Honoré, Bo E.; Hu, Luoja

Working Paper
Estimation of a transformation model with truncation, interval observation and time-varying covariates

Working Paper, No. 2009-16

Provided in Cooperation with:
Federal Reserve Bank of Chicago


This Version is available at:
http://hdl.handle.net/10419/70535
Estimation of a Transformation Model with Truncation, Interval Observation and Time-Varying Covariates

Bo E. Honoré and Luojia Hu

WP 2009-16
Estimation of a Transformation Model with Truncation, Interval Observation and Time–Varying Covariates*

Bo E. Honoré† Luojia Hu‡

First complete version: December 2007.
This Version: September 2009.

Abstract

Abrevaya (1999b) considered estimation of a transformation model in the presence of left–truncation. This paper observes that a cross–sectional version of the statistical model considered in Frederiksen, Honoré, and Hu (2007) is a generalization of the model considered by Abrevaya (1999b) and the generalized model can be estimated by a pairwise comparison version of one of the estimators in Frederiksen, Honoré, and Hu (2007). Specifically, our generalization will allow for discretized observations of the dependent variable and for piecewise constant time–varying explanatory variables.

* This research was supported by the National Science Foundation and the Gregory C. Chow Econometric Research Program at Princeton University. We thank seminar participants at Rice University, Université Paris 1 Panthéon–Sorbonne and the Federal Reserve Bank of Chicago as well as members of Princeton’s Microeconometric Reading Group for comments. The opinions expressed here are those of the authors and not necessarily those of the Federal Reserve Bank of Chicago or the Federal Reserve System.

† Mailing Address: Department of Economics, Princeton University, Princeton, NJ 08544-1021. Email: honore@Princeton.edu.

‡ Mailing Address: Economic Research Department, Federal Reserve Bank of Chicago, 230 S. La Salle Street, Chicago, IL 60604. Email: lhu@frbchi.org.
1 Introduction

The transformation model

\[ h(T_i^*) = g(X_i'\beta) + \varepsilon_i \]  

(1)

is often used to model durations. In models like this it is important to allow for right censoring and sometimes also for left truncation because the samples used in many applications include spells that are in progress at the start of the sample period. See Abrevaya (1999b).

It is also sometimes desirable to allow for the dependent variable to be discretized so that one observes only whether it falls in a particular interval. So the observed duration, \( T_i \), would be \( t \) if \( T_i^* \in (t-1, t] \). See Prentice and Gloeckler (1978) or Meyer (1990). Moreover, in duration models it is often interesting to allow for time–varying covariates, which are not easily directly incorporated into the transformation model (Flinn and Heckman (1982)).

The contribution of this paper is to specify a statistical model that allows for interval observations and time–varying covariates but which simplifies to a model with interval observations from (1) when the covariates are time invariant. We then propose an estimator for the parameters of the model. The estimator can be interpreted as a generalization of the truncated maximum rank correlation estimator proposed in Abrevaya (1999b).

Consider first the transformation model, (1), with strictly increasing \( h(\cdot) \) and \( g(\cdot) \) and with \( \varepsilon_i \) independent of \( X_i \) and continuously distributed with full support. In this model

\[
P(T_i^* > t | X_i) = P(h(T_i^*) > h(t) | X_i) = P(g(X_i'\beta + \varepsilon_i > h(t) | X_i) = 1 - F(h(t) - g(X_i'\beta))
\]

where \( F \) is the CDF for \( \varepsilon_i \). This gives

\[
P(T_i^* > t | X_i, T_i^* > t - 1) = \frac{1 - F(h(t) - g(X_i'\beta))}{1 - F(h(t-1) - g(X_i'\beta))}
\]

where the assumption that \( \varepsilon_i \) has full support guarantees that the denominator is not 0.

When \( 1 - F(\cdot) \) is log–concave (which is implied by the density of \( \varepsilon_i \) being log–concave; see Heckman and Honoré (1990)), the right–hand side is an increasing function of \( g(X_i'\beta) \) and
hence of $X_i' \beta$. See the Appendix. This means that one can write the event $T_i^* > t \mid X_i, T_i^* > t - 1$ in the form $1 \{X_i' \beta > \eta_{it}\}$ for some (possibly infinite) random variable $\eta_{it}$ that is independent of $X_i$ and has CDF $\frac{1 - F(h(t) - \cdot)}{1 - F(h(t - 1) - \cdot)}$. Therefore, if we define

$$Y_{it} \equiv 1 \{T_i^* \in (t - 1, t]\} = 1 \{T_i = t\}$$

then we can write

$$Y_{it} = 1 \{X_i' \beta - \eta_{it} \leq 0\} \quad \text{for } t \text{ such that } \sum_{l \leq t - 1} Y_{il} = 0 \quad (2)$$

In other words, a transformation model with discretized observations of the dependent variable and log–concave errors is a special case of the model

$$Y_{it} = 1 \{X_i' \beta - \eta_{it} \leq 0\} \quad \text{for } t \text{ such that } \sum_{l \leq t - 1} Y_{it} = 0 \quad (3)$$

where the difference between (2) and (3) is that the latter allows for time–varying covariates. Note that this line of argument is valid even if $T_i$ is left truncated.

It is interesting to note that Abrevaya (1999b) also assumes log–concavity of $1 - F(\cdot)$.

Note that although log–concavity implies an increasing hazard for $h(T^*)$, it does not impose such a restriction on $T^*$.

Equation (3) is a cross–sectional version of the model considered by Frederiksen, Honoré, and Hu (2007). It is well understood that estimators of panel data models can be turned into estimators of cross–sectional models by considering all pairs of observations as units in a panel. See, for example, Honoré and Powell (1994), who apply the idea in Honoré (1992)

1The assumption of log–concavity also appears elsewhere in the literature on truncated regression models. See, for example, Lee (1993) and (Honoré 1992).

2Let $\lambda(\cdot)$ denote the hazard for $\varepsilon$. The hazard for $h(T^*)$ is then $-\frac{\partial \log(P(h(T^*) > t \mid X))}{\partial t} = -\frac{\partial \log(1 - F(t - \beta(X' \beta)))}{\partial t} = \lambda(t - g(X' \beta))$. When $1 - F(\cdot)$ is log–concave this is an increasing function of $t$. On the other hand, the hazard for $T^*$ is $-\frac{\partial \log(P(T^* > t \mid X))}{\partial t} = \lambda(h(t) - g(X' \beta)) h'(t)$. The derivative of this with respect to $t$ is $\lambda'(h(t) - g(X' \beta)) h'(t)^2 + \lambda(h(t) - g(X' \beta)) h''(t)$, which can be of either sign.
for panel data censored regression models to all pairs of observations in a cross section. The insights in Frederiksen, Honoré, and Hu (2007) can therefore be used to construct an estimator of $\beta$. We pursue this in Section 3 after discussing the model in more detail in Section 2.

2 The model

Consider a spell with integer-valued duration, $T_i$, that starts at (integer-valued) time $-V_i \leq 0$. Following the discussion above, we model the event that the spell lasts $s$ periods conditional on lasting at least $s - 1$, by the qualitative response model

$$Y_{is} = 1 \{X'_{is}/\beta - \eta_{is} \leq 0\}, \quad (4)$$

where $\eta_{is}$ is independent of $X_{is}$ and the distribution of $\eta_{is}$ is allowed to change over time.

When there is left truncation, one must distinguish between duration time and calendar time. We will index the observables, $Y$ and $X$, by calendar time, and the unobservable $\eta$ by duration time. At first sight, this difference is confusing, but it is necessitated by the fact that the discussion in the previous section implied that one should allow the distribution of $\eta_{is}$ to vary by duration time. On the other hand, it seems natural to denote the first observation for an individual by $t = 1$.

With this notation, we assume that we observe $(Y_{it}, X_{it})$ starting at $t = 1$, where

$$Y_{it} = 1 \{X'_{it}/\beta - \eta_{i,t+V_i} \leq 0\}. \quad (5)$$

With this notation, $\eta$s with the same time subscript will have the same distribution under the class of models discussed above. We will let $T_i$ denote the first time that $Y_{it}$ equals 1. Since $Y_{it}$ is not defined after the end of the spell, and since we want to allow for random right censoring, we assume that we observe $Y_{it}$ from $t = 1$ until, and including, $T_i$ or until a random censoring time $C_i - 1$ (whichever comes first). In other words, we observe $(Y_{it}, X_{it})$ for $t = 1, 2, ..., T_i$ where $T_i = \min \{T_i, C_i - 1\}$. So when an observation is censored, $C_i$ will be
the first time period in which individual $i$ is not observed. We also assume that we observe the presample duration, $V_i$, for each observation.

The statistical assumption on the errors in (5) is that conditional on $V_i$ and on $\{Y_{is} = 0 \text{ for } s < t\}$, $\eta_{i, t+V_i}$ is independent of $(C_i, \{X_{is}\}_{s \leq t})$. As explained in Section 1, if the errors are log-concave and the covariates are time-invariant, this is exactly what is implied by an underlying transformation model for $T_i^*$, where we observe whether a spell that started at time $-V_i$ and was in progress at time $t - 1$ is still in progress at time $t$. Note that when the covariates are time-varying they are not restricted to be strictly exogenous, and that the censoring times can be covariate-dependent, as long as they do not depend on the $\eta$s.

In the next section we will apply the insight of Frederiksen, Honoré, and Hu (2007) to construct an estimator for $\beta$ under these assumptions when the researcher has access to a random sample of individuals.

3 The estimator

The key insight for the construction of the estimator can be easily illustrated if we ignore censoring first (so $T_i = T$ for all $i$).

Let $t_1$ and $t_2$ be arbitrary. Consider the two events $A = \{T_i = t_1, T_j > t_2\}$ and $B = \{T_i > t_1, T_j = t_2\}$ where $t_1 + V_i = t_2 + V_j$. Under the stated assumptions, it then follows immediately from Lemma 1 of (Frederiksen, Honoré, and Hu 2007) that

\[
P(A \mid A \cup B, X_{it_1}, X_{jt_2}, V_i, V_j) =\begin{cases} \frac{1}{2} & \text{if } (X_{it_1} - X_{jt_2})' \beta > 0, \\ \frac{1}{2} & \text{if } (X_{it_1} - X_{jt_2})' \beta = 0, \\ \frac{1}{2} & \text{if } (X_{it_1} - X_{jt_2})' \beta < 0. \end{cases}
\]

This suggests estimating $\beta$ by maximizing

\[
\sum_{i<j} \sum_{t_1=1}^{T_i} \sum_{t_2=1}^{T_j} 1 \{t_1 + V_i = t_2 + V_j\} 
\cdot (1 \{T_i = t_1, T_j > t_2\} \cdot 1 \{(X_{it_1} - X_{jt_2})' \beta > 0\} 
\quad + 1 \{T_i > t_1, T_j = t_2\} \cdot 1 \{(X_{it_1} - X_{jt_2})' \beta < 0\})
\]
(6) is the same as one of the objective functions in (Frederiksen, Honoré, and Hu 2007), except that that paper considers a panel data situation.

It is convenient to rewrite (6) as

$$\sum_{i<j} 1 \{ T_j + V_j > T_i + V_i > V_j \} \cdot 1 \left\{ (X_{iT_i} - X_{j,T_i+V_i-V_j} )' b > 0 \right\} + 1 \{ V_i < T_j + V_j < T_i + V_i \} \cdot 1 \left\{ (X_{iT_i+V_i-V_j} - X_{j,T_j} )' b < 0 \right\}$$  \hfill (7)$$

This has the same structure as the maximum rank correlation estimator developed in Han (1987).

When there is censoring, (6) can be modified to

$$\sum_{i<j} \sum_{t_1=1}^{T_i} \sum_{t_2=1}^{T_j} 1 \{ t_1 + V_i = t_2 + V_j, t_1 < C_i, t_2 < C_j \} \cdot \left( 1 \{ T_i = t_1, T_j > t_2 \} \cdot 1 \left\{ (X_{it_1} - X_{jt_2})' b > 0 \right\} + 1 \{ T_i > t_1, T_j = t_2 \} \cdot 1 \left\{ (X_{it_1} - X_{jt_2})' b < 0 \right\} \right).$$  \hfill (8)$$

And equation (7) can be rewritten as

$$\sum_{i<j} 1 \{ T_j + V_j > T_i + V_i > V_j, T_i < C_i \} \cdot 1 \left\{ (X_{iT_i} - X_{j,T_i+V_i-V_j} )' b > 0 \right\} + 1 \{ V_i < T_j + V_j < T_i + V_i, T_j < C_j \} \cdot 1 \left\{ (X_{iT_j+V_j-V_i} - X_{j,T_j} )' b < 0 \right\}.  \hfill (9)$$

The intuition for the estimator is essentially based on pairwise comparisons. Specifically, we compare an individual $i$ who was observed to fail at time $T_i$ (and thus had a complete duration $T_i + V_i$) to all other observations $j$ that survived up to the same duration. At the true parameter value $\beta$, if the index for individual $i$ at the time he/she failed, $X_{iT_i}^{'} \beta$, is larger than the index for the comparable individual $j$ at the time that corresponds to the same duration, $X_{j,T_i+V_i-V_j}^{'} \beta$, then individual $j$ is likely to survive longer than individual $i$ (that is, $T_j + V_j > T_i + V_i$). Note the set of comparison observations $j$ includes censored spells provided the censoring time occurs after $T_i + V_i - V_j$. The additional inequality in the indicator function, $T_i + V_i > V_j$, ensures that the time at which $j$ is being compared to $i$ is within the sample period (i.e., not truncated).
Again this estimator has the same structure as Han’s (1987) maximum rank correlation estimator and the asymptotic distribution is therefore the one given in Sherman (1993) under the regularity conditions stated there.

4 Asymptotic properties

Consistency and asymptotic normality can be established as in (Sherman 1993), Abrevaya (1999b) or Khan and Tamer (2007).

First note that some normalization of the parameter is needed, since the parameter vector is only identified up to scale. For example, we can normalize the last component of $\beta$ to be 1.

The two key assumptions for consistency of the estimator are (1) at least one component of the explanatory variable $X$ is continuously distributed with full support, and (2) the error $\eta$ has full support. Without the first assumption, the parameter is not identified, since a small change in the parameter value could leave the ranking of the index unchanged. The second assumption on the error guarantees that the set of effective observations that make a nonzero contribution to the objective function is not empty. Both assumptions are standard in the semi-parametric estimation literature.

To establish asymptotic normality results, we need some additional notations. Denote $D_i = 1 \{T_i < C_i\}$, which is an observable variable indicating a complete (uncensored) spell.

The objective function can be rewritten as

$$\frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{j \neq i} D_i \cdot 1 \{T_i + V_i > V_j, T_j + V_j > V_i, T_j + V_j > T_i + V_i\} \cdot 1 \left\{X_i' \tilde{T}_i b > X_j' \tilde{T}_j + V_i - V_j b\right\}. \quad (10)$$

Define the function

$$\tau \left( (t, \bar{t}, d, v, \{x_s\}_{s \leq t}), b \right)$$

$$\equiv E \left[ D_i \cdot 1 \{T_i + V_i > v, t + v > V_i, \bar{t} + v > T_i + V_i\} \cdot 1 \{X_i' \tilde{T}_i b > x_i' \tilde{T}_i + V_i - v b\} \right]$$
\[ +E \left[ d \cdot 1 \left\{ t + v > V_j, T_j + V_j > v, T_j > t + v \right\} \cdot 1 \left\{ x'_i b > X_{j,t+v-V_j} b \right\} \right] \]
\[ = E \left[ D_i \cdot 1 \left\{ T_i + V_i > v, t + v > V_i, T_i + V_i > t + v \right\} \cdot 1 \left\{ X_{iT_i} b > x'_{iT_i + V_i - V_i} b \right\} \right] \]
\[ +E \left[ d \cdot 1 \left\{ t + v > V_i, T_i + V_i > v, T_i > t + v \right\} \cdot 1 \left\{ x'_i b > X_{i,t+v-V_i} b \right\} \right]. \]

Following Theorem 4 of Sherman (1993), we have
\[ \sqrt{n} \left( \hat{\beta} - \beta \right) \overset{\text{d}}{\rightarrow} N(0, 4\Gamma^{-1} \Delta \Gamma^{-1}) \] (11)

where
\[ \Gamma = E \left[ \nabla_2 \tau \left( (T_i, T_i, D_i, V_i, \{X'_{is}\}_i) \right), \beta \right] \]
\[ \Delta = E \left[ \nabla_1 \tau \left( (T_i, T_i, D_i, V_i, \{X'_{is}\}_i) \right), \beta \right] \nabla_1 \tau \left( (T_i, T_i, d_i, V_i, \{X'_{is}\}_i) \right)^T ] \]

with \( \nabla_1 \) and \( \nabla_2 \) denoting the first– and second–derivative operator, respectively.

Following Sherman (1993), we can further express the variance-covariance matrix in terms of “model primitives.” Specifically,

\[ \Delta = V_X \left[ \sum_{s_1, s_2} (X_{s_2} - \mu_{s_1} (X'_{s_2} \beta)) S \left( T, T, D, V, s_1, s_2, X'_{s_2} \beta \right) g_{X'_{s_1} \beta} (X'_{s_2} \beta) \right] \]

and

\[ \Gamma = E_X \left[ \sum_{s_1, s_2} (X_{s_2} - \mu_{s_1} (X'_{s_2} \beta)) (X_{s_2} - \mu_{s_1} (X'_{s_2} \beta))^T S_7 \left( T, T, D, V, s_1, s_2, X'_{s_2} \beta \right) \cdot g_{X'_{s_1} \beta} (X'_{s_2} \beta) \right] \]

where \( X_{s_2} \) is composed of the first \( K - 1 \) coordinates of \( X'_{s_2} \) (the ones corresponding to the piece of \( \beta \) that is not normalized to 1), \( g_{X'_{s_1} \beta} (\lambda) \) is the marginal density of \( X'_{s_1} \beta \),

\[ \mu_{s_1} (\lambda) = E \left[ X_{s_1} | X'_{s_1} \beta = \lambda \right] \]

and

\[ S \left( ((t, \bar{t}, d, v, s_1, s_2), \lambda) \right) = E \left[ A_i (t, \bar{t}, d, v, s_1, s_2) | X'_{s_1} \beta = \lambda \right] \]
and

\[
A_i (t, \tilde{t}, d, v, s_1, s_2) = D_i \cdot 1 \{T_i + V_i > v, t + v > V_i, T_i + V_i = s_1, T_i + V_i - v = s_2\} - d \cdot 1 \{t + v > V_i, T_i + V_i > t + v, T_i = s_2, t + v - V_i = s_1\}
\]

The asymptotic variance matrix can be estimated by plugging in the estimator \(\hat{\beta}\) and calculating sample analogs of \(\Gamma\) and \(\Delta\) using numerical derivatives based on a smoothed version of \(\tau\). See Section 6 for more discussion.

## 5 Relationship to other estimators

The estimator proposed in the previous section is related to a number of existing estimators, and it coincides with some of them in special situations. For example, when \(C_i = 2\) for all \(i\), and with no left truncation (so \(V_i = 0\) for all \(i\)), (5) is a standard discrete choice model, and in that case the objective function in (9) becomes

\[
\sum_{i<j}^{n} 1 \{Y_i > Y_j\} \cdot 1 \{(X_i - X_j)'b > 0\} + 1 \{Y_i < Y_j\} \cdot 1 \{(X_i - X_j)'b < 0\}
\]

which is the objective function for Han’s (1987) maximum rank correlation estimator.

When there is left truncation, and the covariates are time invariant and there is no censoring, the estimator defined by maximizing (9) is the same as the truncated maximum rank correlation estimator in (Abrevaya 1999b). This is most easily seen by noting that without censoring and with time–invariant covariates, (10) becomes

\[
\frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{j \neq i} 1 \{T_i + V_i > V_j, T_j + V_j > V_i, T_j + V_j > T_i + V_i\} \cdot 1 \{(X_i'b > X_j'b)\} = \frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{j \neq i} 1 \{T_i + V_i > V_j, T_j + V_j > V_i\} 1 \{T_j + V_j > T_i + V_i\} \cdot 1 \{(X_i'b > X_j'b)\}.
\]

Except for the difference in notation and the normalization by \(n(n-1)\), this is exactly equation (7) in Abrevaya (1999b).
Khan and Tamer (2007) consider a model with left censoring as well as right censoring, whereas we allow for left truncation as well as right censoring. When there is neither left censoring nor left truncation, and when the covariates are time invariant, the estimator defined by maximizing (9) coincides with the estimator proposed by Khan and Tamer (2007), except that ours applies to discretized durations and theirs to exactly measured durations, and we allow for time–varying covariates. Whether right censoring or right truncation is more interesting depends on the specific application. Left truncation will, for example, be relevant if as in Frederiksen, Honoré, and Hu (2007), the duration of interest is the length of employment on a given job, and one has information on a sample of workers observed between two fixed points in time. In this case the durations are left truncated, because spells that ended before the start of the sampling will not appear in the data, and it is crucial for our approach that one observes the duration of employment in the current job at the start of the sampling. On the other hand, models with both left and right censoring are, for example, useful for estimation of models where the dependent variable is a fraction, which is restricted to be between zero and one, and where both zeros and ones are likely to be observed. See, for example Alan and Leth-Petersen (2006). Both Khan and Tamer (2007) and we allow the censoring points at the right to be observed only when the observation equals a censoring point. Khan and Tamer (2007) also allow the left–censoring point to be unobserved when an observation is not left–censored, whereas we assume that the truncation point is observed for everybody who is not truncated, but not for truncated durations. Both papers assume that one observes the actual duration, and not just the duration from the censoring/truncation point. In the duration contexts we have in mind, this is the most severe assumption.\footnote{See also Heckman and Singer (1986) for a discussion of the effect of different sample schemes on the analysis of duration data.}

The framework here is also closely related to standard statistical duration models with discretized observations. The proportional hazard model can be written as

\[ Z(T) = -x'\beta + \varepsilon, \]

\[ 3 \text{See also Heckman and Singer (1986) for a discussion of the effect of different sample schemes on the analysis of duration data.} \]
where $Z$ is the log integrated baseline hazard and $\varepsilon$ has an extreme value distribution. Prentice and Gloeckler (1978), Meyer (1990) and (Hausman and Woutersen 2005) study a version of this model with interval observations. Meyer (1990) and Hausman and Woutersen (2005) also allow for time–varying explanatory variables and for $\varepsilon$ to be a sum of an extreme value distributed random variable and a random variable that captures unobserved heterogeneity. While the estimation in Meyer (1990) is likelihood–based and hence fundamentally different from ours, the structure of the estimator proposed in Hausman and Woutersen (2005) shares many of the features of the estimator proposed here. The main difference is that theirs is based on a comparison of the integrated hazards rather than just the current index, $X'\beta$. As a result, the approach does not seem to generalize to models with left truncation. On the other hand, log–concavity plays no role in Hausman and Woutersen (2005).

6 Monte Carlo experiment

In this section, we conduct a small scale Monte Carlo study to illustrate the proposed estimation method and investigate its finite sample performance. We also demonstrate how to conduct inference and examine how good an approximation the asymptotic distribution provides for finite samples.

The designs are based on the following:

- All of the designs have two explanatory variables.
- $\beta = (\beta_1, \beta_2)' = (1, 2)'$. The parameter of interest is $\theta = \beta_2/\beta_1$. The fact that this is one dimension greatly simplifies the computations.
- Time–varying intercept $\beta_0 = -4 + (s/10)^{1.2}$. This introduces duration dependence beyond the duration dependence introduced by the shape of $F$ and by the choice of $h(\cdot)$. 
• The time between the start of a spell and the first period of observation is uniformly distributed on the integers between 1 and 5.

• The censoring time is generated as the minimum of ten periods from the start of the spell and $Q$ periods from the start of the sample, where $Q$ is uniformly distributed on the integers between 1 and 8.

We consider a number of designs within this framework.

**Design 1: Dynamic Probit.**

The two explanatory variables are generated by $i.i.d.$ draws from a bivariate normal distribution with zero means, variances equal to 2 and 1, respectively, and covariance equal to 1. In this design, $\eta_{i,t}$ is i.i.d $N(0,4)$.

**Design 2: Transformation Model Hazard.**

This design is set up as a generalization of a transformation model. Specifically, using the notation of Section 1, we assume that $h(u) = \log(u)$, $g(u) = u$, and $\varepsilon \sim N(0,1)$. Using the derivation in Section 1, this yields

$$P(T_i = t) = P(T_i^* < t \mid \{X_{is}\}_{s \leq t}, T_i^* > t - 1) = 1 - \frac{1 - \Phi(\log(t) - X_{it}'\beta)}{1 - \Phi(\log((t - 1) - X_{it}'\beta))}.$$ 

As in Design 1, the two explanatory variables are generated by $i.i.d.$ draws from a bivariate normal distribution with zero means, variances equal to 2 and 1, respectively, and covariance equal to 1.

**Design 3: Feedback.**

Recall that our model does not require the explanatory variables to be strictly exogenous. In this design we therefore allow for feedback from the error $\eta$ to future values of $X$. Specifically, we follow Design 1 except that the explanatory variables are defined by

• $X_{2t} = \eta_{t-1}$ for $t > 1$ and standard normal for $t = 1$.

• $X_{1t} = X_{2t} + N(0,1)$. 

12
**Design 4: Covariate–Dependent Censoring and Truncation.**

Our model allows censoring and truncation to be correlated with explanatory variables. In this design, we follow the basic structure of Design 1 but let censoring be defined by the outcome of a probit with explanatory variable \( X_{1t} \).

**Design 5: Dynamic Probit 2.**

This design is like Design 1 except that

- \( X_{2s} \sim N(0.5 - 0.2s, 1) \).
- \( X_{1s} = 1 \{ X_{2s} + N(0, 1) > 0 \} \)
- \( \eta_{i,s} \sim N(0, 0.1 + (0.15s)) \)

The summary statistics for the five designs are reported in Table 1. For each design, 100,000 observations are drawn from the underlying data–generating process. We then compute the fraction of the sample that is censored, the fraction that is truncated, and the mean and standard deviation of the underlying duration.

Below, we report Monte Carlo results for the point estimates of \( \beta \) as well as for the performance of tests statistics based on the asymptotic distribution in Section 4. To do this, we estimate the components of the variance of the estimators by sample analogs of smoothed versions of the components.\(^4\)

Recall that

\[
\Gamma = E \left[ \nabla_2 \tau \left( (T_i, T_i, D_i, V_i, \{X_{1s}'\}, \beta) \right) \right]
\]

\(^4\)A recent paper by Subbotin (2007) has shown that the nonparametric bootstrap can be used to estimate the quantiles and variance of various maximum rank correlation estimators. The structure of our estimator is essentially the same as that of the maximum rank correlation estimators he considers. We therefore conjecture that the bootstrap could have been used to estimate the variance in our case as well, although this would increase the computational burden.
and
\[
\Delta = E \left[ \nabla_1 \tau \left( (T_i, T_i, D_i, V_i, \{X'_{is} \}) , \beta \right) \right] \nabla_1 \tau \left( (T_i, T_i, D_i, V_i, \{X'_{is} \}) , \beta \right)
\]
where
\[
\tau \left( (t, \bar{t}, d, v, \{x_s \}_{s \leq \bar{t}}), b \right)
= E \left[ D_i \cdot 1 \left\{ T_i + V_i > v, t + v > V_i, \bar{t} + v > T_i + V_i \right\} \cdot 1 \left\{ X'_{iT_i} b > x'_{T_i + V_i - v} b \right\} \right] + E \left[ d \cdot 1 \left\{ t + v > V_i, T_i + V_i > v, \bar{T}_i + V_i > t + v \right\} \cdot 1 \left\{ x' b > X'_{i,t + v - V_i} b \right\} \right].
\]

We then estimate \( \tau \) by the smoothed version
\[
\hat{\tau} \left( (t, \bar{t}, d, v, \{x_s \}_{s \leq \bar{t}}), b \right)
= \frac{1}{n} \sum_{i=1}^{n} D_i \cdot 1 \left\{ T_i + V_i > v, t + v > V_i, \bar{t} + v > T_i + V_i \right\} \cdot \Phi \left( \frac{X'_{iT_i} b - x'_{T_i + V_i - v} b}{h} \right)
+ \frac{1}{n} \sum_{i=1}^{n} d \cdot 1 \left\{ t + v > V_i, T_i + V_i > v, \bar{T}_i + V_i > t + v \right\} \cdot \Phi \left( \frac{x' b - X'_{i,t + v - V_i} b}{h} \right).
\]

Then
\[
\hat{\nabla}_1 \tau \left( (t, \bar{t}, d, v, \{x_s \}_{s \leq \bar{t}}), b \right)
= \frac{1}{nh} \sum_{i=1}^{n} D_i \cdot 1 \left\{ T_i + V_i > v, t + v > V_i, \bar{t} + v > T_i + V_i \right\}
\cdot \phi \left( \frac{X'_{iT_i} b - x'_{T_i + V_i - v} b}{h} \right) \left( \tilde{X}'_{iT_i} - \tilde{x}'_{T_i + V_i - v} \right)
+ d \cdot 1 \left\{ t + v > V_i, T_i + V_i > v, \bar{T}_i + V_i > t + v \right\}
\cdot \phi \left( \frac{x' b - X'_{i,t + v - V_i} b}{h} \right) \left( \tilde{x}'_{t} - \tilde{X}'_{i,t + v - V_i} \right),
\]
and
\[
\hat{\nabla}_2 \tau \left( (t, \bar{t}, d, v, \{x_s \}_{s \leq \bar{t}}), b \right)
= \frac{-1}{nh^2} \sum_{i=1}^{n} D_i \cdot 1 \left\{ T_i + V_i > v, t + v > V_i, \bar{t} + v > T_i + V_i \right\}
\cdot \left( \frac{X'_{iT_i} b - x'_{T_i + V_i - v} b}{h} \right) \phi \left( \frac{X'_{iT_i} b - x'_{T_i + V_i - v} b}{h} \right)
\cdot \left( \tilde{X}'_{iT_i} - \tilde{x}'_{T_i + V_i - v} \right) \left( \tilde{X}'_{iT_i} - \tilde{x}'_{T_i + V_i - v} \right)'.
\]
\[ + d \cdot 1 \{ t + v > V_i, T_i + V_i > v, T_i + V_i > t + v \} \]
\[
(X'_{iTi} b - x'_{iTi+v} b) \phi \left( \frac{x'_{iTi+v} b - X'_{i,t+v-V_i} b}{h} \right)
\]
\[
(\tilde{x}'_t - \tilde{X}'_{i,t+v-V_i}) (\tilde{x}'_t - \tilde{X}'_{i,t+v-V_i})'.
\]
And therefore
\[
\hat{\Gamma} = \frac{1}{n} \sum_{i=1}^{n} \hat{\nabla}^2 \tau \left( (T_i, T_i, D_i, V_i, \{X'_{iis}\}), \hat{\beta} \right)
\]
and
\[
\hat{\Delta} = \frac{1}{n} \sum_{i=1}^{n} \hat{\nabla}^1 \tau \left( (T_i, T_i, D_i, V_i, \{X'_{iis}\}), \hat{\beta} \right) \hat{\nabla}^1 \tau \left( (T_i, T_i, D_i, V_i, \{X'_{iis}\}), \hat{\beta} \right)'.
\]
As mentioned earlier, \( \beta \) is only identified up to scale. One possibility is to normalize one of the coefficients to 1, and hence essentially focus on \( \beta_2/\beta_1 \) or \( \beta_1/\beta_2 \). Unfortunately, this normalization will lead to different MAE and RMSE depending on which of the coefficients is normalized. So if one were to compare different estimators, one might reach different conclusions depending on a seemingly innocent normalization. This is unsatisfactory in models where there is only one parameter. For this reason, it is likely to be better to consider \( \theta = \log (\beta_2/\beta_1) = \log (\beta_2) - \log (\beta_1) \) the parameter of interest. This means that the true parameter is \( \log (2) \approx 0.693 \) for all of the designs. We estimate \( \theta \) by a grid search over the interval between \( -\log (6) \) and \( \log (6) \) with equal grids of size \( \frac{1}{200} \). Since the parameter of interest is one dimensional, the line search is feasible despite the fact that the calculation of the objective function requires \( O (n^2) \) operations. When it is of higher dimension, it would be beneficial to use the method described in Abrevaya (1999a) to calculate the objective function in \( O (n \cdot \log (n)) \) operations. We calculate the variance of \( \theta \) by applying the so-called \( \delta \)-method to (11).

For each design, the Monte Carlo experiment is conducted with 5,000 replications for each of the 5 sample sizes: 100, 200, 400, 800 and 1,600. The results are reported in Tables 2–6. Overall, the results across the designs are broadly consistent with predictions from the asymptotic theory. Some additional remarks are in order.

First, the results illustrate the consistency of the estimator, since both the median ab-
solute error (MAE) and root mean squared error (RMSE) decrease as sample size increases. Moreover, the estimator is close to median unbiased even for small sample sizes.

Second, the theory predicts that the estimator converges to the true parameter value at the rate $\sqrt{n}$. This is borne out in the simulation as the MAE and RMSE decrease toward zero at a rate of approximately $\sqrt{2}$ when the sample size is doubled. For example, a regression of the log of the median absolute error on the log of the sample size (and design dummies) yields a coefficient of $-0.543$ with a standard error of 0.006.

Third, to examine the normality prediction from the asymptotic theory, we estimate the density for $\hat{\theta} - \theta$ and plot the kernel estimate in Figures 1-5. The left–hand side of each figure gives the estimated density of the estimator of $\beta_2/\beta_1$ centered at the true value. They show severe asymmetry in the distribution of the estimator: it tends to be skewed to the left, especially in small samples. As mentioned, this is expected because of the somewhat unnatural normalization. The right–hand side of the figures shows the estimated density of the estimator of log ($\beta_2/\beta_1$), again centered at the true value. There one can see that the distribution becomes more symmetric and closer to normal as sample size increases.

Finally, the asymptotic theory suggests that we can conduct inference using t–tests. Under the null, the test statistic should follow a standard normal distribution. In the simulation, we compute a t–statistic for each of the 5,000 estimates $\hat{\theta}$ and calculate the fraction of times the null is rejected at the 20% level. We focus on tests with (nominal) size of 20% rather than the conventional 5% because the results for the latter are likely to be more erratic for a finite number of simulations. The results are reported for various bandwidths used in the estimation of the asymptotic variance-covariance matrix of the estimator.

In general, the rejection rate is closer to the nominal size of the test when the bandwidth is smaller and the sample size is larger. For example, for the bandwidth (0.05, 0.20) and sample size 1,600, the reject rate is 21.8%, 22.7%, 20.8% and 21.6% for Designs 1 through 4.\footnote{Different bandwidths are used in estimating the matrix $\Delta$ and $\Gamma$. The latter is based on a second derivative, and one would therefore expect it to require a larger bandwidth than the former.} These
are close to being statistically indistinguishable from the nominal size. The performance of
the test under some other combinations of bandwidth and sample size is less encouraging.
The test also performs less well under Design 5. We speculate that this is because of the
discreteness of $x_{i1}$.

The last row reports the reject rates computed using the average of the variance–covariance
matrix estimated over all the bandwidth choices. Overall, the t-test tends to over–reject the
null.

It is interesting to compare our results to a standard logit or probit estimation of (5) where one uses $x_1$, $x_2$ and time dummies as explanatory variables. Since we expect them
to perform comparably, we focus on the logit maximum likelihood estimator. Designs 1, 3
and 4 are all correctly specified probit models, so one would expect the logit estimator to do
well for this design. This is confirmed in panels one, three and four of Table 7. The bias is
small and the MAE and RMSE fall at a rate close to root–$n$. It is less clear what to expect
for Designs 2 and 5. Panel 2 of Table 7 shows that the logit estimator does well for Design
2. It appears to be close to unbiased and its MAE and RMSE fall at a rate close to root–$n$.
One potential explanation for this is that misspecified maximum likelihood estimators often
do well when the explanatory variables are jointly normally distributed. See, for example,
Ruud (1983). Design 5 shows a situation where the logit estimator does relatively poorly.
The bias is quite high, and as a result, the MAE and RMSE do not fall rapidly as the sample
size increases.

7 Conclusion

In this paper we propose a generalization of the transformation model that is appropriate for
studying duration outcome with truncation, censoring, interval observations of the dependent
variable, and time-varying covariates. We develop an estimator for this model, discuss its

6Since our estimator of $\theta$ was calculated by a grid search over the interval between $-\log(6)$ and $\log(6)$,
we censored the logit maximum likelihood estimator of $\beta_2/\beta_1$ to be in the interval between $\frac{1}{6}$ and 6.
asymptotic properties and investigate its finite sample performance via a Monte Carlo study. Overall, the results suggest that the estimator performs well in finite samples, and the asymptotic theory provides a reasonably good approximation to the distribution. We also investigate test–statistics for the estimator. Those require estimation of the asymptotic variance of the estimator. This is somewhat sensitive to different choices of bandwidth. Investigating the optimal bandwidth choice in this case could be an interesting future research topic.

References


8 Appendix

Assume that $H(\cdot)$ is a log–concave function and let

$$f(w) = \frac{H(a_2 - w)}{H(a_1 - w)}$$

where $a_2 > a_1$. Let $w_1 < w_2$ and

$$\Delta a = a_2 - a_1, \quad \Delta w = w_2 - w_1 \quad \text{and} \quad \lambda = \frac{\Delta a}{\Delta a + \Delta w}$$

then

$$a_2 - w_2 = \lambda (a_2 - w_1) + (1 - \lambda) (a_1 - w_2)$$

so by concavity of $\ln (H(\cdot))$,

$$\ln (H(a_2 - w_2)) > \lambda \ln (H(a_2 - w_1)) + (1 - \lambda) \ln (H(a_1 - w_2)). \quad (12)$$

Also

$$a_1 - w_1 = (1 - \lambda) (a_2 - w_1) + \lambda (a_1 - w_2)$$

so

$$\ln (H(a_1 - w_1)) > (1 - \lambda) \ln (H(a_2 - w_1)) + \lambda \ln (H(a_1 - w_2)) \quad (13)$$

Adding (12) and (13) yields

$$\ln (H(a_2 - w_2)) + \ln (H(a_1 - w_1)) > \ln (H(a_2 - w_1)) + \ln (H(a_1 - w_2))$$

and

$$\ln (f(w_2)) - \ln (f(w_1)) = \ln (H(a_2 - w_2)) - \ln (H(a_1 - w_2)) - \ln (H(a_2 - w_1)) + \ln (H(a_1 - w_1)) > 0$$

Hence $f$ is an increasing function.
Table 1: Summary Statistics for the Designs

<table>
<thead>
<tr>
<th></th>
<th>Design 1</th>
<th>Design 2</th>
<th>Design 3</th>
<th>Design 4</th>
<th>Design 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction Truncated</td>
<td>0.260</td>
<td>0.349</td>
<td>0.316</td>
<td>0.260</td>
<td>0.183</td>
</tr>
<tr>
<td>Fraction Censored</td>
<td>0.297</td>
<td>0.100</td>
<td>0.190</td>
<td>0.425</td>
<td>0.543</td>
</tr>
<tr>
<td>Mean Duration</td>
<td>5.317</td>
<td>3.592</td>
<td>4.369</td>
<td>5.317</td>
<td>7.270</td>
</tr>
<tr>
<td>Standard Deviation of Duration</td>
<td>3.422</td>
<td>2.023</td>
<td>2.918</td>
<td>3.422</td>
<td>3.937</td>
</tr>
</tbody>
</table>

Table 2: Results for Design 1

<table>
<thead>
<tr>
<th>Performance of Estimator</th>
<th>$n = 100$</th>
<th>$n = 200$</th>
<th>$n = 400$</th>
<th>$n = 800$</th>
<th>$n = 1600$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
<td>0.721</td>
<td>0.695</td>
<td>0.702</td>
<td>0.693</td>
<td>0.693</td>
</tr>
<tr>
<td>MAE</td>
<td>0.331</td>
<td>0.215</td>
<td>0.155</td>
<td>0.105</td>
<td>0.070</td>
</tr>
<tr>
<td>Mean</td>
<td>0.727</td>
<td>0.706</td>
<td>0.702</td>
<td>0.698</td>
<td>0.694</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.488</td>
<td>0.331</td>
<td>0.227</td>
<td>0.155</td>
<td>0.106</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Significance when testing at 20% level</th>
<th>$n = 100$</th>
<th>$n = 200$</th>
<th>$n = 400$</th>
<th>$n = 800$</th>
<th>$n = 1600$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05, 0.20</td>
<td>0.332</td>
<td>0.240</td>
<td>0.212</td>
<td>0.207</td>
<td>0.218</td>
</tr>
<tr>
<td>0.05, 0.40</td>
<td>0.163</td>
<td>0.152</td>
<td>0.187</td>
<td>0.241</td>
<td>0.271</td>
</tr>
<tr>
<td>0.10, 0.20</td>
<td>0.421</td>
<td>0.323</td>
<td>0.280</td>
<td>0.256</td>
<td>0.246</td>
</tr>
<tr>
<td>0.10, 0.40</td>
<td>0.243</td>
<td>0.232</td>
<td>0.261</td>
<td>0.296</td>
<td>0.302</td>
</tr>
<tr>
<td>0.20, 0.20</td>
<td>0.494</td>
<td>0.382</td>
<td>0.323</td>
<td>0.285</td>
<td>0.260</td>
</tr>
<tr>
<td>0.20, 0.40</td>
<td>0.324</td>
<td>0.301</td>
<td>0.305</td>
<td>0.326</td>
<td>0.319</td>
</tr>
<tr>
<td>0.40, 0.20</td>
<td>0.536</td>
<td>0.423</td>
<td>0.350</td>
<td>0.303</td>
<td>0.269</td>
</tr>
<tr>
<td>0.40, 0.40</td>
<td>0.390</td>
<td>0.347</td>
<td>0.337</td>
<td>0.343</td>
<td>0.329</td>
</tr>
<tr>
<td>average</td>
<td>0.289</td>
<td>0.259</td>
<td>0.261</td>
<td>0.274</td>
<td>0.273</td>
</tr>
</tbody>
</table>
### Table 3: Results for Design 2

<table>
<thead>
<tr>
<th>Performance of Estimator</th>
<th>(n=100)</th>
<th>(n=200)</th>
<th>(n=400)</th>
<th>(n=800)</th>
<th>(n=1600)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
<td>0.718</td>
<td>0.713</td>
<td>0.698</td>
<td>0.693</td>
<td>0.692</td>
</tr>
<tr>
<td>MAE</td>
<td>0.540</td>
<td>0.370</td>
<td>0.255</td>
<td>0.175</td>
<td>0.125</td>
</tr>
<tr>
<td>Mean</td>
<td>0.656</td>
<td>0.712</td>
<td>0.705</td>
<td>0.697</td>
<td>0.695</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.752</td>
<td>0.553</td>
<td>0.392</td>
<td>0.274</td>
<td>0.186</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Significance when testing at 20% level</th>
<th>(n=100)</th>
<th>(n=200)</th>
<th>(n=400)</th>
<th>(n=800)</th>
<th>(n=1600)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05, 0.20</td>
<td>0.423</td>
<td>0.350</td>
<td>0.268</td>
<td>0.236</td>
<td>0.227</td>
</tr>
<tr>
<td>0.05, 0.40</td>
<td>0.164</td>
<td>0.175</td>
<td>0.180</td>
<td>0.224</td>
<td>0.265</td>
</tr>
<tr>
<td>0.10, 0.20</td>
<td>0.515</td>
<td>0.422</td>
<td>0.335</td>
<td>0.280</td>
<td>0.253</td>
</tr>
<tr>
<td>0.10, 0.40</td>
<td>0.269</td>
<td>0.252</td>
<td>0.245</td>
<td>0.273</td>
<td>0.296</td>
</tr>
<tr>
<td>0.20, 0.20</td>
<td>0.578</td>
<td>0.477</td>
<td>0.372</td>
<td>0.302</td>
<td>0.269</td>
</tr>
<tr>
<td>0.20, 0.40</td>
<td>0.359</td>
<td>0.317</td>
<td>0.293</td>
<td>0.303</td>
<td>0.313</td>
</tr>
<tr>
<td>0.40, 0.20</td>
<td>0.620</td>
<td>0.514</td>
<td>0.394</td>
<td>0.317</td>
<td>0.279</td>
</tr>
<tr>
<td>0.40, 0.40</td>
<td>0.426</td>
<td>0.363</td>
<td>0.322</td>
<td>0.319</td>
<td>0.323</td>
</tr>
<tr>
<td>average</td>
<td>0.333</td>
<td>0.299</td>
<td>0.274</td>
<td>0.273</td>
<td>0.275</td>
</tr>
</tbody>
</table>

### Table 4: Results for Design 3

<table>
<thead>
<tr>
<th>Performance of Estimator</th>
<th>(n=100)</th>
<th>(n=200)</th>
<th>(n=400)</th>
<th>(n=800)</th>
<th>(n=1600)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
<td>0.683</td>
<td>0.699</td>
<td>0.693</td>
<td>0.696</td>
<td>0.688</td>
</tr>
<tr>
<td>MAE</td>
<td>0.400</td>
<td>0.263</td>
<td>0.174</td>
<td>0.120</td>
<td>0.085</td>
</tr>
<tr>
<td>Mean</td>
<td>0.691</td>
<td>0.716</td>
<td>0.707</td>
<td>0.700</td>
<td>0.694</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.569</td>
<td>0.405</td>
<td>0.271</td>
<td>0.185</td>
<td>0.124</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Significance when testing at 20% level</th>
<th>(n=100)</th>
<th>(n=200)</th>
<th>(n=400)</th>
<th>(n=800)</th>
<th>(n=1600)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05, 0.20</td>
<td>0.389</td>
<td>0.289</td>
<td>0.214</td>
<td>0.201</td>
<td>0.208</td>
</tr>
<tr>
<td>0.05, 0.40</td>
<td>0.216</td>
<td>0.150</td>
<td>0.143</td>
<td>0.202</td>
<td>0.255</td>
</tr>
<tr>
<td>0.10, 0.20</td>
<td>0.468</td>
<td>0.372</td>
<td>0.288</td>
<td>0.264</td>
<td>0.247</td>
</tr>
<tr>
<td>0.10, 0.40</td>
<td>0.280</td>
<td>0.226</td>
<td>0.225</td>
<td>0.273</td>
<td>0.302</td>
</tr>
<tr>
<td>0.20, 0.20</td>
<td>0.551</td>
<td>0.455</td>
<td>0.345</td>
<td>0.308</td>
<td>0.273</td>
</tr>
<tr>
<td>0.20, 0.40</td>
<td>0.368</td>
<td>0.307</td>
<td>0.287</td>
<td>0.316</td>
<td>0.327</td>
</tr>
<tr>
<td>0.40, 0.20</td>
<td>0.608</td>
<td>0.503</td>
<td>0.384</td>
<td>0.330</td>
<td>0.287</td>
</tr>
<tr>
<td>0.40, 0.40</td>
<td>0.438</td>
<td>0.370</td>
<td>0.325</td>
<td>0.344</td>
<td>0.342</td>
</tr>
<tr>
<td>average</td>
<td>0.326</td>
<td>0.268</td>
<td>0.247</td>
<td>0.266</td>
<td>0.273</td>
</tr>
</tbody>
</table>
Table 5: Results for Design 4

<table>
<thead>
<tr>
<th></th>
<th>Performance of Estimator</th>
<th>Significant when testing at 20% level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n = 100</td>
<td>n = 200</td>
</tr>
<tr>
<td>Median</td>
<td>0.706</td>
<td>0.693</td>
</tr>
<tr>
<td>MAE</td>
<td>0.385</td>
<td>0.260</td>
</tr>
<tr>
<td>Mean</td>
<td>0.704</td>
<td>0.703</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.557</td>
<td>0.397</td>
</tr>
</tbody>
</table>

Table 6: Results for Design 5

<table>
<thead>
<tr>
<th></th>
<th>Performance of Estimator</th>
<th>Significant when testing at 20% level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n = 100</td>
<td>n = 200</td>
</tr>
<tr>
<td>Median</td>
<td>0.711</td>
<td>0.704</td>
</tr>
<tr>
<td>MAE</td>
<td>0.567</td>
<td>0.409</td>
</tr>
<tr>
<td>Mean</td>
<td>0.646</td>
<td>0.710</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.789</td>
<td>0.612</td>
</tr>
</tbody>
</table>
Table 7: Performance of the Logit MLE

<table>
<thead>
<tr>
<th>Design 1</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n = 100$</td>
<td>$n = 200$</td>
<td>$n = 400$</td>
<td>$n = 800$</td>
<td>$n = 1600$</td>
<td></td>
</tr>
<tr>
<td>Median</td>
<td>0.699</td>
<td>0.699</td>
<td>0.692</td>
<td>0.694</td>
<td>0.693</td>
<td></td>
</tr>
<tr>
<td>MAE</td>
<td>0.240</td>
<td>0.165</td>
<td>0.114</td>
<td>0.084</td>
<td>0.059</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.715</td>
<td>0.702</td>
<td>0.696</td>
<td>0.695</td>
<td>0.693</td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>0.369</td>
<td>0.252</td>
<td>0.173</td>
<td>0.123</td>
<td>0.085</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Design 2</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n = 100$</td>
<td>$n = 200$</td>
<td>$n = 400$</td>
<td>$n = 800$</td>
<td>$n = 1600$</td>
<td></td>
</tr>
<tr>
<td>Median</td>
<td>0.696</td>
<td>0.703</td>
<td>0.692</td>
<td>0.689</td>
<td>0.695</td>
<td></td>
</tr>
<tr>
<td>MAE</td>
<td>0.413</td>
<td>0.282</td>
<td>0.199</td>
<td>0.139</td>
<td>0.100</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.691</td>
<td>0.720</td>
<td>0.701</td>
<td>0.694</td>
<td>0.695</td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>0.639</td>
<td>0.443</td>
<td>0.303</td>
<td>0.214</td>
<td>0.149</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Design 3</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n = 100$</td>
<td>$n = 200$</td>
<td>$n = 400$</td>
<td>$n = 800$</td>
<td>$n = 1600$</td>
<td></td>
</tr>
<tr>
<td>Median</td>
<td>0.691</td>
<td>0.692</td>
<td>0.695</td>
<td>0.692</td>
<td>0.694</td>
<td></td>
</tr>
<tr>
<td>MAE</td>
<td>0.302</td>
<td>0.202</td>
<td>0.137</td>
<td>0.098</td>
<td>0.069</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.711</td>
<td>0.710</td>
<td>0.701</td>
<td>0.696</td>
<td>0.695</td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>0.454</td>
<td>0.307</td>
<td>0.209</td>
<td>0.144</td>
<td>0.103</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Design 4</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n = 100$</td>
<td>$n = 200$</td>
<td>$n = 400$</td>
<td>$n = 800$</td>
<td>$n = 1600$</td>
<td></td>
</tr>
<tr>
<td>Median</td>
<td>0.687</td>
<td>0.695</td>
<td>0.696</td>
<td>0.696</td>
<td>0.692</td>
<td></td>
</tr>
<tr>
<td>MAE</td>
<td>0.288</td>
<td>0.201</td>
<td>0.138</td>
<td>0.097</td>
<td>0.067</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.706</td>
<td>0.702</td>
<td>0.701</td>
<td>0.699</td>
<td>0.693</td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>0.438</td>
<td>0.302</td>
<td>0.209</td>
<td>0.146</td>
<td>0.100</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Design 5</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n = 100$</td>
<td>$n = 200$</td>
<td>$n = 400$</td>
<td>$n = 800$</td>
<td>$n = 1600$</td>
<td></td>
</tr>
<tr>
<td>Median</td>
<td>0.884</td>
<td>1.008</td>
<td>1.046</td>
<td>1.055</td>
<td>1.063</td>
<td></td>
</tr>
<tr>
<td>MAE</td>
<td>0.521</td>
<td>0.406</td>
<td>0.361</td>
<td>0.362</td>
<td>0.370</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.761</td>
<td>1.012</td>
<td>1.080</td>
<td>1.082</td>
<td>1.073</td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>0.948</td>
<td>0.685</td>
<td>0.540</td>
<td>0.476</td>
<td>0.428</td>
<td></td>
</tr>
</tbody>
</table>
Figure 1: Density of Estimation Error for Estimator and Its Log for Design 1

Figure 2: Density of Estimation Error for Estimator and Its Log for Design 2
Figure 3: Density of Estimation Error for Estimator and Its Log for Design 3

Figure 4: Density of Estimation Error for Estimator and Its Log for Design 4
Figure 5: Density of Estimation Error for Estimator and Its Log for Design 5
# Working Paper Series

A series of research studies on regional economic issues relating to the Seventh Federal Reserve District, and on financial and economic topics.

<table>
<thead>
<tr>
<th>WP-06-01</th>
<th>U.S. Corporate and Bank Insolvency Regimes: An Economic Comparison and Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Robert R. Bliss and George G. Kaufman</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WP-06-02</th>
<th>Redistribution, Taxes, and the Median Voter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Marco Bassetto and Jess Benhabib</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WP-06-03</th>
<th>Identification of Search Models with Initial Condition Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gadi Barlevy and H. N. Nagaraja</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WP-06-04</th>
<th>Tax Riots</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Marco Bassetto and Christopher Phelan</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WP-06-05</th>
<th>The Tradeoff between Mortgage Prepayments and Tax-Deferred Retirement Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gene Amromin, Jennifer Huang, and Clemens Sialm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WP-06-06</th>
<th>Why are safeguards needed in a trade agreement?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Meredith A. Crowley</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WP-06-07</th>
<th>Taxation, Entrepreneurship, and Wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Marco Cagetti and Mariacristina De Nardi</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WP-06-08</th>
<th>A New Social Compact: How University Engagement Can Fuel Innovation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Laura Melle, Larry Isaak, and Richard Mattoon</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WP-06-09</th>
<th>Mergers and Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Craig H. Furfine and Richard J. Rosen</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WP-06-10</th>
<th>Two Flaws in Business Cycle Accounting</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lawrence J. Christiano and Joshua M. Davis</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WP-06-11</th>
<th>Do Consumers Choose the Right Credit Contracts?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sumit Agarwal, Souphala Chomsisengphet, Chunlin Liu, and Nicholas S. Souleles</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WP-06-12</th>
<th>Chronicles of a Deflation Unforetold</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>François R. Velde</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WP-06-13</th>
<th>Female Offenders Use of Social Welfare Programs Before and After Jail and Prison:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Does Prison Cause Welfare Dependency?</td>
</tr>
<tr>
<td></td>
<td>Kristin F. Butcher and Robert J. LaLonde</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WP-06-14</th>
<th>Eat or Be Eaten: A Theory of Mergers and Firm Size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gary Gorton, Matthias Kahl, and Richard Rosen</td>
</tr>
</tbody>
</table>
Working Paper Series (continued)

Do Bonds Span Volatility Risk in the U.S. Treasury Market?  
A Specification Test for Affine Term Structure Models
Torben G. Andersen and Luca Benzoni  
WP-06-15

Transforming Payment Choices by Doubling Fees on the Illinois Tollway  
Gene Amromin, Carrie Jankowski, and Richard D. Porter  
WP-06-16

How Did the 2003 Dividend Tax Cut Affect Stock Prices?  
Gene Amromin, Paul Harrison, and Steven Sharpe  
WP-06-17

Will Writing and Bequest Motives: Early 20th Century Irish Evidence  
Leslie McGranahan  
WP-06-18

How Professional Forecasters View Shocks to GDP  
Spencer D. Krane  
WP-06-19

Evolving Agglomeration in the U.S. auto supplier industry  
Thomas Klier and Daniel P. McMillen  
WP-06-20

Mortality, Mass-Layoffs, and Career Outcomes: An Analysis using Administrative Data  
Daniel Sullivan and Till von Wachter  
WP-06-21

The Agreement on Subsidies and Countervailing Measures:  
Tying One’s Hand through the WTO.  
Meredith A. Crowley  
WP-06-22

How Did Schooling Laws Improve Long-Term Health and Lower Mortality?  
Bhashkar Mazumder  
WP-06-23

Manufacturing Plants’ Use of Temporary Workers: An Analysis Using Census Micro Data  
Yukako Ono and Daniel Sullivan  
WP-06-24

What Can We Learn about Financial Access from U.S. Immigrants?  
Una Okonkwo Osili and Anna Paulson  
WP-06-25

Bank Imputed Interest Rates: Unbiased Estimates of Offered Rates?  
Evren Ors and Tara Rice  
WP-06-26

Welfare Implications of the Transition to High Household Debt  
Jeffrey R. Campbell and Zvi Hercowitz  
WP-06-27

Last-In First-Out Oligopoly Dynamics  
Jaap H. Abbring and Jeffrey R. Campbell  
WP-06-28

Oligopoly Dynamics with Barriers to Entry  
Jaap H. Abbring and Jeffrey R. Campbell  
WP-06-29

Risk Taking and the Quality of Informal Insurance: Gambling and Remittances in Thailand  
Douglas L. Miller and Anna L. Paulson  
WP-07-01
Working Paper Series (continued)

Fast Micro and Slow Macro: Can Aggregation Explain the Persistence of Inflation? 
Filippo Altissimo, Benoît Mojon, and Paolo Zaffaroni

Assessing a Decade of Interstate Bank Branching 
Christian Johnson and Tara Rice

Debit Card and Cash Usage: A Cross-Country Analysis 
Gene Amromin and Sujit Chakravorti

The Age of Reason: Financial Decisions Over the Lifecycle 
Sumit Agarwal, John C. Driscoll, Xavier Gabaix, and David Laibson

Information Acquisition in Financial Markets: a Correction 
Gadi Barlevy and Pietro Veronesi

Monetary Policy, Output Composition and the Great Moderation 
Benoît Mojon

Estate Taxation, Entrepreneurship, and Wealth 
Marco Cagetti and Mariacristina De Nardi

Conflict of Interest and Certification in the U.S. IPO Market 
Luca Benzoni and Carola Schenone

The Reaction of Consumer Spending and Debt to Tax Rebates — Evidence from Consumer Credit Data 
Sumit Agarwal, Chunlin Liu, and Nicholas S. Souleles

Portfolio Choice over the Life-Cycle when the Stock and Labor Markets are Cointegrated 
Luca Benzoni, Pierre Collin-Dufresne, and Robert S. Goldstein

Nonparametric Analysis of Intergenerational Income Mobility with Application to the United States 
Debopam Bhattacharya and Bhaskar Mazumder

How the Credit Channel Works: Differentiating the Bank Lending Channel and the Balance Sheet Channel 
Lamont K. Black and Richard J. Rosen

Labor Market Transitions and Self-Employment 
Ellen R. Rissman

First-Time Home Buyers and Residential Investment Volatility 
Jonas D.M. Fisher and Martin Gervais

Establishments Dynamics and Matching Frictions in Classical Competitive Equilibrium 
Marcelo Veracierto

Technology’s Edge: The Educational Benefits of Computer-Aided Instruction 
Lisa Barrow, Lisa Markman, and Cecilia Elena Rouse
Working Paper Series (continued)

The Widow’s Offering: Inheritance, Family Structure, and the Charitable Gifts of Women  
*Leslie McGranahan*  
WP-07-18

Demand Volatility and the Lag between the Growth of Temporary and Permanent Employment  
*Sainan Jin, Yukako Ono, and Qinghua Zhang*  
WP-07-19

A Conversation with 590 Nascent Entrepreneurs  
*Jeffrey R. Campbell and Mariacristina De Nardi*  
WP-07-20

*Meredith A. Crowley*  
WP-07-21

Health Capital and the Prenatal Environment: The Effect of Maternal Fasting During Pregnancy  
*Douglas Almond and Bhashkar Mazumder*  
WP-07-22

The Spending and Debt Response to Minimum Wage Hikes  
*Daniel Aaronson, Sumit Agarwal, and Eric French*  
WP-07-23

The Impact of Mexican Immigrants on U.S. Wage Structure  
*Maude Toussaint-Comeau*  
WP-07-24

A Leverage-based Model of Speculative Bubbles  
*Gadi Barlevy*  
WP-08-01

Displacement, Asymmetric Information and Heterogeneous Human Capital  
*Luojia Hu and Christopher Taber*  
WP-08-02

BankCaR (Bank Capital-at-Risk): A credit risk model for US commercial bank charge-offs  
*Jon Frye and Eduard Pelz*  
WP-08-03

Bank Lending, Financing Constraints and SME Investment  
*Santiago Carbó-Valverde, Francisco Rodríguez-Fernández, and Gregory F. Udell*  
WP-08-04

Global Inflation  
*Matteo Ciccarelli and Benoît Mojon*  
WP-08-05

Scale and the Origins of Structural Change  
*Francisco J. Buera and Joseph P. Kaboski*  
WP-08-06

Inventories, Lumpy Trade, and Large Devaluations  
*George Alessandria, Joseph P. Kaboski, and Virgiliu Midrigan*  
WP-08-07

School Vouchers and Student Achievement: Recent Evidence, Remaining Questions  
*Cecilia Elena Rouse and Lisa Barrow*  
WP-08-08
Working Paper Series (continued)

Does It Pay to Read Your Junk Mail? Evidence of the Effect of Advertising on Home Equity Credit Choices  
Sumit Agarwal and Brent W. Ambrose  
WP-08-09

The Choice between Arm’s-Length and Relationship Debt: Evidence from eLoans  
Sumit Agarwal and Robert Hauswald  
WP-08-10

Consumer Choice and Merchant Acceptance of Payment Media  
Wilko Bolt and Sujit Chakravorti  
WP-08-11

Investment Shocks and Business Cycles  
Alejandro Justiniano, Giorgio E. Primiceri, and Andrea Tambalotti  
WP-08-12

New Vehicle Characteristics and the Cost of the Corporate Average Fuel Economy Standard  
Thomas Klier and Joshua Linn  
WP-08-13

Realized Volatility  
Torben G. Andersen and Luca Benzoni  
WP-08-14

Revenue Bubbles and Structural Deficits: What’s a state to do?  
Richard Mattoon and Leslie McGranahan  
WP-08-15

The role of lenders in the home price boom  
Richard J. Rosen  
WP-08-16

Bank Crises and Investor Confidence  
Una Okonkwo Osili and Anna Paulson  
WP-08-17

Life Expectancy and Old Age Savings  
Mariacristina De Nardi, Eric French, and John Bailey Jones  
WP-08-18

Remittance Behavior among New U.S. Immigrants  
Katherine Meckel  
WP-08-19

Birth Cohort and the Black-White Achievement Gap: The Roles of Access and Health Soon After Birth  
Kenneth Y. Chay, Jonathan Guryan, and Bhaskar Mazumder  
WP-08-20

Public Investment and Budget Rules for State vs. Local Governments  
Marco Bassetto  
WP-08-21

Why Has Home Ownership Fallen Among the Young?  
Jonas D.M. Fisher and Martin Gervais  
WP-09-01

Why do the Elderly Save? The Role of Medical Expenses  
Mariacristina De Nardi, Eric French, and John Bailey Jones  
WP-09-02

Using Stock Returns to Identify Government Spending Shocks  
Jonas D.M. Fisher and Ryan Peters  
WP-09-03
Stochastic Volatility
*Torben G. Andersen and Luca Benzoni*

The Effect of Disability Insurance Receipt on Labor Supply
*Eric French and Jae Song*

CEO Overconfidence and Dividend Policy
*Sanjay Deshmukh, Anand M. Goel, and Keith M. Howe*

Do Financial Counseling Mandates Improve Mortgage Choice and Performance? Evidence from a Legislative Experiment
*Sumit Agarwal, Gene Amromin, Itzhak Ben-David, Souphala Chomsisengphet, and Douglas D. Evanoff*

Perverse Incentives at the Banks? Evidence from a Natural Experiment
*Sumit Agarwal and Faye H. Wang*

Pay for Percentile
*Gadi Barlevy and Derek Neal*

The Life and Times of Nicolas Dutot
*François R. Velde*

Regulating Two-Sided Markets: An Empirical Investigation
*Santiago Carbó Valverde, Sujit Chakravorti, and Francisco Rodriguez Fernandez*

Establishments Dynamics, Vacancies and Unemployment: A Neoclassical Synthesis
*Marcelo Veracierto*

The Price of Gasoline and the Demand for Fuel Economy: Evidence from Monthly New Vehicles Sales Data
*Thomas Klier and Joshua Linn*

Estimation of a Transformation Model with Truncation, Interval Observation and Time-Varying Covariates
*Bo E. Honoré and Luojia Hu*