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Great Moderations and U.S. Interest Rates: Unconditional Evidence

James M. Nason and Gregor W. Smith

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Abstract: The Great Moderation refers to the fall in U.S. output growth volatility in the mid-1980s. At the same time, the United States experienced a moderation in inflation and lower average inflation. Using annual data since 1890, we find that an earlier 1946 moderation in output and consumption growth was comparable to that of 1984. To assess the impact of these moderations, we also isolate the 1969–83 Great Inflation using quarterly data since 1947. We then examine the quantitative predictions of a consumption-based asset pricing model for shifts in the unconditional average of U.S. interest rates across these time periods. A central finding is that such shifts probably were related to changes in average inflation rather than to moderations in inflation and consumption growth.

JEL classification: E32, E43, N12

Key words: Great Moderation, asset pricing, interest rate
1. Introduction

The great moderation (GM) generally is defined as a drop in the variance of output growth in the US during the 1980s. This drop was large; by some measures the variance fell by 50 percent. It seems to have been sudden. And it can be dated to early 1984 according to studies by Kim and Nelson (1999) and McConnell and Pérez-Quirós (2000). Another notable fact about the GM is that it coincided with decreases in the volatility of inflation and the average level of inflation. Cecchetti et al (2007) describe how the average US inflation rate rose during the late 1960s then fell during the GM.

We put the 1984 GM in historical perspective by comparing it to an earlier one, the drop in business-cycle volatility after 1945, and to an even earlier immoderation, the increase in volatility in the interwar period. These shifts affected the unconditional means and variances of inflation and real consumption growth, moments which are related to the general level of interest rates, according to asset pricing theory. We use the unconditional consumption-capital-asset pricing model (CCAPM) to predict the effects of these changes on the average interest rate within each period. Studying these unconditional moments has the advantage that we do not need to model the time-series properties (e.g. persistence) or predictability of these growth rates. Thus the moderations provide a new form of evidence on our understanding of interest rates.

Consumption-based asset-pricing theory links the average interest rate to the first and second moments of consumption growth and inflation. High mean inflation is associated with a high average interest rate from the usual Fisher effect. High mean consumption growth also is associated with a high average interest rate, because of risk-adjusting payoffs. Rapid expected consumption growth makes future payoffs less valuable and so lowers average bond prices. The same, standard Euler equation implies that a moderation in consumption growth (a decline in variance) reduces precautionary saving and so leads to a lower average bond price or higher average return. The convexity of bond prices in expected future inflation means that there is an inflation risk premium. A moderation in inflation thus also leads to a higher average interest rate, according to the theory.

Once parameters are estimated, the theory automatically weights the effects of changes
in these first and second moments on the average interest rate. When we input historical changes in moments into the theory, we find that the moments of consumption do not play a significant role. As for inflation, our main finding is that shifts in the average US interest rate in the 20th century probably were due to shifts in average inflation, rather than to those in volatility.

Section 2 provides some research background by reviewing work that identifies the GM, that seeks to explain it, and that measures its economic effects. Section 3 documents the moderations and other changes in moments. Section 4 outlines a standard, asset pricing model and derives the predicted links between average interest rates and the unconditional moments of consumption growth and inflation. Focusing on unconditional moments means that our findings apply whether a moderation is due to a fall in conditional variance or to a fall in persistence. We exploit the breaks in these unconditional moments across time periods to identify preference parameters. Section 5 uses annual data from 1889 to 2006 to estimate parameters, test the asset pricing model, and decompose changes in interest rates into components due to moderations and those due to changes in mean inflation. Section 6 does the same with postwar quarterly data. Section 7 argues that extending the asset pricing model by using alternative pricing kernels based on habit persistence or recursive utility does not alter the conclusions of our study. Section 8 summarizes our findings and offers suggestions for further research.

2. Background: Timing, Explanations, and Effects

The fall in the volatility of US GDP growth during the 1980s has been documented by Kim and Nelson (1999), McConnell and Pérez-Quirós (2000), Blanchard and Simon (2001), and Stock and Watson (2003). Blanchard and Simon (2001) argue that the early postwar period – say from 1947 to 1984 – should be split into two parts, with output volatility falling in the first part of this period then rising in the 1970s. Nevertheless, their measure of volatility gives values throughout this period that are all greater than any measure after 1985. Notably, the 1980s shift coincided with a drop in the mean and variance of the inflation rate. Stock and Watson (2003, 2007), Nason (2006), and Cecchetti et al (2007) examine the moderation in US inflation.
There is also evidence of moderations in other industrialized countries. Stock and Watson (2003) show volatility results for the past 50 years for G7 countries. Cecchetti, Flores-Launes, and Krause (2006) also provide evidence of the international nature of the GM. Evidence is summarized by Armesto and Piger (2005) and Summers (2005). Unlike the US case though, not all of these moderations were sudden. And they varied in timing, and did not generally coincide with changes in the inflation process. Thus it is more difficult to assess their likely impacts.

The volatility of US output growth also fell during the 1940s. As is well-known, some of this may be due to measurement error, as suggested by Romer (1986). (Also see Weir (1986) and Balke and Fomby (1989) on this debate.) Romer (1999) looks at data back to 1886 on unemployment rates, industrial production, and GNP. She finds that pre-World War I volatility is comparable to post-World War II volatility; it is the interwar period that stands out as a period of immoderation. Balke and Gordon’s (1989) method does not yield this result though. We compare the 1980s moderation with the 1940s moderation in the next section, for several different series.

Leading explanations for the 1980s decline in output growth volatility include: (a) changed structure of the economy that adapts better to shocks (e.g. improved inventory management, more efficient energy use, and greater diversification of value-added); (b) monetary policy that stabilized output fluctuations; and (c) good luck (milder shocks).

Cecchetti, Flores-Launes, and Krause (2006) conclude from the international evidence that explanation (a) is the most important, though changes in monetary policy also played a significant role. Kahn, McConnell, and Pérez-Quirós (2002) argue that explanation (a) accounts for the GM in output and (b) for that in inflation. Blanchard and Galí (2007) examine the milder response to oil shocks in the 2000s than in the 1970s in industrialized countries. They contend that a mix of offsetting shocks, labour market flexibility, and monetary policy accounts for the decline. Justiniano and Primiceri (2006) claim that a one-time drop in the volatility of investment-specific TFP was the source of the GM.

Bernanke (2004) supports (b) and argues that some methods attribute effects to milder shocks that reflect the policy environment. Romer (1999) gives an informal assessment of
these long-term changes. Her explanation (p. 43) is that since 1985, “policy has not generated bouts of severe inflation and so has not had to generate bouts of recession to control it.” Other studies explain the moderation in inflation volatility by placing more emphasis on the evolution of policymakers’ beliefs, either tied to continuous testing of the long-run implications of the natural rate hypothesis – as by Cogley and Sargent (2001) – to learning about the validity of the Phillips curve trade-off – as do Sargent, Williams, and Zha (2006) – or a once and for all shift in policy-rule parameters – as by Benati and Surico (2006). Fernández-Villaverde and Rubio-Ramírez (2007) find there was drift in Taylor-rule parameters but that a short sample leads to imprecise parameter estimates.

Other investigators favor explanation (c). Sims and Zha (2006) study the US data from 1959 to 2003. They argue that the most likely model is one with no change in policy parameters but rather changes in shock variances. Ahmed, Levin, and Wilson (2002) argue that good luck played the largest role in the output moderation, while good monetary policy was more important for the change in the inflation process. Benati (2007) studies moderations for the UK and finds that milder shocks are the main explanation. But Benati and Surico (2007) show that it can be difficult to distinguish good luck from good policy.

Taylor (1999) also considers changes in monetary policy as a source of the GM. He estimates the coefficients of a policy rule for the federal funds rate that depends on inflation and output. Like us, he studies data for a long span beginning in the late nineteenth century. He divides this period into policy regimes, such as the classical gold standard period from 1879 to 1914, or the post-1985 period of stronger interest rate response to inflation. Taylor holds the functional form of the policy rule constant across regimes, but allows the parameters to change.

Several studies have looked at the effects of the GM. For example, Fogli and Perri (2006) measure its impact on the US current account with a two-country real business cycle (RBC) model. The RBC model predicts that if a country experiences a greater reduction in income risk than its trading partners (as the US did in the 1980s), it will engage in less precautionary saving. In Fogli and Perri’s study the GM explains about 20
percent of the increase in the US current account deficit since the mid-1980s.

Rudebusch and Wu (2007) look at the impact of the GM on the term structure of interest rates. They estimate a two-factor, no-arbitrage model and ask what varies in the term structure across the samples before and after the moderation. Their answer is that the change is detectable not in the volatility or persistence of their asset pricing factors but rather to the factor related to the pricing of risk. They argue that this change is linked to a break in monetary policy.

Campbell (2005) studies the effect of the GM on profits forecasts from the Survey of Professional Forecasters (SPF). He employs an asset pricing model based on a utility function with habit formation, to help interpret the effects of the moderation in consumption. He also shows that this model is consistent with the observation that there was little decline in stock-market volatility. Campbell (2007) uses SPF surveys to assess whether there was a decline in volatility of unpredictable shocks or of predictable changes (from the SPF) at the time of the GM. He then uses the CCAPM to predict the effects on forecasts of the equity premium.

Lettau, Ludvigson, and Wachter (2006) and Sill (2006) examine whether a drop in macroeconomic risk might explain the 1990s stock-market boom. The idea is that the equity premium may have fallen because of this decline in risk. Lettau, Ludvigson, and Wachter calibrate a model with regime changes in the mean and variance of consumption growth, and learning about those changes. They combine this model of consumption growth with recursive preferences. Sill uses a production-based model and so traces the effect of a decline in TFP shock-volatility on the equity premium. Both papers conclude that these moderations account for a significant part of the boom in equity prices. Nonetheless, these results are rely on auxiliary assumptions about underlying persistence and conditional volatility.

Like these authors we employ asset pricing theory to study moderations, but we focus on the impact on the average interest rate. We study long time spans that may include multiple moderations. However, we do not seek to identify the underlying, exogenous shocks or structure of the economy that led to moderations. Instead, we study an optimality con-
dition that links interest rates, inflation, and consumption growth that is independent of the openness of the US economy and of auxiliary assumptions about the underlying source of moderations. Our approach does share an important idea with Lettau, Ludvigson, and Wachter. They focus on the 1952-2002 period, but also observe a negative correlation between stock prices and GDP growth volatility for the US since 1880. In essence, we study this low-frequency correlation but for the risk-free interest rate. Further, our identification treats bond rates across moderations as returns on different assets.

This paper also is complementary to Taylor’s (1999) evaluation of monetary policy regimes. He looks for changes in the coefficients of a Taylor rule for the federal funds rate and then assesses their likely effects on business cycles. In contrast, we study the impact of moderations on market interest rates. Although the moderations may have been caused by changes in policy rules, we treat the parameters of the Euler equation describing savings as constant over time. Thus, we use the fact that moments changed first to identify the Euler equation parameters and second to measure the likely impact of moderations per se on market interest rates.

We study unconditional moments for a simple reason: moderations are defined in terms of these moments, as documented in the next section. For the 1980s moderation there is an ongoing debate about whether the decline in the unconditional variance of inflation, for example, was due to a decline in the conditional variance or in persistence. Our approach applies either way.

3. Moderations in GDP Growth, Consumption Growth, and Inflation

We begin by briefly documenting moderations in US real GDP growth. Real GDP per capita, denoted $y_t$, at an annual frequency is from Johnston and Williamson (2007). The corresponding growth rate is defined as $g_y = 100(y_t/y_{t-1} - 1)$. We focus on the period since 1889, because we also have consumption data since then. We divide the period into four, non-overlapping sub-periods: 1889-1914, 1915-1945, 1946-1983, and 1984-2006, and index these by $i$, which in this exercise runs from 1 to 4. The number of years in period $i$ is $T_i$. There are $T_1$ observations for 1889-1914, $T_2$ for 1915-1945, and so on. The first two break
dates are chosen arbitrarily to coincide with the beginning of the 1914-1918 War and the end of the 1939-1945 War. The first break date also coincides with the end of the classical gold standard and the founding of the Federal Reserve. The second one of course has been used as a dividing point for a number of assessments of whether business cycles became more moderate in the postwar period. The third break date is set at 1984, because of the statistical evidence of Kim and Nelson (1999) and McConnell and Pérez-Quirós (2000) who identify a drop in volatility then. We do not test for break dates, but just take these dates to be of interest given existing statistical work.

We measure volatility in period $i$ with the standard deviation of the real output growth rate: $\text{sd}_i(g_y)$. This choice has the advantage that it is in the same units as the growth rate itself. Thus comparing it to the mean growth rate, for example, is unaffected by changes in scale, such as quoting the rates in percentages. We measure the sampling variability of $\text{sd}_i(g_y)$ with its standard error, denoted $\text{se}[\text{sd}_i(g_y)]$ and found by dividing the standard deviation by $\sqrt{2(T_i - 1)}$. And finally, we compare volatility from period $i - 1$ to period $i$ with the statistic $\text{sd}^2_{i-1}/\text{sd}^2_i$. We label this statistic $F_y$, because under normality this statistic is distributed $F(T_{i-1}, T_i)$. So we also report the $p$-value from locating the statistic in this density function. Small $p$-values thus provide evidence of GMs.

We also aim to see whether moderations occurred in the growth rates of real consumption expenditures and prices. We specifically focus on real, per capita consumption of nondurables and services and the associated price deflator. The data source is www.econ.yale.edu/~shiller/dat/chapt26.xls from Shiller (1989), who also describes the underlying series that we revise and update. Our data appendix provides details. For consumption growth, $g_c$, and inflation, $\pi$, we study the same time periods and statistics as for $g_y$.

Figure 1 plots the annual data since 1890. The upper panel depicts the growth rates of real output per capita (solid black line) and real consumption per capita (dashed red line). The lower panel includes the inflation rate, measured with the consumption deflator, (solid blue line) and the interest rate (dashed green line).

Table 1 presents the measures of volatility in each time period and the tests for
moderations. Time marches forward from left to right so that columns refer to time periods. The top panel reports the standard deviation of output growth. Taking the potential break dates as given, the first finding is that one cannot reject the hypothesis that the standard deviation of output growth is the same in the first two time periods, 1889-1914 and 1915-1945. The point estimates suggest an immoderation in 1915-1945, but the change is not statistically significant at any conventional level.

The second finding, though, is that one can reject the hypothesis of equal standard deviations across both later break dates, as all p-values are 0.00. Even with these relatively small annual samples, the moderations are easy to detect. At the 1946 break the standard deviation of output growth falls by 53%, while at the 1984 break it falls by 58%. Thus the two moderations also are comparable in scale. We conclude that these two changes are equally worthy of study and that it may be informative to study them jointly.

The second panel of table 1 concerns consumption growth. Changes in the volatility are tested with the statistic $F_c$. Here the point estimates do not indicate an interwar immoderation, but the conclusion is the same: there is no evidence of a break in the volatility in 1915. For the 1946 and 1984 break dates the findings are the same as for real GDP growth. Both dates mark moderations, and the two changes are of comparable scale. The standard deviation of consumption fell by 57% between the second and third periods and by 56% between the third and fourth periods.

The third panel provides information on moderations in inflation. The price level is measured as the consumption deflator, from the extended Shiller data set. If we instead use the CPI from Officer (2007) the numbers are very similar and so are not shown. From the 1889-1914 period to the 1915-1945 period the standard deviation of inflation rises sharply, but the sampling variability is so large that one cannot conclude there was an immoderation, using the test statistic $F_\pi$. But for the next two break dates the story is the same as for income growth and consumption growth. Both dates mark moderations at any level of statistical significance. From the second to the third period the standard deviation fell by 57% while from the third period to the fourth period it fell by 72%. Thus, in this respect too the 1984 GM had a precedent in 1946.
The reader might wonder whether the relatively high volatility in the 1915-1945 period stems from the war years. It does not. When we isolate the inter-war years 1920 to 1940 and reconstruct table 1 the conclusions are unchanged. The standard deviations of output growth and inflation are somewhat smaller for 1920-1940 than for 1915-1945. But the drops in standard deviations from 1920-1940 to 1946-1983 remain similar in scale to those in table 1 and all three tests again yield \(p\)-values of 0.00.

When we examine the possible impact of these moderations on interest rates in the next section, we find that the means also matter. So, for the record, we next tabulate the means of the three growth rates, where for example \(m_i(g_y)\) denotes the sample mean of the growth rate of GDP in period \(i\). Again we also find the associated standard errors. We use them to construct \(t\)-tests of the hypothesis that the mean is the same in two time periods. In calculating the test statistic, we naturally do not assume that the variances are the same in the two periods, given the findings in table 1.

Table 2 contains the evidence on changes in mean growth rates of GDP, consumption, and prices. It can be summarized very simply. Although the average growth rates do vary across time periods, only one change can easily be detected in these annual data and so has a low \(p\)-value: the drop in average inflation between 1946-1983 and 1984-2006. Between these two time periods, the average inflation rate fell by 30\%, with a \(p\)-value of 0.02.

It is possible that there are breaks in either standard deviations or means that we cannot detect because of the limited number of annual observations. To avoid this risk, we next report the same set of statistics, but for quarterly data since 1947. Growth rates are measured as annualized, quarter-on-quarter rates of change, so they are on the same scale as the annual data. (Later we also study the quarterly, year-on-year growth rates.) Output again is real per capita GDP, consumption is real, per capita personal expenditures on nondurables and services, and the price level is the associated deflator. The appendix gives the sources for these data.

Figure 2 shows quarterly US output and consumption growth, inflation, and the interest rate since 1947. The upper panel is devoted to the growth rates of real output per capita (solid black line) and real consumption per capita (dashed red line). The lower
panel offers the inflation rate, measured with the consumption deflator, (solid blue line) and the interest rate (dashed green line).


Table 3 reports that all three standard deviations dropped from 1969:1-1983:4 to 1984:1-2006:3. The standard deviations of the growth rates of real GDP, real consumption, and the price level fell by 55%, 38%, and 52% respectively. There is also evidence of a smaller drop of 22% in the standard deviation of consumption growth at the earlier, 1969 break date.

Table 4 presents the means of these postwar, quarterly growth rates, along with their standard errors. Here the only shifts are in $m_1(\pi)$, the unconditional mean inflation rate. It jumps up by 4.4 percentage points or 189% at the first break, then down by 3.61 percentage points or 54% at the second break. At each break, the $t$-statistic is large and the $p$-value is 0.00.

Our statistical tests for breaks in standard deviations and means reported so far have been parametric. Their use involves the assumption that the underlying samples are not just independent but that each follows a normal distribution. While we do not find evidence of non-normality, the tests for this have low power in small samples. We thus also undertook non-parametric, distribution-free tests based on ranks. To test for breaks in the variance – as in tables 1 and 3 – we used the squared rank test recommended by Conover (1999) and Sprent and Smeeton (2001). To test for breaks in the mean – as in tables 2 and 4 – we used the Wilcoxon-Mann-Whitney $U$-test. Although some $p$-values changed, the conclusions about moderations and changes in means both were unaffected by adopting these methods.

Here then is a brief summary of the findings of this section. First, the standard
deviations of output growth, consumption growth, and inflation fell in 1946 and in 1984. Thus there were two moderations. Second, the mean of inflation in the postwar period first rose, then fell.

According to standard macroeconomic models of asset prices, these breaks in the volatility of consumption growth and inflation have implications for interest rates. We next use these striking facts to look at the effects of moderations on interest rates. By studying the 1946 moderation, and not just the 1984 one, we acquire more evidence. And we try to isolate the effect of changes in the volatility by controlling for postwar shifts in the average inflation rate,

Before turning to the theory we need to recap and extend the notation. Growth rates of real GDP, real consumption, and prices are denoted $g_y$, $g_c$, and $\pi$ respectively. A $t$ subscript, denoting a specific time period, is used only when needed. The historical (sample) unconditional mean and standard deviation of $g_c$, say, in period $i$ are denoted $m_i(g_c)$ and $sd_i(g_c)$ respectively. The corresponding theoretical (population) mean and standard deviation are denoted $\mu_c$ and $\sigma_c$ respectively, while $\sigma_{\pi_c}$ is a population covariance.

4. Asset Pricing Model

Asset prices provide a perspective on the moderations. According to economic theory, the average interest rate is linked to the means and variances of consumption growth and inflation. Shifts in these moments thus predict shifts in average returns and so provide episodes with which to test asset pricing models. Some practitioners such as JPMorgan Chase (2007) have suggested that the moderation in inflation played a role in the decline in long-term interest rates, but to our knowledge this possibility has not been studied formally. Traditional, consumption-based asset pricing theory predicts the opposite effect; an inflation moderation leads to a rise in the general level of interest rates. Thus, our research questions include: Are the historical shifts in the general level of interest rates consistent with the great moderations? Can we use the theory to understand which changes in moments explain changes in interest rates?

We focus on one-year bond returns because they depend on growth rates of consumpt-
tion and prices over a one-year horizon. The long spans of macroeconomic data necessary to study moderations and unconditional moments of interest rates prior to 1950 are available only at annual frequency. Thus the horizon for these bond investments corresponds with the frequency over which we can calculate growth rates. However, we also study the returns on 90-day bills aligned with postwar, quarterly data on consumption and prices.

Asset-pricing theory predicts an effect of moderations on the equity premium. But equity is long-lived, and so modeling this effect requires auxiliary assumptions about whether moderations were anticipated or not, or a model of learning that explains shifts in expectations. So we omit long-term debt and equity because their predicted returns are sensitive to these assumptions. But this is a natural subject for further research.

Our analysis engages the CCAPM because it is written in terms of consumption growth and inflation. One asset for which this model makes price predictions is a one-period, nominal, discount bond. Denote the price of such a bond at time \( t \), maturing at time \( t + 1 \), by \( Q_t \). The net return on this bond is denoted \( r_t \). The gross return (1 plus the net return) is denoted \( R_t \). Suppose there is a CRRA utility function \( u \) with coefficient \( \alpha \) and discount factor \( \beta \). \( E_t \) denotes a conditional expectation at time \( t \), while \( E \) denotes an unconditional expectation. Consumption is denoted \( c_t \) and the price level \( p_t \).

The Euler equation linking this year and next is:

\[
\frac{Q_t}{p_t} u'(c_t) = E_t \beta \frac{1}{p_{t+1}} u'(c_{t+1}).
\]  

Denote gross consumption growth by \( 1 + g_c \) and gross inflation by \( 1 + \pi \). Because \( u'(c_t) = c_t^{-\alpha} \), the one-period bond price, or inverse, gross interest rate, then can be rewritten as:

\[
Q_t = \frac{1}{R_t} = E_t \beta \frac{(1 + g_{ct+1})^{-\alpha}}{(1 + \pi_{t+1})},
\]  

so that, by the law of iterated expectations, its unconditional expectation is:

\[
E \frac{1}{R_t} = \beta E \frac{(1 + g_{ct+1})^{-\alpha}}{(1 + \pi_{t+1})}.
\]  

The function that is being forecasted on right-hand side of this expression opens up in \( g_{ct} \) and \( \pi_t \). From Jensen’s inequality, then, a fall in the variance of either \( g_c \) or \( \pi \) generates a
lower average bond price or a higher average interest rate. That is the qualitative, predicted link between moderations and interest rates. We next try to make it quantitative.

Suppose that \( g_c \) and \( \pi \) in a given time period have population, unconditional means \( \mu_c \) and \( \mu_\pi \), variances \( \sigma^2_c \) and \( \sigma^2_\pi \), and covariance \( \sigma_{c\pi} \). To focus on the effects of changes in the first and second unconditional moments of consumption growth and inflation, we take a second-order, Taylor series expansion of the expectation term on the right-hand side of the unconditional Euler equation (3) around \( (\mu_c, \mu_\pi) \):

\[
E\left(\frac{1 + g_{ct+1}}{1 + \pi_{t+1}}\right)^{-\alpha} \approx \frac{(1 + \mu_c)^{-\alpha}}{(1 + \mu_\pi)} + \frac{(1 + \mu_c)^{-\alpha}}{(1 + \mu_\pi)^3} \sigma^2_\pi + 0.5\alpha(1 + \alpha)\frac{(1 + \mu_c)^{-\alpha-2}}{(1 + \mu_\pi)} \sigma^2_c + \alpha \frac{(1 + \mu_c)^{-\alpha-1}}{(1 + \mu_\pi)^2} \sigma_{c\pi}.
\]

Notice that this approximate decomposition does not rely on any assumptions about the persistence or conditional moments.

The economic interpretation of this key equation is standard. High mean inflation, \( \mu_\pi \) is associated with high average interest rates from the usual Fisher effect. Consumption growth affects interest rates because of risk-adjusting bond prices and payoffs. Thus the more rapid is consumption growth on average the less valuable is a future payoff from holding the bond, so the lower is the bond price and the higher the return. And greater volatility encourages precautionary saving, which leads to a higher bond price or a lower return. The convexity of the bond price in inflation also induces an inflation risk premium; an increase in the volatility of inflation raises the average bond price and so lowers the average interest rate. Thus asset pricing theory links the mean bond price (or inverse of the gross return) to the first and second moments of consumption growth and inflation. The theory also tells us how to weight the means and variances of the two growth rates. And it tells us that shifts in their covariance may be worth investigating as a source of shifts in average interest rates.

We need values for the parameters \( \alpha \) and \( \beta \) in order to predict the effect of moderations. To estimate those values, we exploit the property that the unconditional moment condition
(3) holds within each time period $i$. Thus we estimate and test using sample versions of:

$$\sum_{t=1}^{T_i} \left[ \frac{1}{R_t} - \beta \frac{(1 + g_{ct+1})^{-\alpha}}{1 + \pi_{t+1}} \right] = 0. \quad (5)$$

Estimation by GMM chooses parameter values to minimize the weighted sum of squared deviations from this equation, which involves only unconditional moments.

To see that splitting the sample across the $i$ periods identifies the parameters even though we have only one moment condition (5), imagine substituting the sample version of the approximation (4) into the estimating equations (5) (although we estimate without using the approximation). Observe that the equations in different time periods are not identical as long as at least one moment of consumption growth or inflation changes between periods. Thus we can identify and estimate both parameters as long as there is at least one moderation and so two distinct periods of time. If there is more than one moderation, then the parameters are overidentified, because there are more time periods than parameters, which provides the usual $J$-test.

5. Annual Evidence 1889-2006

The annual interest rate series again comes from the Shiller data set, updated by the authors. Details are given in the appendix. Section 3 showed that there were two breaks in the variances of $g_c$ and $\pi$, in 1946 and 1984. There is a break in the mean of inflation in 1984. Using the sample version of the unconditional asset pricing model (3) in each of the periods 1889-1945, 1946-1983, and 1984-2006 gives three moment conditions. With two parameters to estimate, we then can use GMM and also test the model’s fit based on its overidentification.

Estimation by iterated GMM in annual data gives the following coefficients, with standard errors in parentheses: $\hat{\alpha} = -0.943(3.19)$ and $\hat{\beta} = 0.963(0.057)$. The $J$-test statistic, with 1 degree of freedom is 3.52, with a $p$-value of 0.06. Thus the restrictions across the three time periods would narrowly be accepted at the traditional 5% significance level. But the value of $\hat{\alpha}$ is insignificantly different from zero. Because $\alpha$ cannot be identified with any precision, we next set it equal to zero and re-estimate, finding that $\hat{\beta} = 0.9807(0.0034)$ and $J(2) = 4.40$ with a $p$-value of 0.11.
These findings mean that the two moderations in consumption-growth volatility probably did not contribute to changes in average interest rates. With this part of our research question answered, we set \( \alpha = 0 \) in the approximation (4) to give:

\[
E \frac{1}{(1 + \pi_{t+1})} \approx \frac{1}{(1 + \mu_\pi)} + \frac{\sigma_\pi^2}{(1 + \mu_\pi)^3}.
\]

(6)

so that the approximate asset pricing model is:

\[
E \frac{1}{R_t} \approx \beta \left[ \frac{1}{(1 + \mu_\pi)} + \frac{\sigma_\pi^2}{(1 + \mu_\pi)^3} \right].
\]

(7)

This result gives us a framework in which to investigate the likely impact on interest rates (or, strictly speaking, average bond prices) of (a) the 1946 and 1984 moderations in inflation and, later, (b) shifts in the mean of inflation associated with the 1969-1983 period.

This specialization of the asset pricing model implicitly uses risk-neutrality, because \( \alpha \) is set at zero. But it still involves an ‘inflation risk premium’, in that a higher inflation variance leads to a higher bond price and lower interest rate.

Our evidence on \( \hat{\alpha} \) leads to the conclusion that changes in the moments of consumption growth did not cause changes in average interest rates. The reader might wonder whether using traditional instruments (like lagged variables) in GMM estimation would lead to a precisely estimated, positive \( \hat{\alpha} \). We argue that it would not, for there is a wealth of relatively negative evidence on the CCAPM using this approach, as surveyed by Campbell, Lo, and MacKinlay (1997) and Cochrane (2001). By using a new set of moments, we provide new evidence on the CCAPM that reinforces the finding that it is difficult to find an association between consumption growth and bond returns.

The consumption-volatility term in the Taylor-series expansion of the bond-pricing equation, shown here:

\[
+ 0.5\alpha(1 + \alpha)(1 + \mu_c)^{-\alpha - 2} \frac{\sigma_\pi^2}{(1 + \mu_\pi)^2} \sigma_c^2,
\]

involves \( \alpha \). If changes in \( \sigma_c^2 \) were correlated with changes in the average bond price then the estimator would yield a significant value \( \hat{\alpha} \). This term also shows that small moderations aligned with large changes in average bond price would yield a large value \( \hat{\alpha} \) for the moment condition to fit. Our specific GMM estimator is set up to identify \( \hat{\alpha} \) solely from shifts in
unconditional moments between time periods. So finding that $\alpha$ is insignificantly different from zero is indeed evidence that shifts in the moments of consumption were not related to shifts in average bond price or interest rate.

This finding does not mean that we require the real interest rate to be constant. It could vary within each time period. Or, it could vary in moments across time periods but in a way unrelated to the changes in the moments of consumption growth. The $p$-value for the $J$-test is relatively low (and even lower below in quarterly data), which means that there is variation in average bond prices or interest rates across time periods that is unrelated to the variation in the moments of inflation. Hence our conclusions about the role of inflation moments must be provisional, pending the identification of a better-fitting asset pricing model. However, we do know that such a model must contain a factor other than consumption growth.

We construct and report predicted means – denoted $\hat{m}$ – using the sample version of the inflation-based model (7):

$$\hat{m}_i(R^{-1}) = \hat{\beta} \left[ \frac{1}{1 + m_i(\pi)} + \frac{sd_i^2(\pi)}{(1 + m_i(\pi))^3} \right].$$

(8)

If we (a) ignored Jensen’s inequality and (b) approximated $\ln(1+r_t)$ by $r_t$ (which roughly holds at low interest rates), then we could write:

$$-\ln m_i(R^{-1}) \approx -\ln \frac{1}{1 + m_i(r)} \approx m_i(r),$$

(9)

which shows that the logarithm of the left-hand side of the prediction formula (8) is approximately the mean interest rate. We also report the sample statistic $-\ln m_i(R^{-1})$ to give a variable on the same scale as the average interest rate, while avoiding the two approximation errors. For simplicity we refer to this as the average interest rate. Our key formula (8) continues to be valid during periods of high nominal interest rates.

Table 5 contains the results. The first three rows repeat the sample sizes, standard deviation of inflation, and mean of inflation, from tables 1 and 2. For period $i$, the model’s predicted average bond price is $\hat{m}_i(R^{-1})$ while the corresponding historical mean bond price is $m_i(R^{-1})$. The latter is reported with its standard error. We measure the
sampling variability directly from the data rather than using the estimated model to attach a standard error to $\hat{m}_i(R^{-1})$. The last two rows of table 5 contain the same comparison but for the approximate mean interest rate. The model’s prediction is $-100\ln \hat{m}_i(R^{-1})$ and the historical counterpart is $-100\ln m_i(R^{-1})$. The standard error for the latter is found using the $\delta$-method formula: $\text{se}[-100\ln m_i(R^{-1})] = 100\text{se}[m_i(R^{-1})]/m_i(R^{-1})$.

For comparison with tables 1 and 2, we also tested for breaks in the mean bond price, using the $t$ and $U$ tests. The conclusions were the same with both tests, so table 5 also shows the $t$-statistics and their $p$-values. These show breaks in 1915 and 1945 but not in 1984. These findings do not match up with the timing in tables 1 and 2, where there are moderations in consumption growth and inflation in 1945 and 1984 and a drop in mean inflation in 1984. This discrepancy, like the relatively low $p$-value on the $J$-test statistic, show that there is room for improvement in the model’s fit.

Table 6 translates the numbers from table 5 into predicted and actual changes in the average interest rate at the dates of the two moderations. The first row gives the predicted changes based on the moderations, the changes in $\text{sd}_i(\pi)$ alone. According to the parameterized model, each moderation led to a small increase in the average interest rate. The next row gives the predicted changes based on changes in average inflation, $m_i(\pi)$, alone. Here the theory predicts a much larger effect. The third row gives the combined effect (the effects are not exactly additive but close to that in practice) while the fourth row gives the actual change. For the 1946 moderation, the model’s predicted change is on the same scale as the actual change, and most of the predicted change stems from the change in average inflation.

As for the 1984 moderation, the asset pricing model predicts a fall in the mean interest rate, whereas the actual rate rose. In this case the contribution of the moderation is comparable in scale to the actual interest rate change, but the moderation leads to a small interest rate increase according to the model. Again this predicted effect is much smaller than the predicted effect of the change in mean inflation. We re-examine these findings with quarterly data in the next section.

In the annual-data exercise we did not separate the Great Inflation, due to the limited number of observations. With 15 annual observations, moments from the 1969-1983 period have large standard errors. Turning to quarterly, postwar data allows us to isolate that period yet draw more reliable inferences. And the greater number of observations also allows us to estimate the parameters solely in postwar data.

The three time periods are 1947:1-1968:4, 1969:1-1983:4, and 1984:1-2006:3. The interest rate applies to the 3-month T-bill. The interest rate and the growth rates are annualized, so the results are directly comparable to those of the previous section. The same GMM estimator yields these parameters estimates, with standard errors in parentheses: \( \hat{\alpha} = -1.95(19.7) \) and \( \hat{\beta} = 0.9496(0.357) \). The \( J \)-statistic is 6.61 with a \( p \)-value of 0.01. Hence the stability of the pricing model across the three time periods is rejected at the 1% significance level.

When we estimate using different combinations of the three time periods, we find that \( \hat{\alpha} \) is small compared to its standard error in all cases. Thus the rejection under the \( J \)-test stems from small, but significant changes in \( \hat{\beta} \) across these periods. But since the level of \( \beta \) does not affect the model’s prediction for changes in the mean interest rate at moderations, we proceed to document the predictions. We find no significant \( \hat{\alpha} \). Lettau and Ludvigson (2007) reach a similar, negative conclusion from estimating and testing with unconditional Euler equations and portfolio returns in postwar US data. In fact, they find no role for consumption within various extensions of the CCAPM (including those we discuss in the next section). Therefore we use the same approximation formula (7) based on the mean and variance of the inflation rate. When we re-estimate with \( \alpha \) set at 0 we find \( \hat{\beta} = 0.9889(0.002) \), so we use this value to find the predicted average bond prices and (roughly) interest rates in each time period.

Tables 7 contains the results. Again we report the \( t \)-test statistic and its \( p \)-value for the test of a break in the average bond price between periods. This test reveals significant breaks (with \( p \)-values of 0.00) at 1969 and 1984. In the macroeconomic quantities, tables 3 and 4 show breaks at both dates in two variables: the standard deviation of consumption
growth and the mean inflation rate. In the quarterly data, then, the timing of breaks seems to be aligned between bond prices and fundamentals. However, if the break in $\sigma_c$ were driving the break in the mean bond price, that would lead to a role for $\alpha$ in the pricing equation, since it is differences in unconditional means that the estimator tries to match. Since we do not find a significant positive $\alpha$, it seems more likely that the two breaks in average bond prices were driven by breaks in the moments of inflation.

Table 8 shows predicted and actual changes between periods in the approximate mean interest rate. This time both predicted interest-rate changes are on the same scale as the actual changes, though the theory under-predicts the rise in 1969 and over-predicts the fall in 1984. The predictions for 1984 are improved, by comparison with the exercise with annual data in table 6, because the quarterly data provide enough observations for us to isolate the Great Inflation.

A key virtue of asset pricing theory is that it describes the joint effects of means and variances on the interest rate. Once again we find that the lion’s share of explanation is done by $m_{i}(\pi)$, while the 1984 moderation – the drop in $sd_{i}(\pi)$ – affects only the second decimal place of the interest rate. Thus, the asset pricing model (3) attributes relatively minor effects to the 1984 moderation in inflation. It seems more likely that changes in average inflation instead explain most of the changes in average interest rates, through the traditional Fisher effect.

In postwar quarterly data we also studied interest rates at 1-year maturity. The data appendix provides details of two different interest rate series we adopted. One of these, the quarterly average of the 1-year treasury constant-maturity rate, begins in 1953. Making a virtue of necessity, this sample thus begins after the 1951 Treasury-Federal Reserve Accord that released the Federal Reserve from any obligation to support the price of federal government debt. We aligned these interest rates with the corresponding year-to-year growth rates in quarterly consumption and prices.

The conclusions from these data are the same as those documented in the tables. Year-to-year growth rates of output, consumption, and prices are more moderate after 1983. Average inflation is significantly higher during 1969-1983 than before or after. In the
asset pricing model, we cannot reject the hypothesis that \( \alpha = 0 \) at traditional significance levels. With one interest rate series, \( \hat{\alpha} \) is negative and with the other it is positive. And, changes in mean inflation drive most of the changes in predicted, average interest rates.

Finally, our central result that changes in the mean of the inflation rate mattered more than inflation moderations is in no way rigged into the setup of the approximate asset pricing equation (7) used to obtain the predictions. Inspection of this equation shows that large changes in the variance will lead to large changes in bond prices and interest rates, especially if the mean inflation rate is small. Historically, the inflation moderations simply were not large enough (nor the mean small enough) for this to have happened.

7. Preferences

A variety of contributors to asset pricing theory and macroeconomics have worked with preferences different from the ones we have adopted so far. We next examine two generalizations that have been successfully used in macroeconomics and finance.

7.1 External Habit

One of the most widely-used revisions begins with a utility function in which individual consumption is assessed by comparison to aggregate consumption or to past consumption, a reference level sometimes referred to as ‘habit.’ Abel (1990) called this feature of utility ‘catching up with the Joneses.’ This theory has been promising in helping economists understand a range of features of asset pricing phenomena, such as the equity premium. But we next show that engaging these preferences do not affect our results about moderations and interest rates.

The measure of habit can be current, lagged, or a mixture, and can scale consumption by subtraction or multiplication. We first use Abel’s (1999) version, which measures habit as a mixture of current and lagged consumption and enters it multiplicatively into the utility function. The utility function in a given time period now is:

\[
u = \frac{1}{1 - \alpha} \left( \frac{c_t}{s_t} \right)^{1-\alpha}, \tag{10}\]

in which \( s_t \) is a reference stock of current and past aggregate consumption, given by:

\[
s_t = c_t^{\delta_0} c_{t-1}^{\delta_1},
\]

which consumers take as given or external. Thus marginal utility becomes:

\[
u'(c_t) = s_t^{\alpha - 1} c_t^{-\alpha} = c_t^{\delta_0(\alpha - 1) - \alpha} c_{t-1}^{\delta_1(\alpha - 1)}.
\]

The effect of current consumption on utility depends on the value of past consumption, an indicator of habit persistence. To simplify notation, we label \( a \equiv \delta_0(\alpha - 1) - \alpha \) and \( b = \delta_1(\alpha - 1) \). Then the unconditional Euler equation becomes:

\[
E \frac{1}{R_t} = \beta E \left( \frac{1}{1 + g_{ct+1}} \right)^a \left( \frac{1}{1 + g_{ct}} \right)^b \frac{1}{1 + \pi_{t+1}}.
\]

We label \( \sigma_{c,c-1} \equiv \text{cov}(g_{ct}, g_{ct-1}) \) and \( \sigma_{\pi,c} \equiv \text{cov}(\pi_t, g_{ct-1}) \). Then the second-order Taylor series approximation of this function of \( \{g_{ct+1}, g_{ct}, \pi_{t+1}\} \) contains the same terms as the original version (4) without habit, albeit with a different interpretation of the coefficients, and two new terms in \( \sigma_{c,c-1} \) and \( \sigma_{\pi,c} \). The comparison shows: (a) the same weight on \( \mu_\pi \) and \( \sigma_\pi^2 \); (b) for a given coefficient on \( \mu_c \), a different, predicted effect of \( \sigma_c^2 \); and (c) two new covariances. However the autocovariance of consumption growth is tiny historically, a reflection of the near-random-walk property of consumption. When we investigate all these changes statistically, we find little quantitative change and no change in our main economic conclusion.

Galí (1994) uses a setup in which habit is measured only by current, aggregate consumption raised to the risk aversion coefficient scaled by a habit parameter. For the reference stock process (11), it means \( \delta_1 = 0 \). Looking at the expressions (12) and (13) shows that in this case the predictions for asset prices are identical to those of an economy with standard CRRA preferences, not just very similar as in the general case. Although there is an additional utility function parameter, Galí’s habits model has implications for unconditional mean interest rates that are identical to the standard ones.

Campbell and Cochrane (1999) use a different functional form for utility, but again allow for external habit so that utility depends on aggregate consumption. Their setup
has success in matching asset pricing moments such as the equity premium and a range of cyclical features of stock prices. They observe that models with external habits and random-walk consumption sometimes lead to large swings in the risk-free interest rate. However, their model is parameterized to have a constant, real interest rate (p. 213, equation 12), given by $-\ln \beta + \alpha \mu_c - 0.5\alpha(1 - \phi)$, in which $\phi$ is the coefficient in an AR(1) process for the stock of external habit. This takes the form we have already considered, with the exception of the new, last term. They use a long span of annual data and postwar quarterly data to calibrate their model, and adopt $\phi = 0.87$. It is possible that this varies over different time periods, and this would be worth studying. However, great moderations are not described in terms of changes in the persistence properties of the reference level of consumption, which is a feature of these external-habit preferences.

These examples take the reference level of consumption as given, and so are sometimes referred to as embodying ‘external habit.’ Another possibility is ‘internal habit’ in which the consumer takes into account the impact of the current consumption choice on the stock of habit and hence future utility. In this case asset prices depend on the expected value of future consumption growth $x_{t+1}$, in addition to $x_t$ and $x_{t-1}$. But since the unconditional mean of each of these terms is the same (and consumption growth has little autocorrelation) the predictions for average interest rates do not change significantly. We omit the details, but again this modification would not affect our findings.

### 7.2 Recursive Preferences

Our second example of alternative preferences is again widely used in asset pricing: Epstein-Zin preferences that allow separate measurement of risk aversion and intertemporal substitution. Applications include Epstein and Zin (1991) of course and, more recently, Bansal and Yaron (2004) and Lettau, Ludvigson, and Wachter (2006).

The pricing kernel now depends in part on $R_{wt}$, the gross return on wealth (e.g. on an asset that pays a dividend equal to real consumption). Again $\beta$ is the discount factor and $\alpha$ the coefficient of relative risk aversion. The intertemporal elasticity of substitution is $1/(1 - \rho)$. Define $\gamma \equiv (1 - \alpha)/\rho$. The expression for the unconditional mean, nominal,
The bond price is:

\[
E \frac{1}{R_t} = E \left[ \frac{\beta^\gamma (1 + g_{ct+1})^{\gamma (\rho - 1)} R_{wt+1}^{\gamma - 1}}{1 + \pi_{t+1}} \right]
\]  

(14)

Thus, the canonical unconditional Euler equation (3) is a special case when \( \alpha = 1 - \rho \) so that \( \gamma = 1 \).

This extension is interesting to explore because of its use in recent studies. In addition, it implies that the previous pricing kernel and unconditional GMM estimation were subject to omitted variables bias, with \( R_{wt} \) missing. Lettau, Ludvigson, and Wachter show a correlation between consumption volatility and the stock market, in quarterly data since 1952 and in annual data since the nineteenth century, and for the US and for other countries. This low-frequency correlation then leads them to formally estimate and test a model of the equity premium in the 1990s, where they find a significant role for the consumption moderation. Here we study a low-frequency correlation using long spans of time and average, risk-free interest rates, and again we estimate and test using only unconditional moments.

With three regimes (two moderations) we can just identify the three parameters \( \{\beta, \alpha, \rho\} \) using the unconditional mean. Thus we cannot test over-identifying restrictions, but we can ask whether consumption growth or the market return has a significant impact on the average bond price. That also tells us whether controlling for the market return leads to a finding that consumption growth moderations matter for average interest rates.

We mimic Epstein and Zin and represent \( R_{wt} \) with a real market return \( R_{mt} \). In the long annual sample, the real return is calculated using the S&P composite index. The value-weighted NYSE index serves to measure the real return in quarterly data. Both series include dividends.

Estimation in annual data yields insignificant values for \( \hat{\gamma} \) and \( \hat{\rho} \). The findings in quarterly, postwar data are similarly negative. Thus, we cannot reject the hypothesis that both parameters are zero, at any reasonable level of statistical significance. The information from the moderations does not suggest a low-frequency or cross-regime correlation between the bond return and the stock-market return that would identify \( \hat{\gamma} \). Further work
might examine measures of the return on wealth that include the return to human wealth, but meanwhile this approach does not overturn our main conclusions.

There are many other examples of ‘exotic’ preferences, featuring a discount factor that depends on the level of consumption, quasi-geometric discounting, or disappointment aversion. Backus, Routledge, and Zin (2004) provide a complete survey. For adopting one of these utility models to change our findings, one would need to find (a) changes in the predictions for unconditional mean interest rates on bonds that (b) are driven by a macroeconomic moment that shifts during GMs. We have not come across examples that satisfy these requirements, but studying the observable implications of these utility functions is an active research area.

8. Conclusions

The US Great Moderation of the 1980s is an ideal episode with which to improve our understanding of interest rate history. Its timing has been studied extensively; it seems to have been a sharp break, and it was accompanied by a shift in the inflation process. We combine this perspective with a long span of data that includes the comparable 1946 moderations in real consumption growth and inflation. Consumption-based asset pricing theory, using assumptions about the utility function, links average, nominal interest rates simultaneously to the means and variances of consumption growth and inflation. Because we base predictions on unconditional moments, our findings apply whether these moderations were due to decreases in unconditional variances or to decreases in persistence. A central finding is that shifts in twentieth-century US interest rates probably were due to a traditional source – the average level of inflation through the Fisher effect – rather than to shifts in the volatility of consumption growth or inflation.

The p-values of tests of overidentification (based on the stability of the asset pricing model) are low for the long span of annual data and for the postwar quarterly data. That finding gives scope for further analysis of moderations and U.S. interest rates. But moderations in consumption growth do not seem to be the source of changes in average interest rates. There also is scope for further study using moderations in other countries,
though we have not pursued that because there is so far less consensus about the timing than in the US case. And moderations in output in other countries often fail to coincide with moderations in inflation, which makes it difficult to use long spans of data.

It also would be interesting to study evidence of moderation in household consumption, even though long time spans of disaggregated consumption data may not be available. Our use of unconditional moments is appropriate whether moderations were caused by a change in conditional variance or in persistence. But other properties of asset prices (such as the shape of the term structure of interest rates) may be sensitive to this distinction and so provide evidence on these two separate changes. Finally, we have taken the moderations as given and focused on their effects. Future research could explore whether the factors that explain the 1984 moderation were responsible for the moderation of 1946.
Appendix: Data Sources and Definitions

**Annual Data:** Real output per capita, $y$, is from Johnston and Williamson (2007). Real consumption of nondurables and services per capita, $c$, and the deflator for personal consumption expenditures, $p$, are from www.econ.yale.edu/~shiller/dat/chapt26.xls which is described by Shiller (1989) and revised and updated by us. The interest rate, $r$, is a synthetic series constructed from 6-month rates, denoted $r_6$ using the following formula:

$$r = 100 \left[ \frac{1}{(1 - r_6(\text{January})/200)(1 - r_6(\text{July})/200)} - 1 \right],$$

which is found in Shiller (1989, p. 444). For 1889-2004 the underlying series comes from the Shiller data set. Data prior to 1939 are 4-6-month commercial paper rates from Macaulay (1938). For 1939 to 1997 the interest rate is the 6-month commercial paper rate from the Federal Reserve Bulletin; for 1998-2004, it is the rate on 6-month certificates of deposit on the secondary market, from the Federal Reserve; for 2005 and 2006 we update the series with the 6-month certificate of deposit rate. The real, stock-market return is on the S&P composite index, from the Shiller data set.

**Quarterly Data:** Output, $y$, is real per capita chained GDP; consumption, $c$, is real, per capita consumption of nondurables and services (NDS); the price level is the associated deflator. Chained NDS expenditures are computed with a Fisher ideal chain index. All data are seasonally adjusted at annual rates. The source of the NIPA data is the Bureau of Economic Analysis via the FRED database at the Federal Reserve Bank of St. Louis. The population measure is found at www.bea.gov, NIPA Table 7.1. The interest rate, $r$, is the three-month T-bill rate, from FRED series tb3m, averaged from monthly data. The real, stock-market return is the nominal return on the value-weighted NYSE index, from CRSP, deflated using the consumption deflator.

In the postwar data we also studied quarterly observations on 1-year interest rates. The two alternative ways of measuring this rate were: (a) the synthetic 1-year rate, for 1947:1-1964:2 from the 4 to 6-month commercial paper rate from January Federal Reserve Bulletins 1948-1965; for 1964:2-2006:4 6-month certificate of deposit rate from FRED and the Federal
Reserve; and (b) the 1-year treasury constant-maturity rate, quarterly average 1953:2-2006:4, from the Board of Governors of the Federal Reserve System H.15 Selected Interest Rates.
References


Figure 1: Annual US Growth Rates and Interest Rates

- GDP growth
- Consumption growth
- Inflation rate
- Interest rate

Year: 1900 1920 1940 1960 1980 2000

GDP growth and consumption growth have fluctuated significantly over the years. Inflation rates have also varied, with peaks and troughs, while interest rates reflect similar patterns, although they appear to be more volatile.
### Table 1: Great Moderations
Annual GDP Growth, Consumption Growth, and Inflation

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<tbody>
<tr>
<td>$T_i$</td>
<td>25</td>
<td>31</td>
<td>38</td>
<td>23</td>
</tr>
<tr>
<td>$sd_i(g_y)$</td>
<td>5.78</td>
<td>7.57</td>
<td>3.58</td>
<td>1.50</td>
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<tr>
<td>$se[sd_i(g_y)]$</td>
<td>0.83</td>
<td>0.98</td>
<td>0.42</td>
<td>0.23</td>
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<td>$F_{y/(p)}$</td>
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<td>4.46/(0.00)</td>
<td>5.75/(0.00)</td>
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<td>$sd_i(g_c)$</td>
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<td>$se[sd_i(g_c)]$</td>
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<td>0.59</td>
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<td>0.13</td>
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<td>$F_{c/(p)}$</td>
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<td>5.31/(0.00)</td>
<td>5.16/(0.00)</td>
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<td>$sd_i(\pi)$</td>
<td>2.88</td>
<td>7.78</td>
<td>3.36</td>
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<tr>
<td>$se[sd_i(\pi)]$</td>
<td>0.42</td>
<td>1.00</td>
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<td>0.14</td>
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<td>$F_{\pi/(p)}$</td>
<td>0.14/(0.99)</td>
<td>5.33/(0.00)</td>
<td>13.2/(0.00)</td>
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</tbody>
</table>

Notes: $T_i$ is the number of observations in period $i$; $g_y$ is the percentage growth rate of real GDP per capita, $g_c$ is the percentage growth rate of real consumption of nondurables and services per capita, $\pi$ is the inflation rate in the consumption deflator; $sd$ is a standard deviation and $se$ the associated standard error; $F$ is the test statistic for equality of standard deviations between the current and previous periods and $p$ is the associated $p$-value.
Table 2: Means Across Moderations  
Annual GDP Growth, Consumption Growth, and Inflation

<table>
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<td>$T_i$</td>
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<td>38</td>
<td>23</td>
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<tr>
<td>$m_i(g_y)$</td>
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<td>$se[m_i(g_y)]$</td>
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<td>1.51</td>
<td>0.72</td>
<td>0.30</td>
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<td>$t_y/(p)$</td>
<td>1.4/(0.16)</td>
<td>-1.17/(0.24)</td>
<td>0.79/(0.43)</td>
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</tr>
<tr>
<td>$m_i(g_c)$</td>
<td>2.00</td>
<td>1.74</td>
<td>2.18</td>
<td>2.00</td>
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<td>$se[m_i(g_c)]$</td>
<td>0.98</td>
<td>0.92</td>
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<td>$t_c/(p)$</td>
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<tr>
<td>$m_i(\pi)$</td>
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<td>1.56</td>
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</tr>
<tr>
<td>$t_\pi/(p)$</td>
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<td>1.36/(0.18)</td>
<td>-2.4/(0.02)</td>
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</tr>
</tbody>
</table>

Notes: $T_i$ is the number of observations in period $i$; $g_y$ is the percentage growth rate of real GDP per capita, $g_c$ is the percentage growth rate of real consumption of nondurables and services per capita, $\pi$ is the inflation rate in the consumption deflator; $m$ is a sample mean and $se$ the associated standard error; $t$ is the test statistic for equality of means between the current and previous periods and $p$ is the associated $p$-value.
Figure 2: Quarterly US Growth Rates and Interest Rates

![Graph showing GDP and Consumption Growth (%) from 1950 to 2000 with corresponding inflation and interest rates.](image-url)

- GDP growth
- Consumption growth
- Inflation rate
- Interest rate

Year:
- 1950
- 1960
- 1970
- 1980
- 1990
- 2000

GDP and Consumption Growth (%):
- -10
- -5
- 0
- 5
- 10
- 15

Inflation and Interest Rates (%):
- 0
- 5
- 10
- 15
Table 3: Postwar Great Moderations
Quarterly GDP Growth, Consumption Growth, and Inflation

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_i$</td>
<td>87</td>
<td>60</td>
<td>91</td>
</tr>
<tr>
<td>$sd_i(g_y)$</td>
<td>4.84</td>
<td>4.70</td>
<td>2.08</td>
</tr>
<tr>
<td>$se[sd_i(g_y)]$</td>
<td>0.37</td>
<td>0.43</td>
<td>0.15</td>
</tr>
<tr>
<td>$F_{y/(p)}$</td>
<td>1.06/(0.40)</td>
<td>5.13/(0.00)</td>
<td></td>
</tr>
<tr>
<td>$sd_i(g_c)$</td>
<td>2.73</td>
<td>2.13</td>
<td>1.32</td>
</tr>
<tr>
<td>$se[sd_i(g_c)]$</td>
<td>0.21</td>
<td>0.20</td>
<td>0.10</td>
</tr>
<tr>
<td>$F_{c/(p)}$</td>
<td>1.65/(0.02)</td>
<td>2.58/(0.00)</td>
<td></td>
</tr>
<tr>
<td>$sd_i(\pi)$</td>
<td>2.46</td>
<td>2.60</td>
<td>1.26</td>
</tr>
<tr>
<td>$se[sd_i(\pi)]$</td>
<td>0.19</td>
<td>0.24</td>
<td>0.09</td>
</tr>
<tr>
<td>$F_{\pi/(p)}$</td>
<td>0.89/(0.68)</td>
<td>4.25/(0.00)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: $T_i$ is the number of observations in period $i$; $g_y$ is the percentage growth rate of real GDP per capita, $g_c$ is the percentage growth rate of real consumption of nondurables and services per capita, $\pi$ is the inflation rate in the consumption deflator (all at annual rates); $sd$ is a standard deviation and $se$ the associated standard error; $F$ is the test statistic for equality of standard deviations between the current and previous periods and $p$ is the associated $p$-value.
### Table 4: Means Across Postwar Moderations
Quarterly GDP Growth, Consumption Growth, and Inflation

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_i$</td>
<td>87</td>
<td>60</td>
<td>91</td>
</tr>
<tr>
<td>$m_i(g_y)$</td>
<td>2.39</td>
<td>1.77</td>
<td>2.04</td>
</tr>
<tr>
<td>se[$m_i(g_y)$]</td>
<td>0.52</td>
<td>0.50</td>
<td>0.22</td>
</tr>
<tr>
<td>$t_y/(p)$</td>
<td>-0.78/(0.42)</td>
<td>0.43/(0.67)</td>
<td></td>
</tr>
<tr>
<td>$m_i(g_c)$</td>
<td>2.04</td>
<td>1.97</td>
<td>1.94</td>
</tr>
<tr>
<td>se[$m_i(g_c)$]</td>
<td>0.29</td>
<td>0.23</td>
<td>0.14</td>
</tr>
<tr>
<td>$t_c/(p)$</td>
<td>-0.16/(0.87)</td>
<td>-0.11/(0.91)</td>
<td></td>
</tr>
<tr>
<td>$m_i(\pi)$</td>
<td>2.31</td>
<td>6.68</td>
<td>3.07</td>
</tr>
<tr>
<td>se[$m_i(\pi)$]</td>
<td>0.26</td>
<td>0.28</td>
<td>0.13</td>
</tr>
<tr>
<td>$t_\pi/(p)$</td>
<td>10.2/(0.00)</td>
<td>-9.9/(0.00)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: $T_i$ is the number of observations in period $i$; $g_y$ is the percentage growth rate of real GDP per capita, $g_c$ is the percentage growth rate of real consumption of nondurables and services per capita, $\pi$ is the inflation rate in the consumption deflator (all at annual rates); $m$ is a sample mean and se the associated standard error; $t$ is the test statistic for equality of means between the current and previous periods and $p$ is the associated $p$-value.
Table 5: Annual Interest-Rate Predictions

\[ \hat{m}_i(R^{-1}) = 0.9807 \left[ \frac{1}{(1 + m_i(\pi))} + \frac{sd_i^2(\pi)}{(1 + m_i(\pi))^3} \right] \]

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_i )</td>
<td>25</td>
<td>31</td>
<td>38</td>
<td>23</td>
</tr>
<tr>
<td>( sd_i(\pi) )</td>
<td>2.88</td>
<td>7.78</td>
<td>3.36</td>
<td>0.93</td>
</tr>
<tr>
<td>( m_i(\pi) )</td>
<td>0.64</td>
<td>2.43</td>
<td>4.48</td>
<td>3.10</td>
</tr>
<tr>
<td>( \hat{m}_i(R^{-1}) )</td>
<td>0.9752</td>
<td>0.9629</td>
<td>0.9396</td>
<td>0.9513</td>
</tr>
<tr>
<td>( m_i(R^{-1}) )</td>
<td>0.9548</td>
<td>0.9707</td>
<td>0.9497</td>
<td>0.9469</td>
</tr>
<tr>
<td>( t_{R^{-1}/(p)} )</td>
<td>3.72/(0.00)</td>
<td>-3.17/(0.00)</td>
<td>-0.39/(0.70)</td>
<td></td>
</tr>
<tr>
<td>(-100 \ln \hat{m}_i(R^{-1}))</td>
<td>2.51</td>
<td>3.78</td>
<td>6.23</td>
<td>4.99</td>
</tr>
<tr>
<td>(-100 \ln m_i(R^{-1}))</td>
<td>4.62</td>
<td>2.97</td>
<td>5.16</td>
<td>5.46</td>
</tr>
</tbody>
</table>

Notes: \( sd_i(\pi) \) and \( m_i(\pi) \) are the standard deviation and mean of the inflation rate; \( \hat{m}_i(R^{-1}) \) is the predicted mean bond price; \( m_i(R^{-1}) \) is the mean historical bond price; \( t \) is the test statistic for equality of means between the current and previous periods and \( p \) is the associated p-value; \(-100 \ln \hat{m}_i(R^{-1})\) is approximately the predicted mean interest rate, and \(-100 \ln m_i(R^{-1})\) is the corresponding historical mean. Standard errors for the historical moments are in parentheses.

Table 6: Predicted and Actual Changes in Mean Interest Rates

<table>
<thead>
<tr>
<th>Source</th>
<th>1946</th>
<th>1984</th>
</tr>
</thead>
<tbody>
<tr>
<td>( sd_i(\pi) )</td>
<td>0.47</td>
<td>0.09</td>
</tr>
<tr>
<td>( m_i(\pi) )</td>
<td>2.00</td>
<td>-1.33</td>
</tr>
<tr>
<td>( sd_i(\pi), m_i(\pi) )</td>
<td>2.45</td>
<td>-1.23</td>
</tr>
<tr>
<td>Actual</td>
<td>2.19</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Notes: Contributions to changes use the levels and formula from table 5.
Table 7: Quarterly Interest-Rate Predictions

\[ \hat{m}_i(R^{-1}) = 0.9889 \left( \frac{1}{(1 + m_i(\pi))} + \frac{sd_i^2(\pi)}{(1 + m_i(\pi))^3} \right) \]

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_i )</td>
<td>87</td>
<td>60</td>
<td>91</td>
</tr>
<tr>
<td>( sd_i(\pi) )</td>
<td>2.46</td>
<td>2.60</td>
<td>1.26</td>
</tr>
<tr>
<td>( m_i(\pi) )</td>
<td>2.31</td>
<td>6.68</td>
<td>3.07</td>
</tr>
<tr>
<td>( \hat{m}_i(R^{-1}) )</td>
<td>0.9671</td>
<td>0.9275</td>
<td>0.9596</td>
</tr>
<tr>
<td>( m_i(R^{-1}) )</td>
<td>0.9751</td>
<td>0.9299</td>
<td>0.9535</td>
</tr>
<tr>
<td>( t_{R^{-1}/(p)} )</td>
<td>-13.03/(0.00)</td>
<td>6.17/(0.00)</td>
<td></td>
</tr>
<tr>
<td>(-100 \ln \hat{m}_i(R^{-1}))</td>
<td>3.34</td>
<td>7.52</td>
<td>4.12</td>
</tr>
<tr>
<td>(-100 \ln m_i(R^{-1}))</td>
<td>2.52</td>
<td>7.27</td>
<td>4.76</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td>(0.34)</td>
<td>(0.22)</td>
</tr>
</tbody>
</table>

Notes: \( sd_i(\pi) \) and \( m_i(\pi) \) are the standard deviation and mean of the inflation rate; \( \hat{m}_i(R^{-1}) \) is the predicted mean bond price; \( m_i(R^{-1}) \) is the historical mean bond price; \( t \) is the test statistic for equality of means between the current and previous periods and \( p \) is the associated \( p \)-value; \(-100 \ln \hat{m}_i(R^{-1})\) is approximately the predicted mean interest rate, and \(-100 \ln m_i(R^{-1})\) is the corresponding historical mean. Standard errors for the historical moments are in parentheses.

Table 8: Predicted and Actual Changes in Mean Interest Rates

<table>
<thead>
<tr>
<th>Source</th>
<th>1969</th>
<th>1984</th>
</tr>
</thead>
<tbody>
<tr>
<td>( sd_i(\pi) )</td>
<td>-0.01</td>
<td>0.05</td>
</tr>
<tr>
<td>( m_i(\pi) )</td>
<td>4.19</td>
<td>-3.45</td>
</tr>
<tr>
<td>( sd_i(\pi), m_i(\pi) )</td>
<td>4.18</td>
<td>-3.40</td>
</tr>
<tr>
<td>Actual</td>
<td>4.75</td>
<td>-2.51</td>
</tr>
</tbody>
</table>

Notes: Contributions to changes use the levels from table 7.