

Davies, James B.; Zhang, Jie; Zeng, Jinli

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# Optimal tax mix in a two-sector growth model with transitional dynamics

James B. Davies  
University of Western Ontario

Jie Zhang  
Victoria University of Wellington

Jinli Zeng  
National University of Singapore

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## Abstract

This paper examines the problem of optimal tax mix analytically in a two-sector growth model with transitional dynamics. Tax revenue is required to provide a pure public good. The key problems are: over-consumption of leisure under labor income or consumption taxes; and under-investments in human and physical capital under income taxes. Without investment subsidies, consumption taxes do better than uniform income taxes, but can be improved on locally via positive taxation of physical capital income and a negative tax on labor income. With subsidies the first best can be achieved in a system where: (i) consumption and labor income taxes are either zero or of the same rate but opposite signs; (ii) physical capital income taxes are used either exclusively or more heavily than labor income taxes when their rates are below 100%; and (iii) investment subsidy rates equal income tax rates for both forms of capital, respectively. In any given circumstances, a range of alternative tax mixes may provide equivalent results. This result, combined with practical constraints, may help to explain the variety of tax mixes observed across countries.

*JEL classification:* E60; H20; O40

*Keywords:* Growth; Transitional dynamics; Optimal taxation; Subsidies

Correspondence:

James Davies  
Department of Economics  
Faculty of Social Science  
The University of Western Ontario  
London, Ontario  
CANADA, N6A 5C2

# 1. Introduction

Recent literature has incorporated endogenous human capital in perfect foresight dynamic optimal tax analyses. (See e.g. Bull, 1993, and Jones et al., 1993, 97.) This literature has extended the zero long-run capital income tax results of Judd (1985) and Chamley (1986), obtained with only physical capital, to show that in the steady state optimal tax rates on human as well as physical capital income, and on consumption, are zero (e.g., Jones et al., 1993, 97). The optimal program calls for taxes to be high in the short-run, with a large surplus being built up that can be used to finance government expenditure in the long-run as tax rates go to zero.

While the insights of the dynamic optimal tax literature have been path-breaking, tax rates falling toward zero in the long-run are in sharp contrast to what we observe. In the real world tax systems show more stability over time. Recent papers in the optimal tax literature have provided a variety of reasons why taxes should not disappear in the long-run.<sup>1</sup> Attention is thus turning to the optimal design of more stable tax systems. In order to establish a benchmark in this area we think it is useful to ask what the optimal design of taxes would be if governments were limited to the choice of stationary tax rates. That is the task we set ourselves in this paper.

While it is arbitrary to limit the choice of tax regimes to stationary ones, it is important to note that *all* of the dynamic optimal tax discussion relies on arbitrary assumptions. The literature assumes that governments can commit to a future trajectory of taxes, sidestepping the time inconsistency problem; that lump sum taxes (e.g. an initial capital levy) are not available; and that tax rates lie within certain bounds. In this context, investigating the implications of constant tax rates is a natural additional path to explore.

There is a significant amount of previous literature that, rather than solving the optimal constant tax structure problem, has asked computationally which of certain alternative stationary tax regimes is superior for welfare or growth. (See e.g. Summers, 1981; Auerbach, and Kotlikoff, 1987;

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<sup>1</sup>Jones et al. (1993, 97) identify upper limits on tax rates, revenue constraints, pure profits arising from productive government spending, or inclusion of capital in the social planner's (not households') preferences as reasons for taxes to persist in the long run. Aiyagari (1995) shows that incomplete markets and borrowing constraints can produce the same result. Judd (1997), and Guo and Lansing (1999) find that imperfect competition may even motivate negative capital income taxes that persist in the long run.

and Pecorino, 1993, 94.) The answer is that consumption taxes usually dominate either wage taxes or uniform income taxes. In this paper we ask whether, when the range of choice is widened, it will still be best to rely wholly on a consumption tax, or whether a *mix* of taxes would be better. This question has empirical relevance since most countries depend on a rich mix of taxes rather than on a narrow revenue source.<sup>2</sup>

This paper investigates the optimal stationary tax mix in an endogenous growth model with both physical and human capital and endogenous leisure. In this model, the government may employ a full range of alternative tax instruments to finance public consumption, and it may also subsidize investments.<sup>3</sup> A novel feature of this paper is that we are able to derive, analytically, optimal tax schemes that apply not only in the steady state but also in transition. This transparency results from simple assumptions on preferences and technology. We assume log utility, a Cobb-Douglas production function, and full depreciation of all capital in one period. While these assumptions clearly limit the generality of our results, they allow a full characterization of the dynamic path of the two-sector growth model and an explicit representation of the welfare function. They also allow us to handle readily investment subsidies for both physical and human capital. The results provide a number of insights that would not emerge so strongly from less simple models.

In agreement with previous work, in a world *without* human or physical investment subsidies, our model shows that a pure consumption tax is superior to a uniform income tax, both in welfare terms and also in terms of growth rates. However, if the uniformity of income taxes is relaxed, one can do better. Welfare and growth can be improved if, starting from a pure consumption tax, one levies a tax on capital income and a subsidy on labor income. This might seem to be rather academic, since in the real world we observe substantial positive tax rates on labor. However, we show that if investment subsidies are introduced a system that dominates a consumption tax,

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<sup>2</sup>This fact is reflected in our Table 4. See also Mendoza et al. (1994, Tables 1-3), or Krusell et al. (1996, Table 1).

<sup>3</sup>In the standard Ramsey framework, tax revenues are used for lump-sum transfers to individuals. In practice, however, a large proportion of tax revenue is used to provide public goods and services in many countries. According to the Barro-Wolf data set in Barro (1991), most of the OECD countries spend more government revenue on the provision of goods and services than transfers net of social security. Due to this fact, we focus on the extreme case where there are no lump-sum transfers in the main analysis but we will discuss in Section 3 (and Appendix J) whether our main results hold in the case where the tax revenue, net of subsidies, is used for lump-sum transfers only.

without unrealistic tax rates, can be designed.

In a world with exogenous human capital it is well known that a pure consumption tax is equivalent to a Hall-Rabushka tax, that is a tax on labor and business income that allows immediate expensing of capital. When human capital is endogenous, the equivalence continues to hold as long as full costs of human capital investment are also immediately deductible. We refer to such a scheme, which subsidizes investment at the same rate it taxes income, a “modified Hall-Rabushka tax”. Since it is possible to improve on a pure consumption tax it is of course also possible to improve on the modified Hall-Rabushka scheme. This is done by increasing the tax rate on capital and reducing it on labor income, but keeping the investment subsidy rates the same. In the resulting system physical capital is taxed more heavily than labor; costs of physical investment are not fully deductible against capital income taxes; and human capital investment is subsidized at a higher rate than labor income is taxed. If we keep in mind the substantial subsidies to human capital investment that are delivered outside tax systems, this package is remarkably similar to what is observed in the United States and many other countries.

When investment subsidies are allowed it is not only possible to improve on consumption taxes, but the first-best can also be achieved. There is a continuum of first-best schemes, with the following characteristics: (i) if consumption and labor income taxes are non-zero they are of the same rate but opposite signs, (ii) the tax rate on physical capital income exceeds that on labor income when both are below 100%, (iii) subsidy rates on investments equal income tax rates, for both forms of capital. In such schemes the effects of consumption and labor income taxes on labor-leisure choice cancel out, and intertemporal distortions are avoided by subsidizing the costs of investment at the same rate at which benefits are taxed (as is also the case in the modified Hall-Rabushka scheme).

While the real world certainly exhibits subsidies on investments, and cases where effective tax rates on particular forms of consumption or income are negative, generally tax rates are positive. We interpret this as resulting from forces outside the model, such as the need to combat evasion and avoidance. Thus, while the first-best results are instructive they are not a practical guide to policy. We argue that when real-world constraints on tax and subsidy rates are taken into account

the best system that can be achieved may be one that starts from a modified Hall-Rabushka scheme and adjusts capital and labor income tax rates upward and downward respectively, as described above.

The remainder of this paper is organized as follows. Section 2 introduces the model and derives the main results. The rankings of different tax schemes in terms of growth and welfare are given in Section 3. Application of the results to the interpretation of real-world tax systems is discussed in Section 4. Section 5 gives some concluding remarks. Unless provided in the text, proofs are relegated to the appendices.

## 2. The basic model and results

Our model has an infinite number of periods ( $t = 0, 1, 2, \dots$ ) and a constant population with measure one. Agents are identical and infinitely lived. Each agent is endowed with one unit of time per period, which is allocated among leisure  $Z_t$ , production  $L_t$ , and education  $S_t$ . There are a publicly provided consumption good, and a private good that can be consumed, invested in human capital through education, or used to form new physical capital through a one-for-one conversion.

The representative agent's preferences are defined over private consumption  $C_t$ , leisure  $Z_t$ , and public consumption  $G_t$  as

$$U_0 = \sum_{t=0}^{\infty} \rho^t (\ln C_t + i \ln Z_t + \beta \ln G_t), \quad \beta > 0, \quad 0 < \rho < 1, \quad i = 0, 1, \quad (1)$$

where  $\beta$  measures the taste for public consumption,  $\rho$  is the subjective discount factor, and  $i$  indicates whether leisure is elastic. If  $i = 0$  leisure is inelastic while if  $i > 0$  leisure is elastic; both treatments of leisure are seen in the literature on optimal taxation. For notational simplicity, positive values of  $i$  are normalized to unity.<sup>4</sup> We view elastic leisure as a general case so that when we do not specify otherwise leisure is elastic.

The private good is produced according to:

$$Y_t = AK_t^\epsilon (H_t L_t)^{1-\epsilon}, \quad A > 0, \quad 0 < \epsilon < 1, \quad (2)$$

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<sup>4</sup>The essence of the results is unaffected if  $i$  takes other positive values.

where  $Y_t$  is output,  $A$  a productivity parameter,  $K_t$  physical capital,  $L_t$  labor,  $H_t$  human capital (skill), and  $\epsilon$  the share parameter that measures the importance of physical capital relative to effective labor.

Human capital or skills accumulate through education:

$$H_{t+1} = A_H Q_t^\alpha (S_t H_t)^{1-\alpha} + (1 - \delta_H) H_t, \quad A_H > 0, \quad 0 < \alpha < 1, \quad 0 \leq \delta_H \leq 1, \quad (3)$$

where  $Q_t$  is the private investment of goods in education,  $S_t$  the time input in learning,  $A_H$  a productivity parameter,  $\delta_H$  the rate of human capital depreciation, and  $\alpha$  the share parameter that measures the importance of physical inputs relative to the effective units of time inputs. This two-sector growth model with production and education is similar to that in Lucas (1988).

## 2.1. Competitive equilibrium

We assume that government expenditures on public consumption  $G_t$  are funded by flat-rate taxes. The tax instruments we consider include a consumption tax (at a rate  $\tau_{ct}$ ), a labor (human capital) income tax ( $\tau_{lt}$ ), a physical capital income tax ( $\tau_{kt}$ ), an education subsidy ( $s_{qt}$ ), and a physical capital investment subsidy or investment tax credit ( $s_{kt}$ ). An individual's budget and time constraints are given by:

$$(1 + \tau_{ct})C_t = (1 - \tau_{lt})w_t H_t L_t + (1 - \tau_{kt})r_t K_t - (1 - s_{qt})Q_t - (1 - s_{kt})[K_{t+1} - (1 - \delta_K)K_t], \quad (4)$$

$$Z_t = 1 - L_t - S_t, \quad (5)$$

where  $w_t$  and  $r_t$  are, respectively, the wage rate and physical capital rental rate, and  $\delta_K \in [0, 1]$  is the rate of physical capital depreciation. Perfect competition implies  $w_t = (1 - \epsilon)Y_t/(H_t L_t)$  and  $r_t = \epsilon Y_t/K_t$ .

It is well known that full depreciation of capital in one period and the use of Cobb-Douglas functions for technologies and preferences allow an analytical solution for individuals' choices in such a two-sector growth model.<sup>5</sup> In order to investigate optimal taxation analytically, we assume

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<sup>5</sup>See, e.g., King, Plosser, and Rebelo (1988) and Devereux and Love (1994). These assumptions are also used in a recent paper by Devarajan, Xie, and Zou (1998) to analyze public capital in the steady state equilibrium.

full depreciation of capital in one period, that is  $\delta_H = \delta_K = 1$ .<sup>6</sup> We will further exploit the advantage of these assumptions to derive a welfare function that applies in the initial and all future periods given the initial states by considering the adjustment of the two types of capital in transition. Thus, the two-sector growth model here will be able to generate optimal taxation analytically for both the short run and the long run. In addition to being of interest for their own sake, the qualitative results should help to guide the direction of future simulations using models of tax mix with partial depreciation of capital and more complex functional forms.

The government budget constraint is given by

$$G_t = \tau_{ct}\bar{C}_t + \tau_{lt}w_t\bar{H}_t\bar{L}_t + \tau_{kt}r_t\bar{K}_t - s_{qt}\bar{Q}_t - s_{kt}\bar{K}_{t+1}, \quad (6)$$

where the upper bar on a variable refers to the variable's aggregate or average value with population of unit mass. Eq. (6) means that the net tax revenue for public consumption equals taxes from private consumption  $\bar{C}_t$ , labor income  $w_t\bar{H}_t\bar{L}_t$ , and capital income  $r_t\bar{K}_t$ , minus subsidies on investments in human and physical capital,  $s_{qt}\bar{Q}_t$  and  $s_{kt}\bar{K}_{t+1}$ . With identical agents,  $\bar{C}_t = C_t$ ,  $\bar{H}_t\bar{L}_t = H_tL_t$ ,  $\bar{K}_t = K_t$ , and  $\bar{Q}_t = Q_t$  in equilibrium.

Define  $\Theta_t = (\tau_{ct}, \tau_{lt}, \tau_{kt}, s_{qt}, s_{kt}, G_t, \bar{H}_t, \bar{K}_t, w_t, r_t)$ , a vector that includes the policy variables, the average capital stocks, the wage rate, and the interest rate. Given  $\Theta$  and initial capital stocks  $H_0$  and  $K_0$ , the representative agent chooses  $\{C_t, S_t, L_t, H_{t+1}, K_{t+1}\}_{t=0}^{\infty}$  to maximize (1) subject to the education technology (3) and the individual's budget and time constraints (4) and (5). Observe that average human and physical capital,  $\bar{H}_t$  and  $\bar{K}_t$ , affect individuals' welfare via  $G_t$  by (1) and (6). The Lagrangian for this problem is:

$$\mathcal{L} = \sum_{t=0}^{\infty} \rho^t \left\{ \ln \frac{(1 - \tau_{lt})w_t H_t L_t + (1 - \tau_{kt})r_t K_t - (1 - s_{qt})Q_t - (1 - s_{kt})K_{t+1}}{1 + \tau_{ct}} \right. \\ \left. + \ln(1 - L_t - S_t) + \beta \ln G_t - \eta_t [H_{t+1} - A_H Q_t^\alpha (S_t H_t)^{1-\alpha}] \right\}, \quad (7)$$

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<sup>6</sup>This model can also be viewed as an overlapping generations model with two-period lived altruistic agents. Corresponding to this alternative interpretation of the model, agents are born with zero human capital and parents invest in children's education; physical capital investment is bequests to children. The length of a period is then about 30 years. In this case, full depreciation of capital per period is natural for human capital and is quite realistic for physical capital.



where  $\eta_t$  is the Lagrange multiplier. The first-order conditions for (7) are

$$L_t : \frac{1}{1 - L_t - S_t} = \frac{(1 - \tau_{lt})w_t H_t}{(1 + \tau_{ct})C_t}, \quad (8)$$

$$Q_t : \frac{1 - s_{qt}}{(1 + \tau_{ct})C_t} = \eta_t \frac{\alpha H_{t+1}}{Q_t}, \quad (9)$$

$$S_t : \frac{1}{1 - L_t - S_t} = \eta_t \frac{(1 - \alpha)H_{t+1}}{S_t}, \quad (10)$$

$$H_{t+1} : \eta_t = \frac{\rho(1 - \tau_{lt+1})w_{t+1}L_{t+1}}{(1 + \tau_{ct+1})C_{t+1}} + \eta_{t+1} \frac{\rho(1 - \alpha)H_{t+2}}{H_{t+1}}, \quad (11)$$

$$K_{t+1} : \frac{1 - s_{kt}}{C_t} = \frac{\rho(1 - \tau_{kt+1})r_{t+1}}{C_{t+1}}. \quad (12)$$

Eq. (8) equates the loss in utility from working for an additional hour (less leisure) to the gain in utility from earning more income for private consumption. By (9) and (10), the marginal utility forgone from investing an additional unit of goods, or hour, in human capital is equal to the marginal utility obtained from the subsequent increase in human capital. In (11), the present value of the next period's human capital is equal to the gain in utility from higher earnings and higher ability in learning later. Eq. (12) says that the loss in utility from investing in physical capital now will be compensated by the gain in utility from increasing future capital income.

With the log utility function, the Cobb-Douglas production function or education technology, full depreciation of capital per period, and the balanced government budget, agents expect stationary rates of taxes and subsidies:  $(\tau_c, \tau_k, \tau_l, s_k, s_q)$ . Let  $\bar{Y}_t = A\bar{K}_t^\epsilon(\bar{H}_t\bar{L}_t)^{1-\epsilon}$  with  $\bar{Y} = Y$  in equilibrium. Eqs. (2)-(6) and (8)-(12) with expected stationary rates of taxes and subsidies lead to

$$S_t = S = \frac{\rho(1 - \alpha)(1 - \epsilon)(1 - \tau_l)}{(1 - \epsilon)(1 - \tau_l)(2 - \rho) + \epsilon(1 - \tau_k)(1 - \rho)[1 - \rho(1 - \alpha)]}, \quad (13)$$

$$L_t = L = \frac{(1 - \epsilon)(1 - \tau_l)[1 - \rho(1 - \alpha)]}{(1 - \epsilon)(1 - \tau_l)(2 - \rho) + \epsilon(1 - \tau_k)(1 - \rho)[1 - \rho(1 - \alpha)]}, \quad (14)$$

$$C_t = \left\{ \frac{(1 - \rho)\{(1 - \epsilon)(1 - \tau_l) + \epsilon(1 - \tau_k)[1 - \rho(1 - \alpha)]\}}{(1 + \tau_c)[1 - \rho(1 - \alpha)]} \right\} Y_t \equiv \gamma_c Y_t, \quad (15)$$

$$Q_t = \left\{ \frac{\alpha\rho(1 - \epsilon)(1 - \tau_l)}{(1 - s_q)[1 - \rho(1 - \alpha)]} \right\} Y_t \equiv \gamma_q Y_t, \quad (16)$$

$$K_{t+1} = \left[ \frac{\epsilon\rho(1 - \tau_k)}{1 - s_k} \right] Y_t \equiv \gamma_k Y_t, \quad (17)$$

$$H_{t+1} = A_H \gamma_q^\alpha S^{1-\alpha} H_t^{(1-\alpha)} Y_t^\alpha, \quad (18)$$

$$G_t = [\tau_c \gamma_c - s_q \gamma_q + \tau_l(1 - \epsilon) + \tau_k \epsilon - s_k \gamma_k] \bar{Y}_t \equiv \gamma_g \bar{Y}_t. \quad (19)$$

Expecting stationary rates of taxes and subsidies, households allocate time and income proportionately among the competing uses. These proportional allocations are stationary over time but responsive to taxes. The fractions of time spent on working and learning are lower the higher is the tax rate on labor (human capital) income. But the fractions of time spent on working and learning are higher the higher is the tax rate on physical capital income, reflecting a negative income effect on leisure.<sup>7</sup> Thus, labor income taxes and physical capital income taxes affect the time allocation in opposite directions. These direct effects of income taxes fully offset each other if income tax rates are uniform. Taxes on human (physical) capital income lower its rate of return and hence lower the fraction of output invested in human (physical) capital, while investment subsidies encourage investments. Moreover, income taxes and consumption taxes lower the fraction of income spent on private consumption.

Define  $h_t \equiv H_t/K_t$  and  $\sigma \equiv (1 + \beta)/\{(1 - \rho)[1 - \epsilon\rho(1 - \alpha)]\}$ . Given initial stocks of capital in period 0 (i.e.  $H_0$ ,  $K_0$ , and  $h_0$ ), we obtain the expressions for the representative agent's welfare function  $U_0$  and the growth rate  $\mu_t \equiv Y_{t+1}/Y_t - 1$  as follows (see Appendix A for the derivation):

$$U_0 = \frac{1}{1 - \rho} B(\tau_c, \tau_k, \tau_l, s_q, s_k) + (1 - \epsilon)\sigma \ln h_0 + \frac{(1 + \beta)}{1 - \rho} \ln K_0 + B_0, \quad (20)$$

with  $B_0$  being a constant and

$$B(\tau_c, \tau_k, \tau_l, s_q, s_k) = \ln \gamma_c + \alpha\rho\sigma(1 - \epsilon) \ln \gamma_q + \epsilon\rho\sigma[1 - \rho(1 - \alpha)] \ln \gamma_k + \ln(1 - S - L)$$

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<sup>7</sup>The effect of a rise in the physical capital income tax rate on the fractions of time spent on working and learning can be negative if the tax revenue is used for transfers that vary fully with the tax. In this case,  $G_t$  in (6) would be added to the right-hand side of the private budget constraint, and in equilibrium that constraint would be  $C_t = w_t H_t L_t + r_t K_t - Q_t - K_{t+1}$  after substituting out  $G_t$ . Then, a rise in  $\tau_k$  raises  $C_t$  through reducing  $K_{t+1}$  as in (17). As can be seen in (8), a rise in  $C_t$  in turn reduces the marginal benefit of working for an additional hour (the right-hand side) and hence a decline in  $L$  or  $S$  is needed to reduce the marginal benefit of leisure (the left-hand side). If (i)  $G_t$  is public goods or (ii)  $G_t/Y_t$  is fixed when  $G_t$  is a transfer, then from (4) the rise in  $\tau_k$  reduces  $C_t$  through lowering  $r_t K_t(1 - \tau_k) - K_{t+1} = \epsilon(1 - \rho)(1 - \tau_k)Y_t$  where  $K_{t+1}$  is given in (17) and  $s_k = 0$  is assumed in order to focus on public goods or exogenous transfers.

$$\begin{aligned}
& +\beta \ln \gamma_g + \rho \sigma (1 - \alpha)(1 - \epsilon) \ln S + \sigma (1 - \epsilon)[1 - \rho(1 - \alpha)] \ln L, \\
\ln(1 + \mu_t) = & \left\{ \frac{\alpha(1 - \epsilon)[1 - \epsilon^{t+1}(1 - \alpha)^{t+1}]}{1 - \epsilon(1 - \alpha)} \right\} \ln \gamma_q + \left[ \frac{\alpha\epsilon + \epsilon^{t+1}(1 - \alpha)^{t+1}(1 - \epsilon)}{1 - \epsilon(1 - \alpha)} \right] \ln \gamma_k + \\
& \left\{ \frac{(1 - \alpha)(1 - \epsilon)[1 - \epsilon^{t+1}(1 - \alpha)^{t+1}]}{1 - \epsilon(1 - \alpha)} \right\} \ln S + \left[ \frac{(1 - \epsilon)}{1 - \epsilon(1 - \alpha)} \right] \times \\
& \left[ \alpha + (1 - \epsilon)\epsilon^{t+1}(1 - \alpha)^{t+2} \right] \ln L + (1 - \epsilon)\epsilon^{t+1}(1 - \alpha)^{t+1} \ln h_0 + m_t, \quad (21)
\end{aligned}$$

where  $m_t$  varies with time but is irresponsive to any tax instrument.

Note that since  $h_0$  and  $K_0$  are taken as initially given and  $B_0$  is a constant, any optimal rates of taxes and subsidies derived from maximizing  $U_0$  in (20) are simply from maximizing the stationary function  $B$  by choice of a stationary vector  $(\tau_c, \tau_k, \tau_l, s_k, s_q)$ , which is consistent with agents' expectation. In addition, the indirect utility function  $U_0$  in (20) considers not only welfare in steady state equilibrium but also that in transition toward the steady state, in contrast to some previous papers in the literature (e.g., Devarajan, Xie, and Zou, 1998) that only examine steady-state welfare. Thus optimal taxation derived from (20) applies in both the short run and the long run in this model. Note also that the growth rate,  $\mu_t$ , increases with the number of hours spent on production and education,  $L$  and  $S$ , as well as with the fractions of output invested in human and physical capital,  $\gamma_q$  and  $\gamma_k$ .

To be comparable to the existing literature on optimal taxation, we start with the comparison of consumption taxes and uniform income taxes without subsidies. We then allow subsidies but impose uniformity on income tax rates. Finally, we allow the full use of tax/subsidy instruments without imposing prior restrictions on them. In order to compare the different tax solutions with the first-best, we first provide the social planner's problem and its solution below.

## 2.2. Social planner's solution

Given  $(K_0, H_0)$ , the social planner's problem is

$$\max \sum_{t=0}^{\infty} \rho^t [\ln C_t + \ln Z_t + \beta \ln G_t], \quad (22)$$

by choice of  $(C_t, Q_t, K_{t+1}, G_t, L_t, S_t)$  subject to

$$C_t = Y_t - Q_t - G_t - K_{t+1}, \quad (23)$$

$$Y_t = AK_t^\epsilon (H_t L_t)^{1-\epsilon}, \quad (24)$$

$$H_{t+1} = A_H Q_t^\alpha (S_t H_t)^{1-\alpha}, \quad (25)$$

$$Z_t = 1 - S_t - L_t. \quad (26)$$

The solution is given by (see Appendix B for the derivation):

$$S_t = S \equiv \frac{(1 + \beta)(1 - \epsilon)(1 - \alpha)\rho}{(1 + \beta)(1 - \epsilon) + (1 - \rho)[1 - \epsilon\rho(1 - \alpha)]}, \quad (27)$$

$$L_t = L \equiv \frac{(1 + \beta)(1 - \epsilon)[1 - \rho(1 - \alpha)]}{(1 + \beta)(1 - \epsilon) + (1 - \rho)[1 - \epsilon\rho(1 - \alpha)]}, \quad (28)$$

$$C_t = \frac{1}{\sigma[1 - \rho(1 - \alpha)]} Y_t \equiv \gamma_c Y_t, \quad (29)$$

$$Q_t = \frac{\alpha\rho(1 - \epsilon)}{1 - \rho(1 - \alpha)} Y_t \equiv \gamma_q Y_t, \quad (30)$$

$$G_t = \frac{\beta}{\sigma[1 - \rho(1 - \alpha)]} Y_t \equiv \gamma_g Y_t, \quad (31)$$

$$K_{t+1} = \rho\epsilon Y_t \equiv \gamma_k Y_t, \quad (32)$$

$$H_{t+1} = A_H \gamma_g^\alpha (S H_t)^{1-\alpha} Y_t^\alpha. \quad (33)$$

The solution for  $U_0$  parallels that in (20) with  $L$ ,  $S$ , and the  $\gamma$ 's in  $B$  being defined in (27)-(32). Given the same set of technologies and preferences, the solution derived from (22)-(26) dominates (at least “weakly”) any competitive solution because the constraints (4) and (6) for the latter are more restrictive than (23) for the former.

How does the social planner’s solution compare with a competitive equilibrium in the absence of taxes? Setting  $\tau_l = \tau_k = \tau_c = s_k = s_q = 0$  in (13)-(19) we see that in the no-government solution (i) public goods,  $G_t = 0$ , (ii) schooling and work time,  $S_t$  and  $L_t$  are less than in the first-best, and

therefore leisure,  $Z_t$ , is higher, (iii) the fractions of output devoted to human and physical capital investment,  $\gamma_q$  and  $\gamma_k$ , are the same as in the first-best, and (iv) the fraction of output devoted to private consumption is the same as that devoted to private *plus* public goods in the first-best. Since time spent on working or going to school is less than in the first-best, it is also clear that national income,  $Y_t$ , is below the Pareto optimal level in the no-government solution. Finally, welfare is lower in the no-government equilibrium, as is evident from the fact that the marginal utility of public goods is (positively) infinite at  $G_t = 0$ .

### 2.3. Optimal taxation with uniform income taxes and/or consumption taxes

Let us look at uniform income taxation first. Suppose a uniform proportional income tax at a rate  $\tau_y$  ( $= \tau_l = \tau_k$ ) is the only available tax instrument. In this situation we note that  $\gamma_g = \tau_y$  for budget balance. Also, from (13)-(19) we see that time allocation is unchanged from its (non-optimal) no-government pattern, while the investment rates  $\gamma_q$  and  $\gamma_k$  fall below their levels without taxes. The decline in investments means that, after the first period, national income,  $Y_t$ , is below its zero tax level, which was already suboptimal. We have:

**Proposition 1.** *In the absence of a consumption tax and subsidies, the optimal uniform income tax rate is  $\tau_y^* = \gamma_g^* = \beta / \{\sigma[1 - \rho(1 - \alpha)]\}$ . The ratio of public consumption to output,  $\gamma_g^*$ , is the same as in the first-best.*

**Proof.** Optimizing the uniform income tax entails  $\partial B / \partial \tau_y = 0$  that, together with (13)-(19), leads to  $\tau_y^*$ . Then  $\gamma_g^* = \tau_y^*$  under (19), which is equal to  $\gamma_g$  in (31).  $\square$

The optimal uniform income tax rate, or the ratio of public consumption to output, depends positively on the taste for public consumption ( $\beta$ ) and negatively on the importance of physical capital in production ( $\epsilon$ ), the importance of physical inputs in education ( $\alpha$ ), and the discount factor ( $\rho$ ). Note also that under uniform income taxation, public consumption accounts for the same fraction of output as in the social planner's solution. However, public consumption, like output, is below the Pareto optimal *level*.

Because of the distortions of the uniform income tax, the allocations of time and income are

not the same as in the social planner's solution. Specifically, leisure is higher, while investments in physical and human capital are lower, than Pareto optimal levels. The reason for the under-investment in capital is obvious because uniform income taxation without subsidies lowers the private rate of return on investments in capital. However, the reason for the over-consumption of leisure is less obvious. From (13) and (14), the uniform income tax rate cancels out all the direct tax effects on time allocation. It might therefore appear that the income tax does not distort the choice of leisure. The key, of course, is to compare the choice of leisure with the amount that would be chosen under an optimal lump-sum tax. Since the latter would have only an income effect, and no substitution effect, it would reduce the amount of leisure chosen in any specification where leisure was a normal good, as it is here.

The result that a uniform income tax has *no* impact on the allocation of time, including the choice of leisure, is due to our use of Cobb-Douglas preferences. In this case income and substitution effects of the uniform income tax are equal but opposite in sign. While a more general model would of course be desirable, it would be less tractable. And since labor supply is quite inelastic on average empirically, the Cobb-Douglas case at least has relevance and plausibility.

Next, suppose instead that only a proportional consumption tax ( $\tau_c$ ) is available. In this case, we see from (13)-(19) that proportional time allocations are again unaffected by the tax (leisure,  $Z_t$ , remains above the Pareto optimal level), but in addition the investment rates  $\gamma_q$  and  $\gamma_k$  are now unaffected by the tax rate. Then the optimal tax scheme is given by:

**Proposition 2.** *In the absence of income taxes and subsidies, the optimal consumption tax rate is  $\tau_c^* = \beta$ . Under the optimal tax scheme,  $\gamma_g^* = \beta / \{\sigma[1 - \rho(1 - \alpha)]\}$ .*

**Proof.** The optimal scheme requires  $\partial B / \partial \tau_c = -1 / (1 + \tau_c) + \beta \gamma_c / [\gamma_g (1 + \tau_c)] = 0$ , implying  $\tau_c^* = \beta$  and  $\gamma_g^* = \tau_c^* \gamma_c^* = \beta / \{\sigma[1 - \rho(1 - \alpha)]\}$  under (13)-(19).  $\square$

The optimal consumption tax rate depends only on the importance of public consumption ( $\beta$ ). Note that the ratio of public consumption to income is the same as that in the uniform income tax case and in the social planner's solution. Since the tax revenue as a fraction of output is the same as that in the uniform income tax case and the consumption tax base is smaller than the

income tax base, starting from the initial period the optimal consumption tax rate is higher than the optimal income tax rate ( $\tau_c^* > \tau_y^*$ ). Since the investment rates are higher than under uniform income taxation, the consumption tax economy will enjoy a higher growth rate, and higher output after the initial period.

Now suppose that a consumption tax and a uniform income tax are the only tax instruments:

**Proposition 3.** *In the absence of subsidies, the optimal combined income and consumption tax rates are  $\tau_y^* = 0$  and  $\tau_c^* = \beta$ . Under the optimal tax scheme,  $\gamma_g^* = \beta / \{\sigma[1 - \rho(1 - \alpha)]\}$ .*

**Proof.** The tax solution is obtained from  $\partial B / \partial \tau_c = 0$ ,  $\partial B / \partial \tau_y = 0$ , and (13)-(19). The rest of the proof is similar to that of Proposition 2.  $\square$

Even if a uniform income tax is available, the government should use only the consumption tax when subsidies are not used. Thus the lack of intertemporal distortions under the consumption tax makes it superior to the uniform income tax in the present model. This proposition accords with the result from a large number of existing studies on taxation. However, this result is subject to the constraint of the available tax/subsidy instruments. As we will see below, if subsidies are available, they can correct the investment distortions caused by income taxes. In that case, consumption taxes will not necessarily be better than income taxes.

Now we look at a richer menu of tax/subsidy instruments while maintaining uniformity of income tax rates. Assume that the menu includes a uniform income tax ( $\tau_y$ ), a consumption tax ( $\tau_c$ ), an education subsidy ( $s_q$ ), and an investment tax credit ( $s_k$ ). All elements of this menu find use in the real world and thus it is important to know how well the combinations of the taxes/subsidies can perform. In this case, the size of the net revenue from the combination of the labor income tax and the human capital investment subsidy is  $\tau_y w_t \bar{H}_t \bar{L}_t - s_q \bar{Q}_t$  in (6), or  $[\tau_y(1 - \epsilon) - s_q \gamma_q] \bar{Y}_t$  in (19). Similarly, the size of the net revenue from the combination of the physical capital income tax and the investment tax credit is  $\tau_y r_t \bar{K}_t - s_k \bar{K}_{t+1}$  in (6), or  $[\tau_y \epsilon - s_k \gamma_k] \bar{Y}_t$  in (19). These net tax revenues are positive if the rates of the subsidy and the tax credit are low enough.<sup>8</sup> The optimal

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<sup>8</sup>More precisely, by (16) and (17), the net revenues are positive if  $s_q < [1 - \rho(1 - \alpha)] \tau_y / \{[1 - \rho(1 - \alpha)] \tau_y + \alpha \rho(1 - \tau_y)\}$  and  $s_k < \tau_y / [\tau_y + \rho(1 - \tau_y)]$ . Obviously, these inequalities hold if  $\tau_y = s_q = s_k$  as in Proposition 4 below.

tax schemes in this case are described by:

**Proposition 4.** *The optimal tax schemes with a uniform income tax, a consumption tax, an education subsidy, and an investment tax credit are all the combinations of tax/subsidy rates such that  $\tau_y^* = s_k^* = s_q^* \geq 0$ ,  $0 \leq \tau_c^* \leq \beta$ , and  $1 + \tau_c^* = (1 - \tau_y^*)(1 + \beta)$ . Under all the optimal schemes,  $\gamma_g^* = \beta / \{\sigma[1 - \rho(1 - \alpha)]\}$ .*

**Proof.** See Appendix C.

All the optimal schemes in Proposition 4 provide exactly the same equilibrium solution in (13)-(21) and satisfy the budget constraints (4) and (6), and are thus equivalent. The equivalence is more intuitive when rewriting (4) under the optimal schemes as

$$C_t = (w_t H_t L_t + r_t K_t - Q_t - K_{t+1}) \frac{1 - \tau_y^*}{1 + \tau_c^*}$$

which is the same across the schemes under  $(1 - \tau_y^*) / (1 + \tau_c^*) = 1 / (1 + \beta)$ . Note that the tax schemes here include those in Propositions 2 and 3. Putting it differently, a uniform income tax enriched by investment subsidies can perform just as well as a consumption tax, or as various mixes of both.

The key to the equivalence in Proposition 4 is that the uniform income tax it considers allows immediate expensing of all investments, for both human and physical capital. This form of tax is referred to as a “cash-flow” tax. Since it subsidizes investment costs at the same rate at which their payoffs are taxed, it is intertemporally non-distortionary. On the physical capital side it has been widely advocated as a replacement for the corporate income tax since the recommendations of the Meade Committee in the United Kingdom and the “Blueprints” report in the United States came out.<sup>9</sup>

A cash-flow tax on business income is one of the two central planks of the form of consumption tax advocated by Hall and Rabushka (1995). Since the Hall-Rabushka tax would also fall on labor income at the same rate as on business income, the uniform income tax of Proposition 4 corresponds with a modified Hall-Rabushka scheme, in which there is a deduction for the costs of education and

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<sup>9</sup>See Institute for Fiscal Studies, 1978, and United States Department of the Treasury, 1977. Boadway, Bruce and Mintz (1983, 84) provided a rigorous treatment in a partial equilibrium framework. Lucas (1990) noted that a tax on capital income with an investment tax credit can imitate a capital levy perfectly. We extend this argument to a general equilibrium setting with endogenous leisure and human capital.



training. Note that the proposition indicates that all combinations of this form of Hall-Rabushka tax and a consumption tax that produce the required revenue are equally acceptable.

Note finally that (i)  $\tau_y^* = s_k^* = s_q^* = 0$  and  $\tau_c^* = \beta$  and (ii)  $\tau_c^* = 0$  and  $\tau_y^* = s_k^* = s_q^* = \beta/(1 + \beta)$  are optimal schemes implied by Proposition 4. These two special schemes mean that the *pure* uniform income tax rate is always lower than the *pure* consumption tax rate, because the income tax base is larger than the consumption tax base. For this reason, there are often concerns about whether a switch from an income tax to a consumption tax is ideal since a high consumption tax rate relative to the income tax rate may be harmful. Our result here indicates that even though a switch from an income tax with subsidies to a consumption tax entails a decline in the tax base and a rise in the tax rate, there is neither harm nor gain.

#### 2.4. Optimal taxation with non-uniform income taxes and/or consumption taxes

In this section, we relax the uniformity restriction on income taxes. We start with zero subsidies as in the previous section for ease of comparison. In the first case, we only consider income taxes  $(\tau_l, \tau_k)$ . The government budget constraint in this case is  $\gamma_g = (1 - \epsilon)\tau_l + \epsilon\tau_k$ . Define  $\Lambda_1 = (1 - \epsilon)(2 - \rho)(1 - \tau_l) + \epsilon(1 - \rho)(1 - \tau_k)[1 - \rho(1 - \alpha)]$ . The optimal income taxation is:

**Proposition 5.** *In the absence of a consumption tax, and without subsidies, optimal income tax rates,  $\tau_l$  and  $\tau_k$ , are determined implicitly in:*

$$\left(\frac{1}{1 - \epsilon}\right) \frac{\partial B}{\partial \tau_l} = \frac{\beta}{\gamma_g} - \frac{1 - \rho}{\Lambda_1 - (1 - \epsilon)(1 - \tau_l)} - \frac{\alpha\rho\sigma}{1 - \tau_l} + \frac{\epsilon(1 - \rho)(1 - \tau_k)[1 - \rho(1 - \alpha)]}{\Lambda_1[\Lambda_1 - (1 - \epsilon)(1 - \tau_l)]} - \frac{\epsilon\sigma(1 - \rho)(1 - \tau_k)[1 - \rho(1 - \alpha)]}{\Lambda_1(1 - \tau_l)} = 0, \quad (34)$$

$$\left(\frac{1}{\epsilon}\right) \frac{\partial B}{\partial \tau_k} = \frac{\beta}{\gamma_g} - \frac{(1 - \rho)[1 - \rho(1 - \alpha)]}{\Lambda_1 - (1 - \epsilon)(1 - \tau_l)} - \frac{\sigma\rho[1 - \rho(1 - \alpha)]}{1 - \tau_k} - \frac{(1 - \epsilon)(1 - \rho)(1 - \tau_l)[1 - \rho(1 - \alpha)]}{\Lambda_1[\Lambda_1 - (1 - \epsilon)(1 - \tau_l)]} + \frac{\sigma(1 - \epsilon)(1 - \rho)[1 - \rho(1 - \alpha)]}{\Lambda_1} = 0. \quad (35)$$

In general,  $\tau_l^* \neq \tau_k^*$ .

**Proof.** Differentiating the welfare function in (20) with respect to  $\tau_k$  or  $\tau_l$  provides (34) and (35). With the uniformity  $\tau_l = \tau_k = \tau_y > 0$ , it is easy to verify that each of (34) and (35) leads to a solution for  $\tau_y$  and that the two solutions for  $\tau_y$  are different in general.  $\square$

When income taxes are the only tax instruments, uniformity of the two tax rates is generally not optimal, owing to their asymmetric influences on leisure and investment in human and physical capital. Both taxes distort investments in a similar way, but labor income taxation increases leisure from a already high level while taxation of physical capital income reduces leisure. From (13) and (14), if the discount factor  $\rho$  is smaller then time allocations are more sensitive to income taxes (in terms of percentage changes in  $L$  and  $S$ ). From (16) and (17), percentage changes in investments as fractions of output ( $\gamma_q$  and  $\gamma_k$ ) are simply  $-1/(1 - \tau_l)$  and  $-1/(1 - \tau_k)$ , which are independent of the value of  $\rho$ . Thus, when  $\rho$  is small, distortions on time allocations (especially under labor income taxation) are important, so it may be better to tax physical capital income more heavily than labor income. When  $\rho$  is large, the converse may be true.

In Table 1, we illustrate the implications of Proposition 5 in an example where we set  $\alpha = 0.1$ ,  $\beta = 0.4$ , and  $\epsilon = 0.3$  but allow  $\rho$  to vary from 0.1 to 0.9.<sup>10</sup> When  $\rho$  is small, the physical capital income tax rate is higher than the labor income tax rate; when  $\rho$  has mid values the two tax rates are similar; and when  $\rho$  is large, the labor income tax rate becomes large and the physical capital income tax rate becomes negative.

When a consumption tax is used together with income taxes, the government budget constraint is  $\gamma_g = \tau_c \gamma_c + (1 - \epsilon)\tau_l + \epsilon\tau_k$ . We then have:

**Proposition 6.** *In the absence of subsidies, optimal tax rates on consumption and the two types*

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<sup>10</sup>The parameterization is plausible. Capital's share in output  $\epsilon$  is about 30%. The taste for public consumption  $\beta$  is chosen to have a reasonable ratio of public consumption to output, which also declines in  $\rho$ . The value of  $\alpha$  is less known and many existing studies assume that it is smaller than the capital's share parameter in production, meaning that education is more time intensive and less (physical) capital intensive than production. In addition to  $\alpha = 0.1$ , we also did simulations with  $\alpha = 0.3$  and found similar results. Corresponding to full depreciation of capital, one period here may be 20-30 years. For an annual discounting factor in the range of 0.95 to 0.98, the compounding discounting factor over 30 years may range from 0.2 to 0.8.

of income,  $\tau_c$ ,  $\tau_l$  and  $\tau_k$ , are determined implicitly in

$$\frac{\partial B}{\partial \tau_c} = \left( \frac{1}{1 + \tau_c} \right) \left( \frac{\beta \gamma_c}{\gamma_g} - 1 \right) = 0, \quad (36)$$

$$\begin{aligned} \left( \frac{1}{1 - \epsilon} \right) \frac{\partial B}{\partial \tau_l} &= \frac{\alpha \rho}{\gamma_c [1 - \rho(1 - \alpha)]} - \frac{\alpha \rho \sigma}{1 - \tau_l} + \frac{\epsilon(1 - \rho)(1 - \tau_k)[1 - \rho(1 - \alpha)]}{\Lambda_1 [\Lambda_1 - (1 - \epsilon)(1 - \tau_l)]} - \\ &\quad \frac{\epsilon \sigma(1 - \rho)(1 - \tau_k)[1 - \rho(1 - \alpha)]}{(1 - \tau_l) \Lambda_1} = 0, \end{aligned} \quad (37)$$

$$\begin{aligned} \left( \frac{1}{\epsilon} \right) \frac{\partial B}{\partial \tau_k} &= \frac{\rho}{\gamma_c} - \frac{\sigma \rho [1 - \rho(1 - \alpha)]}{1 - \tau_k} - \frac{(1 - \epsilon)(1 - \rho)(1 - \tau_l)[1 - \rho(1 - \alpha)]}{\Lambda_1 [\Lambda_1 - (1 - \epsilon)(1 - \tau_l)]} + \\ &\quad \frac{\sigma(1 - \epsilon)(1 - \rho)[1 - \rho(1 - \alpha)]}{\Lambda_1} = 0. \end{aligned} \quad (38)$$

Under the optimal scheme,  $\epsilon[1 - \rho(1 - \alpha)]\tau_k^* = -\alpha(1 - \epsilon)\tau_l^*$ ,  $\tau_l^* < 0$ ,  $0 < \tau_k^* < 1$ ,  $\tau_c^* > 0$ , and  $\gamma_g^* = \beta / \{\sigma[1 - \rho(1 - \alpha)]\}$ . The feasible solution  $(\tau_c, \tau_l, \tau_k) = (\beta, 0, 0)$  is not optimal, where  $\partial B / \partial \tau_c = 0$ ,  $\partial B / \partial \tau_l < 0$ , and  $\partial B / \partial \tau_k > 0$ .

**Proof.** See Appendix D.

When both income and consumption taxes are used without uniformity on income taxes, optimal taxation features positive taxes on consumption and physical capital income but negative taxes on labor income. Taxing consumption accords with existing views in the literature. What is surprising here is that physical capital income should be taxed as well, while labor income is subsidized. Taxing consumption avoids investment distortions, taxing physical capital and subsidizing labor income reduce the labor-leisure distortion, and subsidizing labor also raises investment in human capital. This tax mix can thus do better than pure income taxation, pure consumption taxation, or a mix of uniform income taxation and consumption taxation.

We report simulation results based on (36)-(38) in Table 2 for a wide range of the value of the discount factor  $\rho$ . The values of the tax rates on consumption and physical capital income, the subsidy rate on labor income (i.e.  $-\tau_l$  when  $\tau_l < 0$ ), and the ratio of public consumption to output are inversely related to the value of  $\rho$ . When  $\rho$  is large, the income tax (subsidy) rates are quite small and the consumption tax is the main instrument to fund public consumption.

As we found earlier, a pure consumption tax is equivalent to a modified Hall-Rabushka tax that allows a deduction for costs of education, or a revenue neutral combination of the two. This means that the insight from Proposition 6 that taxing physical capital more heavily and labor less heavily effects a welfare improvement can be applied to a different starting point: namely one where there is some element of the modified Hall-Rabushka tax. This observation leads to:

**Proposition 7.** *If the tax and subsidy rates are all constrained to be non-negative, it is possible to improve on a pure consumption tax via a tax/subsidy scheme in which: (i)  $\tau_c, \tau_l, \tau_k, s_k, s_q \geq 0$ , (ii)  $\tau_k > \tau_l$ , (iii)  $\tau_k > s_k$ , and (iv)  $\tau_l < s_q$ .*

**Proof.** Start from an initial system that displays a mix of a pure consumption tax and a modified Hall-Rabushka tax. This system (which is equivalent to a pure consumption tax) will have  $\tau_c > 0$  and  $\tau_k = \tau_l = s_k = s_q > 0$ . Proposition 6 implies that welfare can be improved, relative to this starting point, by increasing  $\tau_k$  and reducing  $\tau_l$ .  $\square$

We argue in Section 4 that this proposition points to a set of relations between tax and subsidy rates that is qualitatively realistic for a range of countries.

Table 3 illustrates Proposition 7 in a simulated example starting with a mix of a 5% consumption tax, a 25% uniform income tax, and 25% subsidy rates on investment in human and physical capital. A parameter configuration that supports this tax mix to be an optimal scheme in Proposition 4 is  $\alpha = 0.1$ ,  $\beta = 0.4$ ,  $\epsilon = 0.3$ , and  $\rho = 0.5$ , implying a 22.47% optimal ratio of tax revenue to output. Note that from Proposition 4, such a tax mix is equivalent to a 40% pure consumption tax ( $\beta$ ), or to a 28.57% modified Hall-Rabushka tax where  $\tau_l = \tau_k = s_k = s_q = \beta/(1 + \beta)$ . Deviations from this tax mix, through raising the physical capital income tax by one percentage point at a time and lowering the labor income tax correspondingly to maintain the 22.47% optimal ratio of tax revenue to output, improve welfare when the physical capital income tax rate is below 36%. Any further increase in the physical capital income tax, together with a decline in the labor income tax, leads to a marginal loss in welfare.

Now we further enlarge the menu of the tax/subsidy instruments. Suppose that the government

can set all tax/subsidy rates  $(\tau_c, \tau_l, \tau_k, s_q, s_k)$  without sign restrictions. In so doing, we consider two versions with elastic or inelastic leisure, corresponding to  $i = 1$  and  $i = 0$  respectively in (1), since both versions regarding leisure are seen in the literature on optimal taxation. First, consider the case with inelastic leisure where time is allocated only between education and production. The first-order condition concerning the allocation of time is

$$S_t : \frac{(1 - \tau_l)w_t H_t}{(1 + \tau_c)C_t} = \eta_t \frac{(1 - \alpha)H_{t+1}}{S_t}.$$

The other first-order conditions are the same as in the case with elastic leisure. Solving the original optimization problem with this modification gives

$$S_t = S \equiv \rho(1 - \alpha), \quad (39)$$

$$L_t = L \equiv 1 - \rho(1 - \alpha). \quad (40)$$

The solution for other variables remains the same. Correspondingly, we have

$$B(\tau_c, \tau_k, \tau_l, s_q, s_k) = \ln \gamma_c + \alpha \rho \sigma (1 - \epsilon) \ln \gamma_q + \rho \epsilon \sigma [1 - \rho(1 - \alpha)] \ln \gamma_k + \beta \ln \gamma_g. \quad (41)$$

When leisure is inelastic, the social planner's problem is

$$\max \sum_{t=0}^{\infty} \rho^t [\ln C_t + \beta \ln G_t], \quad (42)$$

by choice of  $(C_t, Q_t, K_{t+1}, G_t, L_t, S_t)$  subject to (23)-(26) with (26) being replaced by  $S_t + L_t = 1$ , given  $(K_0, H_0)$ . The solution for  $C_t, Q_t, G_t, K_{t+1}$  and  $H_{t+1}$  is the same as that in the case with elastic leisure, while the solution for  $S_t$  and  $L_t$  is the same as that in (39) and (40); see Appendix B for derivation. Note that the time allocation with inelastic leisure is the same in both the decentralized equilibrium and the social planner's problem since now there is no over-consumption of leisure by assumption. The solution for  $U_0$  parallels that in (20) and (41) with the  $\gamma$ 's being defined in (29)-(32).

The optimal taxation with inelastic leisure is given by:

**Proposition 8.** *With inelastic leisure, the Pareto optimal tax/subsidy systems include all combinations of tax/subsidy rates such that (i)  $\tau_l^* = s_q^*$  and  $\tau_k^* = s_k^*$  and (ii)  $(1 + \tau_c^*) = \sigma(1 - \rho)\{(1 -$*

$\epsilon)(1 - \tau_l^*) + \epsilon(1 - \tau_k^*)[1 - \rho(1 - \alpha)]\}$ . In particular,  $\tau_y^* = \tau_l^* = \tau_k^* = s_q^* = s_k^* \geq 0$ ,  $0 \leq \tau_c^* \leq \beta$ , and  $1 + \tau_c^* = (1 + \beta)(1 - \tau_y^*)$ . Under the optimal tax schemes,  $\gamma_g^* = \beta/\{\sigma[1 - \rho(1 - \alpha)]\}$ .

**Proof.** See Appendix E.

The tax schemes in Proposition 8 set equal rates for income taxes and investment subsidies so that the under-investment problem is corrected, and thereby these tax schemes are first-best with inelastic leisure. As a special case in Proposition 8, the results in Proposition 4 with uniform income taxes and consumption taxes are now first-best when leisure is inelastic.

Also, note that when  $\epsilon = 0$ , the model degenerates to a one-capital (human capital) model. Then all combinations of  $\tau_c^* > 0$  and  $\tau_l^* \geq 0$  such that  $s_q^* = \tau_l^*$  and  $\gamma_g^* = \beta(1 - \rho)/\{(1 + \beta)[1 - \rho(1 - \alpha)]\}$  are optimal. When  $\epsilon = 1$ , the model also becomes a one-capital (physical capital) model, and the optimal tax schemes include all combinations of  $\tau_c^* \geq 0$  and  $\tau_k^* \geq 0$  such that  $s_k^* = \tau_k^*$  and  $\gamma_g^* = \beta(1 - \rho)/(1 + \beta)$ . The special case with inelastic leisure and with only physical capital is similar to that in Turnovsky (1996) where the composition of revenues from income and consumption taxes is determined by the degree of public goods congestion. But in Proposition 8 the subsidy-enriched income taxes and the consumption tax are perfect substitutes so that if leisure is inelastic, then first-best taxation includes all possible mixes of consumption and income taxes with subsidies at the same rate.

What happens to optimal taxation if leisure is elastic? In this case, a uniform income tax cannot achieve the first-best, as seen in Proposition 4. Without uniformity of income tax rates, the size of the net revenue from the combination of the labor income tax and the human capital investment subsidy is  $\tau_l w_t \bar{H}_t \bar{L}_t - s_q \bar{Q}_t$  in (6), or  $[\tau_l(1 - \epsilon) - s_q \gamma_q] \bar{Y}_t$  in (19). Similarly, the size of the net revenue from the combination of the physical capital income tax and the investment tax credit is  $\tau_k r_t \bar{K}_t - s_k \bar{K}_{t+1}$  in (6), or  $[\tau_k \epsilon - s_k \gamma_k] \bar{Y}_t$  in (19). These net tax revenues are positive if the rates of the subsidy and the tax credit are low enough. Moreover, from (19)  $\gamma_g = \tau_c + \tau_l(1 - \epsilon) - s_q \gamma_q + \tau_k \epsilon - s_k \gamma_k$ , and hence public consumption may be financed by one or two taxes while the other taxes may be zero or negative. For example, we may have a positive consumption tax but negative income taxes as long as the net revenue is positive for public consumption. Optimal taxation in this case is given

by:

**Proposition 9.** *When all of  $(\tau_c, \tau_k, \tau_l, s_k, s_q)$  may be set freely, without sign restrictions, all combinations of taxes such that  $\tau_c^* = -s_q^* = -\tau_l^*$ ,  $\tau_k^* = s_k^*$ , and*

$$1 - \tau_k^* = (1 - \tau_l^*) \left\{ \frac{\epsilon[1 - \rho(1 - \alpha)] - \beta(1 - \epsilon)}{\epsilon(1 + \beta)[1 - \rho(1 - \alpha)]} \right\} < (1 - \tau_l^*). \quad (43)$$

*are Pareto optimal, that is first-best. Under all the optimal schemes,  $\gamma_g^* = \beta/\{\sigma[1 - \rho(1 - \alpha)]\}$ .*

**Proof.** See Appendix F.

The Pareto optimal tax schemes with elastic leisure have the following important features. First, a uniform income tax with subsidies at the same rate and a consumption tax are first-best with inelastic leisure but not so with elastic leisure. This is because a uniform income tax with subsidies or a consumption tax both cause the familiar static distortion on the goods-leisure margin. Here, setting the consumption tax and labor income tax rates equal but with opposite signs eliminates that distortion.<sup>11</sup>

Second, labor (capital) income taxes and education subsidies (investment tax credits) have the same rate  $\tau_l^* = s_q^*$  ( $\tau_k^* = s_k^*$ ). These equalities of the income tax and subsidy rates, respectively for both types of income, eliminate under-investment in human and physical capital.

Third, the relationship between capital and labor income taxes given by (43), along with the same rate but opposite signs of consumption and labor income taxes, achieves the Pareto optimal ratios of private and public consumption to income. From this and the second point above, the Pareto optimal proportional output allocation is thus reached by the tax schemes in Proposition 9.

Fourth, the relations between labor and physical capital income taxes and between consumption and labor income taxes in Proposition 9 achieve the first-best time allocation by reducing leisure. To see this, let us first divide both the numerators and denominators of the right-hand sides in (13) and (14) by  $(1 - \tau_l)$ , resulting in the factor  $(1 - \tau_k)/(1 - \tau_l)$  in the denominators. Obviously, the higher the factor  $(1 - \tau_k)/(1 - \tau_l)$ , the less the time spent on education and production (or the

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<sup>11</sup>It might appear to do so at the cost of eliminating the labor income and the portion of consumption funded by labor income as a source of net revenue. That conclusion ignores both the subsidy,  $s_q$ , which reduces the net revenue from taxing the return to human capital, and the fact that a portion of labor income is saved, reducing consumption tax revenue.

higher the leisure). Under either zero or uniform income taxes, this factor is unity; but with the relation in Proposition 9, this factor is always less than unity, i.e.  $(1 - \tau_k)/(1 - \tau_l) = \{\epsilon[1 - \rho(1 - \alpha)] - \beta(1 - \epsilon)\}/\{\epsilon(1 + \beta)[1 - \rho(1 - \alpha)]\} < 1$ . For  $\tau_l < 1$  and  $\tau_k < 1$ , it follows that  $\tau_k > \tau_l$ .<sup>12</sup> The idea is to tax income from effective labor more lightly than from physical capital to encourage greater use of time in education and production rather than in leisure, in contrast to the case with inelastic leisure in Proposition 8 where the two income taxes are perfect substitutes. This idea was exemplified earlier in Table 2 and Proposition 6. When consumption taxes are positive, labor income taxes become a net subsidy even though the education subsidy becomes a tax at the same rate according to  $\tau_c = -\tau_l = -s_q$ . This is because the labor income tax base  $1 - \epsilon$  is greater than the education subsidy base  $\alpha\rho(1 - \epsilon)/[1 - \rho(1 - \alpha)]$  in (16).

Furthermore, if  $\tau_k = 0$  and  $\tau_l < 1$  then  $1 - \tau_l > 1$  must hold under the Pareto optimal taxation since  $(1 - \tau_k)/(1 - \tau_l) < 1$ , implying  $\tau_l < 0$  and hence  $\tau_c = -\tau_l > 0$ . Namely, when the tax on physical capital income is zero, the consumption tax is positive and the labor income tax net of education subsidies is negative or a net subsidy for labor income. This net subsidy for labor income, with the presence of consumption taxes, reduces leisure.

Finally,  $\tau_c^* = \tau_l^* = s_q^* = 0$  and  $\tau_k^* = s_k^* = \beta[1 - \epsilon\rho(1 - \alpha)]/\{\epsilon(1 + \beta)[1 - \rho(1 - \alpha)]\}$  is a special Pareto optimal scheme. That is, taxing physical capital income and providing an investment tax credit at the same rate without other tax/subsidy instruments is Pareto optimal when the net tax revenue as a fraction of income is equal to the optimal ratio of public consumption to income  $\gamma_g^*$ . This special scheme reveals some important insights: (i) labor and physical capital income taxes are not symmetric, (ii) consumption taxes are not necessarily better than capital income taxes, and (iii) a switch from physical capital income taxes to either labor income taxes or consumption taxes can be welfare reducing.

The intuition behind the above results can be summarized as follows. The cash flow form of tax on physical capital income ( $\tau_k = s_k$ ) is effectively lump-sum. Sole reliance on this source of

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<sup>12</sup>To be practically relevant, income tax rates should be below 100%. This corresponds to  $\beta < \epsilon[1 - \rho(1 - \alpha)]/(1 - \epsilon)$ . In other words, Proposition 9 has a practical value if the taste for public goods is not too strong relative to private goods (i.e.  $\beta$  is not too large).



revenue allows the first-best to be achieved. However, the first-best outcome can also be attained with non-zero consumption and labor income tax rates and a subsidy on human capital investment, provided that these latter rates are set to avoid any effect on the shadow price of leisure or the rate of return to human capital investment.<sup>13</sup> Provided the latter conditions are met, a wide range of alternative tax/subsidy mixes can achieve the first-best.

According to Proposition 9, the effective tax on human or physical capital income is generally non-zero. For example, when the physical capital income tax is used exclusively and physical capital investment is subsidized at the same rate ( $\tau_k^* = s_k^* > 0$  and  $\tau_c^* = \tau_l^* = s_q^* = 0$ ), we have noted that the net tax revenue is positive. Also, the presence of subsidies on investment leads to different results compared to previous ones without such subsidies. These features of optimal taxation in our model differ from that in Jones, Manuelli, and Rossi (1997) where effective taxes (i.e. taxes net of subsidies) on labor and capital income are zero (in the long run) independently of the details of the tax code with respect to available tax instruments (such as taxes or subsidies, excluding lump sum taxes).

## 2.5. Depreciation allowances

In the last scenario, we consider another feature of the tax code practiced in some countries: a capital depreciation allowance. In addition to all the tax instruments discussed in Proposition 9, suppose the government can also use a capital depreciation allowance  $d_{kt}$  ( $\geq 0$ ).<sup>14</sup> With this instrument, the budget constraints become

$$G_t = \tau_c \bar{C}_t + \tau_l w_t \bar{H}_t \bar{L}_t + \tau_k (r_t - d_{kt}) \bar{K}_t - s_q \bar{Q}_t - s_k \bar{K}_{t+1}, \quad (44)$$

$$(1 + \tau_c) C_t = (1 - \tau_l) w_t H_t L_t + [(1 - \tau_k) r_t + \tau_k d_{kt}] K_t - (1 - s_q) Q_t - (1 - s_k) K_{t+1}. \quad (45)$$

The first-order condition with respect to physical capital investment is

$$K_{t+1} : \frac{1 - s_k}{C_t} = \frac{\rho[(1 - \tau_k)r_{t+1} + \tau_k d_{k(t+1)}]}{C_{t+1}}. \quad (46)$$

<sup>13</sup>Under Proposition 9 the consumption tax base is greater than the labor income tax base net of education subsidy because  $\gamma_c + \gamma_q - (1 - \epsilon) = (1 - \tau_k^*)\epsilon(1 - \rho)/(1 + \tau_c^*) > 0$  for  $1 + \tau_c^* > 0$  and  $\tau_k^* < 1$ .

<sup>14</sup>As will be seen below, the optimal capital depreciation allowance is time dependent. Also, recall that in our model the capital depreciation rate is 100% per period.

The other first-order conditions are the same as (8)-(11).

Let  $y_t \equiv Y_t/K_t = A(L_t h_t)^{1-\epsilon}$  and  $\tilde{d}_{kt} \equiv d_{kt}/y_t$ , where  $\tilde{d}_{kt}$  is the output-capital ratio adjusted capital depreciation allowance. The first-order conditions and the budget constraints lead to the same form of solution for  $U_0$  as in (20) and the solution for other variables as:

$$S_t = S = \frac{\rho(1-\alpha)(1-\epsilon)(1-\tau_l)}{(1-\epsilon)(1-\tau_l) + [1-\rho(1-\alpha)](1+\tau_c)\gamma_c}, \quad (47)$$

$$L_t = L = \frac{(1-\epsilon)(1-\tau_l)[1-\rho(1-\alpha)]}{(1-\epsilon)(1-\tau_l) + [1-\rho(1-\alpha)](1+\tau_c)\gamma_c}, \quad (48)$$

$$C_t = \left\{ \frac{(1-\rho)\{(1-\epsilon)(1-\tau_l) + \epsilon(1-\tau_k)[1-\rho(1-\alpha)]\}}{(1+\tau_c)[1-\rho(1-\alpha)]} + \frac{\tau_k(\tilde{d}_{kt} - \rho\tilde{d}_{k(t+1)})}{1+\tau_c} \right\} Y_t \\ \equiv \gamma_c Y_t, \quad (49)$$

$$K_{t+1} = \left[ \frac{\rho[\epsilon(1-\tau_k) + \tau_k\tilde{d}_{k(t+1)})]}{1-s_k} \right] Y_t \equiv \gamma_k Y_t, \quad (50)$$

$$G_t = [\tau_c\gamma_c - s_q\gamma_q + \tau_l(1-\epsilon) + \tau_k\epsilon - s_k\gamma_k - \tau_k\tilde{d}_{kt}] \bar{Y}_t \equiv \gamma_g \bar{Y}_t. \quad (51)$$

Note that  $\gamma_c$ ,  $S_t$ , and  $L_t$  are time independent if the optimal  $\tilde{d}_{kt}^*$  is time independent.

The optimal tax schemes with the depreciation allowance are given by:

**Proposition 10.** *With all tax instruments available, including depreciation allowances, the Pareto-optimal tax schemes are all the combinations of tax instruments such that  $\tau_c^* = -s_q^* = -\tau_l^*$ ,  $\tau_k^* \geq s_k^*$ ,  $\tilde{d}_{kt}^* = \tilde{d}_k^* \equiv \epsilon(1 - s_k^*/\tau_k^*)$  and*

$$1 - s_k^* = (1 - \tau_l^*) \left\{ \frac{\epsilon[1 - \rho(1 - \alpha)] - \beta(1 - \epsilon)}{\epsilon(1 + \beta)[1 - \rho(1 - \alpha)]} \right\}. \quad (52)$$

*Under all the optimal schemes,  $\gamma_g^* = \beta/\{\sigma[1 - \rho(1 - \alpha)]\}$ .*

**Proof.** See Appendix G.

The optimal tax schemes in Proposition 10 include all the tax schemes in Proposition 9 and share all the features in Proposition 9 except that  $\tau_k^*$  and  $s_k^*$  are not required to be always the same due to the availability of the capital depreciation allowance instrument. From the optimal (output-capital ratio adjusted) capital depreciation allowance  $\tilde{d}_k^* = \epsilon(1 - s_k^*/\tau_k^*)$ , we can see that

the capital depreciation allowance and the investment tax credit are perfect policy substitutes. The government can use either the capital depreciation allowance or the investment tax credit or both, along with other tax/subsidy instruments, to achieve the Pareto optimal outcome. Obviously, if the investment tax credit rate is the same as the tax rate on capital income ( $s_k^* = \tau_k^*$ ), then the capital depreciation allowance instrument is redundant ( $\tilde{d}_k^* = 0$ ). Similarly, if the capital depreciation allowance is set at  $\tilde{d}_k^* = \epsilon$ , the investment tax credit instrument also becomes redundant ( $s_k^* = 0$ ). Note that the output-capital ratio adjusted capital depreciation allowance  $\tilde{d}_k^*$  (hence  $\gamma_c$ ,  $L$ , and  $S$ ) is time independent but the unadjusted  $d_{kt}^*$  is not so; the latter depends positively on the output/capital ratio  $y_t$ .

### 3. Comparison of the tax schemes

The optimal tax schemes with elastic leisure in Section 2 can be ranked according to their resulting growth rates of output and welfare levels. (With inelastic leisure, the optimal tax schemes in Proposition 8 obtain the first-best outcome.) Let  $\mu_{t_i}$  and  $U_i$  be the growth rate and the welfare level, respectively, for the optimal tax schemes in Proposition  $i$ .

We first look at the ranking of growth rates. Since we have not obtained analytical tax solutions for Propositions 5, 6, and 7, they are excluded in the following ranking.

**Proposition 11.** *With elastic leisure,  $\mu_{t_9} = \mu_{t_{10}} > \mu_{t_2} = \mu_{t_3} = \mu_{t_4} > \mu_{t_1}$ .*

**Proof.** See Appendix H.

The intuition of this ranking can be easily understood from the formula for the growth rate of per capita output. The formula indicates that the growth rate depends positively on the time spent on working and learning, ( $S$  and  $L$ ), and on human and physical capital investments, ( $\gamma_q$  and  $\gamma_k$ ). According to Proposition 11, the schemes with more tax/subsidy instruments but less prior restrictions generate faster growth. The reason is simple: more policy instruments and more flexibility can do better to remove the distortions of taxation.

The welfare ranking of the tax schemes across the propositions is given below.

**Proposition 12.** *With elastic leisure,  $U_9 = U_{10} > U_2 = U_3 = U_4 > U_1$ .*

**Proof.** See Appendix I.

The ranking of welfare levels is the same as that of growth rates. The intuition is that for these tax schemes the dynamic inefficiency associated with the growth effects of taxes dominates the welfare ranking: the optimal tax schemes that generate faster growth by encouraging investments of time and income in education and production also make individuals better off. For welfare rankings concerning Propositions 5, 6, and 7, recall our earlier discussions:  $U_9 > U_7 = U_6 > U_5 > U_1$  and  $U_7 = U_6 > U_2 = U_3 = U_4$ . The comparison between  $U_5$  and  $U_2$  is unclear.

The effects of taxes on growth and welfare have recently received a great deal of attention; see, e.g., Barro (1990), Caballé (1998), Cooley and Hansen (1992), Davies and Whalley (1989), Hendrichs (1999), Judd (1987), Lucas (1990), Milesi-Ferretti and Roubini (1998a, 1998b), Rebelo (1991), Stokey and Rebelo (1995), and Trostel (1993), in addition to the work mentioned in the introduction. Our results on the growth and welfare rankings extend this line of research to consider more tax/subsidy instruments and more tax regimes.

Finally, under all the second-best schemes with uniform income taxes or with consumption taxes, and under all the first-best tax schemes, the ratio of public spending (net tax revenue) to income is the same as required by a Pareto optimum. In other words, the ratio of tax revenue to output is the same across all these optimal tax schemes, suggesting that most of the optimal taxation problem in this paper can be reformulated as a Ramsey problem where the tax revenue goes back to individuals as lump-sum transfers rather than public consumption, as shown below.

Let  $T_t$  be a transfer funded by taxes as in (6). Due to the modeling of endogenous growth in our model in contrast to the original Ramsey problem, we modify the assumption of a fixed transfer in the standard Ramsey problem to one where a fixed fraction of output is provided as transfers, i.e.  $\gamma_g = T_t/Y_t$  for all  $t$ . The optimal taxation problem now is to find the least costly tax schemes to finance the transfers. Accordingly, assume  $\beta = 0$  to abstract from public consumption, and add the transfer,  $T_t$ , to the right-hand side of (4). Then the government and the representative agent's budget constraints are respectively

$$\gamma_g Y_t = T_t \equiv \tau_c \bar{C}_t + \tau_l w_t \bar{H}_t \bar{L}_t + \tau_k r_t \bar{K}_t - s_q \bar{Q}_t - s_k \bar{K}_{t+1}, \quad (53)$$

$$(1 + \tau_c)C_t = (1 - \tau_l)w_t H_t L_t + (1 - \tau_k)r_t K_t - (1 - s_q)Q_t - (1 - s_k)K_{t+1} + \gamma_g Y_t. \quad (54)$$

The optimal taxation in the modified Ramsey problem is given by:

**Proposition 13.** *The optimal schemes with a fixed fraction of output as lump-sum transfers,  $\gamma_g$ , are all combinations of taxes such that  $\tau_c^* = -s_q^* = -\tau_l^*$ ,  $\tau_k^* = s_k^*$ , and  $\tau_k^* - \tau_l^* = \gamma_g / [\epsilon(1 - \rho)]$ .*

**Proof.** See Appendix J.

From this proposition, we can see that the main features in Proposition 9 still hold true in the case with transfers accounting for a fixed portion of output. In particular,  $\tau_k^* > \tau_l^*$  so long as  $\gamma_g > 0$ . If transfers were fully adjustable, and there were no public goods, optimal taxation in this model would be trivial: zero tax rates and no transfer.

## 4. Applications

It is interesting to ask what implications our model may have for real-world tax mix or for tax reform proposals. It might appear that governments ought to implement tax systems consistent with our Proposition 9, based on our most general case. At least one aspect of this proposition has a strong echo in the real world. The proposition says that there is a continuum of first-best tax schemes, and we certainly observe a wide variety of tax systems. (This is reflected e.g. in Table 4.) However, we run into trouble with specific prescriptions about tax and subsidy rates. To begin with, both human and physical capital must have subsidies at the same rate at which they are taxed. Tax systems that implement this precise provision are seldom observed. And to make matters worse, wage and consumption tax rates must be equal in absolute terms, but of opposite sign.

To get realistic predictions from our model it must be recognized that there are practical constraints on subsidy and tax rates. The latter stem, e.g., from the need to limit evasion and avoidance. Key aspects to take note of include: (i) marginal revenue losses due to evasion and avoidance likely increase with tax rates, as discussed e.g. by Boadway et al. (1994), and (ii) subsidies delivered through the tax system, or negative tax rates, give agents incentives to exaggerate subsidized expenditures or income. Another important point to note is that very sizeable subsidies in kind are

provided in the form of public schools and publicly supported colleges and universities. It may be that these subsidies are to an extent exogenous to the design of the tax system, and greater than would be justified on efficiency grounds (e.g. externalities and borrowing constraints).

Summing up real-world constraints, tax designers may have to accept that (i) tax rates must be positive, (ii) subsidies delivered via the tax system cannot be too large, and (iii) substantial subsidies to human capital investment will be delivered outside the tax system. These considerations put us far outside the world of Proposition 9. But they do not preclude the design of an attractive tax system. We will show this first by demonstrating that they do not prevent the implementation of a consumption tax approach, and then that it is possible to go further and improve on the latter. When we do so we arrive at a system that, qualitatively, is similar to the tax systems observed in the United States and many other countries.

Consider first how a tax designer aiming at the consumption tax approach could work within the above constraints. Note that he could not just adopt a pure consumption tax because the existence of the human capital subsidy is a deviation from that approach. Recall, however, that a pure consumption tax is equivalent to a modified Hall-Rabushka tax. Suppose that direct subsidies to education exceeded those justified on efficiency grounds by, say, 30 % points. One could then, in effect, implement a modified Hall-Rabushka scheme by taxing both labor and physical capital income at 30 %, allowing *no* subsidy to human capital investment in the tax system, and delivering a 30% subsidy to physical capital investment. This approach would only differ from a 30% modified Hall-Rabushka tax by providing its subsidy to human capital investment *outside* the tax system. Now, it could be that this system would be sufficient to collect the required revenue, but there might be a shortfall. If so, any revenue shortfall could be corrected by adding a pure consumption tax.

The outcome described bears some similarity to tax systems that are observed in practice. However, the uniformity of tax burdens on human and physical capital, and the extent of the generosity towards physical capital investment do not appear realistic. Consider then the implications of Proposition 7. That proposition tells us that, starting from the mixed consumption tax/modified

Hall-Rabushka tax, we can raise welfare by increasing the tax on physical capital income and reducing that on labor income. We then end with a tax/subsidy system that is remarkably realistic: (i) consumption, labor income, and physical capital income are all taxed at a positive rate; (ii) physical capital is taxed at a higher rate than labor income; (iii) direct costs of education and training are not subsidized via the tax system, and (iv) physical capital income is taxed at a higher rate than that at which physical capital investment is subsidized.<sup>15</sup>

It is interesting to relate this discussion to the range of tax mixes observed across the G7 countries, as shown in Table 4. The table shows that the two countries with the lowest overall tax burden, Japan and the U.S., make relatively little use of consumption taxes compared with the other countries. In terms of the above discussion this might be explained as follows. Not only do these two countries have relatively small overall revenue requirements, but like the other G7 countries they also have highly developed public school systems and strong public support for higher education. Thus, if one wanted to take a consumption tax approach in these countries, given the need to offset a high subsidy rate to human capital investment from outside the tax system, one would rely more on the Hall-Rabushka element, and less on the pure consumption tax element than in countries with a higher overall revenue requirement. Even after the welfare-raising adjustment of increasing the tax on physical capital and reducing that on labor, one would expect a lower tax on consumption than in the other G7 countries.

Note some of the implications of the above discussion for tax reform. We do not conclude that the object of reform should be a consumption tax approach, accomplished either via a pure consumption tax or the modified Hall-Rabushka scheme. This means that our analysis does not, for example, support a strict cash-flow approach to the taxation of business income. And it rejects the suggestion that labor income and physical capital income should be taxed at the same rate,

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<sup>15</sup>This tax/subsidy system corresponds fairly well to what is observed in the United States and other wealthy English-speaking countries, Japan, and many smaller countries. It does not correspond fully to what is observed in many European countries, however, where e.g. capital income taxes are lighter than labor income taxes. Consider the following four points in turn. (i) That the various taxes are levied at significant positive rates is clear from Table 4. (ii) Among the G-7 countries Mendoza et al. (1994, Tables 2 and 3) show that capital income taxes have traditionally been much higher than labor income taxes in the U.S., the U.K., Canada, and Japan. (iii) Deductions or credits for tuition are typically absent or partial. And (iv) since interest and depreciation allowances do not cover the full costs of capital, capital income is taxed at a rate higher than the effective subsidy on physical investment.

indicating instead that physical capital should be taxed more heavily than human capital.

Finally, we should note that tax rates on physical capital income have been falling recently in many countries, and may already be below those on labor income in some cases. Does this mean that governments are ignoring the logic of our Proposition 7? An alternative explanation for their behavior lies in open economy considerations. Our analysis has been for a closed economy. In the real world there is considerable international capital mobility, and the sensitivity of capital movements to taxes is increasing. The reduction in capital income tax rates could thus just be the result of tax competition, rather than representing a welfare-improving trend.

## 5. Concluding remarks

In this paper we have investigated the optimal mixes of various tax/subsidy instruments and the tax treatment of capital incomes in a simple endogenous growth model with both physical and human capital accumulation where taxes are used to fund the provision of pure public consumption goods. We derived analytically the optimal rates of taxes and subsidies that apply not only in the steady state but also in transition toward the steady state. Several interesting results were obtained. If leisure is inelastic a range of tax mixes are equivalent. A pure consumption tax is first-best, but cash-flow taxes on human or physical capital (investment costs subsidized at the same rate income is taxed) can also be used since they are non-distortionary. If leisure is elastic, but subsidies cannot be used, a consumption tax does better than a uniform income tax, but can be improved on by taxing consumption less and introducing a positive tax on physical capital and a negative tax on labor income. Finally, if subsidies are allowed, the first-best can be achieved in a range of tax schemes that have the following features: (i) if consumption and labor income taxes are non-zero they are of the same rate but opposite signs, (ii) the tax rate on physical capital income exceeds that on labor income when both are below 100%, (iii) subsidy rates on investments equal income tax rates, for both forms of capital, that is both kinds of capital are subject to cash-flow taxes. Under all the optimal schemes studied, public goods account for the same fraction of output. We can assume, alternatively, that government revenue is used to pay a lump-sum transfer that is set



as a pre-specified proportion of output without disturbing our results.

As discussed in Section 4 we believe these results may help us to understand real-world tax systems, especially if a few practical considerations outside the model are taken into account. One of these is that the marginal revenue losses due to evasion or avoidance likely increase with tax rates, implying that a *mix* of taxes has an advantage, *ceteris paribus*. Another is that negative tax rates or very high subsidies are likely to cause exaggeration of the subsidized forms of income or expenditure by taxpayers. Further, large in-kind subsidies to education are provided outside the tax system. These considerations imply that we should not expect the ideal scheme we identified above (which, e.g., cannot have positive taxes on both consumption and labor income) to be observed in the real world. But this does not mean our model lacks real-world relevance.

We have shown that it is possible to improve on a pure consumption tax by taxing physical capital income at a positive rate and labor income at a negative rate. This might appear to run afoul of the real-world consideration that tax rates should be non-negative, but this problem can be avoided as follows. We have noted that a pure consumption tax is equivalent to a cash-flow tax levied at the same rate on both physical and human capital, that is a modified Hall-Rabushka tax. Thus, one can start from some combination of a consumption tax and a modified Hall-Rabushka tax, rather than a pure consumption tax, and it remains true that an improvement can be achieved by increasing the tax rate on capital and reducing it on labor. The result, we argue is a realistic tax system; one where consumption, labor and physical capital are all taxed, but capital income is taxed more heavily than labor income. A further feature of this system is that subsidies to both education and physical investment are provided, but while the education subsidy rate exceeds the tax rate on labor income the subsidy rate to physical capital is less than the corresponding tax rate. This concatenation of subsidy and tax rates appears representative, qualitatively, of what is found in the United States and many other countries.

## Appendix

**A. Derivation of the value function  $U_0$  and the growth rate  $\mu_t$ .** From (2), (17), and (18), we have  $h_{t+1} = h_\infty^{1-\epsilon(1-\alpha)} h_t^{\epsilon(1-\alpha)}$  where  $h_\infty = [A_H \gamma_k^{-1} \gamma_q^\alpha S^{1-\alpha} A^{-(1-\alpha)} L^{-(1-\alpha)(1-\epsilon)}]^{1/[1-\epsilon(1-\alpha)]}$  is the human-physical capital ratio along the balanced steady-state growth path. Hence  $\ln h_{t+1} = [1 - \epsilon(1 - \alpha)] \ln h_\infty + \epsilon(1 - \alpha) \ln h_t$ . Solve this difference equation:

$$\ln h_{t+j} = [1 - \epsilon^j(1 - \alpha)^j] \ln h_\infty + \epsilon^j(1 - \alpha)^j \ln h_t. \quad (\text{A.1})$$

From (2) and (17), we get  $Y_{t+1} = AL^{1-\epsilon} h_{t+1}^{1-\epsilon} K_{t+1} = AL^{1-\epsilon} h_{t+1}^{1-\epsilon} \gamma_k Y_t$ , or  $\ln Y_{t+1} = \ln(AL^{1-\epsilon} \gamma_k) + (1 - \epsilon) \ln h_{t+1} + \ln Y_t$  which leads to

$$\ln Y_{t+j} = j \ln(AL^{1-\epsilon} \gamma_k) + (1 - \epsilon) \sum_{i=1}^j \ln h_{t+i} + \ln Y_t, \quad (\text{A.2})$$

where

$$\sum_{i=1}^j \ln h_{t+i} = \left[ j - \frac{\epsilon(1 - \alpha) - \epsilon^{j+1}(1 - \alpha)^{j+1}}{1 - \epsilon(1 - \alpha)} \right] \ln h_\infty + \frac{\epsilon(1 - \alpha) - \epsilon^{j+1}(1 - \alpha)^{j+1}}{1 - \epsilon(1 - \alpha)} \ln h_t,$$

by using (A.1).

Substitute (15) and (19) into

$$U_t = \sum_{j=0}^{\infty} \rho^j [\ln C_{t+j} + \ln(1 - L - S) + \beta \ln G_{t+j}],$$

and then we have

$$U_t = \frac{1}{1 - \rho} [\ln \gamma_c + \ln(1 - L - S) + \beta \ln \gamma_g] + (1 + \beta) \sum_{j=0}^{\infty} \rho^j \ln Y_{t+j}.$$

The remaining steps toward (20) for  $U_0$  are straight forward by setting  $t = 0$  and using (A.2) in the above equation.

For the growth rate, we have  $1 + \mu_t = Y_{t+1}/Y_t = A \gamma_k [L(h_\infty)^{1-\epsilon(1-\alpha)} h_t^{\epsilon(1-\alpha)}]^{1-\epsilon}$ . By (A.1), it is easy to see  $h_t = (h_\infty)^{1-\epsilon^t(1-\alpha)^t} (h_0)^{\epsilon^t(1-\alpha)^t}$ . Substituting  $h_t$  and  $h_\infty$  into  $\mu_t$  and taking logs provide (21).  $\square$

**B. The social planner's problem.** With elastic leisure, the Lagrangian function for the social planner's problem is

$$\begin{aligned} \mathcal{L} = & \sum_{t=0}^{\infty} \rho^t \{ \ln[AK_t^\epsilon (H_t L_t)^{1-\epsilon} - Q_t - G_t - K_{t+1}] \\ & + \ln(1 - S_t - L_t) + \beta \ln G_t + \eta_t [A_H Q_t^\alpha (S_t H_t)^{1-\alpha} - H_{t+1}] \}, \end{aligned} \quad (\text{A.3})$$

where  $\eta_t$  is the Lagrangian multiplier. Then the first-order conditions are

$$S_t : \frac{1}{1 - S_t - L_t} = \eta_t \frac{(1 - \alpha)H_{t+1}}{S_t}, \quad (\text{A.4})$$

$$L_t : \frac{1}{1 - S_t - L_t} = \frac{(1 - \epsilon)Y_t}{L_t C_t}, \quad (\text{A.5})$$

$$Q_t : \frac{1}{C_t} = \eta_t \frac{\alpha H_{t+1}}{Q_t}, \quad (\text{A.6})$$

$$G_t : \frac{1}{C_t} = \frac{\beta}{G_t}, \quad (\text{A.7})$$

$$K_{t+1} : \frac{1}{C_t} = \frac{\rho \epsilon Y_{t+1}}{K_{t+1} C_{t+1}}, \quad (\text{A.8})$$

$$H_{t+1} : \eta_t = \frac{\rho(1 - \epsilon)Y_{t+1}}{H_{t+1} C_{t+1}} + \eta_{t+1} \frac{\rho(1 - \alpha)H_{t+2}}{H_{t+1}}. \quad (\text{A.9})$$

Solving the above equations gives the solution in (27)-(33).

In the case with inelastic leisure, the social planner's problem given in (42) has the first-order condition with respect to  $S_t$  as

$$S_t : \frac{(1 - \epsilon)Y_t}{(1 - S_t)C_t} = \eta_t \frac{(1 - \alpha)H_{t+1}}{S_t}. \quad (\text{A.10})$$

The other first-order conditions are the same as in the case with elastic leisure. The first-order conditions lead to the solution for (42).  $\square$

**C. Proof of Proposition 4.** The first-order conditions are given by:

$$\tau_c : \frac{\beta \gamma_c}{\gamma_g} = 1, \quad (\text{A.11})$$

$$\tau_y : \frac{\beta}{\gamma_g} (1 - \tau_y + s_k \gamma_k - \tau_c \gamma_c + s_q \gamma_q) = 1 + \alpha \sigma \rho (1 - \epsilon) + \epsilon \rho \sigma [1 - \rho (1 - \alpha)], \quad (\text{A.12})$$

$$s_q : \frac{\beta\gamma_q}{\gamma_g} = \alpha\sigma\rho(1 - \epsilon), \quad (\text{A.13})$$

$$s_k : \frac{\beta\gamma_k}{\gamma_g} = \epsilon\rho\sigma[1 - \rho(1 - \alpha)]. \quad (\text{A.14})$$

These equations and the solution for the agent's problem lead to the optimal schemes in Proposition 4. Under the special scheme  $\tau_l^* = 0$ ,  $\tau_k^* = 0$ ,  $s_q^* = 0$ ,  $s_k^* = 0$  and  $\tau_c^* = \beta$ , it is obvious that  $1 + \tau_c^* = (1 + \beta)(1 - \tau_y^*)$ , and that  $\gamma_g^* = \beta/\{\sigma[1 - \rho(1 - \alpha)]\}$ . Let  $\Gamma_c = (1 + \beta)/\{\sigma[1 - \rho(1 - \alpha)]\}$  and  $\Gamma_q = \alpha\rho(1 - \epsilon)/[1 - \rho(1 - \alpha)]$ . Under the other schemes where  $\tau_y^* = \tau_l^* = \tau_k^* = s_k^* = s_q^* > 0$ ,  $1 + \tau_c^* = (1 + \beta)(1 - \tau_y^*)$  and  $0 \leq \tau_c^* < \beta$ , we have (by noting  $\Gamma_c + \Gamma_q + \epsilon\rho = 1$  and  $(1 - \tau_y^*)(1 + \beta) = 1 + \tau_c^*$ ):

$$\begin{aligned} \gamma_g^* &= \frac{\tau_c^*(1 - \tau_y^*)\Gamma_c}{1 + \tau_c^*} - \tau_y^*\Gamma_q + \tau_y^* - \tau_y^*\epsilon\rho \\ &= \frac{\tau_c^*\Gamma_c}{1 + \beta} + \tau_y^*\Gamma_c. \end{aligned}$$

Also,  $1 + \tau_c^* = (1 - \tau_y^*)(1 + \beta)$  implies  $\tau_y^* = (\beta - \tau_c^*)/(1 + \beta)$ . Then,  $\gamma_g^* = \beta/\{\sigma[1 - \rho(1 - \alpha)]\}$ . Since the consumption tax scheme in Propositions 2 and 3 is a special case here, the schemes in Proposition 4 are not Pareto optimal (with higher leisure than in the social planner's solution).  $\square$

**D. Proof of Proposition 6.** Differentiating the welfare function with respect to the three tax rates gives the first-order conditions (36)-(38). Multiply both sides of (38) by  $\epsilon(1 - \tau_k)/[(1 - \epsilon)(1 - \tau_l)]$  and substitute the resulting equation into (37):

$$\begin{aligned} \gamma_c\sigma[1 - \rho(1 - \alpha)]\{\epsilon[1 - \rho(1 - \alpha)] + \alpha(1 - \epsilon)\} = \\ \alpha(1 - \epsilon)(1 - \tau_l) + \epsilon(1 - \tau_k)[1 - \rho(1 - \alpha)]. \end{aligned} \quad (\text{A.15})$$

By  $\beta\gamma_c = \gamma_g$  in (36) and the fact that  $\gamma_g + \gamma_c + \gamma_q + \gamma_k = 1$  where  $\gamma_q$  and  $\gamma_k$  are defined in (16) and (17), we have:

$$1 = (1 + \beta)\gamma_c + \left[ \frac{\rho}{1 - \rho(1 - \alpha)} \right] \{\alpha(1 - \epsilon)(1 - \tau_l) + \epsilon(1 - \tau_k)[1 - \rho(1 - \alpha)]\}. \quad (\text{A.16})$$

Eqs. (A.15) and (A.16) imply

$$1 = (1 + \beta)\gamma_c + \rho\sigma\gamma_c\{\epsilon[1 - \rho(1 - \alpha)] + \alpha(1 - \epsilon)\},$$

which yields  $\gamma_c^* = 1/\{\sigma[1 - \rho(1 - \alpha)]\}$  and hence  $\gamma_g^*$  by noting that  $\beta\gamma_c = \gamma_g$ . Substituting  $\gamma_c^*$  back into (A.15) gives the relation between  $\tau_k^*$  and  $\tau_l^*$ .

For the signs of  $\tau_l^*$  and  $\tau_k^*$ , substitute  $\tau_k = -\alpha(1 - \epsilon)\tau_l/\{\epsilon[1 - \rho(1 - \alpha)]\}$  and  $\gamma_c = 1/\{\sigma[1 - \rho(1 - \alpha)]\}$  into (38) and rearrange terms:

$$-\left\{\frac{\alpha\rho\sigma[1 - \epsilon\rho(1 - \alpha)]\tau_l}{\epsilon[1 - \rho(1 - \alpha)] + \alpha(1 - \epsilon)\tau_l}\right\} = \left(\frac{1 - \rho}{\Lambda_1}\right)\left\{\frac{\beta}{1 - \rho} + \frac{[\alpha + \epsilon(1 - \alpha)(1 - \rho)]\tau_l}{\Lambda_1 - (1 - \epsilon)(1 - \tau_l)}\right\}, \quad (\text{A.17})$$

where  $\Lambda_1 - (1 - \epsilon)(1 - \tau_l) = (1 - \rho)\{(1 - \epsilon)(1 - \tau_l) + \epsilon(1 - \tau_k)[1 - \rho(1 - \alpha)]\} > 0$  for  $\tau_l < 1$  and  $\tau_k < 1$ . Note that  $\tau_l < 1$  or  $\tau_k < 1$  is obviously needed in any optimal tax scheme; otherwise after-tax income from either labor or physical capital would be negative, and hence investment in either human or physical capital would be negative according to (16) and (17) with zero subsidies. Suppose  $\tau_l \geq 0$ . Then the left-hand side of (A.17) is negative but the right-hand side of (A.17) is positive. So  $\tau_l^* < 0$  and consequently  $\tau_k^* > 0$  according to their relation shown above.

Also note that  $\gamma_c = 1/\{\sigma[1 - \rho(1 - \alpha)]\}$  as proved above, and  $\gamma_c = [\Lambda_1 - (1 - \epsilon)(1 - \tau_l)]/\{(1 + \tau_c)[1 - \rho(1 - \alpha)]\}$  by both the definition of  $\Lambda_1$  and equation (15). These two expressions of  $\gamma_c$  plus the relation  $\tau_k^* = -\alpha(1 - \epsilon)\tau_l^*/\{\epsilon[1 - \rho(1 - \alpha)]\}$  imply that

$$\begin{aligned} 1 + \tau_c &= \left[\frac{1 + \beta}{1 - \epsilon\rho(1 - \alpha)}\right]\{(1 - \epsilon)(1 - \tau_l^*) + \epsilon(1 - \tau_k^*)[1 - \rho(1 - \alpha)]\} \\ &= \left[\frac{1 + \beta}{1 - \epsilon\rho(1 - \alpha)}\right][1 - \epsilon\rho(1 - \alpha) - (1 - \epsilon)(1 - \alpha)\tau_l^*] > 1, \end{aligned}$$

since  $\tau_l^* < 0$ . It follows that  $\tau_c^* > 0$ .

At the feasible solution  $(\tau_c, \tau_l, \tau_k) = (\beta, 0, 0)$ , we have  $\gamma_g = \beta\gamma_c$  and

$$\gamma_c = \frac{(1 - \rho)[1 - \epsilon\rho(1 - \alpha)]}{(1 + \beta)[1 - \rho(1 - \alpha)]},$$

$$\Lambda_1 = 1 - \epsilon + (1 - \rho)[1 - \epsilon\rho(1 - \alpha)].$$

Eq. (36) obviously holds in equality. Eq. (37) becomes:

$$\frac{\partial B}{\partial \tau_l} = \frac{-\beta\epsilon[1 - \rho(1 - \alpha)]}{[1 - \epsilon\rho(1 - \alpha)]\{1 - \epsilon + (1 - \rho)[1 - \epsilon\rho(1 - \alpha)]\}} < 0.$$

And (38) becomes:

$$\frac{\partial B}{\partial \tau_k} = \frac{\beta(1-\epsilon)[1-\rho(1-\alpha)]}{[1-\epsilon\rho(1-\alpha)]\{1-\epsilon+(1-\rho)[1-\epsilon\rho(1-\alpha)]\}} > 0.$$

Thus, the solution  $(\tau_c, \tau_l, \tau_k) = (\beta, 0, 0)$  is not optimal.  $\square$

**E. Proof of Proposition 8.** Note that the time allocation is independent of the taxes and is already the first best with inelastic leisure. For Pareto optimal taxation, equating output allocations  $(\gamma_c, \gamma_g, \gamma_q, \gamma_k)$  to those of the social planner's leads to (i) and (ii). More specifically,  $(\gamma_q, \gamma_k)$  is first-best if and only if (i) holds;  $\gamma_c$  is first-best if and only if (ii) holds. And  $\gamma_g$  is first-best under (i) and (ii) because  $\gamma_c + \gamma_g + \gamma_k + \gamma_q = 1$  in either the competitive solution or the social planner's. We can verify that the first-best tax schemes also result when maximizing  $B$  by choice of the tax instruments. The first-order conditions for the optimal taxes are:

$$\tau_c : \left(-1 + \frac{\beta\gamma_c}{\gamma_g}\right) \frac{1}{1+\tau_c} = 0, \quad (\text{A.18})$$

$$s_q : \left(\alpha\sigma\rho(1-\epsilon) - \frac{\beta\gamma_q}{\gamma_g}\right) \frac{1}{1-s_q} = 0, \quad (\text{A.19})$$

$$s_k : \left\{ \epsilon\rho\sigma[1-\rho(1-\alpha)] - \frac{\beta\gamma_k}{\gamma_g} \right\} \frac{1}{1-s_k} = 0, \quad (\text{A.20})$$

$$\tau_k : -\frac{(1-\rho)}{\gamma_c(1+\tau_c)} - \frac{\rho\sigma[1-\rho(1-\alpha)]}{1-\tau_k} + \frac{\beta}{\gamma_g} \left[ \frac{1+\rho\tau_c - s_k(1-\rho)}{(1+\tau_c)(1-s_k)} \right] = 0, \quad (\text{A.21})$$

$$\begin{aligned} \tau_l : & -\frac{(1-\epsilon)(1-\rho)}{\gamma_c(1+\tau_c)[1-\rho(1-\alpha)]} - \frac{\alpha\sigma\rho(1-\epsilon)}{1-\tau_l} \\ & + \frac{\beta}{\gamma_g} \left\{ \frac{(1-\epsilon)[\alpha\rho\tau_c + 1 - \rho(1-\alpha)]}{(1+\tau_c)[1-\rho(1-\alpha)]} + \frac{\alpha\rho(1-\epsilon)s_q}{(1-s_q)[1-\rho(1-\alpha)]} \right\} = 0. \end{aligned} \quad (\text{A.22})$$

It is easy to verify that (i) and (ii) are the solution for the first-order conditions.  $\square$

**F. Proof of Proposition 9.** Let  $B_l \equiv \ln(1-S-L) + \rho\sigma(1-\alpha)(1-\epsilon) \ln S + \sigma(1-\epsilon)[1-\rho(1-\alpha)] \ln L$ .

The first-order conditions for the optimal taxation are given by (A.18)-(A.20) and

$$\tau_k : -\frac{\epsilon(1-\rho)}{\gamma_c(1+\tau_c)} - \frac{\epsilon\rho\sigma[1-\rho(1-\alpha)]}{1-\tau_k} + \frac{\beta}{\gamma_g} \left[ -\frac{\tau_c\epsilon(1-\rho)}{1+\tau_c} + \epsilon + \frac{s_k\gamma_k}{1-\tau_k} \right] + \frac{\partial B_l}{\partial \tau_k} = 0, \quad (\text{A.23})$$

$$\begin{aligned} \tau_l : & - \frac{(1-\epsilon)(1-\rho)}{\gamma_c(1+\tau_c)[1-\rho(1-\alpha)]} - \frac{\alpha\sigma\rho(1-\epsilon)}{1-\tau_l} \\ & + \frac{\beta}{\gamma_g} \left\{ - \frac{\tau_c(1-\epsilon)(1-\rho)}{(1+\tau_c)[1-\rho(1-\alpha)]} + \frac{s_q\gamma_q}{1-\tau_l} + (1-\epsilon) \right\} + \frac{\partial B_l}{\partial \tau_l} = 0, \end{aligned} \quad (\text{A.24})$$

where

$$\frac{\partial B_l}{\partial x} = \Lambda_1 \left[ \frac{\sigma}{1-\tau_l} - \frac{1}{\Lambda_1 - (1-\epsilon)(1-\tau_l)} \right] \frac{\partial(S+L)}{\partial x}, \quad x = \tau_l, \tau_k,$$

with  $\Lambda_1 \equiv (1-\epsilon)(1-\tau_l)(2-\rho) + \epsilon(1-\tau_k)(1-\rho)[1-\rho(1-\alpha)]$ . These conditions and the solution for the agent's problem give the optimal tax rates in Proposition 9. It can be easily verified that under these optimal tax rates, the competitive solution is identical to the social planner's. (The tax solution in Proposition 9 can also be simply derived, without going through the first-order conditions, by choosing taxes/subsidies such that the competitive and the social planner's solutions are the same.)

□

**G. Proof of Proposition 10.** The first-order conditions with respect to  $\tau_c, s_q$  and  $s_k$  are the same as (A.18)-(A.20) and the first-order conditions with respect to  $\tau_l, \tau_k$  and  $\tilde{d}_{kt}$  are respectively

$$\begin{aligned} \tau_l : & - \frac{(1-\epsilon)(1-\rho)}{\gamma_c(1+\tau_c)[1-\rho(1-\alpha)]} - \frac{\alpha\sigma\rho(1-\epsilon)}{1-\tau_l} \\ & + \frac{\beta}{\gamma_g} \left\{ - \frac{\tau_c(1-\epsilon)(1-\rho)}{(1+\tau_c)[1-\rho(1-\alpha)]} + \frac{s_q\gamma_q}{1-\tau_l} + (1-\epsilon) \right\} + \frac{\partial B_l}{\partial \tau_l} = 0, \end{aligned} \quad (\text{A.25})$$

$$\begin{aligned} \tau_k : & (\tilde{d}_{kt} - \epsilon) \left\{ \frac{1-\rho}{\gamma_c(1+\tau_c)} + \frac{\rho\sigma[1-\rho(1-\alpha)]}{1-s_k} + \frac{\beta}{\gamma_g} \left[ \frac{\tau_c(1-\rho)}{1+\tau_c} - \frac{s_k\rho}{1-s_k} - 1 \right] \right\} + \\ & \frac{\partial B_l}{\partial \tau_k} = 0, \end{aligned} \quad (\text{A.26})$$

$$\tilde{d}_{kt} : \left( \frac{1}{\gamma_c} - \frac{\beta}{\gamma_g} \right) \frac{\tau_k}{1+\tau_c} + \frac{\partial B_l}{\partial \tilde{d}_{kt}} = 0, \quad (\text{A.27})$$

where

$$\frac{\partial B_l}{\partial x} = \Lambda_2 \left[ \frac{\sigma}{1-\tau_l} - \frac{1}{\Lambda_2 - (1-\epsilon)(1-\tau_l)} \right] \frac{\partial(S+L)}{\partial x}, \quad x = \tau_l, \tau_k, \tilde{d}_{kt}$$

with  $\Lambda_2 \equiv (1-\epsilon)(1-\tau_l)(2-\rho) + [\epsilon(1-\tau_k)(1-\rho) + \tau_k(\tilde{d}_{kt} - \rho\tilde{d}_{k(t+1)})][1-\rho(1-\alpha)]$ . Solving the above first-order conditions along with the solution for the agent's problem gives the optimal tax

rates in Proposition 10. Again, under these optimal tax rates, the competitive solution is identical to the social planner's solution.  $\square$

**H. Proof of Proposition 11.** By (21), higher  $L$ ,  $S$ ,  $\gamma_k$ , or  $\gamma_q$  means a higher growth rate. Since all tax schemes in Propositions 9 and 10 are Pareto optimal, the result  $\mu_{t_9} = \mu_{t_{10}}$  is obvious. The equality of  $\mu_{t_2}$ ,  $\mu_{t_3}$  and  $\mu_{t_4}$  comes from the fact that all the tax schemes in Propositions 2-4 give the same values of  $\gamma_q, \gamma_k, L$  and  $S$ . The first inequality  $\mu_{t_i} < \mu_{t_j}$ , for  $i = 2, 3, 4$  and  $j = 9, 10$ , holds because, under Propositions 2-4, the values of  $\gamma_q$  and  $\gamma_k$  are the same as those in the socially optimal solution but the values of  $L$  and  $S$  are smaller than their socially optimal values. The second inequality  $\mu_{t_1} < \mu_{t_i}$ , for  $i = 2, 3, 4$ , holds because, compared with the tax schemes in Propositions 2-4, the tax scheme in Proposition 1 leads to the same values of  $L$  and  $S$ , but smaller values of  $\gamma_q$  and  $\gamma_k$ .  $\square$

**I. Proof of Proposition 12.** The equality of  $U_9$  and  $U_{10}$  is due to the fact that all the tax schemes in Propositions 9 and 10 are Pareto optimal. Since the tax schemes in Propositions 2-4 generate the same equilibrium, they lead to the same welfare level. So we have  $U_2 = U_3 = U_4$ . Now we need to show that  $U_i < U_j$ , for  $i = 2, 3, 4$  and  $j = 9, 10$ . From the welfare measure  $B(\tau_c, \tau_k, \tau_l, s_q, s_k, d_{kt})$  in Section 2, we can express the difference in welfare between the competitive equilibrium and social optimum as

$$\begin{aligned} F(\beta) &\equiv B_{\text{competitive}} - B_{\text{social}} \\ &= [1 + \sigma(1 - \epsilon)] \ln \frac{[1 + \sigma(1 - \epsilon)](1 + \beta)}{\sigma(1 - \epsilon) + 1 + \beta} - \sigma(1 - \epsilon) \ln(1 + \beta). \end{aligned} \quad (\text{A.28})$$

When  $\beta = 0$ , it is obvious that  $F(0) = 0$  and  $U_i = U_j$ , for  $i = 2, 3, 4$  and  $j = 9, 10$ . In other words, the two solutions are the same and the optimal tax schemes in Propositions 2-4 are Pareto optimal when there is no public good. It can also be easily shown that for all permissible  $\beta > 0$  we have  $\partial F / \partial \beta < 0$ . As a result,  $F < 0$  if  $\beta > 0$ . That is, if there is a public good, then the optimal tax schemes in Propositions 2-4 lead to a lower level of welfare than in the social planner's solution, and thereby these optimal tax schemes in the competitive economy are the second best schemes. The result  $U_1 < U_i$ , for  $i = 2, 3, 4$ , is implied by the optimal tax schemes in Propositions 2-4.  $\square$



**J. The solution to the Ramsey problem.** Agents optimize subject to (54). The first-order conditions are the same as (8)-(12). Solving these first-order conditions and using the technology and budget constraints, we have

$$S_t = S = \frac{\rho(1-\alpha)(1-\epsilon)(1-\tau_l)}{(1-\epsilon)(1-\tau_l)(2-\rho) + [\gamma_g + \epsilon(1-\tau_k)(1-\rho)][1-\rho(1-\alpha)]}, \quad (\text{A.29})$$

$$L_t = L = \frac{(1-\epsilon)(1-\tau_l)[1-\rho(1-\alpha)]}{(1-\epsilon)(1-\tau_l)(2-\rho) + [\gamma_g + \epsilon(1-\tau_k)(1-\rho)][1-\rho(1-\alpha)]}, \quad (\text{A.30})$$

$$C_t = \left\{ \frac{(1-\rho)\{(1-\epsilon)(1-\tau_l) + \epsilon(1-\tau_k)[1-\rho(1-\alpha)]\} + \gamma_g[1-\rho(1-\alpha)]}{(1+\tau_c)[1-\rho(1-\alpha)]} \right\} Y_t \equiv \gamma_c Y_t, \quad (\text{A.31})$$

$$Q_t = \left\{ \frac{\alpha\rho(1-\epsilon)(1-\tau_l)}{(1-s_q)[1-\rho(1-\alpha)]} \right\} Y_t \equiv \gamma_q Y_t, \quad (\text{A.32})$$

$$K_{t+1} = \left[ \frac{\epsilon\rho(1-\tau_k)}{1-s_k} \right] Y_t \equiv \gamma_k Y_t, \quad (\text{A.33})$$

$$H_{t+1} = A_H \gamma_q^\alpha S^{1-\alpha} H_t^{(1-\alpha)} Y_t^\alpha, \quad (\text{A.34})$$

$$T_t = [\tau_c \gamma_c - s_q \gamma_q + \tau_l(1-\epsilon) + \tau_k \epsilon - s_k \gamma_k] \bar{Y}_t \equiv \gamma_g \bar{Y}_t. \quad (\text{A.35})$$

$$U_0 = \frac{1}{1-\rho} B(\tau_c, \tau_k, \tau_l, s_q, s_k, \gamma_g) + (1-\epsilon)\sigma \ln h_0 + \frac{1}{1-\rho} \ln K_0 + B_0, \quad (\text{A.36})$$

with  $B_0$  being a constant and

$$B(\tau_c, \tau_k, \tau_l, s_q, s_k, \gamma_g) = \ln \gamma_c + \alpha\rho\sigma(1-\epsilon) \ln \gamma_q + \epsilon\rho\sigma[1-\rho(1-\alpha)] \ln \gamma_k + \ln(1-S-L) \\ + \rho\sigma(1-\alpha)(1-\epsilon) \ln S + \sigma(1-\epsilon)[1-\rho(1-\alpha)] \ln L,$$

where we redefine  $\sigma$  as  $\sigma \equiv 1/\{(1-\rho)[1-\epsilon\rho(1-\alpha)]\}$ .

Then the government's optimization problem is to maximize  $B$  subject to

$$\gamma_g = \tau_c \gamma_c - s_q \gamma_q + \tau_l(1-\epsilon) + \tau_k \epsilon - s_k \gamma_k. \quad (\text{A.37})$$

The first-order conditions for the government's optimization problem are

$$\tau_c : (1 + \xi \gamma_c) \frac{1}{1 + \tau_c} = 0, \quad (\text{A.38})$$

$$s_q : [\alpha\sigma\rho(1-\epsilon) + \xi\gamma_q] \frac{1}{1-s_q} = 0, \quad (\text{A.39})$$

$$s_k : \{\epsilon\rho\sigma[1-\rho(1-\alpha)] + \xi\gamma_k\} \frac{1}{1-s_k} = 0, \quad (\text{A.40})$$

$$\tau_k : -\frac{\epsilon(1-\rho)}{\gamma_c(1+\tau_c)} - \frac{\epsilon\rho\sigma[1-\rho(1-\alpha)]}{1-\tau_k} + \xi \left[ \frac{\tau_c\epsilon(1-\rho)}{1+\tau_c} - \epsilon - \frac{s_k\gamma_k}{1-\tau_k} \right] + \frac{\partial B_l}{\partial \tau_k} = 0, \quad (\text{A.41})$$

$$\begin{aligned} \tau_l : & -\frac{(1-\epsilon)(1-\rho)}{\gamma_c(1+\tau_c)[1-\rho(1-\alpha)]} - \frac{\alpha\sigma\rho(1-\epsilon)}{1-\tau_l} \\ & + \xi \left\{ \frac{\tau_c(1-\epsilon)(1-\rho)}{(1+\tau_c)[1-\rho(1-\alpha)]} - \frac{s_q\gamma_q}{1-\tau_l} - (1-\epsilon) \right\} + \frac{\partial B_l}{\partial \tau_l} = 0, \end{aligned} \quad (\text{A.42})$$

where  $\xi$  is the Lagrangian multiplier associated with (A.37) and

$$\frac{\partial B_l}{\partial x} = \Lambda_3 \left[ \frac{\sigma}{1-\tau_l} - \frac{1}{\Lambda_3 - (1-\epsilon)(1-\tau_l)} \right] \frac{\partial(S+L)}{\partial x}, \quad x = \tau_l, \tau_k,$$

with  $\Lambda_3 \equiv (1-\epsilon)(1-\tau_l)(2-\rho) + [\gamma_g + \epsilon(1-\tau_k)(1-\rho)][1-\rho(1-\alpha)]$ . The optimal solution is given by (27)-(33) with  $\beta = 0$ .

The first-order conditions, (A.38)-(A.42), and the solution for the agent's problem give the optimal taxation in Proposition 13.  $\square$

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**Table 1****Simulation results with income taxes only**Parameters:  $\alpha = 0.1$ ,  $\beta = 0.4$ ,  $\epsilon = 0.3$ 

Value of $\rho$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Variables (%)									
$\pi$	12.2	17.7	21.1	23.5	25.5	27.1	28.7	29.9	29.9
$\tau_k$	64.2	47.2	34.7	23.8	13.4	2.5	-9.4	-23.1	-38.6
$\gamma_g$	27.8	26.6	25.2	23.6	21.8	19.8	17.3	14.0	9.4

**Table 2****Simulation results with income taxes and consumption taxes**Parameters:  $\alpha = 0.1$ ,  $\beta = 0.4$ ,  $\epsilon = 0.3$ 

Value of $\rho$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Variables (%)									
$\tau_c$	125.0	99.9	85.4	74.8	66.2	58.8	52.4	46.9	42.6
$\tau_l$	-93.8	-64.3	-47.3	-35.2	-25.7	-17.9	-11.4	-6.1	-2.2
$\tau_k$	24.0	18.3	15.1	12.8	10.9	9.1	7.2	5.1	2.7
$\gamma_g$	27.5	26.4	25.2	23.9	22.5	20.8	18.8	16.0	11.4

**Table 3****Deviations from a tax mix by raising  $\tau_k$  and reducing  $\tau_l$** Parameters:  $\alpha = 0.1$ ,  $\beta = 0.4$ ,  $\epsilon = 0.3$ , and  $\rho = 0.5$ 

Cases	$\tau_c$ (%)	$\tau_l$ (%)	$\tau_k$ (%)	$s_k = s_q$ (%)	$\gamma_g$ (%)	$B/(1 - \rho)$
Hall-Rabushka	0.0	28.57	28.57	28.57	22.47	-11.20947
Consumption tax	40.0	0.00	0.00	0.00	22.47	-11.20947
Tax mix	5.0	25.00	25.00	25.00	22.47	-11.20947
Deviation 1	5.0	24.29	26.00	25.00	22.47	-11.20219
Deviation 2	5.0	23.80	27.00	25.00	22.47	-11.20069
Deviation 3	5.0	23.31	28.00	25.00	22.47	-11.19935
Deviation 4	5.0	22.81	29.00	25.00	22.47	-11.19817
Deviation 5	5.0	22.32	30.00	25.00	22.47	-11.19716
Deviation 6	5.0	21.83	31.00	25.00	22.47	-11.19632
Deviation 7	5.0	21.34	32.00	25.00	22.47	-11.19565
Deviation 8	5.0	20.85	33.00	25.00	22.47	-11.19515
Deviation 9	5.0	20.36	34.00	25.00	22.47	-11.19483
Deviation 10	5.0	19.86	35.00	25.00	22.47	-11.19468
Deviation 11	5.0	19.37	36.00	25.00	22.47	-11.19472

**Table 4**  
**The tax mix in G-7 countries, 1996**

	Tax Types					
	Total Taxes	Personal Income	Corporate Income	Payroll	Consumption	Other
	( % of GDP )					
Japan	28.4	5.7	4.7	9.3	4.4	4.3
U.S.	28.5	10.7	2.7	6.7	4.9	3.5
U.K.	36.0	9.3	3.8	6.1	12.7	4.2
Canada	36.8	13.9	3.3	5.9	9.2	4.6
Germany	38.1	9.4	1.4	14.5	10.6	2.1
Italy	43.2	10.8	4.0	13.1	11.2	4.0
France	45.7	6.4	1.7	18.1	12.5	6.9
OECD Av.	37.7	10.1	3.1	8.4	12.3	3.8

Source: Organization for Economic Co-operation and Development. 1997. *Revenue Statistics of OECD Member Countries 1965-96*. Paris: OECD.; United States. 1999, Central Intelligence Agency. *The World Factbook*. Washington, DC: US Government Printing Office.