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## Firm entry and labor market dynamics

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Firm Entry and Labor Market Dynamics

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**Abstract:** We present a model of aggregate fluctuations in which monopolistic firms face sunk costs to enter the production process and labor markets are characterized by search and matching frictions. Entrants post vacancies and are matched to idle workers. Our specification of sunk costs gives rise to a countercyclical net present value of a vacancy; it is always zero in models where entry is free. The model displays a strong degree of amplification and propagation. The time-varying value of a vacancy has implications for the surplus division between firms and workers over business cycle. In the data, we proxy this division using the ratio of corporate profits to output and workers' compensation to output. We document the cyclical behavior of profit's and labor's shares: Profit's share leads the cycle and is procyclical and more volatile than output. Labor's share inversely leads the cycle and is weakly countercyclical and smoother than output. Our model is consistent with the cross-correlations of both shares and the higher volatility of the share of profits. Regarding propagation and amplification, the model matches the persistence of vacancy creation and two-thirds of the observed volatility of market tightness relative to output.

JEL classification: E24, E32, J32

Key words: search, matching, business cycles, income shares

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# 1 Introduction

Our paper analyzes the role of firm entry and exit, and product creation, in explaining the behavior of labor markets over the business cycle. To this end, we formulate a dynamic general equilibrium model in which monopolistic firms are required to pay a sunk cost of entry and labor markets are characterized by search and matching frictions. Upon entry, firms post vacancies to possibly match with an idle worker and begin production. In our economy, sunk entry costs result in a positive and countercyclical value of a vacant position. It is countercyclical as higher productivity allows firms to use fewer inputs to pay for the sunk cost. We examine the implications of sunk costs and monopolistic competition for (a) the propagation and amplification of technology shocks into the labor market and (b) the cyclical dynamics of firms' and workers' income shares.

Sunk entry costs help propagate and amplify technology shocks into the labor market. They make vacancy postings and job creation adjust more slowly, as entrants react sluggishly to a shock. As in equilibrium the value of a vacant position is equal to the sunk cost, the latter amplify shocks as well: since it is relatively cheaper to enter the market in booms, the posting of vacancies reacts more strongly to a positive technology shock, and vice-versa. Monopolistic competition also plays a role and we show that it leads to more amplification and propagation relative to a perfectly competitive economy. The degree of market power matters because the prospect of higher profits increases the ability of firms to face the cost of entry. In turn, this affects vacancy creation, employment, and labor market conditions more generally.

The core of search and matching models of the labor market (e.g. Mortensen and Pissarides (1994)) is the surplus division between workers and firms. In the data, we proxy this division using two income shares: the ratio of corporate profits to output and the ratio of workers' compensation to output. We use the cyclical dynamics of these two shares as another dimension on which to judge our model. The profits' and labor's shares over the cycle behave rather different. The profits' share leads output, is procyclical, and volatile. The labor's share inversely leads the cycle, is (weakly) countercyclical, and smoother than output. Our model replicates cross-correlation pattern of both shares with

output. Our profits' share is more volatile than the labor's share, but the labor's share is too volatile relative to the data. To our knowledge, this is the first paper that documents the joint behavior of these two shares and attempts to replicate it in a dynamic equilibrium model. We compare the behavior of the income shares in our model relative to a version of Mortensen and Pissarides (1994) to which we add capital accumulation.<sup>1</sup> That model is broadly inconsistent with the cyclical dynamics of income shares: neither share lead output and both are too smooth. Qualitatively, it is consistent with the procyclicality of profits' share and the countercyclicality of labor's share. Quantitatively, it is not. Since vacancies adjust too quickly to changes in aggregate conditions, the correlations of income shares with output are too strong. In our model, the moderate adjustment brought about by sunk costs tame those correlations, generating more realistic dynamics.

Our paper contributes to several branches of the literature on aggregate fluctuations. One branch is the enormous literature that has followed Shimer (2005). He showed that after hitting the Mortensen and Pissarides (1994) model with plausible aggregate shocks, labor market variables barely moved. In the data, those variables are very volatile, in particular market tightness -the ratio of vacancies-to-unemployment. There have been several attempts to increase the amplification of shocks in Shimer's model. For instance, Hagedorn and Manovskii (2007) argue that large non-employment options exist for workers i.e. there is a large value to being unemployed. However, this argument has seemingly implausible implications for the reaction of the unemployment rate to changes in the unemployment benefit (see, for example, Hornstein, Krusell, and Violante (2005)). Another example, Hall (2005) argues that the empirical discrepancy can be explained by real wage rigidity. This argument, however, also has counterfactual implications (see for instance Pissarides (2007) or Haefke and Van Rhens 2007).<sup>2</sup> With our calibration, the model does not match the volatility of market tightness in the data, but it matches more than half the volatility. Given that we have followed Shimer's (2005) calibration quite faithfully, we

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<sup>1</sup>It is similar to that of Andolfatto (1996) and Merz (1995).

<sup>2</sup>Recently, Eyigungor (2007) has introduced match-specificity in capital and capital-embodied technology shocks to improve the business cycle implications of the Mortensen and Pissarides framework.

find this remarkable. Moreover, the model matches the persistence of vacancy creation and the correlation between unemployment and vacancies. Fujita and Ramey (2007) is a study closely related to ours. They focus on sunk costs of entry to explain the propagation of technology shocks. They too have a time-varying asset value of a vacancy. However, it is procyclical which kills the amplification of shocks. While they improve the standard model regarding propagation, they are silent about income shares. Also, they need to rely on the Hagedorn and Manovskii (2007) mechanism to amplify shocks. We have two important additional elements: monopolistic competition, which we show is important; and capital, which allows us to evaluate the model's investment dynamics.

As important as amplification and propagation of shocks are our model's implications for the dynamics of income shares. The literature on the cyclical behavior of the profits' and labor's share is not as large as the one above. Gomme and Greenwood (1995) build a model in which wage payments include net payoffs from contingent claims. The claims are traded between the employer and the employee and partially insure the worker against shocks. They study their model's implication for the cyclical behavior of wages and labor's share. Our model is broader in scope. It relates the ability of the search and matching framework to match the labor market business cycle facts and the income shares. Recently, Ríos-Rull and Choi (2006) note that the empirical impulse response of labor's share to a technology shock is at odds with the standard Mortensen and Pissarides (1994) framework (augmented with technology shocks). We replicate their empirical results and compute the response of the labor's share to a technology shock. The larger amplification and propagation of our firm entry model brings a striking improvement on this dimension as well.

The way we model firm entry is close to Bilbiie, Ghironi, and Melitz (2006) who focus on the propagation mechanism of sunk costs of entry. However, their labor market structure is simple having only a competitive market for hours worked.<sup>3</sup>

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<sup>3</sup>The same can be said for related firm entry models such as Chatterjee and Cooper (1993), Devereaux, Head, and Lapham (1996), and Jaimovich (2008), and Ambler and Cardia (1998). Hopenhayn and Rogerson (1992) build a model with sunk costs of entry but without aggregate shocks.

## 2 The Model Economies

### 2.1 Environment

Our economy is populated by a large extended household comprised of a continuum of members of total mass equal to  $\bar{N}$  and an infinite mass of firms.

Members in the household can either be employed or unemployed. Unemployed agents receive an unemployment benefit while they search for jobs with the hope of finding a job opportunity. This opportunity will allow them to enter into a relationship with a firm, to negotiate a contract that stipulates the retribution for their services, and to produce output during the following period. A fraction  $N_t$  of employed agents works and gets paid the negotiated wage. Members of the household have preferences over a sequence of composite goods over time,  $\{C_t\}_{t=1}^{\infty}$ . The per-period utility function is of the relative risk aversion class. The household's (expected) discounted lifetime utility as of time 0 is given by,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_t)^{1-\sigma}}{1-\sigma} \right], \quad (1)$$

where  $\beta \in (0, 1)$  is the discount factor and  $\sigma > 0$  is the coefficient of relative risk aversion. We assume that each firm produces a differentiated commodity. At each point in time, there is a subset of goods  $X_t \subseteq X$  available to consumers and the composite good is made up of commodities from that subset. The available set is time-varying as not all firms will produce every period. To aggregate over the different commodities, we use a Dixit and Stiglitz (1977) aggregator:

$$C_t = \left( \int_{x \in X_t} [c_t(x)]^{\frac{\gamma-1}{\gamma}} dx \right)^{\frac{\gamma}{\gamma-1}}, \quad (2)$$

where  $\gamma > 1$  is the symmetric elasticity of substitution between commodities. If  $p_t(x)$  is the price of product  $x$ <sup>4</sup>, then the level of  $c_t(x)$  chosen to minimize the cost of acquiring

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<sup>4</sup>This price is written in terms of "money", which in our economy is only used as a convenient unit of account and not valued for facilitating trades or for any other purpose.

$C_t$  given prices  $\{p_t(x)\}$  for all  $x$  is:

$$c_t(x) = \left( \frac{p_t(x)}{P_t} \right)^{-\gamma} C_t, \quad (3)$$

where  $P_t$  is the cost of acquiring one unit of the composite good, or the price index<sup>5</sup>:

$$P_t = \left( \int_{x \in X_t} [p_t(x)]^{1-\gamma} dx \right)^{\frac{1}{1-\gamma}}.$$

Each firm uses one unit of labor to produce its commodity. The job market in our economy is characterized by the existence of search and matching frictions (see Rogerson *et al.* (2005) for a survey of this literature). In order to hire a worker, a firm must post a vacancy and undertake a recruiting expense of  $\omega$  per vacancy posted. Firms and potential workers match in a labor market according to a constant-returns-to-scale matching technology  $M(\bar{N} - N, V)$  given by:

$$M(\bar{N} - N, V) = \frac{(\bar{N} - N)V}{((\bar{N} - N)^\xi + V^\xi)^{\frac{1}{\xi}}} \quad (4)$$

This matching function takes as inputs the total number of unemployed individuals who are searching,  $\bar{N} - N$ , and the total number of vacancies posted by firms,  $V$ . The output is a number of matches  $M$ . Denoting by  $\theta$  the vacancies to unemployment ratio  $\frac{V}{\bar{N} - N}$ , the

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<sup>5</sup> $P$  can be obtained by solving the consumer expenditure minimization problem for constructing one unit of composite good:

$$\begin{aligned} P &= \min_c \int_{x \in X_t} p(x) c(x) dx, \\ \text{s.t. } C &= \left( \int_{x \in X_t} [c(x)]^{\frac{\gamma-1}{\gamma}} dx \right)^{\frac{\gamma}{\gamma-1}} = 1. \end{aligned}$$



probabilities that a vacancy gets filled,  $q_t$ , and that a worker finds a job,  $f_t$  are given by<sup>6</sup>,

$$q_t = \frac{V}{M(N-N,V)} = \frac{1}{(1 + \theta_t^\xi)^{\frac{1}{\xi}}}, \quad (5)$$

$$f_t = \frac{\bar{N}-N}{M(N-N,V)} = \frac{\theta_t}{(1 + \theta_t^\xi)^{\frac{1}{\xi}}}. \quad (6)$$

A match between a firm and a worker results in a wage contract that specifies a wage  $w_t(x)$  paid in exchange of labor services. We assume that firms and workers split the surplus from their relationship according to a Nash bargaining rule. We will be more specific about this rule below after we have fixed some notation regarding workers' and firms' value functions. The relationship between a firm and a worker can break either because the firm ends production or exogenously for any other reason (at rate  $s$ ).

Firms need to pay a sunk cost to begin the goods production process. Opening a firm or starting a new product variety needs a flow  $y^E$  using a linear technology that has capital as its only input, i.e.  $y^E = Z_t K_t^E$ . The productivity shock  $Z_t$  follows a first-order Markov process. Denoting by  $r_t$  the rental rate of capital and noting that one unit of capital produces  $Z_t$  units of the composite good, the sunk cost of entry is  $\frac{r_t y^E}{Z_t}$  or  $r_t K_t^E$  (in units of the composite consumption good). We denote the number of entrants – the number of firms that pay the sunk cost – by  $N_{E,t}$ .

Production of the differentiated commodity involves only labor. Denoting the firm's output of the differentiated product  $x$  by  $y_t^c(x)$ , it is obtained with the following linear technology,

$$y_t^c(x) = Z_t l_t(x) \quad (7)$$

where  $Z_t$  is an aggregate productivity shock with the same process as that of the sunk cost, and  $l_t(x)$  is the labor amount the firm uses, which will equal one if the firm produces and zero otherwise. The firm charges a price equal to  $\rho_t(x)$  and its profits are given by

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<sup>6</sup>We depart from the more frequent Cobb-Douglas specification for the matching function in order to bound the job-finding and vacancy-filling probabilities to be between 0 and 1. This functional form was chosen by Den Haan, Ramey, and Watson (2000).

$$\pi_t(x) = \rho_t(x)Z_t - w_t(x).$$

Finally, the government plays a very limited role in our economy. Its task is solely to tax the household a lump-sum quantity and rebate it in the form of a benefit for the unemployed.

## 2.2 Optimization and Equilibrium

We restrict ourselves to a symmetric equilibrium in which all goods-producing firms charge equal prices,  $\rho_t(x) = \rho_t$ ; demand one unit of labor which gets paid the same wage  $w_t(x) = w_t$ ; and produce the same amount of output,  $y_t^c(x) = y_t^c$ . Given the CES structure of the consumption aggregate, the relative price  $\rho_t$  that firms charge is given by <sup>7</sup>  $N_t^{\frac{1}{\gamma-1}}$  and the per-firm profit is given by,  $\pi_t = \rho_t Z_t - w_t$ . The relevant state vector for the firm is the quadruplet  $(K_t, N_t, V_t, Z_t)'$  with  $K_t = N_{E,t} K_t^E$ . To save on notation, we will write down value functions without being specific about their dependence on the state vector.

Households own a diversified portfolio of firms and as a result firms discount expected future flows taking into account the household's inter-temporal condition. Consequently, a firm's appropriate discount factor between periods  $t$  and  $t + 1$  is,

$$\Delta_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma}. \quad (8)$$

Let  $Q_t$  denote the capital value of a vacancy and  $J_t$  denote the capital value of a filled job. The following two recursive relationships must be satisfied:

$$Q_t = -\omega + (1 - \tau) E_t \Delta_{t+1} [q_t J_{t+1} + (1 - q_t) Q_{t+1}], \quad (9)$$

$$J_t = \pi_t + (1 - \tau) E_t \Delta_{t+1} [(1 - s) J_{t+1} + s Q_{t+1}]. \quad (10)$$

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<sup>7</sup>Given that  $p_t(x) = p_t$  and  $\rho_t = \frac{p_t}{P_t} = \frac{p_t}{\left( \int_{x \in X_t} [p_t]^{1-\gamma} dx \right)^{\frac{1}{1-\gamma}}}$ , implies that  $\rho_t = \frac{p_t}{p_t \left( \int_{x \in X_t} dx \right)^{\frac{1}{1-\gamma}}}$  and as a result,  $\rho_t = \left( \int_{x \in X_t} dx \right)^{\frac{1}{\gamma-1}} = N_t^{\frac{1}{\gamma-1}}$ , as  $N_t$  is the both the fraction of firms producing as well as the number of workers in the goods-producing sector by our assumption of one job per firm.

Equation (9) states that the value of a vacancy (once the entry decision has been made) is the difference between two objects. First, the expected value of entering the labor market and trying to match with a worker. This matching happens with probability  $q_t$ , as long as the firm survives for one period, which happens with probability  $1 - \tau$ . Second, the vacancy cost  $\omega$ . The difference between the two is, again, the value of a vacancy.

The interpretation of equation (10) is analogous: the value of a filled job is the profit flow  $\pi$  plus the expected continuation value of the relationship between the firm and the worker. Conditional on the firm's survival, the relationship ends with probability  $s$  and continues with probability  $1 - s$ .

In equilibrium, the entry of firms occurs until the value of vacancies is equal to the sunk cost,

$$Q_t = r_t K_t^E \tag{11}$$

Due to entry costs, vacant jobs have positive value in equilibrium which in turn leads firms to repost vacancies following separations. The following two equations give the laws of motion for the stock of employment and vacancies:

$$N_{t+1} = (1 - \tau) [(1 - s)N_t + f_t(\bar{N} - N_t)], \tag{12}$$

$$V_{t+1} = (1 - \tau) [(1 - q_t)V_t + sN_t] + N_{E,t}. \tag{13}$$

Employment at time  $t + 1$  is the sum of matches  $(1 - s)N_t$  that were not destroyed either by the death of a firm or other form of separation, and the newly-formed matches  $f_t(\bar{N} - N_t)$  from a previous pool of unemployed people. The total number of vacancies in the economy, given by equation (13), is equal to vacancies that did not get filled in the current period,  $(1 - q_t)V_t$  plus the number of separated matches  $sN_t$ . Of course, we need to include the fraction of firms which continue operating for at least one more period. Finally, we need to add to that total the number of newly created firms  $N_{E,t}$ , each of which posts a vacancy. Both employment and vacancies are predetermined variables.

The household's problem is relatively straightforward. Given its current period resources, it chooses consumption and investment to maximize the expected discounted value of lifetime utility. In addition to wage income and unemployment benefits, the household gets interest from bond holdings as well as a pay-out from its diversified ownership stake in firms. The aggregate dividends firms pay out equal to  $d_t = N_t\pi_t - \omega V_t - Q_t N_{E,t}$ . Finally, the household also gets taxed a lump-sum amount  $T_t$  which the government uses to finance the unemployment benefit program. Denoting by  $W_t$  the household's value function at time  $t$ , the optimization problem can be expressed as:

$$W_t = \max_{C_t, I_t} \frac{C_t^{1-\sigma}}{1-\sigma} + \beta E_t W_{t+1} \quad (14)$$

subject to

$$C_t + I_t = b(\bar{N} - N_t) + w_t N_t + r_t K_t + d_t - T_t, \quad (15)$$

$$K_{t+1} = (1 - \delta) K_t + I_t. \quad (16)$$

The optimal inter-temporal condition is:

$$\beta E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} (r_{t+1} + 1 - \delta) \right] = 1 \quad (17)$$

As was mentioned in the previous section, wages for the employed workers are the result of Nash bargaining between each worker-firm pair. The surplus of the match for the household is captured by the change in welfare derived from having a marginal unemployed person employed. This change is given by  $\frac{\partial W_t}{\partial N_t}$  which in units of the consumption good is  $\frac{\partial W_t}{\partial N_t} C_t^\sigma$ . The surplus for the firm is given by  $J_t - Q_t$ , the difference between the value of a filled job and the value of a vacancy. The Nash bargaining solution when the firm's

bargaining parameter is given by  $\phi$  satisfies the following surplus-splitting rule:

$$\frac{J_t - Q_t}{1 - \phi} = \frac{C_t^\sigma \frac{\partial W_t}{\partial N_t}}{\phi}, \quad (18)$$

which yields the following equation for wages:

$$w_t = (1 - \phi)b + \phi(\rho_t Z_t + \omega) - \phi(1 - \theta_t)(\omega + Q_t - (1 - \tau)E_t \Delta_{t+1} Q_{t+1}) \quad (19)$$

We can now define a symmetric equilibrium for our economy. It is a sequence of prices  $\rho_t, w_t, r_t$ ; a sequence of aggregate quantities  $K_t, C_t, N_t, V_t, N_{E,t}, \pi_t$ ; and a sequence of value functions  $Q_t, J_t, W_t$  such that for any time period  $t$ , the following conditions hold:

1. (*Household Optimization*) Given prices  $\rho, w, r$  the household's optimization results in decision rules for  $C_t$  and  $I_t$  and the value function  $W_t$ .
2. (*Factor Market Clearing*) The interest rate  $r_t$  equates the capital demanded by new entrants  $N_E$  to that supplied by the household, and the wage  $w$  satisfies the Nash bargaining solution given by equation (19).
3. (*Goods Market Clearing*)  $C_t = w_t N_t + N_t \pi_t - \omega V_t - N_{E,t} Q_t$ .
4. (*Firm's Optimization*) Given the demand for a differentiated commodity given by equation (3),  $\rho_t$  is the profit-maximizing price for the monopolist. Aggregate labor demand and vacancies posted by all firms,  $N_{E,t}$ ,  $N_t$  and  $V_t$  satisfy equations (12) and (13), and the vacancy and filled position values satisfy equations (9) and (10).
5. (*Entry Condition*)  $Q_t = r_t K_t^E$ .
6. (*Government*) The government satisfies its budget constraint:  $b(\bar{N} - N_t) = T$ .

## 2.3 Calibration

We calibrate the model to the monthly frequency by assigning values to parameters, so that steady-state moments in the model match those observed in U.S. data. The risk aversion coefficient  $\sigma$  is set to 1.5 which is well within the range of values typically used in studies of aggregate fluctuations. The discount factor  $\beta$  is set to  $0.99^{\frac{1}{3}}$  which implies a steady-state interest rate equal to 4.2% per annum.

We assume that total factor productivity  $Z_t$  follows an AR(1) process with persistence parameter  $\rho_z$  and a zero-mean normally distributed shock with variance  $\sigma_\epsilon^2$ . We set  $\rho_z = 0.964$  and  $\sigma_\epsilon = 0.0052$  which are consistent with the cyclical persistence and variance in the observed quarterly Solow residual. Lacking direct evidence on a reasonable value for the workers' bargaining parameter  $\phi$ , we set it equal to 0.5 to make our results comparable to the existing literature.

We calibrate the exit probability  $\tau$  and the separation rate  $s$  following a procedure similar to that used by Den Haan et al. (2000). Let  $\Sigma$  be the total job separation rate caused either by a firm's death or by any other cause. The rate at which firms exit the market and do not repost vacancies is  $\tau$ , while  $(1-\tau)s$  is the rate at which workers separate from firms but where firms repost vacancies immediately after. Hence,  $\Sigma = \tau + (1-\tau)s$ . The fraction of vacancies that are reposted right after separations is then  $\frac{(1-\tau)s}{\Sigma}$ . Denote this quantity by  $\Omega$ . Note also that  $\Sigma N$  gives the total flow out of employment, and as a result,  $\Omega q \Sigma N$  gives the total number of posted vacancies filled. If we subtract the number of posted vacancies filled from the total flow out of employment, we get the steady-state mass of jobs that is destroyed permanently:  $\Sigma N - \Sigma N \Omega q = \Sigma N (1 - \Omega q)$ . In a steady state, job destruction must equal job creation. The empirical evidence described by Shimer (2005) sets  $\Sigma$  equal to 0.1 at the quarterly frequency which implies  $1 - (1 - 0.1)^{\frac{1}{3}} = 0.035$  at the monthly frequency. Therefore,

$$\Sigma = (1 - \tau)s + \tau = 0.035 \tag{20}$$

Davis *et al.* (1996) report that the job-creation-to-employment ratio in the manufacturing sector is 0.052 quarterly, which implies a value of 0.018 at the monthly frequency. Given a value of  $q = 0.802$  per month,

$$\frac{\text{Job Creation}}{\text{Employment}} = \frac{\Sigma N(1 - \Omega q)}{N} = 0.018 \quad (21)$$

From equations (20) and (21) we can solve for  $s = 0.021$  and  $\tau = 0.014$ .

Consistent with estimates reported by Basu and Fernald (1997), we set  $\gamma = 11$ , which implies a markup of 10 percent. Changing the total mass of workers  $\bar{N}$  only amounts to changing the levels, i.e., the scale of output and the mass of employment, etc., but the unit-free ratios, e.g., unemployment rate, v-u ratio, and consumption-output ratio etc., are not affected. Therefore, a choice of  $\bar{N}$  does not affect any of the second moments and the impulse responses. We just pick  $\bar{N} > 1$  so that the monopolist's price is larger than the resulting price if markets are competitive, which is given by  $\lim_{\gamma \rightarrow \infty} N_t^{\frac{1}{\gamma-1}} = 1$ .

We are left with five parameters to calibrate:  $(b, y^E, \delta, \omega, \xi)$ . We choose five additional moments that the model needs to match in a steady-state. Based on his own calculations, Shimer (2005) documents that the monthly job finding rate is 0.45. Blanchard and Diamond (1989) argue that vacancy postings have an average of 3 weeks, which implies that the vacancy filling rate is  $1 - (1 - 1/3)^4 = 0.802$  per month. Note that the steady state value of market tightness can be written as  $\theta = \frac{f}{q} = 0.56$ . We choose to match the aggregate capital to aggregate output ratio and we set it to a value of 36, which implies a value of 3 at the annual frequency. We set total recruiting costs as a fraction of GDP, given by  $\omega V/Y$ , to 0.015. Finally, a controversial choice is that of the value of the unemployment benefit  $b$ . Much of the literature argues that the value of non work activities is far below what workers produce on the job. However, calibrations such as Hagedorn and Manovskii's (2007) claim much success in terms of the cyclical properties of the model when the outside option for workers is very close to their productivity. Under the interpretation of  $b$  as purely monetary unemployment benefits, we set  $b$  so that the steady

Table 1: Summary of Parameterization

Parameter	Value	Target/Source
$\sigma$	1.500	Prev. work
$\phi$	0.500	Prev. work
$\rho_z$	0.964	NIPA
$\sigma_z$	0.005	NIPA
$\tau$	0.014	$\frac{Job\ Creation}{Employment} = 0.018$
$s$	0.021	$\Sigma = 0.035$
$\gamma$	11	10% markup
$\beta$	$(0.99)^{\frac{1}{3}}$	$r=4.2\%$
$\delta$	0.002	$K/Y = 36$
$b$	0.380	$b/w = 0.42$
$y^E$	3140	$wV/Y = 0.015$
$\omega$	0.516	$f = 0.45$
$\xi$	1.551	$\theta = 0.561$

state replacement ratio  $b/w$  is 0.42 as in Shimer (2005) and Gertler and Trigari (2006). Concluding, to assign values to the vector of parameters  $(b, y^E, \delta, \omega, \xi)$ , we choose the following five moments:  $f = 0.45$ ,  $\theta = 0.56$ ,  $\omega V/Y = 0.015$ ,  $K/Y = 36$  and  $b/w = 0.42$ .

We summarize our parameterization in Table 1.

## 3 Results

### 3.1 The Labor Market: Amplification and Propagation

Having assigned parameter values to the model, we solve it, simulate it, and judge its implications against U.S. data. Our solution technique is standard: we approximate the true solution by a first order approximation around the model's stochastic steady-state. Since the calibration is done at the monthly frequency, we transform the model's output by aggregating its "monthly" data into "quarterly" data. We do this by taking 3-month averages. We obtain a sample of data from the Bureau of Economic Analysis (BEA) and R. Shimer's website on aggregate variables covering the period 1951:Q1-2003:Q4. We transform the model's output and U.S. data in the same way: we de-trend them by



taking logs and applying a Hodrick-Prescott filter.<sup>8</sup> Table 2 shows some statistics for our sample of U.S. data for some selected quantities. We focus on consumption ( $C$ ), investment ( $I$ ), unemployment ( $U$ ), vacancies ( $V$ ), the vacancies-to-unemployment ratio ( $V/U$ ), GDP ( $Y$ ), consumption ( $C$ ) and total factor productivity ( $Z$ ). Regarding labor market variables, the two most salient features are: the high volatility of unemployment, vacancies, and the vacancies-to-unemployment ratio; and the strong negative correlation between unemployment and vacancies. The first of the two has been the object of a large literature spawned by Shimer’s (2005) study, as the discrepancy between the data and the model is large. Other important features are the weak correlation between vacancies and productivity and the stronger correlation between vacancies and output. Other well-known business cycle facts are the lower volatility of consumption and the higher volatility of investment, relative to that of output. Both are highly procyclical.

We display results from two theoretical economies in Tables 3-6: a model labeled *MPK* which stands for Mortensen-Pissarides with capital. Readers should think of it as the same model solved in Shimer (2005), to which we add capital accumulation, we model it in discrete and not in continuous time, and we calibrate it differently. To ease comparison and for the sake of exposition we give a more detailed description of the *MPK* model in an Appendix. The second model is the baseline model with firm entry described above. We label it the “Entry” model.

Table 3 displays the models’ standard deviations for some selected variables. Thanks to research by R. Shimer and others, the inability of the *MPK* model to account for the high volatilities of labor market variables is well-known. The volatility of the  $V/U$  ratio relative to output is less than two. It is 15 in the data. Similar conclusions can be reached about the volatilities of unemployment and vacancies separately. Aggregate consumption is also much smoother than in the data, while the volatility of investment matches quantitatively

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<sup>8</sup>The HP smoothing parameter we use is  $10^5$ , which is larger than the typical choice of 1,600. We choose it for two reasons. First, given the large propagation in our model, much of the dynamics are in lower frequencies than those retained by a smoothing parameter equal to 1,600. And two, it was used by Shimer and therefore it facilitates comparisons.

its empirical counterpart. On the other hand, labor market variables are greatly amplified in the “Entry” model. The volatility of market tightness –  $V/U$  – relative to output increases to 6.6, almost half of what is observed in the data. The volatility of vacancies more than doubles and the volatility of unemployment more than triples relative to  $MPK$ . The volatility of aggregate investment decreases, but relative to output is still as volatile as in the  $MPK$  model. It is worth emphasizing that output in the “Entry” model is defined as  $Y = wN + rK + N\pi$ . In other words, we include all sources of income, irrespective of whether they are used to pay for the sunk cost or not.

Tables 4 and 5 show cross-correlations between a few aggregates for the same two models. The  $MPK$  model succeeds in obtaining a positive relationship between consumption, investment, and vacancies, with output. Additionally, it correctly predicts a negative relationship between unemployment and vacancies, known in the literature as the slope of the Beveridge curve. This relationship is, however, somewhat weaker than in the data. It is only -0.62 in the model but -0.901 in the data. The baseline model with entry (shown in Table 5) roughly matches the correlation between consumption and investment with output. Vacancies are also pro-cyclical but have a low contemporaneous correlation with output (0.244). As firms need to choose whether they enter or not a period before choosing their level of vacancies, vacancies lag the cycle. For the same reason, the  $V/U$  ratio (and as a result the job finding probability) is roughly acyclical. The entry model predicts a Beveridge curve that is about as steep as in the data with a slope of -0.93, approximately what is observed in the data. The reader might be surprised about the business cycle behavior of investment in the “Entry” model. Both the volatility and correlation of investment with output are in line with US data. This might seem puzzling, as all capital in the model is used to pay for sunk costs. Moreover,  $y^E = ZK_E$ , and as a result  $K_E$  falls with a positive innovation to  $Z$  to keep  $y^E$  constant, making the procyclical investment all the more puzzling. However, aggregate capital  $K$  is defined as  $K = N_E K_E$ , and the number of entrants reacts more, relative to its average, than (per-firm) capital.

Finally, we display autocorrelations for the two models and for the same variables we

showed in Table 3. The model with firm entry features a larger degree of propagation into labor market variables than the *MPK* model. A particularly striking case is vacancies. Its first order autocorrelation increases from 0.72 in the *MPK* model to 0.98 in the “Entry” model, just slightly above its empirical value. The reason is that as entrants react sluggishly to TFP shocks, vacancy creation persists for a longer time period. We reach a similar conclusion in the case of market tightness. The autocorrelations for unemployment, consumption, investment and output are roughly of the same magnitude and in line with their empirical counterparts.

### 3.1.1 Producing Differentiated Commodities with Capital and Labor

The model with firm entry improves over *MPK* when it comes to propagating and amplifying technology shocks into the labor market. Here, however, we relax the assumption that the entire capital stock is used to pay for the sunk cost. A type  $x$  commodity is produced according to,

$$y_t^c(x) = Z_t l_t(x)^{1-\alpha} (K_t^c)^\alpha \quad (22)$$

We maintain the assumption that one firm equals one job. If the firm produces  $l_t(x) = 1$  and it is zero otherwise. In the definition of aggregate capital we need to include capital used by commodity-producing firms. As a result,

$$K_t = N_t K_t^c + N_{E,t} K_t^E \quad (23)$$

It is important to note that aggregate capital  $K_t$  is the state variable. Within a given time period, capital is perfectly mobile and therefore the interest rate  $r_t$  is also the rental rate of capital in the commodity-producing sector. It is given by,

$$r_t = \alpha \frac{y_t^c}{K_t^c} \lambda_t \quad (24)$$

In the previous expression  $\lambda_t$  is the marginal cost. By dropping the  $x$  as an argument, we have made implicit in our notation the symmetric nature of our equilibrium definition. Finally, the per-firm profits are given by,

$$\pi_t = \rho_t y_t^c - r_t k_t^c - w_t \quad (25)$$

All other elements of the environment, in particular the functioning of labor markets and the household’s optimization problem, do not change relative to the baseline “Entry” model. Regarding the parameterization, the addition of  $K_t^c$  introduces a new parameter  $\alpha$ , set to be 0.30. However, as we recalibrate the model the numerical values of the other parameters change as a result.

We label this model the “Entry Both” model. Tables 7 - 9 display results for the volatility, persistence, and cross-correlations of variables. Table 7 shows standard deviations for the same variables as Table 3 for the *MPK* model and the model with capital in both sectors. Regarding amplification the latter model loses much compared to the baseline entry model. The volatilities of vacancies and  $V/U$  relative to output are barely above those of the *MPK* model. The larger propagation into the labor market is, on the other hand, intact. This is apparent in Table 8 which shows autocorrelations for the same variables as Table 6. Vacancy creation is still very persistent and most other autocorrelations are larger than in the *MPK* model. When we turn to cross-correlations, displayed on Table 9, we see that the “Entry Both” model retains the strong negative relationship between unemployment and vacancies (-0.859) and the strong counter-cyclicality of the unemployment rate. Consumption and investment are roughly as correlated with output as in the data.

It is informative to compare the implications of both entry models – with and without capital in the commodity-producing sector – for the cyclicalities of  $Q$ .  $Q$ , the net present value of a vacant job, is countercyclical in both models (Tables 5 and 9). Its contemporaneous correlation with  $Z$ , the exogenous TFP process, is -0.350 in the “Entry Both”

model and -0.663 in the baseline firm entry model. However, although the unconditional correlation between  $Q$  and  $Z$  is similar,  $Q$ 's response to an innovation in the TFP process is quite different. Figure 1 shows the impulse response of  $Q$  to a one-standard deviation shock in  $Z$  in the two models. The response in the baseline entry model is to initially fall remaining below the steady-state for a long time period. In the "Entry Both" model the initial response is muted. It falls very gradually until reaches a level below the steady state, at which remains for a considerable time period. As  $Q = rK_E$  in both models, and  $K_E$  must fall roughly in the same magnitude, the difference must be in the behavior of interest rates. In the baseline entry model interest rates initially fall. In the "Entry Both" model, interest rates cannot fall much because they are also related to the marginal product of capital in the commodity-producing sector. In fact, in our calibration they rise compensating for the fall in  $K_E$  and forcing  $Q$  to remain roughly constant in the initial period. Whether interest rates are countercyclical or procyclical is an empirical question albeit not an easy one to answer. Stock and Watson (1999) construct a series of real rates using a measure of expected inflation and nominal bond returns, finding that interest rates are counter-cyclical. King and Rebelo (1999) emphasize the difficulty in standard RBC economies to replicate this fact. Gomme, Ravikumar, and Rupert (2008) find that real S&P 500 returns are (weakly) countercyclical but synthetic returns constructed from NIPA data are not.

### 3.1.2 The Role of Monopolistic Competition

A model with endogenous entry and sunk costs need not include monopolistically competitive goods markets. A time-varying value of a vacancy will arise even if markets are perfectly competitive. In our model economy, this happens as  $\gamma$  approaches  $\infty$ . As a relatively low value for  $\gamma$  is not a standard feature in many search and matching models of the labor market, we assess the sensitivity of the model's predictions to its value. Figure 2 shows two plots. In the upper plot we show, for values of  $\gamma$  ranging from 7 to 90, the first order correlation of vacancy creation (on the right axis) and the standard

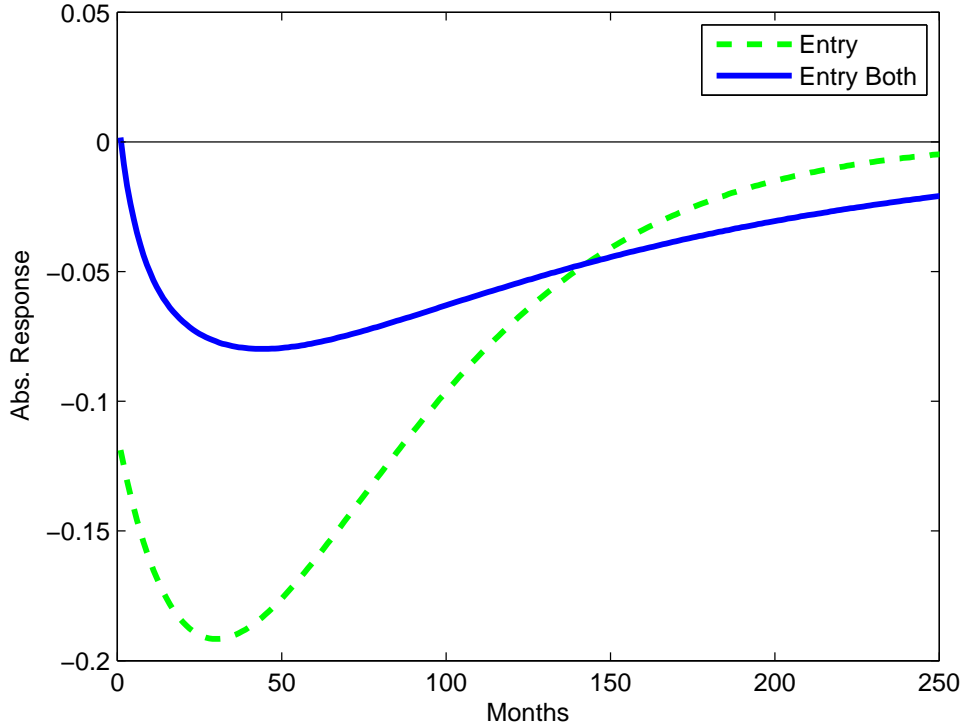


Figure 1: Response of the value of a vacancy  $Q$  in the two entry models after a one standard deviation innovation to the TFP process. Dashed line: baseline entry model; solid line: capital in both sectors.

deviation of  $V/U$  relative to that of the Solow residual (on the left axis). Lower levels of  $\gamma$  (i.e. higher market power) imply larger levels of amplification and propagation. As the prospects of profits rise, entrants react more strongly to higher productivity. The value of vacancy creation has to fall more to satisfy the no-entry condition and as a result, the value of a vacancy is more counter-cyclical. This fact is reflected in the bottom plot of Figure 2, where for the same values of  $\gamma$  we plot the correlation coefficient between the value of a vacancy and the Solow residual.

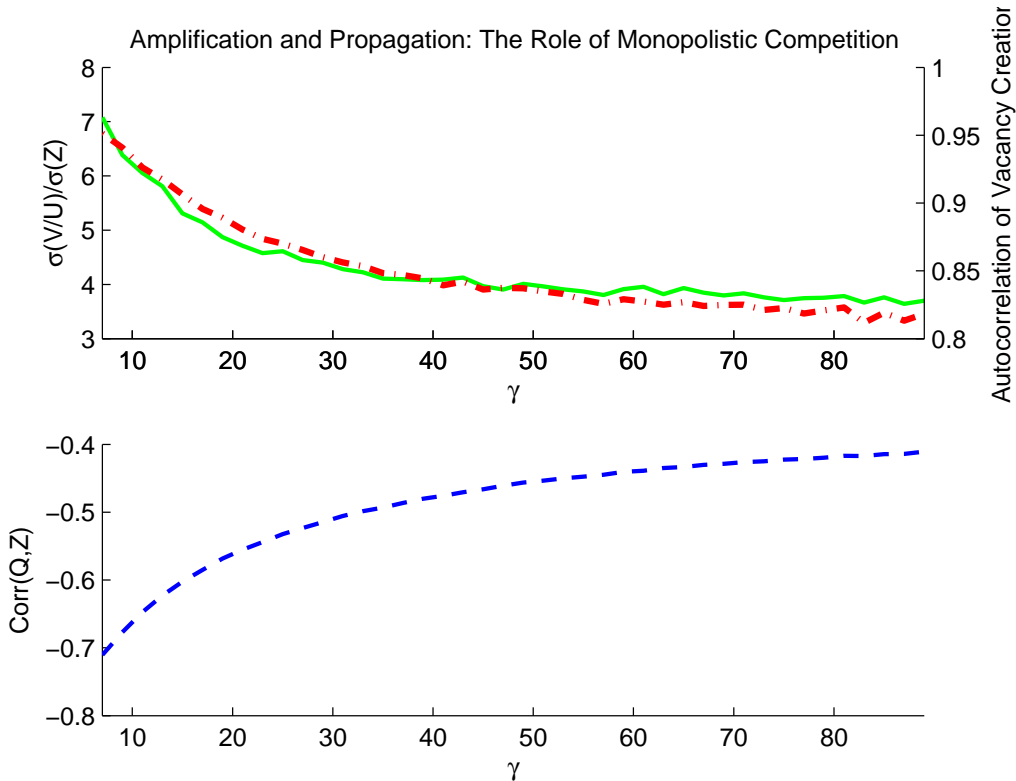


Figure 2: Amplification and propagation as a function of market power ( $\gamma$ ). Upper plot:  $\sigma(V/U)/\sigma(Z)$  (solid line, left axis);  $Corr(V_t, V_{t-1})$  (dashed-dotted line, right axis). Lower plot:  $Corr(Q, Z)$ .

### 3.2 The Cyclical Behavior of Labor’s and Corporate Profits Shares

A novel aspect of our research is a countercyclical  $Q$ . Absent any good measures of the value of a vacancy, at least that we are aware of, we analyze the models’ implications for the behavior of (corporate) profits and wages shares in national income over the business cycle. Very loosely we interpret the behavior of these two variables as an empirical measure of how the surplus division between workers and firms changes over the cycle. Our goal is to analyze if our firm entry models help to explain these shares’ behavior. We label as “profits’ share” the ratio of corporate profits to GDP, and we plot its deviations relative to an HP-trend in Figure 3. We also plot HP-filtered real GDP. Analogously, we label (as much of the literature) the ratio of compensation to employees to GDP as

“labor’s share”. Some statistics of these series are given on the first two rows of Table 10. The profits’ share (first row) is about 4 times more volatile than output, procyclical (the contemporaneous correlation is about 0.5), and it leads the cycle by one quarter (the correlation of output with lagged profits’s share is 0.59). The labor’s share is less volatile than output, (weakly) counter-cyclical with a contemporaneous correlation close to zero, and lags the cycle.

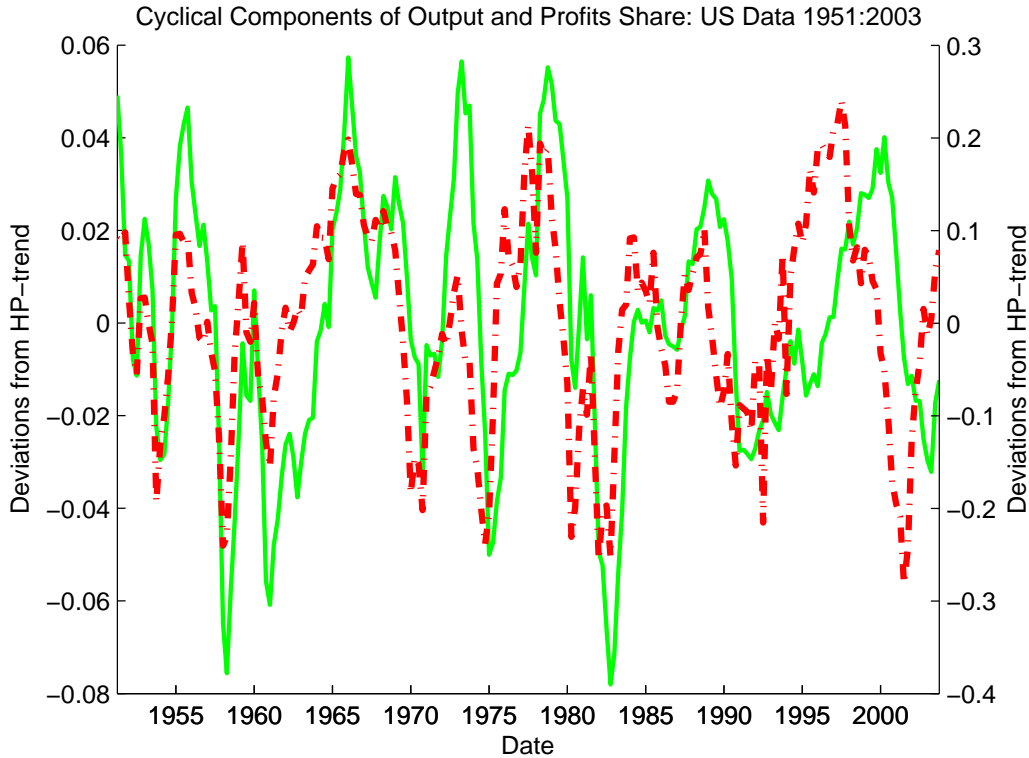


Figure 3: HP-Filtered estimates of output (solid line) and the profits share (dashed-dotted line).

The bottom six rows of Table 10 show the same moments for each of the three models: *MPK*, “Entry”, and “Entry Both”. Consistent with the low amplification mechanism, both shares are too smooth in the *MPK* model. In particular, the labor’s share volatility is about 10 times smoother than its empirical counterpart. Contemporaneous correlations are too strong (close to -1 and 1) although qualitatively *MPK* is consistent with the



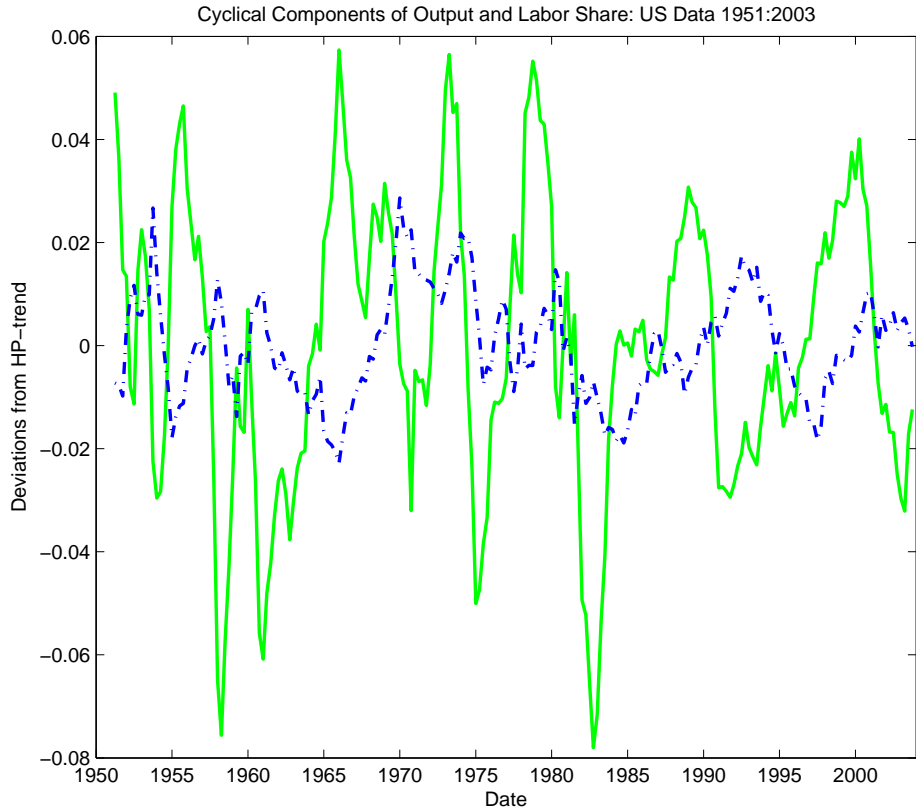


Figure 4: HP-Filtered estimates of output (solid line) and the labor share (dashed- dotted line).

data. However, shares are not consistent with the leading behavior observed in U.S. data: correlations are greatest (in absolute value) at zero lags. The models with firm entry fit the data much better. They are consistent with the leading behavior of the profits share and the inversely leading behavior of the labor’s share. In the “Entry” model both shares are more volatile, the profits’ share has a weak positive contemporaneous correlation with output and the labor’s share has a negative correlation but very close to zero (-0.07). The profits’ share leads the cycle by almost two quarters, and the labor’s share inversely leads the cycle by two quarters. In the “Entry Both” model, the profits’ share is more volatile than output and the labor’s share is less volatile than output. However, the “Entry” model delivers a positive correlation between output and the labor’s share.

Two features are key to understand the intuition behind the leading behavior of the profits' share. After a positive technology shock, wages react slowly and peak several periods after the shock hits. As a result, profits initially rise but fall below steady state with a trough that bottoms several periods later. The second key element is that total output includes that produced in the sunk cost sector, but the share of the commodity-producing output is much larger. As the latter rises with TFP, it reverts to steady-state fairly quickly. Output in the sunk cost sector, represented by  $Q$ , remains below the steady state much longer, causing total output to drop below steady-state after the trough in profits, generating the correlation pattern.

Ríos-Rull and Choi (2007), building on the empirical analysis of Ríos-Rull and Santaella-Llopis (2008) note that an empirical regularity of the labor's share is at odds with US data in a large class of models. In particular they focus on the response of the labor share to a technology shock. We begin by plotting in Figure 5 two series: our empirical measures of the (de-trended) Solow residual and the labor share constructed in a similar manner to Ríos-Rull and Santaella-Llopis (2008). The data are quarterly and the sample spans 1954:1-2003:IV. We fit a bivariate VAR to these two series and we identify a “fundamental” innovation to the technology process by assuming that the labor's share does not affect technology contemporaneously. Specifically we assume that the “structural” representation of the reduced-form VAR takes the following form,

$$lsh_t = \alpha_0 + \beta_0 z_t + \alpha_{11} lsh_{t-1} + \alpha_{12} z_{t-1} + \epsilon_{lsh,t}$$

$$z_t = \alpha_1 + \alpha_{21} lsh_{t-1} + \alpha_{22} z_{t-1} + \epsilon_{z,t}$$

Figure 6 shows the response of labor's share to a one-standard deviation innovation to the technology process. The labor's share falls contemporaneously and starts rising one quarter after the shock. The rise continues for about five years, after which the labor's share slowly returns to its steady-state level.

Besides the “over-shooting” property, the two most noticeable features are the mag-

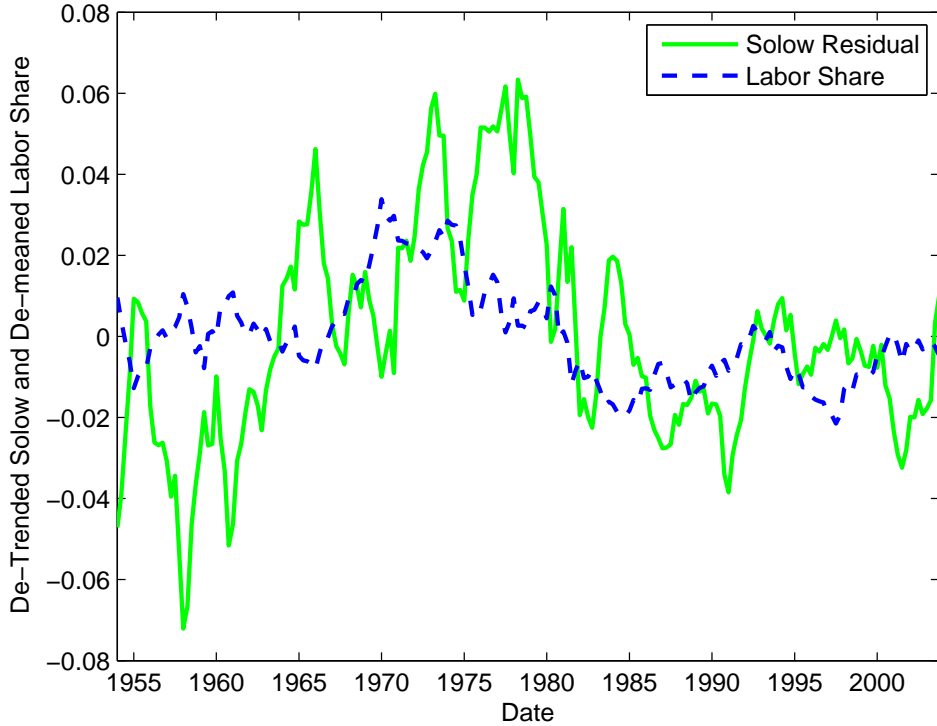


Figure 5: De-measured labor’s share and de-trended Solow Residual; postwar US data.

nititude of the rise (significantly above the steady-state level) and the persistence of the response. In their quest for models that can match this feature of the data, Ríos-Rull and Choi (2008) focus on a family of search and matching models of the labor market in which wages are the result of Nash bargaining. The standard Mortensen and Pissarides (1994) model matches only the initial drop. After the first period, the response of the labor’s share is rather muted when compared to the data. It never rises much above its steady state level and it displays virtually no persistence. This is true even when the model is calibrated to match the volatility of the vacancies-to-unemployment ratio observed in the data. As our framework belongs to the same family of search and matching models we perform a similar analysis.

Figure 7 displays the response of the labor’s share in the three models. We compute the quarterly response as the three-month average of the original monthly response. The first

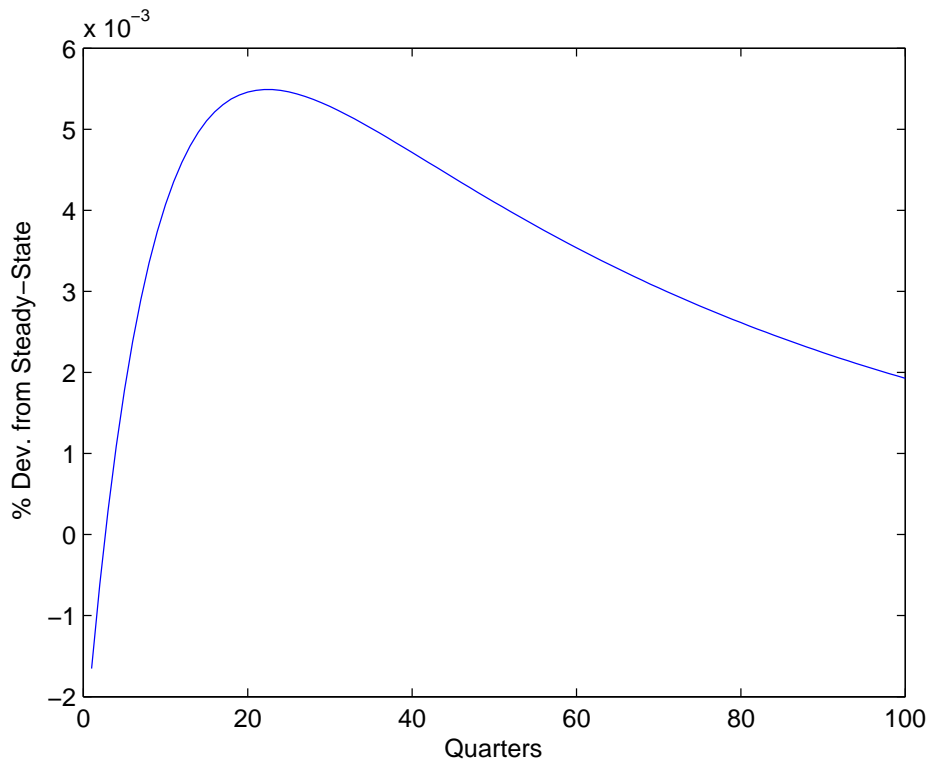


Figure 6: Response of the labor’s share to a one standard deviation (orthogonalized) innovation to technology.

model, labeled MPK, is a version of the one presented in Mortensen and Pissarides (1994) with capital. The second is the firm-entry model we have presented above. As noted before, the response in the MPK model does not display the “over-shooting” property and it is rather short-lived. Deviations from steady state return to zero shortly after the impulse. On the contrary, in the model with entry the labor’s share initially falls, rising significantly above its steady state value until its peak five years after the impulse. It also persists at above-steady-state levels for a long period of time.

## 4 Conclusions

We have presented a model in which monopolistic firms face sunk costs of entry and labor markets are characterized by search and matching frictions. Our specification of sunk

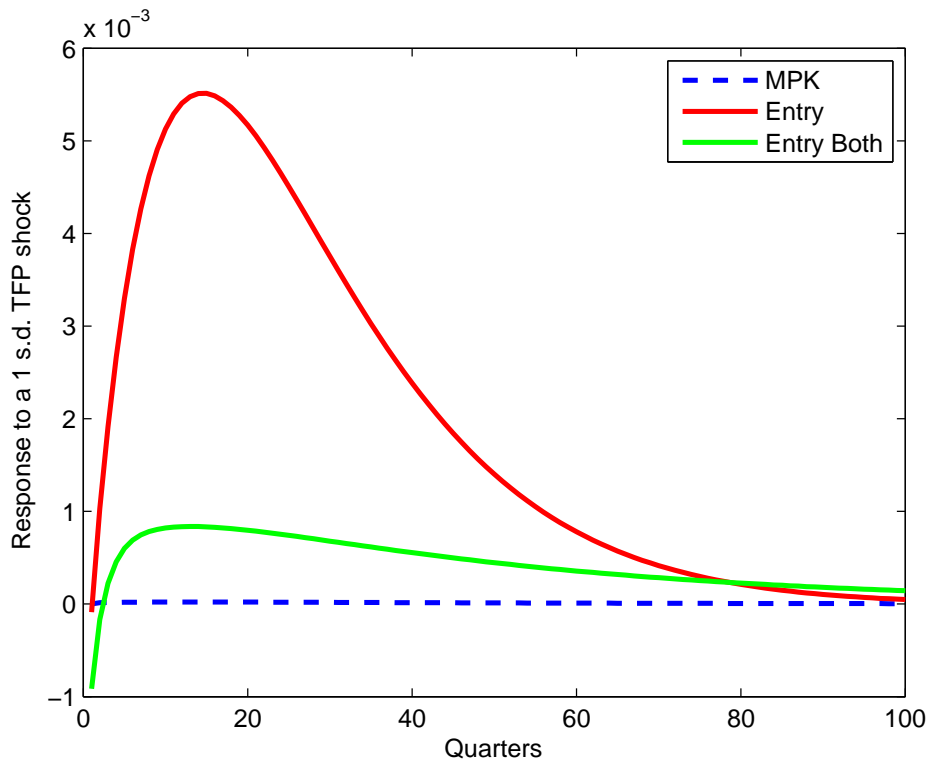


Figure 7: Response of the labor's share to a one standard deviation (orthogonalized) innovation to technology in our three model economies.

costs imply an equilibrium value of vacancy creation that is time-varying. Moreover, it is counter-cyclical amplifying technology shocks and increasing the persistence of labor market variables. The time-varying value of vacancy creation affects the firms' threat point in the wage bargaining process. This has implications for the surplus division between firms and workers over the business cycle. As proxies of how this negotiation takes place, we judge the model on its ability to match the dynamics of two shares: corporate profits to output and workers' compensation to output. We document their empirical regularities: the profits' share leads the cycle, it is more volatile than output, and procyclical. The labor's share inversely leads the cycle, it is smoother, and counter-cyclical. We have shown that quantitatively the model matches the lead-lag pattern and the contemporaneous correlation with output. However, the labor's share is too

volatile, more volatile than output. We contribute to the well-known “Shimer puzzle” by showing that our amplification mechanism is substantial. The volatility of market-tightness relative to output in our model is more than half that observed in the data and the persistence of vacancy creation is about the same.

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## A Appendix: The *MPK* Model

For the sake of exposition we briefly describe the *MPK* model. This model serves as a benchmark framework for many of the results in the text.

The economy is populated by a large household of measure one. Members of the household can be either employed or unemployed. Denote the fraction of those employed at time  $t$  by  $N_t$ . The household's preferences are given by,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_t)^{1-\sigma}}{1-\sigma} \right], \quad (26)$$

A general good  $Y$  is produced using capital  $K$  and labor  $N$  by a single firm employing the following technology,

$$Y = ZK^\alpha N^{1-\alpha} \quad (27)$$

$Z$  represents technology (TFP) that evolves according to:

$$\log(Z_t) = \rho \log(Z_{t-1}) + \epsilon_t \quad (28)$$

The innovation  $\epsilon_t$  is *i.i.d.* The labor market is characterized by search and matching frictions. Unemployed workers search and the firms post vacancies. It costs  $\omega$  to post one vacancy and workers and firms match according to the following matching technology,

$$M(1-N, V) = \frac{(1-N)V}{((1-N)^\xi + V^\xi)^{\frac{1}{\xi}}} \quad (29)$$

The household owns shares in the firm obtaining profits equal to  $\pi_t$ . As a result, the firm discounts the future (between any period  $t$  and  $t+1$ ) using the intertemporal marginal rate of substitution of the household:  $\beta(\frac{C_{t+1}}{C_t})^{-\sigma}$ . The equilibrium value of a vacancy is zero, as firms will enter until there are no gains to made by posting them. Denote by  $q$  the probability that a firm fills a vacancy, by  $s$  the rate at which existing matches between

workers and firms separate, and by  $J$  the capital value of a filled job. In equilibrium,

$$\omega = q_t \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} J_{t+1} \quad (30)$$

and

$$J_t = \pi_t + \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} J_{t+1} (1 - s) \quad (31)$$

We assume wages are negotiated through Nash bargaining, in which firms have a bargaining weight equal to  $\phi$ . Wages are the solution to the following surplus splitting rule,

$$\frac{J_t}{1 - \phi} = \frac{C_t^\sigma \frac{\partial W_t}{\partial N_t}}{\phi}, \quad (32)$$

The interpretation of this expression is analogous to that of the text. As vacancies have zero value in equilibrium the threat point for the firm is zero. For the representative household it is given by the marginal disutility of having one more member unemployed. Finally, the aggregate resource constraint equates total goods produced net of vacancy creation costs to the sum of investment and consumption:

$$Y_t - \omega V_t = C_t + I_t \quad (33)$$

Table 2: Summary Statistics, quarterly U.S. data, 1951:1 to 2003:4

		$C$	$I$	$Y$	$Z$	$V$	$U$	$V/U$
	Std. Dev.	0.015	0.071	0.025	0.015	0.202	0.190	0.382
	Autocorr.	0.945	0.942	0.927	0.893	0.948	0.938	0.946
Corr. Matrix	$C$	1.000	0.454	0.850	0.315	0.607	-0.730	0.684
	$I$		1.000	0.713	0.139	0.781	-0.630	0.726
	$Y$			1.000	0.474	0.846	-0.890	0.889
	$Z$				1.000	0.265	-0.248	0.263
	$V$					1.000	-0.901	0.977
	$U$						1.000	-0.973
	$V/U$							1.000

Table 3: Standard Deviations

	MPK	Entry
$U$	0.875	6.100
$V$	0.875	3.200
$V/U$	1.625	9.600
$C$	0.313	1.600
$I$	4.563	4.300
$Y$	1.000	1.000

Table 4: Cross Correlations: MPK Model

	$C$	$I$	$V/U$	$U$	$V$	$Y$	$Z$
$C$	1.000	0.613	0.765	-0.800	0.497	0.763	0.648
$I$		1.000	0.978	-0.922	0.824	0.979	0.997
$V/U$			1.000	-0.956	0.818	1.000	0.987
$U$				1.000	-0.619	-0.961	-0.929
$V$					1.000	0.809	0.839
$Y$						1.000	0.987
$Z$							1.000

Table 5: Cross Correlations: Baseline Entry Model

	$C$	$I$	$Q$	$V/U$	$U$	$V$	$Y$	$Z$
$C$	1.000	0.510	-0.921	0.686	-0.621	0.842	0.727	0.895
$I$		1.000	-0.162	-0.237	0.326	-0.020	0.955	0.835
$Q$			1.000	-0.872	0.871	-0.969	-0.418	-0.663
$V/U$				1.000	-0.941	0.963	0.017	0.302
$U$					1.000	-0.930	0.075	-0.216
$V$						1.000	0.244	0.516
$Y$							1.000	0.956
$Z$								1.000

Table 6: Autocorrelations

		1	2	3	4
$V/U$	MPK	0.888	0.744	0.616	0.503
	Entry	0.990	0.962	0.920	0.876
$U$	MPK	0.942	0.822	0.693	0.569
	Entry	0.990	0.963	0.922	0.869
$V$	MPK	0.718	0.493	0.373	0.291
	Entry	0.984	0.948	0.900	0.843
$C$	MPK	0.961	0.900	0.836	0.767
	Entry	0.932	0.836	0.742	0.652
$I$	MPK	0.887	0.733	0.595	0.472
	Entry	0.836	0.642	0.478	0.341
$Y$	MPK	0.893	0.749	0.620	0.504
	Entry	0.864	0.691	0.541	0.411
$Z$		0.879	0.726	0.583	0.458

Table 7: Standard Deviations

	MPK	Entry Both
$U$	0.875	1.250
$V$	0.875	0.688
$V/U$	1.625	1.938
$C$	0.313	0.500
$I$	4.563	3.063
$Y$	1.000	1.000

Table 8: Autocorrelations

		1	2	3	4
	MPK	0.888	0.744	0.616	0.503
$V/U$	Entry Both	0.969	0.900	0.814	0.720
	MPK	0.942	0.822	0.693	0.569
$U$	Entry Both	0.974	0.909	0.824	0.732
	MPK	0.718	0.493	0.373	0.291
$V$	Entry Both	0.927	0.816	0.712	0.616
	MPK	0.961	0.900	0.836	0.767
$C$	Entry Both	0.951	0.900	0.836	0.767
	MPK	0.887	0.733	0.595	0.472
$I$	Entry Both	0.902	0.765	0.634	0.511
	MPK	0.893	0.749	0.620	0.504
$Y$	Entry Both	0.894	0.755	0.629	0.514
$Z$		0.879	0.726	0.583	0.458

Table 9: Cross Correlations: Capital in Both Sectors

	$C$	$I$	$Q$	$\theta$	$U$	$V$	$Y$	$Z$
$C$	1.000	0.734	-0.895	0.963	-0.926	0.966	0.835	0.722
$I$		1.000	-0.377	0.702	-0.592	0.883	0.985	0.996
$Q$			1.000	-0.905	0.940	-0.758	-0.513	-0.350
$V/U$				1.000	-0.979	0.929	0.787	0.669
$U$					1.000	-0.859	-0.687	-0.554
$V$						1.000	0.947	0.874
$Y$							1.000	0.983
$Z$								1.000

Table 10: Profits ( $psh_t$ ) and Labor ( $lsh_t$ ) Shares over the Cycle: ( $Y_t$ =Output,  $x_t$ =Share)

	$\sigma(x_t)/\sigma(Y_t)$	$C(x_{t-2}, Y_t)$	$C(x_{t-1}, Y_t)$	$C(x_t, Y_t)$	$C(x_{t+1}, Y_t)$	$C(x_{t+2}, Y_t)$
<b><i>U.S. Data</i></b>						
$x_t = psh_t$	4.065	0.611	0.588	0.505	0.297	0.082
$x_t = lsh_t$	0.372	-0.242	-0.209	-0.14	0.019	0.159
<b><i>MPK</i></b>						
$x_t = psh_t$	0.862	0.760	0.880	0.940	0.750	0.560
$x_t = lsh_t$	0.038	-0.770	-0.880	-0.950	-0.780	-0.590
<b><i>Entry</i></b>						
$x_t = psh_t$	6.241	0.440	0.390	0.310	0.080	-0.030
$x_t = lsh_t$	3.588	-0.280	-0.180	-0.070	0.140	0.200
<b><i>Entry Both</i></b>						
$x_t = psh_t$	1.181	0.290	0.265	0.170	0.143	-0.360
$x_t = lsh_t$	0.476	-0.04	0.03	0.160	0.440	0.620