Dhawan, Rajeev; Jeske, Karsten

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Energy Price Shocks and the Macroeconomy: The Role of Consumer Durables

Rajeev Dhawan and Karsten Jeske

Working Paper 2006-9
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Abstract: So far, the literature on dynamic stochastic general equilibrium models with energy price shocks uses energy on the production side only. In these models, energy shocks are responsible for only a negligible share of output fluctuations. We study the robustness of this finding by explicitly modeling private consumption of energy at the household level in addition to energy use at the firm level to account for total energy use in the economy. Additionally, we distinguish between investment in consumer durables and investment in capital goods. The model economy is calibrated to match total energy use and durable goods consumption as observed in the U.S. data. Simulation results indicate that, despite higher total energy use, this economy has an even smaller proportion of output fluctuations attributable to energy price shocks. Productivity shocks continue to be the primary force behind business cycle fluctuations. The driving force behind our results is that the household now has the flexibility to rebalance its investment portfolio. Specifically, the energy price hike is absorbed by reducing durable goods investment more than investment in capital goods, thereby cushioning the hit to future production at the expense of current consumption. Hence, our model better matches the consumption volatility observed in the data.

JEL classification: E32, Q43

Key words: energy prices, business cycles, durable goods
1 Introduction

As Hamilton and Herrera (2004) and Hamilton (2005) point out, nine out of ten of the U.S. recessions since World War II and every recession since 1973 were preceded by a spike in oil prices. However, when one calculates the dollar share of energy expenditure in the economy\(^1\) and uses the elasticity of output with respect to a given change in energy use, it can only explain a small fraction of the drop in GDP during a typical recession (see Hamilton (2005)). This is also evident in the Dynamic Stochastic General Equilibrium (DSGE) literature that models energy use exclusively on the production side to examine business cycle properties of energy price shocks. For example Kim and Loungani (1992) have shown that energy price fluctuations can only generate a small fraction of the output fluctuations observed in the U.S. data.\(^2\) One conclusion from their research is that output is mainly driven by shocks to total factor productivity (TFP), and - going one step further - all previous recessions would have occurred even without energy price shocks.

Hamilton (2005) conjectures that the key mechanism whereby oil shocks can significantly affect the economy is by disrupting spending by firms and consumers on goods other than energy. Lee and Ni (2002) found that oil price shocks tend to reduce supply in oil-intensive industries but reduce demand in durable goods industries such as autos. Thus, transportation services and energy use are strong complements in the real world. Hence, we construct a DSGE model that explicitly models private consumption of energy, durable goods and non-durable goods (ex energy) at the household level in addition to energy use on the production side.

The paper has two main findings. First, introducing durable goods and household energy consumption actually decreases the relevance of energy price shocks for output volatility, despite increasing total energy consumption in the economy. This is because households now have two margins of adjustment for their investment decision (durable or fixed investment) in response to

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\(^1\)According to the Bureau of Economic Analysis and the Energy Information Administration, between 1970 and 2005, residential energy consumption was on average 4.8 percent of GDP, commercial and industrial 4.0 percent.

\(^2\)Rotemberg and Woodford (1999) study output impulse response functions and show that under imperfect competition the effect of an oil price shock is stronger than under perfect competition. Finn (2000) shows that one can increase the response to an oil price shock even under perfect competition when one models energy use as a function of capacity utilization. However, both papers are silent on the business cycle properties of the model in response to energy shocks. Specifically, they do not report the share of output fluctuations explained by energy price shocks and the other business cycle facts such as volatility of investment, consumption and comovement of these variables.
an exogenous shocks. This additional degree of freedom to rebalance their portfolio is missing in a typical DSGE model with or without energy use when responding to shock (TFP or oil).

In our economy we show that an energy price increase has a larger negative effect on durables than on fixed capital. Even though both capital stocks decrease in response to higher energy prices, the fixed capital drops by less than the stock of durables after households rebalance their portfolio. Most importantly, fixed capital drops less than in a Kim and Loungani type economy which explains why energy accounts for less output fluctuations in our model. Finally, TFP shocks alone account for the majority of output volatility while energy by itself plays almost no role in our model.

Furthermore, in a basic DSGE model without energy use and a single consumption good, volatility of consumption is far lower than the one observed in the data (see Cooley and Prescott (1995)). Our second main result is that introducing durable goods and energy price shocks together raises consumption volatility to a value close to the observed one. Introducing only durable goods but switching off energy price shocks does not produce the desired result. This is again due to the rebalancing effect as the household reduces the hit to future production by reducing spending on durable goods.

Our paper proceeds as follows: Section 2 introduces our model with durable goods. Section 3 explains the parametrization, Section 4 details the solution algorithm we use. In Sections 5 and 6 we go through the numerical results and Section 7 concludes.

2 Model

The representative household gets utility from consuming three types of consumption goods: consumption of nondurables and services excluding energy ($N$), the flow of services from the stock of durables goods ($D$) and energy use ($E_h$). The household uses the following aggregator function to combine these three types of consumption into $C^A$:

$$C^A_t = N^\gamma_t \left( \theta D^\rho_{t-1} + (1 - \theta) E^\rho_{h,t} \right)^{1 - \gamma}$$

where $\theta \in (0, 1)$ and $\rho \leq 1$. With this aggregation function the elasticity of substitution between energy and durable goods is $\frac{1}{1 - \rho}$. We will choose $\rho < 0$, which implies that the durable goods and
energy are complements. This is similar to the aggregator function used by Fernandez-Villaverde and Krueger (2001) and Jeske and Krueger (2005) who use a Cobb-Douglas aggregator between non-durable and durable consumption. We have extended it to include the third type of consumption good, which is energy. The elasticity of substitution between non-durable consumption and the composite of durables and energy goods is one in our model. This feature is motivated by the Ogaki and Reinhart (1998) who found that in the U.S. data the elasticity of substitution between durables and nondurable goods was close to one.\footnote{Similarly, Rupert et. al. (1995) found that the elasticity of substitution between market goods and home production was not significantly different from one.} Notice that the stock of durables from last period enters today’s utility function. That way the timing of durable goods investment is analogous to fixed investment where yesterday’s capital stock $K_{t-1}$ enters today’s production function.

We write the period $t$ utility function as following:

$$u(C_t^A, H_t) = \varphi \log C_t^A + (1-\varphi) \log (1 - H_t)$$

where $\varphi \in (0,1)$ and $H$ denotes hours worked. This log-utility specification is the same as in Kim and Loungani (1992) and Leduc and Sill (2004).

The timing convention is as follows: Households set durable goods stock $D_{t-1}$ in period $t-1$ and this stock will produce the flow of durable good services in period $t$. In other words, the durable goods stock $D_{t-1}$ is a state variable at time $t$. Durable goods depreciate at rate $\delta_d$ per period. Moreover, there are convex adjustment costs for adjusting the stock of durable goods. Thus the durable goods investment $I_{D,t}$ necessary to alter the durable goods stock from $D_{t-1}$ to $D_t$ is:

$$I_{D,t} = D_t - (1 - \delta_d) D_{t-1} + \frac{\omega_{1d}}{1 + \omega_{2d}} \left( \frac{D_t - D_{t-1}}{D_{t-1}} \right)^{1+\omega_{2d}}$$

where $\omega_{1d} \geq 0, \omega_{2d} > 0$. Notice that in steady state adjustment costs will be zero.

Additionally, notice that the variable $C_t^A$ in the utility function does not correspond to consumption observed in the National Income and Product Accounts (NIPA) data. Total real consumption based on NIPA definition is defined as $C_t = I_{D,t} + N_t + E_{h,t}$. This distinction is relevant when we simulate the economy. When we compute second moments and plot impulse responses for consumption we are always referring to this NIPA based $C_t$ of consumption rather
than the aggregator based $C_t^A$.

Following Kim and Loungani (1992), firms produce output by combining three inputs: Labor $H$, capital $K$ and energy $E_f$ according to the following production function:

$$Y_t = Z_{y,t} \left( \eta K_{t-1}^{\psi} + (1 - \eta) E_{f,t}^{\psi} \right)^{\frac{1}{\psi}} H_t^{1-\alpha} \tag{2}$$

where the term $Z_y$ is a TFP shock that follows a stochastic process and $\psi \leq 1$.

Just as for durable goods, there is an adjustment cost for altering the capital stock from $K_{t-1}$ to $K_t$, which implies that capital investment $I_{K,t}$ is

$$I_{K,t} = K_t - (1 - \delta_k) K_{t-1} + \frac{\omega_{1k}}{1 + \omega_{2k}} \left( \frac{K_t - K_{t-1}}{K_{t-1}} \right)^{1+\omega_{2k}} \tag{3}$$

where $\omega_{1k} \geq 0, \omega_{2k} > 0$.

We assume that all of the energy inputs need to be imported as in Kim and Loungani (1992) and Leduc and Sill (2004). The social planner’s problem is then:

$$\max E \sum_{t=0}^{\infty} \beta^t u \left( N_t^\gamma \left( \theta D_{t-1}^\rho + (1 - \theta) E_{h,t}^\rho \right)^{\frac{1-\gamma}{\rho}}, H_t \right)$$

subject to:

$$N_t + I_{D,t} + I_{K,t} + P_t (E_{h,t} + E_{f,t}) = Y_t$$

and equations (1), (2) and (3). \footnote{An alternative way would have been to set up a two sector model as in Baxter (1996) where one sector produces nondurable goods and the other durable goods.} We derive first order conditions in appendix A.

### 3 Calibration

#### 3.1 Preference and technology parameters

One model period corresponds to one quarter in the data. We set $\alpha = 0.36$ and the time preference factor $\beta = 0.99$. These two parameters will remain unchanged for all the model specifications we consider in this paper.

Notice that one cannot calibrate both the elasticity of substitution and the share parameter.
in a CES type production or utility function at the same time by just matching steady state values. Take the example of the CES utility function. In Appendix B we derive equation (31) showing that the steady state ratio of household energy use and durable goods stock is 

\[ \frac{E_h}{D} = \frac{[(1 - \theta)(1 - \beta + \beta \delta_d)]}{(\beta \theta')} \] 

Thus, we cannot calibrate both the share parameter \( \theta \) and CES parameter \( \rho \) at the same time from just the \( E_h/D \) ratio. An analogous result holds for the CES parameters on the production side (\( \psi \) and \( \eta \)). Instead we consider varying degrees of substitutability \( \rho \) and \( \psi \) and match the share parameters \( \theta \) and \( \eta \) in order to generate steady state values observed in the data. As a first guess we use \( \rho = \psi = -0.7 \). This is the same value Kim and Loungani (1992) use in their production function.5

In the economy with durable goods we pick the remaining parameters in order to match moments from the data to steady state values in the model. Using NIPA data from 1970:Q1 to 2005:Q4 we construct series for energy use on the consumption side6 which corresponds to \( E_h \) in the model, consumption of nondurables and services excluding energy (\( N \)) and consumption of durables (\( I_D \)). We will use the ratios \( E_h/N = 0.0780 \) and \( I_D/N = 0.1585 \) based on the NIPA data for our calibration.

We also target the stock of durables to output ratio \( D/Y = 1.23 \), which, according to the Flow of Funds Statistics, is the average ratio of durable goods wealth to GDP between 1970 and 2005, a capital output ratio \( K/Y = 12 \) as is standard in the literature and hours worked \( H = 0.3 \).

We find that the household energy use based on NIPA data is 4.8 percent of GDP and the total energy consumption equals 8.8 percent of GDP between 1970 and 2005.7 Thus the firm energy use is 4 percent of GDP which in conjunction with \( K/Y = 12 \) implies that \( K/E_f = 300.8 \)

We use the above defined six moments \( (E_h/N, I_D/N, D/Y, K/Y, H, K/E_f) \) to calibrate the six parameters \( \gamma, \theta, \eta, \varphi, \delta_d, \delta_k \). Appendix C details how the first order conditions in steady

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5Kim and Loungani also report results for unit-elasticity but as we will see later, even for \( \rho = \psi = -0.7 \) we generate too much volatility in both \( I_D \) and \( I_K \). Going towards more substitutability would increase the volatilities even more.

6We combine Gasoline, Fuel Oil and Other Energy Goods (part of nondurable consumption) and Electricity and Gas (part of PCE Services).

7From Table 1.5 in the Annual Energy Review 2005, Energy Information Administration, we have annual data on total energy use from 1970 to 2001. We extrapolate total energy consumption for the years 2002 to 2005 by assuming the same growth rates in total energy consumption as in household energy consumption based on NIPA data. Without this extrapolation, that is, using only data until 2001, the share of energy is 9.0 percent.

8This is different from the Kim and Loungani (1992) value of 200 which is based on data from 1947 to 1987, while we calibrate our economy to data from 1970 to 2005.
Table 1: Model Parameters

<table>
<thead>
<tr>
<th>parameters</th>
<th>Model E-I</th>
<th>Model E-II</th>
<th>Model ED-I</th>
<th>Model ED-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>-0.7000</td>
<td>-3.0000</td>
<td>-0.7000</td>
<td>-0.7000</td>
</tr>
<tr>
<td>$\psi$</td>
<td>-0.7000</td>
<td>-0.0001</td>
<td>-0.7000</td>
<td>-0.7000</td>
</tr>
<tr>
<td>$\sigma^2_z$</td>
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<td>0.0070</td>
<td>0.0070</td>
<td>0.0082</td>
</tr>
<tr>
<td>$\omega_{1d}$</td>
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<td>0.0000</td>
<td>27.9000</td>
<td>27.9000</td>
</tr>
<tr>
<td>$\omega_{1k}$</td>
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<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\gamma$</td>
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<td>0.7957</td>
<td>0.7957</td>
<td>0.7957</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.9569</td>
<td>0.9999</td>
<td>0.9977</td>
<td>0.9977</td>
</tr>
<tr>
<td>$\eta$</td>
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<td>0.8839</td>
<td>0.9977</td>
<td>0.9977</td>
</tr>
<tr>
<td>$\varphi$</td>
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<td>0.3056</td>
<td>0.3413</td>
<td>0.3413</td>
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<tr>
<td>$\delta_d$</td>
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<td>0.0793</td>
<td>0.0166</td>
<td>0.0166</td>
</tr>
<tr>
<td>$\delta_k$</td>
<td>0.0166</td>
<td>0.0198</td>
<td>0.0166</td>
<td>0.0166</td>
</tr>
</tbody>
</table>

state pin down the six parameter values. Also notice that we can perform this calibration independent of the adjustment cost parameters, since adjustment costs $\omega_{1d} = \omega_{1k} + (D_t - D_{t-1})^{1+\omega_{2d}}$ and $\omega_{1k} = (K_t - K_{t-1})^{1+\omega_{2k}}$ are zero in the steady state. The parameters that reproduce the data moments above together with $\rho = \psi = -0.7$ and zero adjustment costs form the Model ED-I as summarized in Table 1.

In the economy without durable goods we proceed in a similar fashion. Again, ratios $K/E_f$ and $K/Y$ pin down the two parameters $\eta$ and $\delta_k$ on the production side, while the value for $H$ determines the value for $\varphi$ (see Appendix C for the details). We calibrate this economy for two alternative CES parameter values, $\psi = -0.7000$ as above and $\psi = -0.0001$ and call the two specifications Model E-I and Model E-II.9

We take the same $K/E_f$ ratio as in our economy with durable goods, that is, we set $K/E_f = 300$. This puts the energy use on the firm side at 4 percent of GDP which is equal to the figure we calibrated from the NIPA and EIA data.10

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9 This is similar to the Kim and Loungani (1992) model economy but for quarterly rather than annual data.
10 We could have used two alternative calibrations. First, we could have used the same capital to energy ratio that Kim and Loungani used. Their $K/E_f$ of 50 which is based on annual energy consumption would have translated into $K/E_f = 200$ using quarterly data. This implies a steady state value for the energy to output ratio of 6 percent on the firm side, which is well above the value we observe. Going one step further, one can put the entire energy consumption of 8.8 percent of GDP that we generated in our economy with durable goods onto the production side in the Kim and Loungani type economy to better compare the outcomes of the economies with and without durable goods. This requires a capital to energy ratio of $K/E_f = 136$. As Section 5 shows, even with a relatively low energy utilization of 4 percent in the production function, we generate excess volatility in investment. Thus, for these two alternative calibrations the investment volatilities turned out to be even higher.
3.2 Calibration of shocks

Just as Cooley and Prescott (1995), we assume that log-TFP follows an AR(1) process:

\[ z_{y,t} = 0.95z_{y,t-1} + \varepsilon_{z,t} \]

where

\[ \varepsilon_{z,t} \sim iid \sim N(0, \sigma_z^2) \]

with \( \sigma_z = 0.007 \). Furthermore we estimate an energy price ARMA(1,1) process.\(^{11}\) Energy prices refer to the price index of energy (Table 1.5.4 in the BEA, series ‘gasoline, fuel oil, and other energy goods,’ and ‘electricity and gas’) adjusted by the GDP deflator. We use quarterly log energy prices from 1970Q1-2005Q4 to estimate

\[ p_t = \rho_p p_{t-1} + \varepsilon_{p,t} + \rho_\varepsilon \varepsilon_{p,t-1} \]

via Maximum Likelihood\(^{12}\). This procedure yields:

\[ \rho_p = 0.9753 \]
\[ \rho_\varepsilon = 0.4217 \]
\[ \varepsilon_{p,t} \sim iid \sim N(0, (0.0308)^2) \]

3.3 Adjustment costs

In the models without durable goods (E-I and E-II) we abstract from adjustment cost. We also set adjustment costs to zero in the benchmark model ED-I with durable goods. Later we assume that the cost functions are quadratic (\( \omega_{2d} = \omega_{2k} = 1 \)), as in Bruno and Portier (1995), and adjust the proportional part of adjustment costs \( \omega_{1d} \) and \( \omega_{1k} \) in order to match volatilities of durables and capital goods investments in the model to the data. We call this model ED-II. The details are in the Section 5.

\(^{11}\) Notice that Rotemberg and Woodford (1996) use a different process for energy price shocks. They estimate a VAR with two variables, nominal oil price changes and real oil prices and study the effect of exogenous shocks to nominal price changes. As a robustness check we reestimated their VAR and incorporated it in our model. We found that using their shock specification does not change our results.

\(^{12}\) We use the Kalman Filter to write down the likelihood function as described in Hamilton (1994).
4 Solution Algorithm

We use the methodology put forward by Collard and Juillard (2001). From the first order in Appendix A conditions, we derive eleven conditions guiding the dynamic behavior of eleven variables \( N, D, E_h, H, W, E_f, K, R, Y, I_d, I_k \) plus two equations for the shocks.

We then run the program Dynare Version 3.0 to generate a second order approximation for the policy function (see Collard and Juillard (2001) for the methodological details). To generate second order moments for each of the specifications we consider we simulate 1000 economies each 144 quarters long, which is the same length as the data series from 1970:Q1 to 2005:Q4.

5 Numerical Results

Table 2 details the standard deviations of HP-filtered series for both the data and the model simulations. The first set of numbers are for simulations when both the TFP and energy shocks are present. The next panel is for only the energy shocks and the last panel for only the TFP shock.

Looking at the columns for model E-I and E-II (simple DSGE model without durable goods), in the version with both shocks we generate output volatility close to that in the data, though consumption volatility is far below the data value, whereas the investment volatility is slightly above its empirical target. Model simulations with only energy price shocks can account for only about 15 percent of output fluctuations in E-I and 21 percent in E-II specifications. In each case more than 90 percent of output fluctuations are generated by TFP shocks alone. We thus replicate the main result from Kim and Loungani (1992), that energy price shocks do not play a major role in accounting for output fluctuations. Total factor productivity is still the driving force. Moreover, consumption volatility is well below its empirical target. The model accounts for only 31 percent of the target standard deviation of consumption and thus less than 10 percent of its variance. As previous research has pointed out,\(^{13}\) in this simple RBC type model, households are doing to good a job in smoothing consumption.

In the economy with durable goods we first report the results without adjustment costs (ED-I) in Table 2. With both shocks present, consumption volatility is almost equal to the data value.

\(^{13}\)See for example Cooley and Prescott (1995).
and thus much higher than in the economy without durable goods (E-I and E-II). The model
ED-I also generates volatility for output very close the one observed in the data. Furthermore,
the household energy use is much more volatile in the model than in the data. It appears that
our initial guess for the elasticity of substitution between durables and energy could be too high.
Moreover, the model generates excess volatility for both durable goods and fixed investment.
Notice that this happens despite the fact that the volatility of the sum of the two is below
its target. To explain this artifact, let's examine the impulse response function of investment
variables to an energy price shock displayed in Figure 1.

The top left panel displays a one time, one standard deviation positive shock to $\varepsilon_{p,t}$, i.e., an
increase in energy prices. Notice that $P_t$ increases for two periods which is due to the ARMA(1,1)
structure of the energy price process. The sum of investment in durables and fixed capital
($I_D + I_K$) in the top left panel reacts as expected, i.e., it falls for two periods mirroring the rise
in energy prices followed by a reversion back to the steady state after period 2, which is the
expected response of investment to an energy price shock.

Apart from the direct effect that energy prices have on investment, in the first period after the
shock there must be an additional effect because investment in durables ($I_D$) drops dramatically
whereas investment in fixed capital ($I_K$) rises for one period before it falls to values below steady
state fixed investment. A look at the first order conditions explains why this happens. In the
absence of adjustment costs, equations (5) and (7) in Appendix A yield

$$
E^{t+1} \Lambda_{t+1} [R_{t+1} + 1 - \delta_k] = E^{t+1} u_1 (C_{t+1}, H_{t+1}) N_{t+1}^{\gamma} \\
\times (\theta D_{t}^{\rho} + (1 - \theta) E_{h,t+1}^{\rho}) \frac{1 - \gamma}{1 - \gamma} \left(1 - \gamma\right) \theta D_{t}^{\rho-1} \\
+ E^{t+1} \Lambda_{t+1} [1 - \delta_d] (4)
$$

that is, in terms of time $t+1$ utility, the return of fixed capital must equal that of durable goods.
The energy share in the CES part of the utility function is $1 - \theta = 0.0431$. This value is much
higher than the energy share in the CES part of the production function $1 - \eta = 0.0023$. Thus,
the percentage drop in $R_{t+1}$ due to higher energy prices and lower firm energy use is smaller than
the drop in marginal utility from durables.\footnote{This assumes that the percentage drop in $E_{f,t}$ is roughly equal to that in $E_{h,t}$, which is confirmed by the impulse responses to the energy price shock in Figure 3.} In order to equalize the difference in returns, the
household rebalances its portfolio. It increases the fixed capital stock $K$ and further decreases the durables stock $D$. This leads to the large drop in durables investment and a one period increase in fixed capital investment that’s large enough to offset the negative effect from higher energy prices on investment. In subsequent periods, both investment series are below their steady state values, i.e., the line for $I_K$ falls below zero, too. Since $K$ is high enough and $D$ is low enough to align the returns of durables and fixed capital, the rebalancing in subsequent periods is small enough not to reverse the sign of the investment deviations from steady state, i.e., we observe the direct negative effect of an energy price hike in both investment series.

In the case of a shock to productivity both investment series move in the same direction (see Figure 2). Both investment types go up in response to a positive productivity shock. The response in durables investment is muted in the first period, which is due to the fact that productivity has a direct effect only on the production function and not the utility function. Thus, in order to equalize the two sides in equation (4) the jump in fixed capital investment is larger than in durables investment. The impulse responses for the other important model variables are in Figure 3. The plots are again for a positive productivity shock and an energy shock that increases the energy price.

Even in this basic durable goods model ED-I with excess volatility in investment, the proportion of output volatility explained with pure energy shocks is only about 14 percent. Despite the explicit modeling of durable goods, energy prices are not accounting for a sizeable share of output fluctuations. This is an astonishing result, because the total energy use in the ED-I economy is more than twice as high as in the economy without durable goods (both E-I and E-II), yet the relevance of energy price shocks for output volatility has diminished.\footnote{We performed sensitivity analysis by decreasing the degree of substitutability in the production function by setting $\psi = -2.0$. We found that lowering this parameter led to an even lower level of output fluctuation (8 percent) attributable to energy price shocks.}

As pointed out above, Model ED-I is off in three important dimensions. It has excess volatility in durables and fixed investment as well as household energy use. Consequently, we make the parameters in the adjustment cost functions $\omega_{1d}$ and $\omega_{1k}$ non-zero to reduce volatility in investment. Moreover, we make durable goods and household energy use less substitutable (reduce $\rho$), which will curb the volatility of $E_h$. Also, we increase the standard deviation $\sigma_z$ of innovations to productivity in order to match the empirical target for output volatility. Our aim is to exactly
match the output and two investment volatilities in the data. Specifically, we pick $\omega_{1d} = 0.66$, $\omega_{1k} = 27.9$, $\sigma_z = 0.0082$ with $\rho = -3.0$. We call this new parametrization Model ED-II (see Table 2 for the volatilities).

The reduced investment volatilities are consistent with the impulse response functions in Figure 4 where we see that the adjustment costs indeed muted the investment response to the energy shock. A one standard deviation shock to productivity has a smaller effect on durables investment than a one standard deviation shock to energy prices. The initial drop in $I_D$ in response to a shock to energy price $P$ is about three times larger than the increase in $I_D$ in response to a shock to productivity. For fixed investment $I_K$ it is the reverse: a shock to productivity generates an increase in fixed capital investment about six times larger than the drop in response to an energy price hike. The same mechanism that drove the investment variables impulse response functions in Model ED-I works here, too, though it is muted by the adjustment costs. Energy shocks still have a larger effect on durables investment, since household energy consumption has a larger share in the utility function than firm energy use has in the production function. Likewise, a productivity shock has a direct effect only on the production function, which creates a large response in the fixed capital investment series. The return to durables is only indirectly affected, thus the response in durables investment after a productivity shock is smaller than that of fixed capital investment.

In Table 2, consumption volatility is close to the data in the new Model ED-II but most importantly, all three subcomponents of consumption match their data volatility numbers reasonably well. We achieve this by breaking the link between the consumption aggregator $C^A$, which is the series that consumers want to smooth, and measured consumption $C^{NIPA}$. Since the service coming out of the stock of durables $D_{t-1}$ enters the consumption aggregator, households can smooth $C^A$ despite large fluctuations in measured consumption $C^{NIPA}$ coming from fluctuations in durables investment. Nadenichek (1999) applies this trick in a different context. He shows that in an international business cycle model durable goods generate less comovement in measured consumption, again because of the volatility of durable goods investment. Our result, though, is special in the sense that model ED-II with only a TFP shock still generates only about 50 percent of the desired consumption fluctuation. Only the inclusion of energy price shocks drives the volatility of consumption to 70 percent of its empirical target. Durable goods
alone don’t generate consumption volatility, mostly because the return of durables is not directly affected by TFP. Mostly energy price shocks drive the fluctuations in durables investment.

6 Stochastic Properties of Shocks

We also use the model to back out the implied TFP shocks from the data. To this end, we use two data series, output $Y_t$ and energy prices $P_t$, and use the Kalman Filter to generate shocks $\varepsilon_{z,t}$ and $\varepsilon_{p,t}$ as well as all the remaining variables of the model.

Figure 6 reports the series for output, TFP and energy prices. By construction the output and energy price series are identical in the model and data (except for scaling) because those were the two series used in the Kalman Filter. According to the model, output and TFP are very strongly correlated (with a correlation of 0.935). Each recession since 1973 is accompanied by a sharp drop in TFP. At the same time the most recent increase oil prices since 2002 has not caused a recession thanks to strong productivity growth. The effect of high energy prices in the most recent episode is that they curb output growth, rather than cause a recession. Specifically, the peak in productivity in the year 2005 is only about 0.7 percentage points below the one in 2000 but because of high energy prices, recent output is barely above the trend compared to being 4 percent above the trend in 2000.

If our main conclusion is that TFP shocks rather than energy shocks drive output fluctuations, an objection would be that parts of TFP are affected by energy prices. For example, one could come up with a model where energy price hikes make all factors less productive. Put differently, while in Figure 6 it looks like TFP rather than energy prices are responsible for recessions, TFP itself was driven down by energy prices in each recession. Thus, the argument that the recessions would have happened even without energy price hikes is no longer valid.

For this reason, we study some of the statistical properties of the two shocks generated by the Kalman Filter. If the hypothesis of energy price hikes lowering TFP was correct, the Kalman Filter generates a negative correlation between the two shocks, i.e., a price hike has a negative effect on productivity. Figure 7 plots cross-correlations between the two shocks. Energy price shocks and TFP shocks are positively correlated with a contemporaneous correlation of about 0.27. This implies that the above mentioned hypothesis is not only wrong, the story actually
goes in the reverse direction: an energy price hike is associated with an increase in total factor productivity.\textsuperscript{16}

Figure 8 plots cross-correlations of TFP and Energy prices with each other. The contemporaneous correlation of $Z_{y,t}$ and $P_t$ is essentially zero, again refuting the hypothesis that energy price shocks drive TFP. According to the lower panel, an energy price spike is associated with a trough in TFP four to five quarters down the road, but at the same time, according to the top panel, a peak in TFP is also associated with a spike in energy prices six quarters down the road.

7 Conclusion

The main conclusion from our work is that energy price shocks are not a major factor for business cycle fluctuations even when incorporating three distinct categories of consumption: durables, nondurable goods and energy consumption. With explicit modeling of durable goods we give the household an additional margin of adjustment in its aggregate investment decision. Thus, in response to an exogenous shock the household not only decides how much to invest in total but also rebalances its portfolio mixture of durable goods and fixed capital. Energy shocks indeed cause a disruption in durable goods investment but at the same time the disruption in fixed capital investment is smaller than in a Kim and Loungani (1992) type of economy with only one type of investment. Therefore, the household can cushion the drop in output by adjusting on the durable goods margin instead of fixed capital. Consequently, TFP remains the driving force behind output fluctuations.

Modeling durable goods with energy price shocks significantly increases the consumption volatility in our model to about 70 percent of the desired level. This is an improvement over a simple DSGE type model without durable goods but energy price shocks that only reproduces 30 percent of the consumption volatility in the data. Again the rebalancing effect is the key to generating this result.

For future research it will be interesting to see how this rebalancing effect works in the presence of money and explicit monetary policy rules. The objective will be to find the optimal

\textsuperscript{16}Notice also that there appears to be some serial correlation in TFP shocks according to the top panel. It’s possible that we have to model TFP as an ARMA(1,1) process the way we did it for energy price shocks. This, together with the fact that output is much more persistent in the data than the model (see Figure 5) is a reason to revisit the parameter estimates in the TFP process in future research.
monetary policy following an oil shock given the state of the real economy. Another avenue of future research would be to improve the importance of energy price shocks by introducing multiple sectors of production with frictions in the movement of labor between sectors as in Hamilton (1988).

References


Table 2: Volatilities in the data versus model

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Data based on quarterly NIPA data from the BEA from 1970:Q1 to 2005:Q4. Notice that there are no quarterly data on firm energy use. Simulation results are averages over 1000 simulations each with length 144 quarters.
Table 3: Correlations with output in the data versus model

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Data based on quarterly NIPA data from the BEA from 1970:Q1 to 2005:Q4. Notice that there are no quarterly NIPA data on firm energy use. Simulation results are averages over 1000 simulations each with length 144 quarters.
Figure 1: Investment variables: Impulse Response Functions to an energy price shock in Model ED-I
Figure 2: Investment variables: Impulse Response Functions to a TFP shock in Model ED-I
Figure 3: Impulse Response Functions in Model ED-I
Figure 4: Impulse Response Functions in Model ED-II
Figure 5: Cross Correlations with Output: Model ED-II (solid line) vs. Data (dotted).
Figure 6: Detrended output compared to Productivity and Energy Prices.
Figure 7: Model: Cross-correlations of shocks.
Figure 8: Model: Cross-correlations of TFP and Energy Prices.
Appendix

A First order conditions:

- Nondurables:
  \[ \beta' u_1 (C_t^A, H_t) \gamma N_t^{\gamma - 1} \left( \theta D_t^{p-1} + (1 - \theta) E_{h,t}^p \right)^{\frac{1-\gamma}{\rho}} = \beta' \Lambda_t \]
  where
  \[ u_1 (C_t^A, H_t) = \frac{\varphi}{C_t^A} \]
  Thus:
  \[ \Lambda_t = \varphi \gamma N_t^{\gamma - 1} \]

- Durables:
  \[ \beta' \Lambda_t \left[ 1 + \frac{\omega_{1d}}{D_{t-1}} \left( \frac{D_t - D_{t-1}}{D_{t-1}} \right)^{\omega_{2d}} \right] = E \beta^{t+1} u_1 (C_{t+1}^A, H_{t+1}) N_{t+1}^\gamma \]
  \[ \times \left( \theta D_t^p + (1 - \theta) E_{h,t+1}^p \right)^{\frac{1-\gamma}{\rho} - 1} (1 - \gamma) \theta D_t^{p-1} \]
  \[ + E \beta^{t+1} \Lambda_{t+1} \left[ 1 - \delta_d + \omega_{1d} \left( \frac{D_{t+1} - D_t}{D_t} \right)^{\omega_{2d}} \frac{D_{t+1}}{D_t^2} \right] \]
  Thus:
  \[ \Lambda_t \left[ 1 + \frac{\omega_{1d}}{D_{t-1}} \left( \frac{D_t - D_{t-1}}{D_{t-1}} \right)^{\omega_{2d}} \right] = \beta E u_1 (C_{t+1}^A, H_{t+1}) N_{t+1}^\gamma \]
  \[ \times \left( \theta D_t^p + (1 - \theta) E_{h,t+1}^p \right)^{\frac{1-\gamma}{\rho} - 1} (1 - \gamma) \theta D_t^{p-1} \]
  \[ + E \beta \Lambda_{t+1} \left[ 1 - \delta_d + \omega_{1d} \left( \frac{D_{t+1} - D_t}{D_t} \right)^{\omega_{2d}} \frac{D_{t+1}}{D_t^2} \right] \]

Plug in for \( \Lambda \):

\[ 1 + \frac{\omega_{1d}}{D_{t-1}} \left( \frac{D_t - D_{t-1}}{D_{t-1}} \right)^{\omega_{2d}} = \beta E \frac{u_1 (C_{t+1}^A, H_{t+1}) N_{t+1}^\gamma \left( \theta D_t^p + (1 - \theta) E_{h,t+1}^p \right)^{\frac{1-\gamma}{\rho} - 1} (1 - \gamma) \theta D_t^{p-1}}{u_1 (C_t^A, H_t) \gamma N_t^{\gamma - 1} \left( \theta D_t^{p-1} + (1 - \theta) E_{h,t}^p \right)^{\frac{1-\gamma}{\rho}}}
\]
\[ + E \frac{\varphi \gamma N_{t+1}^{-1} \left[ 1 - \delta_d + \omega_{1d} \left( \frac{D_{t+1} - D_t}{D_t} \right)^{\omega_{2d}} \frac{D_{t+1}}{D_t^2} \right]}{\varphi \gamma N_t^{-1}} \]
\[ = \beta E \left\{ \frac{\varphi}{C_{t+1} N_{t+1}^\gamma \left( \theta D_t^p + (1 - \theta) E_{h,t+1}^p \right)^{\frac{1-\gamma}{\rho}} \gamma N_{t+1}^{-1}}{\varphi \gamma N_t^{\gamma - 1}} \right\} \]
\[ \times \left( \theta D_t^p + (1 - \theta) E_{h,t+1}^p \right)^{-1} (1 - \gamma) \theta D_t^{p-1} \]
\[ + \beta E \frac{N_t}{N_{t+1}} \left[ 1 - \delta_d + \omega_{1d} \left( \frac{D_{t+1} - D_t}{D_t} \right)^{\omega_{2d}} \frac{D_{t+1}}{D_t^2} \right] \]
Further simplification yields:

\[ 1 + \frac{\omega_{1d}}{D_{t-1}} \left( \frac{D_t - D_{t-1}}{D_{t-1}} \right)^{\omega_{2d}} = \beta \frac{(1 - \gamma)^\theta}{\gamma} E \left\{ N_t \left( \theta D_{t}^\rho + (1 - \theta) E_{h,t+1}^\rho \right)^{-1} D_{t-1}^{\rho-1} \right\} \]

\[ + \beta \left[ 1 - \delta_d + \omega_{1d} \left( \frac{D_{t+1} - D_t}{D_t} \right)^{\omega_{2d}} \frac{D_{t+1}}{D_t^2} \right] E \frac{N_t}{N_{t+1}} \]

\[ = \beta \frac{(1 - \gamma)^\theta}{\gamma} E \left\{ N_t \left( \theta D_{t} + (1 - \theta) E_{h,t+1}^\rho D_{t}^{1-\rho} \right)^{-1} \right\} \]

\[ + \beta \left[ 1 - \delta_d + \omega_{1d} \left( \frac{D_{t+1} - D_t}{D_t} \right)^{\omega_{2d}} \frac{D_{t+1}}{D_t^2} \right] E \frac{N_t}{N_{t+1}} \]  (6)

- **Energy on consumer side:**

\[ \beta' u_1 \left( C^A_t, H_t \right) N_t^\gamma \left( \theta D_{t-1}^\rho + (1 - \theta) E_{h,t}^\rho \right)^{\frac{\rho-1}{\gamma}} \left( 1 - \gamma \right) (1 - \gamma) E_{h,t}^{\rho-1} = P_t \beta' \Lambda_t \]

- **Hours worked:**

\[ -\beta' u_2 \left( C^A_t, H_t \right) = \beta' \Lambda_t W_t \]

where

\[ u_2 \left( C^A_t, H_t \right) = -\frac{1 - \varphi}{1 - H_t} \]

and

\[ W_t = (1 - \alpha) Z_{y,t} \left( \eta K_{t-1}^\psi + (1 - \eta) E_{f,t}^\psi \right)^{\frac{\psi}{\alpha}} H_t^{-\alpha} \]

Thus:

\[ 1 = \frac{\varphi}{1 - \varphi} \gamma W_t N_t^{-1} (1 - H_t) \]

- **Capital:**

\[ \beta' \Lambda_t \left[ 1 + \frac{\omega_{1d}}{K_{t-1}} \left( \frac{K_t - K_{t-1}}{K_{t-1}} \right)^{\omega_{2d}} \right] = E \beta' \Lambda_{t+1} \left[ R_{t+1} + 1 - \delta_k \right] \]

\[ + \omega_{1k} \left( \frac{K_{t+1} - K_t}{K_t} \right)^{\omega_{2k}} \frac{K_{t+1}}{K_t^{2}} \]  (7)

where the real interest rate is given by:

\[ R_{t+1} = Z_{y,t+1} \left( \eta K_t^\psi + (1 - \eta) E_{f,t+1}^\psi \right)^{\frac{\psi-1}{\alpha}} H_{t+1}^{1-\alpha} \eta K_t^{\psi-1} \]  (8)

- **Firm’s energy use:**

\[ P_t = Z_{y,t} \left( \eta K_{t-1}^\psi + (1 - \eta) E_{f,t}^\psi \right)^{\frac{\psi-1}{\alpha}} H_t^{1-\alpha} (1 - \eta) \alpha E_{f,t}^{\psi-1} \]

- **Resource constraint:**

\[ N_t + D_t - (1 - \delta_d) D_{t-1} + K_t - (1 - \delta_k) K_{t-1} = Z_{y,t} \left( \eta K_{t-1}^\psi + (1 - \eta) E_{f,t}^\psi \right)^{\frac{\psi}{\alpha}} H_t^{1-\alpha} - P_t (E_{h,t} + E_{f,t}) \]

We rearrange the above conditions and add the shock processes and definitions of output and investment to get 13 equations to be fed into Dynare:
• Resource constraint
\[ N_t + I_{d,t} + I_{k,t} = Y_t - P_t (E_{h,t} + E_{f,t}) \] (9)

• Investment in durables
\[ I_{d,t} = D_t - (1 - \delta_d) D_{t-1} + \frac{\omega_{1d}}{1 + \omega_{2d}} \left( \frac{D_t - D_{t-1}}{D_{t-1}} \right)^{1+\omega_{2d}} \] (10)

• Investment in capital
\[ I_{k,t} = K_t - (1 - \delta_k) K_{t-1} + \frac{\omega_{1k}}{1 + \omega_{2k}} \left( \frac{K_t - K_{t-1}}{K_{t-1}} \right)^{1+\omega_{2k}} \] (11)

• Nondurables vs. Energy:
\[ P_t = \frac{(1 - \gamma)(1 - \theta)}{\gamma} N_t \left( \theta D_{t-1}^\rho + (1 - \theta) E_{h,t}^\rho \right)^{-1} E_{h,t}^{-\rho} \] (12)

• Labor supply:
\[ N_t = \frac{\varphi \gamma}{1 - \varphi} W_t (1 - H_t) \] (13)

• Wage equation:
\[ W_t = (1 - \alpha) Z_{y,t} \left( \eta K_{t-1}^\psi + (1 - \eta) E_{f,t}^\psi \right)^\alpha H_t^{1 - \alpha} \] (14)

• Interest rates:
\[ R_t = Z_{y,t} \left( \eta K_{t-1}^\psi + (1 - \eta) E_{f,t}^\psi \right)^\alpha H_t^{1 - \alpha} \alpha \eta K_t^{\psi-1} \] (15)

• Firm’s energy use:
\[ P_t = Z_{y,t} \left( \eta K_{t-1}^\psi + (1 - \eta) E_{f,t}^\psi \right)^\alpha H_t^{1 - \alpha} (1 - \eta) \alpha E_{f,t}^{\psi-1} \] (16)

• Output:
\[ Y_t = Z_{y,t} \left( \eta K_{t-1}^\psi + (1 - \eta) E_{f,t}^\psi \right)^\alpha H_t^{1 - \alpha} \] (17)

• Capital Euler equation
\[ 1 + \frac{\omega_{1k}}{K_{t-1}} \left( \frac{K_t - K_{t-1}}{K_{t-1}} \right)^{\omega_{2k}} = \beta E \left\{ \frac{N_t}{N_{t+1}} \left[ 1 + R_{t+1} - \delta_k + \omega_{1k} \left( \frac{K_{t+1} - K_t}{K_t} \right)^{\omega_{2d}} \frac{K_{t+1}}{K_t^2} \right] \right\} \] (18)

• Durables Euler Equation:
\[ 1 + \frac{\omega_{1d}}{D_{t-1}} \left( \frac{D_t - D_{t-1}}{D_{t-1}} \right)^{\omega_{2d}} = \beta \frac{(1 - \gamma) \theta}{\gamma} E \left\{ \frac{N_t}{N_{t+1}} \left( \theta D_t + (1 - \theta) E_{h,t+1}^\rho D_t^{1-\rho} \right)^{-1} \right\} \]
\[ + \beta \left[ 1 - \delta_d + \omega_{1d} \left( \frac{D_{t+1} - D_t}{D_t} \right)^{\omega_{2d}} \frac{D_{t+1}}{D_t^2} \right] E \frac{N_t}{N_{t+1}} \] (19)

• Productivity shock:
\[ \log Z_{y,t} = \rho z \log Z_{y,t-1} + \varepsilon_{z,t} \] (20)
• Energy prices:
\[
\log P_t = \rho_p \log P_{t-1} + \varepsilon_t + \rho_e \varepsilon_{t-1} \tag{21}
\]

B Construct steady state

This section details how to derive steady state values for all endogenous variables given the parameters.

• Resource Constraint
\[
N + \delta_d D + \delta_k K = Y - P (E_h + E_f) \tag{22}
\]

• Nondurables vs. Energy
\[
P = \frac{(1 - \gamma) (1 - \theta)}{\gamma} \frac{\delta d}{\delta d} D + (1 - \theta) E_h^{\rho - 1} E_f^{\rho - 1} \tag{23}
\]

• Labor supply
\[
N = \frac{\gamma}{1 - \varphi} W (1 - H) \tag{24}
\]

• Wage equation
\[
W = (1 - \alpha) \frac{Y}{H} \tag{25}
\]

• Interest rates
\[
R = Z_y \left( \eta K^{\psi} + (1 - \eta) E_f^{\psi} \right)^{1 - \alpha} H^{1 - \alpha} \eta K^{\psi - 1}
\]
\[
= Y \left( \eta K^{\psi} + (1 - \eta) E_f^{\psi} \right)^{-1} \alpha \eta K^{\psi - 1} \tag{26}
\]

• Firm’s energy use
\[
P = Z_y \left( \eta K^{\psi} + (1 - \eta) E_f^{\psi} \right)^{1 - \frac{\alpha}{\psi}} H^{1 - \alpha} (1 - \eta) \alpha E_f^{\psi - 1}
\]
\[
= Y \left( \eta K^{\psi} + (1 - \eta) E_f^{\psi} \right)^{-1} (1 - \eta) \alpha E_f^{\psi - 1} \tag{27}
\]

• Output
\[
Y = Z_y \left( \eta K^{\psi} + (1 - \eta) E_f^{\psi} \right)^{1 - \frac{\alpha}{\psi}} H^{1 - \alpha} \tag{28}
\]

• Capital Euler Equation
\[
1 = \beta (1 + R - \delta_k)
\]

and thus:
\[
R = \frac{1}{\beta} - 1 + \delta_k \tag{29}
\]

• Durables Euler Equation
\[
1 = \beta \frac{(1 - \gamma) \theta}{\gamma} N \left( \theta D^{\rho} + (1 - \theta) E_h^{\rho} \right)^{-1} D^{1 - \rho} + \beta (1 - \delta_d) \tag{30}
\]
Solve for steady state. As always:

\[ R = \frac{1}{\beta} - 1 + \delta_k \]

From the interest rate and firm energy use equations we get:

\[ \frac{R}{P} = \frac{\eta}{1 - \eta} \left( \frac{K}{E_f} \right)^{\psi-1} \]

Call the capital energy ratio \( \kappa_{ke} \), then

\[ \kappa_{ke} = \left( \frac{R}{P} \frac{1 - \eta}{\eta} \right)^{\frac{1}{\psi-1}} \]

which is determined by parameters. Call \( \kappa_{Ef} \) the firm energy use to output ratio then

\[
\begin{align*}
P &= Y \left( \eta K^{\psi} + (1 - \eta) E_f^{\psi} \right)^{\psi-1} (1 - \eta) \alpha E_f^{\psi-1} \\
&= \left( \eta \left( \frac{E_f \kappa_{ke}}{Y} \right)^{1-\psi} + (1 - \eta) \frac{E_f}{Y} \right)^{-1} (1 - \eta) \alpha \\
&= \left( \eta \left( \frac{E_f \kappa_{ke}}{Y} \right)^{1-\psi} + (1 - \eta) \frac{E_f}{Y} \right)^{-1} (1 - \eta) \alpha \\
&= \left( \eta \kappa_{ke}^{1-\psi} \kappa_{Ef} + (1 - \eta) \kappa_{Ef} \right)^{-1} (1 - \eta) \alpha \\
&= \left( \eta \kappa_{ke}^{1-\psi} + (1 - \eta) \right)^{-1} (1 - \eta) \alpha \kappa_{Ef}^{-1}
\end{align*}
\]

Thus:

\[
\kappa_{Ef} = \frac{(1 - \eta) \alpha}{\left( \eta \kappa_{ke}^{1-\psi} + (1 - \eta) \right) P}
\]

which is again only determined by parameters. Also notice that capital output ratio \( \kappa_K = \frac{K}{Y} = \frac{E_f \kappa_{ke}}{Y} = \kappa_{Ef} \kappa_{ke} \).

From the output equation

\[ Y = Z_y \left( \eta K^{\psi} + (1 - \eta) E_f^{\psi} \right)^{\psi} H^{1-\alpha} \]

Divide through by \( Y \) to get

\[ 1 = Z_y \left( \eta \left( \frac{K}{Y} \right)^{\psi} + (1 - \eta) \left( \frac{E_f}{Y} \right)^{\psi} \right)^{\psi \alpha} \left( \frac{H}{Y} \right)^{1-\alpha} \]

Thus:

\[ 1 = Z_y \left( \eta \kappa_K^{\psi} + (1 - \eta) \kappa_{Ef}^{\psi} \right)^{\psi \alpha} \kappa_H^{1-\alpha} \]

Thus:

\[ \kappa_H = (Z_y)^{-\frac{1-\alpha}{\psi \alpha}} \left( \eta \kappa_K^{\psi} + (1 - \eta) \kappa_{Ef}^{\psi} \right)^{-\frac{1-\alpha}{\psi(1-\alpha)}} \]

Also, the steady state wage rate is determined solely by parameters. It is the labor share times
output to hours ratio:

\[ W = \frac{1 - \alpha}{\kappa_H} \]

From the consumer durables vs. nondurables equation:

\[ 1 = \beta \frac{(1 - \gamma) \theta}{\gamma} N (\theta D^\rho + (1 - \theta) E_h^\rho)^{-1} D^{\rho-1} + \beta (1 - \delta_d) \]

Nondurables vs. energy:

\[ P = \frac{(1 - \gamma)(1 - \theta)}{\gamma} N (\theta D^\rho + (1 - \theta) E_h^\rho)^{-1} E_h^{\rho-1} \]

Solve for \((\theta D^\rho + (1 - \theta) E_h^\rho)^{-1}\):

\[ (\theta D^\rho + (1 - \theta) E_h^\rho)^{-1} = P \frac{\gamma}{(1 - \gamma)(1 - \theta)} E_h^{1-\rho} N^{-1} \]

Plug into the previous equation

\[ 1 = \beta \frac{(1 - \gamma) \theta}{\gamma} N P \frac{\gamma}{(1 - \gamma)(1 - \theta)} E_h^{1-\rho} N^{-1} D^{\rho-1} + \beta (1 - \delta_d) \]

\[ = \beta P \frac{\theta}{(1 - \theta)} \left( \frac{E_h}{D} \right)^{1-\rho} + \beta (1 - \delta_d) \]

Thus:

\[ \frac{E_h}{D} = \left[ \frac{1 - \beta + \beta \delta_d}{\beta \theta P (1 - \theta)} \right]^{1-\rho} \quad (31) \]

Next, write the Nondurables vs. Energy equation as:

\[ P = \frac{(1 - \gamma)(1 - \theta)}{\gamma} N (\theta D^\rho E_h^{1-\rho} + (1 - \theta) E_h)^{-1} \]

\[ = \frac{(1 - \gamma)(1 - \theta)}{\gamma} N \left( \theta \left( \frac{E_h}{D} \right)^{1-\rho} + (1 - \theta) \frac{E_h}{D} \right)^{-1} \]

Thus:

\[ \frac{N}{D} = \frac{\gamma P}{(1 - \gamma)(1 - \theta)} \left( \theta \left( \frac{E_h}{D} \right)^{1-\rho} + (1 - \theta) \frac{E_h}{D} \right) \]

Also notice that

\[ \frac{E_h}{D} = \frac{\kappa_{Eh}}{\kappa_D} \quad \text{and} \quad \frac{N}{D} = \frac{\kappa_N}{\kappa_D} \]

Next, rewrite the budget constraint as:

\[ 1 - P \kappa_{Ef} - \delta_k \kappa_K = \kappa_N + \delta_d \kappa_D + P \kappa_{Eh} \]

\[ = \kappa_D \left[ \frac{\kappa_N}{\kappa_D} + \delta_d + \frac{P \kappa_{Eh}}{\kappa_D} \right] \]

Then:

\[ \kappa_D = \frac{1 - P \kappa_{Ef} - \delta_k \kappa_K}{\kappa_N + \delta_d + \frac{P \kappa_{Eh}}{\kappa_D}} \]
There is one equation left, we have not used so far: The labor supply equation. It will make the link between output ratios $\kappa$ and the level variables:

$$N = \frac{\varphi \gamma}{1 - \varphi} W (1 - H)$$

Divide by $Y$:

$$\kappa_N = \frac{\varphi \gamma W}{1 - \varphi Y} - \frac{\varphi \gamma W \kappa_H}{1 - \varphi}\
= \frac{\varphi \gamma}{1 - \varphi} \frac{1 - \alpha}{H} - \frac{\varphi \gamma W \kappa_H}{1 - \varphi}$$

Thus:

$$\frac{\varphi \gamma}{1 - \varphi} \frac{1 - \alpha}{H} = \kappa_N + \frac{\varphi \gamma}{1 - \varphi} W \kappa_H$$

Solve for $H$:

$$H = \frac{\varphi \gamma}{1 - \varphi \kappa_N + \frac{\varphi \gamma}{1 - \varphi} W \kappa_H}$$

Via $H$ and $\kappa_H$ we determine $Y$ which gives us all other steady state variables because we computed the output ratios for each variable.

C Calibration

Economy with durable goods We will use the steady state relationships from Appendix B to pin down six parameter values. We set targets for steady state values of ratios $E_h/N$, $I_D/N$, $D/Y$, $K/E_f$ and $K/Y$ and hours worked $H$. Then we use the first order conditions in the steady state to solve for the six remaining parameters $\gamma$, $\theta$, $\eta$, $\varphi$, $\delta_k$ and $\delta_d$.

From the firm energy use equation (27) we derive:

$$1 = \frac{Y}{E_f} \left( \eta K^\psi + (1 - \eta) E_f^\psi \right)^{-1} (1 - \eta) E_f^\psi \alpha$$

$$= \left( \frac{K}{Y} \right)^{-1} K E_f \left( \eta \left( \frac{K}{E_f} \right)^\psi + (1 - \eta) \right)^{-1} (1 - \eta) \alpha$$

This equation pins down $\eta$.

Notice that the non-linear root-finding problem is well-behaved. Specifically can we prove that the right hand side is monotone and the values for $\eta = 0$ and $\eta = 1$ are on opposite sides of 1. If $\eta = 1$ then the RHS is zero. If $\eta = 0$ then the RHS is $\left( \frac{K}{Y} \right)^{-1} K E_f \alpha$. This is than 1, because $E_f/Y < \alpha$, that is, the energy share of output cannot be larger than $\alpha$, which is the expenditure share of energy and capital combined. For example in a realistic calibration with $K/Y = 12$ and $K/E_f = 200$ with $\alpha = 0.36$, the RHS is equal to 6. Also notice that the RHS is monotonically
decreasing if.

\[
0 > -\left( \eta \left( \frac{K}{E_f} \right)^{\psi} + 1 - \eta \right) - \left( \left( \frac{K}{E_f} \right)^{\psi} - 1 \right) (1 - \eta) \alpha
\]

\[
= -\eta \left( \frac{K}{E_f} \right)^{\psi} - 1 + \eta - \left( \frac{K}{E_f} \right)^{\psi} (1 - \eta) \alpha + (1 - \eta) \alpha
\]

\[
= -\left( \frac{K}{E_f} \right)^{\psi} [1 + (1 - \eta) \alpha] - (1 - \eta) (1 - \alpha)
\]

This is obviously the case as long as \( \eta \in [0, 1] \).

Next, from equation (26) we derive the steady state interest rate:

\[
R = \left( \frac{K}{Y} \right)^{-1} \left( \eta + (1 - \eta) \left( \frac{K}{E_f} \right)^{-\psi} \right)^{-1} \alpha \eta
\]  

(33)

Given interest rate \( R \), the capital Euler equation (29) pins down depreciation of physical capital:

\[
\delta_k = R - \frac{1}{\beta} + 1
\]  

(34)

Also, the nondurables to output ratio is:

\[
\frac{N}{Y} = \frac{D N}{Y D} = \frac{D N}{Y I_d} \delta_d
\]  

(35)

From the resource constraint (22):

\[
\frac{N}{Y} + \delta_d \frac{D}{Y} + \delta_k \frac{K}{Y} = 1 - \left( \frac{E_h}{Y} + \frac{E_f}{Y} \right)
\]

Thus:

\[
\delta_d \left[ \frac{D N}{Y I_d} + \frac{D}{Y} \right] = 1 - \delta_k \frac{K}{Y} - \frac{E_h N}{N Y} - \frac{E_f K}{K Y}
\]

Thus:

\[
\delta_d = \frac{1 - \delta_k \frac{K}{Y} - \frac{E_f K}{K Y}}{\frac{D N}{Y I_d} + \frac{D}{Y} + \frac{E_h}{N Y I_d}}
\]  

(36)

From the durables Euler equation (30) we derive:

\[
1 = \beta \frac{(1 - \gamma)}{\gamma} N (\theta D^\rho + (1 - \theta) E_h^\rho)^{-1} D^{\rho - 1} + \beta (1 - \delta_d)
\]  

(37)

From equation (23) we get:

\[
1 = \frac{(1 - \gamma)}{\gamma} \frac{(1 - \theta)}{\gamma} N (\theta D^\rho + (1 - \theta) E_h^\rho)^{-1} E_h^{\rho - 1}
\]  

(38)

Solve this for \( \frac{1 - \gamma}{\gamma} \):

\[
\frac{1 - \gamma}{\gamma} = (1 - \theta)^{-1} N^{-1} (\theta D^\rho + (1 - \theta) E_h^\rho) E_h^{-\rho + 1}
\]  

(39)
and plug into (37): 

\[ 1 = \beta \frac{\theta}{1 - \theta} \left( \frac{D}{E_h} \right)^{\rho - 1} + \beta (1 - \delta_d) \]

We can solve this for \( \theta \) : 

\[ \frac{1 - \theta}{\theta} = \beta \left( \frac{D}{E_h} \right)^{\rho - 1} \]

Thus: 

\[ \theta = \frac{1 - \beta (1 - \delta_d)}{1 - \beta (1 - \delta_d) + \beta \left( \frac{D}{E_h} \right)^{\rho - 1}} \] (40)

Moreover in (39) we can solve for \( \gamma \) : 

\[ \frac{1 - \gamma}{\gamma} = (1 - \theta)^{-1} \frac{E_h}{N} \left( \theta \left( \frac{D}{E_h} \right)^{\rho} + 1 - \theta \right) \]

Thus: 

\[ \gamma = \frac{1 - \theta}{1 - \theta + \frac{E_h}{N} \left( \theta \left( \frac{D}{E_h} \right)^{\rho} + 1 - \theta \right)} \] (41)

Finally, from the wage equation (25) and labor supply equation (24) we get: 

\[ \frac{N}{Y} = \frac{\varphi \gamma}{1 - \varphi} \frac{1 - \alpha}{H} (1 - H) \]

Solve for \( \varphi \) : 

\[ \frac{1 - \varphi}{\varphi} = \frac{Y}{N} \frac{1 - \alpha}{H} (1 - H) \]

Thus: 

\[ \varphi = \frac{1}{1 + \left( \frac{N}{Y} \right)^{-1} \gamma (1 - \alpha) \frac{1 - H}{H}} \] (42)

**Calibration in the economy without durable goods** Notice that in an economy without durable goods, first order conditions on the production side are identical to those in the economy with durable goods. Hence, we determine parameters \( \eta \) and \( \delta_k \) the same way as above. The only remaining parameter is \( \varphi \). Without the inclusion of durables investment we get

\[ \varphi = \frac{1}{1 + \left( \frac{C^A}{Y} \right)^{-1} (1 - \alpha) \frac{1 - H}{H}} \] (43)

where - via the resource constraint - we can deduce the consumption to output ratio from the target moments:

\[ \frac{C^A}{Y} = 1 - \frac{E_f}{Y} - \delta_k \frac{K}{Y} = 1 - \left( \frac{K}{E_f} \right)^{-1} \frac{K}{Y} - \delta_k \frac{K}{Y} \] (44)