

Barlevy, Gadi

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Federal Reserve Bank of Chicago

**A Leverage-based Model of  
Speculative Bubbles (Revised)**

*Gadi Barlevy*

WP 2011-07

# A Leverage-based Model of Speculative Bubbles (Revised)\*

Gadi Barlevy  
Economic Research Department  
Federal Reserve Bank of Chicago  
230 South LaSalle  
Chicago, IL 60604  
e-mail: gbarlevy@frbchi.org

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## Abstract

This paper examines whether theoretical models of bubbles based on the notion that the price of an asset can deviate from its fundamental value are useful for understanding phenomena that are often described as bubbles, and which are distinguished by other features such as large and rapid booms and busts in asset prices together with high turnover in asset ownership. In particular, I focus on risk-shifting models similar to those developed in Allen and Gorton (1993) and Allen and Gale (2000). I show that such models could explain these phenomena, and discuss under what conditions booms and speculative trading would emerge. In addition, I show that these models imply that speculative bubbles can be associated with low rather than high premia on loans, in accordance with observations on credit conditions during episodes in which asset prices boomed and crashed.

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# 1 Introduction

The large fluctuations in U.S. equity and housing prices over the past decade have led to renewed interest in the phenomenon of asset bubbles. Among academic economists, the term “bubble” is usually defined as an asset whose price differs from that asset’s fundamental value, i.e. the expected discounted value of its dividends. Since neither the rise nor fall in equity and housing prices were accompanied by obvious changes of comparable magnitude in the value of productive assets or home ownership, many economists suspect these episodes involved a departure between asset prices and fundamentals. However, among non-economists, what made these episodes stand out was not some presumption that assets may have traded at a price different from fundamentals, but the spectacular rise and fall in asset prices over relatively short periods and the seemingly frenzied trade in such assets.<sup>1</sup> This paper explores whether theoretical models of bubbles as deviations from fundamentals can capture these distinctive features. That is, are these theoretical models useful for understanding the episodes most associated with the term “bubble” even though they were designed to capture a conceptually distinct notion from the features that distinguish these episodes?

In what follows, I restrict my attention to a particular class of models that give rise to equilibrium bubbles. These models, known as agency or risk-shifting models of bubbles, were developed by Allen and Gorton (1993) and Allen and Gale (2000). The key feature of these models is that traders buy assets not with their own wealth but with funds provided by others. If borrowers face limited liability, they would be willing to pay more for assets than the expected value of the dividends they yield, since borrowers can default and shift any losses they incur to their lenders if these purchases turn out to be unprofitable.

To be sure, there are other models that give rise to equilibrium asset bubbles besides risk-shifting models. One example are monetary models, in which the existence of a bubble facilitates transfers between agents. Perhaps the best known example of such models are overlapping generations models following Samuelson (1958), Diamond (1965), and Tirole (1985). In these models, a bubble allows for intergenerational transfers as each generation buys assets from the generation that preceded it and then sells these same assets to the next generation of agents.<sup>2</sup> Another example are “greater-fool” models of bubbles, where agents buy assets they believe to be overvalued because they expect to profitably resell these assets to other agents who value the asset differently. These models include the work of Harrison and Kreps (1978) and Scheinkman and Xiong (2003), where agents think others are wrong about the underlying process for dividends, and

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<sup>1</sup>This is reflected in how the term “bubble” is defined among non-economists. Merriam-Webster.com defines it as “a state of booming economic activity (as in a stock market) that often ends in a sudden collapse,” while the New York Times Online Guide to Essential Knowledge defines it as a “market in which the price of an asset continues to rise because speculators believe it will continue to rise even further, until prices reach a level that is not sustainable; panic selling begins and the price falls precipitously.”

<sup>2</sup>Monetary models can arise in other settings as well. For example, Kocherlakota (1992) and Santos and Woodford (1997) offer examples of bubbles in models with a finite number of infinitely-lived agents based on Bewley (1980). Both go on to show that if agents face borrowing limits, bubbles can emerge on assets available in zero net supply. Since the historical episodes of so-called bubbles all involve assets available in positive net supply, this particular theory of bubbles seems less relevant here.

Allen, Morris, and Postlewaite (1993) and Conlon (2004), where agents are not sure what others know about dividends. The reason I focus specifically on risk-shifting models is that in these models credit allows bubbles to arise, in contrast to other models of bubbles where credit plays no essential role.<sup>3</sup> Since many of the historical episodes suspected to be bubbles were associated with easy access to credit, models of bubbles where credit is essential for bubbles would appear to be especially relevant for capturing these episodes.

Previous work on risk-shifting models by Allen and Gorton (1993) and Allen and Gale (2000) largely focused on showing that equilibrium bubbles are logically possible. However, their analysis leaves open the question of whether such models can capture the salient features of historical episodes often described as bubbles. For example, do these models generally predict booms and busts in asset prices, or do they only imply that assets are overvalued? Do these models generally predict the frenzied asset trading that we observe during these episodes, or do they imply what Hong and Sraer (2011) dub “quiet” bubbles whereby assets are overvalued but traded infrequently? Are these models consistent with the observation that these episodes tend to occur in periods of low interest rates and seemingly relaxed credit conditions? And do these models have other implications about episodes in which asset prices deviate from fundamentals?

This paper seeks to answer these questions. I show that risk-shifting models can give rise to both overvaluation and boom-bust patterns, but that the two are driven by distinct forces. The extent to which assets are overvalued depends on the total amount agents can borrow against the assets they purchase: The more funds traders can access, the more overvalued the asset becomes. By contrast, the rate at which asset prices appreciate over time depends on whether trading the asset is perceived to be risky: Rapid price appreciation can only occur if agents expect that there is a significant chance they will be unable to profit from selling the asset in the future. Even though rapid appreciation is associated with the asset becoming increasingly overvalued with time, the rate of price appreciation is not due to a secular increase in the forces that drive overvaluation, i.e. a rise in the amount of resources that traders can borrow over time. I also show that risk-shifting models can generate asset churning or what Harrison and Kreps (1978) refer to as speculative behavior, meaning that agents buy assets with the hope of profiting by selling them in the future. Whether bubbles are “quiet” and trade hands infrequently or “noisy” and churn often depends on both the extent to which the asset is overvalued and how risky agents perceive trading to be.

Finally, I show that risk-shifting models imply that overvaluation and speculation can be associated with low interest rates charged to borrowers, and somewhat paradoxically, especially to the borrowers who engage in the riskiest behavior. This result arises for several reasons. First, as risky assets become more overvalued, expected profits from trading these assets falls. This leads fewer agents to buy them, and so lenders expect a smaller fraction of their borrowers will default and can therefore charge lower rates. The emergence of speculation similarly exposes lenders to lower risk of default, but for a different reason: If borrowers

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<sup>3</sup>Of course, credit can be introduced into these other models of bubbles, but even if it amplifies underlying bubbles, credit is not what allows them to emerge. Caballero and Krishnamurthy (2006) and Farhi and Tirole (2011) incorporate credit in overlapping generations models, while Simsek (2011) incorporates it in the Harrison and Kreps (1978) model.

intend to sell their assets, some of the expected losses from the collapse in asset prices will be shifted on to the next buyer of the assets. A third reason is that lenders in the model try to steer speculators towards contracts with features that limit the losses from default incurred by the lender, e.g. smaller loans, and must charge low interest rates to draw speculators to these contracts. Hence, speculators may be offered credit at particularly low rates. More generally, the model predicts the occurrence of bubbles will be associated with the use of certain financial contracts designed to minimize the risk for lenders. Building on these insights, Barlevy and Fisher (2011) adapt the risk-shifting model to the case of the housing market, and confirm that cities where house prices appreciated especially quickly tended to rely on mortgages that were used sparingly elsewhere and are consistent with the types of credit arrangements predicted by their model.

The paper is structured as follows. Section 2 examines a one-period model, which illustrates the forces that lead to overvaluation and the connection between the extent of overvaluation and equilibrium interest rates. Section 3 extends this model to two periods so that we can examine price dynamics, speculation, and the relationship between speculation and equilibrium interest rates. In both of these sections, I exogenously restrict the set of contracts lenders can offer. Section 4 considers endogenous contracting, and examines what type of contracts will be offered when speculative bubbles arise. Section 5 concludes.

## 2 One-Period Model

I begin with a one-period model that illustrates how risk-shifting allows assets to trade above their fundamental value in equilibrium. This model is similar to the one-period model in Allen and Gale (2000). One important difference is that they assume agents can borrow as much as they like at the going interest rate, while in my model agents borrow a finite amount of resources in equilibrium. This distinction allows me to study the role of the amount traders can borrow in determining asset prices and interest rates.

Consider an economy where there is a risky asset, available in fixed supply, which cannot be sold short. Such supply restrictions are essential, or else supply would turn infinite if the price were ever above its fundamental value. The fixed supply of the asset can be viewed as a technological constraint on asset production. More generally, all I need is that the supply schedule be upward sloping, so supplying more units is increasingly costly. Short sales restrictions can arise from similar informational frictions to those I consider, e.g. agents who sell assets short cannot be trusted to deliver the asset they sell short or to replicate its payoffs. In what follows, I take short sales restrictions as given rather than derive them.

The risky asset pays a dividend  $d$  at the end of the period that is binomially distributed, specifically

$$d = \begin{cases} D > 0 & \text{with probability } \epsilon \\ 0 & \text{with probability } 1 - \epsilon \end{cases}$$

For example, the asset could represent a claim on the profits of a firm with a patent that may or may not pan out, and  $D$  represents the profits to the firm if the patent is successful. It will be natural to interpret

patents as succeeding independently of one another, so that information about aggregate outcomes will not be informative about any individual asset. Under this interpretation, the price implications that emerge from the model apply to individual assets rather than to the aggregate portfolio of all assets.

Since the supply of the asset is fixed, I can normalize the number of assets to 1. At the beginning of the period, all shares are endowed to agents henceforth known as original owners. These could be the inventors who come up with patentable ideas. A second group of agents, henceforth known as potential buyers, can buy these assets from their original owners. These potential buyers must finance at least part of their purchases with funds acquired from a third and final group of agents, henceforth known as creditors. Creditors can buy the assets as well, but will never prefer to do so in equilibrium. For simplicity, suppose potential buyers own no resources and must finance all of their purchases with funds acquired from creditors.

For bubbles to arise, some potential buyers must use the funds they secure for something other than purchasing risky assets. Otherwise, creditors would refuse to lend when assets are overvalued. I therefore allow some potential buyers to do something else with the resources they borrow. Specifically, I assume they can use their funds to operate a productive technology if they wish. Formally, suppose there are two types of potential borrowers, whom lenders cannot tell apart when they provide them with funds:

- Agents with access to a productive technology, whom I henceforth call entrepreneurs.
- Agents without access to a productive technology, whom I henceforth call non-entrepreneurs.

The production technology available to entrepreneurs can be summarized as follows. Entrepreneurs can produce  $R > 1$  units of output per unit of input, up to a fixed capacity of the input that I set to 1. The fact that  $R > 1$  will lead creditors to fund these agents and try to reap some of these returns for themselves. Since entrepreneurs can employ no more than one unit of resources profitably, lenders will never lend more than this amount to any single borrower. As noted earlier, this distinguishes the model from Allen and Gale (2000), where agents can borrow unlimited amounts.

**Remark 1:** Entrepreneurial activity need not correspond to physical production. Entrepreneurs could equally achieve the return  $R$  by buying assets and managing them more productively, or by buying assets as part of some arbitrage strategy others have yet to recognize, or by buying undervalued assets on the basis of private information. Indeed, Allen and Gorton (1993) model profitable borrowers as agents who trade assets on private information. Interpreting entrepreneurial activity this way makes the case for asymmetric information more plausible: All borrowers use the funds they borrow similarly, namely to buy assets, and lenders with little knowledge about asset markets may not be able to gauge *a priori* whether the assets borrowers propose to buy can be profitably deployed or whether borrowers are trying to profit at the lender's expense. When trading strategies are not transparent, as is true for some hedge funds, lenders will not even know which assets agents are buying. The latter case comes closest to the environment I analyze. ■

Let  $n$  denote the number of non-entrepreneurs and  $\phi$  denote the fraction of all potential buyers who are non-entrepreneurs, implying there must be  $(\phi^{-1} - 1)n$  entrepreneurs. Note that a change in  $n$  holding  $\phi$  fixed amounts to changing the total number of potential buyers, not just the number of non-entrepreneurs.

The market for funds operates as follows. There is a large number of potential creditors, ensuring the market will be competitive. Creditors offer contracts, and agents choose which contract to enter. Creditors cannot monitor what borrowers do with the funds once they obtain them or observe anything about the assets borrowers purchase. As suggested above, this can reflect limited information about what borrowers buy or an inability by lenders to assess the fundamentals of the assets their borrowers are buying. For now, I follow Allen and Gale (2000) in allowing creditors to only offer a particular type of debt contract, although later on in Section 3 I allow creditors to design contracts optimally and show that under some conditions, such a contract is optimal. In particular, I assume creditors lend out one unit of resources per borrower – they would never agree to lend more – for  $1 + r$  units at the end of the period.

**Remark 2:** The assumption that lenders cannot monitor what agents do with their funds may be somewhat plausible in equity markets when traders follow opaque strategies, but it is unsuited for the housing market, where lenders appraise the houses borrowers purchase and can observe whether they sell them. However, Barlevy and Fisher (2010) show that the risk-shifting model can be adapted to such environments. They model a housing market where some individuals derive high utility from homeownership because they can customize a home to their own tastes. These individuals serve the same role as entrepreneurs here: They are profitable borrowers who will not default on their loans. If preferences are private information, lenders will be unable to distinguish these buyers from those who buy for speculative reasons. Barlevy and Fisher show that the latter buyers can push the price of a house above its fundamental value, i.e. the social value of an additional house when housing supply is fixed. Since all agents buy the same asset, observing the asset is not an issue. The key is that there is other relevant information lenders cannot observe. ■

Let  $p$  denote the price of one share of the risky asset. The expected payoff from taking the one unit of resources agents can borrow and using it to buy  $1/p$  shares of the risky asset is given by

$$\epsilon \cdot \max\left(\frac{D}{p} - (1 + r), 0\right) \tag{1}$$

Hence, non-entrepreneurs can profit from borrowing and buying assets if  $p < D/(1 + r)$ , and earn nothing otherwise. Entrepreneurs who secure funds can instead use them to initiate production, earning

$$\max(R - (1 + r), 0) \tag{2}$$

Entrepreneurs will therefore produce if

$$\max(R - (1 + r), 0) > \epsilon \cdot \max\left(\frac{D}{p} - (1 + r), 0\right).$$

The payoffs in (1) and (2) assume default is costless to the borrower. More generally, a borrower may incur costs from not paying his obligation, e.g. stigma or loss of access to future credit. Suppose there were



a constant utility cost of default,  $k$ . In what follows, I consider the limiting case where  $k \rightarrow 0$ , so that I can continue to use the payoffs in (1) and (2) without having to keep track of  $k$ . All of my results extend to the case where  $k$  is strictly positive but small. At the same time, taking the limit as  $k$  tends to 0 is not equivalent to simply setting  $k = 0$ . This is because when  $k > 0$ , agents will refuse to borrow if they expect to default with certainty, e.g. a non-entrepreneur will never take a loan where  $(1+r)p > D$ . This property will be preserved in the limit as  $k \rightarrow 0$ . When  $k = 0$ , though, agents would be willing to take out loans they expect to default on with certainty. Looking at the limit as  $k \rightarrow 0$  thus rules out additional equilibria that can arise when  $k = 0$  but which are not robust to the introduction of small default costs.

An equilibrium for this economy corresponds to a price  $p$  for the asset and an interest rate  $r$  for borrowing that ensure the following conditions:

1. Demand for the asset from potential buyers is equal to the amount original owners are willing to sell
2. Creditors earn zero expected profits on the resources they lend out, and cannot earn positive profits by offering some other contract that borrowers strictly prefer.

Turning first to the asset market, supply for the asset is straightforward: The original owners value the asset at its expected payoff,  $\epsilon D$ , and so supply is 0 if  $p < \epsilon D$ , any amount in  $[0, 1]$  if  $p = \epsilon D$ , and 1 if  $p > \epsilon D$ . Demand for the asset is the total amount of resources buyers spend on the asset divided by the price  $p$ . Since in equilibrium each borrower can obtain one unit of resources, the total amount of wealth buyers bring to the asset market will depend on the number of agents who intend to buy assets. This will depend on the number of potential buyers, i.e. the  $n$  non-entrepreneurs and  $(\phi^{-1} - 1)n$  entrepreneurs, and on the return  $R$  on entrepreneurial production that determines what entrepreneurs want to do with the funds they obtain. For a fixed  $\phi$ , the equilibrium  $p$  and  $r$  will thus be functions of  $n$  and  $R$ .

Figure 1 illustrates how equilibrium depends on  $(n, R)$ . For each  $n$ , there is a cutoff  $R^*(n)$  such that if the return to entrepreneurial activity  $R < R^*(n)$ , production will not be sufficiently profitable to entice entrepreneurs given what lenders must charge to break even on their loans. Hence, in equilibrium, all agents who borrow would opt to buy assets. Lenders know this, and will only lend if the asset is not overvalued. For low values of  $R$ , then, the unique equilibrium is  $p = \epsilon D$ . Given that all borrowers purchase assets, lenders set  $1+r \geq 1/\epsilon$  to ensure they recover the entire dividend  $d$  and break even on their loans.

If the return to entrepreneurial activity is high, i.e. if  $R > R^*(n)$ , production will be profitable enough to entice entrepreneurs in equilibrium. In this case, lenders will compete to fund entrepreneurs. Since they cannot tell borrowers apart, though, they will have to take on non-entrepreneurs as well. Competition among lenders will drive  $1+r$  down until expected profits to lenders are zero. This implies  $1+r$  will be below  $1/\epsilon$ . Recall from (1) that non-entrepreneurs would be willing to buy risky assets as long as  $(1+r)p < D$ . When  $1+r < 1/\epsilon$ , the maximum amount they would pay for an asset will exceed the expected dividend

$\epsilon D$ . Whether the equilibrium  $p$  in fact exceeds  $\epsilon D$  depends on the number of non-entrepreneurs  $n$ . With only few buyers, i.e. when  $n$  is small, there will not be enough demand to raise the price above  $\epsilon D$ . But as more borrowers vie for the fixed supply of assets, which is the only way they can profit at the expense of lenders, they will bid up the price of the asset above  $\epsilon D$ . Thus, a bubble can emerge in equilibrium.

**Remark 3:** A few comments are in order on whether  $p > \epsilon D$  ought to be called a “bubble.” First, since borrowers who buy risky assets value the option to default, they would not view themselves as overpaying for the asset. One might therefore argue for including the option value to default as part of the fundamentals. But the original motivation for the way economists define a bubble presumably reflected a concern that a mispriced asset would convey the wrong signal to agents, e.g. potential producers of the asset might supply too much of it if it were overpriced. Hence, the fundamental value should reflect the gains to creating an additional asset. Since the borrower’s option value to default cancels the lender’s expected loss and adds nothing to social value, it should not be folded into the fundamental value. Another argument against using the term bubble is that lenders in the model effectively invest in two assets, risky assets purchased by non-entrepreneurs and production projects run by entrepreneurs. Since the two are inextricably bundled for lenders, who cannot invest in one without the other, they can arguably be viewed as a single, properly valued asset that offers no excess return to lenders. But this treats assets as coming in fixed proportions. If additional risky assets could be produced,  $p > \epsilon D$  would again convey the wrong signal, justifying treating the risky asset as a bubble. Finally, my model differs in several ways from other models of bubbles such as monetary models. For example, the equilibrium  $p$  here is unique, so if a bubble exists, there is no additional equilibrium with  $p = \epsilon D$ . Also, a bubble does not raise aggregate wealth, but merely redistributes resources among agents. Some may view these implications as contrary to what the notion of a bubble ought to capture. But the conventional definition for a bubble does not generically deliver these implications, and I only use the term bubble in the sense that price differs from fundamentals. ■

Formally, the equilibrium asset price  $p$  and the rate charged to borrowers  $r$  can be summarized as functions of  $(n, R)$  as follows, where the proof is provided in an Appendix.

**Proposition 1:** For each  $n$ , there exists a cutoff  $R^*(n)$  such that if  $R < R^*(n)$ , equilibrium is given by

$$p = \epsilon D, \quad r \geq 1/\epsilon - 1$$

and if  $R > R^*(n)$  the equilibrium is given by

$$\begin{aligned}
 p = \epsilon D, \quad r &= \frac{(1-\epsilon)\phi}{1-(1-\epsilon)\phi} && \text{if } n < \epsilon D \\
 p = n, \quad r &= \frac{(1-\epsilon)\phi}{1-(1-\epsilon)\phi} && \text{if } \epsilon D < n < (1-(1-\epsilon)\phi)D \\
 p = (1-(1-\epsilon)\phi_n^*)D, \quad r &= \frac{(1-\epsilon)\phi_n^*}{1-(1-\epsilon)\phi_n^*} && \text{if } n > (1-(1-\epsilon)\phi)D
 \end{aligned}$$

where  $\phi_n^*$  is the unique value below  $\phi$  that solves the quadratic equation

$$\phi_n^* = \frac{(1 - (1 - \epsilon) \phi_n^*) D}{(\phi^{-1} - 1) n + (1 - (1 - \epsilon) \phi_n^*) D}.$$

By comparison, Allen and Gale (2000) show that when agents can borrow without limit, assets must trade above fundamentals. Hence, they essentially focus on the case where  $n$  is large, i.e. the amount borrowed against assets is large, and  $p$  exceeds  $\epsilon D$ . Since my model allows for other cases, it offers additional insights on what determines the extent of overvaluation and how interest rates and asset prices are related.

These insights can be seen from comparative statics on how the equilibrium  $p$  and  $r$  respond to changes in various variables. Figure 2 illustrates how  $p$  and  $r$  vary with  $n$  when  $R > R^*(n)$ . I focus on these values for  $R$  since according to Proposition 1, the number of non-entrepreneurs  $n$  has no impact on the equilibrium  $p$  and  $r$  when  $R < R^*(n)$ . The solid lines trace out  $p$  and  $r$  for different values of  $n$ . The dashed lines show the same relationship when we raise the dividend  $d$  in the high state from  $D$  to  $D + \Delta$  for  $\Delta > 0$ .

Consider first how the price of the asset  $p$  varies with  $n$ . For low values of  $n$ , asset prices accord with standard asset pricing theory:  $p$  equals the fundamental value of the asset and is independent of  $n$ . For intermediate values of  $n$ , asset prices are determined according to what Allen and Gale (1994) describe in a different context as “cash-in-the-market pricing.” That is, the price of the asset is the ratio between the cash buyers bring to the asset market and the quantity of assets owners wish to sell.<sup>4</sup> At these intermediate values of  $n$ , the price of the asset is independent of the fundamentals: News that the expected dividend is higher will have no impact on the price of the asset. At still higher values of  $n$ , the price of the asset depends on both  $n$  and the fundamentals. The price is still set by cash-in-the-market pricing, but the amount of cash in the asset market depends on the expected payoff to buying assets, which in turn depends on fundamentals. Risk-shifting models thus admit the possibility that asset prices are governed by fundamentals, by the amount agents can collectively borrow against assets, or by both.<sup>5</sup> The key implication for my purposes, though, is that the extent to which assets are overvalued – the gap between  $p$  and  $\epsilon D$  – depends on the total amount traders can borrow against the asset. Intuitively, the more resources borrowers bring to the asset market, the more they will bid up the price as they compete to buy scarce risky assets in order to profit at the expense of their lenders. It may be tempting to infer from this that to generate growing bubbles in a dynamic version of the model, we need the amount agents collectively borrow to rise over time. As I discuss

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<sup>4</sup>Allen and Gale (1994) develop a model where agents choose between cash and assets in advance, and then a random number of agents are hit with an immediate need for liquidity and must sell their assets for cash. If more agents are hit with liquidity shocks than expected, assets trade *below* their fundamental value, with the price equal to the ratio of cash held by liquid agents and assets held by illiquid agents. By contrast, here cash corresponds to the amount of resources agents can borrow and then use to buy risky assets, and assets trade *above* their fundamental value.

<sup>5</sup>Chancellor (2000) argues that these different explanations for what determines asset prices date back to at least the 18th Century and John Law, who is often remembered for his role in the Mississippi Bubble. Chancellor writes: “John Law, who was also a land-bank projector, went even further and claimed that the price was simply the result of the interaction of supply and demand. Applied to the stock market, Law’s idea suggests that share prices are determined by liquidity (the supply of new funds to the market) rather than a reflection of inherent values.” (p45)

in the next section, this conjecture turns out to be incorrect: Price appreciation is not driven by the growth of new funds into the asset market, but by the inherent riskiness of asset trading.

Another insight of the model concerns the relationship between interest rates and asset prices. The reason assets can trade above their fundamental value is that borrowers are willing to overpay for the assets they buy because they can shift some of the risk from buying assets to lenders. Lenders will in turn charge borrowers a premium above the risk-free rate – which here is equal to zero – to cover these expected losses. This would suggest that bubbles should be associated with higher borrowing premia. But as evident from Figure 2 where  $p$  and  $r$  move in opposite directions as we vary  $n$ , more overvaluation will not necessarily be associated with higher premia. This observation is important, since the historical episodes described as bubbles often coincide with looser credit conditions. One of the key insights of the paper is that in risk-shifting models, overvaluation and speculation may in fact be associated with lower rates charged to borrowers, or more precisely with lower premia on borrowers over the risk free rate.<sup>6</sup>

To understand why overvaluation can be associated with low borrowing premia, observe that the more the asset is overvalued, the smaller the expected profits from buying risky assets. If this leads fewer agents to buy risky assets, lenders will be exposed to less default risk and can charge lower rates. In the model, once expected profits from buying risky assets turn to zero, only a strict fraction of non-entrepreneurs will buy assets in equilibrium. This is because I consider the limiting case where the cost of default  $k$  tends to 0. Since borrowers must not default when  $d = D$ , the debt obligation per asset  $(1 + r)p$  cannot exceed  $D$ . For  $n$  sufficiently large, this cannot be true if all non-entrepreneurs borrow and buy assets. Hence, as  $n$  rises, only a fraction of non-entrepreneurs can buy assets. While this particular mechanism for pushing non-entrepreneurs away from buying risky assets is somewhat special, the basic argument is more general: Higher asset prices reduce the profitability of buying risky assets and push potential borrowers to other activities. If borrowers have outside opportunities with strictly positive returns, the fraction of borrowers who gamble on risky assets will necessarily decline as the asset becomes increasingly more overvalued.<sup>7</sup>

To be sure, the intuition that lenders ought to charge higher premia when assets are overvalued has some merit. Consider what happens if we held the number of entrepreneurs fixed but reduced the number of non-entrepreneurs, i.e. we let both  $n$  and  $\phi$  tend to zero. From Proposition 1,  $p$  will tend to  $\epsilon D$  and  $r$  will tend to 0. That is, there will no bubble and loan rates will tend to the risk-free rate. If we instead let  $n$  and  $\phi$  both be bounded away from zero,  $r$  will be strictly positive, and, if  $n$  and  $R$  are sufficiently large, the price will exceed fundamentals. In other words, if the absence of a bubble corresponds to the absence

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<sup>6</sup>In all the comparative static exercises I consider, the risk-free rate is constant (and equal to 0). The predictions I consider for interest rates thus ignore any possible connections between asset prices and the risk-free rate. In particular, my analysis has nothing to say about how policies or shocks that drive down risk-free rates are related to bubbles.

<sup>7</sup>That implication in Figure 2 that  $r$  is globally non-increasing in  $n$  is not robust, and stems from the assumption that  $d$  has a bimodal distribution. If  $d$  had more than two possible realizations, a higher  $n$  could imply agents default in a larger number of states of the world, and interest rates would rise with  $n$  in some regions. Still, a higher  $n$  would eventually drive borrowers away from risky assets and would lead to lower borrowing premia.

of borrowers seeking to profit from shifting risk, borrowing premia will indeed be higher when the asset is overvalued than when it is not. But in general it is not true that lenders will not be concerned about risk-shifting when asset prices equal fundamentals. For example, if we lower  $n$  but hold  $\phi$  fixed, the price of the asset will equal its fundamental value, but non-entrepreneurs will still default if  $d = 0$ . Likewise, if we lower  $R$ , lenders will expect all borrowers to buy risky assets. In this case, the equilibrium interest rate will be especially high, since default risk will be at its highest given there will be no profitable borrowers to offset those who buy risky assets. Whether borrowing premia are high or low in the absence of a bubble is ambiguous, unlike the implication that more overvaluation must eventually lead to lower premia.

In the next section, I build on the one period model to address dynamic issues that are the ultimate interest of this paper. But the static model is itself useful for illustrating some features of risk-shifting models, e.g. the cash-in-the-market pricing aspect of these models, the role of the amount agents can borrow against assets in generating overvaluation, and the interest rate implications of these models.

### 3 Dynamics: a Two-Period Model

Since the distinguishing features of the historical episodes often cited as bubbles – asset prices boom and then crash, agents tend to churn assets – are dynamic, we need to move beyond a one period model to assess the relevance of risk-shifting models for understanding these episodes. This section extends the static model in the previous section to a dynamic environment. Although previous work on risk-shifting has considered dynamic models, these were limited in important ways. For example, Allen and Gorton (1993) and Allen and Gale (2000) consider dynamics, but both force agents to sell assets at exogenous dates rather than trade strategically. This not only sidesteps the question of whether traders will choose to churn assets in equilibrium, but can matter for price dynamics. For example, as I discuss below, Allen and Gorton (1993) find that the growth rate of asset prices is indeterminate, whereas here the price path is generically unique.

To keep things simple, I allow for only two periods, which are indexed by  $t \in \{1, 2\}$ . I try to retain as much of the structure of the one-period model above as possible. As in the static model, there is a fixed supply of risky assets that cannot be sold short and which is held by original owners at the beginning of period 1. These assets continue to pay out a single stochastic dividend  $d$  after all trade is done, which now corresponds to the end of date 2. The dividend  $d$  is equal to  $D$  with probability  $\epsilon$  and 0 otherwise. For now, suppose the dividend is only revealed when it is paid out. For reasons that will become clear below, I eventually allow  $d$  to be revealed before trade occurs at date 2.

Agents who can purchase these assets arrive in periods 1 and 2. I assume the date at which potential buyers arrive is exogenous, and that if they wish to buy assets, they must do so in the period in which they arrive. I could assume all potential buyers arrive in period 1 and then let them time their actions, but this

only complicates the analysis without adding much insight.<sup>8</sup> As in the static model, potential buyers own no resources and must secure resources from a third group, creditors. All three groups are risk-neutral, and none discount between periods. Neither assumption is essential.

As in the static model, I assume two types of potential buyers – entrepreneurs with an alternative use for funds and non-entrepreneurs who can only buy risky assets. Let  $n_t$  denote the number of non-entrepreneurs in period  $t$ . Suppose non-entrepreneurs make up a fraction  $\phi$  of potential borrowers in each period, so the number of entrepreneurs in date  $t$  is  $(\phi^{-1} - 1)n_t$ . Entrepreneurial production is similar to the static model: They can convert one unit of input into  $R > 1$  units of output, up to a limit of one unit of input. In line with the static model, I assume entrepreneurs must invest in the period in which they arrive, and that  $R$  only accrues when  $d$  is paid out, i.e. at the end of period 2.

Given entrepreneurs can only make payments at the end of date 2, I restrict creditors to debt contracts that provide agents with one unit of resources upon arrival and come due at the end of date 2, i.e. borrowers arriving at date  $t$  must repay  $1 + r_t$  at the end of date 2. I return to the question of whether these contracts are optimal in Section 4. Letting  $p_t$  denote the price of the asset at date  $t$ , an equilibrium is a path  $\{(p_t, r_t)\}_{t=1}^2$  such that in each period (1) the market for assets clears, and (2) creditors earn zero expected profits and no creditor can earn positive profits by offering a contract that borrowers strictly prefer.

By design, the two-period and one-period models only differ in terms of the way asset markets operate. These differences can be summarized as follows. First, in the two-period model original owners who do not sell their assets at date 1 have another opportunity to sell at date 2. Second, agents who bought assets at date 1 can choose whether to sell them at date 2. Neither feature arises with only one period. Nevertheless, equilibrium in the two-period model sketched out above is identical to the equilibrium of a one-period model in which the number of potential buyers is the same as the total number of buyers across the two periods.

**Proposition 2:** In equilibrium,  $p_1 = p_2 = p$  and  $r_1 = r_2 = r$  where  $p$  and  $r$  correspond to the price and interest rate in the one-period model with  $n = n_1 + n_2$ . There is no speculation in equilibrium: Agents who buy the asset in date 1 do not resell the asset in date 2.

The reason the dynamic model delivers the same outcome as a static model in which all potential borrowers arrive in the same period is that the original owners can choose when to sell their assets, and they act to ensure the price does not appreciate over time. In particular, if  $p_2$  were to exceed  $p_1$ , the original owners would wait to sell the asset in period 2. But since non-entrepreneurs who show up at date 1 want to buy the asset, the market wouldn't clear. Hence, the price of the asset must be the same in both periods. This in turn rules out equilibrium speculation, since agents who buy the asset at date 1 can only benefit from

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<sup>8</sup>In particular, allowing agents to time their actions either results in a corner solution where all agents act in the same period, or it introduces additional endogenous variables – the fraction who choose a given action in the first period – as well as additional equilibrium conditions – specifically, indifference conditions about when to act.

selling it for a strictly higher price at date 2. Only original owners agree to sell assets, and rather than sell them all in one period as in the static model, they now liquidate their holdings over two periods.<sup>9</sup>

Before I turn to the question of how to modify the model to potentially allow for price booms and speculative behavior, it is worth pointing out that the result of Proposition 2 does not depend on how  $n_1$  compares with  $n_2$ . That is, even though in the static model a larger number of traders will lead to more overvaluation, it need not follow that an increase in the number of traders between dates 1 and 2 will lead to a growing bubble and rising asset prices. This is because in the dynamic model, the arrival of more traders at date 2 will push up the price of the asset at date 2, but will also raise the price in period 1 as original owners hold out for a higher price before agreeing to sell their assets. This observation demonstrates that price appreciation, if and when it emerges, is due to distinct forces from those responsible for overvaluation.<sup>10</sup>

Since the simplest extension of the one-period model fails to generate asset price booms and speculative behavior, we need to enrich the model in some way for these to possibly arise. One obvious direction is to allow for uncertainty, especially given that the reason prices fail to grow in the model above is that agents can perfectly anticipate conditions at date 2. To gain some insight on what type of uncertainty is needed, let us reinterpret the equilibrium restriction that  $p_1 = p_2$  in the deterministic model as a restriction on the extent of overvaluation. Note that the fundamental value of the asset  $f_t$  in each period  $t$  is just  $\epsilon D$ . The fact that  $p_1 = p_2$  in equilibrium thus implies that the bubble term  $b_t = p_t - f_t$  satisfies

$$\begin{aligned} b_1 &= p_1 - \epsilon D \\ &= p_2 - \epsilon D = b_2 \end{aligned}$$

That is, in the absence of uncertainty, the equilibrium bubble term must be equal in the two periods. Recall that I assumed no discounting, so the risk-free interest rate is zero. Hence, in the absence of uncertainty, the bubble term grows at the risk-free rate. But this is precisely the Blanchard and Watson (1982) insight that with rational agents, a deterministic bubble must grow at the risk-free rate.<sup>11</sup> Blanchard and Watson go on to argue that if the bubble might not survive into the next period, then conditional on not bursting, the bubble will grow faster than the risk-free rate. Intuitively, if the asset is overvalued, selling it is profitable for an agent. If there is a risk the asset will not remain overvalued next period, then there must be some

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<sup>9</sup>More generally, prices must leave original owners indifferent as to when they sell. This implies prices must grow at rate  $\beta^{-1} - 1$  where  $\beta$  denotes the time-discount factor. The reason prices are constant in my model is that I assume no discounting, i.e.  $\beta = 1$ . If  $\beta < 1$ , prices would rise, but creditors would demand at least  $\beta^{-1} - 1$  interest on one-period loans, so speculation remains unprofitable. Ignoring discounting is thus inconsequential: It is not responsible for the absence of speculation, and all of my results on price booms can be interpreted as applying to price appreciation in excess of the risk-free rate.

<sup>10</sup>Note that if the original owners were forced to sell their assets at date 1, as in Allen and Gale (2000),  $p_1$  and  $p_2$  would be determined by cash-in-the-market pricing, and  $n_2 > n_1$  would in fact imply price appreciation. This reinforces the point that letting agents trade strategically is important for understanding price dynamics in these models, since otherwise dynamic models will artificially take on the properties of related static models.

<sup>11</sup>Blanchard and Watson's result was partly anticipated by Diamond (1965), who showed that the price of an intrinsically worthless bubbly asset must grow at the risk-free rate. However, since the asset he considered had no inherent value, it was not immediately obvious from his analysis whether the restriction applies to the total price of the asset or the bubble component.

offsetting reward that compensates those who wait to sell. This compensation accrues as capital gains if the bubble survives. Hence, the type of uncertainty needed to generate a growing bubble involves uncertainty about whether the bubble component will survive from one period to the next.

While Blanchard and Watson allow the bubble to burst exogenously, in my model overvaluation arises endogenously and its collapse must similarly arise endogenously. To motivate the particular way I let the bubble burst, note that there is already a sense in which a bubble bursts even in the one-period model: Once  $d$  is revealed, borrowers can no longer use the asset to earn rents from lenders. If we allowed agents to trade the asset after  $d$  was revealed but before it materializes, the asset could only trade at  $p = d$ . That is, the bubble must burst once  $d$  is revealed. Suppose then that  $d$  is revealed before the asset market opens at date 2 with some probability  $q > 0$ . In the state of the world where  $d$  is revealed, the price of the asset will either rise to  $D$  or fall to 0, but either way the bubble term will vanish. In particular, the bubble can burst even as the price of the asset rises. This may seem odd to readers used to monetary bubbles where the fundamental value of the asset is zero and the bursting of a bubble is equivalent to a price collapse. But when the fundamental value of the asset can vary over time, there is nothing that requires asset prices to fall when bubbles burst. If  $d$  is revealed to equal  $D$ , the owner of the asset will experience a capital gain, but he will still incur some loss as the option to sell the asset at this high price becomes worthless; there is no difference between selling the asset for  $p = D$  and consuming the dividend  $D$  directly. The option to sell assets is only valuable if agents can sell at an inflated price, which is why an agent who holds on to the asset must be compensated for the risk that  $d$  will be revealed early regardless of what value  $d$  assumes.

Although I focus on the arrival of news on  $d$  as the triggering event for the bubble bursting, there can be other triggering events. For example, I could have assumed that the number of traders who arrive at date 2 is random. Specifically, suppose no traders show up in date 2 with probability  $q$ , and with probability  $1 - q$  the number of traders is  $n_2$ . This will be nearly identical to the case I consider, with the only difference being the price that clears the market if the bubble bursts. The reason I use early revelation of  $d$  as the triggering event is that this makes it clear that price appreciation can occur independently of how many traders arrive at date 2.

In characterizing the equilibrium path  $\{p_t, r_t\}_{t=1}^2$  for the case where  $d$  can be revealed early, it will be useful to restrict some of the parameters in advance to reduce the number of cases to consider. First, I assume that the return to entrepreneurial production  $R$  is large, specifically

$$R > 1 + \frac{1 - \epsilon}{1 - \phi(1 - \epsilon)} \quad (3)$$

This restriction ensures that entrepreneurial activity is sufficiently profitable that entrepreneurs will prefer to produce in equilibrium. Second, I assume that the total number of traders  $n_1 + n_2$  is not too large, i.e.

$$n_1 + n_2 < (1 - (1 - \epsilon)\phi)D \quad (4)$$

This restriction limits how much non-entrepreneurs can bid up the price of the asset, and ensures that in equilibrium they can earn positive profits pretending to be entrepreneurs and buying risky assets.



Proposition 3 below characterizes the equilibrium for all pairs  $(n_1, n_2) \in \mathbb{R}_{++}^2$  given (3) and (4). The proof of the proposition is contained in the Appendix. Here, I describe the equilibrium heuristically. Since (3) ensures entrepreneurs will produce in equilibrium, all agents will be able to secure funding. All  $n_1$  non-entrepreneurs who arrive at date 1 will then borrow and buy risky assets. If  $d$  is revealed, only entrepreneurs will want to borrow at date 2. Agents who bought assets at date 1 will either default or repay, depending on the realization of  $d$ . If  $d$  remains uncertain, non-entrepreneurs who arrive at date 2 will borrow to buy risky assets. The only question in this case is at what prices assets trade and who sells assets at date 2.

To tackle these questions, start with period 2. At the beginning of the period, assets can be held by both original owners who have yet to sell and non-entrepreneurs who bought assets in period 1. If  $d$  is revealed, the equilibrium price will equal  $d$  and trade will be indeterminate. If  $d$  remains uncertain, non-entrepreneurs who arrive at date 2 will want to buy assets, so demand will equal  $n_2/p_2$ . Graphically, this demand schedule corresponds to a hyperbola. As for the supply of assets, since all  $n_1$  non-entrepreneurs who arrive at date 1 buy assets,  $n_1/p_1$  shares will be owned by agents who arrived at date 1 while the remaining  $1 - n_1/p_1$  shares will be held by original owners. The latter would sell their assets for a price of  $\epsilon D$  or above. As for a trader who bought assets in period 1, let  $r_1$  denote the interest rate he faces. If he held on to his assets, his expected profits would equal

$$\epsilon \left( \frac{D}{p_1} - (1 + r_1) \right)$$

If he instead sold his assets at date 2, he would earn

$$\frac{p_2}{p_1} - (1 + r_1)$$

Comparing the two, the agent would be willing to sell the asset if

$$p_2 \geq \epsilon D + (1 - \epsilon)(1 + r_1)p_1 \tag{5}$$

Agents who bought assets at date 1 thus require a higher price to sell than original owners, since they must both cover their debt obligation and be compensated for the rents they earn at their lenders' expense. The supply of the asset is thus an upward sloping step function, where the steps correspond to the reservation prices of different agents. Figure 3 illustrates the supply and demand curves for the asset if all non-entrepreneurs face the same interest rate  $r_1$  at date 1, and the different ways these curves can intersect.<sup>12</sup> These intersections correspond to different equilibria, which can be partitioned as follows:

- a. At least some original owners hold on to assets until the end of date 2. In this case,  $p_2 = \epsilon D$ .
- b. All original owners sell by date 2, but no non-entrepreneur who bought at date 1 sells at date 2.

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<sup>12</sup>Figure 3 only illustrates the different ways supply and demand can intersect, *not* that different equilibria correspond to shifts of the demand curve against a stable supply curve. To the contrary, in equilibrium  $p_1$  depends on the expectation of  $p_2$ , so changes in  $n_2$  can affect the supply curve. In addition, as I discuss later in the text, when only a fraction of those who bought assets at date 1 sell at date 2, two distinct rates  $r_{1*} < r_1^*$  will be offered in equilibrium. Figure 3 thus fails to depict supply correctly for case (c), although the set of equilibria can still be partitioned into the distinct cases (a)-(d) in the text.

- c. All original owners sell by date 2, and some non-entrepreneurs who bought at date 1 sell at date 2.
- d. All original owners and all non-entrepreneurs who bought at date 1 sell at date 2.

Proposition 3 shows that the set  $\{(n_1, n_2) \in \mathbb{R}_{++}^2 : n_1 + n_2 < (1 - (1 - \epsilon)\phi)D\}$  can be partitioned into four regions each uniquely associated with one of these equilibria. This partition is illustrated graphically in Figure 4, where region A corresponds to the values  $(n_1, n_2)$  for which the equilibrium is of type (a), region B corresponds to the values  $(n_1, n_2)$  for which the equilibrium is of type (b), and so on. The precise expressions for equilibrium asset prices and interest rates in these cases are given in the Appendix.

**Proposition 3:** Suppose  $R > 1 + \frac{1 - \epsilon}{1 - (1 - \epsilon)\phi}$ . Then for  $(n_1, n_2) \in \mathbb{R}_{++}^2$  such that  $n_1 + n_2 < (1 - (1 - \epsilon)\phi)D$ , an equilibrium exists and is almost surely unique.

Armed with the characterization of equilibrium in Proposition 3, we can now return to the questions posed in the Introduction: When does the model generate booms and busts in asset prices? When does it generate speculation? What does it imply about speculative bubbles and interest rates?

I begin with the implications for asset prices. As in the static model, as long as the total number of non-entrepreneurs  $n_1 + n_2$  exceeds the threshold  $\epsilon D$ , the price of the asset will exceed the fundamental value as long as  $d$  remains uncertain. Since  $d$  is always uncertain at date 1, it follows that  $p_1 > \epsilon D$ . Moving on to period 2, if  $d$  is revealed before trade commences, the price will equal  $d$ . Hence, the price starts out somewhere between  $\epsilon D$  and  $D$  and either falls to 0 or rises to  $D$ . The model can therefore generate a collapse in asset prices, specifically if  $d$  is revealed to be low. Compared with the case where  $p_1$  is equal to fundamentals, the decline in price will be larger in absolute terms the more overvalued is the asset, and would be larger in proportional terms if the lower realization for  $d$  were positive rather than 0. Hence, the presence of a bubble implies a bigger price collapse in response to bad news about the asset.

If  $d$  remains uncertain at the beginning of date 2, the proof of Proposition 3 implies the equilibrium price  $p_2$  will be higher than price  $p_1$ , i.e. if the bubble doesn't burst, the asset will become more overvalued. In other words, price booms are also possible. The rate at which the price appreciates depends on what happens in period 1. If  $n_1/p_1 < 1$ , some original owners continue to hold the asset into period 2. But since  $n_1 > 0$ , some original owners must have sold their assets in period 1. Original owners must therefore be indifferent between selling in period 1 and waiting to sell the asset in period 2, i.e.

$$p_1 = q\epsilon D + (1 - q)p_2 \tag{6}$$

Defining a bubble  $b_t$  as the gap between the price and the fundamentals  $p_t - f_t$ , we can rewrite (6) as

$$\epsilon D + b_1 = q\epsilon D + (1 - q)(\epsilon D + b_2)$$

which reduces to

$$b_2 = \frac{1}{1 - q}b_1$$

As long as not all original owners sell the asset in period 1, then, the growth rate of the bubble term – which determines the rate growth rate in the price of the asset – is uniquely determined: The bubble either collapses to 0 if  $d$  is revealed, or grows at rate  $q/(1 - q)$  otherwise.<sup>13</sup> If all of the original owners sell the asset at date 1, so that  $n_1/p_1 = 1$ , the rate at which the bubble grows is unique, but will no longer equal  $q/(1 - q)$ . Since the original owners must agree to sell their holdings at date 1, condition (6) now becomes

$$p_1 \geq q\epsilon D + (1 - q)p_2$$

which tells us that

$$b_2 \leq \frac{1}{1 - q}b_1$$

The risk that the bubble bursts thus governs the maximal rate at which the bubble can grow, even if the exact rate depends on  $n_1$  and  $n_2$ . Hence, a necessary condition for a price boom is that agents believe they face a risk that the asset may cease to be overvalued, i.e.  $q > 0$ . At first, this may seem to contradict conventional wisdom that asset market booms are often associated with excessive optimism about future asset prices. However, there is some evidence that at the very least such exuberance is not universally shared. For example, Temin and Voth (2004) document how Hoare’s Bank traded shares in the South Sea Company in 1720 even though they viewed it as risky given the terms at which they were willing to lend to others against the same asset. More recently, Citigroup CEO Charles Prince’s infamously stated in the *Financial Times* on July 9, 2007 that “When the music stops, in terms of liquidity, things will be complicated. But as long as the music is playing, you’ve got to get up and dance. We’re still dancing.” More work is necessary to gauge how aware agents are of the risks they face during supposed bubble episodes.

Next, I turn to the implications of the model for speculative behavior: When would agents churn assets, buying in one period and selling in another? Unlike my results for price appreciation, here the work of Blanchard and Watson (1982) offers little guidance. In particular, in their analysis individuals are always indifferent between selling and holding on to the asset, and so their model says little on whether agents actively trade assets. By contrast, my model is specific about whether speculative behavior will occur. In particular, speculation can occur, but only under certain conditions, namely in equilibria of type (c) and (d) when some agents who buy the asset at date 1 also sell the asset at date 2. Figure 4 reveals that such equilibria are associated with certain parameters, specifically high values of  $n_1$ ,  $n_2$ , or both. Essentially, market clearing will require agents who bought assets at date 1 will to sell at date 2 if either a large fraction of those who own the asset at date 2 bought at date 1, or if demand for assets at date 2 is sufficiently large that it cannot be supplied by the original owners alone. Since higher values of  $n_1$  and  $n_2$  also imply more overvaluation, this suggests speculation is more likely for more overvalued assets.

The existence of equilibrium speculation also requires that  $q$  be large, i.e. that there be enough risk that the bubble will burst early. Indeed, recall from Proposition 2 that when  $q = 0$ , speculative behavior can be

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<sup>13</sup>Note the contrast to Allen and Gorton (1993), where asset price growth is indeterminate. This is because they assume assets are indivisible and the number of buyers and sellers is always equal. In this case, any price between the reservation price of the seller and the reservation price of the buyer will clear the market. But this result is not robust.

ruled out. Formally, the role of  $q$  is reflected in the fact that the boundary of region (B) in Figure 4 varies with  $q$ . As we let  $q$  tend to 0, region (B) will encompass all of  $\{(n_1, n_2) \in \mathbb{R}_{++}^2 : n_1 + n_2 < (1 - (1 - \epsilon)\phi)D\}$ , and so speculation will not occur. Intuitively, the greater the risk that the bubble will collapse, the lower the price of the asset must be in period 1 to induce original owners to sell. At a lower price, non-entrepreneurs who buy the asset in period 1 can afford to buy more assets, and hence they command a greater share of the assets at date 2. If demand for the assets at date 2 is high enough, some of these non-entrepreneurs may have to sell their assets for the market to clear. Since  $q$  is related to the rate of price appreciation, this implies that other things equal, speculation is more likely to occur with faster asset price appreciation. Bubbles in risk-shifting models are thus not inextricably linked to speculative behavior; speculation is more likely to raise the more assets are overvalued and the more rapidly asset prices grow.<sup>14</sup>

Lastly, I turn to the implications of the model for interest rates. In equilibria of type (b) in which there is no speculative behavior, non-entrepreneurs hold on to their assets until the end of date 2, then repay in full if  $d = D$  and default if  $d = 0$ . The probability of recouping anything from non-entrepreneurs is  $\epsilon$ . Since entrepreneurs always repay their debt, the requirement that lenders earn zero expected profits yields

$$r_1 = \frac{(1 - \epsilon)\phi}{1 - (1 - \epsilon)\phi} \quad (7)$$

By contrast, in equilibria of type (d) in which non-entrepreneurs who buy at date 1 sell their assets at date 2 if the bubble survives, non-entrepreneurs repay their debt if either the bubble persists into period 2, which occurs with probability  $1 - q$ , or if the bubble bursts but  $d = D$ , which occurs with probability  $q\epsilon$ . Hence, the probability of recouping anything from them is  $(1 - q) + q\epsilon$ . The zero profit condition now implies

$$r_1 = \frac{q\phi(1 - \epsilon)}{1 - q\phi(1 - \epsilon)} \quad (8)$$

Speculative behavior thus lowers the premia charged to borrowers: Traders who buy assets at date 1 but sell them at date 2 shift the risk if  $d$  turns out to be low to whoever buys the asset from them (who in turn shift the risk to their lenders, who charge these losses to date-2 entrepreneurs). Hence, speculative bubbles in which assets trade hands multiple times reduce the risk of default for lenders who finance asset purchases early on. This is distinct from the result in the static model that more overvaluation will draw in fewer agents to buy risky assets and thus lead to lower interest rates. That said, the growth rate of the bubble is positively related to the probability the bubble bursts early,  $q$ , while the risk premium is negatively to  $q$ . Rapid asset price growth will therefore be associated with only slightly lower premia.

**Remark 4:** A few remarks are in order about equilibria of type (c), in which only some traders who bought assets at date 1 sell them at date 2. In this case, two distinct contracts will be offered in equilibrium. This is because if only one contract were offered, the fact that a strict fraction of agents sell at date 2 implies that those who bought at date 1 are indifferent under this contract between holding and selling their assets.

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<sup>14</sup>Hong and Sraer (2011) also examine whether equilibrium bubbles are likely to involve churning, although in a different model of bubbles. They focus on the role of the payoff structure to the asset rather than the factors I consider.

But in that case, lenders could earn positive profits by charging an infinitesimally lower rate to induce non-entrepreneurs to sell. The only possible equilibrium is one in which some borrowers are offered a low interest rate that corresponds to the rate in equilibria of type (d) in which all agents who buy assets at date 1 sell them at date 2, and any remaining borrowers are offered a higher rate that corresponds to the rate in equilibria of type (b) in which all agents who buy assets at date 1 hold on to them at date 2. Although borrowers prefer lower rates, they will be unable to convince lenders to offer them such contracts, and low-interest loans will be rationed. Formally, in equilibrium lenders who offer a high rate must believe that if they offered the lower rate, there is some probability non-entrepreneurs who borrow from them would hold on to their assets at date 2. Lenders will therefore find it unprofitable to lend at the low rate. These beliefs are sequentially rational, since the proof of Proposition 3 shows that at the lower interest rate, non-entrepreneurs who buy assets at date 1 are indifferent between holding and selling them at date 2. A similar issue arises in the next section, where lenders offer different contracts to entrepreneurs and non-entrepreneurs in equilibrium. This outcome is sustained by beliefs among lenders that non-entrepreneurs might accept the contract aimed at entrepreneurs if only offered that contract. ■

To recap, this section shows that dynamic models of risk shifting can give rise to some of the distinguishing features associated with historical episodes often branded as bubbles, e.g. booms and busts, speculative behavior, and low borrowing rates. Moreover, since these features only arise in certain conditions, the model yields testable implications that in principle could be used to gauge the plausibility of risk shifting models. For example, asset prices can only appreciate at a rate commensurate with the perceived risk of the bubble bursting. The collapse in asset prices must be linked to specific events that imply the asset can no longer be used to profit at the expense of lenders. Speculative behavior will only emerge when assets are significantly overvalued and when there is rapid price appreciation. Evidence against these predictions should call into question the relevance of risk-shifting models for explaining boom-bust episodes.

## 4 Optimal Contracts

Up to now, I have allowed creditors to offer only certain types of contracts. In this section, I relax this assumption and allow lenders to choose which contracts to offer. This serves two purposes. First, it offers a robustness exercise: Can speculative bubbles still emerge when lenders can design contracts optimally? Second, it reveals what types of contracting arrangements might emerge in these models when speculative bubbles arise. To preview my results, I find that endogenizing contracts makes it harder but not impossible for bubbles to occur: Lenders have a strong incentive to design contracts so as to avoid funding agents who plan to buy overvalued assets, but for some parameter values and some environments they will be unable to avoid such agents altogether. In these cases, lenders will design contracts to minimize losses from such agents, and the purchases of overvalued assets will tend to be financed with certain types of contracts.

I analyze the contracting problem using the two-period model from the previous section. As in that

section, I limit attention to the case where  $n_1$ ,  $n_2$ , and  $R$  satisfy (3) and (4), so that in equilibrium entrepreneurs are better off producing and non-entrepreneurs can earn rents by buying risky assets. Analyzing the choice of contracts requires me to be more explicit about what agents know than I have been so far. In line with my focus on debt contracts, I consider a costly state verification model as in Townsend (1979) and Gale and Hellwig (1985) that can give rise to such contracts. That is, I assume lenders cannot observe what borrowers do with their funds or anything about the assets they buy, but at the end of date 2, after  $d$  is paid out and output from production is realized, they can pay to learn what the agent did as well as their cash holdings. Agents cannot consume their cash prior to the audit. The auditing cost is assumed to equal a fraction  $\lambda \in (0, 1)$  of the borrower’s cash holdings. I impose this specification for analytical convenience, but it can also be interpreted to mean that is easier to audit agents with fewer resources to hide.

One important difference from the traditional costly-state verification model is that here borrowers differ *ex-ante* and not just *ex-post* after receiving funding. Lenders might therefore want to audit all borrowers regardless of their report to deter non-entrepreneurs from borrowing in the first place. This will not be optimal if the fraction of non-entrepreneurs  $\phi$  is small and the audit cost  $\lambda$  is large: Screening out non-entrepreneurs requires auditing with certainty, so auditing costs per borrower will be  $\lambda R$ , while the benefit of excluding non-entrepreneurs is proportional to  $\phi$ . I assume  $\phi$  is small and  $\lambda$  large in what follows.

Even when lenders opt not to audit, they might be able to identify entrepreneurs by designing the transfers in their contracts strategically. As the model is specified, the only way agents who chose to produce can prove themselves is if their cash holdings exceed the maximal cash holdings of agents who did not produce. If that were the case, lenders could avoid non-entrepreneurs altogether by requiring borrowers to transfer a large amount of resources at the end of date 2 that only an agent who produced could afford. A borrower who fails to do this would be audited and all of his cash holdings seized. The lender could then transfer resources back to the borrower to attain any desired repayment rate. Lenders could thus avoid taking on non-entrepreneurs, and consequently bubbles would not arise in the market for risky assets.

This way to screen out non-entrepreneurs only works under certain conditions. First, agents cannot engage in “window-dressing” through hidden borrowing that allows them to make payments they could not otherwise afford. But even if window-dressing is impossible, as I henceforth assume, producers do not always earn more cash than agents who chose not to produce. In particular, their cash holdings at the end of date 2 equal  $R$ . The most non-entrepreneurs who arrived at date  $t$  can have by the end of date 2 is  $D/p_t$ . If the latter exceeds  $R$ , lenders will not be able to force producers to reveal themselves, and may have to take on non-entrepreneurs if they wish to lend to entrepreneurs. Since I am interested in whether it is possible for bubbles to arise under endogenous contracting, I restrict attention to values of  $n_1$ ,  $n_2$ , and  $R$  that imply  $D/p_1$  and  $D/p_2$  exceed  $R$  so as not to rule out bubbles *a priori*. This requires two restrictions. First, to ensure the return to production  $R$  isn’t too large, I replace (3) with the more restrictive condition

$$\frac{1}{\epsilon} > R > 1 + \frac{1 - \epsilon}{1 - \phi(1 - \epsilon)} \tag{9}$$

Second, to ensure the price of the asset isn't too high in either period, I limit the total number of non-entrepreneurs by replacing (4) with

$$n_1 + n_2 < (1 - q) D/R + q\epsilon D \quad (10)$$

It is easy to verify that the second inequality in (9) together with (10) imply (4), so these restrictions define a subset of the set defined by (3) and (4).<sup>15</sup> The next lemma confirms these conditions ensure  $D/p_t > R$ .

**Lemma 1:** If  $n_1$ ,  $n_2$ , and  $R$  satisfy (9) and (10), then  $D/p_1 > R$  and  $D/p_2 > R$  in any equilibrium.

To describe the lender's contracting problem, I rely on the revelation principle which holds that any contract can be replicated with a direct mechanism in which borrowers report their private information and these reports trigger transfers between the two parties and an audit strategy at the end of date 2. For brevity, I only rigorously discuss the setup for a contract offers to those who arrive at date 1. The contract for an agent who arrives at date 2 can be readily inferred from my discussion. Figure 5 summarizes the reports agents must make at each point in time depending on the reports they previously issued.

The first report an agent must make occurs when he first arrives at date 1. This report, denoted  $\hat{\theta}_1$ , reflects whether the agent is an entrepreneur ( $\hat{\theta}_1 = e$ ) or a non-entrepreneur ( $\hat{\theta}_1 = n$ ). The contract then stipulates a transfer  $x_1(\hat{\theta}_1)$  from the creditor to the borrower. Since agents own no resources,  $x_1(\hat{\theta}_1) \geq 0$ .

The next instance in which the agent has private information comes at the end of period 1. At this point, the agent must issue a second report  $\hat{\theta}_2$  on what he did with the funds he obtained. Given my parametric assumptions above, individuals will have strict preferences on what to do with their funds, so I can assume agents allocate funds to only one activity. I therefore force the agent to report that he either initiated entrepreneurial activity ( $\hat{\theta}_2 = e$ ), bought assets ( $\hat{\theta}_2 = b$ ), or did nothing ( $\hat{\theta}_2 = \emptyset$ ). Since an agent cannot truthfully report both  $\hat{\theta}_1 = n$  and  $\hat{\theta}_2 = e$ , I assume an agent who reports  $\hat{\theta}_1 = n$  can only report  $\hat{\theta}_2 \in \{b, \emptyset\}$ . Let  $x_2(\hat{\theta}_1, \hat{\theta}_2)$  denote the transfer from the lender to the borrower at this stage. If the agent reports that he used his funds, i.e.  $\hat{\theta}_2 \in \{e, b\}$ , he would have no resources to transfer if he were truthful, so  $x_2(\hat{\theta}_1, \hat{\theta}_2) \geq 0$ . If the agent reports he did nothing, the constraint becomes  $x_1(\hat{\theta}_1) + x_2(\hat{\theta}_1, \hat{\theta}_2) \geq 0$ .

The next instance in which the agent may have private information is at date 2, after the asset market clears but before  $d$  and  $R$  are paid out. At this point, an agent who reported buying assets, i.e.  $\hat{\theta}_2 = b$ , must issue a report  $\hat{\theta}_3$  whether he sold his assets before  $d$  was revealed ( $\hat{\theta}_3 = p_2$ ), sold them after learning  $d$  is high ( $\hat{\theta}_3 = D$ ), sold them after learning  $d$  is low ( $\hat{\theta}_3 = 0$ ), or held on to them ( $\hat{\theta}_3 = h$ ). An agent who reported  $\hat{\theta}_2 \in \{e, \emptyset\}$  would have nothing to report if he were truthful, so without loss of generality I only let him report  $\hat{\theta}_3 = \emptyset$ . Let  $x_3(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3)$  denote the net transfer from the lender to the borrower.

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<sup>15</sup>It is also easy to check that these restrictions allow for the case  $n_1 + n_2 > \epsilon D$  which is necessary for a bubble to emerge.

Finally, at the end of date 2, the agent must issue one last report  $\hat{y}$  related to his earnings. If the initial transfer  $x_1(\hat{\theta}_1) > 0$ , it will be convenient to set  $\hat{y}$  to earnings divided by  $x_1(\hat{\theta}_1)$ . If  $x_1(\hat{\theta}_1) = 0$ , the agent cannot do anything anyway, so I can restrict him to reporting  $\hat{y} = 0$ . When  $x_1(\hat{\theta}_1) > 0$ , the report  $\hat{y}$  can take on five possible values:  $R - 1$ , if the agent chose to produce;  $p_2/p_1 - 1$ , if the agent bought and sold assets before learning  $d$ ;  $D/p_1 - 1$ , if  $d = D$  and the agent held on to the assets or if  $d$  was revealed early; 0, if the agent did nothing; and  $-1$ , if  $d = 0$  and the agent did not sell the asset before  $d$  was revealed. I let the agent report any of these  $\hat{y}$  regardless of previous reports. In particular, I need to let an agent who bought assets but pretended to produce come clean if he is unable to pay because  $d = 0$ . After these reports, the contract specifies a probability  $\sigma_{\hat{y}}(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{y})$  of auditing the agent. I require monitoring to be non-stochastic, i.e.  $\sigma_{\hat{y}} \in \{0, 1\}$ . The reason is that stochastic monitoring raises distracting technical issues which I remark on below. However, with suitable modifications to address these issues, my results carry over to the case of stochastic audits as well. The contract then stipulates a net transfer  $x_{\hat{y}}(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{y})$  if there is no audit and a net transfer  $x_y(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{y}, y)$  if there is an audit. Since the lender may want back-and-forth payments after the last report to verify an agent's type, in principle I should allow for one additional transfer after  $x_{\hat{y}}(\cdot)$ . Rather than introduce additional notation to capture such a transfer, I simply point out this transfer in the one instance where it will be necessary.

Now that I've described contracts, I can proceed to characterize which contracts will be used in equilibrium. Since such contracts must be optimal for the lender, I begin with some observations about optimal contracts. First, suppose a borrower announces he has no cash at the end of date 2, i.e.

$$x_1(\hat{\theta}_1) + x_2(\hat{\theta}_1, \hat{\theta}_2) + x_3(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3) + \hat{y}x_1(\hat{\theta}_1) = 0 \quad (11)$$

In this case, it will be optimal for the lender to audit the borrower. The reason is that auditing is costless if the agent is telling the truth, and only serves to punish agents who pretend to have zero cash holdings. Hence, there is no reason not to audit the agent and grab all of his cash if he is untruthful.<sup>16</sup>

Second, if agents cannot use hidden borrowing to make payments they cannot otherwise afford, a lender should use the size and timing of transfers to limit the misreports agents can make. In particular, he can check whether an agent who reports doing nothing, i.e.  $\hat{\theta}_2 = \emptyset$ , is truthful by asking him to transfer some resources at the end of date 1. This would only be feasible if the true  $\theta_2 = \emptyset$ . Subsequent transfers can then be used to replicate any desired terminal payoff for this type. Likewise, the lender can verify whether the agent truly sold his assets early at a positive price, i.e.  $\hat{\theta}_3 \in \{p_2, D\}$ , by asking him to transfer an amount in excess of  $x_1(\hat{\theta}_1)$  at the beginning of date 2. Lastly, the lender can verify whether the agent earned  $D$  per asset by requiring an agent who makes this report to transfer  $D/p_1$ , either early or late in date 2. The latter case is the one instance in which I need to allow an additional transfer after the agent reports  $\hat{y}$ .

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<sup>16</sup>Since auditing agents with zero cash holdings is costless, it is optimal to audit such a borrower with certainty, even if the lender can monitor with any probability between 0 and 1. In particular, the argument for stochastic monitoring as a way to reduce expected monitoring costs does not apply given the audit is costless when the agent is truthful.



Third, it will be optimal for the lender to audit the agent and seize all of his cash if the agent is ever inconsistent in his reports, e.g. announcing that he bought assets but then reporting  $\hat{y} = R - 1$ . Since  $\lambda < 1$ , auditing and punishing an agent is never costly for the lender, so there is no reason not to do so. This implies that in equilibrium, an agent who reports he chose to produce, i.e.  $\hat{\theta}_2 = e$ , will have to pay some amount if he reports his income is  $R - 1$ , and will be audited and left with no cash otherwise. Likewise, an agent who reports he bought assets, i.e.  $\hat{\theta}_2 = b$ , will have to pay some amount that may depend on what he says he did with the assets he bought, and will be audited and left with no cash otherwise.

We can use these observations to determine when the equilibrium contract will call for an audit. First, without loss of generality, an agent who announces  $\hat{y} = -1$  without contradicting past reports will be audited. This is because we can always take the equilibrium contract and construct an equivalent contract with  $x_2(\hat{\theta}_1, \hat{\theta}_2) = x_3(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3) = 0$  for reports consistent with a report of  $\hat{y} = -1$ , ensuring the agent will have no cash at the end of date 2 if the true  $y = -1$ . This allows the lender to audit at no cost if the agent is truthful while discouraging misreports. If we set  $x_{\hat{y}}(\cdot)$  to ensure the same terminal payoff to a truthful agent as in the original contract, this contract will do at least as well as the original equilibrium contract.

Next, suppose the agent reports  $\hat{y} = 0$  or  $\hat{y} = D/p_1 - 1$ . Since the lender can directly verify these reports, neither will be audited in equilibrium. The lender cannot verify  $\hat{y} = p_2/p_1 - 1$  in the same way, since an agent who sells his assets early at price  $D$  can pass himself as this type. Since lenders prefer non-entrepreneurs sell their assets if  $d$  is not revealed, they may want to audit such reports to reward agents. For now, I take no stand on whether this report will be audited, although below I show that in equilibrium it will not.

Finally, suppose the agent reports  $\hat{y} = R - 1$ , and suppose this report were audited. In that case, a lender could always offer the same contract without auditing agents who report  $\hat{\theta}_2 = e$  and  $\hat{y} = R - 1$ . To attract entrepreneurs, the lender could charge an agent who reports he produced slightly less than the original contract, ensuring they would continue to prefer production. While others may now be able to pass themselves off as entrepreneurs, if  $\phi$  is close to 0 and  $\lambda$  is close to 1, the lender would save more on auditing costs than lose in expected collections. Hence, auditing these reports could not have been an equilibrium.<sup>17</sup>

Given these audit rules, we can characterize equilibrium contracts as debt contracts. That is, an agent who arrives at date 1 reports his type  $\hat{\theta}_1 \in \{e, n\}$ , receives  $x_1^{\hat{\theta}_1}$  resources, and commits to a repayment schedule. An agent who reports  $\hat{\theta}_1 = e$  will be expected to produce, and so as noted above would be asked

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<sup>17</sup>A similar argument can be used to show there can be no equilibrium where lenders audit self-reported producers with positive probability, since the cost of a marginal increase in the audit probability exceeds the benefit for small  $\phi$ . The problem with allowing for stochastic monitoring is that when lenders can monitor with arbitrarily small probability, not auditing reports of  $\hat{y} = R - 1$  is not an equilibrium either. The reason is that a lender could pick off entrepreneurs by auditing such reports with a small probability, making these contracts less appealing to non-entrepreneurs. This reveals a non-existence problem akin to the one in Rothschild and Stiglitz (1976). Various ways to deal with this non-existence have been proposed. One example is to use a different equilibrium concept, such as the one proposed by Wilson (1977) in which the equilibrium contract cannot be improved upon only by contracts that remain profitable when unprofitable contracts are withdrawn. Under this definition, the equilibrium contract is the same as in the nonstochastic monitoring case. Mimra and Wambach (2011) provide a game-theoretic foundation for Wilson equilibria, so one could appeal to their argument to support this equilibrium.

to pay back some amount  $(1 + r_1^e) x_1^e$  at the end of date 2 or else be audited. An agent who reports  $\hat{\theta}_1 = n$  will be expected not to produce. In that case, the amount he owes depends on when he pays. If he waits until the end of period 2, meaning he reported holding on to his assets ( $\hat{\theta}_3 = h$ ), he will owe the lender  $x_1^n(1 + r_1^{n,\text{late}})$  and will be audited otherwise. If he pays at the beginning of date 2, meaning he reported selling assets for a positive price, he would owe  $x_1^n(1 + r_1^{n,D})$  if he sold at price  $D$  and a potentially different amount  $x_1^n(1 + r_1^{n,p_2})$  if he sold at price  $p_2$ . If he pays earlier still at date 1, meaning he reported doing nothing ( $\hat{\theta}_2 = \emptyset$ ), he would owe  $x_1^n(1 + r_1^{n,\emptyset})$ . That is, the borrower either receives  $x_1^e$  with a requirement to repay at rate  $r_1^e$  at the end of date 2, or he receives  $x_1^n$  with a schedule of rates  $\{r_1^{n,\emptyset}, r_1^{n,D}, r_1^{n,p_2}, r_1^{n,\text{late}}\}$ . This description abstracts from any back-and-forth transfers prior to settlement. However, these transfers only affect constraints that these rates must satisfy to ensure agents report truthfully. That is, these transfer do not deny that a contract can be summarized with a payment schedule, only that the schedule cannot be arbitrary. An important observation is that since agents who report  $\hat{\theta}_1 = n$  must at some point make a transfer that an agent who produced cannot, either because of timing or size, we need not worry about whether an agent who plans to produce will try to take the contract aimed at someone who reports  $\hat{\theta}_1 = n$ . The relevant incentive constraints are on agents who do not produce and might try to pass themselves as entrepreneurs, or to misreport when they sold assets or for how much. I apply this result repeatedly below.

By a similar argument, an agent who arrives at date 2 receives a simple debt contract depending on his report  $\hat{\theta}_1 \in \{e, n\}$  that specifies an initial transfer  $x_2^{\hat{\theta}_1}$  and an amount he must repay at the end of period 2, but which might also reward him if he shows he did nothing with the funds he received.

The fact that lenders will not audit agents who report they chose to produce and pay lenders back implies that non-entrepreneurs can ensure themselves positive expected profits by pretending to be entrepreneurs and keeping the assets they buy, since if  $d = D$  they can afford any payment an entrepreneur can. Hence, lenders who want to fund entrepreneurs must provide rents to non-entrepreneurs. However, this need not imply that bubbles will arise. Recall that even when non-entrepreneurs buy risky assets, the price of the asset will not exceed  $\epsilon D$  if  $n_1 + n_2$  is small. But the next proposition reveals a more striking result: Once lenders can design contracts optimally, a bubble will not arise regardless of the number of non-entrepreneurs.

**Proposition 4:** When creditors are free to choose any contract  $\{x_1, x_2, x_3, x_y, x_{\hat{y}}, \sigma_{\hat{y}}\}$ , the equilibrium price of the asset  $p_1 = p_2 = \epsilon D$ , i.e. bubbles will not emerge.

The explanation for this result is as follows. If there were a bubble, creditors would be better off paying non-entrepreneurs not to speculate by setting  $r_1^{n,\emptyset} < 0$  than letting non-entrepreneurs buy overvalued assets.<sup>18</sup> Intuitively, letting a non-entrepreneur buy an overvalued asset is a costly way for the creditor to provide him with rents: When the asset is overvalued, the creditor ends up paying some resources to whoever the non-entrepreneur buys the asset from beyond the rents he provides the non-entrepreneur.

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<sup>18</sup>Alternatively, a lender could set  $x_1^n = 0$  and then transfer some positive amount of resources to the agent later on.

While it is true that paying agents not to speculate can eliminate bubbles, in practice this is likely to be a dubious proposal, since such payments could draw in people who have no intention to speculate if they receive no payment. We can capture this in the model by introducing an additional group of agents who own no resources but cannot participate in the asset market for some reason, e.g. high entry costs. By assumption, these agents cannot buy risky assets to take advantage of lenders, and would never borrow at a positive interest rate. But they would gladly accept a contract that gives them free resources. If this group were large enough, paying non-entrepreneurs not to speculate would incur losses for creditors.

Formally, adding these agents imposes the following restriction on contracts: If the cumulative transfers from the creditor to the borrower are positive, then the borrower must be left with zero cash holdings at the end of the period, since otherwise agents with no intention to buy risky assets will accept the contract. Since an agent with zero cash holdings will be audited in equilibrium, this restriction can be formalized as follows. First, there can be no transfer of resources from the creditor to the agent without auditing, i.e.

$$x_1(\hat{\theta}_1) + x_2(\hat{\theta}_1, \hat{\theta}_2) + x_3(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3) + x_{\hat{y}}(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{y}) \leq 0 \quad (12)$$

Second, if there is a positive transfer of resources from the creditor to the agent with auditing, then the terminal cash holdings of the agent must be 0, i.e.

$$x_1(\hat{\theta}_1) + x_2(\hat{\theta}_1, \hat{\theta}_2) + x_3(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3) + x_y(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, y) > 0 \quad (13)$$

implies

$$x_1(\hat{\theta}_1) + x_2(\hat{\theta}_1, \hat{\theta}_2) + x_3(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3) + x_y(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, y) + y \cdot x_1(\hat{\theta}_1) = 0 \quad (14)$$

This restriction rules out paying agents not to speculate: In that case  $y = 0$ , and (13) would contradict (14). Cumulative transfers can only be positive if the agent buys risky assets, holds them until  $d$  is revealed or paid out, and  $d$  turns out to be 0. In that case, the agent is unable to pay the borrower. But certainly lenders cannot offer loans with negative interest rates. Similar restrictions apply to date-2 contracts.

Restriction (13) and (14) implies creditors who wish to fund entrepreneurs must take on non-entrepreneurs and let them buy overvalued assets. The remainder of this section analyzes equilibrium for this case.

Since (9) ensures entrepreneurs always choose to produce in equilibrium, they will be fully funded, i.e.  $x_1^e = x_2^e = 1$ . Otherwise, suppose a lender scaled up  $x_t^e$  and  $x_t^n$  proportionally but charged entrepreneurs who produce slightly more than under the original contract, a small enough increase that they would still prefer this new contract. Since entrepreneurs who produce cannot pass themselves off as non-entrepreneurs, they will choose the contract designed for them. Non-entrepreneurs will prefer the contract offered to them given that under the original equilibrium they preferred the contract offered them when entrepreneurs were charged a lower rate. Since scaling up both loans and leaving interest rates fixed would leave profits at zero, charging entrepreneurs higher rates ensures positive profits. Hence,  $x_t^e < 1$  cannot be an equilibrium.

The only remaining terms of the contracts offered to entrepreneurs are the rates  $r_1^e$  and  $r_2^e$  due at the end of period 2. Since lenders earn zero expected profits in equilibrium, the amount collected from entrepreneurs

must equal the expected losses on non-entrepreneurs. This, in turn, depends on the terms offered to non-entrepreneurs. To solve for these, I begin with a preliminary result. Let  $V_t(\theta_1, \hat{\theta}_1)$  denote the expected utility to an agent of type  $\theta_1$  who announces his type as  $\hat{\theta}_1$  and then behaves optimally. My next lemma shows that in equilibrium, a non-entrepreneur will be indifferent about what type he announces:

**Lemma 2:** In equilibrium,  $V_t(n, n) = V_t(n, e)$

Lemma 2 implies that the equilibrium contract offered to non-entrepreneurs solves the problem of maximizing the expected revenue from non-entrepreneurs while leaving the non-entrepreneur with the same utility as if he reported  $\hat{\theta}_1 = e$ . For suppose the equilibrium contract did not solve this problem, i.e. there exists a contract  $x'$  with identical terms to entrepreneurs and which leaves non-entrepreneurs with the same utility  $V_t(n, e)$  but collects more revenue from them. Suppose the lender offered a contract  $x''$  equivalent to  $x'$  but which charged non-entrepreneurs slightly less so that their utility under  $x''$  was higher than  $V_t(n, e)$ . This allows the lender to also charge entrepreneurs slightly less than under the original equilibrium contract without inducing non-entrepreneurs to report  $\hat{\theta}_1 = e$ . Since the original contract earned zero profits,  $x'$  would generate positive profits if it attracted both types given it collects more from non-entrepreneurs but the same amount from entrepreneurs. Since the profits from  $x''$  can be made arbitrarily close to the profits from  $x'$  when it attracts both types, it follows that the original contract could not have been an equilibrium. Essentially, the creditor would like to grab as much resources from non-entrepreneurs to make his contract as attractive as possible to entrepreneurs.<sup>19</sup>

Applying this insight, the contract for agents who show up at date 2 amounts to a loan size  $x_2^n$  and a repayment rate  $r_2^n$  due at the end of date 2 that satisfy

$$\max_{x_2^n, r_2^n} x_2^n [\epsilon (1 + r_2^n) - 1] \quad (15)$$

subject to

- i.  $\epsilon \left( \frac{D}{p_2} - (1 + r_2^n) \right) x_2^n = \epsilon \left( \frac{D}{p_2} - (1 + r_2^e) \right)$
- ii.  $R - (1 + r_2^e) \geq \epsilon \left( \frac{D}{p_2} - (1 + r_2^n) \right) x_2^n$
- iii.  $r_2^n \geq 0$

Constraint (i) corresponds to the requirement that  $V_2(n, n) = V_2(n, e)$ . Constraint (ii) ensures that entrepreneurs prefer to produce than accept the contract intended for non-entrepreneurs and buy risky assets. It will be automatically satisfied given the second inequality in (9) which implies  $R$  is large enough to induce entrepreneurs to produce. As discussed above, we do not need to worry about entrepreneurs pretending to be non-entrepreneurs to get a low interest rate but then producing, since transfers can be designed to

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<sup>19</sup>Contracts that earn zero profits and feature such cross-subsidization are sometimes known as Wilson-Miyazaki-Spence contracts, following Wilson (1977), Miyazaki (1977) and Spence (1978). As I discuss in footnote 17 above, these contracts arise in my framework for related reasons to these papers, namely that my assumptions preclude certain types of contracts that would break these equilibria. However, these papers appeal to a different notion of equilibrium to rule out such contracts, while here they are ruled out by other aspects of the economic environment that restrict what contracts lenders can offer.

detect such misreporting. Constraint (iii) follows from the requirement that rates cannot be negative given (13) and (14). Solving this maximization problem reveals the following:

**Proposition 5:** In an equilibrium where  $p_2 > \epsilon D$ , non-entrepreneurs will be offered a contract in which  $x_2^n = \frac{D - (1 + r_2^e)p_2}{D - p_2} < 1$  and  $r_2^n = 0$ , while entrepreneurs will be offered a contract where  $x_2^e = 1$  and  $r_2^e$  solves

$$(1 - \phi)r_2^e + x_2^n\phi(\epsilon r_2^e - (1 - \epsilon)) = 0.$$

To understand this result, observe that lenders would always want to lend less to non-entrepreneurs, but they need to charge a lower rate on such loans to ensure non-entrepreneurs choose these contracts which may not make them so profitable. When non-entrepreneurs earn relatively smaller profits, lenders do not need to lower rates too much to attract non-entrepreneurs to smaller loans. In particular, if the asset is overvalued, lenders will find it optimal to offer non-entrepreneurs the smallest loans they can, which are bounded by the fact that interest rates cannot be negative. This reveals a distinct reason for why speculative bubbles might be associated with low premia for borrowing: In order to steer agents who intend to buy risky assets towards contracts that incur smaller losses for lenders, these loans must carry low rates. This is different from the explanations presented in the previous two sections, in which lower premia were due to the fact that speculation and overvaluation made loans less risky and so allowed lenders to charge lower premia.

**Remark 5:** The fact that lenders do not need to worry about entrepreneurs shifting into smaller low-interest loans to finance production plays an important role behind Proposition 5. If entrepreneurs who produce could pass themselves off as non-entrepreneurs, e.g. by using hidden borrowing to meet any transfer, we would need to add a constraint to (15) to insure the entrepreneur does not prefer the smaller loan:

$$R - (1 + r_2^e) \geq (R - (1 + r_2^n))x_2^n$$

Adding this constraint implies the equilibrium contract will depend how  $D/p_2$  compares to  $R$ . Intuitively, the optimal contract depends on which type earns more per each dollar invested, an agent who produced or an agent who bought risky assets. If  $D/p_2 < R$ , an agent who chose to produce values increasing his loan more, and will not be attracted to a contract where the interest rate reduction was tailored to keep an agent intent on buying risky assets interested in a smaller contract.<sup>20</sup> If  $D/p_2 > R$ , an agent who bought risky assets values increasing the scale of his loan more. In that case, in equilibrium both types will be offered the same contract, i.e.  $x_2^n = x_2^e = 1$  and  $r_2^e = r_2^n = \frac{(1 - \epsilon)\phi}{1 - (1 - \epsilon)\phi}$ . Hence, there are conditions that lead to a pooling equilibrium in which both types receive the same contract as I assumed so far, namely when lenders cannot pay agents not to buy overvalued assets and when borrowers can engage in hidden borrowing so that lenders cannot use the size and timing of transfers to restrict borrowers. ■

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<sup>20</sup>Although earlier I imposed parameter restrictions that would ensure  $D/p_2 > R$ , this was because I assumed agents could not engage in hidden borrowing, allowing the lender to detect agents who did not produce. Once agents can engage in hidden borrowing, lenders can no longer screen borrowers this way and bubbles can occur even if  $D/p_2 < R$ .

Finally, I turn to the contracts offered in period 1. A few observations allow me to simplify the characterization of these contracts. As in Section 3, in equilibrium lenders must believe the non-entrepreneurs they lend to will either hold or sell their assets at date 2 with certainty if  $d$  is not revealed. For lenders who expect a borrower to hold on to his assets, the equilibrium contract can be described as a simple *long-term* debt contract  $\{x_1^n, r_1^{n,\text{late}}\}$  in which the agent receives  $x_1^n$  upon arrival and is asked to repay  $x_1^n (1 + r_1^{n,\text{late}})$  at the end of date 2 or else be audited and be stripped of his cash holdings. To see this, denote the original loan amount by  $\hat{x}_1^n$ . Since rates must be nonnegative, agents will prefer buying assets to doing nothing, so we can ignore the rate for repayment at date 1. Denote the remaining rates by  $\{\hat{r}_1^{n,D}, \hat{r}_1^{n,p_2}, \hat{r}_1^{n,\text{late}}\}$ . Construct a new contract  $\{x_1^n, r_1^{n,\text{late}}\}$  where  $x_1^n = \hat{x}_1^n$  and  $r_1^{n,\text{late}} = q\hat{r}_1^{n,D} + (1-q)\hat{r}_1^{n,\text{late}}$ . If the borrower holds on to his assets, he will only repay if  $d = D$ . Under the original contract, he would have expected to pay  $q\hat{r}_1^{n,D} + (1-q)\hat{r}_1^{n,\text{late}}$  with probability  $\epsilon$ . The same is true for the new contract. If the agent prefers to sell his assets under the new contract if  $d$  is not revealed, the original contract could not have been optimal, since the new contract recoups the same expected amount but with a larger probability. Since all agents are indifferent between the new contract and the original equilibrium contract, and the trading strategy of non-entrepreneurs is unchanged, the contract  $\{x_1^n, r_1^{n,\text{late}}\}$  is equivalent to the original equilibrium contract.

Next, consider a borrower who intends to sell his assets. Here the equilibrium contract can be represented using a simple *short-term* debt contract  $\{x_1^n, r_1^{n,\text{early}}\}$  in which the agent receives  $x_1^n$  upon arrival and is asked to repay  $x_1^n (1 + r_1^{n,\text{early}})$  at the beginning of date 2 or else be audited and be stripped of all of his cash holdings. To see this, denote the terms of the equilibrium contract by  $\hat{x}_1^n$  and  $\{\hat{r}_1^{n,D}, \hat{r}_1^{n,p_2}, \hat{r}_1^{n,\text{late}}\}$ . Construct a new contract  $\{x_1^n, r_1^{n,\text{early}}\}$  where  $x_1^n = \hat{x}_1^n$  and  $r_1^{n,\text{early}} = \frac{1-q}{q\epsilon+(1-q)}\hat{r}_1^{n,p_2} + \frac{q\epsilon}{q\epsilon+(1-q)}\hat{r}_1^{n,\text{late}}$ . If the borrower intends to sell his assets, this contract would ensure the same expected repayment. But a borrower who accepts a short-term contract would necessarily sell the asset early at date 2. Hence, the short-term contract  $\{x_1^n, r_1^{n,\text{short}}\}$  is equivalent to the original contract, since all agents are indifferent between this and the original equilibrium contract and it has no effect on the trading strategy of non-entrepreneurs.

We can therefore derive the terms for non-entrepreneurs using a problem analogous to (15) in which the contract involves a loan size  $x_1^n$  and a single interest rate, either  $r_1^{n,\text{early}}$  or  $r_1^{n,\text{late}}$  depending on what the lender believes the borrower will do with the asset at date 2. Solving these reveals the following:

**Proposition 6:** Non-entrepreneurs who arrive at period 1 will receive the following contracts:

1. Non-entrepreneurs who intend to keep their assets will receive a long-term contract with  $x_1^n < 1$  and  $r_1^{n,\text{late}} = 0$  in any equilibrium where  $p_1 > \epsilon D$ . If  $p_1 = \epsilon D$ , the value of  $x_1^n$  is not uniquely determined, but a pooling contract where  $x_1^n = x_1^e = 1$  and  $r_1^{n,\text{late}} = r_1^e$  is an equilibrium.
2. Non-entrepreneurs who intend to sell their assets will receive a short-term contract with  $x_1^n < 1$  and  $r_1^{n,\text{early}} = 0$  in any equilibrium where  $p_1 > q\epsilon D + (1-q)p_2$ . If  $p_1 = q\epsilon D + (1-q)p_2$ , the value of  $x_1^n$  is not uniquely determined, but a contract where  $x_1^n = x_1^e = 1$  and  $r_1^{n,\text{early}} = r_1^e$  is an equilibrium.

Date-1 contracts thus resemble date-2 contracts: When the profits non-entrepreneurs earn are not too large, i.e. when  $p_1$  is sufficiently high, lenders will find it profitable to offer small loans despite having to charge lower rates. The main difference with contracts at date 2 is that there is now another dimension that lenders can exploit, namely the maturity of the loan. Requiring lenders to repay early forces them to sell the asset, which the lender prefers if  $d$  remains uncertain so the asset remains overvalued.

**Remark 6:** This reason the contract offered to non-entrepreneurs can be represented as a short-term loan is that they can always sell their assets at the beginning of date 2, even if the bubble bursts. More generally, agents may find it difficult to sell the asset early. For example, if the bubble bursts not because  $d$  is revealed but, as suggested in the previous section, because the number of traders at the beginning of date 2 turns out to be low, in equilibrium agents may have to wait until the end of date 2 to sell. In this case, the equilibrium contract will correspond to a backloaded contract that rewards the borrower for paying early without necessarily penalizing him too severely for paying late. ■

As in Section 3, there can be equilibria in which non-entrepreneurs all sell their assets, all hold on to their assets, or some hold and some sell if  $d$  remains uncertain at the beginning of date 2. In the mixed case, both contracts will be offered. At first, this might seem surprising, since lenders would like non-entrepreneurs to sell their assets, and can ensure they do so by offering a short maturity contract. The reason the two contracts can co-exist is that from Lemma 2, we know that in equilibrium non-entrepreneurs are indifferent between taking the contract offered to them and the one offered to entrepreneurs, where the latter is by necessity a long-term contract. Hence, lenders cannot be sure that if they offered a short-term contract it will be accepted by any non-entrepreneurs they lend to, incurring expected losses for lenders. In line with Remark 4, in equilibrium lenders who offer long-term contracts must believe there is some probability that if they offered a different contract, non-entrepreneurs will switch to the contract geared to entrepreneurs. The same reasoning also explains why lenders do not simply offer the contract only offer the contract  $\{x_1^e, r_1^e\}$  and lend to entrepreneurs only: They cannot be sure non-entrepreneurs won't also accept these contracts.

To recap, allowing lenders to design contracts makes it more difficult, but not impossible, for bubbles to arise. This is because lenders will try to avoid entering into financial arrangements with agents who intend to buy overvalued assets, and even if these arrangements are unavoidable, they would try to dissuade borrowers from using the funds they receive to buy overvalued assets. To the extent that agents can use contracts to achieve this, endogenous contracting can prevent bubbles. However, if agents who buy overvalued assets can pass themselves off as types to whom it is profitable to lend, and if lenders cannot just pay agents not to speculate, bubbles will emerge.<sup>21</sup> In these cases, lenders would want to offer contracts that minimize the losses they incur from speculators. Specifically, the contracts that emerge involve lower rates on smaller loans with possibly backloaded payments. Barlevy and Fisher (2010) build on this last observation and show that in a risk-shifting model adapted to the housing market there is a similar tendency for speculative

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<sup>21</sup>Equally important is my assumption that transfers cannot be contingent on  $d$ , or else a lender could concentrate payments in states where speculation would be profitable, eliminating the incentive to buy risky assets without affecting entrepreneurs.

bubbles to be associated with contracts that encourage early repayment, and find that such contracts were indeed used heavily in cities with house price booms but sparingly elsewhere. But the same observation may explain why empirically lenders lend at low premia to the very people most active in speculation.

## 5 Conclusion

The distinguishing feature of equity and housing markets over the past decade has been the rapid run-up in the prices of these assets followed by an equally sharp collapse in these prices. While various pundits have readily taken to referring to these episodes as bubbles, to economists who reserve the term “bubble” to mean a deviation between the price of an asset and its underlying value it should be much less obvious whether this notion is appropriate for characterizing these episodes. In particular, the alternative explanation for why asset prices are so volatile is that the underlying fundamentals are themselves highly volatile. This is particularly true in models where different agents value the same asset differently for whatever reason. In that case, if the price of the asset was equal to the value that the marginal trader assigns to the asset, shocks that change the identity of the marginal trader can result in especially violent swings in asset prices, a theme explored in recent work by Geanakoplos (2009), Burnside, Eichenbaum, and Rebelo (2011), and Albagli, Hellwig, and Tsyvinski (2011). Thus, one could potentially understand these episodes without having to appeal to the notion that asset prices are unhinged from the underlying valuation that some agent attaches to them. Yet the question of whether asset prices reflect fundamentals or not is important. If asset prices fail to reflect their underlying value, they signal to potential producers of the asset to produce a different amount of these assets than is socially optimal, i.e. than the value of providing the marginal trader with another asset. In that case, there may be a role for policy intervention that would not have a parallel when asset prices reflect volatile fundamentals. It is therefore important to gauge whether episodes often branded as bubbles correspond to theoretical models of bubbles as economists define them.

This paper contributes to this question by exploring whether a model in which asset prices deviate from fundamentals can capture the key features of boom-bust episodes in asset markets. I show that a model in which the deviation between asset prices and fundamentals is due to risk-shifting concerns can generate some of the key aspects of these episodes – booms and busts, high turnover in asset positions, and low premia charged to those who borrow in order to purchase assets. Moreover, this model is potentially falsifiable: These phenomena do not arise in general, but only under certain conditions. The model also implies that to the extent that a boom-bust pattern reflects a speculative bubble, agents who borrow to speculate would be offered certain types of contracts. These results suggest theoretical models where prices deviate from fundamentals may have some potential relevance for understanding the various historical episodes casually described as bubbles, and offer preliminary insights that may be applied to further explore the potential importance of these models even when we cannot estimate the fundamental value of an asset and directly infer whether an asset is overvalued.



## Appendix

**Lemma A1:** In any equilibrium,  $p \geq \epsilon D$ .

**Proof:** Suppose  $p < \epsilon D$ . None of the original owners would agree to sell assets. Thus, demand for assets must be zero to achieve equilibrium, so the expected utility to a non-entrepreneur must be zero in equilibrium. However, suppose a lender were to offer to lend out one unit of resources at rate  $r = \frac{1}{\epsilon} - 1$ . Since  $p < \epsilon D$ , it follows that

$$(1+r)p < \frac{1}{\epsilon}(\epsilon D) = D$$

This implies a non-entrepreneur could earn positive profits, and since he earns zero in equilibrium, he would accept this contract. The expected profits to the lender on this loan would equal

$$\epsilon \left( \frac{1}{\epsilon} \right) - (1 - \epsilon) = \epsilon$$

If entrepreneurs also chose this contract, the expected profits to the lender would be  $\epsilon$  if the entrepreneur chose to buy assets, and  $\min\left(\frac{1}{\epsilon}, R\right) - 1 > 0$  if he chose to produce. Thus, even if the contract attracted entrepreneurs, it would ensure positive expected profits to the lender. Since no lender can expect to earn positive expected profits in equilibrium, it follows that  $p < \epsilon D$  cannot be an equilibrium.

**Proof of Proposition 1:** The proof mostly involves tediously verifying that the prices in the statement of the proposition are the only possible equilibrium outcomes at various values of  $R$  and  $n$ .

I begin by showing that there exists a cutoff  $R^*(n)$  such that the only possible equilibrium when  $R < R^*(n)$  is one where  $p = \epsilon D$  and  $r \geq 1/\epsilon - 1$ . As a first step, I argue that there exists a cutoff  $R^*(n)$  such that if  $R < R^*(n)$ , there cannot be an equilibrium in which  $(1+r)p < D$ .

Suppose there was such an equilibrium. Then a borrower could earn strictly positive expected profits by buying assets and defaulting if  $d = 0$ , since they would earn strictly positive profits if  $d = D$ . The expected returns to lending to an agent who buys an asset would be negative as a result. In particular, expected profits would be  $\epsilon(1+r) - 1$ . But  $(1+r) < D/p$  by assumption, and  $D/p \leq 1/\epsilon$  from lemma 2. Hence,  $\epsilon(1+r) - 1 < 0$ .

Since lenders must make zero expected profits in equilibrium, and since non-entrepreneurs strictly prefer to borrow and buy assets if  $(1+r)p < D$ , then this outcome can only be an equilibrium if at least some entrepreneurs also borrow but then use the employ they borrow to produce. Given he could earn positive expected profits from buying assets, an entrepreneur will be willing to produce only if

$$R \geq \frac{\epsilon D}{p} + (1 - \epsilon)(1 + r) \tag{16}$$

The expected profits to a lender if all entrepreneurs opt to produce while all non-entrepreneurs opt to buy risky assets are given by

$$(1 - \phi)r + \phi[\epsilon(1 + r) - 1] \quad (17)$$

Here I use the fact that with a vanishing cost of default, entrepreneurs will only opt to produce if  $R \geq 1 + r$ , so the lender can collect  $r$  from entrepreneurs. Note that the expected profits of the lender are at most (17). This is because with  $(1 + r)p < D$ , all non-entrepreneurs would buy the assets, and so the fraction of borrowers who engage in production is at most  $1 - \phi$ . Given  $\epsilon(1 + r) - 1 < 0$  while  $r > 0$ , expected profits are decreasing in  $\phi$ , and hence (17) is an upper bound.

Define

$$r^* = \frac{(1 - \epsilon)\phi}{1 - (1 - \epsilon)\phi} \quad (18)$$

The rate  $r = r^*$  ensures that if a lender recovers  $r$  from non-entrepreneurs when  $d = D$  and  $r$  from each entrepreneur he lends to, he would exactly break even. It follows that in any equilibrium where  $(1 + r)p < D$ , we must have  $r \geq r^*$  for lenders to be able to earn zero profits. This implies that whatever the equilibrium price  $p$  is, we cannot have both  $(1 + r)p < D$  and

$$\begin{aligned} R &< \frac{\epsilon D}{p} + (1 - \epsilon)(1 + r^*) \\ &= \frac{\epsilon D}{p} + \frac{1 - \epsilon}{1 - (1 - \epsilon)\phi} \end{aligned} \quad (19)$$

If both conditions are met, lenders will not be able to charge enough and still induce entrepreneurs to produce. The last step of the argument is to use this observation to arrive at a cutoff  $R^*(n)$  by considering possible equilibrium prices  $p$  for different values of  $n$ . In particular, the demand for assets by non-entrepreneurs when  $(1 + r)p < D$  is equal to  $n/p$ . Assuming entrepreneurs all employ their resources in production provides a lower bound on the price that can prevail in equilibrium as a function of  $n$ . Substituting in this lower bound provides an expression  $R^*(n)$  such that if  $R < R^*(n)$ , then  $R$  will necessarily be lower than (19), and hence that there can be no equilibrium where  $(1 + r)p < D$ .

**Case (i):**  $n \leq \epsilon D$ . In this case, the lowest possible price that can prevail in equilibrium is  $p = \epsilon D$ . Indeed, this would necessarily be the price if all entrepreneurs chose to produce rather than buy assets, since then demand for the asset is less than 1 and so some original owners would have to hold on to their assets. Thus, to ensure there cannot be an equilibrium in which no entrepreneur is willing to produce, we need to ensure  $R$  is lower than the value of the bound in (19) when  $p = \epsilon D$ , i.e.

$$R^*(n) = 1 + \frac{1 - \epsilon}{1 - (1 - \epsilon)\phi} \quad (20)$$

**Case (ii):**  $\epsilon D < n \leq (1 - (1 - \epsilon)\phi)D$ . In this case, the lowest possible price than can prevail in equilibrium is  $p = n$ . Again, this would necessarily be the price if all entrepreneurs chose to produce rather than buy assets. In that case, all non-entrepreneurs would buy assets given  $(1 + r)p < D$ , but their

demand would exceed supply if  $p < n$  and would fall short of supply if  $p > n$ . To ensure there cannot be an equilibrium in which no entrepreneur is willing to produce, we need to ensure  $R$  is lower than the value of the bound when  $p = n$ , i.e.

$$R^*(n) = \frac{\epsilon D}{n} + \frac{1 - \epsilon}{1 - (1 - \epsilon)\phi} \quad (21)$$

**Case (iii):**  $n > (1 - (1 - \epsilon)\phi)D$ . In this case, if  $(1 + r)p < D$ , the same argument as in case (ii) would imply that the lowest price that can prevail is  $p = n$ . However, substituting this in yields a lower bound that is strictly less than  $1 + r^*$ . Since entrepreneurs can earn positive expected profits when  $(1 + r)p < D$  but will earn zero profits from investing when  $R < 1 + r^*$ , we cannot sustain an equilibrium whenever  $R < 1 + r^*$ , i.e.

$$R^*(n) = \frac{1}{1 - (1 - \epsilon)\phi} \quad (22)$$

Summarizing, if  $R < R^*(n)$  as given by (20), (21), and (22), we are assured of obtaining a contradiction if we assume  $(1 + r)p < D$ . Thus, for such values of  $R$ , the only candidate equilibrium is one where  $(1 + r)p \geq D$ . I next argue that this implies  $p = \epsilon D$ . For suppose not, i.e.  $p > \epsilon D$ . In this case, expected profits to a lender from lending to an agent who buys assets will be negative:

$$\epsilon \min\left(\frac{D}{p}, (1 + r)\right) - 1 = \frac{\epsilon D}{p} - 1 < 0$$

Since  $R < R^*(n)$ , entrepreneurs would not choose to produce if  $r \geq r^*$ . But if  $r < r^*$ , expected profits to lenders would be negative, which cannot be an equilibrium. The only candidate equilibrium that remains is thus  $p = \epsilon D$ . It is easy to check that any combination of  $p = \epsilon D$  and  $(1 + r)p \geq D$  is indeed an equilibrium. If  $(1 + r)p = D$ , agents are indifferent about trading assets. If instead  $(1 + r)p > D$ , then no agent will borrow in equilibrium given there is a vanishingly small cost of default and assets will never trade.

Next, I turn to the case where  $R > R^*(n)$ . Here, the analysis is explicitly conditional on the value of  $n$ .

**Case (i):**  $n < \epsilon D$ . I first argue that there cannot be an equilibrium in which  $p > \epsilon D$ . For suppose there was such an equilibrium. At this price, all original owners would choose to sell their assets. But given  $n < \epsilon D$ , there are not enough non-entrepreneurs to buy up all these assets. Hence, at least some entrepreneurs must purchase assets for this to be an equilibrium. At the same time, if  $p > \epsilon D$ , then some entrepreneurs would have to choose to produce in equilibrium. Otherwise, lenders would earn an expected loss on those who buy assets, since

$$\epsilon \cdot \min\left(\frac{D}{p} - 1, r\right) - (1 - \epsilon) < \epsilon \cdot \min\left(\frac{1}{\epsilon} - 1, r\right) - (1 - \epsilon) \leq 0$$

Since some agents would have to borrow to buy up assets, lenders would need to earn positive profits on some agents to ensure expected profits are zero as required in equilibrium. Hence, the payoff to entrepreneurs

must be identical regardless of whether they produce or buy assets, i.e.

$$\max\left(\epsilon\left(\frac{D}{p} - (1+r)\right), 0\right) = \max(R - (1+r), 0) \quad (23)$$

Next, I argue the payoffs can only be equal if  $r > r^*$ . For suppose  $r \leq r^*$ . Recall that for  $n < \epsilon D$ , the cutoff  $R^*(n) = 1 + r^*/\phi > 1 + r^*$ , implying  $R - (1+r) > 0$ , so the right-hand side of (23) is equal to  $R - (1+r)$ . At the same time, since  $R^*(n) = 1 + (1-\epsilon)(1+r^*)$  and since  $R > R^*(n)$ , we have

$$\begin{aligned} R &> 1 + (1-\epsilon)(1+r) \\ &> \frac{\epsilon D}{p} + (1-\epsilon)(1+r) \end{aligned}$$

It therefore follows that

$$R - (1+r) > \max\left(\epsilon\left(\frac{D}{p} - (1+r)\right), 0\right)$$

which contradicts (23). Hence,  $r > r^*$ .

I now consider two cases for  $p$ . First, suppose  $\epsilon D < p < D/(1+r^*)$ . In this case, I argue that since the equilibrium  $r$  exceeds  $r^*$ , a lender could earn positive expected profits. In particular, suppose the lender offered to lend one unit of resources at a rate  $r^*$ . Since this is less than the equilibrium rate  $r$ , any agent who can earn positive profits at this rate would strictly prefer this contract to the equilibrium contract. Since  $R > R^*(n)$ , the lender could collect  $1 + r^*$  on any entrepreneur who chooses to produce. Moreover, since  $R > R^*(n) > 1 + r^*$  and since  $R > R^*(n) > \frac{\epsilon D}{p} + (1-\epsilon)(1+r)$ , all entrepreneurs would prefer to produce. Non-entrepreneurs would choose to buy assets, since  $(1+r^*)p < D$ . But the lender can recover  $r^*$  from them if  $d = D$ . Hence, expected profits to the lender are given by

$$(1-\phi)r^* + \phi[\epsilon(1+r^*) - 1]$$

which is 0 by construction. However, since both entrepreneurs and non-entrepreneurs strictly prefer this contract, a lender could earn strictly positive profits by charging a little above  $r^*$ . Hence, there cannot exist an equilibrium with  $p$  in this range.

Next, suppose  $p \geq D/(1+r^*)$ . Since we argued  $r > r^*$ , it follows that  $(1+r)p > (1+r^*)p \geq D$ . But since we are considering the limiting case where the cost of default  $k \rightarrow 0$ , non-entrepreneurs will not borrow to buy assets, since they know they will default with certainty. In that case, lenders can earn a strictly positive profit by charging just slightly above  $r^*$ : non-entrepreneurs would not borrow since they would then default with certainty, while entrepreneurs would strictly to borrow from this lender and to use the funds for production. Hence, these prices cannot be equilibrium prices either.

The only remaining candidate equilibrium then is  $p = \epsilon D$ . It is easy to confirm that there is an equilibrium where  $p = \epsilon D$  and  $r = r^*$ . At lower values of  $r$ , lenders earn negative profits. At higher values of  $r$ , lenders could earn a positive expected profit by charging a lower rate.

**Case (ii)**  $\epsilon D < n < (1 - (1 - \epsilon)\phi)D$ . A similar argument as in Case (i) can be used to establish that there cannot be an equilibrium in which  $p > n$ . I next need to argue that there cannot be equilibria in which  $\epsilon D \leq p < n$ . Such an equilibrium would require that not at least some non-entrepreneurs prefer not to buy assets, i.e. expected profits from buying assets must be zero. This implies

$$1 + r \geq \frac{D}{p} > \frac{D}{n}$$

Given  $n < (1 - (1 - \epsilon)\phi)D$ , this implies

$$1 + r > \frac{1}{1 - (1 - \epsilon)\phi} = 1 + r^*$$

But by the same argument as in Case (i), we know that a lender who charges a rate between  $r^*$  and  $r$  can earn strictly positive profits: Since  $(1 + r^*)p < D$ , all non-entrepreneurs would choose to buy the asset if  $r$  was sufficiently close to  $r^*$ , and since  $R > R^*(n)$ , all entrepreneurs would choose to produce. Expected profits to the lender would then be positive. Hence, this cannot be an equilibrium. The only candidate equilibrium is thus  $p = n$ . It is easy to confirm that there is an equilibrium where  $p = n$  and  $r = r^*$ , and that no other  $r$  can be an equilibrium.

**Case (iii)**  $n > (1 - (1 - \epsilon)\phi)D$ . I first argue that  $(1 + r)p \geq D$ . For suppose not, i.e.  $(1 + r)p < D$ . This would imply all non-entrepreneurs would want to buy the asset. Since demand cannot exceed the total available stock of the asset, it must be the case that  $p \geq n$ , implying  $p > (1 - (1 - \epsilon)\phi)D$ . Since lenders cannot charge  $r < r^*$  and earn positive expected profits when all non-entrepreneurs choose to buy assets, then in equilibrium  $r \geq r^*$ . But then  $(1 + r)p > D$ , a contradiction. Hence, in any equilibrium, we must have  $(1 + r)p \geq D$  and non-entrepreneurs must earn zero expected profits.

Next, I argue that  $p > \epsilon D$ . For suppose  $p = \epsilon D$ . Then  $1 + r \geq D/p = 1/\epsilon > r^*$ . But we know that a lender can guarantee himself positive expected profits by charging an interest rate a little above  $r^*$ . Since  $p > \epsilon D$ , all original owners will prefer to sell the asset, so some potential buyers have to buy the asset. Since we know they will not buy the asset if they expect to default with certainty, it follows that  $(1 + r)p = D$ .

Next, I argue  $r \leq r^*$ . For suppose  $r > r^*$ . If a lender were to set  $r$  just above  $r^*$ , entrepreneurs would strictly prefer to buy assets. Even if all non-entrepreneurs choose to buy assets, profits will be positive. Since  $R > R^*(n) = 1 + r^*$ , it follows that entrepreneurs can earn strictly positive profits from producing. Since expected profits from buying assets are zero, all entrepreneurs will produce in equilibrium.

Let  $n^* \in [0, n]$  denote the number of non-entrepreneurs who choose to buy assets in equilibrium. Define

$$\phi^* = \frac{n^*}{(\phi^{-1} - 1)n + n^*} \tag{24}$$

The zero profit condition for lenders that must hold in equilibrium implies

$$r = \frac{(1 - \epsilon)\phi^*}{1 - (1 - \epsilon)\phi^*}$$

Since  $p = (1 + r)D$ , we have  $p = (1 - (1 - \epsilon)\phi^*)D$ . Since I argued above that  $p > \epsilon D$ , we know all original owners sell the asset. Since only non-entrepreneurs buy the asset, equilibrium requires  $p = n^*$ . Substituting in  $p = (1 - (1 - \epsilon)\phi^*)D$  in for  $n^*$  in (24) yields the quadratic equation

$$\phi^* = \frac{(1 - (1 - \epsilon)\phi^*)D}{(\phi^{-1} - 1)n + (1 - (1 - \epsilon)\phi^*)D}$$

At  $n = (1 - (1 - \epsilon)\phi)D$ , this equation collapses to  $\phi^* = \phi$ . For  $n > (1 - (1 - \epsilon)\phi)D$ , there are two roots. Differentiating the formula for quadratic roots reveals that one root must be below  $\phi$  and one above  $\phi$ . The latter root cannot correspond to an equilibrium, since it would imply more agents buy assets than there are entrepreneurs. Hence, the equilibrium corresponds to the unique root whose value lies below  $\phi$ . This root is monotonically decreasing in  $n$ , and its limit as  $n \rightarrow \infty$  is zero. Hence,  $\lim_{n \rightarrow \infty} (p, r) = (D, 0)$ . ■

**Proof of Proposition 2:** For the case where  $R > 1 + \frac{1 - \epsilon}{1 - (1 - \epsilon)\phi}$  and both  $n_1$  and  $n_2$  are less than  $(1 - (1 - \epsilon)\phi)D$ , we can appeal to Proposition 3 for the case of  $q = 0$ . Confirming the equilibrium when either  $n_1$  or  $n_2$  exceed  $(1 - (1 - \epsilon)\phi)D$ , when  $R$  is between  $1 + \frac{1 - \epsilon}{1 - (1 - \epsilon)\phi}$  and  $R^*(n_1 + n_2)$ , and when  $R < R^*(n_1 + n_2)$  rely on similar arguments to those in the proof of Proposition 1. ■

**Proof of Proposition 3:** I first argue that the restrictions (3) and (4) ensure that in equilibrium, entrepreneurs will produce in both periods and that non-entrepreneurs in period 1 buy risky assets. I then use these observations to set up and solve for the equilibrium for all  $(n_1, n_2)$  in the set

$$\mathcal{N} \equiv \{(n_1, n_2) \in \mathbb{R}_{++}^2 : n_1 + n_2 < (1 - (1 - \epsilon)\phi)D\} \quad (25)$$

Note that if  $d$  is revealed before agents trade the asset at date 2, the asset will not trade at any price other than  $d$ . Hence, in the asset market, there are two prices that need to be solved – the price of assets in date 1, denoted  $p_1$ , and the price of the asset in date 2 if  $d$  is not already revealed, denoted  $p_2$ .

First, I note that lemma A1 extends to the two period model, i.e.  $p_1 \geq \epsilon D$  and  $p_2 \geq \epsilon D$ .

Next, I argue that in any equilibrium,  $\frac{p_2}{p_1} \leq \frac{1}{1 - q}$ . It will suffice to show that  $p_1 \geq (1 - q)p_2 + q\epsilon D$  in equilibrium, since this inequality implies

$$\frac{p_2}{p_1} \leq \frac{1}{1 - q} \left( 1 - \frac{q\epsilon D}{p_1} \right) \leq \frac{1}{1 - q}$$

Suppose to the contrary that  $p_1 < (1 - q)p_2 + q\epsilon D$ . In that case, all original owners would prefer to wait and sell in period 2 than to sell in period 1. For the asset market to clear in period 1, no agent must want to buy assets at this date. I now use this to generate a contradiction. Since  $p_1 \geq \epsilon D$ , the inequality  $p_1 < (1 - q)p_2 + q\epsilon D$  implies  $(1 - q)p_2 + q\epsilon D > \epsilon D$  and so  $p_2 > \epsilon D$ . The fact that  $p_2 > \epsilon D$  implies all original owners would wish to sell their assets at date 2 if  $d$  were not revealed. Market clearing then requires that agents who arrive at date 2 be willing to buy assets at this date, so  $(1 + r_2)p_2 \leq D$ . In addition, we

have  $p_2 = (1 - q)p_2 + qp_2 > (1 - q)p_2 + q\epsilon D > p_1$ , so  $p_2 > p_1$ . Since  $(1 + r_1)p_1 \geq D$  to ensure no agent wants to buy the asset at date 1, we have

$$1 + r_1 \geq \frac{D}{p_1} > \frac{D}{p_2} \geq 1 + r_2$$

where the middle inequality uses the fact that  $p_2 > p_1$ . Now, since date 2 is equivalent to the static model given none of the original owners sold at date 1, and since we assume  $R > 1 + \frac{1 - \epsilon}{1 - \phi(1 - \epsilon)} \geq R^*(n)$  and  $n_2 \leq n_1 + n_2 < (1 - (1 - \epsilon)\phi)D$ , it follows that  $1 + r_2 \geq 1 + r^*$  where

$$r^* \equiv \frac{(1 - \epsilon)\phi}{1 - (1 - \epsilon)\phi}$$

since charging less than  $r^*$  would yield lenders negative expected profits to the lender. Since  $r_1 > r_2$ , it follows that  $r_1 > r^*$ . But now we can obtain a contradiction. If a lender at date 1 were to offer an interest rate  $r_1$  just above  $r^*$ , he could earn strictly positive expected profits: All agents would borrow, entrepreneurs to produce and non-entrepreneurs to buy assets. From the static model, expected profits to the lender would be positive if  $r_1$  exceeded  $r^*$  and non-entrepreneurs held on to their assets to maturity. The fact that agents might sell their assets at date 2 must only make expected profits to the lender weakly higher, since we are considering the limiting case where  $k \rightarrow 0$  so that agents only sell their assets if they can pay back their debt. Hence,  $p_1 < (1 - q)p_2 + q\epsilon D$ . could not have occurred in equilibrium.

I now use the bound on  $p_2/p_1$  above to argue that if  $R > 1 + r^*$ , entrepreneurs in both periods will prefer to initiate production than to buy risky assets when they are charged an interest rate  $r_1 \leq r^*$ . For date 2, this follows from the analysis of the static model. In particular, we have

$$\begin{aligned} R &> 1 + \frac{1 - \epsilon}{1 - (1 - \epsilon)\phi} \\ &= 1 + (1 - \epsilon)(1 + r^*) \\ &\geq \frac{\epsilon D}{p_2} + (1 - \epsilon)(1 + r_1) \end{aligned}$$

Recall that in the proof of Proposition 1, I argued that this condition implies an entrepreneur prefers investing to buying the assets and holding them until the end of the period. Hence, entrepreneurs in period 2 prefer to produce if charged no more than  $r^*$ . As for date 1, the expected profits from production are equal to  $R - (1 + r_1)$  while the expected profits from buying an asset are given by

$$q\epsilon \left( \frac{D}{p_1} - (1 + r_1) \right) + (1 - q) \max \left\{ \frac{p_2}{p_1} - (1 + r_1), \epsilon \left( \frac{D}{p_1} - (1 + r_1) \right) \right\}$$

If the second term is equal to  $\epsilon \left( \frac{D}{p_1} - (1 + r_1) \right)$ , the choice is equivalent to the static model and we know the entrepreneur prefers to produce if  $r_1 \leq r^*$ . If the second term is equal to  $\epsilon \left( \frac{D}{p_1} - (1 + r_1) \right)$ , then the entrepreneur will prefer to produce if

$$R - (1 + r_1) > q\epsilon \left( \frac{D}{p_1} - (1 + r_1) \right) + (1 - q) \left( \frac{p_2}{p_1} - (1 + r_1) \right)$$

or, rearranging, if

$$R > q \frac{\epsilon D}{p_1} + (1 - q) \frac{p_2}{p_1} + q(1 - \epsilon)(1 + r_1) \quad (26)$$

But we know  $\frac{\epsilon D}{p_1} \leq 1$  and  $\frac{p_2}{p_1} \leq \frac{1}{1 - q}$ , so that a sufficient condition for (26) is

$$R > 1 + q(1 + r_1)$$

But if  $R > 1 + (1 + r^*)$ , it will also exceed  $1 + q(1 + r^*)$  for  $q \in (0, 1)$  and  $r^*$  positive. Hence, entrepreneurs will also prefer to produce than buy assets and selling them at date 2 if  $d$  remains uncertain.

Lastly, I argue that equilibrium interest rates are at most  $r^*$  in both periods. In period 2, if lenders charge above  $r^*$ , a lender could charge an interest rate just above  $r^*$  and guarantee himself positive profits by attracting both entrepreneurs and non-entrepreneurs, just as in the static model. At date 1, if lenders charged more than  $r^*$ , a lender could once again make positive expected profits charging just above  $r^*$  and attracting both types. This contract would yield them a profit if non-entrepreneurs held on to their assets at date 2, and even higher profits if non-entrepreneurs sold their assets at date 2. This is because in the limiting case where  $k \rightarrow 0$ , agents will not sell their assets unless they can repay their debt in full, so lenders recover the full debt obligation if agents sell the asset. Combined with the results above, it follows that entrepreneurs will engage in production in equilibrium.

Next, I argue that in equilibrium, all non-entrepreneurs will buy assets in period 1. If some non-entrepreneurs chose not to buy assets at date 1, it must be because  $(1 + r_1)p_1 \geq D$ . But since I just showed that  $r \leq r^*$  in equilibrium, it follows that  $p_1 \geq (1 + r^*)D = (1 + (1 - \epsilon)\phi)D$ . Since  $n_1 + n_2 < (1 + (1 - \epsilon)\phi)D$ , at this value for  $p_1$  we have  $n_1/p_1 < 1$  and so not all original owners can sell their assets at date 1. Since  $p_1 \geq (1 + (1 - \epsilon)\phi)D > \epsilon D$ , these owners must prefer to sell their assets than holding them to maturity. Hence, those who didn't sell in period 1 must sell them in period 2 if  $d$  is not revealed beforehand. Since original owners must be indifferent between selling at date 1 and waiting until date 2, it follows that  $p_1 = (1 - q)p_2 + \epsilon D$ . Since  $p_1 > \epsilon D$ , this implies  $p_2 > p_1 \geq (1 + (1 - \epsilon)\phi)D$ . Total demand for the asset over the two periods,  $\frac{n_1}{p_1} + \frac{n_2}{p_2}$ , is thus less than 1, implying some original owners must never sell their asset. But holding on to assets is dominated given  $p_1$  and  $p_2$  exceed  $\epsilon D$ . It follows that all non-entrepreneurs buy assets in period 1, and so  $n_1/p_1$  assets will trade hands.

Armed with these results, I can move to analyze what happens in periods 2. If  $d$  is revealed at the beginning of the period, non-entrepreneurs no longer have an opportunity to earn information rents. The value of the asset will then trade at its true value  $d$ , and entrepreneurs can secure funds at zero interest to initiate production. If  $d$  instead remains uncertain, non-entrepreneurs who arrive at date 2 could potentially earn information rents by buying the asset. They would have to buy these assets from either the original owners, who value the assets at  $\epsilon D$ , or non-entrepreneurs who bought the assets at date 1 and who value the assets at  $\epsilon D + (1 - \epsilon)(1 + r_1)p_1$ . Hence, four types of equilibria are possible in this state of the world: (a) at least some of the original owners hold on to them until maturity; (b) all original owners who own



assets at date 2 sell them, but none of those who bought them back in period 1 sell them; (c) all original owners prefer to sell at date 2 and some but not all of those who bought the asset back in period 1 are willing to sell; and (d) all those who own the asset at date 2 sell it. Each of these cases yield a system of equations that pins down  $p_1$  and  $p_2$ .

Before laying out these equations and solving for the equilibrium values of  $p_1$  and  $p_2$ , I derive some preliminary results for case (c) in which only some of those who buy assets in period 1 sell them in period 2, and which involves some important subtleties. I first argue that traders who buy the risky asset at date 1 must not randomize on whether to sell these assets at date 2. For suppose traders did randomize. This would mean such traders are indifferent between holding on to the assets for an expected profit of  $\epsilon \left( \frac{D}{p_1} - (1 + r_1) \right)$  and selling for an expected profit of  $\frac{p_2}{p_1} - (1 + r_1)$ . Suppose a lender in period 1 were to offer a slightly lower interest rate than  $r_1$  to the same trader. Since the trader was just indifferent about selling the asset at  $r_1$ , he would strictly prefer to sell at a lower rate. But this would allow the lender to earn higher profits, since the loss in interest rate can be infinitesimally small while the probability of recovering nearly  $1 + r_1$  jumps up discretely. It follows that in equilibrium there can only be two types of traders at date 1: Those who will definitely sell the asset at date 2 and those who will definitely hold on to it.

Next, I argue that in equilibrium the two types of traders cannot receive the same interest rate. For suppose they did. Let  $\mu$  denote the fraction of agents who buy assets at date 1 that will definitely sell at date 2. Further, let  $\pi_0$  and  $\pi_1$  denote the expected profits from lending to those who will hold and those who will sell, respectively. Expected profits to the lender are given by

$$(1 - \phi) r_1 + \phi ((1 - \mu) \pi_0 + \mu \pi_1)$$

If the interest rate is the same for both types, then  $\pi_1 > \pi_0$ . But then a lender can earn positive expected profits by offering a rate slightly below  $r_1$ : given some buyers are willing to sell the asset at date 2, at a slightly lower rate all buyers would prefer to sell, yielding the lender an expected profit that is arbitrarily close to

$$(1 - \phi) r_1 + \phi \pi_1$$

which is strictly higher than the original equilibrium profits. So borrowers cannot all be charged a common rate in an equilibrium of type (c).

How can an equilibrium with more than one interest rate? Since all borrowers prefer lower interest rates to higher ones, in equilibrium only a fixed number of lower interest rate contracts can be offered, and lenders must only be willing to offer a high rate contracts to all remaining borrowers. I then need to verify under what conditions it will indeed be optimal for lenders to behave this way. Since the model assumes no agent can credibly signal whether or not they are an entrepreneur, contracts must be assigned at random. Thus, in equilibrium, any lender must believe that there is a probability  $\phi$  that any agent he lends to is a non-entrepreneur, regardless of the interest rate he charges. Given this, the first observation we can make

is that the rates charged to non-entrepreneurs who will definitely sell at date 2 if  $d$  is yet to be revealed must ensure expected profits to the lender are zero. Otherwise, a lender could offer a slightly lower rate to these borrowers, which would ensure non-entrepreneurs who borrow will strictly prefer to sell the asset at date 2 and thus will behave the same way. Likewise, entrepreneurs would continue to borrow in order to produce. But this would insure slightly lower profits, so profits originally must have been zero for this to be an equilibrium. This implies that in equilibrium, non-entrepreneurs who sell the asset at date 2 receive a single rate  $r_{1*}$  that satisfies

$$(1 - \phi) r_{1*} + \phi [(1 - q + q\epsilon)(1 + r_{1*}) - 1] = 0$$

or, rearranging,

$$1 + r_{1*} = \frac{1}{1 - q\phi(1 - \epsilon)}$$

Next, I argue that  $r_{1*}$  must also leave non-entrepreneurs just indifferent between holding it and selling it at date 2, i.e.

$$p_2 = \epsilon D + (1 - \epsilon)(1 + r_{1*})p_1$$

For suppose that the non-entrepreneurs who sell their holdings in period 2 strictly prefer to sell at  $r_{1*}$ . Then a lender could earn positive expected profits by offering all agents who in equilibrium receive a rate above  $r_{1*}$  a new contract with a rate that is just a little above  $r_{1*}$ . Since non-entrepreneurs would buy the asset and sell it, they would act in the same way as those receiving the rate  $r_{1*}$ . Entrepreneurs would continue to borrow and produce. Since expected profits were zero at  $r_{1*}$ , they would be strictly positive at a higher rate. The only way to ensure lenders who offer a high rate are unwilling to offer other contracts is if the rate at which traders are just indifferent about selling at date 2 yields zero expected profits. Finally, since lenders must earn zero expected profits in total, they must also earn zero expected profits from lending at the high rate  $r_1^*$ . Since these borrowers never sell the asset, the interest rate is the same as in the static model, i.e.

$$1 + r_1^* = \frac{1}{1 - \phi(1 - \epsilon)}$$

In sum, the equilibrium in case (c) implies there will be two distinct rates offered,  $r_{1*}$  and  $r_1^*$ , and the larger the amount of assets that date 1 buyers sell to new buyers at date 2, the larger will be the fraction of agents receiving low rates in period 1.

We are now in a position to provide equations associated with the different cases (a) - (d) above. In case (a), the price in period 2 must be  $\epsilon D$ , or else all original owners would want to sell the asset. But then  $p_1 = \epsilon D$ , or else all of the original owners would have sold at date 1. But in that case, all the only agents who can sell the asset in period 2 would have a reservation price that exceeds  $\epsilon D$ , making it impossible for the asset market to clear at  $p_2 = \epsilon D$ . Hence, case (a) implies

$$\begin{aligned} p_1 &= \epsilon D \\ p_2 &= \epsilon D \end{aligned} \tag{27}$$

In case (b), all original owners must sell the asset at either date 1 or date 2, so total demand must equal the original stock of the asset, 1. In addition, since demand is positive in both periods, the original owners must be indifferent between selling at date 1 and waiting to sell at date 2. Hence, the prices  $p_1$  and  $p_2$  must satisfy the system of equations

$$\begin{aligned}\frac{n_1}{p_1} + \frac{n_2}{p_2} &= 1 \\ (1 - q)p_2 + q\epsilon D &= p_1\end{aligned}\tag{28}$$

To ensure that none of those who bought the asset at date 1 wish to sell it, we must check that the solution to (28) satisfies  $p_2 \leq \epsilon D + (1 - \epsilon)(1 + r_{1*})p_1$ , i.e.  $p_2$  does not exceed the reservation price of agents who bought the asset at date 1. The reservation price depends on  $r_{1*}$  since an agent who sells the asset will receive the rate  $r_{1*}$ .

In case (c), some but not all of those who bought the asset at date 1 sell it at date 2. From the analysis above, we know that the agents who buy the asset at date 1 and sell at date 2 are offered a rate  $r_{1*}$  at date 1 and are indifferent about selling at date 2. In addition, either all of the original owners sell their assets at date 1, implying  $n_1/p_1 = 1$ , or else the original owners must be indifferent between the two periods. These conditions can be summarized as follows:

$$\begin{aligned}p_2 &= \epsilon D + (1 - \epsilon)(1 + r_{1*})p_1 \\ p_1 &= \max\{(1 - q)p_2 + q\epsilon D, n_1\}\end{aligned}\tag{29}$$

To ensure that not all of those who bought the assets at date 1 sell in date 2, we need to verify that at the prices that solve the system (29),  $n_2/p_2 < 1$ .

Finally, in case (d), all of those who own the asset at the beginning of date 2 sell it. In this case, buyers who show up at date 2 must buy up all existing shares of the asset. Once again, either some of the original owners sell in either period, requiring them to be indifferent between the two periods, or else the original owners buy up all of the assets at date 1, implying  $n_1/p_1 = 1$ . The two conditions are thus

$$\begin{aligned}p_2 &= n_2 \\ p_1 &= \max\{n_1, (1 - q)n_2 + q\epsilon D\}\end{aligned}\tag{30}$$

The rest of the proof shows that we can partition the set  $\mathcal{N}$  into distinct regions where the unique equilibrium in each region corresponds to a different case. These regions are illustrated graphically in Figure A1 and identified in the analysis below.

First, case (a) is the unique equilibrium whenever  $n_1 + n_2 < \epsilon D$ , which corresponds to region A in Figure A1. This is because at  $p_1 = p_2 = \epsilon D$ , the prices that must prevail in this equilibrium, all non-entrepreneurs in both periods would buy the asset. But if  $n_1 + n_2 > \epsilon D$ , the sum of demands in the two periods exceeds 1,

and so we cannot have an equilibrium in which some of the original owners hold on to the asset to maturity. By contrast, it is easy to confirm that  $p_1 = p_2 = \epsilon D$  is an equilibrium when  $n_1 + n_2 < \epsilon D$ . One can verify that none of the other cases can be supported as equilibria in this region: The solution to the system of equations (28) require that either  $p_1 < \epsilon D$  or  $p_2 < \epsilon D$  or both, which cannot occur in equilibrium, while the systems of equations (29) and (30) yield solutions for  $p_1$  and  $p_2$  at which total demand for the asset over the two periods  $\frac{n_1}{p_1} + \frac{n_2}{p_2}$  is less than 1, implying not all original owners sell the asset as required in these two cases.

Next, consider case (b). We already ruled out that this can be an equilibrium in region A. The two equilibrium conditions can be reduced to a single polynomial in  $p_2$  which has only one root that exceeds  $\epsilon D$ . Hence, there is a unique pair  $(p_1, p_2)$  given values  $(n_1, n_2)$ . Recall that to verify that these prices are consistent with an equilibrium of type (b), we need to verify that the implied  $p_2$  is not so high that agents who bought at date 1 wish to sell their assets, i.e.  $p_2 \leq \epsilon D + (1 - \epsilon)(1 + r_{1*})p_1$ . Since  $p_1 = (1 - q)p_2 + q\epsilon D$ , we can substitute for  $r_{1*}$  and  $p_1$  to get

$$p_2 \leq \epsilon D + \frac{(1 - \epsilon)[(1 - q)p_2 + q\epsilon D]}{1 - (1 - \epsilon)q\phi}$$

which, upon rearranging, yields

$$p_2 \leq \frac{1 + q(1 - \epsilon)(1 - \phi)}{1 - (1 - \epsilon)(1 - q(1 - \phi))}\epsilon D$$

Inspecting the system of equations for  $n_1$  and  $n_2$  in case (b) shows that  $p_2$  is increasing in both  $n_1$  and  $n_2$ . For  $n_1 = 0$ , the highest value of  $n_2$  for which this condition holds is given by  $n_2 = \frac{1 + (1 - \epsilon)q(1 - \phi)}{1 - (1 - \epsilon)(1 - q(1 - \phi))}\epsilon D$ .

For  $n_2 = 0$ , the highest value of  $n_1$  for which this condition holds is given by  $n_1 = \frac{1 - (1 - \epsilon)q\phi}{1 - (1 - \epsilon)(1 - q(1 - \phi))}\epsilon D$ .

This establishes that the region in which there exists an equilibrium consistent with case (b) is bounded, and that  $(n_1, n_2)$  outside this region are inconsistent with an equilibrium where case (b) holds. This region is shown graphically as region B in Figure A1. Since values in the interior of region B imply  $p_2 < \epsilon D + (1 - \epsilon)(1 + r_{1*})p_1$  given  $p_2$  is increasing in both  $n_1$  and  $n_2$ , there can be no equilibria associated with cases (c) or (d) inside region B.

Next, we move to case (c). It will help to examine separately equilibria in which  $(1 - q)p_2 + q\epsilon D$  exceeds  $n_1$  and equilibria in which it does not. If  $(1 - q)p_2 + q\epsilon D > n_1$ , then we have two equations and two unknowns, which have a unique solution:

$$\begin{aligned} p_2 &= \frac{1 + q(1 - \epsilon)(1 - \phi)}{1 - (1 - \epsilon)(1 - q(1 - \phi))}\epsilon D \\ p_1 &= \frac{1 - (1 - \epsilon)q\phi}{1 - (1 - \epsilon)(1 - q(1 - \phi))}\epsilon D \end{aligned}$$

One restriction that needs to be satisfied is that  $n_1 < (1 - q)p_2 + q\epsilon D = \frac{1 - (1 - \epsilon)q\phi}{1 - (1 - \epsilon)(1 - q(1 - \phi))}\epsilon D \equiv \bar{n}_1^*$ . At the same time, we need to verify that  $n_2 < p_2$ , or else demand by non-entrepreneurs who arrive at

date 2 would exceed the total supply of the asset. This implies  $n_2 < \frac{1+q(1-\epsilon)(1-\phi)}{1-(1-\epsilon)(1-q(1-\phi))}\epsilon D \equiv \bar{n}_2^*$ . Hence, an equilibrium consistent with case (c) in which  $n_1 < (1-q)p_2 + q\epsilon D$  is confined to values  $\{(n_1, n_2) : (n_1, n_2) \notin A \cup B, n_1 < \bar{n}_1^* \text{ and } n_2 < \bar{n}_2^*\}$ . This set is illustrated graphically as region  $C_1$  in Figure A1. Note that the equilibrium prices  $p_1$  and  $p_2$  do not vary with  $n_1$  and  $n_2$  in this region.

Next, consider case (c) where  $n_1 > (1-q)p_2 + q\epsilon D$ . In this case, the system of equations is given by

$$\begin{aligned} p_2 &= \epsilon D + \frac{1-\epsilon}{1-q\phi(1-\epsilon)}n_1 \\ p_1 &= n_1 \end{aligned}$$

As before, there are two restrictions that need to be satisfied to ensure these prices are indeed an equilibrium. First, we need to confirm that  $n_1 > (1-q)p_2 + q\epsilon D$ . Substituting in the value of  $p_2$  yields  $n_1 > \bar{n}_1^*$  as defined above. Second, we again need to make sure  $n_2 < p_2$  so demand by non-entrepreneurs at date 2 would exceed the total supply of the asset. In this case, this condition implies

$$n_2 < \epsilon D + \frac{1-\epsilon}{1-q\phi(1-\epsilon)}n_1$$

This defines the region  $C_2 = \left\{ (n_1, n_2) : n_1 > \bar{n}_1^*, n_2 < \epsilon D + \frac{1-\epsilon}{1-q\phi(1-\epsilon)}n_1 \right\}$  depicted in Figure A1, which is the only region in which an equilibrium that accords with case (c) arises.

Finally, we consider case (d). Again, we separately consider the case where  $(1-q)n_2 + q\epsilon D$  exceeds  $n_1$  and the case where the opposite is true. If  $(1-q)n_2 + q\epsilon D > n_1$ , then we have  $p_1 = (1-q)n_2 + q\epsilon D$  and  $p_2 = n_2$ . We need to confirm that  $n_1 < (1-q)n_2 + q\epsilon D$ . Since all the agents who buy the asset in period 1 must be willing to sell it in period 2, we need to verify that

$$p_2 \geq \epsilon D + (1-\epsilon)(1+r_{1*})p_1$$

Substituting in for  $p_1$ ,  $p_2$ , and  $r_{1*}$  yields

$$n_2 \geq \epsilon D + \frac{(1-\epsilon)((1-q)n_2 + q\epsilon D)}{1-q\phi(1-\epsilon)}$$

which, upon rearranging, yields

$$n_2 \geq \frac{1+q(1-\epsilon)(1-\phi)}{1-(1-\epsilon)(1-q(1-\phi))}\epsilon D \equiv \bar{n}_2$$

This corresponds to region  $D_1$  in Figure A1, which has no overlap with region  $C_1 \cup C_2$ . Thus, this type of equilibrium cannot occur in these regions. Finally, we have case (d) where  $n_1 > (1-q)n_2 + q\epsilon D$ . Again, to ensure that all the agents who buy the asset in period 1 are willing to sell it in period 2 requires that  $p_2 \geq \epsilon D + (1-\epsilon)(1+r_{1*})p_1$ . Substituting in for  $p_1$ ,  $p_2$ , and  $r_{1*}$  yields

$$n_2 \geq \epsilon D + \frac{(1-\epsilon)n_1}{1-q\phi(1-\epsilon)}$$

Hence, this region, denoted  $D_2$  in Figure A1, is given by

$$\left\{ (n_1, n_2) : \epsilon D + \frac{(1-\epsilon)n_1}{1-q\phi(1-\epsilon)} \leq n_2 \leq \frac{n_1 - q\epsilon D}{1-q} \right\}$$

and this region does not overlap with  $C_1 \cup C_2$ . The uniqueness of equilibrium is thus established for all  $(n_1, n_2) \in \mathcal{N}$ .

The last results in the proposition can now be established. Define

$$\bar{n}_1(n_2) = \max \left\{ n_1^*, \frac{1-q\phi(1-\epsilon)}{1-\epsilon} (n_2 - \epsilon D) \right\}$$

where recall  $\bar{n}_1^* = \frac{1-(1-\epsilon)q\phi}{1-(1-\epsilon)(1-q(1-\phi))} \epsilon D$ . In this case, if  $n_1 + n_2 > \epsilon D$  and  $n_1 < \bar{n}_1(n_2)$ , then we must be in region  $B$ ,  $C_1$ , or  $D_1$ . In any of these cases, we have

$$p_1 = q\epsilon D + (1-q)p_2$$

or, rearranging,

$$p_1 - \epsilon D = (1-q)(p_2 - \epsilon D)$$

as stated in the proposition. If instead  $n_1 + n_2 > \epsilon D$  and  $n_1 < \bar{n}_1(n_2)$ , then we must be in region  $C_2$  or  $D_2$ . In this case, we have

$$p_1 = n_1 > q\epsilon D + (1-q)p_2$$

and subsequently

$$p_1 - \epsilon D > (1-q)(p_2 - \epsilon D)$$

Finally, as either  $n_1$  or  $n_2$  rises, we eventually hit region  $C_1$ ,  $C_2$ ,  $D_1$  or  $D_2$ . In any of these regions, agents who buy the asset at date 1 prefer to sell the asset at date 2. ■

**Proof of Lemma 1:** Observe first that even under a general contract, the fact that lenders lose a finite amount per non-entrepreneur who buys risky assets implies a bound on the amount lenders can collect from entrepreneurs and still earn zero expected profits. This bound can be applied in the same way as in the proof of Proposition 3 to show that any parameters  $n_1$ ,  $n_2$ , and  $R$  which satisfy (3) and (4) will imply that in equilibrium entrepreneurs will prefer to initiate production and non-entrepreneurs can earn strictly positive profits from buying the asset.

I first argue that  $D/p_1 \geq R$ . As noted in the proof of Proposition 3, Lemma A1 extends to the dynamic case to show that  $p_1 \geq \epsilon D$  and  $p_2 \geq \epsilon D$ . If  $p_1 = \epsilon D$ , since the second inequality in (3) implies  $\frac{D}{R} > \epsilon D$  and  $p_1 = \epsilon D$ , it follows that  $\frac{D}{p_1} > R$ . Moreover, since  $p_1 = \epsilon D$  implies  $p_2 = \epsilon D$ , or else there would be excess demand for the asset at date 1, it further follows that  $\frac{D}{p_2} > R$ .

The only relevant case therefore is if  $p_1 > \epsilon D$ . In this regard, note that a necessary condition for  $p_1 > \epsilon D$  is that  $n_1 + n_2 > \epsilon D$ . This is because given (9), only non-entrepreneurs would be willing to buy the asset, and so total demand for the asset is at most  $\frac{n_1}{p_1} + \frac{n_2}{p_2}$ . The latter is in turn bounded above by  $\frac{n_1 + n_2}{\epsilon D}$  given  $p_1$  and  $p_2$  are bounded below by  $\epsilon D$ . When  $n_1 + n_2 < \epsilon D$ , this means the total amount sold is less than the number of assets, and so some original owners must hold on to the asset in all states of the world. This requires  $p_1 = p_2 = \epsilon D$ , a contradiction.

Next, I argue that for  $n_1 + n_2 > \epsilon D$ , we have  $p_1 \leq n_1 + n_2$ . If  $p = \epsilon D$ , the statement follows trivially. If  $p_1 > \epsilon D$ , the original owners can do strictly better than holding the assets they own to maturity. Hence, they must either sell with certainty at date 1, or wait and sell with certainty if the price is even higher at date 2. Hence, if  $d$  remains uncertain, all owners will sell with certainty. This requires that

$$\frac{n_1}{p_1} + \frac{n_2}{p_2} \geq 1$$

Since  $p_2 > p_1$ , it follows that

$$\frac{n_1 + n_2}{p_1} > \frac{n_1}{p_1} + \frac{n_2}{p_2} \geq 1$$

and hence  $p_1 < n_1 + n_2$ , as claimed. By (10) we have  $p_1 < n_1 + n_2 < (1 - q) \frac{D}{R} + q\epsilon D$ . By the second inequality in (9),  $\frac{D}{R} \geq \epsilon D$ , and so  $(1 - q) \frac{D}{R} + q\epsilon D \leq \frac{D}{R}$ . It follows that  $p_1 \leq D/R$ , as we wanted to show.

The last step is to show that if  $n_1 + n_2 > \epsilon D$ , then  $p_2 \leq D/R$ . If  $p_2 = \epsilon D$ , the statement follows from the second inequality in (9). If  $p_2 > \epsilon D$ , then  $p_1 > \epsilon D$ , or else there would be excess demand for the asset at date 1 given the asset is worth at least  $\epsilon D$  regardless of whether  $d$  is revealed, so all agents would strictly prefer to buy it at a price of  $p_1 = \epsilon D$ . Moreover, since the original owners can wait to sell, the fact that some agents will sell at date 1 requires that  $p_1 \geq q\epsilon D + (1 - q)p_2$ . But this implies

$$\begin{aligned} p_2 &\leq \frac{p_1 - q\epsilon D}{1 - q} \\ &\leq \frac{n_1 + n_2 - q\epsilon D}{1 - q} \\ &\leq \frac{(1 - q) D/R}{1 - q} = \frac{D}{R} \end{aligned}$$

which establishes that  $D/p_2$  also exceeds  $R$ . ■

**Proof of Proposition 4:** The proof is by contradiction. Suppose there was an equilibrium where the price of the asset at date 2 exceeded  $\epsilon D$ , i.e.  $p_2 > \epsilon D$ . Let  $x_2^n$  denote the amount of resources non-entrepreneurs receive upon arrival at date 2 under the equilibrium contract, and denote the amount they would have to repay if  $d = D$  by  $(1 + r_2^n) x_2^n$ . The expected payoff to a non-entrepreneur from buying the asset would then equal

$$x_2^n \epsilon \left( \frac{D}{p_2} - (1 + r_2^n) \right)$$

while the expected payoff to the lender in this case is

$$x_2^n (\epsilon (1 + r_2^n) - 1)$$

Summing together yields

$$x_2^n \left( \frac{\epsilon D}{p_2} - 1 \right)$$

This expression is negative when  $p_2 > \epsilon D$ . Hence,

$$x_2^n \epsilon \left( \frac{D}{p_2} - (1 + r_2^n) \right) < x_2^n (1 - \epsilon (1 + r_2^n)) \quad (31)$$

Consider a creditor who offered a contract  $\hat{x}$  in which non-entrepreneurs receive nothing upon arrival, i.e.  $\hat{x}_2^n = 0$  and then paid the agent  $\epsilon \left( \frac{D}{p_2} - (1 + r_2^n) \right) x_2^n$  at the end of date 2, which is just the expected payoff from buying the asset under the original contract. From (31), we know that lenders are strictly better off from this arrangement, since they incur smaller losses. Since (31) is a strict inequality, and since the first inequality in (9) ensures entrepreneurs strictly prefer production to buying risky assets, it will be possible to increase the payment to non-entrepreneurs a little to make them strictly better off and to lower the interest charged to entrepreneurs in a way that makes them better off but still induces them to choose the contract intended for them and to use the funds they borrow for production. Hence, if  $p_2 > \epsilon D$ , no agents buy the asset at date 2. Since any original owners who still hold the asset at date 1 would prefer to sell at a price of  $\epsilon D$  than hold on to the asset, the only possible equilibrium is one in which all the original owners sell the asset at date 1.

Next, we move to date 1. First, consider whether there can be an equilibrium in which the asset is purchased at date 1. Suppose  $p_1 > \epsilon D$ . In that case, we know that  $(1 + r_1) p_1 \geq p_1 > \epsilon D$ , so that any agents who buy the asset at date 1 would only agree to sell the asset at date 2 for a price above  $\epsilon D$ . But we just argued that if  $p_2 > \epsilon D$ , there cannot be an equilibrium in which agents buy the asset at date 2 in equilibrium. Hence, non-entrepreneurs who buy the asset at date 1 expect to hold on to the asset until  $d$  is revealed. But by a similar argument to before, if  $p_1 > \epsilon D$ , then creditors can earn positive expected profits by giving non-entrepreneurs zero resources up front and then paying them after date 2 a little above their expected profits from buying risky assets. Hence, the only possible equilibrium is one where  $p_1 = \epsilon D$ .

However, if  $p_1 = \epsilon D$ , there cannot be an equilibrium where  $p_2 > \epsilon D$ . If there were such an equilibrium, all original owners will hold on to their asset until date 2 and sell if  $p_2 > \epsilon D$ , i.e. if  $d$  was not revealed. But we know that if  $p_2 > \epsilon D$ , there will not be any buyers at date 2 in equilibrium. Hence, the market will not clear at date 2. It follows that not only is  $p_1 = \epsilon D$  in equilibrium, but  $p_2 = \epsilon D$ . ■

**Proof of Lemma 2:** Suppose not, i.e. a non-entrepreneur strictly prefers to announce  $n$  than to announce  $e$ . It is easy to check that in equilibrium,  $r_t^e > 0$  for both  $t \in \{1, 2\}$ : non-entrepreneurs will incur a cost on creditors that must be made up to ensure creditors earn zero expected profits. Consider a lender who only



offers the following contract  $\{\tilde{x}_t^e, \tilde{r}_t^e\}$  aimed at entrepreneurs:

$$\begin{aligned}\tilde{x}_t^e &= x_t^e = 1 \\ \tilde{r}_t^e &= r_t^e - \varepsilon\end{aligned}$$

where  $\varepsilon \in (0, r_t^e)$  so the interest rate remains positive. Entrepreneurs will clearly prefer this contract to the equilibrium contract. Non-entrepreneurs originally strictly preferred the contract they received in equilibrium to the contract  $\{1, r_t^e\}$ , and hence for  $\varepsilon$  small enough will also prefer it to  $\{1, \tilde{r}_t^e\}$ . Hence, this contract will only attract entrepreneurs, and for  $\varepsilon$  small enough the expected profits to the creditor who offers it will be strictly positive. Hence, the original contract could not have been an equilibrium. ■

**Proof of Proposition 5:** Consider the maximization problem

$$\max_{x_2^n, r_2^n} x_2^n [\varepsilon(1 + r_2^n) - 1]$$

subject to

- i.  $\varepsilon \left( \frac{D}{p_2} - (1 + r_2^n) \right) x_2^n = \varepsilon \left( \frac{D}{p_2} - (1 + r_2^e) \right)$
- ii.  $R - (1 + r_2^e) \geq \varepsilon \left( \frac{D}{p_2} - (1 + r_2^n) \right) x_2^n$
- iii.  $r_2^n \geq 0$

First, we can substitute from constraint (1) to rewrite constraint (2) as

$$R - (1 + r_2^e) \geq \varepsilon \left( \frac{D}{p_2} - (1 + r_2^e) \right)$$

or, after rearranging,

$$R \geq \frac{\varepsilon D}{p_2} + (1 - \varepsilon)(1 + r_2^e)$$

Using the proof of Proposition 1, we know that in equilibrium  $1 + r_2^e \leq \frac{1}{1 - (1 - \varepsilon)\phi}$ , or else the lender would make strictly positive profits. But (9) implies

$$R > 1 + \frac{1 - \varepsilon}{1 - (1 - \varepsilon)\phi} \geq \frac{\varepsilon D}{p_2} + (1 - \varepsilon)(1 + r_2^e)$$

so constraint (2) is automatically satisfied in equilibrium and can therefore be ignored. Rearranging constraint (1) yields

$$\varepsilon(1 + r_2^n)x_2^n = \frac{\varepsilon D}{p_2}(x_2^n - 1) + \varepsilon(1 + r_2^e)$$

Substituting this into the objective function yields

$$\left( \frac{\varepsilon D}{p_2} - 1 \right) x_2^n - \frac{\varepsilon D}{p_2} + \varepsilon(1 + r_2^e)$$

When  $p_2 > \varepsilon D$ , this expression is decreasing in  $x_2^n$ , so the optimal contract reduces  $x_2^n$ . From constraint (1), this implies  $r_2^n$  will be as small as possible. Since constraint (3) implies  $r_2^n \geq 0$ , it follows that the optimal contract involves  $r_2^n = 0$ . From constraint (1), this implies

$$x_2^n = \frac{D - (1 + r_2^e)p_2}{D - p_2}$$

The condition on  $r_2^\epsilon$  follows from the fact that lenders earn zero expected profits in equilibrium. ■

**Proof of Proposition 6:** Consider first an agent who intends to hold on to his asset. In this case, the contract is given by

$$\max_{x_1^n, r_1^{n,\text{late}}} x_1^n \left[ \epsilon \left( 1 + r_1^{n,\text{late}} \right) - 1 \right]$$

subject to

$$\begin{aligned} \text{i.} \quad & \epsilon \left( \frac{D}{p_1} - \left( 1 + r_1^{n,\text{late}} \right) \right) x_1^n = \epsilon \left( \frac{D}{p_1} - \left( 1 + r_1^\epsilon \right) \right) \\ \text{ii.} \quad & r_1^{n,\text{late}} \geq 0 \end{aligned}$$

where we know from the proof of Proposition 5 that we can omit the constraint that the entrepreneur must prefer to produce than buy assets. Since this problem is identical to the period 2 problem, the same argument implies that when  $p > \epsilon D$ , in equilibrium  $r_1^{n,\text{late}} = 0$  and

$$x_2^n = \frac{D - (1 + r_2^\epsilon) p_2}{D - p_2}$$

When  $p_1 = \epsilon D$ , profits are independent of  $x_1$ , and so any pair  $(x_1^n, r_1^n)$  that yields expected utility  $V_1(n, e)$  to the agent is an equilibrium.

Next, consider an agent who intends to sell his asset if  $d$  remains uncertain. In this case, the equilibrium contract solves

$$\max_{x_1^n, r_1^{n,\text{early}}} x_1^n \left[ (q\epsilon + 1 - q) \left( 1 + r_1^{n,\text{early}} \right) - 1 \right]$$

subject to

$$\begin{aligned} \text{i.} \quad & \left[ q\epsilon \left( \frac{D}{p_1} - 1 - r_1^{n,\text{early}} \right) + (1 - q) \left( \frac{p_2}{p_1} - 1 - r_1^{n,\text{early}} \right) \right] x_1^n = V_1(n, e) \\ \text{ii.} \quad & r_1^{n,\text{early}} \geq 0 \end{aligned}$$

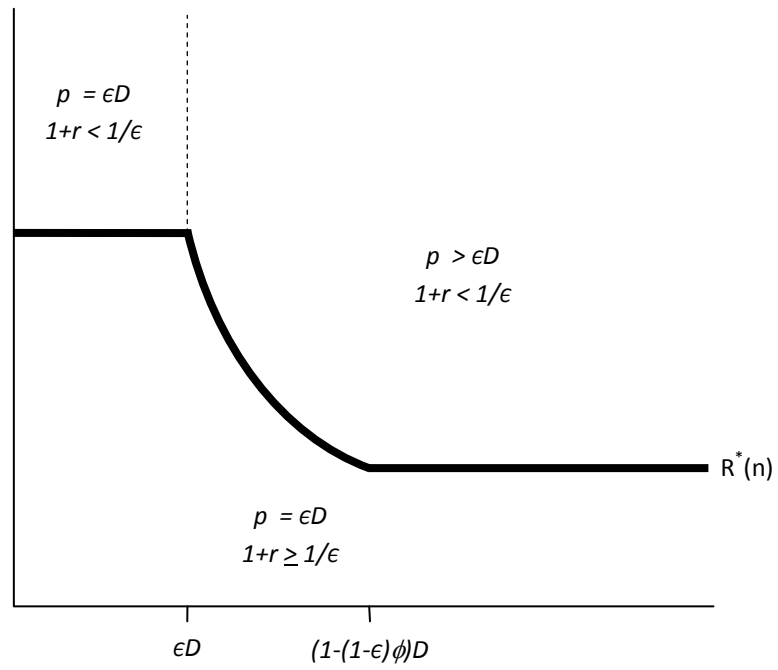
Rearranging constraint (1) yields

$$\left[ \frac{q\epsilon D + (1 - q) p_2}{p_1} \right] x_1^n - V_1(n, e) = (q\epsilon + 1 - q) \left( 1 + r_1^{n,\text{early}} \right) x_1^n$$

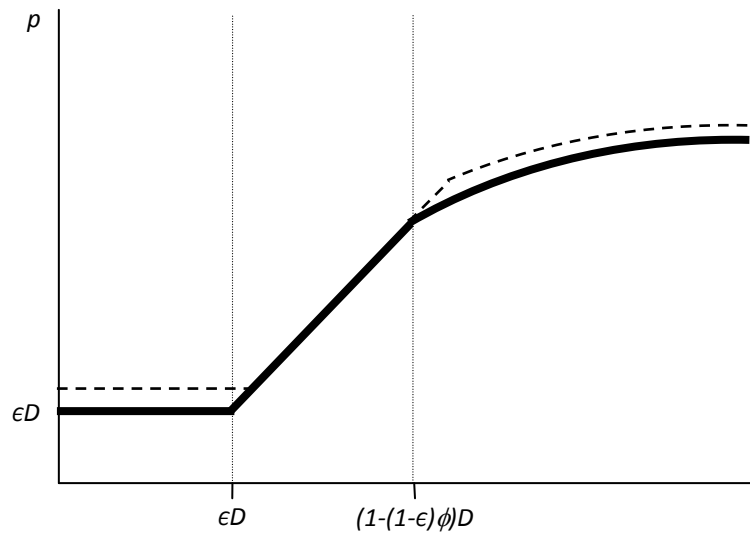
So the objective function can be written as

$$\left[ \frac{q\epsilon D + (1 - q) p_2}{p_1} - 1 \right] x_1 - V_1(n, e)$$

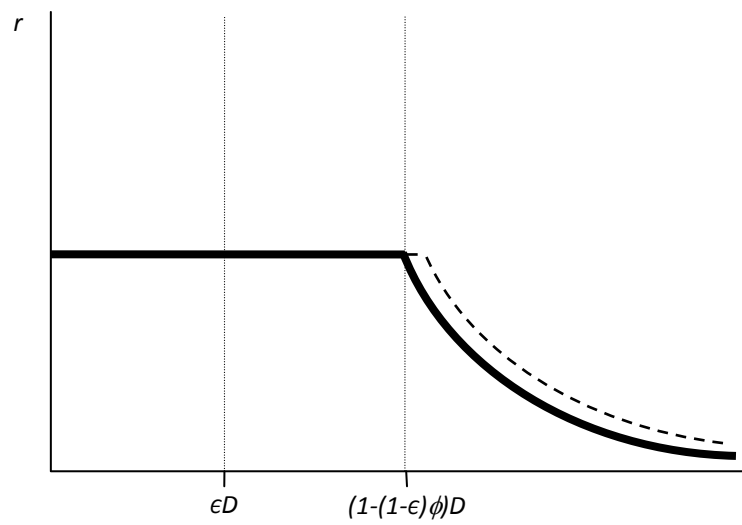
When  $p_1 > q\epsilon D + (1 - q) p_2$ , this expression is decreasing in  $x_1^n$ , so the optimal contract reduces  $x_1^n$  and sets  $r_1^{n,\text{early}} = 0$ . Since  $p_1 \geq q\epsilon D + (1 - q) p_2$  or else the asset market would not clear at date 1, in the only other case profits are independent of  $x_1$ , and so any pair  $(x_1^n, r_1^n)$  that yields expected utility  $V_1(n, e)$  to the agent is an equilibrium. ■



**Figure 1:** Characteristics of equilibrium  $p$  and  $r$  for different  $(n,R)$  pairs

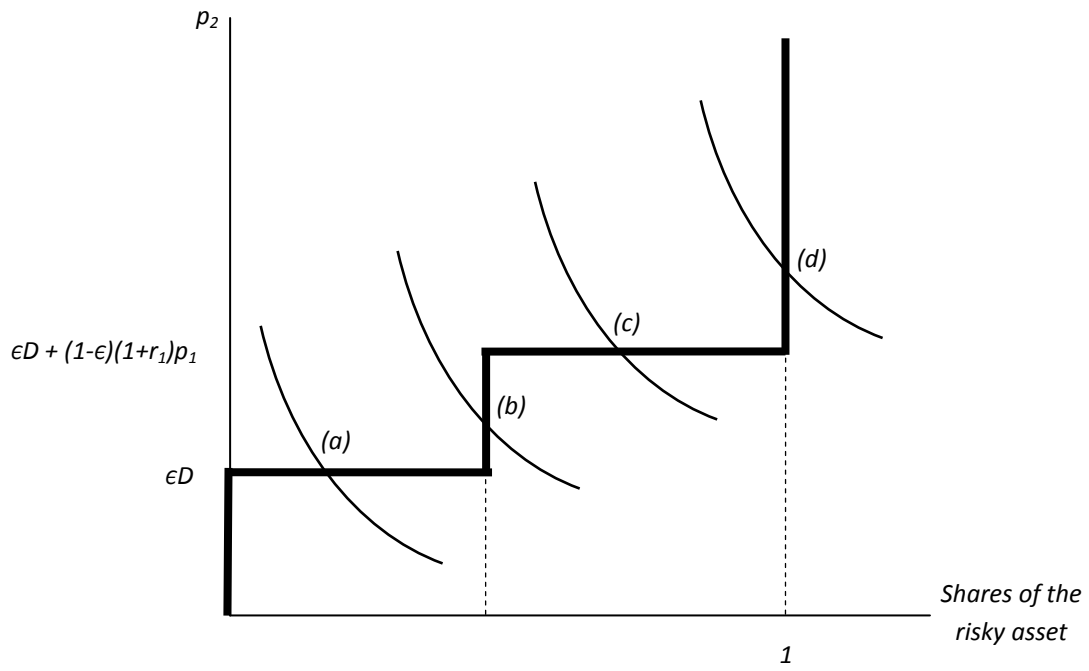


**a. Equilibrium price of the asset**

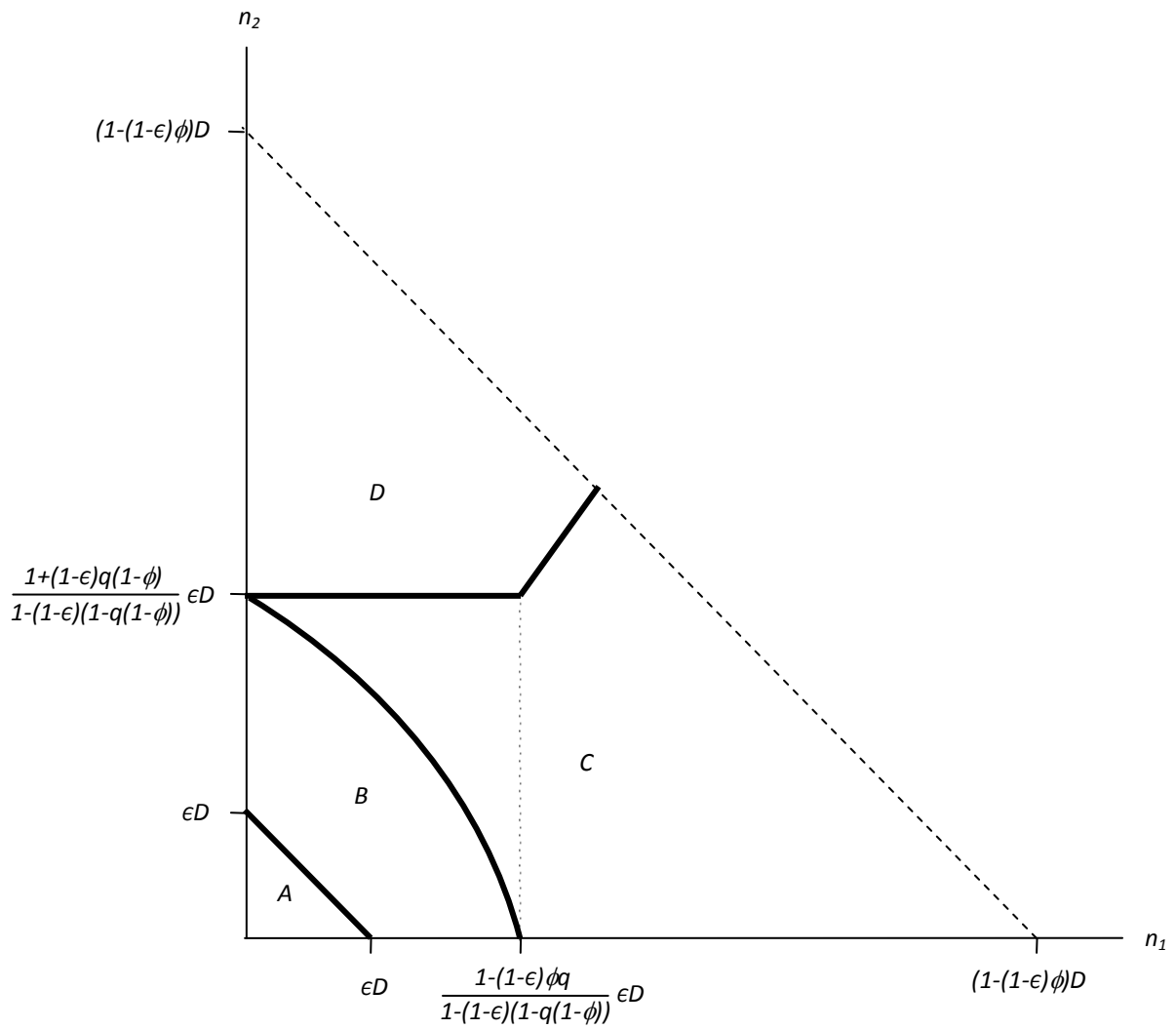


**b. Equilibrium interest rate on loans**

**Figure 2:** Equilibrium  $p$  and  $r$  as functions of  $n$  and  $D$  when  $R > R^*(n)$

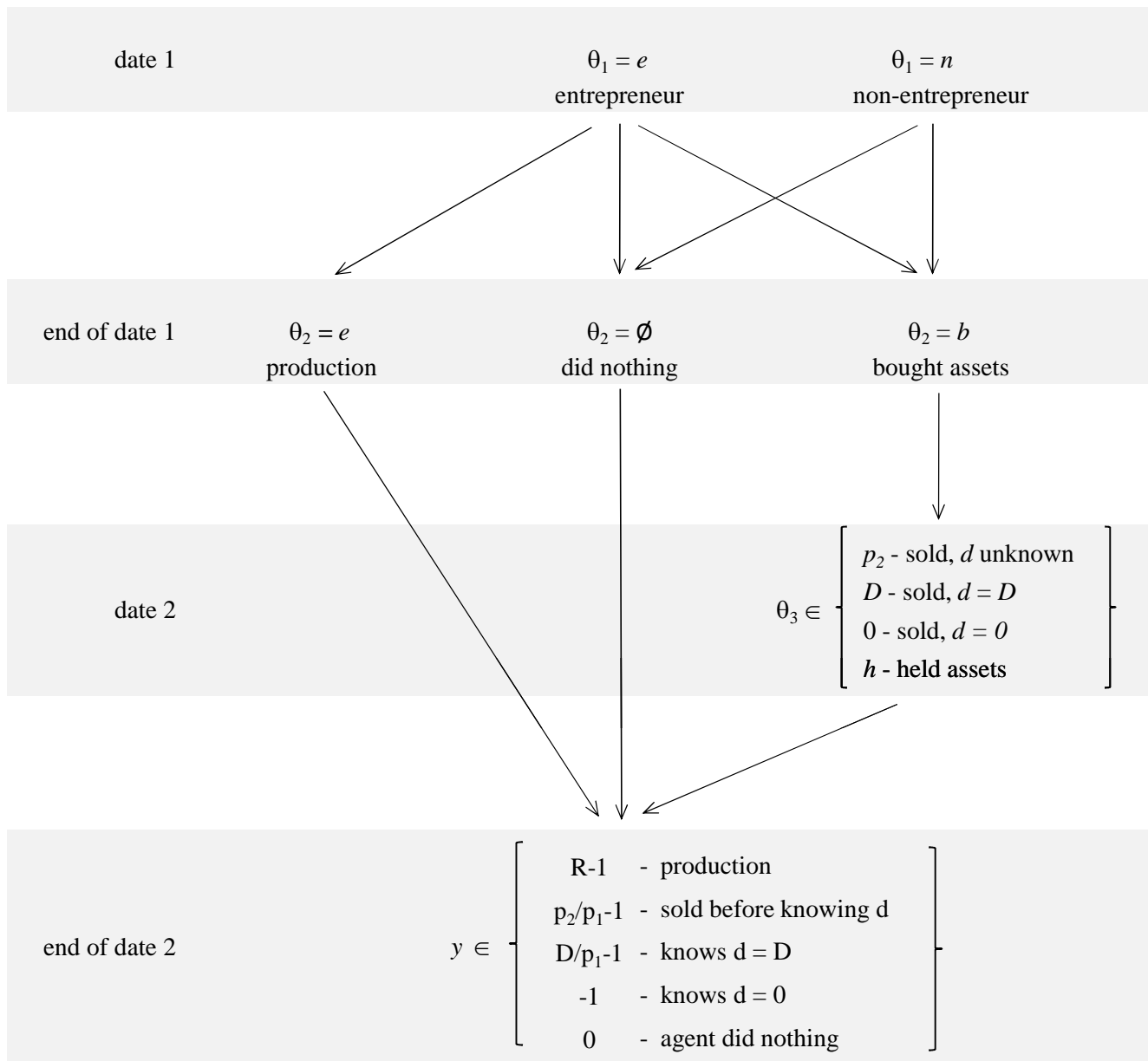


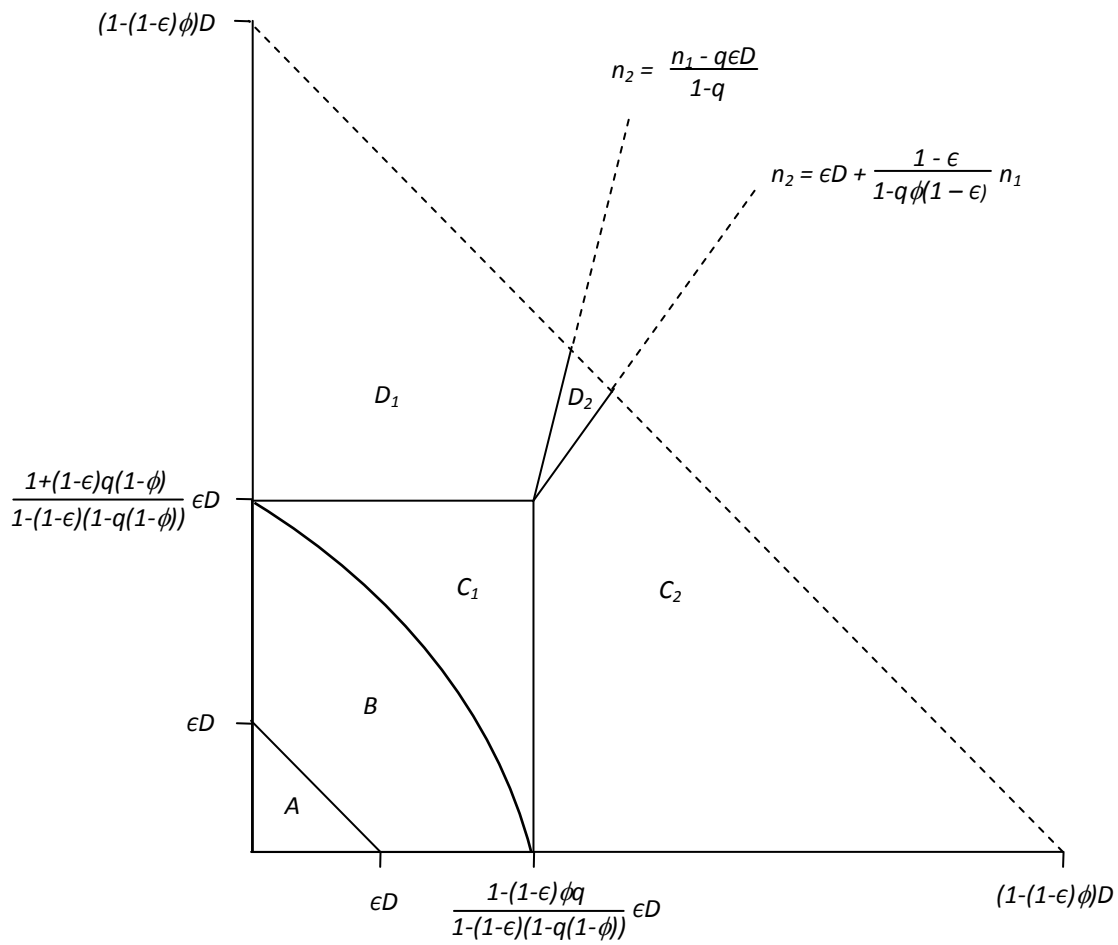
**Figure 3:** Equilibrium in period 2



**Figure 4:** Regions corresponding to different type of equilibrium in period 2

**Figure 5: Reports Mandated by Contract**





**Figure A1:** Equilibrium types for different pairs  $(n_1, n_2)$



## References

- [1] Albagli, Elias, Christian Hellwig, and Aleh Tsyvinski, 2011. "A Theory of Asset Prices Based on Heterogeneous Information" Cowles Foundation Discussion Papers 1827, Yale University.
- [2] Allen, Franklin and Gary Gorton, 1993. "Churning Bubbles" *Review of Economic Studies*, 60(4), October, p813-836.
- [3] Allen, Franklin and Douglas Gale, 1994. "Limited Market Participation and Volatility of Asset Prices" *American Economic Review*, 84(4), September, p933-55.
- [4] Allen, Franklin and Douglas Gale, 2000. "Bubbles and Crises" *Economic Journal*, 110(460), January, p236-255.
- [5] Allen, Franklin, Stephen Morris, and Andrew Postlewaite, 1993. "Finite Bubbles with Short Sales Constraints and Asymmetric Information" *Journal of Economic Theory*, 61(2), December, p206-29.
- [6] Barlevy, Gadi and Jonas Fisher, 2011. "Mortgage Choice and Housing Speculation" Federal Reserve Bank of Chicago Working Paper.
- [7] Bewley, Truman, 1980. "The Optimum Quantity of Money" *Models of Monetary Economics*, eds. J. Kareken and N. Wallace. Minneapolis, Minnesota: Federal Reserve Bank, p169-210.
- [8] Blanchard, Olivier and Mark Watson, 1982. "Bubbles, Rational Expectations, and Financial markets" *Crisis in the Economic and Financial Structure: Bubbles, Bursts, and Shocks*, ed. P. Wachtel. Lexington, Massachusetts: Lexington Books, p295-315.
- [9] Burnside, Craig, Martin Eichenbaum, and Sergio Rebelo, 2011. "Understanding Booms and Busts in Housing Markets" Mimeo, Northwestern University.
- [10] Caballero, Ricardo and Arvind Krishnamurthy, 2006. "Bubbles and Capital Flow Volatility: Causes and Risk Management" *Journal Monetary of Economics*, 53(1), January, p35-53.
- [11] Chancellor, Edward, 2000. *Devil Takes the Hindmost*. London: Penguin Books.
- [12] Diamond, Peter, 1965. "National Debt in a Neoclassical Growth Model" *American Economic Review*, 55(5), Part 1, December, p1126-50.
- [13] Farhi, Emmanuel and Jean Tirole, 2011. "Bubbly Liquidity" *Review of Economics Studies*, forthcoming.
- [14] Geanakoplos, John, 2010. "The Leverage Cycle" *NBER Macroeconomics Annual*, 24(1), p1-66.
- [15] Harrison and David Kreps, 1978. "Speculative Investor Behavior in a Stock Market with Heterogeneous Expectations" *Quarterly Journal of Economics*, 92(2), May, p323-36.
- [16] Hong, Harrison and David Sraer, 2011. "Quiet Bubbles" Mimeo, Princeton University

- [17] Kocherlakota, Narayana, 1992. "Bubbles and Constraints on Debt Accumulation" *Journal of Economic Theory*, 57 (1), p245—56
- [18] Mimra, Wanda and Achim Wambach, 2011. "A Game-Theoretic Foundation for the Wilson Equilibrium in Competitive Insurance Markets with Adverse Selection" CESifo Working Paper 3412.
- [19] Miyazaki, Hajime, 1977, "The Rat Race and Internal Labor Markets", *Bell Journal of Economics*, 8(2), p394-418.
- [20] Rothschild, M., Stiglitz, J., 1976. "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information." *Quarterly Journal of Economics*, 90 (4), p629–649.
- [21] Samuelson, Paul, 1958. "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money" *Journal of Political Economy*, 66, p467-82.
- [22] Santos, Manuel and Michael Woodford, 1997. "Rational Asset Pricing Bubbles" *Econometrica*, 65(1), January, p19-57.
- [23] Scheinkman, Jose and Wei Xiong, 2003, "Overconfidence and Speculative Bubbles," *Journal of Political Economy*, 111(6), December, p1183—1219.
- [24] Simsek, Alp, 2011. "'Belief Disagreements and Collateral Constraints" Mimeo, Harvard University.
- [25] Spence, Michael, 1978. "Product Differentiation and Performance in Insurance Markets", *Journal of Public Economics*, 10(3), p427-447.
- [26] Temin, Peter and Hans-Joachim Voth, 2004. "Riding the South Sea Bubble" *American Economic Review*, 94 (5), December, p1654-1668.
- [27] Tirole, Jean, 1985. "Asset Bubbles and Overlapping Generations" *Econometrica*, 53(6), November, p1499-1528.
- [28] Wilson, Charles, 1977. "A Model of Insurance Markets with Incomplete Information." *Journal of Economic Theory*, 16 (2), p167–207.

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