four first-order conditions yields the following optimal quantities (derived labor demands):

\[
\begin{align*}
  \tilde{q}_1(\cdot) &= 4-4w_1-\beta^2(2c-2w_1+w_2+w_4)+2\beta(-1+2c-2w_1+w_2+w_3+w_4) \\
  \tilde{q}_2(\cdot) &= 2(1-c-w_2)+\beta(-3+c+w_1-w_2+\beta(1+c-w_1+w_2-w_3)+w_3+w_4) \\
  \tilde{q}_3(\cdot) &= 4-4w_3-\beta^2(2c+w_2-2w_3+w_4)+2\beta(-1+2c+w_1+w_2-2w_3+w_4) \\
  \tilde{q}_4(\cdot) &= 2(1-c-w_4)+\beta(-3+c+w_1+2w_2+w_3-w_4+\beta(1+c-w_1-w_3+w_4)).
\end{align*}
\]
The labor unions' wage bills are given by
\[ U_A (\cdot) = w_1 \hat{q}_1 (\cdot) + w_2 \hat{q}_2 (\cdot), \text{ and} \]
\[ U_B (\cdot) = w_3 \hat{q}_3 (\cdot) + w_4 \hat{q}_4 (\cdot). \]

The unions set wages to maximize their wage bills. Solving the four first-order conditions yields the following optimal wage rates:
\[ w_1^{C1si} = w_3^{C1si} = \frac{(2-\beta)(4+\beta(3+c))}{16+(-12+\beta)^3}, \text{ and} \]
\[ w_2^{C1si} = w_4^{C1si} = \frac{2(1-\beta)(4+\beta(-2+c)-4c)}{16+(-12+\beta)^3}. \]

Using the results for \( w_1^{C1si}, w_2^{C1si}, w_3^{C1si} \) and \( w_4^{C1si} \), we obtain the following optimal union wage bills
\[ U_A^{C1si} = U_B^{C1si} = \frac{(2-\beta)((\beta^4-8\beta^3+20\beta^2+16\beta-32)^2+(18\beta^3-4\beta^4+12\beta^2-96\beta+64)c-(22\beta^3-5\beta^4+2\beta^3-80\beta+64))}{(16-12\beta+\beta^2)^2(-2-3\beta+\beta^2)}, \]
quantities
\[ q_1^{C1si} = q_3^{C1si} = \frac{-8+\beta(2-6c+\beta(4-\beta+3c))}{(16+(-12+\beta)^3)(-2+(-3+\beta)^3)}, \]
\[ q_2^{C1si} = q_4^{C1si} = \frac{(-2+\beta)(8+\beta^2(-1+c)-8c-6\beta c)}{2(-32-24\beta+50\beta^2-15\beta^3+\beta^4)}, \]
and profits
\[ \pi_1^{C1si} = \pi_3^{C1si} = \frac{8+\beta^2(4+3c)+\beta(-2+6c)^2}{(16-12\beta+\beta^2)^2(-2-3\beta+\beta^2)^3}. \]

Two symmetric cross-border mergers (\( M^{C2s} \)):
\{13,24\}  
Firms' profit functions are given by
\[ \pi_{13}(\cdot) = (1 - q_1 - \beta(q_2 + q_3 + q_4) - w_1) q_1 + (1 - q_3 - \beta(q_1 + q_2 + q_4) - w_3) q_3, \text{ and} \]
\[ \pi_{24}(\cdot) = (1 - q_2 - \beta(q_1 + q_3 + q_4) - w_2 - c) q_2 + (1 - q_4 - \beta(q_1 + q_2 + q_3) - w_4 - c) q_4. \]

Solving the four first-order conditions yields the optimal quantities:
\[ \hat{q}_1 (\cdot) = \frac{2(-1+w_1)+\beta^2(2c-w_1+w_2-w_3+w_4)-\beta(-2+2c-2w_1+w_2+2w_3+w_4)}{-4+4\beta+8\beta^2}, \]
\[ \hat{q}_2 (\cdot) = \frac{2(-1+c+w_2)-\beta(-2+w_1-2w_2+w_3+2w_4+\beta(2c-w_1+w_2-w_3+w_4))}{4(-1+\beta)(1+2\beta)}, \]
\[ \hat{q}_3 (\cdot) = \frac{2(-1+w_3)-\beta(-2+2c+2w_1+w_2-2w_3+w_4)+\beta^2(2c-w_1+w_2-w_3+w_4)}{-4+4\beta+8\beta^2}, \text{ and} \]
\[ \hat{q}_4 (\cdot) = \frac{2(-1+c+w_4)-\beta(-2+w_1+2w_2+w_3-2w_4+\beta(2c-w_1+w_2-w_3+w_4))}{4(-1+\beta)(1+2\beta)}. \]
The labor unions’ wage bills are given by

\[ U_A(\cdot) = w_1\hat{q}_1(\cdot) + w_2\hat{q}_2(\cdot), \quad \text{and} \]

\[ U_B(\cdot) = w_3\hat{q}_3(\cdot) + w_4\hat{q}_4(\cdot). \]

The unions set wages to maximize their wage bills. Solving the four first-order conditions yields the optimal wage rates

\[ w_1^{C2s} = w_3^{C2s} = \frac{2(1-\beta)(4-3\beta)(-3+c)}{(-4+\beta)(-4+3\beta)}, \quad \text{and} \]

\[ w_2^{C2s} = w_4^{C2s} = \frac{2(1-\beta)(4-3\beta)(-2+\beta)c}{(-4+\beta)(-4+3\beta)}. \]

and the union wage bills

\[ U_A^{C2s} = U_B^{C2s} = \frac{-2(4-3\beta)^2(-2+\beta)(\beta^2) + 2(4-3\beta)^2(-2+\beta)(\beta^2)c + (1-\beta)(-32+\beta^2(30+(-14+\beta)\beta)c)^2}{(4-3\beta)^2(-4+\beta)(1+2\beta)}. \]

Finally, quantities and firm profits are given by

\[ q_1^{C2s} = q_3^{C2s} = \frac{8+\beta(-2-3\beta+6+(-4+\beta)\beta)c}{2(4-\beta)(1+2\beta)(4-3\beta)}, \]

\[ q_2^{C2s} = q_4^{C2s} = \frac{8-8c-\beta(2+3\beta+4+(-7+\beta)\beta)c}{2(4-\beta)(1+2\beta)(4-3\beta)}, \]

\[ \pi_{13}^{C2s} = \frac{(1+\beta)(8-2+3\beta+6+(-4+\beta)\beta)c^2}{2(4-3\beta)^2(-4+\beta)(1+2\beta)^2}, \quad \text{and} \]

\[ \pi_{24}^{C2s} = \frac{(1+\beta)(8-1+c+\beta(2+3\beta+4+(-7+\beta)\beta)c^2}{2(4-3\beta)^2(-4+\beta)(1+2\beta)^2}. \]

One asymmetric cross-border merger \((M^{C1a})\): \(\{14,2,3\}\)  

Firms’ profit functions are given by

\[ \pi_{14}(\cdot) = (1-q_1 - \beta(q_2 + q_3 + q_4) - w_1) q_1 + (1-q_4 - \beta(q_1 + q_2 + q_3) - w_4 - c) q_4, \]

\[ \pi_2(\cdot) = (1-q_2 - \beta(q_1 + q_3 + q_4) - w_2 - c) q_2, \quad \text{and} \]

\[ \pi_3(\cdot) = (1-q_3 - \beta(q_1 + q_2 + q_4) - w_3) q_3. \]

Solving the four first-order conditions yields the optimal quantities:

\[ \hat{q}_1(\cdot) = \frac{2-2w_1+\beta(-3+3c-w_1+w_3+\beta(-1+c-w_1+w_2+w_3)+2w_4)}{2(2+\beta-4\beta^2+\beta^3)}, \]

\[ \hat{q}_2(\cdot) = \frac{-4(-1+c+w_2)+\beta((-2+\beta)c-\beta(w_1-2w_2+w_3)+2(-1+w_1-2w_2+w_3+w_4))}{2(4+(-4+\beta)(-1+\beta)c)}, \]

\[ \hat{q}_3(\cdot) = \frac{4-2w_3-\beta^2(c+w_1-2w_3+w_4)+2\beta(-1+2c+w_1+w_2-2w_3+w_4)}{2(4+(-4+\beta)(-1+\beta)c)}, \quad \text{and} \]

\[ \hat{q}_4(\cdot) = \frac{\beta(-3+2w_1+w_3-\beta(-1+w_2-w_3-w_4)-w_4)-2(1+c+w_4)}{2(2+\beta-4\beta^2+\beta^3)}. \]
The labor unions’ wage bills are given by

\[ U_A(\cdot) = w_1\hat{q}_1(\cdot) + w_2\hat{q}_2(\cdot), \]
\[ U_B(\cdot) = w_3\hat{q}_3(\cdot) + w_4\hat{q}_4(\cdot). \]

The unions set wages to maximize their wage bills. Solving the four first-order conditions yields the following optimal wage rates:

\[ w_{1}^{C_1a} = \frac{-64c\beta + 96\beta + 136\beta^2 - 132\beta^3 + 28\beta^4 - 8c\beta^2 + 50c\beta^3 - 13c\beta^4 - 128}{7\beta^4 - 96\beta^3 + 240\beta^2 - 256}, \]
\[ w_{2}^{C_1a} = \frac{128c + 64\beta - 32c\beta + 128\beta^2 + 106\beta^3 + 21\beta^4 - 120c\beta^2 + 74c\beta^3 - 12c\beta^4 - 128}{7\beta^4 - 96\beta^3 + 240\beta^2 - 256}, \]
\[ w_{3}^{C_1a} = \frac{-32c\beta + 64\beta + 128\beta^2 + 106\beta^3 + 21\beta^4 - 120c\beta^2 + 74c\beta^3 - 12c\beta^4 - 128}{7\beta^4 - 96\beta^3 + 240\beta^2 - 256}, \]
\[ w_{4}^{C_1a} = \frac{128c + 96\beta - 32c\beta + 136\beta^2 - 132\beta^3 + 28\beta^4 - 128c\beta^2 + 82c\beta^3 - 15c\beta^4 - 128}{7\beta^4 - 96\beta^3 + 240\beta^2 - 256}. \]

Optimal quantities are given by

\[ q_{1}^{C_1a} = \frac{-192\beta + 256c\beta - 304\beta^2 + 288\beta^3 - 22\beta^4 - 33\beta^5 + 7\beta^6 - 80c\beta^2 - 244c\beta^3 + 162c\beta^4 - 23c\beta^5 - c\beta^6 + 256}{-14\beta^4 + 248\beta^3 - 1262\beta^2 + 2084\beta^3 + 416\beta^4 - 3008\beta^2 + 512\beta + 1024}, \]
\[ q_{2}^{C_1a} = \frac{216\beta^3 - 384c\beta - 704\beta^2 - 512c + 196\beta^4 - 108\beta^5 + 14\beta^6 + 736c\beta^2 + 168c\beta^3 - 38c\beta^4 + 120c\beta^5 - 11c\beta^6 + 512}{-14\beta^4 + 262\beta^3 - 1496\beta^2 + 3112\beta^3 - 640\beta^4 - 4480\beta^3 + 2048\beta^4 + 2048\beta + 2048}, \]
\[ q_{3}^{C_1a} = \frac{384c\beta - 704\beta^2 + 216\beta^3 + 108\beta^4 + 14\beta^5 - 32c\beta^2 - 38c\beta^3 + 184c\beta^4 - 12c\beta^5 - 3c\beta^6 + 512}{-14\beta^4 + 262\beta^3 - 1496\beta^2 + 3112\beta^3 - 640\beta^4 - 4480\beta^3 + 2048\beta^4 + 2048\beta + 2048}, \]
\[ q_{4}^{C_1a} = \frac{-256c - 192\beta - 64\beta^3 - 304\beta^2 + 288\beta^3 - 22\beta^4 - 33\beta^5 + 7\beta^6 + 80c\beta^2 - 244c\beta^3 + 162c\beta^4 - 23c\beta^5 - c\beta^6 + 256}{-14\beta^4 + 248\beta^3 - 1262\beta^2 + 2084\beta^3 + 416\beta^4 - 3008\beta^2 + 512\beta + 1024}. \]

We can use the results for wage rates and profits to calculate the union wage bills \( U_A^{C_1a} = w_{1}^{C_1a} \cdot q_{1}^{C_1a} + w_{2}^{C_1a} \cdot q_{2}^{C_1a} \) and \( U_B^{C_1a} = w_{3}^{C_1a} \cdot q_{3}^{C_1a} + w_{4}^{C_1a} \cdot q_{4}^{C_1a} \) (explicit derivations are omitted here for reasons of space). Finally, the profits of the unmerged firms 2 and 3 are immediately given by \( \pi_{2}^{C_1a} = (q_{2}^{C_1a})^2 \) and \( \pi_{3}^{C_1a} = (q_{3}^{C_1a})^2 \). The merged firm earns a profit of

\[ \pi_{14}^{C_1a} = \frac{-2(\beta^2 - 1)(c - 1)(-7\beta^5 + 26\beta^4 + 48\beta^3 - 240\beta^2 + 64\beta + 256)^2 + \phi^2}{4(-1 + \beta)(16 + (-12 + \beta)\beta)^2(-2 + (-3 + \beta)\beta)^2(-16 + \beta(-12 + 7\beta)^2), \]

where

\[ \phi(\beta) = \sigma\beta^7 + 188 \cdot 192\beta^6 - 160 \cdot 640\beta^5 - 218 \cdot 880\beta^4 + 219 \cdot 136\beta^3 + 159 \cdot 744\beta^2 - 98 \cdot 304\beta - 65 \cdot 536, \]
\[ \sigma(\beta) = 12\beta^5 + 213\beta^4 - 4653\beta^3 + 27968\beta^2 - 64620\beta + 7568. \]

**Two asymmetric cross-border mergers** (\( M^{C_2a} \): \{14,23\}) Firms’ profit functions are given by

\[ \pi_{14}(\cdot) = (1 - q_1 - \beta(q_2 + q_3 + q_4) - w_1)q_1 + (1 - q_4 - \beta(q_1 + q_2 + q_3) - w_4 - c)q_4, \]
\[ \pi_{23}(\cdot) = (1 - q_2 - \beta(q_1 + q_3 + q_4) - w_2 - c)q_2 + (1 - q_3 - \beta(q_1 + q_2 + q_4) - w_3)q_3. \]
Solving the four first-order conditions yields the optimal quantities:

\[ \hat{q}_1 (\cdot) = \frac{2(-1+w_1)+\beta^2(-w_1+w_2+w_3-w_4)-\beta(-2+3c-2w_1+w_2+w_3+2w_4)}{-4-4\beta+8\beta^2}, \]
\[ \hat{q}_2 (\cdot) = \frac{2(-1+c+w_2)+\beta(2+c-w_1+2w_2+w_3-w_4)+\beta^2(w_1-w_2+w_3+w_4)}{-4-4\beta+8\beta^2}, \]
\[ \hat{q}_3 (\cdot) = \frac{2(-1+w_3)-\beta(-2+3c+w_1+2w_2-2w_3+w_4)+\beta^2(w_1-w_2-w_3+w_4)}{-4-4\beta+8\beta^2}, \]
\[ \hat{q}_4 (\cdot) = \frac{\beta^2(-w_1+w_2+w_3-w_4)+2(-1+c+w_4)+\beta(2+c-2w_1-w_2-w_3+2w_4)}{-4-4\beta+8\beta^2}. \]

The labor unions’ wage bills are given by

\[ U_A (\cdot) = w_1 \hat{q}_1 (\cdot) + w_2 \hat{q}_2 (\cdot), \]
\[ U_B (\cdot) = w_3 \hat{q}_3 (\cdot) + w_4 \hat{q}_4 (\cdot). \]

The unions set wages to maximize their wage bills. Solving the four first-order conditions yields the optimal wage rates

\[ w_1^{C_{2a}} = w_3^{C_{2a}} = \frac{2+\beta(-2+c)}{4-\beta}, \text{ and} \]
\[ w_2^{C_{2a}} = w_4^{C_{2a}} = \frac{2-2\beta-2c+\beta c}{4-\beta}. \]

Firms produce the following optimal quantities

\[ q_1^{C_{2a}} = q_3^{C_{2a}} = \frac{4-\beta^2(2+c)+\beta(-2+4c)}{4(4+3\beta-9\beta^2+2\beta^3)}, \text{ and} \]
\[ q_2^{C_{2a}} = q_4^{C_{2a}} = \frac{4-4c-2\beta(1+c)+\beta^2(-2+3c)}{(-4+\beta)(-4-4\beta+8\beta^2)}. \]

The wage bills of the unions are then given by

\[ U_A^{C_{2a}} = U_B^{C_{2a}} = \frac{\beta^2(-2+c)^2+12\beta(-1+c)-2\beta^2c^2+4(2(-2+c)c)}{2(-4+\beta)(1-\beta)(1+2\beta)}. \]

Finally, firms earn the following profits

\[ \pi_{14}^{C_{2a}} = \pi_{23}^{C_{2a}} = \frac{3c^2\beta^4-16c^2\beta^3+2c^2\beta^2+8c^2+4c^3+16c^3+12c^2-16c^2-4\beta^4-16\beta^3-12\beta^2+16\beta+16}{-32\beta^3+256\beta^3-488\beta^3-184\beta^2+320\beta+128}. \]

With the explicit solutions to industry structures, we can sketch the proofs of our paper. Since all proofs involve large expressions which are hard to include in the text, we restrict ourselves to outlining the relevant comparisons and calculations which need to be performed. All expressions used for these calculations are stated in the previous part of the Appendix.
Proof of Lemma 1  In this Lemma, we show that industry structures involving no or only one merger are always dominated by at least one industry structure involving two mergers for \( \beta \in (0, 1) \) and \( c \in (0, \pi(\beta)) \). Thus, we need to compare the profit levels of the decisive owners in the relevant market structures.

- **No merger** \((M^0)\): We can show that \( M^{D2} \) dom \( M^0 \) by considering \( \pi_{12}^{D2} - (\pi_1^0 + \pi_2^0) > 0 \) and \( \pi_{34}^{D2} - (\pi_3^0 + \pi_4^0) > 0 \).

- **One domestic merger** \((M^{D1})\): We can show that \( M^{D2} \) dom \( M^{D1} \) or \( M^{C2s} \) dom \( M^{D1} \) by considering \( \pi_{34}^{D2} - (\pi_3^{D1} + \pi_4^{D1}) > 0 \) and \( (\pi_{13}^{C2s} + \pi_{24}^{C2s}) - (\pi_{12}^{D1} + \pi_3^{D1} + \pi_4^{D1}) > 0 \).

- **One symmetric cross-border merger between the efficient plants** \((M^{C1se})\): We can show that \( M^{C2s} \) dom \( M^{C1se} \) by considering \( (\pi_{24}^{C2s}) - (\pi_2^{C1se} + \pi_4^{C1se}) > 0 \).

- **One symmetric cross-border merger between the inefficient plants** \((M^{C1si})\): We can show that \( M^{C2s} \) dom \( M^{C1si} \) by considering \( (\pi_{13}^{C2s} - (\pi_1^{C1si} + \pi_3^{C1si}) > 0 \).

- **One asymmetric cross-border merger** \((M^{C1a})\): We can show that \( M^{C2s} \) dom \( M^{C1a} \) by considering \( (\pi_{13}^{C2s} + \pi_{24}^{C2s}) - (\pi_{14}^{C1a} + \pi_2^{C1a} + \pi_3^{C1a}) > 0 \).

Proof of Proposition 1

- **Part (i).** The ranking presented is unique. It is sufficient to show that \( w_C^{\beta a} - w_E^{\beta a} = w_I^{C2s} - w_I^{C2a} = \frac{(2-\beta)bc}{(1-\beta)(4-3\beta)} \) and \( w_E^{C2s} - w_I^{C2s} = \frac{2(1-\beta)c}{4-3\beta} \) are positive. Obviously, this is the case for \( \beta \in (0, 1) \) and \( c \in (0, \pi(\beta)) \).

- **Part (ii).** Since \( w_I^{C2s} - w_I^{C2a} > 0 \), it is sufficient to show that \( w^{D2} - w_I^{C2s} > 0 \) holds for \( \beta \in (0, 1) \).

- **Part (iii).** To rank \( w_E^{C2s}, w_E^{C2a} \) and \( w^{D2} \), it is useful to establish the relations bilaterally. First, we can show that there exists a critical value \( c_1(\beta) \) such that \( w_E^{C2s} - w^{D2} > 0 \) if \( c > c_1(\beta) \). Second, there exists a critical threshold \( c_2(\beta) \) such that \( w^{D2} - w_E^{C2s} > 0 \) if \( c > c_2(\beta) \). A comparison of these thresholds yields \( c_1(\beta) > c_2(\beta) > 0 \). Finally, we can establish that \( w_E^{C2s} - w^{D2} < 0 \) and \( w^{D2} - w_E^{C2s} > 0 \) if \( c_1(\beta) > c > c_2(\beta) \).
Proof of Lemma 2  It can be easily checked from the solutions of firm specific output derived previously in this Appendix that $Q^{C2a} = Q^{C2s}$. The inequality $Q^{C2a} - Q^{D2} > 0$ reduces to 
\[
\frac{\beta(c-2)}{(-4+\beta)(2+\beta)} > 0,
\]
which holds given the restrictions on parameters for all $\beta \in (0,1)$ and $c \in (0, \overline{c}(\beta))$. Equivalently, the inequality $Q^0 - Q^{D2} > 0$ reduces to $\frac{\beta(2+3\beta+2\beta^2)(2-c)}{8+32\beta+28\beta^2+12\beta^3} > 0$. Since $c < \overline{c}(\beta)$, the inequality is fulfilled for all $\beta \in (0,1)$. For the last part of Lemma 2, we consider the inequality $Q^0 - Q^{C2s} > 0$. It is easily confirmed that the expression on the LHS changes its sign at $\beta = 1/2$. More specifically, the difference $Q^0 - Q^{C2s}$ is positive for $\beta < 1/2$ and negative for $\beta > 1/2$.

Proof of Proposition 2  From Lemma 1, we know that the only candidates for equilibrium industry structures are those structures involving two mergers. In order to determine the EIS, we need to compare bilaterally the profits of the decisive owners in each of the two-merger industry structures against those of the other two structures.

**Equilibrium** $M^{D2}$: $M^{D2}$ is the equilibrium industry structure if and only if $M^{D2} \text{ dom } M^{C2s}$ and $M^{D2} \text{ dom } M^{C2a}$, i.e. $\pi_{12}^{D2} + \pi_{34}^{D2} - (\pi_{13}^{C2s} + \pi_{24}^{C2s}) > 0$ and $\pi_{12}^{D2} + \pi_{34}^{D2} - (\pi_{14}^{C2a} + \pi_{23}^{C2a}) > 0$.

Consider first $M^{D2} \text{ dom } M^{C2s}$. Substituting the results for profits derived in this Appendix yields an expression which is quadratic in $c$:

\[
\frac{1}{4(-2\beta^2+\beta+1)(3\beta^3-10\beta^2-16\beta+32)} \delta_1(c,\beta) > 0,
\]

where $\delta_1(c,\beta) = r_1c^2 + s_1c + t_1$ and

\[
\begin{align*}
t_1 &= 252\beta^6 - 384\beta^5 - 572\beta^4 + 896\beta^3 + 320\beta^2 - 512\beta, \\
s_1 &= 384\beta^5 - 252\beta^6 + 572\beta^4 - 896\beta^3 - 320\beta^2 + 512\beta, \text{ and} \\
r_1 &= 2\beta^9 - 15\beta^8 + 24\beta^7 + 103\beta^6 - 316\beta^5 + 381\beta^4 + 512\beta^3 - 1728\beta^2 + 512\beta + 768.
\end{align*}
\]

We see that

\[
\frac{1}{4(-2\beta^2+\beta+1)(3\beta^3-10\beta^2-16\beta+32)} > 0
\]

for all $\beta \in (0,1)$. Additionally, we can unambiguously determine the signs of the rest of the terms. We have that $t_1 < 0$ for all $\beta \in (0,1)$ such that $\delta_1(\beta, c) < 0$ when $c \to 0$. Further, $s_1 > 0$ and $r_1 > 0$ for $\beta \in (0,1)$. 

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The existence of a unique critical value $\bar{c}(\beta) > 0$ such that $\delta_1(\bar{c}(\beta), \beta) = 0$ while $\delta_1(c, \beta) > 0$ for all $c > \bar{c}(\beta)$ follows from noting that $\partial^2 \delta_1 / \partial c^2 = 2r_1 > 0$. Solving the quadratic equation

$$\frac{1}{4(-2\beta^2 + \beta + 1)(3\beta^2 - 10\beta^2 - 16\beta + 32)} \delta_1(c, \beta) = 0$$

yields two real roots, of which only one is feasible and given by

$$\bar{c}(\beta) := 2 \left( \sqrt{-\frac{\varpi(\beta)}{(\rho(\beta))^2} + \frac{(4 - 3\beta)^2(-1 + \beta)(1 + \beta)(8 + 7\beta)}{\rho(\beta)}} \right)$$

where

$$\varpi(\beta) = 126\beta^{15} - 1137\beta^{14} + 2666\beta^{13} + 2809\beta^{12} - 24332\beta^{11} + 52332\beta^{10} + 12100\beta^9 - 265636\beta^8 + 267360\beta^7 + 380080\beta^6 - 614272\beta^5 - 147968\beta^4 + 454656\beta^3 - 20480\beta^2 - 98304\beta,$$

and

$$\rho(\beta) = 2\beta^9 - 15\beta^8 + 24\beta^7 + 103\beta^6 - 316\beta^5 + 381\beta^4 + 512\beta^3 - 1728\beta^2 + 512\beta + 768.$$ 

Thus, there exists a threshold $\bar{c}(\beta)$, which indicates that $\delta_1(c, \beta) > 0$ for all $c > \bar{c}(\beta)$.

Inspection of this threshold function reveals that $\lim_{\beta \to 0} \bar{c}(\beta) = \lim_{\beta \to 1} \bar{c}(\beta) = 0$. Furthermore, in the relevant interval of $2(0, 1)$, $\bar{c}(\beta)$ is a concave function which reaches a global maximum at $\bar{\beta} \approx 0.494$.

Finally, we need to ensure that this solution is feasible with respect to Assumption 1. Thus, define $\Delta c_1(\beta) := \bar{c}(\beta) - \bar{c}(\beta)$. We need to show that $\Delta c_1(\beta) > 0$ for at least some values of $\beta$. We can easily check that $\lim_{\beta \to 0} \Delta c_1(\beta) = 2 - \sqrt{2}$ and $\lim_{\beta \to 1} \Delta c_1(\beta) = 0$. Looking for a numerical solution for $\Delta c_1(\beta) = 0$, we find that $\Delta c_1(\beta) = 0$ for $\beta \approx 0.351 \equiv \bar{\beta}$. Thus, for all $\beta < \bar{\beta}$, $\bar{c}(\beta) < \bar{c}(\beta)$. Hence, we know that there exists a feasible threshold $\bar{c}(\beta)$ such that $M^{D2}$ $\text{dom } M^{C2a}$ whenever $\bar{c}(\beta) < c \leq \bar{c}(\beta)$.

Next, we examine $M^{D2}$ $\text{dom } M^{C2a}$, i.e. $\pi_{D2}^{C2a} + \pi_{D4}^{C2a} - (\pi_{14}^{C2a} + \pi_{23}^{C2a}) > 0$. We can use the expressions derived in this Appendix, so that we obtain for the LHS of the inequality

$$\frac{1}{4(-2\beta^2 + 2\beta + 8)^2(-2\beta^2 + \beta + 1)} \delta_2(c, \beta) > 0,$$

where $\delta_2(c, \beta) = r_2c^2 + s_2c + t_2$ and

$$r_2 = 120\beta + 53\beta^2 - 4\beta^3 - \beta^4 + 48,$$

$$s_2 = 32\beta + 28\beta^2 - 32\beta^3 - 28\beta^4,$$

and

$$t_2 = -32\beta - 28\beta^2 + 32\beta^3 + 28\beta^4.$$
We can easily see that
\[
\frac{1}{4(-\beta^2+2\beta+8)^2(-2\beta^2+\beta+1)} > 0
\]
for \( \beta \in (0, 1) \). By the same reasoning as above, we can establish that \( t_2 < 0 \) for all \( \beta \in (0, 1) \) such that \( \delta_2(\beta, c) < 0 \) when \( c \to 0 \), while \( s_2 > 0 \) and \( r_2 > 0 \) for \( \beta \in (0, 1) \). Thus it must be that \( \delta_2(c, \beta) \) is a convex function since \( \partial^2 \delta_2(c, \beta)/\partial c^2 = 2r_2 > 0 \) and there must exist a critical threshold \( c^+(\beta) \) such that \( \delta_2(c^+(\beta), \beta) = 0 \) and \( \delta_2(c, \beta) > 0 \) for \( c > c^+(\beta) \). Solving \( \delta_2(c, \beta) = 0 \) we obtain two real solutions, of which only one is feasible and given by
\[
c^+(\beta) := \frac{2(-8\beta^3+7\beta^2+7\beta^4)}{48+120\beta+53\beta^2-4\beta^3-7\beta^4} + \sqrt{\frac{96\beta+340\beta^2+248\beta^3-259\beta^4-381\beta^5-953\beta^6+37\beta^7+148\beta^8}{(-48-120\beta-53\beta^2+4\beta^3+\beta^4)^2}}.
\]
Thus, there exists a threshold \( c^+(\beta) \), which indicates that \( \delta_2(c, \beta) > 0 \) for \( c > c^+(\beta) \). Inspecting \( c^+(\beta) \) in more detail, we find that \( \lim_{\beta \to 0} c^+(\beta) = \lim_{\beta \to 1} c^+(\beta) = 0 \), and that \( c^+(\beta) \) is a concave function in the relevant parameter range \( \beta \in (0, 1) \), which reaches a global maximum at \( \beta \approx 0.483 \).

Again, we need to determine that this threshold is feasible, i.e. that \( c^+(\beta) < \bar{c}(\beta) \) for at least some values of \( \beta \). Thus, define \( \Delta c_2(\beta) := \bar{c}(\beta) - c^+(\beta) \), with \( \lim_{\beta \to 0} \Delta c_2(\beta) = 2 - \sqrt{2} \) and \( \lim_{\beta \to 1} \Delta c_2(\beta) = 0 \). It can be verified that \( \Delta c_2(\beta) > 0 \) for all \( \beta < \hat{\beta} \), with \( \hat{\beta} \approx 0.367 \). Thus, \( M^{D2} \) \textit{dom} \( M^{C2a} \) for \( c^+(\beta) < c < \bar{c}(\beta) \).

To establish that \( M^{D2} \) is an equilibrium in a given parameter range, we need to show that \( M^{D2} \) \textit{dom} \( M^{C2s} \) and \( M^{D2} \) \textit{dom} \( M^{C2a} \) at the same time. From the above considerations, we know that \( c \) must not be too small for \( M^{D2} \) dominating the other structures. Specifically, we have derived two thresholds on \( c \), which determine when \( M^{D2} \) dominates either \( M^{C2s} \) (\( c > \bar{c}(\beta) \)) or \( M^{C2a} \) (\( c > c^+(\beta) \)). To derive when \( M^{D2} \) dominates the other structures \textit{at the same time}, we compare \( \bar{c}(\beta) \) to \( c^+(\beta) \) to determine which one of the thresholds is tighter on \( c \). Moreover, for this comparison we only need to consider the feasible parameter range \( 0 < \beta < \hat{\beta} \), since \( \hat{\beta} < \hat{\beta} \). To this aim, we inspect the expression \( \Delta c_3(\beta) := \bar{c}(\beta) - c^+(\beta) \). Mathematical manipulation reveals that \( \Delta c_3(\beta) \) is always positive for \( \beta < \hat{\beta} \). Consequently, for \( c > \bar{c}(\beta) \), \( M^{D2} \) \textit{dom} \( M^{C2s} \) and \( M^{D2} \) \textit{dom} \( M^{C2a} \).

**Equilibrium \( M^{C2s} \):** For two symmetric cross-border mergers to be the equilibrium result of the merger formation process, we need to determine when \( \pi_{13}^{C2s} + \pi_{24}^{C2s} - (\pi_{12}^{D2} + \pi_{34}^{D2}) > 0 \) and \( \pi_{13}^{C2s} + \pi_{24}^{C2s} - (\pi_{14}^{C2a} + \pi_{23}^{C2a}) > 0 \).
**Proposition 3**: From the previous case we can deduce that for \( \beta < \tilde{\beta} \) and \( c < \bar{c}(\beta) \), it holds that \( M^{C_{2s}} \) dom \( M^{D_2} \). Considering the range \( \beta > \tilde{\beta} \), we find that \( M^{C_{2s}} \) dom \( M^{D_2} \) for \( c < \bar{c}(\beta) \).

**Equilibrium \( M^{C_{2a}} \)**: To establish when two symmetric cross-border mergers dominate the two asymmetric cross-border mergers, it must hold that \( \pi_{13}^{C_{2a}} + \pi_{24}^{C_{2a}} - (\pi_{14}^{D_2} + \pi_{34}^{D_2}) > 0 \). Substitution of the expressions derived in this Appendix and simplifying yields

\[
r_4 c^2 > 0,
\]

where

\[
r_4 = \frac{\beta(\beta-2)(8\beta^2-\beta^3-24\beta+16)}{4(1-\beta)(3\beta^2-16\beta+16)^{\frac{3}{2}}}.
\]

Inspection of this term yields that \( \lim_{\beta \to 0} r_4 = 0 \). The sign of \( r_4 \) cannot be determined unambiguously. A numerical solution yields that \( r_4 > 0 \) for \( \beta < 0.91262 \equiv \tilde{\beta} \) and \( r_4 < 0 \) otherwise. Therefore, we can immediately conclude that \( M^{C_{2s}} \) dom \( M^{C_{2a}} \) if \( \beta \in (0, 0.91262) \). As there is no restriction on \( c \), it must only hold that \( c < \bar{c}(\beta) \).

Proof of Proposition 3. We define global welfare as the sum of consumer surplus, firm profits and union wage bills. More specifically, denote global welfare in structure \( M^r \) to be

\[
W^r = V^r - \sum_{i=1}^{4} (p_i^r q_i^r) + \sum_{i=1}^{4} \pi_i^r + U_A^r + U_B^r,
\]

where \( V^r \) denotes the utility function of a representative consumer in industry structure \( M^r \) \((r = 0, D_1, D_2, C1se, C1si, C2s, C1a, C2a) \) and, following Singh and Vives (1984), is defined
as
\[ V^r = \sum_{i=1}^{4} q_i^r - \frac{1}{2} \left( \sum_{i=1}^{4} (q_i^r)^2 + 2\beta \sum_{i=1}^{4} \sum_{j=1}^{4} q_i^r q_j^r \right) + z, \]
where \( z \) denotes the outside numeraire good with a normalized price to 1, and \( i, j = 1, 2, 3, 4 \); \( i \neq j \).

The welfare expressions for each industry structure can be calculated using the expressions presented in this Appendix. Through the relevant comparisons of the welfare expressions, the relation established in Proposition 3 and graphically presented in Figure 3 can be derived.
References


Appendix B

In order to obtain an interior solution in each industry structure, we derive a sufficient condition (which implies an upper bound on cost parameter $c$), which ensures that all plants produce a positive output in every possible industry structure. Most importantly, we need to inspect the incentives of labor unions in structures $M^{D1}$ and $M^{D2}$, i.e. in industry structures with uniform wages, to raise the wage above some critical level such that the inefficient plant ceases production and exits the market.

The intuition for this consideration is simple. With a uniform wage, a union is constrained in its wage choice. It can thus decide to set an intermediate uniform wage and obtain wage bill revenue from employment in both plants of the merged firm, or set a high wage, at which the merged firm closes down the inefficient plant. Obviously, the union only receives wage bill revenue from the efficient plant, but can consequently demand a higher wage rate. The higher is the non-labor cost $c$, the more we move into the direction of such a corner solution.

We begin our analysis by considering industry structure $M^{D1}$, where we distinguish between the interior solution (four-firm case), which we have derived in Appendix A, and the corner solution (three-firm case). For expositional purposes, denote by $U_{j}^{D1}|_{F=4}$ and $U_{j}^{D1}|_{F=3}$ the wage bill of union $j = A, B$ for the respective four- and three-firm cases.

Derivation of the three-firm case in structure $M^{D1}$

When plant 2 does not produce a positive output, profit functions of the firms are given by

\[
\pi_1(\cdot) = (1 - q_1 - \beta(q_3 + q_4) - w_A)q_1,
\]
\[
\pi_3(\cdot) = (1 - q_3 - \beta(q_1 + q_4) - w_3)q_3,
\]
\[
\pi_4(\cdot) = (1 - q_4 - \beta(q_1 + q_3) - w_4 - c)q_4.
\]

Solving the three first-order conditions of the firms’ profit maximization problems, we obtain the following quantities

\[
\hat{q}_1(\cdot) = \frac{2 - \beta - \beta c + \beta w_3 + \beta w_4 - (2 + \beta)w_A}{2(2 + \beta - \beta^2)},
\]
\[
\hat{q}_3(\cdot) = \frac{2 - \beta - \beta c - (2 + \beta)w_3 + \beta w_4 + \beta w_A}{2(2 - \beta)(1 + \beta)},
\]
\[
\hat{q}_4(\cdot) = \frac{2 - \beta - 2c - \beta c + \beta w_3 - (2 + \beta)w_4 + \beta w_A}{2(2 - \beta)(1 + \beta)}.
\]
Labor unions maximize their wage bills

\[ U_A(\cdot) = w_A \tilde{q}_1(\cdot), \text{ and} \]
\[ U_B(\cdot) = w_3 \tilde{q}_3(\cdot) + w_4 \tilde{q}_4(\cdot), \]

by simultaneously setting their wage rates. Solving the first-order conditions yields the following optimal wage rates for the three-firm case:

\[ w_{D1}^A \big|_{F=3} = \frac{4-\beta^2+\beta c}{8-(-4+\beta)\beta}, \quad (6) \]
\[ w_{D1}^3 \big|_{F=3} = \frac{4\beta-6\beta^2+c\beta^2+16}{32+16\beta-4\beta^2}, \quad (7) \]
\[ w_{D1}^4 \big|_{F=3} = \frac{-4c-2\beta+c+4}{32+16\beta-4\beta^2}, \quad (8) \]

and the union wage bills

\[ U_{D1}^A \big|_{F=3} = \frac{(2+\beta)(4+\beta(-\beta+c))^2}{8(2-\beta)(1+\beta)(-8+(-4+\beta)\beta)^2}, \quad (9) \]

We are now able to derive a condition such that all four plants will produce a positive output. To this aim, we examine the incentives of labor union A (or more generally, the labor union setting a uniform wage rate) to have two rather than one plant in its country producing a positive output. Naturally, union A will only prefer to set a low wage and have two plants active if and only if \( U_{D1}^A \big|_{F=4} - U_{D1}^A \big|_{F=3} > 0 \). Using (4) and (9) and simplifying, the LHS is a u-shaped function which has two roots along the real axis. Solving for the two roots, we obtain one feasible solution

\[ \tau(\beta) := \frac{2\lambda(\beta)}{\mu(\beta)} - \sqrt{\frac{\psi(\beta)}{[\mu(\beta)]^2}}, \]

with

\[ \lambda(\beta) = -512-1920\beta-2112\beta^2+1016\beta^4+60\beta^5-244\beta^6+62\beta^7+33\beta^8-14\beta^9+\beta^{10}, \]
\[ \mu(\lambda) = -512-1792\beta-1472\beta^2+1120\beta^3+1736\beta^4+36\beta^5-400\beta^6+46\beta^7+40\beta^8-13\beta^9+\beta^{10}, \]
\[ \psi(\beta) = 262144+2359296\beta+8716288\beta^2+16285696\beta^3+13783040\beta^4-1695744\beta^5 \]
\[ -12696576\beta^6-6645760\beta^7+3405056\beta^8+3706368\beta^9-275520\beta^{10}-971520\beta^{11} \]
\[ -18640\beta^{12}+151872\beta^{13}+332\beta^{14}-13864\beta^{15}+832\beta^{16}+564\beta^{17}-57\beta^{18}-8\beta^{19}+\beta^{20}. \]
Note that $\lim_{\beta \to 0} \overline{c}(\beta) = 2 - \sqrt{2}$ and $\lim_{\beta \to 1} \overline{c}(\beta) = 0$. Moreover, $\partial \overline{c}(\beta)/\partial \beta < 0$ holds everywhere.

**Derivation of the three-firm and two-firm cases in structure $M^{D2}$**

Now, we need to consider industry structure $M^{D2}$. Results for the interior solution (four-firm case) have been derived in Appendix A. Since in this case both unions set a uniform wage rate, we also need to consider the incentives for either one union or both unions to raise the wage(s) so high that the inefficient plant(s) is (are) closed down. Thus, we consider now the three- and two-firm corner solutions of industry structure $M^{D2}$. We will derive a condition on $c$ so that both unions will always prefer the interior solution (four plants active) and compare it to the threshold derived from structure $M^{D1}$.

Again, denote by $U_{j}^{D2}|_{F=4}$, $U_{j}^{D2}|_{F=3}$, and $U_{j}^{D2}|_{F=2}$ the wage bill of a union $j = A, B$ in the four-firm, three-firm and two-firm case of industry structure $M^{D2}$, respectively.

**The three-firm case ($q_1, q_2 > 0$ and $q_3 > 0, q_4 = 0$)**

Consider the case when plant 4 in country $B$ is closed down. Firms’ profit functions are then given by

$$\pi_{12}(\cdot) = (1 - q_1 - \beta q_2 - \beta q_3 - w_A)q_1 + (1 - q_2 - \beta q_1 - \beta q_3 - c - w_A)q_2,$$

$$\pi_{3}(\cdot) = (1 - q_3 - \beta q_1 - \beta q_2 - w_B)q_3.$$ 

Solving the three first-order conditions of the firms’ profit maximization problems yields the optimal quantities

$$\hat{q}_1(\cdot) = \frac{4 - 6\beta + 23\beta^2 + 4\beta c - \beta^2 c - 4w_A + 4\beta w_A + 2\beta w_B - 2\beta^2 w_B}{4(1 - \beta)(2 + 2\beta - \beta^2)},$$

$$\hat{q}_2(\cdot) = \frac{4 - 6\beta + 23\beta^2 - 4\beta c + \beta^2 c - 4w_A + 4\beta w_A + 2\beta w_B - 2\beta^2 w_B}{4(1 - \beta)(2 + 2\beta - \beta^2)},$$

$$\hat{q}_3(\cdot) = \frac{2 + 3\beta + 2\beta w_A - 2w_B - 2\beta w_B}{2(2 + 2\beta - \beta^2)}.$$

The reduced wage bills of the unions are then given by

$$U_A(\cdot) = w_A (\hat{q}_1(\cdot) + \hat{q}_2(\cdot)),$$

$$U_B(\cdot) = w_B (\hat{q}_3(\cdot)).$$

Solving both first-order conditions we obtain as optimal wage rates:

$$w_A^{D2}|_{F=3} = \frac{6\beta - 4c - 4\beta^2 + 4\beta^3 + 8}{16 + 16\beta - 2\beta^2},$$

$$w_B^{D2}|_{F=3} = \frac{2\beta + c \beta - \beta^2 + 4}{8 + 8\beta - \beta^2}.$$
Using the results for \( w_{D2}^{F=3} \) and \( w_{B2}^{F=3} \), we obtain the union wage bills

\[
U_{A}^{D2}|_{F=3} = \frac{(6\beta - 4c - 4c\beta - 4\beta^2 + c\beta^2 + 8)^2}{2(2 + 2\beta - \beta^2)(8 + 8\beta - \beta^2)^2}, \quad \text{and}
\]

\[
U_{B}^{D2}|_{F=3} = \frac{(\beta + 1)(2\beta + c\beta - \beta^2 + 4)^2}{(2 + 2\beta - \beta^2)(8 + 8\beta - \beta^2)^2}.
\]

The two-firm case \((q_1, q_3 > 0 \text{ and } q_2, q_4 = 0)\)

When both inefficient plants are inactive, the profit functions of the two remaining firms are given by

\[
\pi_1(\cdot) = (1 - q_1 - \beta q_3 - w_A)q_1, \quad \text{and}
\]

\[
\pi_3(\cdot) = (1 - q_3 - \beta q_1 - w_B)q_3.
\]

Solving the two first-order conditions of the firms’ profit maximization problems, we obtain the solutions for optimal quantities

\[
\tilde{q}_1(\cdot) = \frac{2 - \beta - 2w_A + \beta w_B}{4 - \beta^2}, \quad \text{and}
\]

\[
\tilde{q}_3(\cdot) = \frac{2 - \beta - 2w_B + \beta w_A}{4 - \beta^2}.
\]

The unions’ wage bills are given by

\[
U_A(\cdot) = w_A \tilde{q}_1(\cdot), \quad \text{and}
\]

\[
U_B(\cdot) = w_B \tilde{q}_3(\cdot).
\]

Solving the two first-order conditions, we obtain the optimal wage rates in the two-firm case:

\[
w_{A2}^{D2}|_{F=2} = w_{B2}^{D2}|_{F=2} = \frac{2 - \beta}{4 - \beta},
\]

and the wage bills

\[
U_{A2}^{D2}|_{F=2} = U_{B2}^{D2}|_{F=2} = \frac{2(2 - \beta)}{(\beta - 4)^2(2 + \beta)}.
\]

We can now inspect under which circumstances a labor union finds it beneficial to reduce a wage rate in order to keep an inefficient plant active in the market. The union wage bills for all three cases are summarized in the following table.
To this aim, start with the two-firm case in the lower right corner. In this case, both inefficient plants (2 and 4) are closed down. Now consider the decision by union A. Given that only plant 3 produces a positive output in country B, union A would earn a higher wage bill when both plants 1 and 2 produce a positive output in country A if and only if

$$U^D_A|_{F=3} - U^D_A|_{F=2} > 0.$$

Using expressions (10) and (12), this inequality can be written as

$$\left( \frac{(6\beta-4c-4c\beta^2-c^2+8)^2}{2(2+2\beta-\beta^2)(8+8\beta-\beta^2)^2} \right) - \left( \frac{2(2-\beta)}{(\beta-4)^2(2+\beta)} \right) > 0,$$

where the LHS is quadratic in c. Solving for the roots, we obtain two roots, only one of which is feasible and is given by

$$c^*(\beta) := \frac{2(-4-3\beta+2\beta^2)}{-4-4\beta+\beta^2} - 2 \sqrt{\frac{256+640\beta^2+192\beta^2-416\beta^3-92\beta^4+114\beta^5-20\beta^6+\beta^7}{(2+\beta)(16+12\beta-8\beta^2+\beta^3)^2}}.$$

Now consider the three-firm case, where $q_1, q_3, q_2 > 0$ and $q_4 = 0$. Under which circumstances would union B be willing to deviate in a way such that also plant 4 produces a positive output? This will be the case if and only if

$$U^D_B|_{F=4} - U^D_B|_{F=3} > 0.$$

Using expressions (5) and (11), we can write this inequality as

$$\left( \frac{(\beta+1)(c-2)^2}{(\beta+2)(2\beta+4)(4\beta+2)} \right) - \left( \frac{(\beta+1)(2\beta+c\beta^2+4)^2}{(2+2\beta-\beta^2)(8+8\beta-\beta^2)^2} \right) > 0.$$
Again the LHS is quadratic in $c$ and has two roots, only one is feasible and given by

$$c^{**}(\beta) := 2 \left( \frac{\varphi(\beta)}{\tau(\beta)} - \sqrt{\frac{(-2-6\beta-3\beta^2+2\beta^3)(64+160\beta+104\beta^2-4\beta^3+10\beta^4+\beta^5)}{(-\tau(\beta))^2}} \right)$$

with

$$\varphi(\beta) = 128 + 416\beta + 400\beta^2 + 48\beta^3 - 50\beta^4 + 8\beta^5 - 5\beta^6,$$

$$\tau(\beta) = 128 + 384\beta + 272\beta^2 - 112\beta^3 - 114\beta^4 + 10\beta^5 - \beta^6.$$

As a next step, we can compare $c^*(\beta)$ and $c^{**}(\beta)$. Using the expressions derived above, it can be checked easily that $c^{**}(\beta) < c^*(\beta)$ for $\beta \in (0,1)$. Thus, for $c < c^{**}(\beta)$ a union always has a unilateral incentive to lower its own wage rate to have both plants in its country produce a positive output, independent of the number of active plants in the rival country.

**Comparison of thresholds derived from structures $M^{D1}$ and $M^{D2}$**

We have derived two conditions, one from structure $M^{D1}$ and one from structure $M^{D2}$, which ensure that within a given industry structure both inefficient plants produce a positive output, because the unions will prefer these cases over the cases where inefficient plants do not produce. We can summarize our previous results as follows

<table>
<thead>
<tr>
<th>Structure</th>
<th>$q_1, q_2 &gt; 0$ and $q_3, q_4 &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M^{D2}$</td>
<td>$c &lt; c^{**}(\beta)$</td>
</tr>
<tr>
<td>$M^{D1}$</td>
<td>$c &lt; \tau(\beta)$</td>
</tr>
</tbody>
</table>

Table 2

Thus, to find a sufficient condition such that all plants produce a positive output in all market structures we can compare $c^{**}(\beta)$ and $\tau(\beta)$. It can be shown that $\tau(\beta) < c^{**}(\beta)$ for $\beta \in (0,1)$. Thus, we only consider firm asymmetry within the valid parameter range $c \in (0,\tau(\beta))$ to ensure an interior solution in every possible industry structure.
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