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Discreteness and the Welfare Cost of Labour Supply Tax Distortions

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Abstract

We compare the welfare costs of tax distortions of labour supply in one and two member household discrete and continuous labour supply (leisure consumption) choice models. In the discrete models taxes induce a large response from a subset of the population, while the majority of the population exhibits unchanged behaviour. In contrast the majority of the population reacts to tax changes in continuous models. Models are made comparable by calibrating to the same aggregate uncompensated labour supply elasticities in discrete and continuous models. The welfare costs of similar taxes are significantly different when individuals face discrete labour supply choices than when they choose working hours continuously, and vary with tax rates in different ways. Analysis of results from these models show that discrete choice matters in the assessment of the costs of labour supply tax distortions.

Key Words: Discreteness, labour supply, welfare, taxes, equilibrium.
JEL Classification: D58, D60, H23, H31

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1. INTRODUCTION

The influence that discrete choice in labour supply decisions can have both on the estimation of labour supply elasticities and on the ability of real business cycle models to calibrate to time series on output and employment has been investigated in labour- econometric (Killingsworth (1983)) and real business cycle literature (Hansen (1987)). However, public finance literature has not analyzed how discreteness can influence estimates of the welfare costs of tax distortions of labour supply.[1] In part, this is because the significance of discrete choice in labour supply for welfare cost estimates is an issue that public finance theorists would not be drawn to, since the issues are quantitative rather than qualitative. But the intuition is that it clearly matters, since in a heterogeneous world, tax induced adjustments will occur discretely for a small number of agents, rather than continuously for all.

Here we use numerical simulation techniques to explore the significance of discrete choice for calculations of the welfare cost of tax distortions of labour supply by constructing observationally equivalent discrete and continuous choice labour supply models.[2] The discrete analogue models we use embody varying forms of agent heterogeneity (over share parameters in preferences, substitution elasticities, endowments), while maintaining equivalence to identical single or multiple agent continuous models through similar model generated aggregate uncompensated labour supply elasticities. Issues arise as to how discrete choice is modelled in the with tax case, and we present two alternative formulations. In one, households have proportional claims on revenues generated by the tax, and redistribution occurs. In the other, tax revenues are returned to those who pay the tax; no inter-agent redistribution occurs. Each discrete choice analogue asymptotically approaches a comparable continuous case as the grid over which discrete choice occurs becomes everywhere dense. We compare welfare costs of tax distortions across discrete and continuous formulations both where individuals have the extreme choice of working 40 hours a week or remaining out of work, as well as intermediate cases (35 or 15 hours of week). In the continuous models, labour supply responds continuously to wage rate and tax changes as is conventional in

[1] There seems to be no discussion of this in existing surveys of taxation and labour supply, including Blundell (1992), and the well known earlier pieces on taxes and labour supply by Rosen (1980), Hausman (1984), and MaCurdy, Green, and Paarsch (1990).

[2] See the discussion of the related but different issue of non linear budget constraints and the welfare cost of taxesin Preston and Walker (1992).

the literature.

Results from these exercises are striking. The welfare costs of similar tax distortions are sharply different in discrete and continuous models. For low tax rates, welfare costs are systematically higher in discrete models but for high tax rates the reverse applies. Distortionary costs are convex in the tax rates in continuous models but closer to the linear in discrete models. These findings occur where we use a formulation where redistribution occurs across individuals reflecting equal revenue shares, or where tax revenues are recycled to individuals who pay the taxes. This finding also occurs whether we compare the impacts of taxes under discreteness in labour supply to single representative-agent or heterogenous-agent continuous household labour supply models. Results are robust across a range of labour supply elasticities and elasticities of substitution in consumption.

As an extension to our basic analysis, we also consider a household optimisation model where there is discrete choice for one household member (the secondary worker) and continuity of choice for the other (the primary worker). Welfare costs of tax distortions in two member households cases compare in similar ways across discrete and continuous models for single member households. We conclude that discreteness in modelling labour supply behaviour matters not only for labour supply estimation, but also for calculation of the welfare costs of labour supply tax distortions.

2. DISCRETE AND CONTINUOUS MODELS OF THE WELFARE COST OF LABOUR SUPPLY TAX DISTORTIONS

As we note in our introduction above, the significance of discreteness (and non-linear budget constraints) for the estimation of labour supply elasticities is widely acknowledged in the literature (see Killingsworth (1983) and Hausman (1984)). However, issues raised by discreteness in labour supply are little discussed when it comes to calculations of the welfare costs of taxes. The closest to our discussion here that we are able to find is by Blinder and Rosen (1985) who analyze the consequences of discontinuities in welfare and related benefit programs. They do not, however, use the analytic formulations of the welfare costs of taxes that we suggest here, nor offer the observation that discreteness can substantially affect welfare cost estimates.

2.1. Continuous Labour Supply Models

The conventional representative single agent continuous model of labour supply behaviour in the presence of a labour income tax is our starting point. This is characterized by solutions to the optimization problem

$$\max \quad U(C, L) \tag{1}$$

$$s.t. \quad PC = w\left(1 - t\right)\left(\overline{L} - L\right) + R \tag{2}$$

$$t.w.\ \left(\overline{L} - L\right) = R \tag{3}$$

where C is consumption of goods, L is leisure, $\overline{L}$ is the labour endowment, w is the wage rate, P is the price of the consumption good, t is the labour income tax rate, and R is tax revenue.

With revenues recycled in lump sum form, the effect of a labour income tax is to change the slope of the budget constraint, and hence change labour supply behaviour. However, to generate an optimal solution in the presence of taxes, using the equation representation of the model (equations (1), (2) and (3)) some additional structure is needed. This is because if (1) is maximised subject to (2) and (3), substituting (3) into (2) simply returns the solution (even in the presence of taxes) to the no tax solution. Something else is needed to generate tax distorted optimal outcome.

Adding a tax distorted first order condition

$$\frac{U_c}{U_L} = \frac{P}{w\,(1-t)} \tag{4}$$

as an additional constraint to the optimization problem (1), (2), and (3) forces the with tax solution to be different from no tax solution. Alternatively treating R in (2) as parametric, and requiring it to be consistent with its value in (3), rather than directly substituting between the equations, again forces the slope of the with tax budget constraint to be $\frac{P}{w(1-t)}$, and optimizing behavior in the presence of taxes is different from the no tax equilibrium.

2.2. Discrete choice labour supply models

Discrete choice labour supply models with taxes are more difficult to formulate than for the continuous case above, because with discontinuity in the choice set a first order condition similar to (4) no longer holds. We use two alternative discrete versions of a continuous with tax model which differ in their treatment of tax revenues. Each of these reverts to the analogous continuous case as the discreteness becomes small. In one, tax revenues are distributed between households in fixed proportions, which can result in significant redistribution between households in the event of a tax change, if individuals respond by switching between discrete labour supply values. In the other, taxes are returned in lump sum form to those who pay taxes (the traditional public finance treatment). As will be seen below, this formulation is not wholly satisfactory as it involves choices among infeasible off budget points and so we treat it as our least preferred version.

In both of our discrete labour supply models, we assume an economy with N heterogenous individuals with each of them endowed with time, facing a market wage and goods prices, and maximising utility from consumption and leisure subject to their own budget constraint. Under discrete choice, each has the option of either working long hours (eight hours a day or 40 hours a week) or remaining out of work (negligible hours). Whether an individual chooses to work depends upon their preferences over consumption and leisure. These differ across individuals, and so optimisation will yield $n_1$individuals out of N as choosing the high labour supply regime, and $n_2\,(N - n_1)$ the lower labour supply regime. Each household compares utility across these two regimes and chooses that regime that maximises its utility. Taxes on consumption and labour supply have significant impact on choices for certain individuals. Conventional first order conditions do not guide

optimal choices in discrete models. However, numerical analysis can be used to find the best option for each individuals in the model.

We use two simple models that take account of the impact of discreteness in labour supply behaviour when analysing the impacts of taxes. In the first revenues collected from taxes are redistributed among all individuals in fixed proportions. When tax rates rise some individuals switch labour supply regimes. As they move from a high labour supply regime to a low labour supply regime revenue falls and lower revenues reduce transfers received by individuals. Thus higher tax rates affect the income of all individuals irrespective of whether they change their labour supply regimes. In the second formulation taxes are returned to individuals who pay them in lump sum from with no inter-individual redistribution of revenue. There is still a distortion of relative prices of consumption and leisure, however, and this has impact on choices of labour supply (consumption) and leisure.

The evaluation of high and low labour supply regimes by any household h involves a comparison of utility in the two regimes. In the CES case,

$$UH_h = \left[\alpha_h^{\frac{1}{\sigma_h}} CH_h^{\frac{\sigma_h - 1}{\sigma_h}} + (1-\alpha_h)^{\frac{1}{\sigma_h}} LL_h^{\frac{\sigma_h - 1}{\sigma_h}} \right]^{\#\frac{\sigma_h}{\sigma_h - 1}} \tag{5}$$

$$UL_h = \left[\alpha_h^{\frac{1}{\sigma_h}} CL_h^{\frac{\sigma_h - 1}{\sigma_h}} + (1-\alpha_h)^{\frac{1}{\sigma_h}} LH_h^{\frac{\sigma_h - 1}{\sigma_h}} \right]^{\#\frac{\sigma_h}{\sigma_h - 1}} \tag{6}$$

where UH_h is utility in the high labour supply regime, with CH_h and LL_h as its optimal choice of (high level) consumption and (low level) leisure; UL_h is utility in the low labour supply regime with CL_h and LH_h as it (low level) consumption and (high level) leisure.

In both equations α_h is the CES share parameter on consumption, and σ_h is the elasticity of substitution between consumption and leisure. The higher the value of the consumption share parameter (α_h), the more likely it is an individual will choose the high labour supply regime, and the higher the value of σ_h on average across the whole population the greater the number of people who switch regimes when taxes change. Whether an individual household prefers to remain in a high labour supply (high consumption or low leisure) regime or in a low labour supply (low consumption or high leisure) regime when tax rates change reflects the outcome of utility comparisons.

Thus, if

$$\begin{aligned} UH_h &> UL_h \Longrightarrow \text{household chooses the high labour supply regime} \\ UH_h &< UL_h \Longrightarrow \text{household chooses the low labour supply regime} \end{aligned} \tag{7}$$

In the no tax case consumers spend their full income (the value of endowments) on consumption goods and buy back leisure. The implies that in the discrete case each individual considers two budget points, one for the high labour supply regime and one for the low labour supply regime given by:

$$P.CH_h + w.LL_h = w.W_h \tag{8}$$

$$P.CL_h + w.LH_h = w.W_h \tag{9}$$

where P is prices of consumption goods, w is the wage rates and W_h is the total time endowment which, for simplicity, we later consider to be 70 hours per week.

Output Y in this model is given by a simple linear constant marginal product technology, which by choice of units we can represent as the labour supplied by individuals who are in the two labour supply regimes, i.e..

$$Y = \sum_{h=1}^{n} LH_h + \sum_{h=1}^{n} LL_h \tag{10}$$

Economy wide market clearing implies that total consumption for the economy is equal to total output, which by choice of units also equals the total labour supplied by all individuals. Individual consumption depends on regime choice and economy wide aggregate consumption includes consumption of all individuals, both those who are in higher and lower labour supply regimes. In the no-tax economy consumption of indviduals is given by their labour income.

If we assume that both the price of consumption and the wage rate are unity, then consumption is given by:

$$C = \sum_{h=1}^{n} LH_h + \sum_{h=1}^{n} LL_h \tag{11}$$

The amount of leisure consumed by a given individual is the time endowment less the hours of labour supplied in the high or low labour supply regime.

The imposition of an income tax will modify the budget points and hence the consumption and leisure points in the tax distorted economy are different than those in the no tax equilibrium. These can be represented as:

$$UHT_h = \left[\alpha_h^{\frac{1}{\sigma_h}} CHT_h^{\frac{\sigma_h - 1}{\sigma_h}} + (1-\alpha_h)^{\frac{1}{\sigma_h}} LLT_h^{\frac{\sigma_h - 1}{\sigma_h}} \right]^{\# \frac{\sigma_h}{\sigma_h - 1}} \tag{12}$$

$$ULT_h = \left[\alpha_h^{\frac{1}{\sigma_h}} CLT_h^{\frac{\sigma_h - 1}{\sigma_h}} + (1-\alpha_h)^{\frac{1}{\sigma_h}} LHT_h^{\frac{\sigma_h - 1}{\sigma_h}} \right]^{\# \frac{\sigma_h}{\sigma_h - 1}} \tag{13}$$

Where UHT_h and ULT_h are utilities in high and low labour supply regime in the tax distorted economy, with CHT_h and LLT_h and CLT_h and LHT_h being the corresponding choices for consumption and leisure.

Again the choice of labour supply regimes by an individual depends upon the evaluation of utilities in the presence of taxes, i.e.:

$$\begin{aligned} UHT_h &> ULT_h \Longrightarrow \text{high labour supply regime} \\ UHT_h &< ULT_h \Longrightarrow \text{low labour supply regime} \end{aligned} \tag{14}$$

If t is the marginal income tax rate, the tax distorted budget points for the high and low labour supply regimes take the form:

$$P.CHT_h + w\left(1-t\right).LLT_h = w\left(1-t\right).W_h + \frac{RV}{N} \tag{15}$$

$$P.CLT_h + w\left(1-t\right).LHT_h = w\left(1-t\right).W_h + \frac{RV}{N} \tag{16}$$

Revenue collections, RV, depend on tax rates on labour income of the n_1 and n_2 individuals in high and low labour supply regime, i.e.

$$RV = \sum_{h=1}^{n_1} t.w.LH_h + \sum_{h=1}^{n_2} t.w.LL_h \tag{17}$$

In the discrete choice model an increase in the tax rate affects labour supply only for a limited subset of individuals who switch from their high to low labour supply regime. The majority of individuals still work the same hours as before the imposition of taxes.

Our second formulation of the discrete model for space reasons we only sketch. It retains the same high and low labour supply points as above (40 or 0 in extreme case and 35 and 20 in intermediate case). It also retains the preferences, technology, endowments and the distribution of individuals across share parameters and elasticity of substitution as above. The major difference is that tax revenues are returned back to the same individual who pays the tax with no inter-individual redistribution. Welfare impacts in this model thus reflect a pure tax distortion with no redistribution of revenues as in the first model and in this sense is closer to conventional public finance treatment. However, it involves comparing off budget points in the discrete wise analysis, and in this sense is less satisfactory.

2.3. Change in labour supply and the welfare costs of tax distortions in discrete models

Tax induced changes in labour supply occur in these discrete models when the utility from fewer hours' of work in the presence of taxes is higher than utility of working more hours, $UHT_h < ULT_h$. Taxes change behaviour in only a small section of working population in both discrete model formulation, but almost all individuals in continuous models.

We calculate a switching index for each individual as:

$$S_h = \begin{matrix} 1 \; if \; UHT_h < ULT_h \\ 0 \qquad\qquad otherwise \end{matrix} \tag{18}$$

The total number of households who change their behaviour as a result of a tax change is then given as:

$$T = \sum_{h=1}^{N} S^h \tag{19}$$

Changes in total labour supply are obtained by adding up the difference between high and low labour supply hours of individuals who switch, i.e.:

$$\Delta LS_h = \sum_{h=1}^{N} S_h \left(LH_h - LL_h \right) \tag{20}$$

The element in the parenthesis is positive for individuals who switch, and zero for those who remain in the same labour supply regime.

2.4. Comparable Continuous labour supply model

We construct two different continuous labour supply choice models to evaluate welfare costs of tax distortions and compare to cost estimates from comparable discrete models. The first has a representative consumer, whose aggregate labour endowment is the same as the total endowment of all individuals in the discrete models. The second has N heterogeneous individuals as in discrete models. Using these two models enables us to also evaluate the role of agent heterogeneity in discrete/continuous model comparisons. These discrete and continuous models are made comparable to each other by fixing endowments and model parameters so as to generate the same uncompensated aggregate labour supply elasticities across all models.

As in the discrete case, we assume CES preferences for the representative consumer in continuous models, i.e.

$$U = \left[\delta C^{\frac{\sigma-1}{\sigma}} + (1-\delta) L^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{21}$$

These we maximize, subject to the with tax budget constraint

$$PC = w(1-t)\left(\overline{L} - L\right) + R \tag{22}$$

where revenue collected by the government is $R = t\, w \left(\overline{L} - L\right)$.

The first order condition for utility maximisation is

$$\frac{L}{C} = \left[\frac{(1-\delta) P}{\delta\, w (1-t)} \right]^{\sigma} \tag{23}$$

We again assume a linear technology with constant marginal product of labour

$$Y = \overline{L} - L \tag{24}$$

Aggregate labour supply is given by

$$LS = \overline{L} - L \tag{25}$$

For the heterogenous agent model, we use subscripts h for individual variables and sum over h to obtain aggregate variables for the N heterogeneous individuals.

$$U_h = \left[\delta_h C_h^{\frac{\sigma_h - 1}{\sigma_h}} + (1-\delta_h) L_h^{\frac{\sigma_h - 1}{\sigma_h}} \right]^{\frac{\sigma_h}{\sigma_h - 1}} \tag{26}$$

$$R = \sum_h t\, w \left(\overline{L}_h - L_h\right) \tag{27}$$

and

$$Y = \sum_h \left(\overline{L}_h - L_h\right) \tag{28}$$

for the revenue and output functions respectively.

We numerically calculate arc (uncompensated) labour supply elasticities for both discrete and continuous models using a small perturbation in the wage rate

around base model values. This gives the model labour supply elasticity (evaluated at the base (or initial) household labour supply values) as

$$e = \frac{\Delta LS}{LS}\frac{w}{\Delta w} \tag{29}$$

We iterate on parameters to achieve aggregate model comparability of the form discussed above. This involves generating similar aggregate (uncompensated) labour supply elasticities in both discrete and continuous models, and equal aggregate labour income in the base (no tax) case. We implement this procedure for numerical examples by first specifying and solving the continuous single agent model, and then manipulating distributions of share and elasticity parameters in preferences in the second version of the continuous model and in the two different discrete models until we get identical uncompensated labour supply elasticities across all models.

We use Hicksian equivalent (EV_h) and compensating variations (CV_h) to measure the welfare costs of tax distortions in money metric utility terms. These welfare cost estimates reflect variations in utility that occur because of changes in quantities of goods consumed, and/or hours of leisure taken by individuals. In the case of linear homogenous utility functions, these equivalent and compensating variations can be represented as:

$$EV_h = \left(\frac{U_h^B - U_h^A}{U_h^A}\right) I_h^N \text{ and } CV_h = \left(\frac{U_h^A - U_h^B}{U_h^B}\right) I_h^o \tag{30}$$

where EV_h and CV_h are the equivalent and compensating variations respectively for household h, U_h^A is the utility in the counterfactual (A for after tax changes) and U_h^B is the utility before the tax change. I_h^o and I_h^N are the old and new incomes of individual h.

Economy wide welfare costs reflect variations in aggregate money metric utility measures. AEV_h and ACV_h are the summed (economywide) welfare cost measures as a fraction (or percentage) of economy wide income, given by the expressions

$$AEV_h = \frac{\sum_{h=1}^{H} EV_h}{\sum_{h=1}^{H} I_h^o} \text{ and } ACV_h = \frac{\sum_{h=1}^{H} CV_h}{\sum_{h=1}^{H} I_h^N} \tag{31}$$

These aggregate welfare measures provide the basis for model comparisons of welfare cost estimates for different labour income tax rates.

3. NUMERICAL IMPLEMENTATION

To analyze the relative performance of discrete and continuous models capturing tax distortions of labour supply we use numerical simulation. The numerical specifications used for both discrete and continuous models are displayed in Table 1. The continuous model 1 has a single representative household, while continuous model 2 and discrete models have N individuals. In both continuous models agents choose hours of work in a continuous fashion, while they have limited choices in discrete case. Tax revenues are redistributed equally among household on a fixed share basis in the discrete model 1, our lead formulation for the discrete case. Discrete model 2 represents a case where revenues are returned to individuals who pay taxes and no redistribution occurs. Individuals differ in preference parameters in continuous model 2 and both versions of the discrete models. Each individual has endowments equal to 70 hours per week. The single representative consumer in the continuous model 1 has a comparable aggregate labour endowment of 7000.

We use specifications of the above models to compute welfare costs of labour income tax distortions given by Hicksian equivalent and compensating variations in money metric utility terms. Two values of work hours, 40 and 1[3] hours per week, are assumed for high and low values of labour supply in both discrete models. In the base case, with a linear technology the price of consumption goods and the wage rates are assumed to be unity. Models are, however, made comparable through calibration of uncompensated labour supply elasticities to similar values (reported in Table 1).

[3]For numerical reasons we use 1 as the discrete approximation of 0 for low labour supply in our discrete model computations.

Table 1
Specifications Used in No Tax Base Case for Numerical Models
Exploring the Welfare Cost of Taxes in Comparable
Discrete and Continuous Models

	Discrete Model 1[1]	**Discrete Model 2**[2]	**Continuous Model 1: Single representative agent**	**Continuous Model 2: With heterogeneity**
Treatment of tax revenues	Redistributed equally per capita	Returned to those who pay the tax	Returned to single consumer	
Number of households	100	100	1	100
Labour endowment in hours per week	7000 in aggregate, 70 per household	7000 in aggregate, 70 per household	7000 in aggregate, 140 per household	7000 in aggregate, 70 per household
Discrete choice (1 is the numerical approximation used for zero consumption)	High and low discrete leisure consumption values for each household are 40 and 1;	High and low discrete leisure consumption values for each household are 40 and 1;	Not applicable	Not applicable
Share parameters on leisure in utility function, δ	Uniform distribution across households over the range 0.01 to 0.99	Uniform distribution across households over the range 0.01 to 0.99	0.5	Uniform distribution across households over the range 0.01 to 0.91
Net of tax price of consumption goods, P	1.0	1.0	1.0	1
Gross of tax wage, W	1.0	1.0	1.0	1
Point estimates of aggregate uncompensated labour supply elasticity[3] (evaluated at the no tax equilibrium)	0.305	0.3	0.302	0.33
Elasticity of substitution in consumption, σ (adjusted across models for equivalence)	1.5	05.25	0.67	0.67

[1] In this model tax revenues are redistributed among households using a fixed proportions distribution scheme.
[2] In this model tax revenues are returned to those who pay the tax in lump sum form.
[3] These are obtained by repeated iteration on model parameters, and hence the values are only approximately equal across the models rather than exactly so.

Table 2 reports welfare cost estimates from tax distortions set at different rates for both versions of the continuous and discrete models with the specifications as set out in Table 1. Several interesting results emerge.

The first is that for low tax rates, except for the 5% tax case discussed in footnote 3 to the table, welfare cost estimates for discrete models exceed those of continuous models, while the opposite is true for high tax rates. This is accounted for by the observation that, following Harberger intuition, welfare cost estimates are convex in tax rates in the continuous case; while cost estimtes are closer to linear in the discrete case. The second is that comparing across continuous models 1 and 2, agent heterogeneity has relatively little impact on welfare cost estimates. This indicates that differences in results between discrete model 1 and continuous model 1 (single agent) reflect discreteness more to them agent heterogeneity. Third, comparing across results for discrete model 1 and discrete model 2, welfare costs are higher when taxes have redistribution roles than when they do not as in discrete model 2.

In discrete model 1 by moving from a high to low labour supply regime in response to taxes an individual causes a reduction in aggregate tax collections imposing a loss on all other households because of the per capita distribution of revenues. Even if the individuals who moved were close to indifferent in terms of their utility comparisons across states, they would still impose burdens on all other non-movers approximately equal to the taxes they would otherwise have paid if they had remained in the high labour supply state. This fiscal externality occurs because all individuals receive smaller transfers when some individuals pay less to the government while choosing to work only a few hours. Welfare costs of tax distortions rise as the discreteness narrows and discrete model approached to comparable continuous case.

The conclusion thus seems to be that there can be a significant variation in welfare costs of tax distortions between discrete and continuous models. This occurs because taxes induces a large shift in the labour supply hours for a small number of individuals in discrete case, whereas it causes a small shift of labour hours for the majority individuals in the continuous case.

Table 2
Hicksian welfare costs of tax distortions as a proportion of base case income in the discrete and continuous models specified in Table 1[1]

Tax rates	Continuous model 1		Discrete Model 1		Continuous Model 2		Discrete Model 2[3]	
	EV[2]	CV[2]	EV[2]	CV[2]	EV[2]	CV[2]	EV[2]	CV[2]
0.05	-0.0002	0.0002	-0.0005	0.0005	-0.0002	0.0003	-0.0000	0.0000
0.10	-0.0009	0.0009	-0.0014	0.0014	-0.0007	0.0014	-0.0009	0.0010
0.15	-0.0022	0.0022	-0.0029	0.0029	-0.0016	0.0034	-0.0009	0.0010
0.20	-0.0042	0.0042	-0.0049	0.0050	-0.0031	0.0064	-0.0009	0.0010
0.30	-0.0106	0.0107	-0.0107	0.0109	-0.0098	0.0181	-0.0032	0.0040
0.40	-0.0215	0.022	-0.0189	0.0196	-0.0224	0.0386	-0.0032	0.0040
0.50	-0.0392	0.0408	-0.0298	0.0316	-0.0411	0.0689	-0.0066	0.0092
0.60	-0.0671	0.072	-0.0438	0.0475	-0.0701	0.1174	-0.0110	0.0169
0.70	-0.1122	0.1264	-0.0616	0.0686	-0.111	0.1905	-0.0110	0.0169
0.80	-0.1891	0.2331	-0.0840	0.0961	-0.1752	0.3193	-0.0160	0.0268

[1]Both discrete models have 0-40 hours of discrete labour supply choices.

[2]The tax revenue is redistributed equally among households in discrete model 1 and given back to households in discrete model 2. EVs and CVs reported here are as a proportion of base case aggregate income.

[3]In this model the number of individuals affected by tax changes does not alter for some ranges of taxes, explaining the discrete adjustment of welfare cost estimates.

Table 3
Hicksian welfare costs of tax distortion with less extreme forms of discreteness of labour supply: Hicksian EV and CVs as a fraction of base case GDP

Income tax rates	EV	CV	Number people switching with 35-20 hours	Number people switching with 1-40 hours
0.05	0.0000	0.0000	1	0
0.10	-0.0002	0.0002	2	1
0.15	-0.0006	0.0006	2	1
0.20	-0.0006	0.0006	3	1
0.30	-0.0013	0.0014	5	2
0.40	-0.0033	0.0037	6	2
0.50	-0.0047	0.0052	7	3
0.60	-0.0079	0.0091	9	4
0.70	-0.0118	0.0139	11	4
0.80	-0.0163	0.0197	15	5

Welfare cost estimates of taxes as a fraction of base case income when individuals have intermediate labour supply choices of 35 and 20 hours rather than more extreme choices of 40 and 0 are given in Table 3. These results show that welfare costs are even lower when individuals have less extreme discrete choices than when choices are more extreme.

Sensitivity results on welfare costs of taxes for various labour supply elasticities are given in Table 3 for both continuous and discrete models. In these cases the tax change is the same across discrete and continuous cases, but the value of the elasticity of labour supply to which calibration takes place changes. In these cases, welfare costs increase in the elasticity with the proportional difference in cost estimates between the discrete and continuous models remaining roughly the same.

The implication of all these results taken as a set is thus to suggest that discreteness is important for assessments of the welfare costs of tax distortions in labour supply.

Table 4
Welfare Costs of 10 percent taxes under Alternative Values of Labour Supply Elasticities to which both Models are Jointly Calibrated[1]

	Welfare cost of Income taxes in continuous and discrete labour supply models			
Elasticity	Continuous 100 household model 1		Discrete Model 1	
	EV	CV	EV	CV
0.20	-0.0006	0.0006	-0.0002	0.0002
0.30	-0.0009	0.0009	-0.0007	0.0007
0.50	-0.0016	0.0016	-0.0011	0.0011
0.80	-0.0025	0.0025	-0.0016	0.0017
0.105	-0.0032	0.0032	-0.0014	0.0014
1.550	-0.0038	0.0039	-0.0018	0.0018
2.050	-0.0045	0.0045	-0.0018	0.0018

[1]This table gives the sensitivity of welfare costs for 10 percent tax rates reported in Table 2.a to various labour supply elasticities.

4. A TWO LABOUR TYPE ONE HOUSEHOLD DISCRETE CHOICE MODEL

We have deliberately kept the models used in section 3 simple so as to make our main point, namely, that discreteness matters in the measurement of the welfare costs of taxes affecting labour supply. But more elaborate and, hence more realistic, models are needed to underpin any general conclusions as to the role of discreteness in analyzing tax distortions of labour supply.

One particularly relevant issue is that in two person households with both primary and secondary workers, may be the secondary worker who faces discrete labour supply choices. Secondary workers may be more likely to be involved in choices relating either to full time job or at home production with zero labour supply. Primary workers may remain in continuous labour supply regime and can accept overtime or part-time offers according to the need of the work place. In such models, the likelihood is that discreteness will have less effect on the welfare costs of taxes, since substitution margins involving continuity of labour supply are present for the household as a whole. A household model with two labour types, in which only one is constrained by discrete choice, also raises the related issue of whether high labour supply responsiveness by unconstrained primary workers under household utility maximization over goods and both leisure types largely reflects constraints on secondary workers. Discreteness of choice for secondary workers may thus generate data which suggests high labour supply elasticities for primary workers, even though household preferences towards their leisure are no different from those of secondary workers.

We have constructed a two labour type discrete choice household model to examine some of these issues, where we again assume a uniform distribution of households across either preference parameters or substitution elasticities. In this model, each household, h, has a preference function defined over goods and leisure of the two household members (primary and secondary worker leisure consumption are L_1^h and L_2^h).

$$U^h = U^h\left(C^h, L_1^h, L_2^h\right) \tag{32}$$

We again consider two regimes, which now correspond to high and low labour supply (leisure consumption) of the secondary worker. Again household utility maximisation implies that each household chooses one out of two possible regimes. For analytical convenience, we assume that preferences are additively separable in the discrete and continuous variables. This allows us to determine demands for goods, and the type 2 (continuous) leisure by each household given the values for

the discrete labour supply of the secondary worker.

These conditional demand functions for household consumption and leisure of the secondary worker are obtained by maximizing a nested utility function defined over the continuous variables for each type of leisure and consumption at the first level (and then with discrete leisure and composite consumption-leisure at the household level) given the discrete choice for the secondary worker. The two labour types are assumed to be perfect substitutes in production. Equilibrium is then given by market clearing in the goods market and for labour type 2, given the choice of regime for discrete labour type one, and the linear technology. We focus here on results of this model suppressing algebraic details for space reasons. In the analogue continuous model, the representative household derives utility from a CES function defined over consumption of goods and leisure of household members one and two. Equilibrium in this case is given by household optimizing behaviour subject to budget and revenue constraints, and an appropriate tax distorted first order condition for primary labour.

As with the single labour type model, we parameterize comparable discrete and continuous models where comparability in this case involves the two uncompensated labour supply elasticities for primary and secondary workers rather than, as before, one single elasticity value for aggregate labour supply. We also consider a N household version as continuous model 4 in which each family member is endowed with 70 hours per week and households differ from each other in preference share parameters and elasticities of substitution between consumption and leisure.

These parameterizations are set out in Table 5. While each household member varies labour supply continuously in models 3 and 4, only the primary worker does so in discrete model 3. The secondary worker is subject to choosing discrete numbers of hours. We calibrate the discrete model to labour supply elasticities of 0.15 (approximately) for primary workers, and 0.5 (approximately) for the secondary worker to make it comparable with the two versions of continuous labour supply models. These specifications also allow us to perform analyses of welfare costs of tax distortions similar to those reported in Table 4 in the earlier section. Table 6 reports our results.

Table 5
Specifications Used in Exploring the Welfare Cost of Taxes in Two Member Household Models

Model features	Continuous Single Agent Model 3	Discrete Model Version 3	Continuous Heterogenous Agent Model 4
Number of households	1	100	100
Labour endowments in hours per week	14000	14000 in aggregate, 140 per household	14000 in aggregate, 140 per household
Leisure consumption of primary worker	Continuous variable	Constrained to high and low discrete values for each household; $L^H = 40$; $L^L = 1$	Continuous variable
Leisure consumption of secondary workers	Continuous variable	Continuous variable	Continuous variable
Share parameters, δ, in the utility function	$\delta_0 = 0.462$ $\delta_1 = 0.345$ $\delta_2 = 0.129$	Share parameters at both levels of preferences are distributed across the 100 households	Share parameters at both levels of preferences are distributed across the 100 households
Net of tax price of consumption goods, P	1.0	1.0	1.0
Gross of tax wage, W	1.0	1.0	1.0
Point estimates of aggregate uncompensated labour supply elasticities (evaluated at no tax equilibrium)	0.144[1] 0.510[1]	0.146[1] 0.517[1]	0.144[1] 0.510[1]
Elasticity of substitution in consumption, σ	$\sigma = 0.5$	$\sigma = 0.5$ σ has a distribution of values across households with mean value 1.025	$\sigma = 0.5$ σ has a distribution of values across households

[1]Numerical difficulties in calibrating the discrete two labour type model implies that elasticities across the two models are close but not identical.

Table 6
Welfare Costs of Tax Distortions of Family Labour Supply Using the Discrete and Continuous Models Specified in Table 5[1]

Tax rates	Continuous model 3 - representative agents		Continuous model 4 - discrete agents		Discrete model 3	
	EV	CV	EV	CV	EV	CV
0.05	-0.0003	0.0003	-0.0002	0.0003	-0.0003	0.0003
0.10	-0.0011	0.0011	-0.0010	0.0011	-0.0011	0.0011
0.15	-0.0026	0.0026	-0.0023	0.0026	-0.0025	0.0026
0.20	-0.0049	0.0049	-0.0044	0.0049	-0.0047	0.0049
0.30	-0.0125	0.0126	-0.0111	0.0125	-0.0120	0.0125
0.40	-0.0253	0.0259	-0.0223	0.0256	-0.0243	0.0257
0.50	-0.0457	0.0479	-0.0849	0.0983	-0.0565	0.0598
0.60	-0.0777	0.0843	-0.1176	0.1428	-0.0713	0.0764
0.70	-0.1273	0.1458	-0.1149	0.1410	-0.0604	0.0637
0.80	-0.2484	0.3305	-0.1838	0.2421	-0.0532	0.0565
0.90	-0.3285	0.4893	-0.3034	0.4669	-0.0537	0.0573

[1]Discrete models have 0-40 hours as discrete labour supply choices.

Two major points emerge from analysis of household labour supply models. First, welfare costs of tax distortions are higher in continuous family labour supply models (continuous 3 and 4) for high tax rates but comparable for lower tax rates. Results are more mixed, however, in the discrete case as the welfare cost estimates from discrete model 3 are lower than those from discrete model 1, although higher than in discrete model 2. This suggests that the redistribution impact among households predominates the intra-household labour leisure substitution effect in model 3, in which case some of the effects of taxes on labour income of one family member falls upon that of another family member. Welfare costs between discrete and continuous models differ by up to factor of five across comparable continuous and discrete family labour supply models, again reflecting the fact that the family as a whole is more capable of absorbing tax shocks in continuous rather than in the discrete models.

5. CONCLUSION

This paper discusses the influence of discrete labour supply (or leisure) choice in estimates of the welfare costs of taxes on labour supply, issues which appear to be little discussed in the literature. We construct comparable discrete and continuous models of labour supply calibrated to yield similar point estimates of uncompensated elasticities of aggregate labour supply around a base case equilibrium in the presence of taxes. We discuss four different versions of continuous labour supply models in which individuals or families can choose their labour supply continuously and three different versions of discrete models in which choices are restricted to more extreme and intermediate work hours per week.

Tax distortions in discrete models affect only a subset of individuals or families, who experience large adjustments in their labour supply while the majority still retain their original work hours. This yields only small change for the majority of individuals or families in continuous models. We find that welfare costs of tax distortions differ between continuous labour supply choice models and comparable discrete choice labour supply models. Discreteness matters more than heterogeneity or the redistribution impacts of taxes in these welfare cost calculation. We thus argue that discreteness perhaps needs to be taken account more than in current literature when evaluating the welfare costs of tax distortions on labour supply, as well as other tax effects.

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