Mandelman, Federico S.; Zanetti, Francesco

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Technology Shocks, Employment, and Labor Market Frictions

Federico S. Mandelman and Francesco Zanetti

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Abstract: Recent empirical evidence suggests that a positive technology shock leads to a decline in labor inputs. However, the standard real business cycle model fails to account for this empirical regularity. Can the presence of labor market frictions address this problem without otherwise altering the functioning of the model? We develop and estimate a real business cycle model using Bayesian techniques that allows but does not require labor market frictions to generate a negative response of employment to a technology shock. The results of the estimation support the hypothesis that labor market frictions are responsible for the negative response of employment.

JEL classification: E32

Key words: technology shocks, employment, labor market frictions
Technology Shocks, Employment, and Labor Market Frictions

1 Introduction

A key question in macroeconomics is what driving forces generate aggregate fluctuations. According to the real business cycle (RBC) paradigm initiated by Kydland and Prescott (1982), cycles are generated by persistent shocks to technology; other shocks are either absent or have a minimal role in explaining aggregate fluctuations. A key feature of this theoretical framework is the positive response of employment to technology shocks, as documented by King and Rebelo (2000). Recent empirical evidence, however, conflicts with this prediction. Galí (1999), using long-run restrictions on a structural VAR, where a technology shock is identified as the only shock that affects labor productivity in the long-run, shows that technology shocks have a contractionary effect on employment. In addition, Francis and Ramey (2005), Liu and Phaneuf (2006), Wang and Wen (2007), and Whelan (2004) find that this result is robust to different specifications of the VAR and the measure of productivity used. Moreover, Shea (1998) and Basu, Fernald, and Kimball (2004) find similar evidence by measuring technology with “Solow residuals” derived from microdata. More recently, Canova, López-Salido and Michelacci (2007) and López-Salido and Michelacci (2007) show that a structural VAR model that incorporates job flows also generates a negative response of employment to technology shocks.\footnote{Nonetheless, the debate on this finding is still open. See, among others, Christiano Eichenbaum and Vigfusson (2004), McGrattan (2004), Chari, Kehoe, McGrattan (2005), and Alexopoulos (2006).} On the basis of this stylized fact, the validity of the RBC paradigm could be called into question.

A possible way to reconcile the RBC paradigm with this stylized empirical fact is to amend the standard model such that it generates a negative reaction of employment to a technol-
ogy shock, but still preserves its original functioning. In this spirit, Hairault, Longot and Portier (1997) embed implementation lags in the adoption of new technology into a standard RBC model to make future productivity higher than the current level, thereby decreasing current labor supply for a given increase in labor demand and, consequently, generating a negative response of employment to a technology shock. Francis and Ramey (2005) introduce habit formation in consumption together with adjustment costs on investment, and Leontief technology with variable utilization to match the negative effect of a technology shock on employment. Lindé (2004) observes that if the process for a permanent technology shock is persistent in growth rates, labor inputs fall on impact. More recently, Collard and Dellas (2007), using an international RBC model, show that if the degree of substitution between domestic and foreign goods is low, the reaction of employment to a technology shock is negative. Finally, Wang and Wen (2007) show that a RBC model with firm entry and exit in which firms need time-to-build before earning profits also delivers a negative response of employment to a technology shock. All these works show that by appropriately modifying the standard RBC model, the underlying framework can be revalidated.

Perhaps surprisingly, all of these contributions affect the response of employment in the RBC framework without changing the functioning of the labor market. In principle though, the labor market should be the part of the model most closely related to the reaction of labor to technology shocks. The standard RBC framework assumes perfectly competitive, frictionless, labor markets. Empirical evidence from virtually all the major industrialized countries show that this is rarely the case, as surveyed by Bean (1994), Nickell (1997), and Yashiv (2007). In practice, labor markets are characterized by frictions that prevent the competitive market mechanism from determining labor market equilibrium allocations. Therefore, would labor market frictions be the factor that can generate a negative response of employment to a technology shock? To answer this question, we set up a RBC model that allows, but does not
require, labor market frictions which are modeled like in Blanchard and Galí (2006). We use Bayesian estimation techniques to investigate whether labor market frictions are empirically consistent with the negative response of employment to technology shocks. The findings of this exercise show that the data prefer the version of the model in which labor market frictions generate a negative response of employment to technology shocks.

As mentioned, the presence of labor market frictions in the standard RBC framework may overturn the positive reaction of employment to a technology shock, while leaving the functioning of the model otherwise unchanged; the intuition can be explained as follows. In the standard RBC model, households supply labor until the marginal disutility from supplying an additional unit of labor equates its marginal contribution to production. An increase in productivity induces the household to supply more labor in response to a technology shock. In a labor market characterized by search and matching frictions, workers and firms face a cost in forming a match. Households supply labor until the marginal disutility from supplying an additional unit of labor equates the marginal contribution to production of an extra unit of labor, as in the standard RBC model, net of hiring costs the firm encounters when recruiting an extra worker. Hence, by introducing labor market frictions the optimal choice of labor units also depends on the cost of hiring an additional worker. Hiring costs refer to costs incurred at all stages of recruitment, thereby including the costs of advertising and screening as well as the costs of training and disrupting production. In principle, as Yashiv (2000a,b) point out, hiring costs can be either pro- or counter-cyclical. On the one hand, recessions represent times of low opportunity costs, thereby implying more re-structuring of the workforce -including more hiring- so that the firms have to devote more resources to screening, leading hiring costs to be counter-cyclical. On the other hand, recessions are also times when, due to the high availability of workers looking for jobs, the cost of advertising is low, encouraging hiring costs to be pro-cyclical. In this paper, we internalize this contradiction by allowing hiring costs to
react directly to productivity and leaving the data to decide whether their reaction is pro- or counter-cyclical. Depending on how the cost of hiring reacts to productivity, the response of employment to a technology shock can be either positive or negative. For instance, if hiring costs co-move positively with productivity, a technology shock increases the marginal product of labor, as in the standard RBC model, but it also increases the cost of recruiting an extra worker. If the latter effect dominates the first one, thereby reducing the marginal rate of transformation, employment would react negatively to a technology shock.

Before proceeding, we discuss the context provided by two related studies. As mentioned, Canova, López-Salido and Michelacci (2007) and López-Salido and Michelacci (2007) find empirical support for a decline in labor inputs in response to technology shocks. They show that this evidence is consistent with an extension of the Solow (1960) growth model that incorporates a vintage structure of technology shocks and labor market frictions. Our approach differs from these studies in two ways. First, in our paper we enrich a standard RBC model with labor market frictions and the negative response of labor inputs to technology shocks is solely due to the structure of the labor market. While the afore mentioned papers draw their conclusions on the assumption that part of the existing productive units fail to adopt the most recent technological advances. Second, we estimate the structural parameters of the model using Bayesian estimation techniques and we then use this coherent framework to draw conclusions. We think that the advantage of our approach is that it develops the analysis using a unified, empirically grounded framework where the data establish whether labor market frictions are solely responsible for the results, rather than simply measuring

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2This assumption implies that newly created jobs always embody new technologies while old jobs are incapable of upgrading their technologies. Hence, technology shocks make some firms unprofitable and generate a displacement of workers which triggers what the authors call Schumpeterian creative destruction that ultimately leads to lower employment. In their investigation the key element to generate the finding is the vintage structure of technology shocks. Labor market frictions are used as a convenient feature to internalize job flows into the analysis, but are not primarily responsible for the negative response of employment to technology shocks.
whether the predictions from a calibrated model are consistent with the empirical evidence.

The remainder of the paper is organized as follows. Section 2 lays out the theoretical model, Section 3 describes the solution, data, and estimation, Section 4 presents the role of labor market frictions, and Section 5 concludes.

2 The model

A standard RBC model is enriched to allow for labor market frictions of the Diamond-Mortensen-Pissarides model of search and matching, as in Blanchard and Galí (2006). This framework relies on the assumption that the processes of job search and recruitment are costly for both the firm and the worker.

The economy is populated by a continuum of infinite-living identical households who produce goods by employing labor. During each period, a constant fraction of jobs is destroyed and labor is employed through hiring, which is a costly process. Each household maximizes the utility function:

$$E \sum_{t=0}^{\infty} \beta^t \varepsilon^b_t \left( \ln C_t - \varepsilon^I_t \frac{N_t^{1+\phi}}{1+\phi} \right),$$

where $C_t$ is consumption, $N_t$ is the fraction of household members who are employed, $\beta$ is the discount factor such that $0 < \beta < 1$, and $\phi$ is the inverse of the Frisch intertemporal elasticity of substitution in labor supply such that $\phi \geq 0$. In this model we assume full participation, such that the members of a household can be either employed or unemployed, which implies $0 < N_t < 1$. Equation (1), similar to Smets and Wouters (2003), contains two preference shocks: $\varepsilon^b_t$ represents a shock to the discount rate that affects the intertemporal rate of substitution between consumption in different periods, and $\varepsilon^I_t$ represents a shock to labor supply. Both shocks are assumed to follow a first-order autoregressive process with i.i.d. normal error terms such that $\varepsilon^b_{t+1} = \varepsilon^b_t \rho_b \exp(\eta_{b,t+1})$, where $0 < \rho_b < 0$, $\eta_b \sim N(0, \sigma_b)$,
and, similarly, \( \varepsilon_{t+1}^l = \epsilon_0(\varepsilon_t^l)^{\rho_l} \exp(\eta_{l,t+1}) \), where \( 0 < \rho_l < 0 \), and \( \eta_l \sim N(0, \sigma_l) \). During each period, output, \( Y_t \), is produced according to the production function:

\[
Y_t = A_t N_t, \tag{2}
\]

where \( A_t = \varepsilon_t^a \) is an exogenous technology shock that follows a first-order autoregressive process with i.i.d. normal error terms such that \( \varepsilon_{t+1}^a = \epsilon_0(\varepsilon_t^a)^{\rho_a} \exp(\eta_{a,t+1}) \), where \( 0 < \rho_a < 0 \), and \( \eta_a \sim N(0, \sigma_a) \). During each period total employment is given by the sum of the number of workers who survive the exogenous separation, and the number of new hires, \( H_t \). Hence, total employment evolves according to

\[
N_t = (1 - \delta) N_{t-1} + H_t, \tag{3}
\]

where \( \delta \) is the job destruction rate, and \( 0 < \delta < 1 \). Accounting for job destruction, the pool of household’s members unemployed and available to work before hiring takes place is:

\[
U_t = 1 - (1 - \delta) N_{t-1}. \tag{4}
\]

It is convenient to represent the job creation rate, \( x_t \), by the ratio of new hires over the number of unemployed workers such that:

\[
x_t = H_t/U_t, \tag{5}
\]

with \( 0 < x_t < 1 \), given that all new hires represent a fraction of the pool of unemployed

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\(^3\)As discussed in Smets and Wouters (2003), the inclusion of these structural shocks is a standard procedure necessary to avoid the singularity problem in the model estimation, and allow for a better characterization of the unconditional moments in the data.
workers. The job creation rate, \( x_t \), may be interpreted as an index of labor market tightness. This rate also has an alternative interpretation: from the viewpoint of the unemployed, it is the probability of being hired in period \( t \), or in other words, the job-finding rate. The cost of hiring a worker is equal to \( G_t \) and, as in Blanchard and Galí (2006), is a function of \( x_t \) and the state of technology:

\[
G_t = A_t^\gamma B x_t^\alpha
\]  

(6)

where \( \gamma \) determines the extent to which, if any, hiring costs co-move with technology; \( \alpha \) is the elasticity of labor market tightness with respect to hiring costs; and \( B \) is a scale parameter. Hence, \( \gamma \in \mathbb{R}, \alpha \geq 0 \), and \( B \geq 0 \). As pointed out in Yashiv (2000a,b) and subsequently in Rotemberg (2006) and Yashiv (2006), this general formulation captures the idea that, in principle, hiring costs may be either pro- or counter-cyclical. Note that, given the assumption of full participation, the unemployment rate, defined as the fraction of household members left without a job after hiring takes place, is defined as:

\[
u_t = 1 - N_t.
\]  

(7)

The aggregate resource constraint

\[Y_t = C_t + G_t H_t\]  

(8)

completes the description of the model.

Since the two welfare theorems apply, resource allocations can be characterized by solving the social planner’s problem. The social planner chooses \( \{Y_t, C_t, H_t, G_t, x_t, U_t, N_{t-1}\}_{t=0}^{\infty} \) to maximize the household’s utility subject to the aggregate resource constraints, represented by equations (2)-(8). To solve this problem it is convenient to use equation (8), together with
the other constraints, to obtain the aggregate resource constraint of the economy expressed in terms of consumption and employment. The aggregate resource constraint of the economy can therefore be written as:

\[ A_t N_t = C_t + A_t^\gamma B \frac{[N_t - (1 - \delta)N_{t-1}]^{1+\alpha}}{[1 - (1 - \delta)N_{t-1}]^\alpha}. \] (9)

In this way, the social planner chooses \( \{C_t, N_t\}_{t=0}^\infty \) to maximize the household’s utility (1) subject to the aggregate resource constraint (9). Letting \( \Lambda_t \) be the non-negative Lagrangian multiplier on the resource constraint, the first order condition for \( C_t \) is:

\[ \Lambda_t = \frac{\varepsilon^h}{C_t}, \] (10)

and the first order condition for \( N_t \) is:

\[ \frac{\varepsilon^l N_t^\phi}{\Lambda_t} = A_t - A_t^\gamma B(1 + \alpha) x_t^\alpha + \beta B(1 - \delta) \frac{A_t^{\alpha+1} x_{t+1}^\alpha}{A_t} \left[ (1 + \alpha) x_{t+1}^\alpha - \alpha x_{t+1}^{1+\alpha} \right]. \] (11)

Equation (10) is the standard Euler equation for consumption, which equates the Lagrange multiplier to the marginal utility of consumption. Equation (11) equates the marginal rate of substitution to the marginal rate of transformation. The marginal rate of transformation depends on productivity, \( A_t \), as in the standard RBC model, but also, due to the presence of labor market frictions, on foregone present and future costs of hiring. More specifically, the three terms composing the marginal rate of transformation are the following. The first term, \( A_t \), corresponds to the additional output generated by a marginal employed worker. The second term represents the cost of hiring an additional worker, and the third term captures

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4 To do so, use equation (2) to substitute for \( Y_t \) into equation (8); use equation (3) to substitute for \( H_t \) into equation (8); use equations (3) and (4) into (5) and substitute the outcome into (6) so to obtain an expression of \( G_t \) that can be used into equation (8).
the savings in hiring costs resulting from the reduced hiring needs in period $t+1$. In the standard RBC model only the first term appears.

3 Bayesian Estimation

Equations (2)-(11) describe the behavior of the endogenous variables $\{Y_t, C_t, H_t, G_t, x_t, U_t, u_t, N_{t-1}, \Lambda_t\}$, and persistent autoregressive processes describe the exogenous shocks $\{\varepsilon^b_t, \varepsilon^l_t, \varepsilon^a_t\}$. The equilibrium conditions do not have an analytical solution. For this reason, the system is approximated by loglinearizing equations (2)-(11) around the stationary steady state.\footnote{See the Appendix for the full derivation of the steady state and loglinearized model.} In this way, a linear dynamic system describes the path of the endogenous variables’ relative deviations from their steady-state value, accounting for the exogenous shocks. The solution to this system is derived using Klein (2000), which is a modification of Blanchard and Kahn (1980), and takes the form of a state-space representation. This latter can be conveniently used to compute the likelihood function in the estimation procedure. The Bayesian estimation technique uses a general equilibrium approach that addresses the identification problems of reduced-form models (see Leeper and Zha, 2000). In addition, as stressed by Lubik and Schorfheide (2005), it overcomes the potential misspecification problem in the comparison of DSGE models, and, as pointed out in Fernandez-Villaverde and Rubio-Ramírez (2004), it outperforms GMM and maximum likelihood methods for small data samples. To understand the estimation procedure, define $\Theta$ as the parameter space of the DSGE model, and $Z^T = \{z_t\}_{t=1}^T$ as the data observed. From their joint probability distribution $P(Z^T, \Theta)$ we can derive a relationship between the prior distribution of the parameters $P(\Theta)$ and conditional distribution of the likelihood function $P(Z^T|\Theta)$. Using Bayesian theory, we obtain the posterior distribution of the parameters, $P(\Theta|Z^T)$, as follows: $P(\Theta|Z^T) \propto P(Z^T|\Theta)P(\Theta)$.\footnote{See the Appendix for the full derivation of the steady state and loglinearized model.}
This method updates the \textit{a priori} distribution using the likelihood contained in the data to obtain the conditional posterior distribution of the structural parameters. The posterior density $P(\Theta|Z^T)$ is used to draw statistical inference on the parameter space $\Theta$. Combining the state-space representation, implied by the solution of the linear rational expectation model, and the Kalman filter we can compute the likelihood function. The likelihood and the prior permit a computation of the posterior, that can be used as the starting value of the random walk version of the Metropolis algorithm, which is a Monte Carlo method used to generate draws from the posterior distribution of the parameters.\textsuperscript{6}

**Data** The econometric estimation uses US quarterly data for output, unemployment, and the job finding rate for the sample period 1951:1 through 2004:4. Output is defined as real gross domestic product in chained 2000 dollars taken from the Bureau of Economic Analysis. The unemployment rate is defined as the civilian unemployment rate, and is taken from the Bureau of Labor Statistics. The job finding rate is taken from Shimer (2007). The data for output and consumption are logged and H-P filtered prior to estimation, and the unemployment and job finding rate series are demeaned.

**Calibration** Some parameters are kept fixed from the start of the calculations. This can be seen as a prior that is extremely precise. As in other similar studies,\textsuperscript{7} a first attempt to estimate the model produced implausible values for the discount factor. We thus set the real interest rate to 4 percent annually, a number commonly used in the literature, which

\textsuperscript{6}This paper reports results based on 200,000 draws of such an algorithm. The jump distribution is normalized to one, with covariance matrix equal to the Hessian of the posterior density evaluated at the maximum. The scale factor is chosen in order to deliver an acceptance rate between 20 and 35 percent depending on the run of the algorithm. Convergence of the algorithm is assessed by observing the plots of the moment draws (mean, standard deviation, skewness and kurtosis). Measures of uncertainty are derived from the percentiles of the draws.

pins down the quarterly discount factor $\beta$ to 0.99. Consistent with US data, the steady state value of the job finding rate, $x$, and unemployment rate, $u$, are set equal to 0.7 and 0.05 respectively. This yields a value for the separation rate, $\delta = ux/(1-u)(1-x)$, roughly equal to 0.12, which is in line with Hall (1995). We need to set a value for $B$, which determines the steady state value of hiring costs. Since there is not precise empirical evidence on this parameter, we follow Blanchard and Galí (2006) and choose $B$ so that hiring costs represent one percent of total output, which seems a reasonable upper bound. This implies that $B$ is roughly equal to 0.11. Finally, before proceeding with the estimation, we need to calibrate some parameters in order to address some identification issues. Of special interest is the estimate for the elasticity of hiring cost to technology, $\gamma$. In principle, hiring costs in equation (6) may increase because of high sensitivity of $G_t$ to the labor market tightness, $\alpha$, or to the state of technology, $\gamma$. At the same time, lower values of $\sigma_a$ necessitate higher values of $\gamma$ to explain the volatility of hiring costs and vice-versa. To address these issues, we proceed in two steps. First, we set the parameters characterizing the stochastic process for productivity, which is a valid procedure under the assumption of i.i.d shocks. We use the estimates in King and Rebelo (2000), and calibrate the autoregressive parameter, $\rho_a$, equal to 0.979 and the standard deviation of technology shocks, $\sigma_a$, equal to 0.0072. Second, as detailed below, we set a very precise prior for $\alpha$, and a very flat prior for $\gamma$ with an agnostic prior mean centered at 0.

**Prior Distributions** Figure 1 depicts the prior density (grey line) of the parameters to be estimated: $\{\sigma_b, \sigma_l, \phi, \gamma, \alpha, \rho_b, \rho_l\}$. The first five columns of Table 1 present the mean and standard deviation of the prior distributions, together with their respective densities and ranges. The shapes of the densities are selected to match the domain of the structural parameters, and we deduct the prior mean and distribution from previous studies. The prior
mean for the variance of the stochastic components \( \{\sigma_b, \sigma_l\} \) is in line with previous studies such as Bencivenga (1992); De-Jong, Ingram, and Whiteman (2000); Chang and Schorfheide (2003); and Smets and Wouters (2003) and is equal to 0.002, and 0.010 respectively. They are assumed to have an Inverse Gamma distribution with a degree of freedom equal to 2. We use this distribution because it delivers positive values with a rather large domain. The prior distribution of the autoregressive parameters of the shocks is a Beta distribution that covers the range between 0 and 1, in accordance to the model specification. As is common practice in the Bayesian estimation literature, we want to distinguish between persistent and non-persistent shocks, so we choose a precise mean, that is, a rather strict standard error, which is equal to 0.1. Since the inverse of the Frisch intertemporal elasticity of substitution in labor supply, \( \phi \), and the elasticity of labor market tightness with respect to hiring costs, \( \alpha \), are theoretically restricted to be positive, we consider a Gamma distribution for them. The prior for \( \phi \) is loosely centered at 0.4 which corresponds to a value in between the microeconomic estimates, as in Pencavel (1986), and the relative large values usually observed in the macro literature, as in Rogerson and Wallenius (2007). In setting the prior for \( \alpha \), as suggested in Blanchard and Galí (2006), we exploit a simple mapping between this model and the Diamond-Mortensen-Pissarides specification, and assume a precise prior mean equal to 1 with a standard error equal to 0.05, which is sufficient to capture the range of estimates in the literature. Finally, since the elasticity of technology shocks to hiring costs, \( \gamma \), is allowed to be either positive, negative, or zero, we assume it has a Normal distribution. In order to get a reliable identification of \( \gamma \), and allow for a wide range of possible values, we impose a

\[ H = Zu^{\gamma}V^{1-\gamma} \]

Hence, \( \alpha \) in our paper corresponds to \( \eta / (1 - \eta) \) in their setup. Since the estimates of \( \eta \) are typically very close to 0.5, as surveyed in Petrongolo and Pissarides (2001), we assume a prior mean for \( \alpha \) equal to one, which is also the parameter value used in Blanchard and Galí (2006).
very flat prior with a mean equal to 0 and a standard deviation equal to 7.\footnote{To check the robustness of the results to the assumptions on the prior distribution of $\gamma$, we have estimated the model using different means and standard deviations on the prior of this parameter. This has very little impact on the results, which are available upon request.}

**Estimation results (posterior distributions)** Figure 1 shows the posterior density (black line) together with the mode of the posterior density (red dotted line) of the estimated parameters. The plots show that the marginal posteriors and the priors of the behavioral parameters are different, supporting the presumption that the data are relatively informative about the values of the estimated parameters. The last three columns of Table 1 report the posterior mean and 95\% probability interval of the structural parameters. The posterior mean of the inverse of the Frisch intertemporal elasticity of substitution in labor supply, $\phi$, equals to 0.34, which implies an elasticity of labor supply equal to 2.9. This is consistent with the value suggested by Rogerson and Wallenius (2007) and more generally is in line with the calibrated values used in the macro literature as advocated by King and Rebelo (2000). The posterior mean of the elasticity of hiring costs to labor market tightness, $\alpha$, is 1.01. As shown in Blanchard and Galí (2006), in a decentralized version of this economy, we can interpret this parameter as the ratio between the wage bargaining power of households and firms. Therefore, the estimated unitary value supports the idea that households and firms share their bargaining power equally. This result is in line with the empirical findings in Petrongolo and Pissarides (2001). Of special interest here, of course, is the estimate for the elasticity of hiring costs to technology, $\gamma$. The posterior mean of $\gamma$ is 4.00, which, as detailed below, supports the fact that the data prefer a positive response of hiring costs to technology shocks. In addition, notice that the estimation delivers a sizable reading for $\gamma$ despite its loose prior.

Turning now to the stochastic processes, the posterior mean of the persistence of preference
shocks, \( \rho_b \), is 0.50, while the estimate of the persistence of labor supply shocks, \( \rho_l \), is 0.85. The posterior mean of the volatility of preference shocks, \( \sigma_b \), is 0.0018, and the posterior mean of the volatility of labor supply shocks, \( \sigma_l \), is 0.0087. These values are similar to the estimates in Smets and Wouters (2003, 2007), and Chang, Doh, and Schorfheide (2007).

Figure 2 traces out the estimated model’s implied impulse responses (alongside 95% confidence intervals) of each variable to a one-standard-deviation technology shock.\(^{10}\) The reaction of output and consumption is positive on impact. The reaction of hiring costs, as expected, given the large and positive estimate of \( \gamma \), is positive and. For this reason it is more costly to recruit workers, as explained in more detail below, and consequently employment declines. As employment falls, unemployment rises; this dampens the reaction of the number of hires and of labor market tightness.

Table 2 reports autocorrelation functions of key macroeconomic variables with output based on the mode of the model’s posterior distribution and the data. In general, the model’s results support the empirical evidence. For instance, the model’s simulations deliver a positive contemporaneous correlation of output with consumption and labor market tightness, as well as a negative correlation with the unemployment rate, which is consistent with the data. Moreover, the model matches the sign of correlations at different leads and lags well. Table 3 shows asymptotic (i.e. infinite horizon) forecast error variance decompositions into percentages due to each of the model’s shocks. Similarly to Smets and Wouters (2007), the variance decompositions indicate that in the long run it is mostly two supply shocks, productivity and labor supply innovations, that account for almost all macroeconomic variability. Since \( \sigma_b \) is estimated to be almost zero, preference shocks contribute nothing to the volatility of any variable.\(^{11}\) Instead, technology shocks account for nearly 75 percent of the unconditional

\(^{10}\)The impulse responses of the model to the preference and labor supply shocks are available on a companion appendix to this paper, available from the authors upon request.

\(^{11}\)Since hiring costs represent only one percent of total output, the shock to the stochastic discount factor
variance in detrended output and consumption, which is a result that closely resembles the findings in Kydland and Prescott (1991) and Ireland (2001). Labor supply shocks account for almost all the variation in unemployment and labor market tightness.

4 The Role of Labor Market Frictions

No hiring costs In order to establish a benchmark against which to compare, Table 4 estimates the model imposing $B = 0$, so that the theoretical framework nests the first order conditions of a standard RBC model where labor frictions are absent. To be consistent throughout the estimation exercise, the prior distributions of the parameters are the same as those in the baseline model. Estimation results indicate that the posterior mean of the inverse of the elasticity of labor supply, $\phi$, equals 0.39, and the posterior mean of the autoregressive component of the labor supply shocks is highly persistent. Similarly, the magnitude of the volatility of the shocks is close to that of the unconstrained model. In general, these estimates are similar to those in the model that allow for labor market frictions, and, moreover, are in line with findings in Bencivenga (1992); De-Jong, Ingram, and Whiteman (2000); Ireland (2001, 2004); Chang and Schorfheide (2003); and Zanetti (2008) who estimate standard RBC models.

What lies behind the posterior means of the parameters for the reaction of the variables to technology shocks? Figure 3 traces out the estimated model’s implied impulse responses of each variable to a one-standard-deviation technology shock for both versions of the model, with and without labor frictions. It is immediately noticeable that the reaction of output and consumption is quantitatively the same across the two models, while the reaction of employment is negative in the presence of labor market frictions.12

plays a minimal role in the variance decompositions of the variables and so it is difficult to identify.
12Of course, in the model with labor market frictions, in addition to the reaction of output, consumption,
How can the presence of labor market frictions generate such a striking result? As mentioned, the answer lies in the way hiring costs react to productivity shocks. Here the reaction is determined by the elasticity of hiring costs to a technology shock, which is represented by the parameter $\gamma$. The estimation exercise allows the value of this parameter to be either positive, negative, or equal to zero and leaves the data to choose the preferred value. The estimation suggests that the data prefer $\gamma$ to be positive, such that hiring costs co-move positively with technology shocks (which is also the assumption in the calibrated model of Blanchard and Galí (2006)). To understand how this generates a negative reaction of employment to technology shocks, consider equation (11), which represents the labor market equilibrium condition. A productivity shock would increase the marginal product of labor, the first term on the right hand side of equation (11), as in the standard RBC model, but it would also increase the cost of recruiting an additional worker, the second term on the right hand side of equation (11), and, at the same time, reduce the hiring needs in period $t+1$, the third term on the right hand side of equation (11). The effect on the second term, namely the cost of recruiting an additional worker, dominates the other two and, as a result, the marginal rate of transformation, which is the right hand side of equation (11) is reduced, and therefore generate a negative response of employment to technology shocks. In the standard RBC model, the correspondent equilibrium condition, equivalent to equation (11), is $\varepsilon_t^lN_t^{\phi+1}/\varepsilon_t^b = 1$, which implies a level of employment invariant to technology shocks, which is the result of offsetting income and substitution effects on labor supply. Without capital accumulation, such a result is standard in this class of models, as King and Rebelo (2000) point out. Despite the different reactions of employment to a technology shock, the functioning of the two models is qualitatively similar.
**Hiring costs not reactive to technological shocks**

Turning to the parameter describing the elasticity of hiring costs to technology shocks, \( \gamma \), we now impose the neutral assumption that hiring costs do not react directly to technology shocks. In this way, we determine whether the data prefer the version of the model with hiring costs reacting to technology shocks or a more constrained specification where hiring costs do not directly react to technology. We test which version of the model the data prefer by imposing \( \gamma = 0 \) on the specification of the model. As before, the prior distributions of the parameters are the same as those in the baseline model. Table 4 reports the posterior mean and 95% probability interval of the parameters for the constrained model. The posterior mean of the structural parameters for this constrained specification are reasonably close to those where \( \gamma \) is allowed to differ from zero. In particular, the posterior mean of the inverse of the elasticity of labor supply, \( \phi \), equals 0.34, and the posterior mean of the autoregressive component of the labor supply shocks are highly persistent. Results indicate that the volatility of the stochastic components are of a similar magnitude than the estimates of the unconstrained model. Also in this instance, the posterior mean of \( \alpha \) is almost unitary and equals 1.01. Overall, the similarity of these estimates with those described above suggests that the underlying RBC model is consistently estimated across different model specifications. Figure 4 shows the model’s implied impulse responses of each variable to a one-standard-deviation technology shock for both the constrained model where \( \gamma = 0 \), and the RBC model with labor frictions. Output, consumption, and employment positively react to a technology shock, as in the unconstrained specification. When \( \gamma = 0 \), hiring costs do not directly react to technological innovations. In this case, the effect on the second term on the right hand side of equation (11), namely the cost of recruiting an additional worker, is dominated by the counteracting effect of the two other terms, thus generating a positive response of employment to technology shocks. The positive reaction of employment leads to a positive response in the number of hires and this,
coupled with the negative reaction of unemployment, generates an increase in labor market tightness and, consequently, the cost of hiring increases slightly on impact.

Model Comparison  In order to establish whether the data prefer the unconstrained formulation of the model, the version without labor market frictions \((B = 0)\), or the version in which hiring costs do not directly react to technological innovations \((\gamma = 0)\), we first consider the difference between the log marginal likelihood of each model with respect to the log marginal likelihood of the unconstrained specification. We thus define the marginal likelihood of a model, \(J\), as follows: \(M_J = \int_\Theta P(\Theta|J) P(Z^T|\Theta, J) d\Theta\). Where \(P(\Theta|J)\) is the prior density for model \(J\), and \(P(Z^T|\Theta, J)\) is the likelihood function of the observable data, conditional on the parameter space \(\Theta\) and the model \(J\). The marginal likelihood of a model (or the Bayes factor) is directly related to the predicted density of the model given by:

\[
\hat{p}^{T+m}_{T+1} = \int_\Theta P(\Theta|Z^T, J) \prod_{t=T+1}^{T+m} P(z_t|Z^T, \Theta, J) d\Theta.
\]

Therefore the marginal likelihood of a model also reflects its prediction performance.

Considering that this criterion penalizes overparametrization, models with labor market frictions do not necessarily rank better if the extra friction does not sufficiently help in explaining the data. As from the last row of Table 4, the log marginal likelihood difference between the unconstrained specification and the model with no hiring costs is 24.53. In other words, in order to choose the constrained version over the original formulation, the Bayes factor requires a prior probability over the constrained version \(e^{24.53}\) times larger than over the unconstrained model. This can be accepted as conclusive evidence in favor of the model with labor market frictions, as suggested in Rabanal and Rubio-Ramírez (2005). Refering to the last row of Table 4, the data also prefer the unconstrained version of the model in which estimation results reflect hiring costs that respond pro-cyclically to technology shocks. In fact, the log-difference between the unconstrained specification and the one in which \(\gamma = 0\),
As a final exercise, in line with the RBC tradition and the seminal work by Merz (1995), we determine which version of the model better matches the sample statistics in the data. Here the series are treated in the same way as in the estimation exercise. Table 5 reports measures of volatility for the posterior means relative to output for the series of consumption, $C_t$, and the unemployment rate, $u_t$, in the different models and the data. The model with labor market frictions produces relative standard deviations of the unemployment rate and consumption that are closer to the values in the data, than the model without labor frictions. Overall, the match between models with labor market frictions and data is better than that of alternative specifications. In the models characterized by labor market frictions, as in the data, consumption is always less volatile than output, and the unemployment rate is less volatile than both output and consumption. The ability of the model to replicate the moments in the data could improve if capital accumulation is added into the model’s specification. In fact, as pointed out in King and Rebelo (2000), the presence of capital accumulation is central for the RBC framework to match the cyclical movements that we see in the data. In this study, the model excludes capital in order to maintain the theoretical framework as close as possible to that of Blanchard and Galí (2006) and leave the investigation of the effect of capital open for future research.

5 Conclusion

Recent empirical evidence led by Galí (1999) and supported by several other studies suggests that a positive technology shock leads to a decline in labor inputs. This is the opposite of a key prediction of the standard RBC model, thereby calling the validity of the RBC paradigm into question. This paper has investigated whether the presence of labor market frictions,
which are modeled as in Blanchard and Galí (2006), may rehabilitate the RBC framework. Using Bayesian techniques, we show that data support the presence of labor market frictions as the factor responsible for the negative response of employment to a technology shock.

The findings of this paper support those who suggest that it seems premature to reject the notion that technology shocks are the main driving forces of the business cycle on the ground of the negative response of employment to technology shocks.

6 References


Appendix. The Steady State and Loglinearized System

Equations (2)-(11) in the paper imply that in the absence of shocks, the economy converges to steady state growth path along which all the variables are constant, with \( Y_t = Y, \ C_t = C, \ H_t = H, \ G_t = G, \ x_t = x, \ U_t = U, \) and \( N_t = N \) for all \( t = 0, 1, 2, \ldots \). Equations (2)-(8) can be used to determine \( Y, \ C, \ H, \ G, \ x, \ U, \) and \( u \) such that:

\[
Y = AN,
\]
\[ H = \delta N, \]
\[ U = 1 - (1 - \delta)N, \]
\[ x = \frac{H}{U}, \]
\[ G = A^\gamma B x^\alpha, \]
\[ C = Y - GH, \]
\[ \text{and } u = 1 - N. \]

The values of \( N \) can be determined by solving numerically the steady state equivalent of equation (11), which is

\[ \frac{N^\phi C}{A} = 1 - A^{\gamma-1} B (1 + \alpha)x^\alpha + \beta B (1 - \delta) A^{\gamma-1} [(1 + \alpha)x^\alpha - \alpha x^{1+\alpha}]. \]

Hence, to compute the steady state values, start by calculating \( N \), then calculate \( Y, C, H, G, x, U, \) and \( u \). Equations (2)-(8) can be loglinearized around the steady state to describe how the model’s variables respond to shocks. Let a hat on a variable denotes the logarithmic deviation from its steady state. The loglinear approximation of (2)-(8) yields

\[ 0 = -\hat{Y}_t + \hat{A}_t + \hat{N}_t, \quad \text{(1)} \]
\[ 0 = -\hat{N}_t + (1 - \delta)\hat{N}_{t-1} + \delta \hat{H}_t, \quad \text{(2)} \]
\[ 0 = U \hat{U}_t + (1 - \delta)N\hat{N}_{t-1}, \quad \text{(3)} \]
\[ 0 = -\hat{x}_t + \hat{H}_t - \hat{U}_t, \quad \text{(4)} \]
0 = −\( \hat{G}_t + \gamma \hat{A}_t + \alpha \tilde{x}_t \), \hspace{1cm} (5)

0 = \( \hat{u}_t + \hat{N}_t \), \hspace{1cm} (6)

0 = −\( \hat{Y}_t + (C/Y)\hat{C}_t + (GH/Y) \left( \hat{G}_t + \hat{H}_t \right) \), \hspace{1cm} (7)

and

\[
0 = \text{mrs} \left( \hat{\varepsilon}^t + \phi \hat{N}_t + \hat{C}_t - \hat{A}_t \right) + \text{term}_1 \left[ (\gamma - 1) \hat{A}_t + \alpha \tilde{x}_t \right] \\
+ \text{term}_2 \left[ (1 + \alpha) \alpha x^\alpha \tilde{x}_{t+1} - \alpha (1 + \alpha) x^{1+\alpha} \tilde{x}_{t+1} \right] \\
+ \text{term}_3 \left( \gamma \hat{A}_{t+1} - \hat{A}_t + \hat{C}_t - \hat{C}_{t+1} - \hat{\varepsilon}_t^b + \hat{\varepsilon}_{t+1}^b \right), \hspace{1cm} (8)
\]

where \( \text{mrs} = -(N^\phi C/A) \), \( \text{term}_1 = -A^{\gamma-1} B(1+\alpha)x^\alpha \), \( \text{term}_2 = \beta B(1-\delta)A^{\gamma-1} \), \( \text{term}_3 = \beta B(1-\delta)A^{\gamma-1} \left[ (1 + \alpha) x^\alpha - \alpha x^{1+\alpha} \right] \).
Table 1. Summary statistics for the prior and posterior distribution of the parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Mean</th>
<th>Prior SE</th>
<th>Density</th>
<th>Range</th>
<th>Posterior</th>
<th>2.5%</th>
<th>97.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>0.40</td>
<td>0.15</td>
<td>Gamma</td>
<td>$\mathbb{R}^+$</td>
<td>0.3438</td>
<td>0.1322</td>
<td>0.5367</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0</td>
<td>7</td>
<td>Normal</td>
<td>$\mathbb{R}$</td>
<td>4.003</td>
<td>1.1526</td>
<td>6.873</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1</td>
<td>0.05</td>
<td>Gamma</td>
<td>$\mathbb{R}^+$</td>
<td>1.0126</td>
<td>0.9303</td>
<td>1.0947</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>0.5</td>
<td>0.1</td>
<td>Beta</td>
<td>[0,1]</td>
<td>0.5005</td>
<td>0.3306</td>
<td>0.6603</td>
</tr>
<tr>
<td>$\rho_l$</td>
<td>0.5</td>
<td>0.1</td>
<td>Beta</td>
<td>[0,1]</td>
<td>0.8485</td>
<td>0.7801</td>
<td>0.9214</td>
</tr>
<tr>
<td>$\sigma_b$</td>
<td>0.002</td>
<td>2*</td>
<td>Inv gamma</td>
<td>$\mathbb{R}^+$</td>
<td>0.0018</td>
<td>0.0005</td>
<td>0.0033</td>
</tr>
<tr>
<td>$\sigma_l$</td>
<td>0.01</td>
<td>2*</td>
<td>Inv gamma</td>
<td>$\mathbb{R}^+$</td>
<td>0.0087</td>
<td>0.0076</td>
<td>0.0099</td>
</tr>
</tbody>
</table>

Notes: Results based on 200,000 draws of the Metropolis Algorithm. For the Inverted Gamma function the degrees of freedom are indicated.
Table 2. Descriptive Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data $\text{Corr}(\text{Variable}_{t+j}, Y_t)$</th>
<th>Model $\text{Corr}(\text{Variable}_{t+j}, Y_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-2  -1  0  1  2</td>
<td>-2  -1  0  1  2</td>
</tr>
<tr>
<td>$Y$</td>
<td>0.59 0.84 1.00 0.84 0.59</td>
<td>0.47 0.72 1.00 0.72 0.47</td>
</tr>
<tr>
<td>$u$</td>
<td>-0.31 -0.45 -0.55 -0.56 -0.48</td>
<td>-0.14 -0.24 -0.37 -0.26 -0.15</td>
</tr>
<tr>
<td>$C$</td>
<td>0.68 0.83 0.87 0.70 0.47</td>
<td>-0.47 0.72 1.00 0.71 0.46</td>
</tr>
<tr>
<td>$x$</td>
<td>0.33 0.46 0.55 0.55 0.46</td>
<td>0.09 0.18 0.36 0.26 0.16</td>
</tr>
</tbody>
</table>

Notes: Results based on 200,000 draws of the Metropolis Algorithm. The posterior estimated median is reported.
## Table 3. Variance Decompositions

<table>
<thead>
<tr>
<th>Variable</th>
<th>$a$</th>
<th>$b$</th>
<th>$l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>0.73</td>
<td>0</td>
<td>0.27</td>
</tr>
<tr>
<td>$u$</td>
<td>0.04</td>
<td>0</td>
<td>0.96</td>
</tr>
<tr>
<td>$C$</td>
<td>0.75</td>
<td>0</td>
<td>0.25</td>
</tr>
<tr>
<td>$x$</td>
<td>0.03</td>
<td>0</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Notes: Results based on 200,000 draws of the Metropolis Algorithm. Asymptotic variance decompositions decompose the forecast error variance into percentages due to each of the model’s shocks. The posterior estimated median is reported.
Table 4. Posterior parameter distribution of the constrained specifications

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior</th>
<th>No Hiring Costs ((B = 0))</th>
<th>No reaction to technology ((\gamma = 0))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Posterior 2.5% 97.5%</td>
<td>Posterior 2.5% 97.5%</td>
</tr>
<tr>
<td>(\phi)</td>
<td>0.4</td>
<td>0.3981 0.1644 0.6233</td>
<td>0.3415 0.1308 0.5402</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>0</td>
<td>--- --- ---</td>
<td>--- --- ---</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>1</td>
<td>--- --- ---</td>
<td>1.0132 0.929 1.0966</td>
</tr>
<tr>
<td>(\eta^b)</td>
<td>0.5</td>
<td>0.5022 0.3362 0.6678</td>
<td>0.5011 0.3337 0.6648</td>
</tr>
<tr>
<td>(\eta^l)</td>
<td>0.5</td>
<td>0.8621 0.7996 0.9255</td>
<td>0.8771 0.8192 0.9323</td>
</tr>
<tr>
<td>(\sigma_b)</td>
<td>0.002</td>
<td>0.0018 0.0004 0.0034</td>
<td>0.0019 0.0004 0.0035</td>
</tr>
<tr>
<td>(\sigma_l)</td>
<td>0.010</td>
<td>0.006 0.0049 0.0071</td>
<td>0.0088 0.0077 0.01</td>
</tr>
<tr>
<td>(\log(\hat{L}))</td>
<td>24.53</td>
<td>1.11</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Results based on 200,000 draws of the Metropolis Algorithm. \(\log(\hat{L})\) represents the log marginal likelihood difference between the unconstrained specification and the model under consideration.
Table 5. Moments Comparison

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>Unconstrained</th>
<th>No Hiring Costs</th>
<th>No reaction to technology model ($\gamma = 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_u/\sigma_y$</td>
<td>0.97</td>
<td>0.52</td>
<td>0.50</td>
<td>0.48</td>
</tr>
<tr>
<td>$\sigma_c/\sigma_y$</td>
<td>0.80</td>
<td>0.96</td>
<td>1.00</td>
<td>0.99</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>1.58</td>
<td>1.00</td>
<td>1.08</td>
<td>1.10</td>
</tr>
</tbody>
</table>

Notes: The data are logged, and then HP-filtered, as in the model. Data is treated in the same way as in the estimation exercise, for consistency simulated series are also logged and HP-filtered.
Figure 1. Prior (grey line) and posterior (black line) distribution, and posterior mode (red dashed line) of the estimated parameters of the unconstrained model.
Figure 2. Impulse responses to a one-standard-deviation technology shock (at the estimated median with 95% confidence intervals) of the unconstrained model. Impulse responses are depicted at the estimated median.
Figure 3. Comparison between the unconstrained model (red solid line) and the model with no labor frictions (blue dashed line, $B = 0$). Impulse responses are depicted at the estimated median.
Figure 4. Comparison between the unconstrained model (blue solid line) and model with hiring costs not reacting to technology shocks (red-dashed line, $\gamma = 0$). Impulse responses are depicted at the estimated median.