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**Working Paper**

## Coordination in the El Farol Bar problem: The role of social preferences and social networks

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## **Coordination in the El Farol Bar Problem: The Role of Social Preferences and Social Networks**

*Shu-Heng Chen and Umberto Gostoli*

**Abstract**

In this paper, the authors continue the pursuit of the self-coordination mechanism as studied in the El Farol Bar problem. However, in addition to efficiency (the optimal use of the public facility), they are also interested in the distribution of the public resources among all agents. Hence, they introduce a two-dimensional El Farol Bar problem, to be distinguished from the early one-dimensional one, which has efficiency as the only concern. The authors ask whether it is possible to have self-coordinating solutions to the El Farol Bar problem so that the public resources can be optimally used with neither idle capacity nor incurring congestion and, in the meantime, the resources can be well distributed among all agents. They consider this ideal situation an El Farol version of a "good society". This paper shows the existence of a positive answer to this inquiry, but it requires two elements, which were largely left out in the conventional literature on the El Farol Bar problem. These elements are social networks and social preferences. The authors first show, through cellular automata, that social networks can contribute to the emergence of a "good society". They then show that the addition of some inequity-averse agents can even guarantee the emergence of the "good society".

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**Keywords** El Farol Bar problem; social preferences; social networks; inequity aversion, cellular automata

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# 1 Introduction

The El Farol Bar problem, introduced by Arthur (1994), has over the years become the prototypical model of a system in which agents, competing for scarce resources, inductively adapt their belief models (or hypotheses) to the aggregate environment they jointly create. The bar's capacity is basically a resource subject to congestion, making the El Farol Bar problem a stylized version of the central problem in public economics represented by the efficient exploitation of common-pool resources. Real-world examples of this problem include traffic congestion and the congestion of computer networks. On the one hand, we hope that the resources can be utilized without too much idle capacity left; on the other hand, we do not want them to be overused which leads to congestion. When, for some reasons, solving this problem by means of central intervention is either infeasible or undesirable, then it has to be solved in a bottom-up manner as the El Farol problem describes.

## 1.1 EL Farol Bar Problem: From Efficiency to Equity

In the literature, most of the studies addressed this problem from the perspective of learning; hence, the answers depend on how agents learn. Briefly put, there are two kinds of learning mechanism being studied in the literature. The first one is *best-response learning* (Arthur, 1994; Edmonds, 1999; Fogel et al., 1999; Challet et al., 2004; Atilgan, Atilgan and Demirel, 2008) and the second one is *reinforcement learning* (Bell and Sethares, 1999; Franke, 2003; Zambrano, 2004; Whitehead, 2008).<sup>1</sup> The typical results are as follows. The best-response learning model tends to have fluctuation, sometimes quite severely, around the threshold (switching between the idle and congestion state), but the steady state where the aggregate bar's attendance is always equal to the bar's maximum capacity is very hard to reach. The reinforcement learning model, however, shows that perfect coordination is possible and that it is, indeed, the long-run behavior to which the system asymptotically converges (Whitehead, 2008). However,

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<sup>1</sup> This distinction is suggested by Franke (2003), who distinguished best-response learning from stimulus-response learning (reinforcement learning). The essential feature of the former is to keep track of numerous belief models and to respond to the best of them. Later on, various evolutionary algorithms have been applied to keeping track of these models (see Section 2 for the literature review). Therefore, another way to distinguish these two strands of the literature by using the standard taxonomy, as suggested by Duffy (2006), is *evolutionary algorithms* vs *reinforcement learning*. The former is applied to a large set of forecasting models (beliefs), whereas the latter is applied to a rather small set of actions only.

it is an equilibrium characterized by complete segregation (a bimodal distribution of participation): the population split into a group of agents who always go or frequently go (filling the bar up to its capacity at all times) and another group of agents who seldom go or never go.

This latter result has led to a new problem which has never been addressed in the literature, namely, the *inequity issue*. The group of people who have been discouraged by previous unsuccessful attempts and decide to quit obviously share very little or none of the public resources, which may further make them the disadvantaged class in the society. In fact, if we consider attending a bar as a social-engagement opportunity to gain more information and social connections, then the quitting can imply social exclusion or isolation. Therefore, the El Farol Bar problem is not narrowly just an economic distribution problem, it may become a social segregation problem characterized by a group of people who fully occupy the available public resource and a group of people who are discouraged, ignored and completely marginalized.

In this paper, we continue the pursuit of the self-coordination mechanism of the El Farol Bar problem. However, in addition to the *efficiency* concern (the optimal use of the public facility), we are also interested in the distribution of the public resources among citizens. Hence, we introduce a *two-dimensional El Farol Bar problem*, to be distinguished from the early *one-dimensional one*, which has efficiency as the only concern. We ask whether it is possible to have self-coordinating solutions to the El-Farol Bar problem without the involvement of central or any top-down intervention so that the capacity can be optimally used (with neither idle capacity nor incurring congestion) and, in the meantime, the resources are well distributed among all agents. We may call this ideal situation with both efficiency and equity the *El Farol version of a "good society"*.<sup>2</sup>

## 1.2 Social Networks and Social Preferences

Through agent-based simulation, we shall show in this paper that the answer is *surprisingly yes*<sup>3</sup>, but the likelihood of the emergence of a good

<sup>2</sup> Probably under the impact of the recent financial crisis and the social turbulence, economists are challenged by a very general and fundamental issue on whether our economics can actually help build a *good society* at large. These reflections can be best exemplified by some recent events, including plenary speeches and organized sessions, in the Allied Social Science Association (ASSA) and American Economic Association (AEA) annual meetings (Marangos, 2011; Shiller, 2012).

<sup>3</sup> Surprise in the sense that the generic procedure or algorithm for programming individuals which can lead to the desirable emergent patterns is generally unknown. This

society, in addition to learning, also depends on two other elements which have not been incorporated in the El Farol Bar literature yet. These two additional elements are *social networks* and *social preferences*. Before we move further, let us first make one remark on these two elements. It should come to us as no surprise that these two elements can be significant in agent-based modeling. In fact, more and more agent-based models have taken these two elements explicitly into account, realizing their importance in emergent dynamics. The former one, social networks, is obvious because agent-based modeling relies heavily on interactions, and social network topologies are simply the assumptions on which the interactions are based.<sup>4</sup> The latter one, social preference, is less obvious but can be well expected when agent-based modeling is extended to the areas involving various pro-social behaviors, which have been examined under intensive interdisciplinary studies across evolutionary biology, the humanities and the social sciences (Chen, 2008; Xianyu, 2010).

Nevertheless, recognizing their potential significance does not automatically imply that we can predict what will happen. In the vein of the “new kind of science” or *computational irreducibility* (Wolfram, 2002), we can probably only learn the rest from computer simulation, and it is this part where surprise may show up. In this paper, it is the combined force of social networks and social preferences which can solve the even harder two-dimensional El Farol problem that surprises us.

### 1.2.1 Social Networks

In this paper, we sequentially introduce two modifications to the original El Farol Bar model, both of which represent a step towards the development of a ‘socially oriented’ version of the El Farol Bar problem. Through a series of simulations, we assess the effect of these socially-grounded assumptions

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challenge is well-known in the study of complex systems. Since the El Farol Bar problem has been constantly modeled as a kind of complex adaptive system, and in this paper it will be modeled via cellular automata (Wolfram, 2002), this challenge, therefore, remains.

<sup>4</sup> There are two bodies of literature related to this development. One is network-based agent models, and the other is the agent-based modeling of (social) networks. The former refers to the agent-based models which explicitly involve networks, mainly for the purpose of interactions and decision-making; hence it can also be termed as the network-based decision model. A number of earliest agent-based economic models are of this type (Albin, 1992; Albin and Foley, 1992). More recent surveys can be found in Wilhite (2006). The latter, the agent-based models of networks, considers agent-based models as formation algorithms of networks, to be separated from sociological models, sociophysical models, and game-theoretic models (Eguiluz et al., 2005; Hamill and Gilbert, 2009). In this paper, we are mainly concerned with the first type.

on the macro-dynamics of the El Farol Bar problem and on the kind of equilibria that the system eventually reaches.

The first of these modifications concerns the *structure* of the agents' interaction and is represented by the introduction of a *social network* connecting the agents and through which the agents can access the information regarding their neighbors' choices and strategies. While in the original setup the agents base their decisions on *global information*, represented by the bar's aggregate attendance, a feature that is likely to cause herding behavior, making it very difficult for them to coordinate their activities, we may wonder whether coordination will be improved if, instead, the agents make use of *local information*, represented by the attendance of their closest neighbors. As we shall see later, this alteration motivates the simulations of bar attendance dynamics through cellular automata.

It is found that the introduction of social networks coupled with neighbor-based decision rules, in a form of cellular automata, allows the system to always reach an equilibrium characterized by perfect coordination, that is, a state where the bar's attendance is always equal to the bar's capacity, but there is a great diversity of these equilibria. The one of most interest to us, the "good society" (all the agents going to the bar with equal accessibility), is one of them. The one normally found with reinforcement learning (a group of agents always going and another group always staying at home) is also one of them. These two, however, are not exhaustive; as we shall see, there are many others, which, to the best of our knowledge, have never been found in the literature before. The effect of social networks can then be concisely represented by the resultant empirical distribution over these equilibria.

### 1.2.2 Social Preferences

After having assessed the effect of this first modification, we introduce a second modification concerning the agents' *preferences for fairness*. In the original versions of the El Farol Bar problem the agents did not care about their attendance frequency (that is, how often they were going to the bar): the only thing that mattered to them was to make the right choice, even if it implied staying all the time at home. In this paper we assume, instead, that some or all agents are characterized by a preference for fairness or inequity aversion. With this fairness or inequity-averse preference, agents expect a *fair share* or a *minimum attendance frequency* and take it as one of their satisfying criteria. If their decision rule does not lead to this fair share, they will react upon it and search for changes.

The inequity-aversion preference is one of the essential ideas in the recent literature on the study of pro-social behavior. There is still a heated discussion on the reconciliation between inequity preference and the conventional rational choice theory, but that is beyond the scope of the paper.<sup>5</sup> In this paper, we simply take this preference either as exogenously given or as endogenously evolving. For the former case, the inequity-averse agents are characterized by a parameter, called the *minimum attendance threshold*; for the latter case, the awareness of inequity is endogenously formed through interactions with neighbors in the familiar ‘*keeping-up-with-the-Joneses*’ manner, i.e., the agents’ minimum attendance threshold is represented by the average of their neighbors’ attendance frequencies.

The incorporation of social preference significantly increases the likelihood of the emergence of the “good society” to the extent that its appearance is always the most likely outcome. The likelihood increases with the size (number) of inequity-averse agents and the degree of their inequity-aversion. However, the emergence of the “good society” does not require all agents to be sensitive to inequity. Our simulation shows that for even a minority of them, up to 20% or 25%, having this kind of awareness, the emergence of the good society is already guaranteed.

The remainder of the present paper is organized as follows. In Section 2, we will present a brief review of the literature of the El Farol Bar problem. Section 3 describes our locally-interacted El Farol Bar model, a model with two-dimensional cellular automata, and the resultant adaptive behavior of agents. Section 4 presents the simulation results with respect to different settings of social networks and social preferences. Section 5 provides a simple analysis and a detailed look at the formation of the perfect coordination in light of our simulation results. Section 6 then concludes the paper with remarks on its current limitations, implications and future work.

## 2 Literature Review

In this section, after introducing the original versions of the El Farol Bar problem and of the closely related minority game (Section 2.1), we will consider some of the papers that have extended these two seminal models in various directions. In particular, we will focus on introducing different *learning models* to the El Farol Bar problem (Section 2.2) and *local interaction*

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<sup>5</sup> The interested reader is referred to Gintis (2008).



(Section 2.3) in the Minority Game.<sup>6</sup> In the first case, our aim is to provide a review of the macro-dynamics emerging from the models introduced so far, in order to have a background against which to compare the results obtained with the model we introduce in this paper. On the other hand, the papers that consider local interaction in the Minority Game show how the introduction of novel interaction structures has received considerable attention in previous works and, at the same time, allow us to see the crucial importance of the interaction structure to the macro-dynamics generated by these kinds of models.

## 2.1 The Seminal Models

In the original El Farol Bar problem (Arthur, 1994),  $N$  people decide independently, without collusion or prior communication, whether to go to a bar. Going is enjoyable only if the bar is not crowded, otherwise the agents would prefer to stay home. The bar is crowded if more than  $B$  people show up, whereas it is not crowded, and thus enjoyable, if attendees are  $B$  or fewer. If we denote the agent's decision "to go" by "1" and "not to go" by "0", and the actual number of attendees by  $n$  ( $n \leq N$ ), then the agent's payoff function has the general form (1).

$$U(x, n) = \begin{cases} u_1, & \text{if } x = 0 \text{ and } n > B, \\ u_2, & \text{if } x = 0 \text{ and } n \leq B, \\ u_3, & \text{if } x = 1 \text{ and } n > B, \\ u_4, & \text{if } x = 1 \text{ and } n \leq B. \end{cases} \quad (1)$$

The payoffs have either the order  $u_4 = u_1 > u_2 = u_3$  or the order  $u_4 > u_1 = u_2 > u_3$ .<sup>7</sup>

Arthur assumes that all the agents know the attendance figures in the past  $m$  periods and each of them has a set of  $k$  predictors or hypotheses, in the form of functions that map the past  $m$  periods' attendance figures into next week's attendance. After each period, the predictors' performance indexes are updated according to the accuracy with which the various predictors forecasted the bar's attendance. Then, the agent selects the most accurate predictor and uses the relative forecast to decide whether to go to the bar or to stay at home the next period. Although the competitive process among predictors never comes to rest, it still produces a remarkable statistical regularity: *at the macro level, the number of attendees fluctuates*

<sup>6</sup> Quite surprisingly, examples of the adoption of local interaction in the former and of different learning mechanisms in the latter are much rarer.

<sup>7</sup> These two versions of the payoff inequality will be discussed in Section 2.2.



*around the threshold level  $B$ , while, at the micro level, each agent goes  $B/N$  percent of the times, in the long run.*

Challet and Zhang (1997) proposed an alternative version of the El Farol Bar problem, known as the minority game (MG). The minority game has a very similar structure to the El Farol Bar problem. There is a binary choice: “go” (1) or “not go” (−1). The three key parameters, namely, the number of agents ( $N$ ), the memory size ( $m$ ) and the size of the strategy pool ( $k$ ), are carried on. The maintenance and the use of the strategy pool is also carried out in a similar manner. The basic finding is that the attendance frequency or the size of the winners fluctuate around the threshold, in this case, 50%. Further studies have shown that the degree of the fluctuation, widely used as a measure of efficiency, can further depend on the three parameters collectively.

Despite their similarity, the El Farol Bar problem and the minority game differ on one fundamental point. The minority games always assign the winners to the less crowded side (those who make a less popular choice) and the losers to the more crowded side. Hence, there is always a majority side that makes the wrong choice, while, in the El Farol Bar problem, there is the possibility of hitting exactly the target  $B$ , a situation where all the agents, no matter what they decided, made the right choice.<sup>8</sup>

Compared to the reception of the El Farol Bar problem in economics, the minority game is much better received in econophysics (Challet, Marsili and Zhang, 2005; Coolen, 2005). It has been further extended into a model of financial markets (Challet, Marsili and Zhang, 2005); in particular, when the financial market is considered from its gambling feature, some can be winners only conditional on others being losers. However, as stated in the introductory section, the kind of problem that concerns us is coordination, and good coordination can lead to a win-win situation. Therefore, we take the original El Farol problem in this paper. Nonetheless, given the similarity between the two, some studies of the minority game can certainly shed light on the study of the El Farol Bar problem. As we shall see in Section 2.3,

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<sup>8</sup> Because of this, there comes an additional difference. To ensure that there is always a minority side, in the minority game it is explicitly assumed that  $N$  is an odd number, an assumption which is not made in the El Farol Bar problem. As a result, the two games are characterized by different dynamics of the average long-term payoff per agent, as  $N$  increases. While in the MG the average long-term payoff per agent improves, as  $N$  grows larger, and asymptotically goes to zero from below, in the case of the El Farol problem it stays around zero, as the positive effect of the decreased fluctuation around the threshold is offset by the negative effect of the decreased probability of hitting the threshold, a possibility which in the MG is precluded by construction.

the inclusion of social networks and the use of cellular automata is actually borrowed from what we learn from the minority game.

## 2.2 Learning in the El Farol Bar Problem

Among the studies introducing novel learning mechanisms in the El Farol Bar problem, we can distinguish two lines of development: those that retain *the best-response behavior* of Arthur's El Farol Bar problem and those introducing *reinforcement learning mechanisms*. While one essential difference between the two is that the latter attempts to reduce the agents' cognitive loading of the former, there is an addition concern to distinguish the two, which is pertinent to the information requirement and hence to payoff inequalities (Equation 1).

Two different payoff inequalities have been, implicitly or explicitly, considered in the literature. Based on Equation (1), these two are (a)  $u_4 = u_1 > u_2 = u_3$  and (b)  $u_4 > u_1 = u_2 > u_3$ . The applicability of one of these two inequalities depends on the assumption that *the agents who did not go to the bar can still ascertain whether the bar was crowded or not*. If the information assumption is satisfied, the inequality (a) is more suitable since it clearly distinguishes the payoff to the right decisions (forecasts) ( $u_4$  and  $u_1$ ) from that to the wrong decisions (forecasts) ( $u_2$  and  $u_3$ ). This is the typical setting of works adopting the best-response behavioral model. The payoff, in these models, is represented by the amount by which the strategies' fitness is increased (decreased) after a right (wrong) forecast. To be precise, in some of these models (as the one proposed by Arthur) the strategy fitness is updated by an amount that is inversely proportional to the difference between the strategy's forecast and the actual aggregate attendance.

However, if the information assumption is not satisfied or those agents staying at home have no possibility of ascertaining whether the bar was too crowded or not, then the inequality (b) which assigns the payoff of staying at home independently of the bar attendance ( $u_1 = u_2$ ) becomes more appropriate. This is the typical payoff setting of works adopting reinforcement learning.

In the first line of development, Edmonds (1999) proposes an extension of the El Farol Bar problem where agents can change their strategies set by means of genetic programming and are given the chance to communicate with other agents before making their decision as to whether to go to the El Farol Bar. Simulations show that, as in the original model, the attendance at the bar fluctuates around the threshold level, and does not seem to settle down into any regular pattern. Another work where the agents' strategies are allowed to co-evolve is that of Fogel et al. (1999). In the model they

propose, the agents are endowed with 10 predictors that take the form of autoregressive models with the number of lag terms and the relative coefficients being the variables that evolve over time using evolutionary programming. Their simulations show that the system demonstrates a similar pattern to the original model (Arthur, 1994), except with a larger fluctuation. Other more technical aspects of managing a pool of strategies have been studied by Atilgan, Atilgan and Demirel (2008). Specifically, the differential effects of a kind of slow learning (sticky learning) and a fast learning upon the convergence to the threshold have been found.

Partially due to its computationally demands, the other line of this research has abandoned best-response behavior to adopt the more basic reinforcement learning framework. One of the first works where the best-response behavior of Arthur's original model has been replaced by reinforcement learning is that of Bell and Sethares (1999). In this paper, the authors present an agent-based model where the agents' strategies are represented by an integer  $c$  determining the agents' attendance frequency: if  $c = 2$  the agent goes to the bar once every 2 periods; if  $c = 3$  he goes once every 3 periods, and so on. Every time an agent goes to the bar and has a good time (because the bar was not too crowded), he decreases  $c$  and goes more often, whereas, in the opposite case, he increases  $c$  and goes less often. No change in the attendance frequency takes place if the agent stays at home, as it is assumed that he cannot assess whether he made the right choice or not.

Subsequently, Franke (2003) proposed a reinforcement learning model in which each agent goes to the bar with a probability  $p$ . If the bar is not crowded he increases  $p$ , while if the bar turns out to be too crowded, he decreases  $p$ . If the agent stays at home, a parameter  $u$  determines the extent to which the attendance probability is updated according to the bar's aggregate attendance.

In both Bell and Sethares (1999) and Franke (2003), simulations show that the populations tend to be split in two groups: a group of frequent bar-goers and a group of agents who very seldom go to the bar. This result has been analytically obtained by Whitehead (2008). By applying the Erev and Roth (1998) model of reinforcement learning to the El Farol Bar framework, he shows that the long-run behavior converges asymptotically to the set of pure strategy Nash equilibria of the El Farol stage game.

To sum up, in the best-response models, the aggregate attendance fluctuates around an average value that falls between  $B/N$  and a lower bound represented by the mixed strategy that maximizes the *aggregate payoff*. The point within this range where the average aggregate attendance falls depends on the values assigned to the many parameters characterizing the

best-response behavior and induction process. On the other hand, in the reinforcement learning model, two classes of agents emerge: those who often go and those who seldom go. The learning process will asymptotically lead to a state of perfect coordination with complete segregation, where a fraction  $B/N$  of the population will always go and the fraction  $1 - B/N$  will always stay at home.

### 2.3 Local Interaction in the Minority Game

While the literature on the El Farol Bar problem focused on the effect of the introduction of various learning mechanisms by retaining the interaction structure of the original model, within the literature on the minority game some works have analyzed the effect of different interaction structures, and in particular local interaction, on the system's macro-dynamics.

The literature on the MG with explicit local interactions or social networks can be divided in two strands characterized by two alternative ways in which the agents make use of local information. In the models of the first strand (Kalinowski et al., 2000; Paczuski et al., 2000; Galstyan and Lerman, 2002; Chau, Chow and Ho, 2004; Caridi and Ceva, 2004), the agents make their decision on the basis of the best strategy in the set of strategies with which they are endowed, as in the standard MG setup. However, differently from the latter, the input is not represented by the past aggregate outcomes but by the action taken by the agents' *neighbors* in the past period. In other words, the variable  $m$ , which in the traditional MG represents the number of the past global outcomes, in these models represents the number of agents each agent looks at in order to make a decision (i.e., the network degree). The general result of this setup is that the system can reach levels of coordination higher than the coordination obtained in the standard MG.

In the models characterizing the second strand (Slanina, 2000; Anghel et al., 2004; Lavicka and Slanina, 2007) each agent can decide whether to follow his own strategies, as in the standard MG setup, or *imitate* the action of one of his neighbors. Of course he will decide to imitate if the performance of the best performing agent among his neighbors is better than his own performance. Also in this case, these works show that through imitation it is possible to reduce the system's variance to a level lower than the lowest variance reachable with the standard MG and that the best coordination is reached with networks characterized by small degrees (usually, in the order of 2).

In the model we present in this paper, we develop a behavioral model which integrates these two approaches: in our model, the agents' strategies are represented by lookup tables defining the agents' actions for every

combination of their neighbors' past choices (like the works in the first strand), but these strategies change over time as the strategies of the best performing agents are imitated (with mutation) by the other agents in the population (like the works in the second strand). As we will see in the subsequent sections, this way of introducing social networks and local interaction to the model turns out to have an equally, if not more, dramatic effect in the case of the El Farol Bar problem. Not only does the efficiency (coordination) reached by the system depend on the interaction structure, but local interaction leads to new phenomena such as the emergence of attractors of various lengths and the emergence of different clusters of agents.

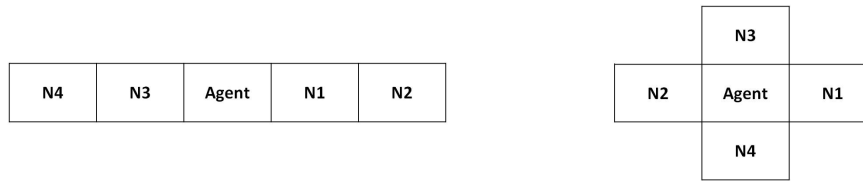
### 3 The Model

#### 3.1 Model Sketch

The review above has pretty much motivated the model to be presented in this section. Let us first give a quick sketch of it. Briefly, in this model, we shall introduce the spatial structure or social networks from the literature on minority games, as reviewed in Section 2.3, to the El Farol Bar problem. As already mentioned in the introductory section, this is our first step to explore the effect of social network topologies on the coordination problem. Hence, we do not consider just one social network, but *two* as a minimum for comparison purposes. Again, as a starting point, we do not complicate the situation with those state-of-the-art networks, but the standard regular network (circular network) and von Neumann networks.

This choice brings us closer to the beginning of agent-based modeling, namely, cellular automata, which are rather locally-based. In our applications, that means agents acquire information and learn, mainly, from their neighbors. This locally-based social learning, like general social learning, significantly reduces the cognitive loading of each individual for maintaining a pool of strategies as assumed by most best-response models reviewed in Section 2.2. In fact, in our setting, the pool of strategies is widely shared and maintained by the entire society of agents, and each agent only holds one for his own sake at each point in time.

With this reduction in cognitive loading, we do not further consider individual-style reinforcement learning since our setting already facilitates social learning. We, however, do allow for the individual's learning but in the usual discovery manner through mutation. The rest of his learning is then through imitation from neighbors. Hence, it is a simple imitation-



**Figure 1** Circular (a) and von Neumann (b) neighborhoods

plus-mutation model for learning. While the agents' motive for imitating and mutating is driven, as usual, by the fitness of their held strategies, we shall later on further differentiate this motive by their preference for equity, and then complete our agent-based El Farol Bar model with both social networks and social preferences.

### 3.2 The Model

In the model we present, we retain the best-response strategies of the original El Farol Bar problem. However, we modify the standard settings by adopting the informational structure introduced by the works on the MG with local interaction. As in the original El Farol Bar problem, we consider a population composed of  $N = 100$  agents and set the attendance threshold  $B/N = 0.6$ . Each agent can 'see' the actions, the strategies and the strategic performances of their neighbors, which are determined by the given social network.

In this paper, we investigate two network typologies as shown in Figure 1: the *circular network* (*circular neighborhood*), where each agent is connected to the two agents to his left and the two agents to his right, and the *von Neumann network* (*von Neumann neighborhood*), with the agents occupying a cell in a bi-dimensional grid covering the surface of a torus. Hence, in each of the two networks, the agent is connected to four neighbors, denoted by  $N1$ ,  $N2$ ,  $N3$  and  $N4$ .

**Representation** Contrary to the prototypical El Farol Bar problem and MG settings, each agent is assigned, at the beginning of the simulation, only one strategy  $z$ , randomly chosen from the whole strategy space. Our representation of the strategy is based on the binary string as normally used in cellular automata. The idea is that each agent will first look at what his four neighbors did in the previous period and only then decide what he will do in the current period, i.e., a mapping from the neighbors' previous decisions to his current decision. Denote the action "going to the bar" by 1 and "staying at home" by 0. Then there are  $2^4$  possible states,

Rule	N1	N2	N3	N4	D
1	0	0	0	0	0
2	0	0	0	1	0
3	0	0	1	0	1
4	0	0	1	1	0
5	0	1	0	0	0
6	0	1	0	1	0
7	0	1	1	0	1
8	0	1	1	1	1

Rule	N1	N2	N3	N4	D
9	1	0	0	0	1
10	1	0	0	1	0
11	1	0	1	0	1
12	1	0	1	1	0
13	1	1	0	0	1
14	1	1	0	1	1
15	1	1	1	0	1
16	1	1	1	1	0

**Figure 2** The agents' strategy (example)

each corresponding to one combination of the decision “1” or “0” made by the four neighbors. Each strategy is composed of 16 rules specifying the action  $D$  the agent has to take in the current period, one rule for each state, as is shown in Figure 2. Each strategy can then be represented by a 16-bit long string. If we fix the numbering order of the 16 states as indicated in Figure 2, then the corresponding 16-bit representation for the strategy exemplified there is simply “0010001110101110”, i.e., an array of the decisions corresponding to each of the sixteen states, respectively. All together, there are  $2^{16}$  possible strategies in the strategy space.

**Fitness** We define the variable  $d_i(t)$  as the action taken by agent  $i$  in period  $t$ : it takes the value 1 if the agent goes to the bar and the value 0 otherwise. Moreover, we define the variable  $s_i(t)$  as the outcome of agent  $i$ 's decision in period  $t$ : it takes the value 1 if the agent took the *right* decision (that is, if he went to the bar and the bar was not crowded or if he stayed at home and the bar was too crowded) and it takes the value 0 if the agent took the *wrong* decision (that is, if he went to the bar and the bar was too crowded or if he stayed at home and the bar was not crowded). The agents are endowed with a memory of length  $m$ . This means that they store in two vectors,  $\mathbf{d}$  and  $\mathbf{s}$  of length  $m$ , the last  $m$  values of  $d$  and  $s$ , respectively. So, at the end of any given period  $t$ , agent  $i$ 's vectors  $\mathbf{d}_i$  and  $\mathbf{s}_i$ , are composed, respectively, of  $d_i(t), d_i(t-1), \dots, d_i(t+1-m)$ , and of  $s_i(t), s_i(t-1), \dots, s_i(t+1-m)$ .

Agent  $i$ 's *attendance frequency* over the most recent  $m$  periods,  $a_i$ , is defined by (2):

$$a_i = \frac{1}{m} \sum_{j=t}^{t+1-m} d_i(j). \quad (2)$$



The attendance frequency's value can go from 1, if the agent always went to the bar, to 0, if the agent never went to the bar, in the last  $m$  periods. Moreover, agent  $i$ 's *decision accuracy rate*,  $f_i$ , is given by (3):

$$f_i = \frac{1}{m} \sum_{j=t}^{t+1-m} s_i(j). \quad (3)$$

The decision accuracy rate can go from 1, if the agent always made the right decision, to 0, if the agent always made the wrong decision, in the last  $m$  periods. We define the duration of agent  $i$ 's current strategy (the number of periods the agent is using his current strategy) as  $r_i$ . In order for the average attendance and the decision accuracy associated with any strategy to be computed, it has to be adopted for a number of periods equal to the agents' memory size  $m$ : so, we can think of  $m$  as the *trial period* of a strategy. We will set this value to 10 for all the agents in all our simulations.

**Inequity-Averse Preference** As mentioned in the introductory section, agents in our model have an inequity-averse preference, which is characterized by a parameter called the *minimum attendance threshold*, denoted by  $\alpha_i$ , that is, a fair share of the access to the public resources or a fair attendance frequency expected by the agent. It can take any value from 0, if the agents do not care about their attendance frequency, to 0.6. We do not consider a higher value than 0.6 because these agents with equity concern do not claim to go with an attendance frequency higher than the threshold  $B/N (= 0.6)$ .

**Learning** Differing from the traditional El Farol Bar problem setup, the agents' strategies are not fixed, but they evolve through both social learning (imitation) and individual learning (mutation). So, the social network plays a role both in the agents' decision process, allowing the agents to gather information regarding their neighbors' choices, and, in the agents' learning process, allowing the agents to imitate their neighbors' strategies. In any given period, an agent  $i$  imitates the strategy of one of his neighbors if the following six conditions are met:

- (a)  $f_i < 1$  and/or  $a_i < \alpha_i$
- (b)  $r_i \geq m_i$

and the agent has at least one neighbor  $j$  for which the following conditions are verified:

- (c)  $f_j > f_i$

$$(d) \ a_j \geq \alpha_i$$

$$(e) \ r_j \geq m_j$$

$$(f) \ z_j \neq z_i$$

Condition (a) is quite obvious. It simply states that the agent will have the tendency to imitate if he is not satisfied with his current situation (strategy). There are two possibilities which may cause this dissatisfaction. First, there are errors in his decision ( $f_i < 1$ ) so there is room for an improvement, and, second, he is not satisfied with his attendance frequency ( $a_i < \alpha_i$ ). Notice that, by this later qualification, the agent may still look for change even though all his decisions are accurate ( $f_i = 1$ ). Condition (b) shows that the agent will not change his strategy frequently and will consider doing so only if the strategy has been tested long enough, i.e., after or upon the completion of the trial period. When imitating neighbors, agent  $i$  will only consider those strategies which not only lead to more accurate outcomes, but also lead to a satisfactory attendance frequency. (Condition (c) and (d)). The above promising strategy should be based on long testing rather than sheer luck (Condition (e)). Finally, agent  $i$  will not imitate the same strategy which he is currently using. Condition (f) is to avoid this repetition.

If the first two conditions are met but at least one of the last four is not, or, alternatively put, if the agent has not yet reached the optimal strategy and in the current period he cannot imitate any of his neighbors, then the agent, with a probability  $p$  ( $p < 1$ ), will mutate a randomly chosen rule on its strategy while with probability  $1 - p$  he will keep using his present strategy. While the imitation process ensures that the most successful strategies are spread in the population, the mutation process ensures that new, eventually better, strategies are introduced over time. Once the agent has adopted a new strategy (either through imitation or mutation) he will reset his memory to zero and will start keeping track of the new strategy's fitness. The agent *stops* both the imitation and the mutation processes if the following two conditions are met:

$$(a) \ f_i = 1$$

$$(b) \ a_i \geq \alpha_i$$

When these two conditions are verified for all the agents, the system reaches the equilibrium: no further change in the agents' behavior takes place after this point as the agents always make the right decision and go to the bar with a satisfying attendance frequency.

### 3.3 Simulation Settings

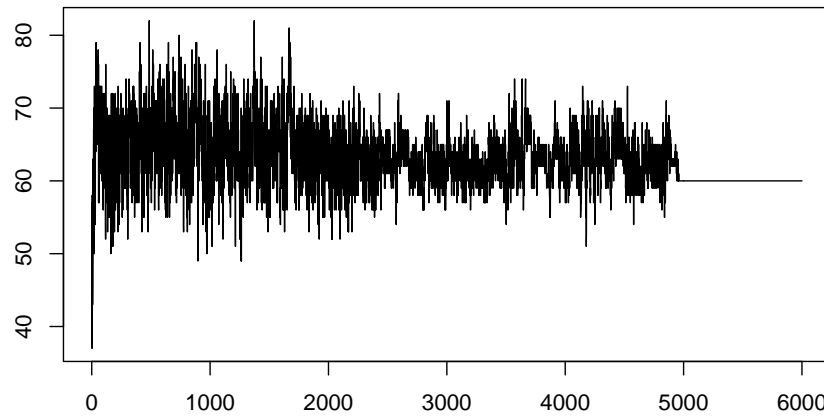
In the next section, we will show the results of simulations based on two versions of the El Farol Bar problem. In the first version (Section 4.1) we introduce the social network only but no inequity-averse preference. In this version, the agents, as in the original model, do not care about their attendance frequency. In this case, the same learning mechanism applies but with the minimum attendance threshold set to 0 ( $\alpha_i = 0, \forall i$ ). Accordingly, the agents decide whether or not to imitate their neighbors only on the basis of the strategies' accuracy rates. In this way, we are able to assess how the outcomes are affected by the introduction of social networks and, in particular, the effect of different network structures on the kinds of equilibria reached by the system, the equilibrium distribution, and, in particular, the emerging likelihood of a "good society".

Then, in Section 4.2, we introduce a second version where all or some agents are inequity averse, in the context of a network structure represented by the von Neumann neighborhood.<sup>9</sup> This version is further differentiated into three scenarios. We first start with a homogeneous population of agents (Section 4.2.1) where all agents are inequity averse with the same threshold, i.e.,  $\alpha_i = \alpha, \forall i$ . We then consider a heterogeneous case (Section 4.2.2) where only a fixed number of agents are inequity averse with a threshold of 0.6. Denote this parameter by  $N_{\alpha,0.6}$ , and the rest of the population are not inequity averse at all. The last scenario (Section 4.3) is the same as the second except that the thresholds of the inequity-averse agents are determined endogenously and socially in a 'keep-up-with-the-Joneses' (KUJ) manner. Denote this number of agents by  $N_{KUJ}$ . Step by step we are moving toward a more natural setting to address the question: *what is the minimum degree of inequity aversion, in terms of  $\alpha, N_{\alpha,0.6}, N_{KUJ}$ , required for the emergence of the "good society" equilibrium?*

## 4 Results of Simulations

Simulation results are presented in this section. The first series of simulations show the contribution of the social network and local interactions to the emergence of the "good society" equilibrium (Section 4.1), and the second series of simulations show the significance of social preference in

<sup>9</sup> In this second set of simulations, we will not consider the circular neighborhood as in this part our aim is to assess the effect of the inequity-averse preference on the equilibrium distribution.



**Figure 3** The attendances dynamics in a typical run (with the von Neumann neighborhood)

the form of inequity aversion (Section 4.2). They will both be presented in histograms (distributions) based on multiple runs.

#### 4.1 Social Networks without Inequity Aversion

We begin with the simulation of the El Farol Bar system with both the circular neighborhood and the von Neumann neighborhood. Each setting is run 1,000 times. The results show that each simulation of both settings always reaches perfect coordination, that is, the state where the bar attendance is always equal to the threshold and, consequently, the agents never make the wrong choice. Figure 3 shows the attendances for a typical run with the von Neumann neighborhood. We can see that, in this example, the equilibrium is reached at around period 5,000.<sup>10</sup>

While the El Farol Bar in both networks eventually converges to the same aggregate outcome (a 60% attendance rate all the time), from the mesoscopic viewpoint, they differ from run to run. To effectively characterize these equilibria at the mesoscopic level, we shall focus on the attendance frequency of agents when the perfect coordination is formed,  $a_i^*$ . In this way, our equilibrium can be represented by the heterogeneity in this attendance frequency over all agents. More precisely, the perfect coordinating

<sup>10</sup> One may wonder about the noticeable discontinuity appearing in this figure. Why does it make the fluctuation suddenly stop? We shall come back to this point in Section 5.

equilibrium of the El Farol Bar problem is given by the set which shows the observed attending frequencies,  $b_i^*$ , and the share of the agents with  $b_i^*$ ,  $\pi_i^*$ .

$$\Xi \equiv \{(b_i^*, \pi_i^*)\}_{i=1}^c \equiv \{(b_1^*, \pi_1^*), (b_2^*, \pi_2^*), \dots, (b_c^*, \pi_c^*)\}, \quad (4)$$

where  $b_1^* > b_2^* > \dots > b_c^*$ .

In (4), “ $c$ ” refers to the number of clusters, and  $\pi_i^*$  is the size of the corresponding cluster. Taking the bimodal perfect coordination equilibrium as an example, we have two clusters of agents, one which always goes ( $b_1^* = 1$ ) and one that never goes ( $b_2^* = 0$ ); 60% of agents are of the first kind, and 40% agents are of the second kind. Hence, this equilibrium is characterized by

$$\Xi_{Bi} \equiv \{(1, 0.6), (0, 0.4)\} \quad (5)$$

Alternatively, the “good society” is an equilibrium characterized as

$$\Xi_G \equiv \{(0.6, 1)\} \quad (6)$$

For convenience, we shall call these equilibria, based on the number of emerging clusters, 1C equilibrium, 2C equilibrium, etc. Hence the “good society” equilibrium,  $\Xi_G$ , is a 1C equilibrium, and the segregated equilibrium,  $\Xi_{Bi}$ , is a 2C equilibrium. Then one way to present our simulation result is to show the histogram of each of these  $C$  equilibria over our 1,000 runs.<sup>11</sup> Figure 4 shows the histogram of the  $C$  equilibria from  $C = 1, 2, \dots, 8$  for both the circular network (CN) and the von Neumann network (vNN).<sup>12</sup>

<sup>11</sup> Here, we use *equilibria* because, except for the 1C equilibrium, we can have multiple equilibria for each  $C$  ( $C \geq 2$ ). For example, for the 2C equilibria, in addition to  $\Xi_{Bi}$  as shown in (5), the other observed 2C equilibrium is:

$$\Xi_2 \equiv \{(1, 0.2), (0.5, 0.8)\} \quad (7)$$

Similarly, for  $C = 3$ , we can have

$$\Xi_{3-1} \equiv \{(0.7, 0.1), (0.6, 0.8), (0.5, 0.1)\} \quad (8)$$

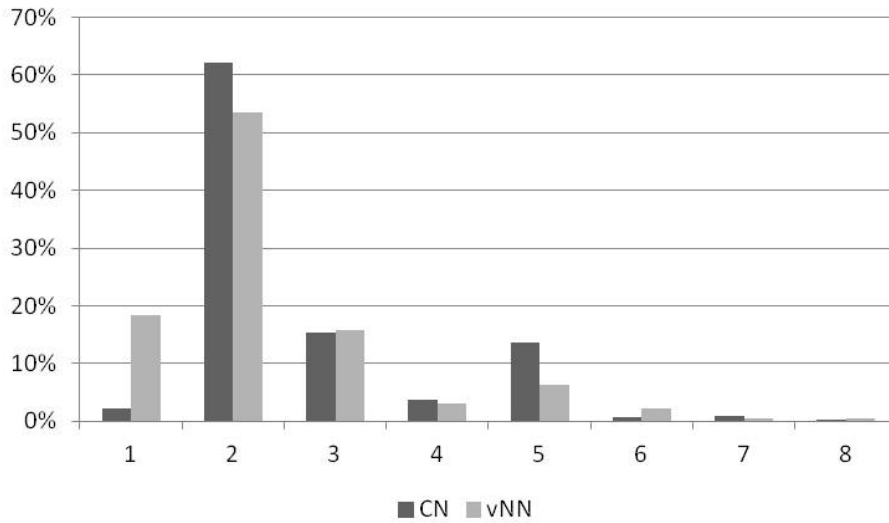
or

$$\Xi_{3-2} \equiv \{(1, 0.4), (0.5, 0.4), (0, 0.2)\}. \quad (9)$$

Furthermore, even for two equilibria having the same  $\{b_i^*\}$ , their  $\{\pi_i^*\}$  can still be different. For example, one alternative for  $\Xi_{3-1}$  is

$$\Xi_{3-3} \equiv \{(0.7, 0.3), (0.6, 0.4), (0.5, 0.3)\}. \quad (10)$$

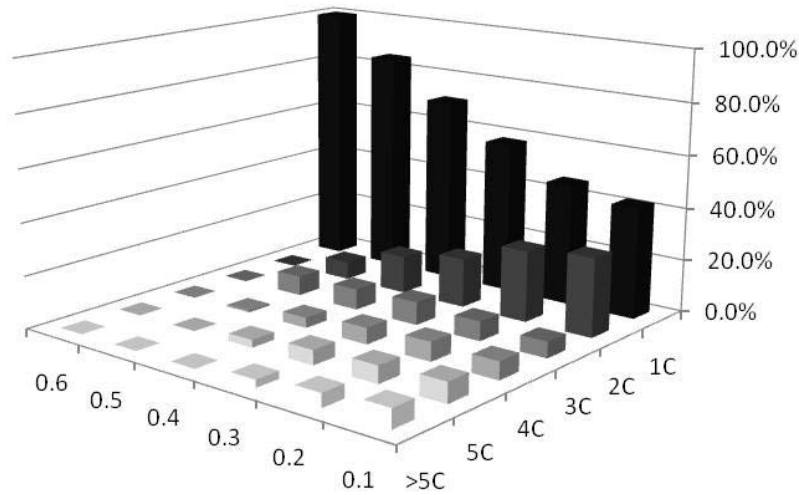
<sup>12</sup> The equilibrium with more than eight clusters of agents has not been found in any of our simulations.



**Figure 4** Histogram of the C Equilibria with the circular (CN) and the von Neumann (vNN) neighborhoods

From Figure 4 we can see that, while the literature on the El Farol Bar problem had identified only one kind of equilibrium, that is,  $\Xi_{Bi}$  in (5), the introduction of social networks leads to the emergence of many different kinds of equilibria. While the 2C equilibria remain the most likely outcome in both networks, with the von Neumann network the system has a non-negligible probability (18%) of reaching the 1C equilibrium (the “good society” equilibrium),  $\Xi_G$ . The fact that the system has relatively good chances to reach the perfectly equitable equilibrium is a quite interesting result considering that, in this version, agents have no minimum attendance thresholds; yet, it is the second most likely equilibrium, with a probability up to almost one third of the probability of the 2C equilibria. Different network structures are, however, characterized by different equilibria distributions: for example, the probability of reaching  $\Xi_G$  declines to only 2% in the circular network.

A finer look at the results further shows that, within the equilibria characterized by the emergence of two clusters (2C), the great majority (over 90%) are represented by the  $\Xi_{Bi}$ . The rest (less than 10%) are represented by  $\Xi_2$  (see footnote 11). The great majority of the 3C equilibria are represented by an equilibrium where some agents never go to the bar, some always go and the rest go with an attendance frequency of 0.5, i.e.,  $\Xi_{3-2}$  in (9). Another relatively frequent outcome is the emergence of the 5C equilibria. Within this case, the great majority is represented by a configuration where,



**Figure 5** Histograms of the C Equilibria for Six Different Homogeneous Thresholds:  $\alpha = 0.1, 0.2, \dots, 0.6$ .

besides the three clusters mentioned for the 3C case, two more groups of clusters, going to the bar respectively with a frequency of 0.4 and 0.6, emerge, such as

$$\Xi_5 \equiv \{(1, 0.58), (0.6, 0.01), (0.5, 0.02), (0.4, 0.01), (0, 0.38)\}. \quad (11)$$

## 4.2 Introducing Inequity-Averse Preference: Exogenous Settings

After seeing the significance of social networks in the emergence of the “good society” equilibrium, in the following sections, we shall examine the effects of the inequity-averse preference. To have a focus, here we only consider the von Neumann network. Hence, in the following sections, agents expect a minimum bar attendance frequency, and, if their actual attendance is below the threshold, they will find a way to change their original decision rule. Within this framework, we further consider two versions of this kind of inequity-averse preference: an *absolute* one which is given *exogenously* (Section 4.2) and a *relative* one which is determined *endogenously* in a “keep-up-with-the-Joneses” manner (Section 4.3).

### 4.2.1 Homogeneous population

For the exogenous settings, we further distinguish the case of a *homogeneous population* from the case of a *heterogeneous population* by addressing two



different but related questions on the role of inequity aversion in the emergence of the “good society”. For the homogeneous setting, since all agents are given the same minimum attendance threshold ( $\alpha_i = \alpha, \forall i$ ), we then look at the relationship between the *likelihood* of the “good society” emerging and the *degree* of inequity aversion (the threshold) in this homogeneous setting.

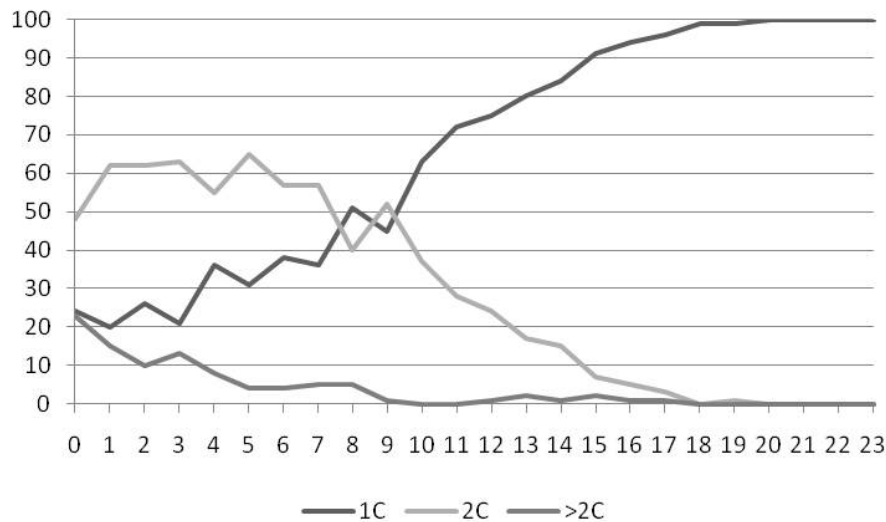
We simulate the El Farol Bar dynamics by assuming  $\alpha = 0.1, 0.2, \dots, 0.6$ . For each parameter value of  $\alpha$ , an El Farol Bar dynamics with the von Neumann network is run 100 times. Figure 5 shows the distribution over various C equilibria as the counterpart of Figure 4. To show how this distribution may change with the increase in  $\alpha$ , the diagram is demonstrated in a three-dimensional version by including the minimum attendance threshold as one of the axes.

From Figure 5, we can see that if all agents have the same threshold of 0.1 (that is, the agents are satisfied if their attendance frequency is equal to or above 0.1), the 1C equilibrium (the “good society”) is already the most likely outcome (around 40%), followed by the 2C equilibrium (around 30%). As we increase the minimum attendance threshold, the frequency of the 1C equilibrium increases while the frequencies of all the others decrease, at different rates. Then, with a minimum threshold level of 0.6, the system reaches the 1C equilibrium 100% of the time, as it is the only equilibrium compatible with this satisfying level.

Hence, as we can see from this simulation, the early bimodal segregation as obtained in many models using reinforcement learning can be significantly changed within a society of agents with a minimal degree of inequity aversion. Presumably, one might have thought that the addition of this inequity aversion might cause the use of the public resource to become harder to coordinate, but this is not necessarily the case. In fact, a more equitable but still well-coordinated outcome emerges. Therefore, in this sense, citizens with the awareness of their rights and striving for that does indeed facilitate the realization of the good society.

#### 4.2.2 Heterogeneous population

From the previous simulation results, we know that if all the agents have the minimum attendance threshold of 0.6, the system will surely converge towards the “good society”. Nonetheless, making all agents have identically strong or weak inequity-averse preferences is a strong assumption. Therefore, it would be interesting to know whether the good society will emerge in a more “natural” setting. In the next simulation, we no longer assume that all agents are inequity averse: some are, but some are not. Without

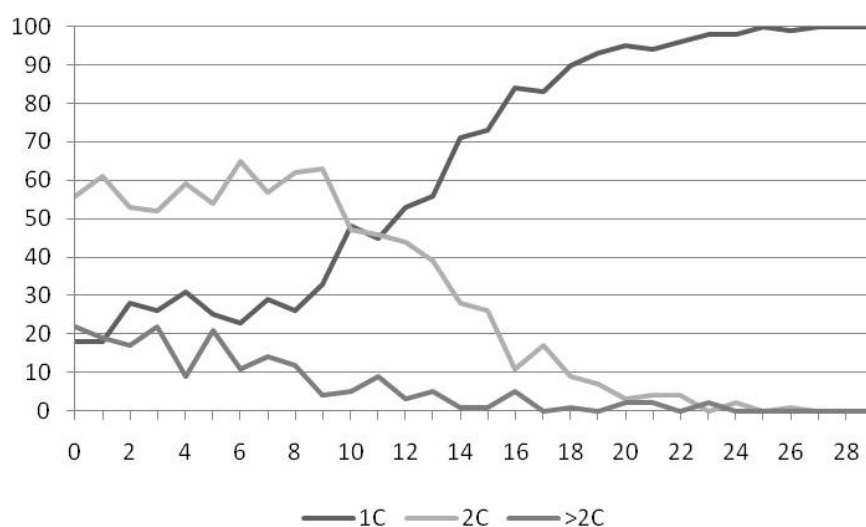


**Figure 6** Evolution of the Distribution over 1C, 2C and Other Equilibria with the Increase in the Number of Inequity-Averse Agents

losing generality, we further assume that those agents with inequity-averse preferences all have a threshold of 0.6. In this setting, we ask a different question: what would be the minimum number of inequity-averse agents required for the emergence of the “good society”?

We want, then, to see how the likelihood of the “good society” equilibrium changes when starting from a population entirely composed of agents with no minimum attendance thresholds, then followed by increasing the number of agents with an  $\alpha$  of 0.6. In this simulation, the *number* of agents with a threshold of 0.6,  $N_{\alpha,0.6}$ , is the key parameter, and, we run each of the parameter settings ( $N_{\alpha,0.6} = 0, 1, 2, \dots$ ) 100 times so as to approximate the distribution by the respective histogram. Figure 6 shows the evolution of the distribution over various  $C$  equilibria. To make this diagram easy to see, we group all equilibria with more than two clusters of agents together, called the *more-than-2C* ( $> 2C$ ) equilibria, and hence only present the distribution over the three major equilibria, i.e., the 1C, 2C and  $> 2C$ .

From Figure 6, as expected, when the number of agents with inequity aversion is small, the bimodal segregation equilibria dominates, but already with just 10 inequity-adverse agents, the 1C equilibria (“good society”) becomes the most likely outcome and that it indeed becomes the unique absorbing state when the population contains at least 20 inequity-adverse



**Figure 7** Evolution of the Distribution over the 1C, 2C and Other Equilibria with the Change in the Number of KUS agents

agents. In other words, it takes only a small minority of the society (around 20%) to lead the system to the perfectly equitable outcome.<sup>13</sup>

The two sequences of simulations above demonstrate two different approaches to manipulate the degree of inequity-aversion, one directly through the threshold ( $\alpha$ ), and one through the number of agents with a threshold of 0.6 ( $N_{\alpha,0.6}$ ). Both show that the “good society” can be the most likely result with only a small degree of inequity aversion:  $\alpha$  is only required to be 0.1 in the first case, and  $N_{\alpha,0.6}$  is only required to be 10 in the second case. The latter case further shows, to surely have the “good society” outcome, that there is no need to have all agents with strong inequity-averse preferences ( $N_{\alpha,0.6} = 100$ ), only one fifth of the population having them is sufficient. This latter result denotes the positive externality of inequity-averse preferences, and hence makes the realization of the “good society” easier than what we thought in the beginning.

### 4.3 Introducing the ‘Keep-up-with-the Joneses’ Behavior

Regarding the significance of the inequity-averse preference, the result will be more convincing if it can emerge endogenously, instead of being imposed exogenously. In the next simulation, we propose an alternative way of forming this inequity-averse preference. The social behavior coined

<sup>13</sup> For this reason, Figure 6 only shows the results up to  $N_{\alpha,0.6} = 23$ .

as “keep up with the Joneses” (KUJ) has recently caught the attention of many economists. It has been extensively incorporated into consumption, asset pricing and macroeconomic models (Gali, 1994; Ljungqvist and Uhlig, 2000). We find that the same idea can be straightforwardly applied to our model, in which agents’ decisions very much depend on those of their neighbors.

Hence, instead of the exogenously given threshold ( $\alpha$ ), agents will now find their own reference based on the attendance frequency averaged over their neighbors, and use that as their own minimum attendance threshold. In other words, the agents characterized by the “keep-up-with-the-Joneses” behavior do not want to be, among those in their neighborhood, those going to the bar with a frequency lower than the average. Since neighbors’ attendance frequencies change over time, this threshold, unlike the previous two settings, is no longer fixed. We call agents in this new setting the KUJ agents.

Like what we have done in Sections 4.2.1 and 4.2.2, we could further distinguish the case with a homogeneous population from the case with a heterogeneous population; in the former, all agents are KUJ agents, whereas in the latter only part of them are KUJ agents. For the former case, it is relatively easy to see that if all the agents in the population try to ‘keep-up-with-the-Joneses’, the system will inevitably end up reaching the “good society” equilibrium, equivalent to the case where all agents have an  $\alpha$  of 0.6 (Section 4.2.1). This is the only state where no agent goes to the bar with a frequency lower than the average (that is, where all the agents go to the bar with the same frequency). So, we will only be interested in the heterogeneous case and pose the same question as the one in Section 4.2.2, i.e., where we inquire about the minimum number of KUJ agents required for the emergence of the “good society”. Therefore, we set the number of KUJ agents as a parameter,  $N_{KUJ}$ , and simulate the El Farol Bar dynamics with  $N_{KUJ} = 0, 1, 2, \dots$ . We run each  $N_{KUJ}$  100 times so as to show the histogram over different  $C$  equilibria.

Figure 7 shows the evolution of the distribution over the 1C, 2C and *more-than-2C* equilibria with the increase in the number of KUJ agents. We can see that, by and large, the evolution of the distribution is similar to the case with a different number of  $N_{\alpha,0.6}$  (Section 4.2.2): in this case, with more than 11 KUJ agents, the 1C equilibrium (“good society”) becomes the most likely outcome. Moreover, it takes only around 25% of the KUJ agents to lead the system to the perfectly equitable outcome, slightly more than the minimum number required for  $N_{\alpha,0.6}$ , but still a small minority.

Rule	N1-N2-N3-N4	Strategy 1	Strategy 2	Rule	N1-N2-N3-N4	Strategy 3	Strategy 4
6	0-1-0-1	0	1	7	0-1-1-0	0	1
11	1-0-1-0	1	0	10	1-0-0-1	1	0
16	1-1-1-1	1	1	16	1-1-1-1	1	1

**Figure 8** Emergent Strategies Characterizing the 1C Equilibrium of the von Neumann Neighborhood

## 5 Looking into the 1C Equilibrium

### 5.1 Consistency between Macro and Micro Emergence

We have shown that perfect coordination, which is not necessarily equitable, can be always achieved in our limited explorations of social networks and social preferences. This result is very *atypical* with respect to what has been found in the existing literature; therefore, it is worth examining the unique dynamics which we experienced in the simulations.

Without losing generality, we shall focus on the case of the 1C equilibrium (“good society”) under the von Neumann network and give an intuitive description of the underlying dynamics leading to it. As we described earlier, our agent-based model is a kind of two-dimensional cellular automata (Wolfram, 2002). The slight difference here is that, instead of being homogeneous, our agents initially follow different strategies, but the subsequent social learning through imitation helps increase the homogeneity among them, and makes them eventually follow the same strategy. In other words, a single strategy becomes the *attractor* under our learning dynamics. This final strategy (attractor) then helps coordinate agents’ attendance in a way that in each single day there are always 60% of agents attending the bar ( $(\frac{n}{N})_t = 0.6, \forall t \geq T^*$ ), and for each agent the attending frequency is 60% of the time ( $r_{i,t} = 0.6, \forall i, t \geq T^*$ ). Fully specifying these attractors can be a daunting task; however, our simulations suggest that the number of the attractors with a non-trivial domain of attraction can be rather limited; as a matter fact, throughout our simulations only four of this kind have been found and they are all shown in Figure 8.

These four strategies are presented in their *effective* version, i.e., the one keeping only the active states. This is so because many states are idle when perfect coordination is achieved with the emergence of rather well-structured social patterns (see Figure 9). The four strategies, as exemplified in Figure 8, all have only three active states left and three corresponding

if-then rules. These three rules together do not require the agent to look at the actions of all four of his neighbors. Take Strategy One as an example. This strategy essentially requires the agent to look at neighbors *N1* or *N3* (shown in bold in Figure 8). If *N1(N3)* attended the bar in the previous period, he will then decide to go (rules 11 and 16); otherwise, he will stay home (rule 6).<sup>14</sup> Alternatively put, the strategy is simply “*do what your neighbors N1 or N3 did in the last period*”. The similar simple interpretation applies to the three other strategies.

These simple rules echo well with the emergent well-structured patterns of bar attendance as the joint force of the upward and downward causation. To see this, Figure 9 presents the snapshot of the social pattern of bar attendance for a typical run at the 1C (“good society”) equilibrium. The specific 1C equilibrium shown in Figure 9 is characterized by Strategy Four (Figure 8). The effective version of Strategy Four bases the decision upon only one of the four neighbors, either *N2* or *N3*. Basically, it imitates what *N2* or *N3* did in the last period. With everyone following this same strategy, in equilibrium each individual will be presented with a periodic cycle involving only three input states, i.e., 0-1-1-0, 1-0-0-1, and 1-1-1-1. They are, respectively, the three antecedents of Rules 7, 10, and 16.<sup>15</sup> The left panel of Figure 9 shows the spatial distribution of the activate rules over the 100 agents for a typical run, and the right panel shows the corresponding attendance distribution.

From the left panel, one can clearly observe the diagonals of the three activated rules, each extending from northeast to southwest, and they are aligned together like a wave which moves one step eastward for each iteration.<sup>16</sup> The right panel demonstrates the same pattern in terms of a wave of diagonals of “0” and “1”. The diagonals of 0 correspond to the diagonals of Rule 10 in the left panel, whereas the diagonals of 1 correspond to those of Rules 7 or 16 there. As these diagonals of rules move eastward, the diagonals of 0 and 1 also move eastward accordingly, and at any point in time only 60% of the agents (gray shaded area in the right panel) attend the bar. At the aggregate level, the bar is then fully utilized.

At the micro level, let us look at the attendance behavior of each individual under the 1C equilibrium. We will continue assuming the equilibrium characterized by Strategy Four as an illustration. Given the fact that each individual will encounter a periodic cycle of his environment (the input

<sup>14</sup> Notice that in equilibrium *N1* and *N3* are well aligned with the same action.

<sup>15</sup> As shall be made clear below, each agent experiences the dynamics of his environment in equilibrium as a 10-period cycle: 16-16-7-7-10-10-7-7-10-10.

<sup>16</sup> This is because in the equilibrium each agent only looks at his neighbor to the west (*N2*) or to the north (*N3*), and imitates their behavior.

10	10	7	7	10	10	16	16	7	7
10	7	7	10	10	16	16	7	7	10
7	7	10	10	16	16	7	7	10	10
7	10	10	16	16	7	7	10	10	7
10	10	16	16	7	7	10	10	7	7
10	16	16	7	7	10	10	7	7	10
16	16	7	7	10	10	7	7	10	10
16	7	7	10	10	7	7	10	10	16
7	7	10	10	7	7	10	10	16	16
7	10	10	7	7	10	10	16	16	7

0	0	1	1	0	0	1	1	1	1
0	1	1	0	0	1	1	1	1	0
1	1	0	0	1	1	1	1	0	0
1	0	0	1	1	1	1	0	0	1
0	0	1	1	1	1	0	0	1	1
0	1	1	1	1	0	0	1	1	0
1	1	1	1	0	0	1	1	0	0
1	1	1	0	0	1	1	0	0	1
1	1	0	0	1	1	0	0	1	1
1	0	0	1	1	0	0	1	1	1

**Figure 9** A snapshot of the rules adopted (left) and the actions taken (right) by the 100 agents once the equilibrium has been reached. In the simulation we would see the diagonals ‘move’ one step eastward in every period.

states), his bar attendance becomes quite regular. In this specific case, each individual will also follow a 10-period cycle as follows: 1-1-1-1-0-0-1-1-0-0 as we can easily see either from the last row (imitating N2) or the last column (imitating N3) of Figure 9. Hence, each individual attends the bar six times every ten days, and not only is the bar fully utilized, but is also equally accessible for each individual.<sup>17</sup>

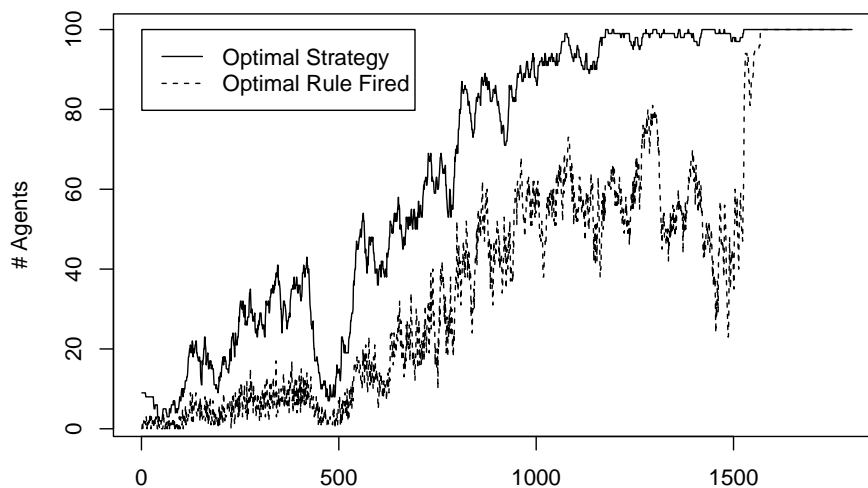
## 5.2 Analysis: Out-of-Equilibrium

The analysis above rests upon the equilibrium only. One more interesting feature of agent-based modeling is out-of-equilibrium analysis. This section, therefore, extends the previous analysis into transition dynamics. We continue the previous example, but now examine how the 1C equilibrium characterized by Strategy 4 is achieved. One can perceive that in this case we initialize the system in the domain of attraction of Strategy 4<sup>18</sup>; hence, one natural thing to look at is the population of agents who actually follow Strategy 4. The solid line of Figure 10 plots the percentage of this population. This figure increases over time but with some degree of fluctuation. Before

<sup>17</sup> It should be pointed out that there are another two periodic cycles also found in our simulation, a 5-period cycle (1-1-1-0-0), and a 10-period cycle (1-1-0-0-1-1-0-1-1-0). All these three cycles can be generated by any of the four strategies shown in Figure 8. Obviously, they all lead to an attendance frequency of 60%.

<sup>18</sup> Of course, since the system is not deterministic, small or large stochastic perturbations may still cause the path to cross the border and also cause travel along the path in another domain of attraction.





**Figure 10** The dynamics of the number of agents adopting the optimal strategy (Strategy 4) and of the number of 'optimal' agents actually adopting one of the three rules of the optimal strategy (in this case, Rules 7, 10 and 16).

it becomes steady, it experiences a significant drop around period 500. Then it comes back and eventually climbs up, and, around period 1,200, it almost successfully drives out all other strategies. At this point, agents are becoming homogeneous by following the same strategy.

However, adopting the same strategy is not sufficient for, when the perfect coordination is reached, many input states will become transient, and there are only three recurrent states left. In Figure 10, we, therefore, also plot the number of agents, shown by the dotted line, who not only adopt the optimal strategy, but actually fire one of the three rules characterizing it, that is, Rule 7, Rule 10 and Rule 16. The number of these agents also rises, but remains far from the number of agents who had adopted Strategy Four (the solid line), and that distance has no clear tendency to be shortened until coming to period 1,500, right before the system *suddenly* reaches the equilibrium. This means that the inputs of the agents are different from those characterizing the equilibrium. Consequently, even if they are already adopting Strategy 4, their behavior is only incidentally determined by the three rules characterizing this strategy as in many periods they end up firing other rules than rules 7, 10 and 16. Then, suddenly, after period 1500,

the system reaches the equilibrium, a state where not only all the agents adopt Strategy 4, but they also use the rules characterizing this strategy.

From Figure 10 we can see that the process leading to the equilibrium is characterized by a stage (in this case lasting up to period 1200) in which the necessary condition for the system to reach the equilibrium (that is, the adoption of Strategy 4 by all the agents) is gradually established by means of a ‘subterranean’ process having no immediate effect on the population’s coordination level. However, once this process comes to an end, the system is ‘ripe’ for the equilibrium: sooner or later, a minor event, such as a mutation, triggers the emergence of coordination that, as we can see from Figure 10, is a very fast process occurring in less than 100 periods, looking much like spontaneous self-ordering (Kauffman, 1993).

## 6 Concluding Remarks

The El Farol Bar problem is a highly abstract model suitable for addressing the fundamental issue of the use and the distribution of public resources. Early studies on this problem have only centered on the efficiency aspect of this issue. The equity part of the issue has been ignored. The contribution of this paper is that we integrate these two aspects and extend the original one-dimensional El Farol Bar problem into a two-dimensional one. While coordination in the two-dimensional El Farol Bar problem can be harder than in the original one-dimensional one, it still can be solved bottom-up.

However, to do so, this paper shows that the bottom-up mechanism crucially depends on the two essential ingredients, namely, social networks and social preferences. We first show that social network topologies matter for the emergence of a “good society” (1C equilibrium), a state where the bar attendance is always equal to its capacity and all the agents go to the bar with the same frequency. This is exemplified by the comparison made between the von Neumann network and the circular network. It is found that it is much easier for the “good society” to emerge under the former (18%) as opposed to the latter (only 2%).

We then show how the introduction of the social preference can further facilitate the emergence of the good society. This depends on a specific group of people who are sensitive to inequity or having inequity-averse preferences. The emergence of the good society can become increasingly likely with the increase in the degree of inequity aversion. Various simulations, however, show that to surely have the “good society” equilibrium the requirement for inequity aversion is rather mild. For example, even a

minority group of agents, like 20% to 25% of the whole population, who are inequity averse, is sufficient.

*Is the result surprising?* Leaving this issue temporally aside, we have to admit that how to find the behavioral rules for each individual so that they can collectively generate the desirable aggregate pattern is in general a very challenging issue for both the sciences and social sciences. This issue has long been studied in various agent-based models, in particular, cellular automata. In this paper, the choice of the social network for the society and the representation of the decision rules for the individuals recast the classical El Farol Bar problem into a familiar environment of cellular automata. When the central planner (model builder) has no single slice of the idea as to what these rules should be, they are then left for the members of society (cellular automata) to find out among themselves. The question, now posed bottom-up, is actually a cruel test for the limit of self-coordination. Despite the possible inherent difficulties, the self-coordination problem may become less hard to solve under some social network topologies and some cultures. This general feature should not be a surprise, but any concretization of it to see how it actually happens is not immediately obvious. While in this paper we have started setting the social preference exogenously, that preference can be generated through a culture of “keeping up with the Joneses” under the given network. In essence, it is the KUJ culture and the network that together facilitate the social search for the “right codes” for a “good society”.

*To what extent, can the findings be generalized?* An abstract model like the one presented here, being far from any realistic settings, certainly has its limitations. The specific analysis and answers which we obtain in this paper may no longer be applicable in other general and realistic settings, but the wonder prompted by this paper remains, i.e., the potential of using social networks, social preferences or cultures to enhance the self-coordination of a society of agents with simple behavioral rules. Hence, from a comparative study viewpoint, one can imagine that, in the case of societies (towns, countries,...) or societies in different times with very different human interaction webs, social preferences and culture, their self-coordination capabilities to solve some “tragedies of commons” may be different, although the “tragedies” may often be solved or institutionalized in a top-down manner, too.

## Appendix: Heterogeneous Strategies in the Multiple Clustering Equilibria

In Section 5, we have provided a simple analysis of the selected strategies in the 1C equilibrium. When the 1C equilibrium emerges, all the agents adopt the same strategy. However, for equilibria with multiple clusters of agents, normally, agents will be heterogeneous in the strategies adopted when the equilibrium is achieved. A brief account of the heterogeneity is given in this appendix.

When the 2C equilibrium emerges, agents are frozen in their state (1 or 0). This means that they only use one rule from their set of 16 rules, as they face the same situation period after period. In general, the equilibrium is characterized by the presence of many different strategies; for example, in one run the equilibrium was characterized by an ecosystem of 18 different strategies. However, if we consider only the rules which happen to be fired by the population of agents, usually only 4 or 5, we observe the presence of few different strategies, usually from 2 to 5 or 6. The vast majority of the agents have the same strategy. As a concrete example, in one run where the equilibrium was characterized by 5 strategies, the number of agents adopting each of these strategies was 83, 10, 4, 2 and 1.

With the number of agents going to the bar in equilibrium equal to 60, this means that in some cases the different behavior observed between the agents going to the bar and those not going was not caused by different strategies, but by different environments which led them to fire different rules of the same strategy. Conversely, agents with the same behavior may have different strategies. However they either fire a different rule because of their different surroundings, or they fire the same rule because they have the same surroundings, prescribing the same action (in the latter case it means that their strategies differ because of differences in the other rules).

When the 3C equilibrium emerges, there are agents going with a frequency of 0.5. However, the general picture is similar to that described for the 2C equilibrium. Although we have agents with a different strategy, the vast majority of agents share the same strategy, but they have a different behavior because of their different environment, i.e. their neighbors' behavior.

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