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Working Paper
Public profit sharing

Research Report, No. 2001-2

Provided in Cooperation with:
Department of Economics, University of Western Ontario

Suggested Citation: Schöb, Ronnie (2001) : Public profit sharing, Research Report, No. 2001-2, The University of Western Ontario, Department of Economics, London (Ontario)

This Version is available at:
http://hdl.handle.net/10419/70419

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Public Profit Sharing*

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May 2001

ABSTRACT
Many countries suffer from persistently high unemployment rates. The scope for labour market reforms is often limited to measures that hurt neither shareholders nor workers. This paper develops a policy proposal, which allows the government to reduce wage costs without changing the income positions as determined in the process of wage negotiations. It is shown that the introduction of public profit sharing, i.e. substituting profit share for social security contributions, can boost employment both in the short run and the long run. Calibrating the model and comparing the results with recent empirical findings about the impact of labour taxation confirm the theoretical findings.

Keywords: public profit sharing, trade unions, unemployment, labour market reform
JEL-classification: E60, J51, C70.

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* A preliminary version of this paper has been presented at seminars at York University and the University of Guelph. I would like to thank participants of these seminars as well as Jim Davies, Michael Reutter and Marcel Thum for many helpful comments. The usual disclaimer applies.
1. Introduction

What a striking contrast: month after month countries such as the USA, Great Britain and the Netherlands report new labour market records, while others like France, Germany, Italy and Spain continuously suffer from high unemployment rates in the range of 8 – 12%, with the prospect of only a small decline in the years to come (cf. OECD 2001). These countries have no reason to turn off the red alert sign.

The reason for this divergence is seen in the existence of rigid labour market institutions in many European countries that are characterized by strong regulatory constraints and collective bargaining arrangements that limit the ability of firms to adjust employment and wages in the face of changing market conditions. Recent empirical research tried to identify the main factors causing high unemployment in Europe.¹ There is common sense about the crucial role of trade unions. The more powerful they are, the more successfully they can raise wages and the more they can press governments to sustain strict employment protection laws and generous welfare benefit systems. However, as the Dutch example shows, unionized labour markets need not necessarily go along with high unemployment. If trade unions are willing to co-operate with employer organisations and the government on a nationwide level, labour market performance can be improved substantially. The second factor is the generous welfare system most European countries have created and sustained. The overall tax burden on labour to finance their welfare systems has created huge distortions that, in combination with other labour market distortions, cause unemployment. Though the short-run effects are generally significant and often long lasting, empirically, it turns out that the long-run effects of labour taxes on unemployment are small in some countries but are reported to be large in others (see Daveri and Tabellini 2000, Reutter 2001). The huge expenditures of the welfare state also contribute significantly to unemployment: generous welfare benefits reduce the cost of becoming unemployed and increase the upward pressure on wages from trade unions.

Many economists thus recommend far-reaching deregulation of the labour market and limiting trade union power to allow wages to adjust downwards. However, these proposals to often neglect the fact that labour market institutions have evolved to smooth out the consequences of other market imperfections. For instance, in an uncertain world, job security legislation as well as collective bargaining can provide workers with insurance against labour income risk – insurance that is not provided by insurance markets (cf. Agell 1999, 2000).

Moreover, collective bargaining has become the main institution where rent-sharing rules are set and distributional conflicts between labour and capital are settled. In Germany, for instance, collective bargaining – the so-called ‘Tarifautonomie’ – is a constitutional right of employer and employee organisations to negotiate labour contracts without undue

governmental interference. This constitutional status allows labour organisations to make these negotiations the forum where the sharing rules for the domestic product between labour and capital are determined, the main distribution key being the wage rate. Restricting or eliminating these constitutional rights of the labour organisations will not be possible without risking social unrest and increasing conflicts between workers and employers. It is thus not very surprising to find strong resistance from trade unions combined with little political pressure to actually reform the German labour market institutions: the Tarifautonomie is sacrosanct for politicians of almost all German parties. For many European countries the picture is pretty much the same. Insider workers, represented by their trade unions, have successfully defended their income positions in the last three decades and will continue to do so, even at the expense of a growing number of unemployed.²

Both the uncertainty about the social cost of liberalizing the labour market and the political constraints make it interesting to focus on labour policy measures which allow labour costs to fall without fundamentally altering the current income position of workers. Any policy aimed at reducing unemployment must thus be Pareto improving in the sense that it makes neither workers nor capital and shareholders worse off. How can this be achieved?

As long as the income position of workers is mainly determined by the wage rate, any reduction in labour cost will make workers worse off. Hence, it is necessary to untangle workers’ income from the wage rate. This can be achieved, in principle, by introducing profit sharing of workers as proposed by Martin Weitzman (1983, 1985): by substituting profit income for wage income, the wage rate and thus labour cost can be reduced without actually affecting the workers’ income. Applying Weitzman’s proposal of a “Share Economy” to unionized labour markets, Pohjola (1987) and Anderson and Devereux (1989) showed that if trade unions and employer organisations bargain over both wages and the profit share of workers, employment would rise. Full employment can be achieved if the wage rate is set equal to the marginal cost of labour.³ This would maximize the surplus that can be shared between shareholders and workers.

Several drawbacks reduce the attractiveness of profit sharing. Holmlund (1989) and Layard and Nickell (1990) showed that the long-run effect of profit sharing might be negligible or even zero. If profit sharing increased employment in the short run, the reservation wage of workers would increase. This in turn would lead trade unions to demand higher wages – a mechanism similar to the long-run mechanism, which shifts the labour tax burden to workers. Furthermore, profit sharing may not be at all favourable for trade unions. Though it will increase the total income of workers it will benefit the unemployed, who will find a new job at the expense of the insider because the profit per worker is declining in the

² For a political economy explanation see Saint Paul (1996, 1997).
³ Profit sharing can actually be considered as a device to introduce efficient bargaining as suggested by McDonald and Solow (1981).
number of workers. If only insiders received profits shares – as proposed by Sinn (1999) – their expected income would rise. However, it would remain unclear as to what extent this compensated for the additional income risk insiders would have to bear instead: the more risk-averse workers are, the less likely it will be that profit sharing will be introduced at a sufficiently large scale.

Private profit sharing may therefore not work. However, there is another possibility to untangle the workers’ income position from the wage cost. As the government takes away a substantial amount of wage income, why not substitute profit shares for labour taxes or social security contributions that the government receive instead of reducing the net-of-tax wage rate, which the workers receive? Such a public profit sharing would leave workers with the same certain take-home income – thus eliminating the disadvantage for insiders of lower income and/or of higher labour income risk – while reducing labour cost and thus promoting employment. This paper shows that indeed public profit sharing can alleviate unemployment both in the short run and – in combination with complementary policy measures – in the long run. Moreover, the paper elaborates on the conditions that must be satisfied to make the introduction of public profit sharing a strictly Pareto-improving and therefore politically feasible policy measure.

The following Section 2 describes the basic model. The allocation mechanism of public profit sharing, exemplified by the replacement of the unemployment insurance contribution rate, is then described in Section 3 for the benchmark case of a constant net-of-tax wage rate. Section 4 introduces wage bargaining between trade unions and firms and analyzes how the net-of-tax wage rate may be affected by profit sharing in the short run. Section 5 discusses the long-run implications. To estimate the magnitude of the employment effects generated by the introduction of public profit sharing, Section 6 calibrates the reform proposal for Germany and relates its findings to the empirical literature on how tax rate changes affect unemployment. Section 7 discusses complementary policy measures, which ensure profit sharing to be successful even in the long run. Section 8 concludes.

2. The model

We consider a small open economy with $M$ domestic firms. Each firm is perfectly specialized in the production of good $X^m, m = 1,..., M$, which is completely exported and sold on the world market. Globally, each firm faces monopolistic competition, whereby the number of firms operating in the world market is fixed.\footnote{In order to model wage negotiations, we need profits accruing to the private sector. Profits exist if the firm can exercise market power in the output market. If the firm faces perfect competition, profits accrue if the technology is linear homogenous with respect to capital, labour and a third fixed factor. The latter model could be easily
shall focus on a representative firm, which produces good $X$ and shall use the index $m$ only where necessary. The good $X$ is produced with capital $K$ and labour $L$ as inputs. Capital is assumed to be perfectly mobile between countries. Labour, by contrast, is internationally immobile. The technology is linear-homogenous and is represented by the production function $X = f(K, L)$. Factors are assumed to be price complements, i.e. if the price of one factor rises, both factor demands decline and vice versa.

The representative firm faces output demand $X^D(p)$, which is decreasing in the output price $p$, measured in units of an imported good, and is assumed to be isoelastic, i.e. $X^D(p) = p^{-\varepsilon}$, with $\varepsilon = -\left(\partial X^D(p)/\partial p\right)$, $p/X$ denoting the output demand elasticity. The closer substitutes for good $X$ on the world market are, the more elastic output demand becomes.

Profit is given by

$$\pi = pX(p) - \tilde{w}L - rK,$$

(2.1)

where the firm considers the world interest rate $r$ and the gross wage rate $\tilde{w}$ as given. Denoting sales revenues with $R \equiv pX(p)$, the profit maximum is given by $R_L = \tilde{w}$ and $R_K = r$, where subindices indicate partial derivatives. Furthermore, a profit maximum requires the output demand elasticity to exceed unity, i.e. $\varepsilon > 1$. In this case the firm will set a price, which exceeds the marginal cost by a constant mark-up factor $\varepsilon/(\varepsilon - 1) > 1$.

The gross wage $\tilde{w}$ is determined by the net-of-tax wage $w$, which is negotiated between a trade union and the firm, plus the labour tax and social security contributions. Denoting the social security contribution rate as $t_s$ and the labour tax rate as $t_L$, and assuming that the social security contributions are fully deductible from the labour tax, the following identity holds:

$$\tilde{w}(1-t_s)(1-t_L) = w.$$

(2.2)

The social security contribution of a worker equals $t_s\tilde{w}$ and the wage tax revenues equal $t_L\tilde{w}(1-t_s)$.

The government levies labour taxes to finance a fixed amount of the public good $G$ and social security contributions to finance unemployment benefit payments. If the government introduces a public profit sharing scheme, unemployment benefit payments may be alternatively financed by a share $\lambda$ of total profits $\pi$. As there are $N$ workers who are either employed or unemployed, unemployment always equals $N - L$. Unemployment benefits $b^{mob}$
are paid to all $N-L$ unemployed. For simplicity we abstract from other social security payments, such as old-age pension and public health care contributions, and we also abstract from other taxes. As the government covers any deficit of the employment agency, the aggregate government budget constraint is given by

$$t_L^* \tilde{w}(1-t_s^*)L + t_s^* \tilde{w}L + \lambda \pi = G + b^{\text{pub}} (N-L). \quad (2.3)$$

Assuming a fixed domestic capital stock $K$, national income $Y$ equals the sum of the income of employed and unemployed workers, domestic capital income, profit income and public revenues. Using (2.1) and (2.3) we have:

$$Y = wL + rK + \pi + G + b^{\text{pub}} (N-L) = pX - r(K-K). \quad (2.4)$$

The model satisfies the usual resource constraint of an open economy. As both private and public goods are imported, the resource constraint is given by $Y = C + G + pX - M$, where $C$ denotes domestic private consumption and $M$ denotes imports, respectively. As imports consists of private and public consumption plus capital import, $M = C + G + r(K-K)$, the resource constraint is consistent with equation (2.4).

The focus of the analysis is on Pareto-improving policy measures that benefit both shareholders and workers. This constrains labour market policy to measures, which ensure that the profit of shareholders and the rents accruing to workers are not decreasing. The utility of a worker is given by $u^w(C,L,G) = u^w(w,1,G)$, and the utility of an unemployed person is given by $u^u(C,L,G) = u^u(b^{\text{pub}}, 0, G)$. The unemployed are looking for work at the given net-of-tax wage rate and they are unhappy about not having it (cf. Clark and Oswald 1994). Thus, unemployment implies that $u^w(w,1,G) > u^u(b^{\text{pub}}, 0, G)$.

3. Reforming the unemployment insurance: a graphical exposition

Recall the initial situation of an economy that is characterized by involuntary unemployment due to excessive labour costs. The government aims at reducing wage costs but is limited to measures that hurt neither shareholders nor workers. This section shows how public profit sharing can meet these requirements and thus functions as a Pareto-improving and hence feasible policy measure to boost employment.

The fundamental mechanism of an introduction of public profit sharing can be illustrated graphically. Figure 1 represents the aggregate labour market of the economy. In the initial equilibrium, the government levies a tax $t_L$ and a social security contribution $t_s^I$. Given the net-of-tax wage rate $w$, the gross wage rate is $w/(1-t_s^I)(1-t_L)$ [cf. equation (2.2)]. The falling curves $R_L$ represent the marginal value product of labour input for given capital stocks and can be interpreted as the short-run (fixed capital stock) labour demand curves. As profit
maximization implies that $R_e = \tilde{w}$, point D indicates the economy’s initial equilibrium where the initial employment level is given by $L^I$ and the capital stock by $K^I$ (which determines the locus of the marginal value product of labour curve). Domestic product is given by the area ADLK below the marginal value product of labour curve. It can be split into several components. Capital income plus profit income is given by the triangle ADC. The gross payroll is given by the rectangle CDLK; gross payroll can be further split into the net-of-tax wage income HILK, tax revenues EFIH and unemployment insurance contributions CDFE.

Figure 1: Public profit sharing and employment

Now consider the case where $t_s$ is reduced to zero while the net-of-tax wage rate is held constant. The assumption of a constant net-of-tax wage rate serves as a benchmark case; it will be dropped in the next section when wage negotiations are introduced into the model. The employment agency is granted a profit share that – at the initial employment level $L^I$ – has to guarantee the unemployment agency revenues equal to the former social security contributions, i.e. $t_s^I \tilde{w}^I L^I = \pi^I + t_s^I \tilde{w}^I L^I$, with $\pi^I$ denoting the original profit. This equation implicitly defines the profit share $\lambda$ that the employment agency receives.

Following the proposal by Sinn (1999), public companies can offer the employment agency preference shares, publicly quoted companies can assign propriatory interest, and unincorporated firms can offer interest-bearing claims entitlements. The benefit entitlements of workers in case of unemployment are unaffected by the change in how the employment agency finances the unemployment insurance system: as before, benefits will be based on (net) wage income and the duration of employment.

Economically, the introduction of public profit sharing could be interpreted as an introduction of a profit tax. However, it differs from a profit tax in two respects. Firstly, public profit sharing comes along with an increase in the profit tax base (by $t_s^I \tilde{w}^I L^I$ due to the elimination of the social security contributions): at a given employment level, public profit sharing thus does not affect private profits at all. Secondly, profit sharing does not require that
all firms have to accept the same profit share $\lambda$. The profit share requires a rule such as revenue neutrality for a given level of employment. Such a rule can then be implemented on a firm or industry level in exactly the same way as discussed with respect to private profit sharing.

As long as the factor input levels remain constant, income distribution does not change. The employment agency receives CDFE, firm owners receive $\pi'$, capital owners $rK'$ and workers $HILK$. As social security contributions are fully tax deductible, the introduction of public profit sharing does not affect tax revenues either. Formally, we have:

$$t_i \tilde{w}^I (1-t_s^I) L^I = \frac{t_i w(1-t_s^I)}{(1-t_s^I)(1-t_l)} L^I = \frac{t_i w}{(1-t_l)} L^A = t_i \tilde{w}^II L^I.$$

However, the gross wage rate falls from $\tilde{w}' = w/(1-t_s^I)(1-t_l)$ to $\tilde{w}'' = w/(1-t_l)$ and the firm will hire more workers. In the short run – at given capital stock $K'$ – employment will rise to $L''$. As capital income remains constant, profit rises by DGF. Hence, both shareholders and the employment agency are better off. In addition, tax revenues rise by FGJI and the net-of-tax wage bill rise by IJML so that domestic income increases. Because the utility of workers exceeds the utility of being unemployed, $u^w(w,1,G) > u^w(b^{mub},0,G)$, all new workers benefit while the incumbent workers are not worse off. Moving from $L^I$ to $L''$ is therefore strictly Pareto improving.

As factors are price complements, the lower gross wage rate leads to an increase in capital demand and in the long run the capital stock will rise. Assuming $K'''$ to be the new equilibrium capital stock, the labour demand curve will shift outwards from $R_L(K',L)$ to $R_L(K''',L)$ and the long run employment level $L'''$ will be even higher than the short run employment level.

Both revenues of the employment agency and tax revenues increase – the former by at least $\lambda \cdot$ DGF, the latter by FGJI. The introduction of the public profit sharing system will thus generate a budget surplus, which can be used to reduce the labour tax rate. This would induce a further increase in profit income, labour income and tax revenues and would thus lead to a further Pareto-improving increase in domestic income.

The results derived in this section crucially depend on the assumption that the net-of-tax wage rate remains constant after the introduction of a public profit sharing scheme. However, both the reduction of social security contributions and the subsequent reduction of the labour tax rate as well as a lower unemployment rate may lead trade unions to demand higher net-of-tax wage rates. To analyse potential feedback effects on the net-of-tax wage rate we therefore endogenize wage setting and analyse the circumstances under which this result prevails.

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5 The long-run change in profits cannot be deducted from Figure 1. However, the short-run increase in profits indicated by the triangle DGF denote the minimum increase in profits in the long-run.
4. Public profit sharing and wage negotiations

So far we have assumed a constant net-of-tax wage rate. However, the introduction of a public profit sharing scheme may affect wage negotiations between the trade union and the firm. We therefore introduce wage negotiations to the model in which the net-of-tax wage rate is determined. After an agreement about the wage rate is reached, firms unilaterally determine the employment level. This is modelled by using the 'right-to manage' model that represents the outcome of the bargaining by an asymmetric Nash bargaining.6

The objective of the trade union is to maximize its $N$ members’ net-of-tax income. Each member supplies one unit of labour if employed, or zero labour if unemployed. The net-of-tax income of a working member depends on the net-of-tax wage rate $w$. If a trade union member is laid off, she will find another job at the average net-of-tax wage rate $\bar{w}$ with probability $(1-u) - u$ indicating the country’s unemployment rate. With probability $u$ she will become unemployed, in which case she receives unemployment benefit payments. As the domestic unemployment rate is exogenously given from the viewpoint of a single trade union, the reservation rate is determined by $b = (1-u)\bar{w} + ub^\omega$. The objective function of the trade union $V^*$ can then be written as $V^* = wL + b(N - L)$. The fall-back position of the trade union is given by $V^0 = bN$, i.e. if the negotiations break down, all members receive their reservation wage $b$.

The firm maximizes its profit $\pi = (1-\lambda)(p(X)X - rK - \tilde{w}L) = (1-\lambda)\pi$. The fall-back position of the firm is given by zero profits, i.e. $D^0 = 0$. Using $V \equiv V^* - V^0$, the Nash bargaining maximand can be written as $\Omega = V^\beta D^{1-\beta}$, with $\beta$ representing the bargaining power of the trade union. The first-order condition with respect to the net-of-tax wage rate is

$$\Omega_w = 0 \iff \beta \frac{V_w}{V} + (1-\beta) \frac{D_w}{D} = 0. \tag{4.1}$$

**Comparative statics**

A change in the social security contribution $t_s$ affects both the trade union’s and the firm’s objective functions. Assuming a constant elasticity of substitution, the labour demand elasticity is given by

$$\eta_{L,\tilde{w}} \equiv \frac{L_{\tilde{w}}}{L} = -\sigma + s(\sigma - \epsilon),$$

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6 Cf. Nash (1950). The ‘right-to-manage’ approach reflects the observation that in most European countries over three-quarters of the workforce earn wages that are covered by collective bargaining in which trade unions and employer organisations agree upon wages only and (in many cases explicitly) delegate the right to determine employment to the firms (cf. e.g. Oswald 1993).
where \( s \) denotes the cost share of labour, \( s \equiv \tilde{w}L/cY \) (cf. Koskela and Schöb 1998). Then, the first-order condition (4.1) can be expressed in explicit terms:

\[
(w-b)(\beta \eta_{L,\bar{w}} + (1-\beta)s(1-\varepsilon)) + w\beta = 0. \tag{4.2}
\]

Total differentiation of equation (4.2) yields

\[
w_{s_t} = -\Omega_{w}^{-1}z(w-b)s_{t_s}, \tag{4.3}
\]

with \( z = [\beta(\sigma-\varepsilon) + (1-\beta)(1-\varepsilon)] \). The first two expressions are negative.\(^7\) As factors are assumed to be price complements we have \( \sigma < \varepsilon \) and \( z < 0 \). Hence the sign of (4.3) equals the negative sign of \( s_{t_s} \). For a CES production technology, the partial derivative of the cost share of labour with respect to the wage rate is given by

\[
 s_{t_s} = s_{\tilde{w}} \frac{\tilde{w}}{(1-t_s)} = \frac{s}{(1-t_s)}(1-s)(1-\sigma) \left\{ \begin{array}{l} < \sigma \Rightarrow \sigma \left\{ > \right\} 1. \tag{4.4} \end{array} \right.
\]

(see Appendix I). For the trade union, the effect of a change in the social security contribution rate on the wage elasticity of labour demand \( \eta_{L,\bar{w}} \) plays a crucial role in the wage negotiations. If labour demand becomes less elastic, fewer workers will be fired when the net-of-tax wage rate increases, while the benefits for those employed remain the same. It becomes more profitable for the trade union to demand higher wages. From condition (4.4), we can infer that if substitutability is low, i.e. \( \sigma < 1 \), the cost share of labour \( s \) increases with the social security contribution rate. A larger share \( s \) implies that a one percent change in the wage rate induces a larger increase in total cost and hence a larger fall in output. This will lead firms to lay off more workers. Hence, if \( s \) increases, labour demand becomes more elastic. Formally, given a constant elasticity of substitution \( \sigma \), the partial derivative of the wage elasticity of labour demand is given by \( \partial \eta_{L,\bar{w}} / \partial s_t = s_{t_s}(\sigma-\varepsilon) \). As labour and capital are price complements, the sign of the change equals the negative sign of \( s_{t_s} \). Hence, the higher the social security contribution rate, the weaker the bargaining position of the trade union as the potential losses of a wage increase go up. The firm, by contrast, will oppose wage increases more strongly when the cost share of labour \( s \) increases as a consequence of an increase in the social security contribution rate, as profits will fall at a higher rate. An increase in \( t_s \) thus weakens the trade union’s bargaining position and strengthens the firm’s bargaining position at the same time: both effects work in the same direction and the net-of-tax wage will fall. If, on the contrary, substitutability is high, e.g. \( \sigma > 1 \), the net-of-tax wage rate will rise. Summing up, we have

\[\text{(The second-order condition is assumed to hold throughout, i.e. } \Omega_{w} = y + (w-b)zs_{w} < 0 \text{ with } y = \beta(1+\eta_{L,\bar{w}}) + (1-\beta)(1-\varepsilon)s.\]

\(^7\)
The public profit share $\lambda$ does not affect the net-of-tax wage income of trade union members directly, i.e. $V_\lambda = 0$. However, a higher public profit share reduces the shareholders’ profit $D$, i.e. $D_\lambda = -\pi$. A higher profit share also implies that the loss in the shareholders’ income due to an increase in the net-of-tax wage increase decreases, i.e. $D_{w_\lambda} = -\pi_{w}$. As can be seen from differentiating equation (4.1) with respect to $\lambda$, these two effects exactly balance. The public profit share therefore has no impact on the negotiated wage rate, i.e. $w_\lambda = 0$. Hence, the only feedback effect we have to consider when public profit sharing is introduced is due to changes of $t_S$.

**Revenue neutrality**

Any surplus the employment agency raises will be transferred to the government. To analyse revenue neutrality, we can therefore focus on the aggregate public budget constraint (2.3), which can be rewritten as

$$GLNbwLtt twLtLtwLtLt govLSLS=\left(-\lambda\pi+\pi_{gov}(N-L)\right) = G.$$  

The first term of the left-hand side denotes the labour tax revenues; the second term denotes the surplus of the employment agency. The sum of both public revenue components must yield the fixed expenditures $G$ for the public good provision. Interpreting revenue neutrality as keeping the public good provision $G$ constant, the introduction of public profit sharing has to meet the following condition:

$$dG = G_{t_S} dt_S + G_{\lambda} d\lambda + G_{w_\lambda} dw = 0.$$  

As the comparative statics result shows, the change in the net-of-tax wage rate depends only on the change of the social security contribution $t_S$ and not on the change of the public profit share $\lambda$, i.e. $dw = w_{t_S} dt_S$. Hence, equation (4.6) can be rewritten as

$$dG = (G_{t_S} + G_{w_\lambda} w_{t_S}) dt_S + G_{\lambda} d\lambda = 0.$$  

Public revenues are always increasing in $\lambda$ as $G_\lambda = \pi$. If we also assume that marginal revenues of social security contributions are positive, $G_{t_S} + G_{w_\lambda} w_{t_S} > 0$, i.e. the economy is on the Laffer-efficient side, it follows immediately that an increase in the public profit share $\lambda$ will result in a revenue-neutral cut in social security contributions:

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8 If the factors were substitutes, the effects would work in the opposite direction and it would not be possible to a priori sign the effect of a labour tax rate increase on the wage negotiations.
The public profit share $\lambda$ does not affect the net-of-tax wage rate nor does it affect the wedge between the net-of-tax wage rate and the gross wage rate. Hence, the total effect of an introduction of a public profit sharing scheme on the gross wage is determined by the direct effect of the reduction of the social security contribution and the induced effect. The effects on the gross wage rate and employment are therefore given by

$$\left. \frac{dt}{d\lambda} \right|_{G=0} = \frac{-\pi}{G_s + G_w w_{t_s}} < 0.$$  

(4.8)

Employment, output and domestic income

Condition (4.5) shows that the net-of-tax wage rate is unaffected if the elasticity of substitution is unity, which is the case for a Cobb-Douglas production function. In this case a reduction of the social security contribution lowers the gross wage rate at the same rate. Thus, the graphical analysis of the previous section perfectly describes the mechanism of a public profit sharing scheme if wages are determined in wage negotiations and the technology is Cobb-Douglas.

If the elasticity of substitution exceeds unity, $\sigma > 1$, the gross wage rate falls even more ($\omega_{t_s} > 0$) and the employment effect is strengthened. By contrast, if $\sigma < 1$, the fall in the gross wage is smaller, as a reduction in the social security contribution leads to an increase in the net-of-tax wage. Nevertheless, irrespective of whether the net-of-tax wage rate increases or not, the introduction of a public profit sharing scheme always results in a higher level of employment as $\omega_{t_s} > -1$. As profits are decreasing in factor prices, profits will increase and so will domestic income. This can be summarized in

**PROPOSITION 1:** If wages are determined in wage negotiations with the reservation wage being fixed, public profit sharing reduces unemployment and increases domestic income.

Welfare

An increase in profits does not imply, however, that the profit income of shareholders increase when the public profit sharing has been introduced. If the net-of-tax wage rate does not
increase, which is the case for $\sigma \geq 1$, the profit will certainly increase in employment as 
profits for any given employment level are not decreasing. As labour rent increases with 
employment, the introduction of public profit sharing will be strictly Pareto improving.

If, by contrast, the net-of-tax wage rate increases, it turns out that at any given employment 
level the shareholders’ profit income will be lower, the higher the share $\lambda$ is. Hence it is not 
clear a priori whether public profit sharing – although it is increasing in employment – is 
Pareto improving. Nevertheless, even when the net-of-tax wage rate increases, it can be 
shown that as long as the public profit share is smaller than the effective tax rate on labour, 
i.e. $\lambda < t_L (1 - t_s) + t_s$, private profits are always increasing in the profit share $\lambda$, i.e. 
$dD/d\lambda > 0$ (see Appendix III). Hence, as long as the reservation wage is unaffected by the 
earnings negotiations, the initial introduction of public profit sharing is strictly Pareto improving. 
The additional surplus will be larger, the easier it is for firms to substitute capital for labour, 
as this discourages trade unions from raising the net-of-tax wage rate after public profit-sharing is introduced.

**PROPOSITION 2:** If wages are determined in wage negotiations with the reservation 
earnings being fixed, public profit sharing is Pareto improving if $\sigma \geq 1$ or, for $\sigma < 1$ the 
public profit share is smaller than the effective tax rate on labour.

### 5. Long-run employment effects

From the viewpoint of a single trade union, the reservation wage $b$ is exogenously given. 
However, profit sharing will reduce unemployment in the short run and thus improve labour 
market conditions. As the unemployment rate $u$ falls the possibility of finding a job increases. 
This will raise the reservation wage and will raise the net-of-tax wage rate as $w_p > 0$. A higher 
net-of-tax wage rate will also affect unemployment benefit payments. Layard, Nickell and 
Jackman (1991) argue that unemployment benefits are – in the long run – proportional to the 
net-of-tax wage rate. Denoting $\gamma$ as the replacement ratio, unemployment benefits are then 
given by $\gamma w$. Hence, the reservation rate is determined by

$$b = (1-u)\overline{w} + u\gamma \overline{w} = (1-u(1-\gamma))\overline{w}.\quad (5.1)$$

As a lower unemployment rate increases the reservation wage, the change in the reservation 
earnings will reduce the efficacy of a public profit sharing scheme to reduce unemployment. To 
see this, consider the symmetric solution where in equilibrium all firms pay the same net-of-
earnings wage rate, i.e. $w^m = \overline{w}$ for all $m$. Solving (4.2) with respect to the net-of-tax wage rate and

---

9 For a detailed derivation of the reservation wage see Layard, Nickell and Jackmann (1991, chapter 2).
substituting the explicit solution for $m$ into equation (5.1), the unemployment rate $u$ is given by

$$u = \frac{-\beta}{(\beta(1-\sigma)(1-s) + s(1-\varepsilon) - \beta)(1-\gamma)}.$$  

(5.2)

Equation (5.2) implicitly defines the long-run unemployment rate as only in the case of $\sigma = 1$, where the cost share of labour is not affected by changes in the social security contribution rate, the domestic unemployment rate is completely determined by exogenous parameters. In general, however, the cost share of labour changes when the social security contribution rate changes. Total differentiation of (5.2) yields: The long-run change of the net-of-tax wage rate is given by

$$\frac{d\tilde{w}}{dt_s} = -\frac{u_s}{u_w} \left(\beta(1-\sigma)(1-s) + s(1-\varepsilon) - \beta\right) + \frac{s_s}{u_w} \left((1-\varepsilon) - \beta(1-\sigma)\right) = -\tilde{w}.$$  

as $u_s = u_w \cdot \tilde{w}/(1-t_s) = \tilde{s} \cdot \tilde{w}$ and $s_s = s_w \cdot \tilde{w}/(1-t_s) = s_w \cdot \tilde{w}$ [cf. equation (4.4)]. The change of the net-of-tax wage rate always equals the negative of the gross wage, i.e. any change in the social security contribution rate falls completely on workers; the gross wage rate does not change:

$$\frac{d\tilde{s}}{dt_s} = \frac{\tilde{s}}{(1-t_s)} + \frac{w}{(1-t_s)} = \tilde{s} - \tilde{s} = 0.$$  

Consequently, the unemployment rate $u$ does not change either. Thus, if the replacement ratio is constant, public profit sharing fails to raise employment in the long run. The dramatic implications this has on the distribution of income can be seen from inspecting Figure 1 again. There, the employment level remains at $L'$ and the gross wage rate remains constant. However, the workers’ income increases by CDFE in the long run because what they paid for social security before the introduction of the public profit sharing they now receive as net wage income. By contrast, as profit income and capital income has not changed either, the profit income that accrues to shareholders is reduced by the same amount CDFE. This may be summarized in a proposition.

**PROPOSITION 3:** If unemployment benefits are proportionate to the net-of-tax wage rate and wages are determined in wage negotiations, public profit sharing will have no long-run effect on employment but change the income distribution in favour of labour income.

This neutrality result crucially depends on the assumption of a constant replacement ratio $b = \gamma \tilde{w}$. Theoretically, this assumption may be questionable for two reasons. First, Blanchard and Katz (1999) argue that the income of the unemployed does not consist of unemployment benefit payments only but also of non-market income, and that the replacement ratio $\gamma$ is
homogenous of degree zero in the wage rate and the non-market income. If the social security contribution drives a wedge between labour income and non-market income, the replacement ratio would decline and, consequently, the long-run employment effect would remain positive – though smaller than the short-run effects.

Second, although unemployment benefits are often paid in proportion to the wage rate (cf. MISSOC 1998), other additional welfare transfers are often cut if the unemployment benefit payments rise. For low-qualified workers in particular, the assumption of constant total public benefit payments is more realistic than the assumption of a constant replacement ratio. The long-run employment effect will be positive as long as there is some fraction of the reservation wage that does not vary proportionately with the net-of-tax wage rate. This can be seen analytically by splitting the reservation income into two components: the first is proportional to the net-of-tax wage rate while the second is a constant.

Even if the employment effect remains positive in the long run, it becomes doubtful that shareowners may not benefit as the introduction of public profit sharing will allow the trade unions to force the shareowner pay part of the bill. As firms will anticipate the long-run consequences on profits they will strongly object to the introduction of a public profit sharing scheme, as they demand at least as high profits as before. There is thus an inherent time consistency problem of any public profit sharing scheme: trade unions would benefit from public profit sharing even if the net-of-tax wage rate remains constant. However, they could gain even more if they raise the net-of-tax wage rate after the profit sharing scheme has been introduced. As this is only possible at the cost of shareholders, public profit sharing could be strictly Pareto improving only if shareholders need not fear net-of-tax wage increases. We will come back to this issue in Section 7.

6. Estimating the employment effect

To exemplify the effects public profit sharing might have, this section provides some estimates for the long-run employment effects of transforming the German unemployment insurance contributions into public profit shares. The contributions to the mandatory German unemployment insurance system currently equal 5.4% of the gross wage including employers’ contribution to the social security system. In our thought experiment, the unemployment insurance contributions will be replaced by a public profit sharing scheme that guarantees that the German employment agency (Bundesanstalt für Arbeit) receives the same revenues if employment stays constant.

In total, we present four estimates as shown in Table 1. The first two estimates show the results of calibrating the model presented above to the case where the trade union credibly commits itself not to take advantage of the introduction of a public profit sharing scheme, i.e.
we consider the case of a constant net-of-tax wage rate. To calculate the pure substitution effect of the 5.4% reduction in the gross wage (Estimate I), we use an aggregate labour demand elasticity for constant output of 0.38 as estimated by Flaig and Rottmann (1998) for the German manufactory sector – an estimate that is in line with other estimates, reported in Hamermesh (1993). It is close to the “best guess” (Hamermesh 1993, p. 135) as most empirical estimates show a constant-output elasticity in the interval of [0.15; 0.75].

Changes in the social contribution rates only affect those 28 millions German workers who pay social security contributions. Hence, the change in employment must be related to these workers instead of the whole workforce. It turns out that employment due to the pure substitution effect of eliminating the unemployment insurance contributions will rise by about 570,000. As 100,000 new jobs lead to a reduction of the official unemployment by roughly 70,000, the substitution effect thus results in a reduction of the standardized unemployment rate (=8.0% in 2000, cf. OECD 2001) by 1.2 percentage points.

\[
\text{Table 1: Estimates of the long-run effects of public profit sharing}
\]

<table>
<thead>
<tr>
<th>Estimate I</th>
<th>Estimate II</th>
<th>Estimate III</th>
<th>Estimate IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial budget surplus in billion DM</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15.3</td>
<td>42.6</td>
<td>21.1</td>
<td>26.0</td>
</tr>
<tr>
<td>Change in the gross wage rate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-6.4</td>
<td>-9.7</td>
<td>-6.9</td>
<td>-7.4</td>
</tr>
<tr>
<td>Change in the unemployment rate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1.4</td>
<td>-5.8</td>
<td>-1.6</td>
<td>-2.2</td>
</tr>
</tbody>
</table>

The introduction of the public profit sharing scheme raises public revenues as discussed in Section 2. We abstract from any additional revenues for the employment agency due to higher profits and focus on the additional tax revenues and the reduced expenditures only, which are equal to 38,000 DM per year per worker (cf. Bach and Spitznagel 1998). We also abstract from additional tax revenues and savings from those taking up jobs without having been officially unemployed before. The initial total budget surplus of a 1.2 percentage point reduction in the unemployment rate is DM 15.3 billion. As a one percentage point reduction of the labour tax rate reduces tax revenues by roughly DM 15 billion, revenue-neutral cuts in the labour tax rate would allow the government to reduce the labour tax rate by 1.2 percentage points. In connection with the elimination of the unemployment insurance contributions this sums up to an overall reduction of the gross wage rate by 6.4 percentage point. The pure substitution effect of the tax rate cuts thus finally leads to a reduction in the standardized
unemployment rate by 1.4 percentage points. These results are reported as Estimate I in Table 1.

The second estimate takes account of the cost reduction effect. Due to lower labour costs, the total cost of production decreases by 3.7 percent (assuming a cost share of labour of 2/3). For an output demand elasticity of $\varepsilon = 1$, this would lead to an initial increase in employment by 1.6 million, which is equivalent to a reduction of the unemployment rate by 3.2 percentage points. This in turn would result in an initial total budget surplus of DM 42.6 billion, which allows the government to reduce the labour tax rate by 5.1 percentage points. A revenue-neutral rebate would thus reduce gross wages by a total of 9.7 percentage points. This would result in the creation of 2.9 million new jobs, which is equivalent to a more than 70% reduction of current unemployment.

These two calibrations assumed a constant net-of-tax wage rate. It might be illustrative to contrast these results with some empirical estimates about the long-run effects changes in the labour tax wedge have on unemployment. Nickell and Layard (1999) regress the log unemployment rate on the total labour tax wedge (among a variety of controls) in a cross-country study with 20 OECD countries over two five-years periods. Using their tax coefficient, a revenue-neutral introduction of a public profit sharing scheme would reduce the gross wage rate by a total of 6.9 percentage points and would result in a fall of the unemployment rate in the long run by 1.6 percentage points, from the current 8% to 6.4% (see Estimate III in Table 1).

Daveri and Tabellini (2000) come to different results. They show that the effects of tax rate changes on unemployment differ between three groups of OECD countries. While they did not find any significant effect for countries like the US, Japan or the Scandinavian countries, the tax wedge effect is more pronounced than reported in Nickell and Layard (1999) in countries like Australia, Belgium, France, Germany, Italy, the Netherlands and Spain. They regress the standardized unemployment rate on the effective tax rate on labour income and report a labour tax coefficient between 0.29 and 0.54 (see their Table 9, p.75). Applying their lowest and highest estimate for this group of countries would indicate a long-run reduction of the unemployment rate in the range of 2.2 to 5.6 percentage points (Estimate IV).

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10 Their tax coefficient of 0.027 (see their Table 15, p. 3053) must be multiplied with the unemployment rate to obtain the change in the unemployment rate when the tax wedge decreases by one percentage point.

7. Implementing public profit sharing

The last section suggests that public profit sharing can reduce unemployment dramatically. However, as pointed out before, there might be only limited scope for public profit sharing to actually become a strictly Pareto-improving measure and hence politically feasible. Any increase in the net-of-tax wage rate is associated with redistribution from profit income to labour income. It is therefore necessary to apply complementary policy measures that ensure public profit sharing to be Pareto improving even in the long run. This section discusses policy options for establishing public profit sharing as a strictly Pareto-improving device in the long run. One possibility is to embed public profit sharing into a ‘compact for employment’.

The Dutch model

In 1982 the Dutch employers’ federation and the trade unions agreed upon working time reduction and wage moderation. Even though it was de facto a bilateral agreement it has actually been a tripartite agreement as the government committed itself at the same time to reduce budget deficits and to reform the social security system. This so-called Wassenaar Agreement marked a change in the labour relations in the Netherlands and proved that corporatist institutions are not necessarily sustaining labour market rigidities. The Dutch model has become a synonym for a corporatist system “with consultation, co-ordination and bargaining over all important issues of socio-economic policy between union federations, employer federations and the government.” (Hartog 1999, p. 484).

A public profit scheme could be implemented within such a tripartite agreement (cf. Schöb 2000). In the first step the labour organisations agree on an upper ceiling for the net-of-tax wage rate or its growth path, respectively. This ceiling does not impose any true constraint on the trade unions as it only ensures that trade unions cannot take advantage of public profit sharing by raising the net-of-tax wage rate, which they won’t do without the introduction of public profit sharing. Fixing the net-of-tax wage rate guarantees the effectiveness of tax policy measures as the whole tax incidence then falls on the producer: a one percentage point reduction in the social security contribution will result in a one percent reduction of the wage cost. The government in turn will abolish the unemployment insurance contribution and will finance the unemployment benefit payments via public profit shares. Furthermore, the government commits itself to using any budget surplus to further reduce the labour tax rate and thus promote further job creation.¹²

¹² For the superiority of reducing labour taxes instead of e.g. source-based capital taxes when the net-of-tax wage is constant, see Koskela and Schöb (1998).
Complementary policy measures

If co-operation is not possible, the introduction of public profit sharing alternatively requires additional policy measures that guarantee its economic efficiency and help increase the probability of gaining political consent. The previous analysis shows that the reservation income $b$ of the trade union members is crucial for the determination of both the efficiency and distributional consequences of introducing public profit sharing. The government must therefore consider complementary policy measures that ensure that the reservation income $b$ does not rise in the long run.

All policy measures, which lower the replacement ratio, will increase the employment effect of public profit sharing while reducing the undesired distributional consequences at the same time. Such policies may comprise the lowering of unemployment benefit payments, the shortening of the unemployment benefit duration, the tightening and stricter enforcement of eligibility rules or the extension of workfare schemes. All these policy measures would reduce the wage pressure due to the introduction of public profit sharing.

Looking at any of these policy measures in isolation, it is clear that they would be strictly opposed by the trade union, as it would make its members worse off. However, if these measures are introduced in conjunction with public profit sharing, the latter will compensate the workers via a lower unemployment rate. Trade union members’ will not be worse off as a reduction of the replacement ratio comes along with greater job security and a higher probability of finding a new job when laid off. Embedding public profit sharing in a broad-based labour market reform thus promises a much higher impact on employment than the empirical estimates quoted above suggest. And it can do so without making any side worse off.

7. Concluding remarks

Public profit sharing aims at reducing the wedge that labour taxes and social security contributions have created between the private and social cost of labour by substituting a non-distorting revenue-raising device for distortionary revenue-raising devices. The stronger the effect of reducing the tax wedge is on the gross wage, the stronger the employment effect will be. As our analysis suggests, the employment effect will be substantial if this can be achieved by either heading for a co-operative solution within a compact for employment or by embedding public profit sharing into a broad set of policy measures which ensure the reform to be both efficient in curing unemployment and strictly Pareto-improving – a prerequisite for political feasibility.

The incentive effects of a public profit sharing scheme are very much the same as those of a private profit sharing scheme. However, there are some distinct advantages of a public profit
sharing scheme. By introducing private profit sharing as suggested by Weitzman (1983), incumbent workers who accept a reduction in their net-of-tax wage rate for a profit share would lose income if the firm hired more workers. Although a private profit sharing scheme where only insiders receive profit shares – as proposed by Sinn (1999) – could avoid this fall in the incumbent workers’ income, they would still be exposed to income risk and it is not clear \textit{a priori} whether the increase in expected income could compensate for bearing additional income risk. Insiders who dominate trade union policy are very likely to impede any private profit sharing scheme.

By contrast, incumbent workers would be indifferent to public profit sharing if they faced no unemployment risk (due to e.g. seniority rules etc.) but would welcome public profit sharing if it reduces their risk of becoming unemployed. Income risk is borne entirely by the public sector, which could easily consolidate idiosyncratic shocks between sectors and intertemporal shocks between booms and busts. There is thus no reason for trade unions to object to the introduction of public profit sharing. Of course, to avoid changing the long-run distribution of income, public profit sharing requires either co-ordination between the government and the labour organisation or implementation within a broad-based labour market reform. But trade unions should be more than happy to give guarantees to shareholders that public profit sharing will not weaken their income position in the long run or to accept any complementary policy measures that provide these guarantees.

High unemployment has persisted in some European countries for nearly three decades now. It is frequently argued that many countries are still reluctant to undertake the necessary labour market reforms because the medicine prescribed “is bitter and hard for many countries to swallow, especially insofar as it appears to raise concerns about equity and appears to threaten some of the rents and privileges of insiders” (Elmeskov et al 1998, p.242). Improving labour market efficiency, however, allows the government to increase the cake that has to be divided between workers and shareowners. Thus there is – at least theoretically – scope for Pareto-improving labour market reforms. To describe labour market reforms as a necessarily bitter and hard to swallow medicine implicitly assumes that the governments lacks a sufficient number of policy instruments to deal with both efficiency and equity considerations. This papers argues – in line with Orzag and Snower (2000) – that this trade-off is no exogenous constraint on policy making and identifies public profit sharing as at least one promising policy instrument to boost employment without affecting existing income positions negatively. Indeed, public profit sharing can benefit all parties who are directly or indirectly affected by the labour market outcome. However, the implementation of such a policy requires politicians to be open to new ideas and willing to experiment with a new type of employment policy.
Appendix I

The partial derivation of the cost share of labour \( s \) with respect to the gross wage yields

\[
    s_w = \frac{\tilde{w}L_w cX + LcX - (c_z X + cX_\tilde{w})\tilde{w}L}{c^2 X} = \frac{s}{\tilde{w}} \left( \eta_{L,\tilde{w}} + 1 - \frac{c_\tilde{w} \tilde{w}}{c} \frac{X_\tilde{w} \tilde{w}}{X} \right)
\]

(A.1)

with \( c \) denoting per unit cost of production. From Shephard’s Lemma follows that \( c_w = L/X \). Hence the third term in brackets simply yields the cost share of labour:

\[
    \frac{c_\tilde{w} \tilde{w}}{c} = s.
\]

The last term in brackets on the right-hand side can be split into

\[
    \frac{X_\tilde{w} \tilde{w}}{X} = \frac{\tilde{w}}{X} (X_L L_w + X_K K_w) = \frac{X_L L}{X} \frac{L_w \tilde{w}}{L} + \frac{X_K K}{X} \frac{K_w \tilde{w}}{K}.
\]

(A.2)

Applying the first-order conditions from profit maximization, \(((\epsilon - 1)/\epsilon) p X_L (K, L) = \tilde{w} \) and \(((\epsilon - 1)/\epsilon) p X_K (K, L) = r \), it follows that \( X_L = \tilde{w}/c \) and \( X_K = r/c \). Substituting into (A.2) and using the definitions \( \eta_{L,\tilde{w}} = -\sigma + s(\sigma - \epsilon) \) und \( \eta_{K,\tilde{w}} = s(\sigma - \epsilon) \), we obtain

\[
    \frac{X_\tilde{w} \tilde{w}}{X} = s\eta_{L,\tilde{w}} + (1-s)\eta_{K,\tilde{w}} = -s\epsilon.
\]

Hence, equation (A-1) can be written as

\[
    s_w = \frac{s}{\tilde{w}} \left( \eta_{L,\tilde{w}} + 1 + s\epsilon \right) = \frac{s}{\tilde{w}} (1-s)(1-\sigma).
\]

(A.3)

Appendix II

From equation (4.3), using the explicit formulation for the second-order condition and (4.4), it follows that

\[
    \omega_{y_s} = \frac{w_s (1-t_s)}{w} = -\frac{(w-b)z \gamma_s (1-t_s)}{(y+(w-b)z)w} = -\frac{(w-b)z \gamma_s \tilde{w}}{y+(w-b)z \gamma_s \tilde{w}}.
\]

Condition \( \omega_{y_s} > -1 \) holds if \( y < 0 \). Calculating the net-of-tax-wage rate from the first-order condition (4.2) yields:

\[
    w = y^{-1} (y - \beta)b.
\]

As \( w > b \) and \( \beta > 0 \), it follows immediately from inspection that \( y < 0 \). Hence, \( \omega_{y_s} = w_s (1-t_s)/w > -1 \). Q.E.D.
Appendix III

The change in the private profits $D$ is given by

\[
\frac{dD}{d\lambda} = -\pi + (1 - \lambda)\pi_t \frac{dt_s}{d\lambda}.
\]

Substituting in the revenue-neutrality condition (4.8), this can be rewritten as

\[
\frac{dD}{d\lambda} = -\pi + (1 - \lambda)\pi_t \frac{-\pi}{G_t + G_w w_s}.
\]  

(A.4)

Using the explicit derivatives of the budget constraint

\[
G_t + G_w w_s = \tilde{w}(1 - t_s)^{-1}L \left[ 1 - \lambda + \left[ t_L (1 - t_s) + t_s + \frac{b^\text{gov}}{\tilde{w}} \right] \eta_{L,\tilde{w}} \right] (1 + \omega_t) - (1 - t_L)\tilde{w}L\omega_t
\]

allows us to rewrite (A.4) as:

\[
\frac{dD}{d\lambda} = -\pi \frac{\omega_t (1 - \lambda) + (1 + \omega_t) \left[ t_L (1 - t_s) + t_s + \frac{b^\text{gov}}{\tilde{w}} \right] \eta_{L,\tilde{w}} - (1 - t_L)(1 - t_s)\omega_t}{\left[ 1 - \lambda + \left[ t_L (1 - t_s) + t_s + \frac{b^\text{gov}}{\tilde{w}} \right] \eta_{L,\tilde{w}} \right] (1 + \omega_t) - (1 - t_L)\tilde{w}L\omega_t}.
\]

Notice that the denominator is always positive as we assume positive marginal tax revenue throughout. If the elasticity of substitution is $\sigma = 1$, it follows from (4.4) that $\omega_t = 0$. Hence,

\[
\left. \frac{dD}{d\lambda} \right|_{\sigma = 1} = -\pi \frac{1 \left[ t_L (1 - t_s) + t_s + \frac{b^\text{gov}}{\tilde{w}} \right] \eta_{L,\tilde{w}}}{\left[ 1 - \lambda + \left[ t_L (1 - t_s) + t_s + \frac{b^\text{gov}}{\tilde{w}} \right] \eta_{L,\tilde{w}} \right]} > 0.
\]

(A.5)

If the elasticity of substitution is $\sigma > 1$, it follows from (4.4) that $\omega_t > 0$ and $\eta_{L,\tilde{w}} < -1$. Rearranging the numerator shows that

\[
\frac{dD}{d\lambda} = -\pi \left[ -\lambda \omega_t + \left[ 1 - (1 - t_L)(1 - t_s) \right] \omega_t + (1 + \omega_t) \left[ t_L (1 - t_s) + t_s + \frac{b^\text{gov}}{\tilde{w}} \right] \eta_{L,\tilde{w}} \right]
\]

\[
= -\pi \left[ -\lambda \omega_t + \left[ t_L (1 - t_s) + t_s \right] \omega_t + \left[ t_L (1 - t_s) + t_s + \frac{b^\text{gov}}{\tilde{w}} \right] \eta_{L,\tilde{w}} + \omega_t \left[ t_L (1 - t_s) + t_s + \frac{b^\text{gov}}{\tilde{w}} \right] \eta_{L,\tilde{w}} + \omega_t \left[ b^\text{gov} - \frac{b^\text{gov}}{\tilde{w}} \eta_{L,\tilde{w}} \right] \right] > 0
\]

Finally, if the elasticity of substitution is $\sigma < 1$, it follows from (4.4) that $\omega_t < 0$. In this case a positive sign of (A.5) requires

\[
-\omega_t \left[ (1 - \lambda - (1 - t_L)(1 - t_s)) - (1 + \omega_t) \left[ t_L (1 - t_s) + t_s + \frac{b^\text{gov}}{\tilde{w}} \right] \right] \eta_{L,\tilde{w}} > 0.
\]
As the second term is always positive, a sufficient condition for (A.5) to be positive is

\[ \lambda < t_L (1 - t_S) + t_S. \]

References


