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## How to talk to multiple audiences

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**How to Talk to Multiple Audiences**

**by**

Maria Goltsman and Gregory Pavlov

**Research Report # 2008-1**

**October 2008**



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# How to Talk to Multiple Audiences\*

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October 20, 2008

## Abstract

We analyze the performance of various communication protocols in a generalization of the Crawford-Sobel (1982) model of cheap talk that allows for multiple receivers. We find that whenever the sender can communicate informatively with both receivers by sending private messages, she can communicate informatively by sending public messages. In particular, it is possible that informative communication with one or both receivers is impossible in private, but possible in public. When the sender is allowed to send both public and private messages, it is possible for the sender to combine the commitment provided by public communication with the flexibility provided by private communication and transmit more information to the receivers than under either private or public communication scenarios. When the players can communicate through a mediator and the receivers are biased in the same direction, it is optimal for the sender to communicate with the receivers through independent private noisy communication channels. It is in general optimal to take advantage of pooling the sender's truth-telling constraints across the receivers when they are biased in the opposite directions.

*Keywords:* Communication; Information; Mechanism design; Cheap talk; Long Cheap talk; Multiple audiences

*JEL classification:* C72; C78; D74; D82

## 1 Introduction

The problem of communicating effectively with several parties with diverse interests arises in many contexts. A firm's disclosure of information about the demand for its product may be simultaneously observed by the capital market, shareholders and competitors (Newman and Sansing (1993), Gigler (1994)). A government bureaucrat may need to communicate with many policymakers with different policy preferences (Johns (2007)). During deliberations of a committee, each member discloses his private information to the other members in order to come to a joint decision (Austen-Smith (1990), Li, Rosen and Suen (2001), Austen-Smith and Feddersen (2006) and others), or a sponsor of a

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proposal wishes to convince the committee to accept it (Caillaud and Tirole (2007)). A lobbyist interacts with the members of the associated group and government officials (Ainsworth and Sened (1993)), or with two separate legislative bodies (Board and Dragu (2008)). A politician's statements are observed by the voters and by leaders of other countries (Levy and Razin (2004), Kurizaki (2007)).<sup>1</sup>

In all those settings, the informed party (the sender) faces the problem of selecting the most effective communication mode. An immediate question is whether the sender would prefer to make public announcements or to communicate privately with each receiver. Also, it is possible that the sender could strictly improve upon both purely public and purely private communication by making some statements in public and some in private, and if so, one can ask which statements should be made in public and which in private. Another question is whether the sender would benefit from adopting a more complex communication protocol (e.g. multi-stage communication, or communication through a mediator), and what the optimal protocol would look like. The issue of comparison between public and private communication has been addressed by some of the papers listed above.<sup>2</sup> However, the answers they provide are specific to the particular settings they study, and do not shed light on the question of whether we should expect communication to be public or private in other contexts.<sup>3</sup> The only attempt to compare public and private communication in a general model of costless communication (cheap talk) with multiple receivers, which we discuss later, has been made by Farrell and Gibbons (1989).<sup>4</sup>

In this paper we analyze the performance of various communication protocols (private and public communication, combined private and public communication, as well as mediated and multi-stage communication) in a model which is a natural extension of the classic cheap-talk model by Crawford and Sobel (1982) to the setting with multiple receivers. The sender privately observes a realization of the state of the world from a continuum of possible states, and communicates with two receivers, each of whom chooses an action from the real line. In order to focus on purely informational externalities between the receivers, we assume that each receiver's payoff depends only on his own action and the state, and that the sender's payoff depends on the actions of both receivers and the state and is separable in the two actions.

In Section 3 we compare public and private communication. In the private communication game, the sender can send individual private messages to the receivers; in the public communication game, the sender's messages are commonly observed by both receivers. For private communication, we show that the amount of information the sender transmits to a given receiver is the same as in the game between the sender and this receiver alone (Proposition 1). As in the model of Crawford and

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<sup>1</sup>A related strand of literature studies signaling models with multiple audiences. For example, a firm's choice of financial structure is simultaneously observed by the capital market and competitors (Gertner, Gibbons and Scharfstein (1988)), or by the capital market and a regulator (Spiegel and Spulber (1997)).

<sup>2</sup>Newman and Sansing (1993), Gigler (1994), Kurizaki (2007); Hagenbach and Koessler (2008) study public and private communication on a network.

<sup>3</sup>A different strand of literature compares private and public contracts in a multilateral contracting environment (e.g. McAfee and Schwartz (1994), Segal (1999)). See also Koessler and Martimort (2008) for a setting with non-transferable utility.

<sup>4</sup>Their analysis is extended for a setting with verifiable information by Koessler (2008) and Ozmen (2004).

Sobel (1982), the amount of information revealed to a receiver depends on the extent to which the preferences of the receiver diverge from the preferences of the sender. In the public communication game, the sender’s set of strategies is more restricted than under the private communication scenario, and it is no longer possible to reveal different information to different receivers. But restricting the set of strategies can sometimes be a good thing, because it allows the sender to commit not to tell each receiver a different lie. The fact that the messages are publicly observed by both receivers thus forces the sender to find a compromise between possibly conflicting incentives for misrepresentation of information to different receivers. We show that the amount of information the sender transmits to the receivers in the public communication game is the same as in the game between the sender and a single representative receiver whose preferences are ‘between’ the preferences of the receivers (Proposition 2).

Whether the sender would like to communicate with the receivers privately or publicly depends on the extent to which the preferences of the receivers and the preferences of the representative receiver are different from the preferences of the sender. For example, consider the case when both receivers are so biased from the perspective of the sender that no informative communication can be sustained as an equilibrium of the private communication game. If the receivers are biased in the same direction, then the representative receiver will be biased enough to preclude the possibility of informative communication in the public communication game. However if the receivers are biased in the opposite directions, then the preferences of the representative receiver may be close enough to the preferences of the sender so that informative communication can be sustained as an equilibrium of the public communication game (the case of ‘mutual discipline’). Section 3.3 (Proposition 3) contains a more detailed discussion of these cases, as well as the other possibilities.

In Section 4 we study communication when the sender can send both private and public messages to the receivers (the ‘combined’ communication protocol). We identify and characterize two possible classes of equilibria, monotonic and nonmonotonic. In monotonic equilibria, the sender’s public announcement partitions the state space into intervals, and the private announcements are used to provide further information to the receivers individually. In nonmonotonic equilibria, the sender’s public announcement divides the state space into subsets which are not intervals. We show that both types of equilibria of the combined communication game often allow the sender to transmit more information to the receivers than under either the private or the public communication scenarios (Propositions 4 and 5).

One reason for the superiority of combined communication is that it has the advantages of both private and public communication. The sender can make use of the commitment the public announcements provide, and at the same time reveal different information to different receivers via private messages. Another, more subtle reason why combined communication is valuable is that it gives flexibility in selecting over different continuation equilibria at the private communication stage, which is valuable for providing incentives for information revelation at the public communication stage. For example, the sender may benefit from revealing less information than maximally possible at the private communication stage (Example 1); when the receivers are identical, the sender may

benefit from providing the receivers with different information at the private communication stage (Example 2).

In Section 5 we study mediated communication and communication under multi-stage protocols. The starting point here is the revelation principle (Myerson (1982)), which states that the outcome of any communication protocol can be replicated by the procedure whereby the sender makes a secret report to a neutral trustworthy mediator, who then makes a private non-binding recommendation (possibly stochastic) to each receiver of what action to take. When the receivers are biased in the same direction, we show that it is optimal for the sender to communicate with the receivers through independent private noisy communication channels (Proposition 6). In this case we also show that there exists an unmediated protocol which implements the optimal mediation rule (Proposition 7). When the receivers are biased in the opposite direction, we show that it is in general optimal to take advantage of pooling the sender's truth-telling constraints across the receivers (Lemma 8 and Example 3).

The paper most related to ours is Farrell and Gibbons (1989). They compare private and public communication in a cheap-talk model where there are two possible states of the world and each receiver has two possible actions. For this model, Farrell and Gibbons introduce the classification of the equilibria of private and public communication games that we use in Section 3.3. However, in our model, unlike theirs, the possible cases can be conveniently interpreted as depending on whether the sender's audience is polarized or homogeneous, extremist or moderate. Also the Farrell and Gibbons model is not rich enough to address some interesting questions. In particular, because their model has only two states, the sender has only two possibilities in a pure-strategy equilibrium: either to reveal the truth completely, or to reveal nothing at all. On the other hand, in our model it is possible to have a situation where the sender communicates some information under either communication protocol, but the informativeness of the statements differs across protocols. Finally, the Farrell and Gibbons model is not well-suited for studying combined private and public communication.

In coincident work, Koessler and Martimort (2008) provide a partial comparison of private and public communication. They do not allow for combining private and public communication, or analyze the optimal communication protocol.

## 2 Environment

There are three players, one sender and two receivers. The sender observes the state of the world  $\theta \in \Theta = [0, 1]$ , while the receivers do not observe  $\theta$ . The common prior over the states of the world is a continuous distribution  $F$  on  $\Theta$ . Each receiver  $i$  can choose an action  $a_i \in \mathbb{R}$ .

We assume that the utility function of the sender is  $u(a_1, a_2, \theta) = -l_1(|a_1 - \theta|) - l_2(|a_2 - \theta|)$ ,  $l'_i(x) > 0$ ,  $l''_i(x) > 0, \forall x > 0$ , and the utility function of receiver  $i$  is  $v_i(a_i, \theta) = -L(|a_i - \theta - b_i|)$ ,  $L'(x) > 0$ ,  $L''(x) > 0, \forall x > 0$ , where  $b_i \in \mathbb{R}$ . Given these preferences, the sender's most preferred actions in state  $\theta$  are  $a_1 = a_2 = \theta$ ; receiver  $i$ 's most preferred action is  $a_i = \theta + b_i$ . The utility of each party in state  $\theta$  decreases in the distance from the preferred action(s) given  $\theta$  to the action(s) that

is(are) actually taken. A special case are the quadratic preferences ( $l_1(x) = l_2(x) = L(x) = x^2$ ), which are assumed in many applications.<sup>5</sup>

Before the receivers take their actions the sender can send them payoff-irrelevant messages (cheap talk). We consider three ways to organize communication: public communication, private communication and combined communication. When communication is *public*, the sender is allowed to send only messages that are publicly observed by both receivers. When communication is *private*, the sender is allowed to send individual messages to each receiver. When communication is *combined*, the sender can send both public and private messages. In Section 5 we discuss more complicated communication arrangements: communication through a mediator and communication where the receivers actively participate in the conversation (long cheap talk). The aim of the paper is to compare equilibria of various modes of communication and find the communication arrangement that maximizes the sender's ex ante utility.

**Definition 1** For an equilibrium of a given game with  $K$  receivers, a function  $a : \Theta \rightarrow \Delta(\mathbb{R}^K)$  is called the equilibrium outcome function if the probability distribution over the actions of the receivers for the sender of type  $\theta$  in this equilibrium is given by  $a(\theta) \in \Delta(\mathbb{R}^K)$ .

When we say that  $a(\theta) = a \in \mathbb{R}^K$  we mean that type  $\theta$  gets the vector of actions  $a$  for sure.

### 3 Pure modes of communication

#### 3.1 Private Communication

In this section we consider Bayesian-Nash equilibria of the following *private communication game*. At the first stage, after observing the state  $\theta$  the sender sends two messages,  $m_1$  and  $m_2$ , to the receivers. Receiver  $i$  is able to observe only the message  $m_i$ . At the second stage, receivers independently choose their actions  $a_1$  and  $a_2$ . A sender's strategy in this game maps the states into probability distributions over pairs of messages. Receiver  $i$ 's strategy maps the messages into actions.<sup>6</sup>

We show that the sender will communicate with each receiver in private as he would in a model where only that receiver is present. Let us introduce the following game (the *Crawford-Sobel (CS) game*). There are two players, one sender and one receiver. The sender privately observes the state  $\theta \in [0, 1]$  distributed according to  $F(\theta)$ . He can send one payoff-irrelevant message to the receiver, who then takes an action  $a \in \mathbb{R}$ . The utility functions of both parties depend on the state and the receiver's action. Given this definition we show the following.

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<sup>5</sup>See for example Gilligan and Krehbiel (1989), Grossman and Helpman (2001), Krishna and Morgan (2001), Stein (1989).

<sup>6</sup>Note that equilibrium strategy of each receiver is pure given our assumptions. After any equilibrium message  $m_i$ , receiver  $i$  solves

$$\min_{a_i} \int_{\theta} L(|a_i - \theta - b_i|) dF_{m_i}(\theta)$$

where  $F_{m_i}$  is the posterior distribution of  $\theta$  following message  $m_i$ . The solution is unique by strict convexity of  $L$ .

**Proposition 1** (i) Suppose there exists an equilibrium of the private communication game with an outcome function  $a(\theta)$ . Then for  $i = 1, 2$  there exists an equilibrium of the CS game between the sender with utility function  $-l_i(|a_i - \theta|)$  and the receiver with utility function  $-L(|a_i - \theta - b_i|)$  with the outcome function  $a_i(\theta) := \text{marg}_{a_i} a(\theta)$ .

(ii) Suppose for  $i = 1, 2$  there exists equilibria of the two CS games with payoffs as in (i) with outcome functions  $a_i(\theta)$ . Then there exists an equilibrium of the private communication game with the outcome function  $a(\theta) = (a_1(\theta), a_2(\theta))$ .

The proposition follows from the assumption that there is no interaction between the actions of the two receivers in the sender's utility function, and that receiver  $i$  only cares about the state and his own action.

Crawford and Sobel (1982) characterize the equilibria of the CS game under more general assumptions on preferences. They prove that if  $b_i = 0$ , there exists an equilibrium where the state is completely revealed to the receiver. If  $b_i \neq 0$ , any equilibrium is characterized by a finite sequence of cutoff types  $0 = \theta_0 < \theta_1 < \dots < \theta_N = 1$  such that the equilibrium outcome function is constant on each interval  $(\theta_{i-1}, \theta_i)$ . If there exists an equilibrium of size  $N$ , then there also exist equilibria with any size smaller than  $N$ . As a consequence, for any fixed value of  $b_i$ , there exists an equilibrium with the maximal number of actions.<sup>7</sup>

In our model, this translates into the fact that any equilibrium generates two interval partitions of  $[0, 1]$ , each partition corresponding to an equilibrium of the CS game with receiver  $i$ . For any values of  $b_1$  and  $b_2$ , there exists an equilibrium where each receiver takes the maximal possible number of actions. We will call this equilibrium the most informative one.

### 3.2 Public Communication

In this section we consider Bayesian-Nash equilibria of the following *public communication game*. At the first stage, after observing the state  $\theta$  the sender sends a message  $m$  that is observed by both receivers. At the second stage, the receivers choose their actions  $a_1$  and  $a_2$ . A sender's strategy in this game maps the states into probability distributions over messages. Receiver  $i$ 's strategy maps the messages into actions  $a_i$ .

**Lemma 1** *In every equilibrium of the public communication game after any equilibrium message the actions of the receivers are related as follows,*

$$a_2 - a_1 = b_2 - b_1 \tag{1}$$

The result follows from the fact that message  $m$  is publicly observed by both receivers, and thus they have identical posterior distributions of the states. Given this result we can characterize the equilibria of the public communication game.

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<sup>7</sup>Lemma 12 in Appendix reviews the CS game equilibria for the case when  $F$  is uniform and payoffs are quadratic.



**Proposition 2** Let  $l_1 \equiv l_2 \equiv l$ , and  $\frac{b_1+b_2}{2} \neq 0$ .

(i) Any equilibrium of the public communication game is characterized by a sequence of cutoff types  $0 = \theta_0 < \theta_1 < \dots < \theta_N = 1$  such that the equilibrium outcome  $a(\theta)$  is a constant action pair on every interval  $(\theta_k, \theta_{k+1})$  for  $i = 1, 2$ .

(ii) There is an equilibrium of the public communication game characterized by cutoff types  $0 = \theta_0 < \theta_1 < \dots < \theta_N = 1$  if and only if the CS game between the sender with utility function  $-l(|a - \theta|)$  and the receiver with utility function  $-L(|a - \theta - \frac{b_1+b_2}{2}|)$  has an equilibrium with the same cutoff types.

To establish part (i) of the result we show that there can be only a finite number of action pairs in any equilibrium when  $\frac{b_1+b_2}{2} \neq 0$ . Given the fact that all action pairs lie on the same line (Lemma 1) the proof turns out to be very similar to the proof of an analogous property for the CS game equilibria.<sup>8</sup> We also show that the set of types which get the same equilibrium action pair must form an interval, and these intervals form a partition of the state space.

Thus the structure of the public communication game equilibria is very similar to the structure of the CS game equilibria. This connection is further strengthened in part (ii) of the result, where we establish that in the public communication game the sender behaves essentially as if she is facing a single representative receiver with the bias equal to the average of the biases of the two receivers ( $\bar{b} = \frac{b_1+b_2}{2}$ ).<sup>9</sup>

**Remark 1.** When  $b_1 + b_2 = 0$ , there exists an equilibrium of the public communication game where every state is revealed truthfully by the sender. This is true regardless of the absolute value of  $b_1$  and  $b_2$ . To see this note that if the sender claims that the state is  $\theta$  and the receivers expect her to be truthful, the optimal action of receiver 1 equals  $\theta + b_1$  and the optimal action of receiver 2 equals  $\theta + b_2 = \theta - b_1$ . Hence if the sender reports the state  $\theta$  truthfully, her utility is  $-2l(|b_1|)$ ; if she misreports the state to be  $\theta + \Delta$ , her utility is  $-l(|\Delta + b_1|) - l(|\Delta - b_1|)$ . Because  $l$  is strictly convex and increasing, the utility from telling the truth is higher than utility from any misreporting ( $\Delta \neq 0$ ).

### 3.3 Comparison between private and public communication

For the remainder of the section we assume that  $F$  is uniform on  $[0, 1]$ , so that assumption (M) of Crawford and Sobel is satisfied. Under this assumption in the private communication game the maximal number of distinct actions that receiver  $i$  can take in equilibrium is a nonincreasing

<sup>8</sup>Lemma 1 in Crawford and Sobel (1982).

<sup>9</sup>To study communication with more than two receivers and to allow asymmetry in the preferences of the sender regarding the receivers one can use the following utility function for the sender:

$$u(a_1, \dots, a_K, \theta) = - \sum_{i=1}^K w_i (a_i - \theta)^2 \text{ where } w_1, \dots, w_K \geq 0 \text{ and } \sum_{i=1}^K w_i = 1.$$

It is easy to show that in the public communication game the sender behaves as if she is facing a single representative receiver with a bias  $\bar{b} = \sum_{i=1}^K w_i b_i$ .

function of  $|b_i|$ .<sup>10</sup> In the public communication game the receivers always get the same message, so the number of distinct actions taken in equilibrium is the same for both receivers. This number is a nonincreasing function of  $\bar{b} = \frac{b_1+b_2}{2}$ . We conclude that there exist a threshold  $b^* \in \mathbb{R}_+$  such that there exists a private communication equilibrium where receiver  $i$  takes at least two different actions if and only if  $|b_i| \leq b^*$ , and there exists a public communication equilibrium where any receiver takes at least two different actions if and only if  $|\bar{b}| \leq b^*$ . In the public communication game all equilibria are ex ante Pareto ranked, with the most informative equilibrium being the best for the sender and both receivers. In the private communication game more informative communication between sender and receiver  $i$  leads to higher ex ante utility for both players. Hence in the private communication game the equilibrium which is the most informative with each of the receivers is best for all players. We will say that private communication is better than public if the ex ante Pareto optimal equilibrium with private communication gives higher utility to the sender than the ex ante Pareto optimal equilibrium with public communication.

We will discuss the conditions under which information transmission is possible with public and private communication. This question was addressed before by Farrell and Gibbons (1989) (hereafter ‘FG’) in a setting where the sender observes realization of one of the two possible states of the world, and each of the two receivers has a choice between two possible actions. As in our model, the payoffs to each receiver are independent of the action of the other receiver. Focusing on pure strategy equilibria, Farrell and Gibbons provide conditions for existence of separating equilibria in the private communication game and in the public communication game. To facilitate comparison with the FG model, we use a classification of the cases similar to theirs.

**1. No communication** ( $|b_1|, |b_2|, |\bar{b}| \geq b^*$ ). This is the case when under the private communication and the public communication scenarios the only possible equilibria are uninformative (babbling) equilibria. No communication occurs in the FG model when the sender wants to convince each receiver that one particular state has occurred regardless of the true state of the world. In our model this case occurs under similar circumstances: either both of the receivers are very biased in the same direction, or the receivers are very biased in the opposite directions and the magnitude of one of the biases is much larger than that of the other. In this case, private and public communication are equivalent in terms of welfare. See Figure 1 for the range of parameters when this case, as well as the other cases, occurs.

**2. Subversion** ( $|b_i| < b^*, |b_j|, |\bar{b}| \geq b^*$ ). There exist informative private communication equilibria with only one of the receivers, and all public communication equilibria are babbling. In the FG model, this case occurs when the sender has an incentive to reveal information to one of the receivers, but it is subverted by her desire to convince the other receiver that one particular state has occurred regardless of the truth. In our model, this case occurs when only one of the receivers has a bias which allows to sustain informative private communication. Unlike in the FG model, the sender reveals to this receiver only some but not all the information unless the preferences of the sender and this receiver are completely aligned, which happens when the receiver’s bias is zero. The

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<sup>10</sup>See Section 5 in Crawford and Sobel (1982).

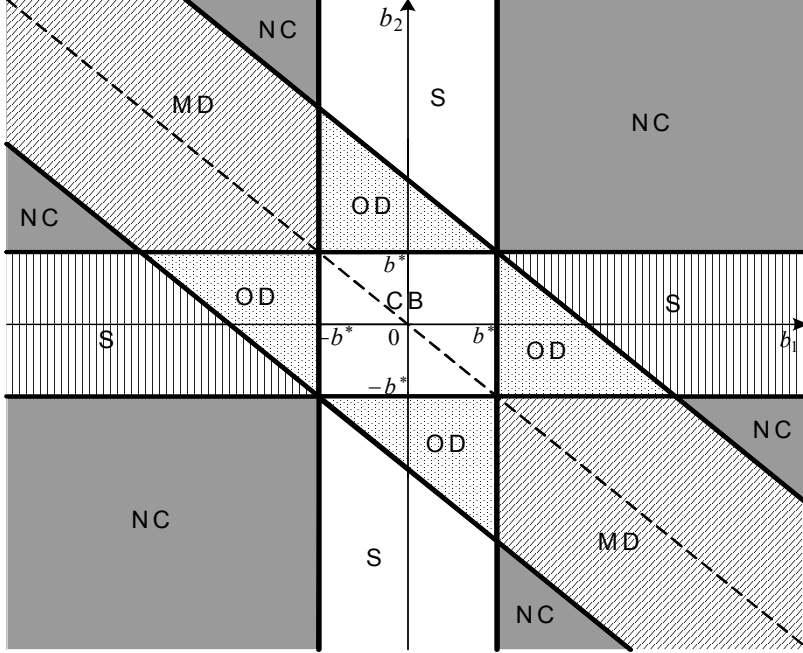


Figure 1: Classification of private and public communication equilibria: NC = ‘no communication’, S = ‘subversion’, MD = ‘mutual discipline’, OD = ‘one-sided discipline’, CB = ‘communication with both’.

other receiver is biased to the extent that the magnitude of the average bias is prohibitively large, and no informative public communication is possible. In this case, private communication is better than public.

**3. Mutual discipline** ( $|b_1|, |b_2| \geq b^*, |\bar{b}| < b^*$ ). There are no informative private communication equilibria with either of the receivers, but there are informative public communication equilibria. In the FG model, ‘mutual discipline’ occurs when under private communication the sender wants to convince each receiver that one particular state has occurred regardless of the true state of the world, but the states the sender wants the receivers to believe in are different. Hence in the public communication scenario these countervailing incentives may result in an existence of a separating equilibrium. In our model this situation occurs when the receivers are biased in the opposite directions, but the magnitudes of their biases are of comparable sizes, so that the absolute value of the resulting average bias is not prohibitively high. In particular, as mentioned in Section 3.2, if the biases are different in sign but equal in absolute value, then a fully separating equilibrium is feasible when communication is public. In this case, public communication is better than private.

**4. One-sided discipline** ( $|b_i| \geq b^*, |b_j|, |\bar{b}| < b^*$ ). There exist informative private communication equilibria with only one of the receivers, as well as informative public communication equilibria. In the FG model, this occurs when the incentive of the sender to reveal the information to one of the receivers is stronger than her desire to convince the other receiver that one particular state has occurred regardless of the truth. In our model, one of the receivers has a bias which allows to sustain informative private communication. The absolute value of the other receiver’s bias is high enough

to preclude the possibility of informative private communication with him, but not high enough to prevent public communication. The welfare comparison between public and private communication in this case in general is ambiguous. In Proposition 3 below we perform the comparison when the distribution of the state is uniform and the payoffs are quadratic.

**5. Communication with both** ( $|b_1|, |b_2|, |\bar{b}| < b^*$ ). There exist both informative private communication equilibria with each of the receivers and informative public communication equilibria. In the FG model, if the sender has incentives to reveal the true state of the world to each receiver privately, then there also exists a separating equilibrium under the public communication scenario. Similarly, the existence of informative communication equilibria with each of the receivers in our model implies that some non-trivial information revelation is possible under the public communication scenario. This is because moderate values of the receivers' biases result in a moderate value of the average bias. Unlike in the FG model, the sender reveals to the receivers only some but not all the information. Thus the outcomes of the private and public communication equilibria are not equivalent in our model, unless the biases of the receivers exactly coincide. The welfare comparison between public and private communication in this case in general is ambiguous. See Proposition 3 below for the results with uniform distribution and quadratic payoffs.

Neither in the FG model, nor in our model it is possible to have 'mutual subversion,' i.e. a case where there exist informative private communication equilibria with each of the receivers but there are no informative public communication equilibria. This case becomes possible if one goes beyond the cheap talk model and allows the sender to make certifiable statements (see Koessler (2007), Ozmen (2004)).

Farrell and Gibbons study only pure strategy equilibria, but under some circumstances in their model there are interesting mixed strategy equilibria as well. For example, there are mixed strategy public communication equilibria which support some information transmission in cases when neither informative public communication, nor informative private communication with either receiver is possible (the 'no communication' case).<sup>11</sup> In contrast, in our model all private and public communication equilibria are essentially equivalent to partitional pure-strategy equilibria.

If the utilities of all players are quadratic ( $l_i(x) = L(x) = x^2$ ), then we can say more about the welfare comparison in case of 'one-sided discipline' and 'communication with both'. The results are summarized by the following proposition.

**Proposition 3** *Let  $l_i(x) = L(x) = x^2$ . Then:*

1. *If  $|b_1|, |b_2|, |\bar{b}| > \frac{1}{4}$ , neither public nor private communication game have an informative equilibrium. The ex ante utility of all players is the same in both cases.*
2. *If  $|b_i| < \frac{1}{4}$  and  $|b_j|, |\bar{b}| > \frac{1}{4}$ , 'subversion' occurs, i.e. there exist informative private communication equilibria with only one of the receivers, and all public communication equilibria are babbling. Private communication is better than public.*

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<sup>11</sup>One of the types of the sender mixes between a 'fully revealing' message and a 'pooling' message, while the second type always sends the 'pooling' message. The mixing probabilities are chosen so that the posterior after the 'pooling' message makes one of the receivers indifferent between his actions, so it is possible to choose a mixed strategy for this receiver to support such an equilibrium. The details are available upon request.

3. If  $|\bar{b}| < \frac{1}{4}$  and  $|b_1|, |b_2| > \frac{1}{4}$ , ‘mutual discipline’ occurs, i.e. there are no informative private communication equilibria with either of the receivers, but there are informative public communication equilibria. Public communication is better than private.

4. If  $|b_i|, |\bar{b}| < \frac{1}{4}$  and  $|b_j| > \frac{1}{4}$ , ‘one-sided discipline’ occurs, i.e. there exist informative private communication equilibria with only one of the receivers, and there are informative public communication equilibria. There exist continuous functions  $\bar{B} : [-\frac{1}{4}, \frac{1}{4}] \rightarrow [\frac{1}{4}, \frac{1}{2}]$ ,  $\underline{B} : [-\frac{1}{4}, \frac{1}{4}] \rightarrow [-\frac{1}{2}, -\frac{1}{4}]$  such that public communication is better than private if and only if  $b_{-i} \in [\underline{B}(b_i), -\frac{1}{4}] \cup [\frac{1}{4}, \bar{B}(b_i)]$ .

5. If  $|b_1|, |b_2|, |\bar{b}| < \frac{1}{4}$ , ‘communication with both’ occurs, i.e. there exist informative private communication equilibria with each of the receivers, as well as informative public communication equilibria. Public communication is better than private if  $|b_1|, |b_2| \in \left(\frac{1}{2N(N+1)}, \frac{1}{2N(N-1)}\right)$  for some  $N = 2, \dots$ .

To illustrate the case of ‘one-sided discipline,’ consider  $b_1 = 0$  and  $b_2 \in (\frac{1}{4}, \frac{1}{2})$ . The best private communication equilibrium involves full revelation of information to receiver 1,  $a_1(\theta) = \theta$  for every  $\theta \in [0, 1]$ , and uninformative communication with receiver 2,  $a_2(\theta) = \frac{1}{2} + b_2$  for every  $\theta \in [0, 1]$ . The ex ante utility of the sender in this private communication equilibrium is  $-\frac{1}{12} - (b_2)^2$ .<sup>12</sup>

The best public communication equilibrium is of size  $N = 2$ , since  $\bar{b} = \frac{1}{2}b_2 \in (\frac{1}{8}, \frac{1}{4})$ . The sender sends two public messages:  $m$  if  $\theta \in [0, \frac{1}{2} + b_2)$  and  $m'$  if  $\theta \in [\frac{1}{2} + b_2, 1]$ . Following the public messages, both receivers take actions equal to the expected state corrected by their biases, which result in the equilibrium outcome  $(a_1(\theta), a_2(\theta)) = (\frac{1}{4} + \frac{1}{2}b_2, \frac{1}{4} + \frac{3}{2}b_2)$  if  $\theta \in [0, \frac{1}{2} + b_2)$  and  $(a_1(\theta), a_2(\theta)) = (\frac{3}{4} + \frac{1}{2}b_2, \frac{3}{4} + \frac{3}{2}b_2)$  if  $\theta \in [\frac{1}{2} + b_2, 1]$ . The ex-ante utility of the sender in this public communication equilibrium is  $-\frac{1}{24} - \frac{3}{2}(b_2)^2$ .<sup>13</sup>

Compared to the best private communication equilibrium, in the best public communication equilibrium more information is transmitted to receiver 2, but less to receiver 1. The comparison between the two equilibria depends on the size of these two effects. When  $b_2$  is close to  $\frac{1}{2}$ , then the public communication equilibrium is not very informative, since the message  $m$  is sent by most types. As a result, the first effect is weaker than the second, so the public communication equilibrium is worse for the sender ex ante than the private communication equilibrium. As  $b_2$  becomes smaller, the public communication equilibrium becomes more informative, and the first effect becomes stronger. For small enough  $b_2$  (namely,  $b_2 > (12)^{-\frac{1}{2}} \approx 0.289$ ) private communication is better for the sender than public.

To illustrate the case of ‘communication with both’, consider  $b_1 = 0$  and  $b_2 \in (\frac{1}{6}, \frac{1}{4})$ . It is possible to support informative private communication of size  $N = 2$  with receiver 2 and full revelation with receiver 1, while the best public communication equilibrium is also of size  $N = 2$ . When  $b_2$  is close to  $\frac{1}{4}$ , communication with receiver 2 is very uninformative in the private communication equilibrium, because the same message is sent to this receiver by almost all types. As a consequence, public communication is better than private for the sender if  $b_2$  is high. As  $b_2$  decreases, private communication becomes better relative to public, and it is straightforward to verify that private

<sup>12</sup>See Lemma 12 in the Appendix.

<sup>13</sup>See Lemma 13 in the Appendix.

communication is better than public as long as  $b_2 \in \left(\frac{1}{6}, \frac{1}{\sqrt{24}}\right)$ .

When  $b_2 \in \left(\frac{1}{12}, \frac{1}{6}\right)$ , the most informative private communication equilibrium is still of size  $N = 2$  with receiver 2, while the best public communication equilibrium is of size  $N = 3$ . Similarly to the previous case, private communication is better than public communication when  $b_2$  is close to  $\frac{1}{4}$ , and vice versa when  $b_2$  is close to  $\frac{1}{6}$ . The story repeats itself if we consider  $b_2$  below  $\frac{1}{12}$ : public and private communication keep alternating as the best communication arrangements. Finally, if  $b_2 = 0$ , then both private and public communication equilibria result in full information revelation and are thus equivalent.

## 4 Combined Communication

### 4.1 Preliminaries

In this section we consider the game where the sender can send both public and private messages. One can imagine various timings for the combined communication game. The sender may first send a public message and then send private messages, or she could first send private messages and then a public message. Public and private messages could also be sent simultaneously. It is not hard to see that the set of the outcomes of Bayesian Nash equilibria in all of the above variants of the model will be the same. When we discuss the structure of the equilibria below we will have the first timing in mind: first public messages, then private messages.

Formally, we consider Bayesian-Nash equilibria of the following *combined communication game*. At the first stage, after observing the state  $\theta$  the sender sends a message  $m$  that is observed by both receivers. Then the sender sends two private messages,  $m_1$  and  $m_2$ , to the receivers. Receiver  $i$  is able to observe only the message  $m_i$ . At the second stage, receivers independently choose their actions  $a_1$  and  $a_2$ . A sender's strategy in this game maps the states into probability distributions over messages. Receiver  $i$ 's strategy maps messages into actions.<sup>14</sup>

Because it is always possible to sustain uninformative communication at the public stage, it is clear that for every equilibrium of the private communication game, there exists an equilibrium of the combined communication game that has the same outcome function. Similarly, combined communication always can replicate public communication. As a result, combined communication cannot be worse than private or public communication. We will be interested in whether combined communication can strictly improve on both.

Farrell and Gibbons (1989) do not study combined communication. Moreover, for every equilibrium of the combined communication game in their framework that we were able to find, there exists an equilibrium of either the private communication game or the public communication game that is equivalent or Pareto dominates it. Given the positive results for our model we present in this section, this suggests that the FG model is not well-suited for studying combined communication.

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<sup>14</sup>In this section we focus on equilibria with deterministic outcome functions. Contrary to the public and private communication scenarios, in case of combined communication there may exist mixed strategy equilibria with a non-degenerate random outcome function. The characterization of such mixed strategy equilibria is a work in progress at the moment.

We begin characterizing the structure of the combined communication equilibria by noticing that the private messages partition the state space into intervals.

**Lemma 2** *Suppose  $m$  is a public message sent in a combined communication equilibrium, and suppose  $\Theta(m)$  is the set of types that send  $m$ . Then*

$$\forall \theta, \theta' \in \Theta(m), \theta < \theta', a_i(\theta) = a_i(\theta') = a_i \Rightarrow a_i(\theta'') = a_i \quad \forall \theta'' \in (\theta, \theta') \cap \Theta(m)$$

for  $i = 1, 2$ . That is, in all combined communication equilibria, conditional on every public message, the equilibrium outcome for each receiver has the interval partition structure.

It turns out that a similar result cannot be proved for the public messages because, as will be shown below, there are combined communication equilibria where the public messages divide the state space into subsets that are not intervals.

Another property of any equilibrium of the combined communication game is given by the following lemma.

**Lemma 3** *Let  $a_1(\theta), a_2(\theta)$  be equilibrium outcome functions of some combined communication equilibrium. Then  $\theta' > \theta$  implies that either  $a_1(\theta') \geq a_1(\theta)$ , or  $a_2(\theta') \geq a_2(\theta)$ , or both.*

Hence it is natural to distinguish the following two classes of combined communication equilibria.

**Definition 2** *A combined communication equilibrium is called monotonic if for every  $\theta' > \theta$ ,*

$$a_i(\theta') \geq a_i(\theta) \text{ for every } i = 1, 2.$$

*combined communication equilibrium is called nonmonotonic if there exists  $\theta' > \theta$  and  $j \in \{1, 2\}$  such that*

$$a_j(\theta') < a_j(\theta).$$

Before proceeding further, we present a necessary condition for the existence of informative combined communication equilibria in terms of the biases of the receivers.

**Lemma 4** *If there exists a combined communication equilibrium with informative private communication with receiver  $i$ , then  $|b_i| < \frac{1}{2}$ .<sup>15</sup>*

## 4.2 Monotonic equilibria

First we show that the equilibria from this class have the interval partition structure.

**Lemma 5** *Any monotonic combined communication equilibrium is interval partitional.*

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<sup>15</sup>In general,  $|b_i|$  must be smaller than half of the length of the support of  $\theta$ .

Hence in the monotonic equilibria the sender first makes public announcements which partition the state space into intervals, and then further refines the information of the receivers by privately communicating with each of them. Notice that all public communication equilibria and all private communication equilibria belong to this class. However, despite the fact that the monotonic equilibria have an intuitive structure, a full characterization of all such equilibria is hard. Instead we settle on deriving a set of necessary conditions for the existence of monotonic equilibria with both informative communication at the public stage and informative communication at the private stage with at least one of the receivers.

**Lemma 6** (i) *Suppose  $F$  is uniform. If there exists a monotonic combined communication equilibrium with informative private communication with receiver  $i$ , then  $|b_i| < \frac{1}{4}$ ;*

(ii) *Suppose  $F$  is uniform and  $l_1(x) = l_2(x) = x^2$ . If there exists an informative monotonic combined communication equilibrium, then  $|\bar{b}| < \frac{1}{4}$ .*

These necessary conditions are easy to interpret. Every monotonic equilibrium contains both a public communication stage and a private communication stage. Thus it is natural that both the conditions for the existence of an informative public communication equilibrium (as in Proposition 2) and the conditions for the existence of an informative private communication equilibrium with at least one of the receivers must be satisfied (as in Proposition 1).

Next we consider an example of a monotonic equilibrium which performs strictly better than any public or private communication equilibrium.

**Example 1** *Suppose  $F$  is uniform,  $l_1(x) = l_2(x) = x^2$ , and  $(b_1, b_2) = (0, \frac{1}{4})$ . Consider the following strategy. The sender sends two public messages: ‘Low’ if  $\theta \in [0, x)$  and ‘High’ if  $\theta \in [x, 1]$ , where  $x = \sqrt{3} - 1 \approx 0.732$ . After both public messages, the sender sends an uninformative message to receiver 2. The sender sends a fully revealing message to receiver 1 following the message ‘High’ and an uninformative message following the message ‘Low’.*

*First we show that this communication arrangement constitutes an equilibrium. Following the public message ‘High’ and the consequent revelation of the true state of the world  $\theta$ , receiver 1 takes the action equal to the state of the world  $\theta$  (since  $b_1 = 0$ ). Following the public message ‘High’, receiver 2 takes an action equal to the expected state conditional on  $\theta \in [x, 1]$  corrected by his bias  $b_2 = \frac{1}{4}$ . Following the public message ‘Low’ both receivers take actions equal to the expected state conditional on  $\theta \in [0, x)$  corrected by their respective biases. Hence the equilibrium outcome function is as follows:*

$$(a_1(\theta), a_2(\theta)) = \begin{cases} (\frac{1}{2}x, \frac{1}{2}x + \frac{1}{4}) & \text{if } \theta \in [0, x) \\ (\theta, (\frac{1}{2}x + \frac{1}{2}) + \frac{1}{4}) & \text{if } \theta \in [x, 1] \end{cases},$$

$$\text{i.e., } (a_1(\theta), a_2(\theta)) \approx \begin{cases} (0.366, 0.616) & \text{if } \theta \in [0, x) \\ (\theta, 1.116) & \text{if } \theta \in [x, 1] \end{cases}$$

*Let us check incentive compatibility for the sender. Type  $x$  is indifferent between sending the public message ‘Low’ and sending the public message ‘High’ (and consequently communicating with*



receiver 1) if

$$\begin{aligned} -\left(\frac{1}{2}x - x\right)^2 - \left(\frac{1}{2}x + \frac{1}{4} - x\right)^2 &= \max_{\hat{\theta} \in [x, 1]} -\left(\hat{\theta} - x\right)^2 - \left(\frac{1}{2}x + \frac{3}{4} - x\right)^2 \\ &= -\left(\frac{1}{2}x + \frac{3}{4} - x\right)^2 \end{aligned}$$

Solving for  $x$  we get  $x = \sqrt{3} - 1$  as claimed above. It is straightforward to calculate the ex-ante utility of the sender in this equilibrium:  $\frac{1}{4}\sqrt{3} - \frac{9}{16} \approx -0.129$ .

The best private communication equilibrium in this example involves full revelation of information to receiver 1 ( $a_1(\theta) = \theta$  for every  $\theta \in [0, 1]$ ), and no information revelation to receiver 2 ( $a_2(\theta) = \frac{3}{4}$  for every  $\theta \in [0, 1]$ ). The ex-ante utility of the sender in this private communication equilibrium is  $-\frac{7}{48} \approx -0.146$ , which is smaller than the utility in the above combined communication equilibrium.

The best public communication equilibrium is of size  $N = 2$ , since  $\bar{b} = \frac{1}{8} \in (\frac{1}{12}, \frac{1}{4})$ . The sender sends two public messages:  $m$  if  $\theta \in [0, \frac{3}{4})$  and  $m'$  if  $\theta \in [\frac{3}{4}, 1]$ . Following the public messages, both receivers take actions equal to the expected state corrected by their biases, which result in the equilibrium outcome function  $(a_1(\theta), a_2(\theta)) = (\frac{3}{8}, \frac{5}{8})$  if  $\theta \in [0, \frac{3}{4})$  and  $(a_1(\theta), a_2(\theta)) = (\frac{7}{8}, \frac{9}{8})$  if  $\theta \in [\frac{3}{4}, 1]$ . The ex-ante utility of the sender in this public communication equilibrium is  $-\frac{13}{96} \approx -0.135$ , which is smaller than the utility in the above combined communication equilibrium.

Clearly the outcome of this combined communication equilibrium cannot be replicated by any public communication equilibrium, because receiver 1 must have more precise information than receiver 2. The reason why this outcome cannot be replicated by a private communication equilibrium is more subtle. Suppose there was a private communication equilibrium that resulted in the same outcome functions. Then type  $x$  would induce the actions  $(a_1, a_2) = (x, 1.116)$ . But this type has a profitable deviation: it could achieve a higher utility by sending to receiver 1 a message which induces action  $x$ , and to receiver 2 a message which induces action  $\frac{1}{2}x + \frac{1}{4} \approx 0.616$ . This deviation is unavailable to the sender in the combined communication game, because at the public stage it is made common knowledge whether the state is above or below  $x$ . As a result, the sender can induce receiver 2 to take the desirable action  $\frac{1}{2}x + \frac{1}{4}$  only if it is bundled together with receiver 1's action  $\frac{1}{2}x \approx 0.366$ . This illustrates the role of having the public communication stage, which is to reduce the number of deviations available to the sender.

Another unusual feature of this combined communication equilibrium is that it prescribes uninformative communication with receiver 1 after the public message 'Low' despite the fact that the preferences of the sender and of receiver 1 are perfectly aligned ( $b_1 = 0$ ). Assume for a moment that the sender fully reveals the state of the world to receiver 1 after every public message. In this case, the public messages carry useful information only for receiver 2, and thus every combined communication equilibrium is equivalent to some private communication equilibrium. However, from the analysis in Section 3 we know that it is impossible to sustain any information revelation to receiver 2 because his bias is too high (equal to  $\frac{1}{4}$ ).

An alternative way to see why it might make sense to forego the possibility of revealing information to receiver 1 after the public message ‘Low’ is as follows. Given that receiver 2 has a high positive bias, the sender is tempted to pretend to be a low type. Hence in order to sustain some information revelation with receiver 2 we need to reduce the sender’s desire of being perceived as a low type. The above combined communication equilibrium achieves this by handicapping the sender in her ability to communicate with receiver 1 following the message ‘Low’ (by prescribing uninformative communication). This feature will also be present in the analysis of the nonmonotonic combined communication equilibria.<sup>16</sup>

Next we generalize the message of Example 1. We show that for a range of parameters it is possible to construct a monotonic combined communication equilibrium which outperforms all public communication equilibria from the ex-ante perspective of the sender.

**Proposition 4** *Suppose  $F$  is uniform and  $l_i(x) = x^2$ . For any  $b_2 \in (-\frac{1}{2}, \frac{1}{2}) \setminus \{0\}$ , there exists  $\varepsilon(b_2) > 0$  such that whenever  $|b_1| \leq \varepsilon(b_2)$ , there exists a monotonic combined communication equilibrium that gives the sender strictly higher ex-ante utility than the best public equilibrium.*

The proof works as follows. We show that whenever there exists a public communication equilibrium of size  $N$ , it is possible to construct a combined communication equilibrium where the sender sends  $N$  public messages which partition the state space into intervals. Consequently, following one of the public messages, an informative private message is sent to one of the receivers. The constructed combined communication equilibrium is shown to be strictly better for the sender than the original public communication equilibrium.

Constructing monotonic equilibria which are better for the sender than the private communication equilibria is more complicated. It turns out that the private communication equilibria sometimes result in the highest possible payoff among all possible equilibria (including combined communication equilibria). This statement is made precise in Lemma 9 in Section 5.

### 4.3 Nonmonotonic equilibria

In the nonmonotonic combined communication equilibria the public messages divide the state space into subsets which are not intervals. Let us consider an example of a nonmonotonic equilibrium which performs strictly better than any public or private communication equilibrium or any monotonic equilibrium.

**Example 2** *Suppose  $F$  is uniform,  $l_i(x) = x^2$ , and  $(b_1, b_2) = (\frac{1}{4}, \frac{1}{4})$ . Consider the following strategy. The sender sends two public messages: ‘Outside’ if  $\theta \in [0, x) \cup [z, 1]$  and ‘Inside’ if*

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<sup>16</sup>Note that it is impossible to transmit any information to receiver 2 if the sender sends a fully revealing message to receiver 1 following the message ‘Low’ and an uninformative message following the message ‘High’. The outcome in such an equilibrium would be as follows:

$$(a_1(\theta), a_2(\theta)) = \begin{cases} (\theta, \frac{1}{2}x + \frac{1}{4}) & \text{if } \theta \in [0, x) \\ (\frac{1}{2}x + \frac{1}{2}, (\frac{1}{2}x + \frac{1}{2}) + \frac{1}{4}) & \text{if } \theta \in [x, 1] \end{cases} .$$

Solving for  $x$ , we get  $x = 1$ . Hence the resulting equilibrium is equivalent to the best private communication equilibrium which involves full information revelation to receiver 1 and uninformative communication with receiver 2.

$\theta \in [x, z]$ , where  $x \approx 0.021$  and  $z \approx 0.932$ . The sender sends an uninformative message to receiver 2 after both messages. Following the message ‘Outside’ the sender reveals to receiver 1 whether  $\theta \in [0, x)$  or  $\theta \in [z, 1]$ , and sends an uninformative message following the message ‘Inside’.

Let us show that this communication arrangement constitutes an equilibrium. Following the public message ‘Outside’ and the consequent private communication, receiver 1 takes the action equal to the expected state conditional on  $\theta \in [0, x)$  or on  $\theta \in [z, 1]$  corrected by his bias  $b_1 = \frac{1}{4}$ . Following the public message ‘Outside’ receiver 2 takes the action equal to the expected state conditional on  $\theta \in [0, x) \cup [z, 1]$  corrected by his bias  $b_2 = \frac{1}{4}$ . Following the public message ‘Inside’ both receivers take actions equal to the expected state conditional on  $\theta \in [x, z)$  corrected by their respective biases. Hence the equilibrium outcome is as follows:

$$(a_1(\theta), a_2(\theta)) = \begin{cases} \left( \left( \frac{1}{2}x \right) + \frac{1}{4}, \left( \frac{x}{x+1-z} \left( \frac{1}{2}x \right) + \frac{1-z}{x+1-z} \left( \frac{1}{2}z + \frac{1}{2} \right) \right) + \frac{1}{4} \right) & \text{if } \theta \in [0, x) \\ \left( \left( \frac{1}{2}x + \frac{1}{2}z \right) + \frac{1}{4}, \left( \frac{1}{2}x + \frac{1}{2}z \right) + \frac{1}{4} \right) & \text{if } \theta \in [x, z) , \\ \left( \left( \frac{1}{2}z + \frac{1}{2} \right) + \frac{1}{4}, \left( \frac{x}{x+1-z} \left( \frac{1}{2}x \right) + \frac{1-z}{x+1-z} \left( \frac{1}{2}z + \frac{1}{2} \right) \right) + \frac{1}{4} \right) & \text{if } \theta \in [z, 1] \end{cases}$$

$$\text{i.e., } (a_1(\theta), a_2(\theta)) \approx \begin{cases} (0.261, 0.991) & \text{if } \theta \in [0, x) \\ (0.727, 0.727) & \text{if } \theta \in [x, z) . \\ (1.216, 0.991) & \text{if } \theta \in [z, 1] \end{cases}$$

Let us check incentive compatibility for the sender. Type  $x$  is indifferent between the ‘low’ strategy of sending the public message ‘Outside’, with the consequent revelation to receiver 1 that her type is in  $[0, x)$ , and the ‘intermediate’ strategy of sending the public message ‘Inside’ if

$$\begin{aligned} & - \left( \frac{1}{2}x + \frac{1}{4} - x \right)^2 - \left( \frac{x}{x+1-z} \left( \frac{1}{2}x \right) + \frac{1-z}{x+1-z} \left( \frac{1}{2}z + \frac{1}{2} \right) + \frac{1}{4} - x \right)^2 \\ & = - \left( \frac{1}{2}x + \frac{1}{2}z + \frac{1}{4} - x \right)^2 - \left( \frac{1}{2}x + \frac{1}{2}z + \frac{1}{4} - x \right)^2 \end{aligned}$$

Type  $z$  is indifferent between the ‘high’ strategy of sending the public message ‘Outside’, with the consequent revelation to receiver 1 that her type is in  $[z, 1]$ , and the ‘intermediate’ strategy of sending the public message ‘Inside’ if

$$\begin{aligned} & - \left( \frac{1}{2}z + \frac{3}{4} - z \right)^2 - \left( \frac{x}{x+1-z} \left( \frac{1}{2}x \right) + \frac{1-z}{x+1-z} \left( \frac{1}{2}z + \frac{1}{2} \right) + \frac{1}{4} - z \right)^2 \\ & = - \left( \frac{1}{2}x + \frac{1}{2}z + \frac{1}{4} - z \right)^2 - \left( \frac{1}{2}x + \frac{1}{2}z + \frac{1}{4} - z \right)^2 \end{aligned}$$

Let  $d := z - x$ . Straightforward but tedious calculations yield  $x = \frac{1}{2}(1-d) \left( 1 + \frac{(1+d)}{(1-d)^2 - 4d} \right)$ , where  $d$  is the root of the following polynomial:  $d^4 - \frac{37}{3}d^3 + 41d^2 - \frac{55}{3}d - \frac{26}{3}$ . There exists a root  $d \approx 0.910$ , which yields  $x \approx 0.021$  and  $z \approx 0.932$ . The ex-ante utility of the sender in this equilibrium is approximately  $-0.266$ .

The best private communication equilibrium, the best public communication equilibrium, and the best monotonic combined communication equilibrium (by Lemma 6) are all babbling equilibria

$(a_1(\theta) = a_2(\theta) = \frac{3}{4}$  for every  $\theta \in [0, 1]$ ). The ex-ante utility of the sender is  $-\frac{7}{24} \approx -0.292$ , which is smaller than the utility in the above nonmonotonic combined communication equilibrium.

Clearly the outcome of this combined communication equilibrium cannot be replicated by any public communication equilibrium, because receiver 1 must have more precise information than receiver 2. The outcome cannot be replicated by some private communication equilibrium either: type  $x$  ( $\approx 0.021$ ) could send to receiver 1 a message which induces action  $(\frac{1}{2}x) + \frac{1}{4} \approx 0.261$ , and to receiver 2 a message which induces action  $(\frac{1}{2}x + \frac{1}{2}z) + \frac{1}{4} \approx 0.727$  instead of  $\left(\frac{x}{x+1-z}(\frac{1}{2}x) + \frac{1-z}{x+1-z}(\frac{1}{2}z + \frac{1}{2})\right) + \frac{1}{4} \approx 0.991$ , which is inconsistent with the outcome of the combined communication equilibrium. Alternatively, just note that the equilibrium outcome of receiver 2 is nonmonotonic in the state, which cannot happen in the private communication equilibrium.

Let us outline the logic behind the constructed equilibrium. Since both receivers have high positive biases, the sender is tempted to pretend to be a low type. Moreover, the biases are so high so that no information revelation can be sustained in a private or a public equilibrium. To support informative communication, we either need to reduce the sender's desire of being perceived as a low type, or to raise the attractiveness of being perceived as a high type. Our equilibrium does both of those things. First, compare the following two strategies of the sender: the 'intermediate' strategy of sending the public message '*Inside*' versus the 'low' strategy of sending the public message '*Outside*' with the consequent revelation to receiver 1 that her type is in  $[0, x)$ . The 'intermediate' strategy results in identical actions by both receivers  $(\frac{1}{2}x + \frac{1}{2}z) + \frac{1}{4} \approx 0.727$ . The 'low' strategy results in a low action by receiver 1  $((\frac{1}{2}x) + \frac{1}{4} \approx 0.261)$  which is relatively more attractive for many of the sender's types than the actions resulting from the 'intermediate' strategy. But the attractiveness of the 'low' strategy is not overwhelming, because the low action of receiver 1 is bundled together with a relatively high action by receiver 2  $\left(\left(\frac{x}{x+1-z}(\frac{1}{2}x) + \frac{1-z}{x+1-z}(\frac{1}{2}z + \frac{1}{2})\right) + \frac{1}{4} \approx 0.991\right)$ . Next, compare the 'intermediate' strategy of sending the public message '*Inside*' versus the 'high' strategy of sending the public message '*Outside*' with the consequent revelation to receiver 1 that her type is in  $[z, 1]$ . The 'high' strategy results in a very high action by receiver 1  $((\frac{1}{2}z + \frac{1}{2}) + \frac{1}{4} \approx 1.216)$ , which is less attractive than the actions induced by the 'intermediate' strategy unless the sender's type is very close to 1. However, the 'high' strategy is made more attractive because the high action of receiver 1 comes bundled together with a relatively low action by receiver 2  $\left(\left(\frac{x}{x+1-z}(\frac{1}{2}x) + \frac{1-z}{x+1-z}(\frac{1}{2}z + \frac{1}{2})\right) + \frac{1}{4} \approx 0.991\right)$ .

An unusual feature of this equilibrium is that it prescribes no information revelation to receiver 2 following the public message '*Outside*', although the sender could have communicated to him whether the state is in  $[0, x)$  or in  $[z, 1]$ . The sender does reveal this information privately to receiver 1, and, since both receivers have identical biases, she could have done so to receiver 2. But it is easy to see that if this information is to be revealed to receiver 2 as well, then the construction breaks down and the resulting equilibrium is equivalent to an uninformative equilibrium.<sup>17</sup> Hence, as in the analysis of the monotonic combined communication equilibrium in Example 1, the key

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<sup>17</sup>The resulting equilibrium is equivalent to a public communication equilibrium of size  $N = 3$ , but we know that the public communication game has only uninformative equilibria.

ingredient which allows to sustain informative communication in this equilibrium is that we handicap the sender in her ability to communicate with one of the receivers.

To some extent, this feature of the equilibrium is a familiar one: in many dynamic settings the parties want to commit to ex post inefficient outcomes for some states of the world in order to support outcome functions which are Pareto superior in the ex ante sense. In our situation, commitment in the literal sense is not required, because the uninformative outcome at the private stage of communication is self-enforcing. It is interesting to contrast our findings with the movement in the cheap talk literature which aims at coming up with an equilibrium refinement that picks the most informative equilibrium.<sup>18</sup> In our environment, as well as in other settings where there are several communication stages, there is a strong rationale against refining away the uninformative and other less informative outcomes.<sup>19</sup>

Equilibria of this sort can also be naturally sustained in the environments where the sender is unable to communicate privately with some of the receivers. For example, a firm may have an ability to hold a private meeting with a lender (or with a union), but be unable to communicate privately with numerous equity holders. Suppose that the firm is able to schedule a private meeting with the lender, and the equity holders observe whether the meeting is scheduled but do not know what is discussed at the meeting. The firm schedules a meeting only in the extreme situations, i.e., when the business conditions are either very good or very bad, and at the meeting the firm reveals to the lender which one is the case. Thus the scheduling of the meeting plays the role of the public message ‘*Outside*’, and the absence of the meeting plays the role of the public message ‘*Inside*’. The assumption that no private meeting between the firm and the equity holders takes place is non-controversial because it is plausible to assume that the lender could always send a spy there.

This example also demonstrates that it may be beneficial for the sender to engage the receivers in a simultaneous conversation rather than to talk to each of them independently even when the receivers are completely identical. Indeed, as we mentioned above, no information can be revealed if the sender communicates privately with each of the receivers, while in our combined communication equilibrium some information is transmitted. The key feature of our equilibrium is that the sender’s message strategy with receiver 1 differs from the message strategy with receiver 2, and thus the receivers are induced to take different actions. If there is just a single receiver, then by Lemma 1 of Crawford and Sobel (1982) all equilibria have an interval partition structure and thus the outcome function is deterministic at almost every state. However, below we show how to enrich a model with a single receiver to sustain information transmission using the idea of our example.

Assume that the sender and the single receiver with the bias of  $\frac{1}{4}$  have a fair coin. The sender sends two messages: ‘*Outside*’ if  $\theta \in [0, x) \cup [z, 1]$  and ‘*Inside*’ if  $\theta \in [x, z)$ . Following the message ‘*Outside*’ a coin is flipped. In case of ‘heads’ the sender reveals to the receiver whether  $\theta \in [0, x)$  or  $\theta \in [z, 1]$ , in case of ‘tails’ no further information is revealed. It is straightforward to see that this constitutes an equilibrium with the same values of  $x$  and  $z$  as above. Following ‘heads’ the receiver behaves as receiver 1 from our example, and following ‘tails’ he assumes the identity of receiver 2.

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<sup>18</sup>See for example Chen, Kartik and Sobel (2008) and references therein.

<sup>19</sup>See for example Aumann and Hart (2003), Krishna and Morgan (2004).

Instead of an access to a coin, we could assume that the receiver is allowed to participate in the conversation with the sender, and thus they can perform a jointly controlled lottery which replicates the coin<sup>20</sup>. We continue the discussion of the benefits of such conversations in Section 5.

Next we generalize the message of Example 2. We show that for a range of parameters it is possible to construct a nonmonotonic combined communication equilibrium.

**Proposition 5** *Suppose  $F$  is uniform,  $l_i(x) = L(x) = x^2$ . For any  $b_2 \in \mathbb{R}$ , there exists  $\varepsilon(b_2) > 0$  such that whenever  $|b_1| \leq \varepsilon(b_2)$  there exists a non-trivial nonmonotonic combined communication equilibrium.*

In the proof of this result we show that there always exists a nonmonotonic combined communication equilibrium of the same form as in Example 2 as long as the preferences of one of the receivers are closely aligned with the preferences of the sender. This is a surprising finding, because in both public equilibria and monotonic combined communication equilibria it is possible to communicate some information to an extremely biased receiver only in the situations of ‘mutual discipline’, i.e., when the large bias of one receiver is countervailed by a large bias in the opposite direction of another receiver. More specifically, informative public equilibria, as well as informative monotonic combined communication equilibria, exist only if the average bias is small enough (Proposition 2 and Lemma 6). In the constructed nonmonotonic equilibria the public signals are informative for any value of the average bias.

We do not claim that the constructed nonmonotonic equilibria are generally better for the sender than other communication arrangements. However, Example 2 shows that there are situations when this is the case.

## 5 Mediated communication and long cheap talk

### 5.1 Mediated communication

In this section we introduce the possibility of mediated communication, whereby the players communicate with a neutral trustworthy party (the mediator) who then sends back private messages to the players. The mediator does not know the state of the world and does not have the power to impose what actions the players are to take.

The value of studying mediated communication is twofold. First, it is interesting to find out when it is beneficial to invite an outside mediator to facilitate communication between the players. Second, according to the revelation principle (Myerson (1982)), any equilibrium outcome of any communication protocol (mediated or unmediated) can be replicated by the procedure whereby the sender secretly reports the state of world to a neutral trustworthy mediator, who then makes non-binding private recommendations (possibly stochastic) to each receiver of what action to take. Hence, when looking for the optimal (according to some criterion) communication protocol, it is

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<sup>20</sup>See Aumann and Hart (2003) for a discussion of jointly controlled lotteries.

enough to optimize within this class. After that one can check whether the outcome can be replicated by some unmediated communication protocol.

Formally, a mediation rule is a family  $(p(\cdot|\theta)_{\theta \in \Theta})$ , where for each  $\theta \in \Theta$ ,  $p(\cdot|\theta)$  is a probability distribution on the space of action pairs  $\mathbb{R}^2$ . Given a mediation rule, the game proceeds as follows. At the first stage, after observing the state  $\theta$ , the sender privately reports a state  $\hat{\theta}$  to the mediator. Upon hearing the report from the sender, the mediator selects the individual recommended actions  $a_1$  and  $a_2$  according to  $p(\cdot|\hat{\theta})$  and privately announces them to each receiver. The revelation principle implies that without loss of generality reporting the true state should be optimal for the sender, and obeying the mediator's recommendation should be optimal for each receiver. The mediation rules that have an equilibrium where the sender always reports the truth and each receiver always obeys the recommendation will be called *incentive compatible*.<sup>21</sup>

We are looking for incentive compatible mediation rules that maximize the ex ante utility of the sender, and focus on the case when  $F$  is uniform and the payoffs are quadratic.

**Definition 3** An *optimal mediation rule*  $p = (p(\cdot|\theta))_{\theta \in \Theta}$  is a family of probability distributions on  $\mathbb{R}^2$  that solves the following problem:

$$\max_{p(\cdot|\theta)_{\theta \in \Theta}} \int_{\mathbb{R}^2 \times \Theta} \left( - (a_1 - \theta)^2 - (a_2 - \theta)^2 \right) dp(a_1, a_2|\theta) d\theta$$

subject to

$$\theta = \arg \max_{\hat{\theta} \in \Theta} \left[ \int_{\mathbb{R}^2 \times \Theta} \left( - (a_1 - \theta)^2 - (a_2 - \theta)^2 \right) dp(a_1, a_2|\hat{\theta}) \right], \forall \theta \in \Theta; \quad (IC - S)$$

$$a_i = E_{\theta}[\theta|a_i] + b_i, \forall a_i \in \mathbb{R}, i = 1, 2. \quad (IC - R)$$

The constraints  $(IC - S)$  say that the sender should find it optimal to tell the truth. The constraints  $(IC - R)$  state that each receiver has no incentive to deviate from the action that is recommended to him by the mediator. The right-hand side of the equality is the expectation of  $\theta$  given the recommendation  $a_i$  corrected by the bias of receiver  $i$ , which is the action that maximizes the payoff of receiver  $i$  when the mediator recommends  $a_i$ . Given  $p(a_1, a_2|\theta)$  and the unconditional distribution of  $\theta$ ,  $E_{\theta}[\theta|a_i]$  is determined uniquely up to a zero-measure subset of  $\mathbb{R}$ .

**Proposition 6** (i) Let  $(b_1, b_2) \in (-\frac{1}{2}, 0)^2$ . The optimal mediation rule is characterized by two sequences of cutoff types  $0 = \theta_{1,0} < \theta_{1,1} < \dots < \theta_{1,N_1} = 1$  and  $0 = \theta_{2,0} < \theta_{2,1} < \dots < \theta_{2,N_2} = 1$  where  $N_i$  is such that  $|b_i| \in \left[ \frac{1}{2}(N_i)^{-2}, \frac{1}{2}(N_i - 1)^{-2} \right)$ , and by two numbers  $\mu_1, \mu_2 \in [0, 1]$ . If  $\theta \in [0, \theta_{i,1})$ , the mediation rule recommends action 0 to both receivers. If  $\theta \in [\theta_{i,k}, \theta_{i,k+1})$ ,  $k = 1, \dots, N_i - 1$ , receiver  $i$  is recommended action 0 with probability  $\mu_i$ , and action  $a_{i,k} = \frac{1}{2}(\theta_{i,k} + \theta_{i,k+1}) + b_i$  with probability  $1 - \mu_i$ .

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<sup>21</sup>The incentive compatible mediation rules are sometimes called *communication equilibria* (Forges (1990), Myerson (1991)).

(ii) Let  $b_i \in (-\frac{1}{2}, 0)$  and  $b_j = 0$ . The optimal mediation rule makes recommendations to receiver  $i$  as in the rule described in (i). The mediation rule makes fully revealing recommendations to receiver  $j$ , i.e., it recommends action  $\theta + b_j$  for every  $\theta \in \Theta$ .

(iii) Let  $(b_1, b_2) \in (0, \frac{1}{2})^2$ . The mediation rule is characterized by two sequences of cutoff types  $0 = \theta_{1,0} < \theta_{1,1} < \dots < \theta_{1,N_1} = 1$  and  $0 = \theta_{2,0} < \theta_{2,1} < \dots < \theta_{2,N_2} = 1$  where  $N_i$  is such that  $|b_i| \in [\frac{1}{2}(N_i)^{-2}, \frac{1}{2}(N_i - 1)^{-2})$ , and by two numbers  $\mu_1, \mu_2 \in [0, 1]$ . If  $\theta \in [\theta_{i,N_i-1}, 1]$ , the mediation rule recommends action 1 to both receivers. If  $\theta \in [\theta_{i,k}, \theta_{i,k+1})$ ,  $k = 0, \dots, N_i - 2$ , receiver  $i$  is recommended action 1 with probability  $\mu_i$ , and action  $a_{i,k} = \frac{1}{2}(\theta_{i,k} + \theta_{i,k+1}) + b_i$  with probability  $1 - \mu_i$ .

(iv) Let  $b_i \in (0, \frac{1}{2})$  and  $b_j = 0$ . The optimal mediation rule makes recommendations to receiver  $i$  as in the rule described in (iii). The mediation rule makes fully revealing recommendations to receiver  $j$ , i.e., it recommends action  $\theta + b_j$  for every  $\theta \in \Theta$ .

(v) Let  $b_1 = b_2 = 0$ . The optimal mediation rule makes fully revealing recommendations to both receivers, i.e., it recommends to receiver  $j$  action  $\theta + b_j$  for every  $\theta \in \Theta$ .

The only role of the optimal mediation rule is to introduce noise into communication. Noise makes the messages to the receivers less informative. But the presence of noise also helps to relax the incentive compatibility constraints for the sender, which makes it easier to motivate her to transmit more information.

To see how it works, consider  $b_1 = b_2 = \frac{1}{4}$ . Since both receivers have high positive biases, the sender is tempted to pretend to be a low type. This is the reason why there are no informative equilibria either in the private communication game or in the public communication game. In the optimal mediation rule, both receivers take action 1 following the report that the state is above  $\frac{5}{6}$ , and otherwise take action 1 with probability  $\frac{1}{10}$  and action  $\frac{5}{12}$  with probability  $\frac{9}{10}$ . Since the sender is risk-averse, the presence of noise following the announcement that the type is in the lower interval reduces her desire of being perceived as a low type, and thus allows to sustain informative communication.<sup>22</sup>

Mediation rules similar to the one in Proposition 6 appeared in the literature on cheap talk games before. For the CS model with a single receiver, Blume, Board and Kawamura (2007) introduced the mediation rule which is otherwise identical to ours, and Goltsman et al (2008) proved its optimality. Thus in the cases described in Proposition 6 the optimal mediation rule with two receivers is equivalent to the twice-replicated optimal mediation rule with a single receiver. It can thus be implemented with the help of two mediators, such that mediator  $i$  is allowed to communicate only with the sender and receiver  $i$  (in particular, the recommendation of mediator  $i$  has to be independent of that of mediator  $j \neq i$  and of the sender's report to mediator  $j \neq i$ ). The mediation rules that can be implemented in this fashion will be called *private mediation rules*. In particular, note that all equilibrium outcomes of the private communication game can be replicated with private mediation rules.

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<sup>22</sup>It does not matter whether randomizations for each receiver are independent or correlated, because the sender's payoff is separable in the actions of the receivers.



Besides the  $(IC - S)$  and  $(IC - R)$  constraints, the incentive compatible private mediation rules satisfy the condition that the sender has to report the state of world truthfully to each of the two mediators. If the optimal mediation rule is private, this means that there are no benefits from pooling together the sender's incentive constraints across receivers. Proposition 6 shows that this is the case when the receivers' biases are of the same sign and of moderate magnitude.

For settings with a single receiver whose absolute value of the bias is greater than  $\frac{1}{2}$ , Goltsman et al (2008) have shown that the optimal mediation rule involves no information transmission. In our model we can show an analogous result when the receivers' biases are identical.

**Lemma 7** *Let  $b_1 = b_2 \in \mathbb{R} \setminus (-\frac{1}{2}, \frac{1}{2})$ . The optimal mediation rule recommends to receiver  $i$  a constant action  $\frac{1}{2} + b_i$  for every  $\theta \in \Theta$ .*

For the cases when the receivers' biases are of the same sign that are not covered by Proposition 6 and Lemma 7, the optimal mediation rule is unknown. We conjecture that the optimal mediation rule also belongs to the class of private rules and is equivalent to the twice-replicated optimal mediation rule for the model with a single receiver.

When the receivers' biases are of the opposite sign, the optimal mediation rule is unlikely to be private, because it may now be valuable to pool the sender's truthtelling constraints across receivers. Consider, for example, the case of 'mutual discipline' discussed in Section 3.3: the receivers are biased in the opposite directions, the magnitudes of their biases are large, but are of comparable sizes. There are no informative equilibria of the private communication game, because the sender always wants to pretend to have a low type with one receiver and a high type with another receiver. However, if the sender's messages are publicly observed by both receivers, then she can no longer tell each receiver a different lie. The sender is thus forced to find a compromise between her countervailing incentives, which allows to sustain informative communication.

One special class of mediation rules, which takes full advantage of pooling the sender's truthtelling constraints across receivers, is when the mediator recommends actions to each of the receivers publicly rather than in a private manner. Such mediation rules will be called *public mediation rules*.<sup>23</sup> Note that all equilibrium outcomes of the public communication games can be achieved with public mediation rules. Also note that since the action recommendations are made publicly, the receivers' posterior distributions over states of the world must be the same. Hence the receivers' incentive constraints  $(IC - R)$  imply that all equilibrium actions of the receivers in this case coincide up to a constant as in the case of the equilibria of the public communication game, i.e.  $a_2 - b_2 = a_1 - b_1$ .<sup>24</sup> Moreover, similarly to the case of equilibria of the public communication game the public mediation rules can be shown to be equivalent (from the point of view of the sender) to a mediation rule between the sender and a single receiver with a bias equal to the average of the two biases, i.e.  $\bar{b} = \frac{b_1 + b_2}{2}$ .<sup>25</sup>

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<sup>23</sup>Our definition of public mediation rules differs from the one in Lehrer and Sorin (1997). We assume that the sender submits to the mediator a report about the state of the world, while Lehrer and Sorin (1997) allow for more general reports.

<sup>24</sup>See Lemma 1.

<sup>25</sup>See Proposition 2.

**Lemma 8** *Let  $b_1 + b_2 = 0$ . The optimal mediation rule is a fully revealing public mediation rule, i.e., it recommends to receiver  $i$  action  $\theta + b_i$  for every  $\theta \in \Theta$ .*

We do not know whether the optimal mediation rules belong to the class of public rules for other values of the receivers' biases. However it is possible to show that the ex ante payoff of the sender from the optimal public mediation rule is higher than from the optimal private mediation rule when the receivers' biases are of the opposite sign and are close in absolute values.<sup>26</sup>

Next we show that in some cases neither private nor public mediation rules are optimal. We present an example of a monotonic equilibrium of the combined communication game which performs better than any private or public mediation rule.

**Example 3** *Let  $(b_1, b_2) = (\frac{1}{40}, -\frac{11}{40})$ . Consider the following strategy. The sender sends two public messages: 'Low' if  $\theta \in [0, x)$  and 'High' if  $\theta \in [x, 1]$ , where  $x \approx 0.261$ . The sender sends an uninformative message to receiver 2 after both public messages. Following the message 'Low' the sender to receiver 1 whether  $\theta \in [0, t)$  or  $\theta \in [t, x)$ , where  $t \approx 0.180$ , and sends an uninformative message following the message 'High'.*

*This communication arrangement constitutes an equilibrium, and the ex ante utility of the sender is approximately  $-0.146$ . The ex-ante payoffs of the sender from the best private and the best public mediation rule are approximately  $-0.151$  and  $-0.149$ , respectively.<sup>27</sup>*

The optimal mechanism for the case described in Example 3 is not known. However the fact that the given monotonic equilibrium performs better than any private or public mediation rule suggests that the optimal mechanism must both take advantage of pooling the sender's truth-telling constraints across receivers, as well as transmit some of the information to the receivers in a private manner.

## 5.2 Unmediated communication protocols

In this section we discuss whether it is possible to implement optimal mediation rules by some communication schemes between the players without the use of mediator. We begin with listing the circumstances when the optimal mediation rules can be implemented as equilibria of the communication games described in Section 3.

**Lemma 9** *(i) Let  $b_1$  and  $b_2$  be of the same sign and either  $|b_i| = \frac{1}{2}(N_i)^{-2}$  for some  $N_i \in \mathbb{N}^*$  or  $b_i = 0$  for  $i = 1, 2$ . The optimal mediation rule is outcome equivalent to the most informative equilibrium of the private communication game.*

*(ii) Let  $b_1 = b_2 \in \mathbb{R} \setminus (-\frac{1}{2}, \frac{1}{2})$ . The optimal mediation rule is outcome equivalent to any babbling equilibrium of any communication game.*

*(iii) Let  $b_1 + b_2 = 0$ . Then the optimal mediation rule is outcome equivalent to the most informative equilibrium of the public communication game.*

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<sup>26</sup>The proof is available upon request.

<sup>27</sup>See Appendix for calculations.

Parts (ii) and (iii) follow directly from Lemmas 7 and 8. Part (i) of the result follows from the fact that the optimal mediation rule in Proposition 6 becomes deterministic for such values of the biases. Note, however, that when the optimal mediation rule in Proposition 6 is stochastic, then it is not possible to implement it as an equilibrium of any communication protocol from Sections 3 and 4. Since each receiver’s best response is always a singleton,<sup>28</sup> the randomization must be performed by the sender, but there can be at most a single type of the sender that is indifferent between any two given actions of receiver  $i$ .<sup>29</sup>

Let us now turn to more complicated protocols with active participation of the receivers. A general model of such protocols, or ‘long cheap talk’, was introduced by Aumann and Hart (2003). We show that with the help of long cheap talk it is indeed possible to implement the optimal mediation rule described in Proposition 6.

**Proposition 7** *Let  $(b_1, b_2) \in (-\frac{1}{2}, 0]^2 \cup [0, \frac{1}{2})^2$ . The optimal mediation rule from Proposition 6 can be achieved with long cheap talk.*

The intuition for the result is as follows. From the discussion following Proposition 6, we know that the optimal mediation rule belongs to the class of private rules, i.e. can be achieved with the help of two independent mediators, one for each receiver. The optimal mediation rule between the sender and receiver  $i$  can be thought of as a noisy communication channel: the sender reports the state of the world, but receiver  $i$  observes a signal which is only stochastically related to the true state of the world. It is possible to use receiver  $j$  to provide such noisy communication device between the sender and receiver  $i$ . Moreover, this can be done in such a way that receiver  $j$  does not learn anything about the state of the world while facilitating communication between the other players.

This arrangement is very similar to the one by Forges (1988), who studied the question of when the mediation rules in sender-receiver games can be achieved without a mediator and just with the help of a correlation device. In the proof of Proposition 7 receiver  $j$  plays a role of the correlation device in the communication between the sender and receiver  $i$ .

We do not claim that the optimal mediation rule presented in Proposition 6 is unique. Indeed it is possible to show that there exist a continuum of private mediation rules that are optimal. Some of them allow for alternative implementation via long cheap talk. Krishna and Morgan (2004) constructed two-stage cheap talk equilibria for the CS model which correspond to optimal mediation rules as long as the receiver’s bias is sufficiently small. This result can be used to implement optimal mediation rules in our model as long as the receivers’ biases are of the same sign and are sufficiently small.<sup>30</sup>

The constructions discussed so far do not work when the optimal mediation rule does not belong to the class of private rules. In this case one can use the following general result adapted

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<sup>28</sup>See footnote 2.

<sup>29</sup>Mitusch and Strausz (2005) emphasize that the advantage of using mediator comes from the possibility to implement stochastic outcomes without imposing constraints that the sender must be indifferent between the receivers’ actions.

<sup>30</sup>See Theorem 1 in Krishna and Morgan (2004) and Theorem 3 in Goltsman et al (2008) for details.

from Proposition 2 in Forges (1990). Note that a help of correlation device is in general needed for it to work.

**Proposition 8** *Consider any equilibrium of any communication protocol (mediated or unmediated) such that each receiver using only a finite number of actions. There exists a universal communication protocol and a correlated equilibrium of this protocol such that the players receive the same interim payoffs as in the original equilibrium.*

## 6 Conclusion

We have analyzed communication via various protocols between the sender and two receivers in a natural extension of the framework of Crawford and Sobel (1982). Throughout the paper we have assumed that the payoffs of each receiver are independent of the action of the other receiver, and that the sender's payoff is separable in the actions of the two receivers. Hence the only thing that links two otherwise 'separable' problems of information transmission (one between the sender and receiver 1, and the other between the sender and receiver 2) is the state of the world which is privately known by the sender.

We have identified several means by which the incentives for information transmission can be affected by simultaneous communication with both receivers in this environment. In Section 3 we have shown that using public announcements has a commitment value, because it reduces the number of deviations available to the sender. In Section 4 we have shown that under combined communication scenario it may be beneficial to reveal less information at the private communication stage in order to improve incentives for information revelation at the public communication stage. In Section 5 we have shown that it may be beneficial to use noisy communication channels, which can be replicated using multi-stage plain conversation protocols between the players.

In the environments where our 'separability' assumptions do not hold, one can expect the following additional effects to come into play. Relaxing payoff interdependencies between the receivers will bring in an element of strategic interaction at the action choice stage. The sender will have to take into account that her message announcements induce a particular information structure into the game to be played between the receivers (which might have multiple equilibria). Relaxing the separability of the sender's payoff in the actions of the receivers will complicate the receivers' inference problem when the messages are private.

There are several related topics for future research that we find interesting. First, one can analyze communication with multiple receivers in an environment where messages are costly. To the best of our knowledge, the existing models of signalling with multiple audiences do not allow for the possibility of private or combined communication. Another avenue for future research is studying communication through other realistic communication channels (for example, using private messages which become publicly known with some probability, or using the 'blind carbon copy' option for private communication). One can also extend our model to allow for communication between the receivers, or for an endogenous choice between communication modes by the sender. Finally, one

can consider a model with many receivers and multidimensional private information.

## 7 Appendix

### 7.1 Proofs of Section 3

**Proof of Proposition 1.** Let  $M_i \subseteq \mathbb{R}$  be the set of messages that can be sent to receiver  $i$ .

(i) Let  $m : \Theta \rightarrow \Delta(M_1 \times M_2)$  be an equilibrium strategy of the sender and  $\alpha_i : M_i \rightarrow \mathbb{R}$  the corresponding equilibrium strategy of receiver  $i$  in the private communication game. Consider the strategy profile for the CS game between the sender and receiver  $i$  where the sender follows the strategy  $m_i(\theta) = \text{marg}_{M_i} m(\theta) = \int_{M_{-i}} dm(\theta)$  and receiver  $i$  follows the same strategy  $\alpha_i(\theta)$  as in the equilibrium of the private communication game.

This strategy profile results in the desired outcome function, and next we show that it constitutes an equilibrium of the CS game with receiver  $i$ . Receiver  $i$  has no profitable deviations following any message  $m_i \in M_i$ , because his posterior over  $\Theta$  following any message is the same as in the original equilibrium of the private communication game. Now assume the sender of type  $\theta$  has a profitable deviation  $m'_i \in M_i$  that results in action  $a'_i$ . Then

$$\begin{aligned} & l_i(|a'_i - \theta|) + \int_{M_1 \times M_2} l_{-i}(|\alpha_{-i}(m_{-i}) - \theta|) dm(\theta) \\ & < \int_{M_i} l_i(|\alpha_i(m_i) - \theta|) dm_i(\theta) + \int_{M_1 \times M_2} l_{-i}(|\alpha_{-i}(m_{-i}) - \theta|) dm(\theta) \\ & = \int_{M_1 \times M_2} [l_i(|\alpha_i(m_i) - \theta|) + l_{-i}(|\alpha_{-i}(m_{-i}) - \theta|)] dm(\theta) \end{aligned}$$

which contradicts the fact that  $(m, \alpha_1, \alpha_2)$  is an equilibrium of the private communication game.

(ii) Let  $m_i : \Theta \rightarrow \Delta(M_i)$  an equilibrium strategy of the sender and  $\alpha_i : M_i \rightarrow \mathbb{R}$  the corresponding equilibrium strategy of receiver  $i$  in the CS game between the sender and receiver  $i$ . Consider the strategy profile for the public communication game where the sender follows the strategy  $m(\theta) = m_1(\theta) \times m_2(\theta)$  and receiver  $i$  follows the same strategy  $\alpha_i(\theta)$  as in the equilibrium of the CS game.

This strategy profile results in the desired outcome function, and next we show that it constitutes an equilibrium of the private communication game. Receiver  $i$  has no profitable deviations following any message  $m_i \in M_i$ , because his posterior over  $\Theta$  following any message is the same as in the original equilibrium of the CS game. Assume the sender of type  $\theta$  has a profitable deviation  $(m'_1, m'_2)$ .

Then

$$\begin{aligned}
& \int_{M_1} l_1(|\alpha_1(m'_1) - \theta|) dm_1(\theta) + \int_{M_2} l_2(|\alpha_2(m'_2) - \theta|) dm_2(\theta) \\
&= \int_{M_1 \times M_2} [l_1(|\alpha_1(m'_1) - \theta|) + l_2(|\alpha_2(m'_2) - \theta|)] dm(\theta) \\
&< \int_{M_1 \times M_2} [l_1(|\alpha_1(m_1(\theta)) - \theta|) + l_2(|\alpha_2(m_2(\theta)) - \theta|)] dm(\theta) \\
&= \int_{M_1} l_1(|\alpha_1(m_1(\theta)) - \theta|) dm_1(\theta) + \int_{M_2} l_2(|\alpha_2(m_2(\theta)) - \theta|) dm_2(\theta)
\end{aligned}$$

Hence there exists  $i \in \{1, 2\}$  such that

$$\int_{M_i} l_i(|\alpha_i(m'_i) - \theta|) dm_i(\theta) < \int_{M_i} l_i(|\alpha_i(m_i(\theta)) - \theta|) dm_i(\theta)$$

which contradicts the fact that  $(m_i, \alpha_i)$  is an equilibrium of the CS game. ■

**Proof of Lemma 1.** After a given message  $m$ , receiver  $i$  solves

$$\min_{a_i} \int_{\theta} L(|a_i - \theta - b_i|) dF_m(\theta)$$

where  $F_m$  is the posterior distribution of  $\theta$  following message  $m$ . The solution  $a_i$  clearly belongs to  $[b_i, 1 + b_i]$  and is unique by strict convexity of  $L$ . The first-order conditions are

$$\int_0^{a_i - b_i} L'(a_i - \theta - b_i) dF_m(\theta) = \int_{a_i - b_i}^1 L'(\theta + b_i - a_i) dF_m(\theta)$$

The above condition depends only on the difference  $a_i - b_i$ , so it implies (1). ■

Before proving Proposition 2 we need two preliminary lemmas.

**Lemma 10** *In every equilibrium of the public communication game, if  $(a_1, a_2) \in \text{support}(a(\theta))$ ,  $(a_1, a_2) \in \text{support}(a(\theta''))$  for some  $\theta < \theta''$ , then  $(a_1, a_2) \in \text{support}(a(\theta'))$ ,  $\forall \theta' \in (\theta, \theta'')$ .*

**Proof.** Suppose  $(a_1, a_2) \in \text{support}(a(\theta))$ ,  $(a_1, a_2) \notin \text{support}(a(\theta'))$  for some  $\theta' \in (\theta, \theta'')$ . Then there must exist  $(a'_1, a'_2) \in \text{support}(a(\theta'))$ ,  $(a'_1, a'_2) \neq (a_1, a_2)$ , that type  $\theta'$  weakly prefers to  $(a_1, a_2)$ :

$$\begin{aligned}
& l_1(|a_1 - \theta|) + l_2(|a_2 - \theta|) \leq l_1(|a'_1 - \theta|) + l_2(|a'_2 - \theta|); \\
& l_1(|a'_1 - \theta'|) + l_2(|a'_2 - \theta'|) \leq l_1(|a_1 - \theta'|) + l_2(|a_2 - \theta'|)
\end{aligned}$$

So, adding the inequalities and substituting (1),

$$\begin{aligned}
& l_1(|a_1 - \theta|) + l_2(|a_1 + b_2 - b_1 - \theta|) - (l_1(|a_1 - \theta'|) + l_2(|a_1 + b_2 - b_1 - \theta'|)) \\
& \leq l_1(|a'_1 - \theta|) + l_2(|a'_1 + b_2 - b_1 - \theta|) - (l_1(|a'_1 - \theta'|) + l_2(|a'_1 + b_2 - b_1 - \theta'|))
\end{aligned}$$

or

$$\int_{\theta}^{\theta'} \int_{a_1}^{a'_1} l_1''(|\tilde{a} - \tilde{\theta}|) d\tilde{a}d\tilde{\theta} + \int_{\theta}^{\theta'} \int_{a_1+b_2-b_1}^{a'_1+b_2-b_1} l_2''(|\tilde{a} - \tilde{\theta}|) d\tilde{a}d\tilde{\theta} \geq 0$$

Since  $l_i'' > 0$ ,  $\theta' > \theta$ , and  $(a_1, a_2) \neq (a'_1, a'_2)$ , this implies that  $a_1 < a'_1$  and, by (1),  $a_2 < a'_2$  as well.

Let us show that type  $\theta'' > \theta'$  strictly prefers  $(a'_1, a'_2)$  to  $(a_1, a_2)$ . The difference in the cost of actions  $(a'_1, a'_2)$  and  $(a_1, a_2)$  for type  $\theta''$  is

$$\begin{aligned} & l_1(|a'_1 - \theta''|) + l_2(|a'_2 - \theta''|) - (l_1(|a_1 - \theta''|) + l_2(|a_2 - \theta''|)) \\ &= l_1(|a'_1 - \theta'|) + l_2(|a'_2 - \theta'|) - (l_1(|a_1 - \theta'|) + l_2(|a_2 - \theta'|)) \\ & - \int_{\theta'}^{\theta''} \int_{a_1}^{a'_1} l_1''(|\tilde{a} - \tilde{\theta}|) d\tilde{a}d\tilde{\theta} - \int_{\theta'}^{\theta''} \int_{a_2}^{a'_2} l_2''(|\tilde{a} - \tilde{\theta}|) d\tilde{a}d\tilde{\theta} \\ & < l_1(|a'_1 - \theta'|) + l_2(|a'_2 - \theta'|) - (l_1(|a_1 - \theta'|) + l_2(|a_2 - \theta'|)) \leq 0 \end{aligned}$$

where the first inequality follows from  $a_1 < a'_1$ ,  $a_2 < a'_2$  and  $\theta'' > \theta'$ , and the second inequality follows from type  $\theta'$  preferring  $(a'_1, a'_2)$  to  $(a_1, a_2)$ . Consequently, type  $\theta''$  strictly prefers  $(a'_1, a'_2)$  to  $(a_1, a_2)$ , which contradicts  $(a_1, a_2) \in \text{support}(a(\theta''))$ . ■

**Lemma 11** *Let  $l_1 \equiv l_2 \equiv l$ , and  $b_1 + b_2 \neq 0$ . Then the set of actions induced in any equilibrium of the public communication game is finite.*

**Proof.** Suppose  $(a_1, a_2)$  and  $(a'_1, a'_2)$  are two distinct equilibrium action pairs in the public communication game. By Lemma 1, all equilibrium action pairs lie on the line  $a_2 = a_1 + b_2 - b_1$ , so without loss of generality assume  $a_1 < a'_1$ ,  $a_2 < a'_2$ . By continuity of  $l(\cdot)$ , there exists type  $\theta$  which is indifferent between  $(a_1, a_2)$  and  $(a'_1, a'_2)$ :

$$l(|a_1 - \theta|) + l(|a_2 - \theta|) = l(|a'_1 - \theta|) + l(|a'_2 - \theta|)$$

Denote

$$g(x) = l(|x|) + l(|x + b_2 - b_1|) \tag{2}$$

for  $x \in \mathbb{R}$ . Then the indifference condition for type  $\theta$  can be rewritten as  $g(a_1 - \theta) = g(a'_1 - \theta)$ . One can easily verify that  $l' > 0$ ,  $l'' > 0$  implies that  $g$  is strictly convex and reaches a global minimum at  $x = -\frac{b_2 - b_1}{2}$ , and  $g(x) = g(y)$  if and only if  $|x + \frac{b_2 - b_1}{2}| = |y + \frac{b_2 - b_1}{2}|$ . Hence

$$\theta = \frac{a_1 + a'_1}{2} + \frac{b_2 - b_1}{2}.$$

Lemma 10 implies that  $(a_1, a_2) \notin \text{support}(a(\tilde{\theta}))$  for any  $\tilde{\theta} > \theta$ , and  $(a'_1, a'_2) \notin \text{support}(a(\tilde{\theta}))$  for any  $\tilde{\theta} < \theta$ . Hence the most preferred action of receiver 1 when the state is  $\theta$ , which is equal to  $\theta + b_1$ , is in the interval  $[a_1, a'_1]$ :

$$\theta + b_1 = \frac{a_1 + a'_1}{2} + \frac{b_1 + b_2}{2} \in [a_1, a'_1]$$

Hence

$$a'_1 - a_1 \geq |b_1 + b_2|$$

On the other hand, all equilibrium actions belong to  $[b_1, 1 + b_1]$ , which gives the result. ■

Now we are ready to prove Proposition 2.

**Proof of Proposition 2.** (i) By Lemma 11 there is a finite number of equilibrium action pairs. Since by Lemma 1 all equilibrium action pairs lie on the line  $a_2 = a_1 + b_2 - b_1$ , there is only a finite number of types which are indifferent between any pair of the equilibrium action pairs. Hence Lemma 10 implies the result.

(ii) Let  $(a_1, a_2)$  be a pair of actions chosen by type  $\theta$  in a given equilibrium of the public communication game, and  $(a'_1, a'_2)$  be a pair of actions that is chosen by some other type. Then it must be the case that

$$l(|a_1 - \theta|) + l(|a_2 - \theta|) \leq l(|a'_1 - \theta|) + l(|a'_2 - \theta|)$$

Substituting (1):

$$l(|a_1 - \theta|) + l(|a_1 + b_2 - b_1 - \theta|) \leq l(|a'_1 - \theta|) + l(|a'_1 + b_2 - b_1 - \theta|) \quad (3)$$

Consider the function  $g$  introduced in the proof of Lemma 11. Then (3) can be rewritten as  $g(a_1 - \theta) \leq g(a'_1 - \theta)$ . It is easy to verify that  $g(x) \leq g(y)$  if and only if  $\left|x + \frac{b_2 - b_1}{2}\right| \leq \left|y + \frac{b_2 - b_1}{2}\right|$ . So (3) holds if and only if

$$\left|a_1 + \frac{b_2 - b_1}{2} - \theta\right| \leq \left|a'_1 + \frac{b_2 - b_1}{2} - \theta\right| \quad (4)$$

Now consider the CS model with a receiver with the loss function  $L\left(\left|a_i - \theta - \frac{b_1 + b_2}{2}\right|\right)$ . Let us check that this model has an equilibrium characterized by the same cutoffs. Indeed, if the sender follows the same strategy as in the original equilibrium, the receiver after given message will take the action equal to  $a_1 - b_1 + \frac{b_1 + b_2}{2} = a_1 + \frac{b_2 - b_1}{2}$ , where  $a_1$  is the action of receiver 1 in the original equilibrium. By (4), the sender's original strategy is the best response to these actions of the receivers. ■

Before proving Proposition 3, we review some standard results for the CS game. Let  $F$  be uniform on  $[0, 1]$ , the utility function of the sender be  $-(a - \theta)^2$  and the utility function of the receiver be  $-(a - \theta - b)^2$  where  $b \in \mathbb{R}$ .

**Lemma 12** *Every equilibrium of the CS game is characterized by a sequence of cutoff types  $0 = \theta_0 < \theta_1 < \dots < \theta_N = 1$  such that*

$$\theta_k = 2k(N - k)b + \frac{k}{N} \text{ for every } k = 0, \dots, N,$$



and  $N$  is an integer from  $\{1, \dots, N(b)\}$  where

$$N(b) = \left\lceil -\frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{2}{|b|}} \right\rceil.$$

The outcome function is

$$a(\theta) = \frac{1}{2} (\theta_{k-1} + \theta_k) + b \text{ for } \theta \in (\theta_{k-1}, \theta_k)$$

The ex ante utilities of the receiver and the sender in the equilibrium of size  $N$  are

$$-\frac{1}{12} N^{-2} - \frac{1}{3} b^2 (N^2 - 1)$$

and

$$-\frac{1}{12} N^{-2} - \frac{1}{3} b^2 (N^2 - 1) - b^2,$$

respectively.

**Proof.** See Section 4 in Crawford and Sobel (1982). ■

**Lemma 13** *The ex ante utilities of each receiver and the sender in the most informative public communication equilibrium are*

$$-\frac{1}{12} (N(\bar{b}))^{-2} - \frac{1}{3} (\bar{b})^2 \left( (N(\bar{b}))^2 - 1 \right)$$

and

$$-\frac{1}{6} (N(\bar{b}))^{-2} - \frac{1}{3} (\bar{b})^2 \left( (N(\bar{b}))^2 - 1 \right) - (b_1)^2 - (b_2)^2,$$

respectively, where  $N(\bar{b}) = \left\lceil -\frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{2}{|\bar{b}|}} \right\rceil$  and  $\bar{b} = \frac{b_1 + b_2}{2}$ .

**Proof.** By Proposition 2, the equilibrium cutoff types are the same as in the CS game with a receiver with bias  $\bar{b}$ . Using Lemma 12, we can calculate ex ante utility of receiver  $i$  in an equilibrium of size  $N$ .

$$\begin{aligned} -E \left( (a_i - \theta - b_i)^2 \right) &= - \sum_{k=1}^N \int_{\theta_{k-1}}^{\theta_k} \left( \frac{1}{2} (\theta_{k-1} + \theta_k) - \theta \right)^2 d\theta = - \frac{1}{12} \sum_{k=1}^N (\theta_k - \theta_{k-1})^3 \\ &= - \frac{1}{12} \sum_{k=1}^N \left( \frac{1}{N} + 2\bar{b}(1 + N - 2k) \right)^3 = - \frac{1}{12} N^{-2} - \frac{1}{3} (\bar{b})^2 (N^2 - 1) \end{aligned}$$

The ex ante utility of the sender is

$$\begin{aligned}
& -E\left((a_1 - \theta)^2\right) - E\left((a_2 - \theta)^2\right) \\
= & -E\left((a_1 - \theta - b_1)^2\right) - (b_1)^2 - E\left((a_2 - \theta - b_2)^2\right) - (b_2)^2 \\
= & -\frac{1}{6}N^{-2} - \frac{1}{3}(\bar{b})^2(N^2 - 1) - (b_1)^2 - (b_2)^2
\end{aligned}$$

By Lemma 12, the most informative equilibrium is of size  $N(\bar{b}) = \left\lceil -\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{2}{|\bar{b}|}} \right\rceil$ . ■

**Proof of Proposition 3.** Define  $f(b) = -\frac{1}{12}(N(b))^{-2} - \frac{1}{3}b^2(N(b)^2 - 1)$ , where  $N(b)$  is the number of distinct actions in the most informative equilibrium of the CS game where the receiver's bias is  $b$ . By Lemmas 12 and 13, private communication is better than public if and only if

$$\Delta(b_1, b_2) := f(b_1) + f(b_2) - 2f(\bar{b}) > 0$$

1.  $\Delta(b_1, b_2) = 0$ .
2.  $\Delta(b_1, b_2) = -\frac{1}{12}(N(b_i))^{-2} - \frac{1}{3}(b_i)^2(N(b_i)^2 - 1) + \frac{1}{12} > 0$ .
3.  $\Delta(b_1, b_2) = -\frac{1}{6} + \frac{1}{6}(N(\bar{b}))^{-2} + \frac{2}{3}(\bar{b})^2(N(\bar{b})^2 - 1) < 0$ .
4. Without loss of generality, fix  $b_1 \in (-\frac{1}{4}, \frac{1}{4})$  and assume  $b_2 > \frac{1}{4}$  (the case when  $b_2 < \frac{1}{4}$  is symmetric). This implies  $\bar{b} \in (0, \frac{1}{4})$  and  $b_2 \in (\frac{1}{4}, \frac{1}{2} - b_1)$ . We show that private communication outperforms public communication iff

$$b_2 > B(b_1) = \sqrt{\frac{1}{12} - 2f(b_1)} - b_1.$$

The ex-ante payoff of the sender from private talk is equal to the one from public talk when

$$\Delta(b_1, b_2) = f(b_1) - \frac{1}{12} - 2f\left(\frac{b_1 + b_2}{2}\right) = 0.$$

Note that the right hand side is strictly increasing in  $b_2$  since  $\frac{b_1 + b_2}{2} > 0$ . Hence, for every  $b_1$  there can be at most one  $b_2$  where this equality is satisfied. We will show that is satisfied in the following region:  $\frac{1}{12} \leq \frac{b_1 + b_2}{2} \leq \frac{1}{4}$  and  $b_2 > \frac{1}{4}$ .

Note that in such case  $N\left(\frac{b_1 + b_2}{2}\right) = 2$ , and thus

$$2f\left(\frac{b_1 + b_2}{2}\right) = -\frac{1}{24} - \frac{1}{2}(b_1 + b_2)^2.$$

Substituting this expression into the above equality and solving for  $b_2$  as a function of  $b_1$  gives:

$$b_2(b_1) = \sqrt{\frac{1}{12} - 2f(b_1)} - b_1.$$

Note that  $\frac{1}{12} \leq \frac{b_1 + b_2(b_1)}{2} \leq \frac{1}{4}$  if and only if

$$\frac{1}{6} \leq \sqrt{\frac{1}{12} - 2f(b_1)} \leq \frac{1}{2}$$

which can be rewritten as

$$-\frac{1}{12} \leq f(b_1) \leq \frac{1}{36}.$$

These inequalities are always satisfied since the range for  $f$  is  $[-\frac{1}{12}, 0]$ . Finally, note that  $b_2(b_1) > \frac{1}{4}$  if and only if

$$\frac{1}{96} - \frac{1}{4}b_1 - \frac{1}{2}(b_1)^2 > f(b_1) \tag{5}$$

which can be rewritten as

$$\left(\frac{1}{96} + \frac{1}{12}(N(b_1))^{-2}\right) - \frac{1}{4}b_1 + \frac{1}{3}\left(N(b_1)^2 - \frac{5}{2}\right)(b_1)^2 > 0.$$

Note that  $N(b_1) \geq 2$  since  $|b_1| < \frac{1}{4}$ , and thus

$$\left(\frac{1}{96} + \frac{1}{12}(N(b_1))^{-2}\right) - \frac{1}{4}b_1 + \frac{1}{3}\left(N(b_1)^2 - \frac{5}{2}\right)(b_1)^2 \geq \frac{1}{4}\left(\frac{1}{24} - b_1\right)$$

Hence the inequality is satisfied when  $b_1 \leq \frac{1}{24}$ .

Consider  $b_1 \in (\frac{1}{12}, \frac{1}{4})$ . Then  $N(b_1) = 2$  and inequality (5) in this case can be rewritten as

$$\frac{1}{2}\left(\frac{1}{4} - b_1\right)^2 > 0,$$

which is always true.

Consider  $b_1 \in (\frac{1}{24}, \frac{1}{12})$ . Then  $N(b_1) = 3$  and inequality (5) in this case can be rewritten as

$$\frac{17}{864} - \frac{1}{4}b_1 + \frac{13}{6}(b_1)^2 > 0,$$

which is always true.

5. Notice that we have  $N(b_1) = N(b_2) = N$ , and

$$|\bar{b}| \leq \frac{1}{2}|b_1| + \frac{1}{2}|b_2| \leq \frac{1}{2N(N-1)}$$

where the first inequality is by triangle inequality. Thus there exists a public talk equilibrium of size  $N$ . Hence

$$\Delta(b_1, b_2) = -\frac{1}{6}(N^2 - 1)(b_1 - b_2)^2 < 0$$

unless  $b_1 = b_2$ . ■

## 7.2 Proofs of Section 4

**Proof of Lemma 2.** Conditional on any public message, the agents have the same posterior on  $\Theta$ , so the proof of Lemma 10 goes through. ■

**Proof of Lemma 3.** Suppose  $\theta' > \theta$ . From incentive compatibility for the sender:

$$\begin{aligned} -l_1(|a_1(\theta) - \theta|) - l_2(|a_2(\theta) - \theta|) &\geq -l_1(|a_1(\theta') - \theta|) - l_2(|a_2(\theta') - \theta|); \\ -l_1(|a_1(\theta') - \theta'|) - l_2(|a_2(\theta') - \theta'|) &\geq -l_1(|a_1(\theta) - \theta'|) - l_2(|a_2(\theta) - \theta'|) \end{aligned}$$

Add up and rearrange to get

$$\begin{aligned} (l_1(|a_1(\theta') - \theta|) - l_1(|a_1(\theta) - \theta|)) - (l_1(|a_1(\theta') - \theta'|) - l_1(|a_1(\theta) - \theta'|)) \\ + (l_2(|a_2(\theta') - \theta|) - l_2(|a_2(\theta) - \theta|)) - (l_2(|a_2(\theta') - \theta'|) - l_2(|a_2(\theta) - \theta'|)) \geq 0 \end{aligned}$$

Hence

$$\int_{\theta}^{\theta'} \left( \int_{a_1(\theta)}^{a_1(\theta')} l_1''(|\tilde{a} - \tilde{\theta}|) d\tilde{a} + \int_{a_2(\theta)}^{a_2(\theta')} l_2''(|\tilde{a} - \tilde{\theta}|) d\tilde{a} \right) d\tilde{\theta} \geq 0$$

Note that  $l_i''(|x|) > 0$  for every  $x$  and  $i = 1, 2$ . Hence, we cannot have both  $a_1(\theta') < a_1(\theta)$  and  $a_2(\theta') < a_2(\theta)$ . ■

**Proof of Lemma 4.** Fix a public message  $m$ , following which there is informative private communication with receiver  $i$ . Then, following this message, the sender is able to induce at least two different actions of this receiver,  $a_i$  and  $a'_i$  such that  $a_i < a'_i$ . By Lemma 2 the equilibrium outcome of every receiver conditional on a given public message is partitional. Hence there must exist  $\theta_1, \theta_2, \theta_3$  and  $\theta_4$  such that  $0 \leq \theta_1 \leq \theta_2 \leq \theta_3 \leq \theta_4 \leq 1$ , and action  $a_i$  is chosen by the types from  $[\theta_1, \theta_2]$  and action  $a'_i$  is chosen by the types from  $[\theta_3, \theta_4]$ . This implies that  $a_i \in [\theta_1 + b_i, \theta_2 + b_i]$  and  $a'_i \in [\theta_3 + b_i, \theta_4 + b_i]$ .

Type  $\theta_2$  must prefer  $a_i$  to  $a'_i$ :  $-l_i(|a_i - \theta_2|) \geq -l_i(|a'_i - \theta_2|)$ . Because  $l_i$  is strictly increasing, this implies  $\frac{1}{2}a'_i + \frac{1}{2}a_i \geq \theta_2$ . Thus

$$b_i \geq a_i - \theta_2 \geq -\frac{1}{2}(a'_i - a_i) \geq -\frac{1}{2}(\theta_4 - \theta_1) \geq -\frac{1}{2}$$

Type  $\theta_3$  must prefer  $a'_i$  to  $a_i$ :  $-l_i(|a'_i - \theta_3|) \geq -l_i(|a_i - \theta_3|)$ . Because  $l_i$  is strictly increasing, this implies  $\frac{1}{2}a'_i + \frac{1}{2}a_i \leq \theta_3$ . Thus

$$b_i \leq a'_i - \theta_3 \leq \frac{1}{2}(a'_i - a_i) \leq \frac{1}{2}(\theta_4 - \theta_1) \leq \frac{1}{2}.$$

■

**Proof of Lemma 5.** Suppose the equilibrium is not partitional, i.e.  $\exists i \in \{1, 2\}, \theta, \theta' \in [0, 1], \theta'' \in$

$(\theta, \theta')$  such that  $a_i(\theta) = a_i(\theta') = a_i$ ,  $a_i(\theta'') \neq a_i$ . Suppose  $a_i(\theta'') < a_i$  (the opposite case is treated similarly). Then  $a_i(\theta'') < a_i(\theta) = a_i(\theta')$ , which is a contradiction to the equilibrium being monotonic. ■

**Proof of Lemma 6.** (i) There must exist a public signal such that there is further communication with receiver  $i$ . If following this public signal receiver  $i$  takes an infinite number of different actions in equilibrium, then  $b_i = 0 \in [-\frac{1}{4}, \frac{1}{4}]$ . Suppose receiver  $i$  takes a finite number of actions in equilibrium, say, action  $\underline{a}_i$  is taken when  $\theta \in (x, y)$  and  $\bar{a}_i$  is taken when  $\theta \in (y, z)$ . Because  $F$  is uniform,  $\underline{a}_i = \frac{1}{2}x + \frac{1}{2}y + b_i$ ,  $\bar{a}_i = \frac{1}{2}y + \frac{1}{2}z + b_i$ .

Since the utility of the sender is separable in the actions of the two receivers, she can optimize over private messages to be sent to receiver  $i$  independently of which messages she plans to send to receiver  $j$ . Thus type  $y$  is indifferent between  $\underline{a}_i$  and  $\bar{a}_i$  if  $l_i(|\underline{a}_i - y|) = l_i(|\bar{a}_i - y|)$ , which implies that

$$y = \frac{1}{2}(\underline{a}_i + \bar{a}_i) = \frac{1}{4}x + \frac{1}{2}y + \frac{1}{4}z + b_i,$$

or

$$y = \frac{1}{2}x + \frac{1}{2}z + 2b_i$$

Note that we need  $z - y \geq 0$ , which implies  $\frac{1}{4}(z - x) \geq b_i$ . Hence,  $\frac{1}{4} \geq b_i$ .

Note that we also need  $y - x \geq 0$ , which implies  $b_i \geq -\frac{1}{4}(z - x)$ . Hence,  $b_i \geq -\frac{1}{4}$ .

(ii) If communication with both receivers is informative, then

$$\left| \frac{1}{2}b_1 + \frac{1}{2}b_2 \right| \leq \frac{1}{2}|b_1| + \frac{1}{2}|b_2| \leq \frac{1}{4},$$

where the first inequality is by triangle inequality, and the second inequality follows from the proof of (i).

Assume there is informative communication at the public stage, as well as informative private communication only with receiver 1 (the argument for receiver 2 is similar). Then there exist  $a, x, y, z, c$  such that  $0 \leq a \leq x \leq y \leq z \leq c \leq 1$  and receiver 1 gets informed whether  $\theta \in (x, y)$  or  $\theta \in (y, z)$ , whether receiver 2 gets informed whether  $\theta \in (a, y)$  or  $\theta \in (y, c)$ . This implies

$$(a_1(\theta), a_2(\theta)) = \begin{cases} (\frac{1}{2}x + \frac{1}{2}y + b_1, \frac{1}{2}a + \frac{1}{2}y + b_2) & \text{if } \theta \in (x, y) \\ (\frac{1}{2}y + \frac{1}{2}z + b_1, \frac{1}{2}y + \frac{1}{2}c + b_2) & \text{if } \theta \in (y, z) \end{cases}$$

Type  $y$  is indifferent between two action profiles  $(a_1, a_2)$  and  $(a'_1, a'_2)$  if

$$y = \frac{(a'_1 - a_1)}{(a'_1 - a_1) + (a'_2 - a_2)} \left( \frac{a_1 + a'_1}{2} \right) + \frac{(a'_2 - a_2)}{(a'_1 - a_1) + (a'_2 - a_2)} \left( \frac{a'_2 + a_2}{2} \right)$$

Hence

$$y = \frac{(z - x)}{(z - x) + (c - a)} \left( \frac{1}{4}x + \frac{1}{2}y + \frac{1}{4}z + b_1 \right) + \frac{(c - a)}{(z - x) + (c - a)} \left( \frac{1}{4}a + \frac{1}{2}y + \frac{1}{4}c + b_2 \right),$$

or

$$y = \frac{(z-x)}{(z-x)+(c-a)} \left( \frac{1}{2}x + \frac{1}{2}z \right) + \frac{(c-a)}{(z-x)+(c-a)} \left( \frac{1}{2}a + \frac{1}{2}c \right) + 2B,$$

where  $B := \frac{(z-x)}{(z-x)+(c-a)}b_1 + \frac{(c-a)}{(z-x)+(c-a)}b_2$ .

Note that we need  $z - y \geq 0$ , which implies

$$\frac{1}{4} \left( (z-x) + \frac{(c-a)}{(z-x)+(c-a)} (z+x-a-c) \right) \geq B$$

If  $z+x-a-c \leq 0$ , then we have  $\frac{1}{4}(z-x) \geq B$ , and hence  $\frac{1}{4} \geq B$ . If  $z+x-a-c > 0$ , then using  $\frac{(c-a)}{(z-x)+(c-a)} \leq 1$  we have  $\frac{1}{4}(2z-a-c) \geq B$ , and hence  $\frac{1}{4} \geq B$ .

Note that we also need  $y - x \geq 0$ , which implies

$$B \geq -\frac{1}{4} \left( (z-x) - \frac{(c-a)}{(z-x)+(c-a)} (z+x-a-c) \right)$$

If  $z+x-a-c \geq 0$ , then we have  $B \geq -\frac{1}{4}(z-x)$ , and hence  $B \geq -\frac{1}{4}$ . If  $z+x-a-c < 0$ , then using  $\frac{(c-a)}{(z-x)+(c-a)} \leq 1$  we have  $B \geq -\frac{1}{4}(a+c-2x)$ , and hence  $B \geq -\frac{1}{4}$ . Thus

$$\begin{aligned} \left| \frac{1}{2}b_1 + \frac{1}{2}b_2 \right| &= \frac{1}{2} \left| \frac{(c-a)-(z-x)}{(c-a)}b_1 + \frac{(z-x)+(c-a)}{(c-a)}B \right| \\ &\leq \frac{1}{2} \frac{(c-a)-(z-x)}{(c-a)} |b_1| + \frac{1}{2} \frac{(z-x)+(c-a)}{(c-a)} |B| \\ &\leq \frac{1}{2} \frac{(c-a)-(z-x)}{(c-a)} \frac{1}{4} + \frac{1}{2} \frac{(z-x)+(c-a)}{(c-a)} \frac{1}{4} = \frac{1}{4} \end{aligned}$$

where the equality follows from the definition of  $B$ , the first inequality follows from the triangle inequality and the definitions of  $a, c, x$  and  $z$ , the second inequality uses the facts that  $|b_1| \leq \frac{1}{4}$  (follows from part (i)) and  $|B| \leq \frac{1}{4}$  (derived above). ■

To prove Proposition 4 we construct the following monotonic equilibrium of the combined communication game. Consider a sequence  $0 = \theta_0 < t < \theta_1 < \dots < \theta_N = 1$ . We say that  $(\theta_0, t, \theta_1, \dots, \theta_N)$  constitute a ‘type-I’ equilibrium of size  $N$  if there exists an equilibrium of the following form:

- 1) At the public stage, the sender announces an element of a partition  $[\theta_k, \theta_{k+1}]$ ,  $k = 0, \dots, N-1$ ;
- 2) At the private stage, if  $\theta \in [0, \theta_1]$ , the sender announces to receiver 1 whether  $\theta \in [0, t]$  or  $\theta \in [t, \theta_1]$ .

**Lemma 14** *Type-I equilibrium of size  $N$  takes the following form:*

$$(a_1(\theta), a_2(\theta)) = \begin{cases} \left( \frac{1}{2}t + b_1, \frac{1}{2}\theta_1 + b_2 \right) & \text{if } \theta \in (0, t) \\ \left( \frac{1}{2}(t + \theta_1) + b_1, \frac{1}{2}\theta_1 + b_2 \right) & \text{if } \theta \in (t, \theta_1) \\ \left( \frac{1}{2}(\theta_{k-1} + \theta_k) + b_1, \frac{1}{2}(\theta_{k-1} + \theta_k) + b_2 \right) & \text{if } \theta \in (\theta_k, \theta_{k+1}) \end{cases}$$

for  $k = 2, \dots, N$ , where

$$\theta_1 = \frac{\left( 1 - \sqrt{1 - (1 - \frac{5}{8}(N-1)^2)((1+N(N-1)(b_1+b_2))(1+(N-1)(N-2)(b_1+b_2)) - 8(N-1)^2b_1^2)} \right)}{1 - \frac{5}{8}(N-1)^2},$$

$$\theta_k = \frac{k-1}{N-1} + (N-k)(k-1)(b_1 + b_2) + \frac{N-k}{N-1}\theta_1 \text{ for } k = 2, \dots, N.$$

**Proof.** Define a sequence of actions for receiver 1  $\underline{a}_{1,1} < \bar{a}_{1,1} < a_{1,2} < \dots < a_{1,N}$ , and for receiver 2  $a_{2,1} < a_{2,2} < \dots < a_{2,N}$ . The equilibrium conditions of the receivers are given in the statement of the Lemma. The equilibrium conditions for the sender:

$$t = \frac{1}{2}(\underline{a}_{1,1} + \bar{a}_{1,1}),$$

$\theta_1$  is such that

$$-(\bar{a}_{1,1} - \theta_1)^2 - (a_{2,1} - \theta_1)^2 = -(a_{1,2} - \theta_1)^2 - (a_{2,2} - \theta_1)^2,$$

and

$$\theta_k = \frac{1}{4}(a_{1,k} + a_{2,k} + a_{1,k+1} + a_{2,k+1}) \text{ for every } k = 2, \dots, N-1.$$

Solving for  $t$  gives:  $t = \frac{1}{2}\theta_1 + 2b_1$ . Solving for the equilibrium cutoffs  $(\theta_2, \dots, \theta_N)$  gives

$$\theta_k = \frac{k-1}{N-1} + (N-k)(k-1)(b_1 + b_2) + \frac{N-k}{N-1}\theta_1 \text{ for } k = 2, \dots, N.$$

Thus the indifference condition for type  $\theta_1$  becomes:

$$\begin{aligned} & - \left( \left( \frac{3}{4}\theta_1 + 2b_1 \right) - \theta_1 \right)^2 - \left( \left( \frac{1}{2}\theta_1 + b_2 \right) - \theta_1 \right)^2 \\ & = - \left( \left( \frac{1}{2}(\theta_1 + \theta_2) + b_1 \right) - \theta_1 \right)^2 - \left( \left( \frac{1}{2}(\theta_1 + \theta_2) + b_2 \right) - \theta_1 \right)^2 \end{aligned}$$

where

$$\theta_2 = \frac{1}{N-1} + (N-2)(b_1 + b_2) + \frac{N-2}{N-1}\theta_1.$$

After rearranging:

$$-\frac{5}{16}(\theta_1)^2 + (b_1 + b_2)\theta_1 - 4b_1^2 - b_2^2 = -\frac{1}{2}(\theta_2 - \theta_1)^2 - (b_1 + b_2)(\theta_2 - \theta_1) - b_1^2 - b_2^2.$$

Substituting out  $\theta_2 - \theta_1$  and simplifying gives:

$$\left( \frac{1}{(N-1)^2} - \frac{5}{8} \right) (\theta_1)^2 - \left( \frac{2}{(N-1)^2} \right) \theta_1 + \left( \frac{1}{(N-1)^2} + (N-2)N(b_1 + b_2)^2 + 2(b_1 + b_2) - 8b_1^2 \right) = 0.$$

Solving for  $\theta_1$  yields a solution given in the statement of the Lemma. ■

**Lemma 15** *Let  $b_1 = 0$  and  $b_2 \in (-\frac{1}{2}, 0)$ . If there exists a public communication equilibrium of size  $N$  then there exists type-I equilibrium of size  $N$ .*

**Proof.** Note that to show that type-I equilibrium of size  $N$  exists it is enough to demonstrate that  $\theta_{k+1} - \theta_k \geq 0$  for  $k = 1, \dots, N-1$  and  $\theta_1 \geq 0$ .

*Case 1.* Let  $N = 2$ . Here we need to show  $\theta_1 \in (0, 1)$ .

By Lemma 14:  $\theta_1 = \frac{8}{3} \left( 1 - \sqrt{1 - \frac{3}{8}(1 + 2b_2)} \right)$ . Hence we need  $b_2 \in \left(-\frac{1}{2}, \frac{5}{16}\right)$  which is always satisfied since  $b_2 \in \left(-\frac{1}{2}, 0\right)$ .

*Case 2.* Let  $N > 2$ . Note that by Lemma 14:

$$\theta_{k+1} - \theta_k = \frac{1}{N-1} (1 - \theta_1) + (N - 2k) b_2 \text{ for } k = 1, \dots, N - 1.$$

Since  $b_2 < 0$  it is enough to show that  $\theta_2 - \theta_1 \geq 0$ , or  $1 + (N - 1)(N - 2)b_2 \geq \theta_1$ .

Note that  $1 - \frac{5}{8}(N - 1)^2 < 0$  so by Lemma 14 we need to show that

$$\begin{aligned} & \sqrt{1 - \left(1 - \frac{5}{8}(N - 1)^2\right) \left((1 + N(N - 1)b_2)(1 + (N - 1)(N - 2)b_2)\right)} \\ & \leq 1 - \left(1 - \frac{5}{8}(N - 1)^2\right) (1 + (N - 1)(N - 2)b_2) \end{aligned}$$

or

$$(1 - N(N - 1)b_2)(1 + (N - 1)(N - 2)b_2) \geq \left(1 - \frac{5}{8}(N - 1)^2\right) (1 + (N - 1)(N - 2)b_2)^2.$$

By Lemma 13 a necessary condition for a public communication equilibrium of size  $N$  to exist is  $1 + N(N - 1)b_2 > 0$ . Since  $b_2 \in \left(-\frac{1}{2}, 0\right)$  this implies  $1 + (N - 1)(N - 2)b_2 > 0$ , and thus the above inequality can be simplified to

$$0 \geq (N - 1)^2 \left(2b_2 - \frac{5}{8}(1 + (N - 1)(N - 2)b_2)\right)$$

which is always true since  $b_2 \in \left(-\frac{1}{2}, 0\right)$ .

Now let us show that  $\theta_1 \geq 0$ . By Lemma 14 we need to show that

$$1 \leq \sqrt{1 - \left(1 - \frac{5}{8}(N - 1)^2\right) \left((1 + N(N - 1)b_2)(1 + (N - 1)(N - 2)b_2)\right)}$$

or

$$0 \leq (1 + N(N - 1)b_2)(1 + (N - 1)(N - 2)b_2)$$

which is always true. ■

**Lemma 16** *Let  $b_1 = 0$  and  $b_2 \in \left(-\frac{1}{2}, 0\right)$ . Assume there exist a public communication equilibrium of size  $N$  and a type-I equilibrium of size  $N$ . Then the latter yields a higher payoff to the sender than the former.*

**Proof.** By Lemma 19 it is enough to show that the equilibrium payoff of type  $\theta = 1$  in the type-I equilibrium of size  $N$  yields higher payoff than the public communication equilibrium of size  $N$ . By



Lemma 13 the payoff type  $\theta = 1$  the public communication equilibrium of size  $N$  is

$$- \left( \left( \frac{1}{2} \theta_{N-1}^* + \frac{1}{2} \right) - 1 \right)^2 - \left( \left( \frac{1}{2} \theta_{N-1}^* + \frac{1}{2} + b_2 \right) - 1 \right)^2$$

where

$$\theta_{N-1}^* = \frac{N-1}{N} + (N-1)b_2.$$

Rearranging gives

$$-\frac{1}{2} (\theta_{N-1}^* - 1)^2 - b_2 (\theta_{N-1}^* - 1) - b_2^2.$$

By Lemma 14 the payoff type  $\theta = 1$  the type-I equilibrium of size  $N$  is

$$- \left( \left( \frac{1}{2} \theta_{N-1} + \frac{1}{2} \right) - 1 \right)^2 - \left( \left( \frac{1}{2} \theta_{N-1} + \frac{1}{2} + b_2 \right) - 1 \right)^2$$

where

$$\theta_{N-1} = \frac{N-2}{N-1} + (N-2)b_2 + \frac{1}{N-1}\theta_1.$$

Rearranging gives

$$-\frac{1}{2} (\theta_{N-1} - 1)^2 - b_2 (\theta_{N-1} - 1) - b_2^2.$$

Hence we need to show that

$$-\frac{1}{2} (\theta_{N-1} - 1)^2 - b_2 (\theta_{N-1} - 1) - \left( -\frac{1}{2} (\theta_{N-1}^* - 1)^2 - b_2 (\theta_{N-1}^* - 1) \right) > 0,$$

or

$$(\theta_{N-1}^* - \theta_{N-1}) (\theta_{N-1}^* + \theta_{N-1} - 2 + 2b_2) > 0.$$

Note that  $\theta_{N-1}^* + \theta_{N-1} - 2 + 2b_2 < 0$ , so it is enough to show  $\theta_{N-1}^* - \theta_{N-1} < 0$ . Also note that

$$\theta_{N-1}^* - \theta_{N-1} = \frac{1}{N-1} \left( \frac{1}{N} + (N-1)b_2 - \theta_1 \right).$$

Hence we need to show  $\frac{1}{N} + (N-1)b_2 < \theta_1$ .

*Case 1.* Let  $N = 2$ . Here we need to show that  $\frac{1}{2} + b_2 < \frac{8}{3} \left( 1 - \sqrt{1 - \frac{3}{8}(1 + 2b_2)} \right)$ , or  $0 < \left( \frac{3}{8} \right)^2 \left( \frac{1}{2} + b_2 \right)^2$ , which is true since  $b_2 \in \left( -\frac{1}{2}, 0 \right)$ .

*Case 2.* Let  $N > 2$ . Note that  $1 - \frac{5}{8}(N-1)^2 < 0$  so by Lemma 14 we need to show that

$$\begin{aligned} & \sqrt{1 - \left( 1 - \frac{5}{8}(N-1)^2 \right) (1 + N(N-1)b_2) (1 + (N-1)(N-2)b_2)} \\ & > 1 - \frac{1}{N} \left( 1 - \frac{5}{8}(N-1)^2 \right) (1 + N(N-1)b_2) \end{aligned}$$

or

$$\begin{aligned} & (1 + N(N-1)b_2)(1 + (N-1)(N-2)b_2) \\ & > \frac{2}{N}(1 + N(N-1)b_2) - \frac{1}{N^2} \left(1 - \frac{5}{8}(N-1)^2\right) (1 + N(N-1)b_2)^2. \end{aligned}$$

By Lemma 13 a necessary condition for a public communication equilibrium of size  $N$  to exist is  $1 + N(N-1)b_2 > 0$ . Thus

$$N^2(1 + (N-1)(N-2)b_2) > 2N - \left(1 - \frac{5}{8}(N-1)^2\right) (1 + N(N-1)b_2)$$

or  $1 + N(N-1)b_2 > 0$  which was shown above to hold. ■

**Proof of Proposition 4.** Let  $b_2 \in (-\frac{1}{2}, 0)$ . By Lemmas 15 and 16 when  $b_1 = 0$  there exists type-I equilibrium which yields a strictly higher payoff than the best public communication equilibrium.

By Lemma 14 the cutoff values defining the type-I equilibrium are continuous functions of  $b_1$ . Hence for every  $b_2 \in (-\frac{1}{2}, 0)$  a type-I equilibrium exists and yields a strictly higher payoff than the best public communication equilibrium whenever  $b_1$  is close enough to 0.

It is straightforward to prove an analogous statement for  $b_2 \in (0, -\frac{1}{2})$  using a ‘type-Ib’ equilibrium of the following kind. Consider a sequence  $0 = \theta_0 < \theta_1 < \dots < \theta_{N-1} < t < \theta_N = 1$ .

- 1) At the public stage, the sender announces an element of a partition  $[\theta_k, \theta_{k+1}]$ ,  $k = 0, \dots, N-1$ ;
- 2) At the private stage, if  $\theta \in [\theta_{N-1}, 1]$ , the sender announces to receiver 1 whether  $\theta \in [\theta_{N-1}, t]$  or  $\theta \in [t, 1]$ .<sup>31</sup> ■

To prove Proposition 5 we construct a nonmonotonic equilibrium of the combined communication game similar to the one in Example 2. Consider a pair  $(x, z)$  such that  $0 < x < z < 1$ . We say that  $(x, z)$  constitute a ‘type-II’ equilibrium if there exists an equilibrium of the following form:

- 1) At the public stage, the sender announces whether  $\theta \in [0, x) \cup [z, 1]$  or  $\theta \in [x, z)$ ;
- 2) At the private stage, if  $\theta \in [0, x) \cup [z, 1]$ , the sender announces to receiver 1 whether  $\theta \in [0, x)$  or  $\theta \in [z, 1]$ .

**Lemma 17** *Type-II equilibrium takes the following form:*

$$(a_1(\theta), a_2(\theta)) = \begin{cases} \left( \left(\frac{1}{2}x\right) + b_1, \left(\frac{x}{x+1-z}\left(\frac{1}{2}x\right) + \frac{1-z}{x+1-z}\left(\frac{1}{2}z + \frac{1}{2}\right)\right) + b_2 \right) & \text{if } \theta \in [0, x) \\ \left( \left(\frac{1}{2}x + \frac{1}{2}z\right) + b_1, \left(\frac{1}{2}x + \frac{1}{2}z\right) + b_2 \right) & \text{if } \theta \in [x, z) \\ \left( \left(\frac{1}{2}z + \frac{1}{2}\right) + b_1, \left(\frac{x}{x+1-z}\left(\frac{1}{2}x\right) + \frac{1-z}{x+1-z}\left(\frac{1}{2}z + \frac{1}{2}\right)\right) + b_2 \right) & \text{if } \theta \in [z, 1] \end{cases},$$

where  $x = (1-d)\left(\frac{1}{2} + \frac{2(1+d)b_1}{(d^2-6d+1)}\right)$ ,  $z = x + d$ , and  $d$  solves the following equation:

$$(1+d) \left( (1-3d) - 16b_1^2 \frac{(3-d)(3d^2-6d-1)}{(d^2-6d+1)^2} - 64b_1b_2 \frac{1}{(d^2-6d+1)} \right) = 0.$$

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<sup>31</sup>The details are available upon request.

**Proof.** The equilibrium conditions of the receivers are given in the statement of the Lemma. The equilibrium conditions for the sender are given by the indifference condition for type  $x$ :

$$\begin{aligned} & - \left( \frac{1}{2}x + b_1 - x \right)^2 - \left( \frac{x}{x+1-z} \left( \frac{1}{2}x \right) + \frac{1-z}{x+1-z} \left( \frac{1}{2}z + \frac{1}{2} \right) + b_2 - x \right)^2 \\ = & - \left( \frac{1}{2}x + \frac{1}{2}z + b_1 - x \right)^2 - \left( \frac{1}{2}x + \frac{1}{2}z + b_2 - x \right)^2 \end{aligned} \quad (6)$$

and the indifference condition for type  $z$ :

$$\begin{aligned} & - \left( \frac{1}{2}z + \frac{1}{2} + b_1 - z \right)^2 - \left( \frac{x}{x+1-z} \left( \frac{1}{2}x \right) + \frac{1-z}{x+1-z} \left( \frac{1}{2}z + \frac{1}{2} \right) + b_2 - z \right)^2 \\ = & - \left( \frac{1}{2}x + \frac{1}{2}z + b_1 - z \right)^2 - \left( \frac{1}{2}x + \frac{1}{2}z + b_2 - z \right)^2 \end{aligned} \quad (7)$$

Denote  $d = z - x$ , and subtract (7) from (6):

$$\frac{1}{4} (1-d) (2x + d - 4b_1 - 1) + d \left( 1 - \frac{2}{1-d}x + 2b_2 \right) = 2(b_1 + b_2) d.$$

Solving for  $x$  gives:

$$x = (1-d) \left( \frac{1}{2} + \frac{2(1+d)b_1}{(d^2 - 6d + 1)} \right).$$

Substitute  $x$  back into equation (6) and simplify to get

$$\frac{d+1}{(d^2 - 6d + 1)^2} \left( \begin{aligned} & -3d^5 + 37d^4 + (-126 + 48b_1^2) d^3 + (74 - 240b_1^2 - 64b_1b_2) d^2 \\ & + (-15 + 272b_1^2 + 384b_1b_2) d + (1 + 48b_1^2 - 64b_2b_1) \end{aligned} \right) = 0$$

which can be rewritten as

$$(1+d) \left( (1-3d) - 16b_1^2 \frac{(3-d)(3d^2 - 6d - 1)}{(d^2 - 6d + 1)^2} - 64b_1b_2 \frac{1}{(d^2 - 6d + 1)} \right) = 0.$$

■

**Proof of Proposition 5.** First we show that there exists a type-II equilibrium when  $b_1 = 0$  and  $b_2 \in \mathfrak{R}$ .

When  $b_1 = 0$  the equilibrium conditions (given in Lemma 17) simplify to:

$$x = \frac{1}{2} - \frac{1}{2}d, \quad z = \frac{1}{2} + \frac{1}{2}d, \quad \text{where } d \text{ solves } (1+d)(1-3d) = 0.$$

The only feasible solution is  $d = \frac{1}{3}$ , which gives  $(x, z) = \left( \frac{1}{3}, \frac{2}{3} \right)$ . Hence,

$$(a_1(\theta), a_2(\theta)) = \begin{cases} \left( \frac{1}{6}, \frac{1}{2} + b_2 \right) & \text{if } \theta \in \left[ 0, \frac{1}{3} \right) \\ \left( \frac{1}{2}, \frac{1}{2} + b_2 \right) & \text{if } \theta \in \left[ \frac{1}{3}, \frac{2}{3} \right) \\ \left( \frac{5}{6}, \frac{1}{2} + b_2 \right) & \text{if } \theta \in \left[ \frac{2}{3}, 1 \right] \end{cases}.$$

Note that this equilibrium is outcome-equivalent to the private communication equilibrium, where with the first receiver there is talk of size  $N(b_1) = 3$ , and there is babbling with the second

receiver ( $N(b_2) = 1$ ).

The left-hand side of the equation which determines  $d$  (given in Lemma 17) is continuously differentiable in  $(d, b_1)$  in an open neighborhood of  $(\frac{1}{3}, 0)$  and has a nonzero partial derivative in  $d$  at  $(\frac{1}{3}, 0)$ . Hence, for  $b_1$  close to zero there exists a feasible type-II equilibrium. Moreover, notice that whenever  $b_1 \neq 0$  this equilibrium involves nontrivial transmission of information to the second receiver:

$$\left( \frac{x}{x+1-z} \left( \frac{1}{2}x \right) + \frac{1-z}{x+1-z} \left( \frac{1}{2}z + \frac{1}{2} \right) \right) - \left( \frac{1}{2}x + \frac{1}{2}z \right) = \frac{1}{2} - \frac{1}{1-d}x = -\frac{2(1+d)b_1}{d^2 - 6d + 1}$$

This expression is not equal to 0 whenever  $b_1 \neq 0$ . ■

### 7.3 Proofs of Section 5

To characterize incentive compatible rules we rely on techniques from Goltsman et al (2008). Let  $\alpha_i(\hat{\theta}) = \int_{\mathbb{R}} a_i dp(a_1, a_2 | \hat{\theta})$  and  $\sigma_i^2(\hat{\theta}) = \int_{\mathbb{R}} (a_i - \alpha_i(\hat{\theta}))^2 dp(a_1, a_2 | \hat{\theta})$  be the conditional expectation and the variance of  $a_i$  given a message  $\hat{\theta}$ . Then an expected payoff of the sender of type  $\theta$  who reported a message  $\hat{\theta}$  in the mediation rule  $p$  is

$$\begin{aligned} U(\theta, \hat{\theta}) &: = \int_{\mathbb{R}^2} \left( -(a_1 - \theta)^2 - (a_2 - \theta)^2 \right) dp(a_1, a_2 | \hat{\theta}) \\ &= -\left( \alpha_1(\hat{\theta}) - \theta \right)^2 - \left( \alpha_2(\hat{\theta}) - \theta \right)^2 - \sigma_1^2(\hat{\theta}) - \sigma_2^2(\hat{\theta}). \end{aligned}$$

Hence, the truthtelling conditions for the sender ( $IC - S$ ) can be written as follows:

$$\theta = \arg \max_{\hat{\theta} \in \Theta} \left[ -\left( \alpha_1(\hat{\theta}) - \theta \right)^2 - \left( \alpha_2(\hat{\theta}) - \theta \right)^2 - \sigma_1^2(\hat{\theta}) - \sigma_2^2(\hat{\theta}) \right], \quad \forall \theta \in \Theta$$

Let  $U(\theta) := U(\theta, \theta)$  be the payoff of type  $\theta$  who reports his type truthfully, and  $U = E_{\theta} [U(\theta)]$  be the ex ante payoff of the sender.

**Lemma 18**  $\{\alpha_1(\theta), \alpha_2(\theta), \sigma_1^2(\theta), \sigma_2^2(\theta)\}_{\theta \in \Theta}$  is incentive compatible for the sender if and only if

- (i)  $\alpha_1(\theta) + \alpha_2(\theta)$  is non-decreasing;
- (ii)  $-\sigma_1^2(\theta) - \sigma_2^2(\theta) = U(\theta) + (\alpha_1(\theta) - \theta)^2 + (\alpha_2(\theta) - \theta)^2$ ,  
and  $U(\theta) = U(0) + \int_0^{\theta} 2 \left( \alpha_1(\tilde{\theta}) + \alpha_2(\tilde{\theta}) - 2\tilde{\theta} \right) d\tilde{\theta}$ .

**Proof. (Only If)**

- (i) From incentive compatibility for the sender for every  $\theta, \theta' \in \Theta$  we have

$$\begin{aligned} & -(\alpha_1(\theta) - \theta)^2 - (\alpha_2(\theta) - \theta)^2 - \sigma_1^2(\theta) - \sigma_2^2(\theta) \\ \geq & -(\alpha_1(\theta') - \theta)^2 - (\alpha_2(\theta') - \theta)^2 - \sigma_1^2(\theta') - \sigma_2^2(\theta') \end{aligned}$$

and

$$\begin{aligned} & -(\alpha_1(\theta') - \theta')^2 - (\alpha_2(\theta') - \theta')^2 - \sigma_1^2(\theta') - \sigma_2^2(\theta') \\ \geq & -(\alpha_1(\theta) - \theta')^2 - (\alpha_2(\theta) - \theta')^2 - \sigma_1^2(\theta) - \sigma_2^2(\theta). \end{aligned}$$

Adding up and rearranging we get

$$(\theta' - \theta) ((\alpha_1(\theta') + \alpha_2(\theta')) - (\alpha_1(\theta) + \alpha_2(\theta))) \geq 0$$

(ii) Note that we can express

$$-\sigma_1^2(\theta) - \sigma_2^2(\theta) = U(\theta) + (\alpha_1(\theta) - \theta)^2 + (\alpha_2(\theta) - \theta)^2.$$

By the generalized Envelope Theorem (Corollary 1 in Milgrom and Segal (2002)) we have

$$U(\theta) = U(0) + \int_0^\theta 2(\alpha_1(\tilde{\theta}) + \alpha_2(\tilde{\theta}) - 2\tilde{\theta}) d\tilde{\theta}.$$

(If) We need to show that for every  $\theta, \theta' \in \Theta$ ,

$$U(\theta) - \left( -(\alpha_1(\theta') - \theta')^2 - (\alpha_2(\theta') - \theta')^2 - \sigma_1^2(\theta') - \sigma_2^2(\theta') \right) \geq 0.$$

Notice that

$$\begin{aligned} & -(\alpha_1(\theta') - \theta')^2 - (\alpha_2(\theta') - \theta')^2 - \sigma_1^2(\theta') - \sigma_2^2(\theta') \\ = & -(\alpha_1(\theta') - \theta')^2 - (\alpha_2(\theta') - \theta')^2 - \sigma_1^2(\theta') - \sigma_2^2(\theta') \\ & + 2\alpha_1(\theta')(\theta - \theta') + 2\alpha_2(\theta')(\theta - \theta') - 2(\theta)^2 + 2(\theta')^2 \\ = & U(\theta') - \int_\theta^{\theta'} 2(\alpha_1(\theta') + \alpha_2(\theta') - 2\tilde{\theta}) d\tilde{\theta}. \end{aligned}$$

So

$$\begin{aligned} & U(\theta) - U(\theta') + \int_\theta^{\theta'} 2(\alpha_1(\theta') + \alpha_2(\theta') - 2\tilde{\theta}) d\tilde{\theta} \\ = & \int_\theta^{\theta'} 2((\alpha_1(\theta') + \alpha_2(\theta')) - (\alpha_1(\tilde{\theta}) + \alpha_2(\tilde{\theta}))) d\tilde{\theta} \geq 0. \end{aligned}$$

■

Next we derive a useful relation between the ex-ante expected payoff of the sender ( $U$ ) and the expected payoffs of the lowest type of the sender ( $U(0)$ ) and of the highest type of the sender ( $U(1)$ ).

**Lemma 19** *If a mediation rule  $p$  is incentive compatible, then*

$$U = \frac{1}{3}U(0) + \frac{1}{3}(b_1 + b_2) - \frac{2}{3}\left((b_1)^2 + (b_2)^2\right) = \frac{1}{3}U(1) - \frac{1}{3}(b_1 + b_2) - \frac{2}{3}\left((b_1)^2 + (b_2)^2\right).$$

**Proof.** First notice that using the results of Lemma 18:

$$\begin{aligned} U &= E\left(- (a_1 - \theta)^2 - (a_2 - \theta)^2\right) = \int_0^1 U(\theta) d\theta \\ &= U(0) + \int_0^1 \int_0^\theta 2\left(\alpha_1(\tilde{\theta}) + \alpha_2(\tilde{\theta}) - 2\tilde{\theta}\right) d\tilde{\theta} d\theta \\ &= U(0) + 2 \int_0^1 (\alpha_1(\theta) + \alpha_2(\theta))(1 - \theta) d\theta - \frac{2}{3}. \end{aligned} \tag{8}$$

Next, notice that the incentive constraints for receiver  $i$  imply

$$E[a_i] = E[\theta] + b_i = \frac{1}{2} + b_i \text{ for } i = 1, 2. \tag{9}$$

Moreover,

$$\begin{aligned} E[(a_1 + a_2)\theta] &= E[a_1 E(\theta | a_1)] + E[a_2 E(\theta | a_2)] \\ &= E[a_1(a_1 - b_1)] + E[a_2(a_2 - b_2)] = E[a_1^2] + E[a_2^2] - b_1 E[a_1] - b_2 E[a_2] \\ &= E[a_1^2 + a_2^2] - \frac{1}{2}(b_1 + b_2) - \left((b_1)^2 + (b_2)^2\right). \end{aligned} \tag{10}$$

Using (9) and (10) we can rewrite (8) as follows

$$\begin{aligned} U &= U(0) + 2 \int_0^1 (\alpha_1(\theta) + \alpha_2(\theta))(1 - \theta) d\theta - \frac{2}{3} \\ &= U(0) + 2E[a_1 + a_2] - 2E[(a_1 + a_2)\theta] - \frac{2}{3} \\ &= U(0) - 2E[a_1^2 + a_2^2] + 3(b_1 + b_2) + 2\left((b_1)^2 + (b_2)^2\right) + \frac{4}{3}. \end{aligned} \tag{11}$$

On the other hand

$$\begin{aligned} U &= E\left(- (a_1 - \theta)^2 - (a_2 - \theta)^2\right) = -E[a_1^2 + a_2^2] + 2E[(a_1 + a_2)\theta] - 2E[\theta^2] \\ &= E[a_1^2 + a_2^2] - (b_1 + b_2) - 2\left((b_1)^2 + (b_2)^2\right) - \frac{2}{3}. \end{aligned} \tag{12}$$

Combine (11) and (12) to get

$$E[a_1^2 + a_2^2] = \frac{1}{3}U(0) + \frac{4}{3}(b_1 + b_2) + \frac{4}{3}\left((b_1)^2 + (b_2)^2\right) + \frac{2}{3}.$$

Hence

$$U = \frac{1}{3}U(0) + \frac{1}{3}(b_1 + b_2) - \frac{2}{3}\left((b_1)^2 + (b_2)^2\right).$$

Finally, notice that

$$U(1) = U(0) + \int_0^1 2 \left( \alpha_1(\tilde{\theta}) + \alpha_2(\tilde{\theta}) - 2\tilde{\theta} \right) d\tilde{\theta} = U(0) + 2(b_1 + b_2)$$

which implies

$$U = \frac{1}{3}U(1) - \frac{1}{3}(b_1 + b_2) - \frac{2}{3} \left( (b_1)^2 + (b_2)^2 \right).$$

■

**Proof of Proposition 6.** (i) Let  $(b_1, b_2) \in (-\frac{1}{2}, 0)^2$ . Constraints  $(IC - R)$  imply

$$\begin{aligned} a_{i,0} &= \frac{\theta_{i,1} \left( \frac{1}{2}\theta_{i,1} \right) + \mu_i (1 - \theta_{i,1}) \left( \frac{1}{2}\theta_{i,1} + \frac{1}{2} \right)}{\theta_{i,1} + \mu_i (1 - \theta_{i,1})} + b_i; \\ a_{i,k} &= \frac{1}{2} (\theta_{i,k} + \theta_{i,k+1}) + b_i, \quad k = 1, \dots, N_i - 1. \end{aligned}$$

Constraints  $(IC - S)$  can be shown to imply

$$\theta_{i,k} = \frac{1}{2} (a_{i,k-1} + a_{i,k}), \quad k = 1, \dots, N_i - 1.$$

It is straightforward but tedious to show that the unique solution satisfying  $a_{i,0} = 0$  is as follows:

$$\begin{aligned} \theta_{i,k} &= 2|b_i|k^2 - (2|b_i|N_i^2 - 1) \frac{2k-1}{2N_i-1}, \quad k = 1, \dots, N_i; \\ a_{i,k} &= |b_i|k - 2|b_i|k(N_i - k) + \frac{(2 - |b_i|)k}{2N_i - 1}, \quad k = 0, \dots, N_i - 1; \\ \mu_i &= 1 - \frac{1 - 2|b_i|}{4(1 - |b_i|)} \left( \frac{1}{N_i - 1} - \frac{1}{N_i} - \frac{2 - |b_i|}{|b_i|N_i - 1} + \frac{2 - |b_i|}{|b_i|N_i - |b_i| + 1} \right). \end{aligned}$$

This mechanism has a property  $U(0) = 0$ , and thus, by Lemma 19, it yields the highest possible ex ante utility of the sender  $U$ .

The argument for the cases (ii)-(v) is similar.<sup>32</sup> ■

**Proof of Lemma 7.** Consider the case  $b_1 = b_2 = b \in [\frac{1}{2}, +\infty)$  (the argument for the other case is symmetric). First note that this mediation rule is incentive compatible. Assume there exists a mediation rule  $\{\alpha_1(\theta), \alpha_2(\theta), \sigma_1^2(\theta), \sigma_2^2(\theta)\}_{\theta \in \Theta}$  which gives the sender a strictly higher ex ante utility than the constant rule. By Lemma 19 the payoff of the highest type of the sender must be from this mediation rule must be higher than from the constant rule, i.e.,

$$-(\alpha_1(1) - 1)^2 - (\alpha_2(1) - 1)^2 - \sigma_1^2(1) - \sigma_2^2(1) > -\left(\frac{1}{2} + b - 1\right)^2 - \left(\frac{1}{2} + b - 1\right)^2 \quad (13)$$

---

<sup>32</sup>A mechanism of the same form appears in Proposition 9 in Blume, Board and Kawamura (2007) and in Theorem 2 in Goltsman et al (2008).

By part (i) of Lemma 18 and equation (9) in Lemma 19 we must have

$$\alpha_1(1) + \alpha_2(1) \geq E(\alpha_1(\theta) + \alpha_2(\theta)) = E(a_1 + a_2) = 1 + 2b.$$

This together with the Jensen inequality gives

$$-(\alpha_1(1) - 1)^2 - (\alpha_2(1) - 1)^2 \leq -2 \left( \frac{\alpha_1(1) + \alpha_2(1)}{2} - 1 \right)^2 \leq -2 \left( \frac{1}{2} + b - 1 \right)^2.$$

Since  $\sigma_1^2(1), \sigma_2^2(1) \geq 0$  we get a contradiction with equation (13). ■

**Proof of Lemma 8.** Notice that by the Jensen inequality the ex ante payoff of the sender can be bounded above as follows

$$U = E \left( -(a_1 - \theta)^2 - (a_2 - \theta)^2 \right) \leq - (E(a_1 - \theta))^2 - (E(a_2 - \theta))^2 = -(b_1)^2 - (b_2)^2$$

Consider a mediation rule that recommends to receiver  $i$  action  $\theta + b_i$  for every  $\theta \in \Theta$ . This rule is incentive compatible and the ex ante payoff of the sender from this rule achieves the above upper bound. ■

### Calculations for Example 3.

First we show that this communication arrangement constitutes an equilibrium. Following the public message ‘Low’ and the consequent private communication receiver 1 takes an action equal to the expected state conditional on  $\theta \in [0, t)$  or on  $\theta \in [t, x)$  corrected by his bias  $b_1 = \frac{1}{40}$ . Following the public message ‘Low’ receiver 2 takes an action equal to the expected state conditional on  $\theta \in [0, x)$  corrected by his bias  $b_2 = -\frac{11}{40}$ . Following the public message ‘High’ both receivers take actions equal to the expected state conditional on  $\theta \in [x, 1]$  corrected by their respective biases. Hence the equilibrium outcome is as follows:

$$(a_1(\theta), a_2(\theta)) = \begin{cases} \left( \frac{1}{2}t + \frac{1}{40}, \frac{1}{2}x - \frac{11}{40} \right) & \text{if } \theta \in [0, t) \\ \left( \frac{1}{2}(t+x) + \frac{1}{40}, \frac{1}{2}x - \frac{11}{40} \right) & \text{if } \theta \in [t, x) \\ \left( \frac{1}{2}(x+1) + \frac{1}{40}, \frac{1}{2}(x+1) - \frac{11}{40} \right) & \text{if } \theta \in [x, 1] \end{cases}.$$

Let us check incentive compatibility for the sender. Type  $t$  is indifferent between a strategy of sending the public message ‘Low’, with a consequent revelation to receiver 1 that her type is in  $[0, t)$ , and a strategy of sending the public message ‘Low’, with a consequent revelation to receiver 1 that her type is in  $[t, x)$

$$-\left( \frac{1}{2}t + \frac{1}{40} - t \right)^2 - \left( \frac{1}{2}x - \frac{11}{40} - t \right)^2 = -\left( \frac{1}{2}(t+x) + \frac{1}{40} - t \right)^2 - \left( \frac{1}{2}x - \frac{11}{40} - t \right)^2$$

Type  $x$  is indifferent between a strategy of sending the public message ‘Low’, with a consequent revelation to receiver 1 that her type is in  $[t, x)$ , and a strategy of sending the public message ‘High’



if

$$\begin{aligned} & - \left( \frac{1}{2}(t+x) + \frac{1}{40} - x \right)^2 - \left( \frac{1}{2}x - \frac{11}{40} - x \right)^2 \\ = & - \left( \frac{1}{2}(x+1) + \frac{1}{40} - x \right)^2 - \left( \frac{1}{2}(x+1) - \frac{11}{40} - x \right)^2 \end{aligned}$$

The solution is  $x = \frac{8}{3} - \frac{1}{30}\sqrt{5209} \approx 0.261$  and  $t = \frac{83}{60} - \frac{1}{60}\sqrt{5209} \approx 0.180$ .

To calculate the ex ante payoff of the sender in this equilibrium we first calculate the payoff of the highest type. The payoff of type  $\theta = 1$  is

$$- \left( \frac{1}{2}(x+1) + \frac{1}{40} - 1 \right)^2 - \left( \frac{1}{2}(x+1) - \frac{11}{40} - 1 \right)^2 = \frac{17}{360}\sqrt{5209} - \frac{5677}{1440} \approx -0.534$$

Using Lemma 19 the ex ante payoff of the sender is approximately  $-0.146$ .

By Proposition 6 in the best private mediator the sender of type  $\theta = 1$  gets action 1 from receiver 1, and a lottery between actions 0 and  $\frac{3}{10}$  with probabilities  $\frac{2}{17}$  and  $\frac{15}{17}$  from receiver 2.

$$U(1) = -(1-1)^2 - \frac{2}{17}(0-1)^2 - \frac{15}{17}\left(\frac{3}{10}-1\right)^2 = -0.55$$

Using Lemma 19 the ex ante payoff of the sender from the best private mediator is approximately  $-0.151$ .

Since the average bias is  $-\frac{1}{8}$ , by Lemma 9 we know that the best public mediator is deterministic. By Proposition 6 the size of partition is  $N = 2$  and the cutoff type  $\theta_1$  is equal to  $\frac{1}{4}$ . Thus in the best public mediator the sender of type  $\theta = 1$  gets action  $\frac{13}{20}$  from receiver 1, and action  $\frac{7}{20}$  from receiver 2.

$$U(1) = - \left( \frac{13}{20} - 1 \right)^2 - \left( \frac{7}{20} - 1 \right)^2 = -0.545$$

Using Lemma 19 the ex ante payoff of the sender from the best private mediator is approximately  $-0.149$ .

**Proof of Lemma 9.** (i) Note that  $\mu_i$  in the proof of Proposition 6 is equal to 0 when  $|b_i| = \frac{1}{2}(N_i)^{-2}$  for some  $N_i = 1, 2, \dots$ . Thus the mediation rule in Proposition 6 is equivalent to the most informative equilibrium of the CS game between the sender and receiver  $i$  described in Lemma 12.

(ii) The optimal mediation rule in Lemma 7 is equivalent to the babbling outcome of any communication protocol.

(iii) The optimal mediation rule in Lemma 8 is equivalent to the outcome of the most informative equilibrium of the public communication game. ■

**Proof of Proposition 7.** We construct a communication protocol that implements the optimal mediation rule when  $(b_1, b_2) \in (-\frac{1}{2}, 0)^2$  (the other cases are similar). Take  $N_j$ ,  $\mu_i$ , and  $(\theta_{j,1}, \dots, \theta_{j,N_j-1})$  to be the same as in Proposition 6 for  $j = 1, 2$ .

The protocol has three stages:

1. For  $i = 1, 2$ , receiver  $i$  produces  $2N_j - 2$  independent draws from the uniform distribution on  $[0, 1]$  (call them  $x_1, \dots, x_{N_j-1}, y_1, \dots, y_{N_j-1}$ ) and a draw from the Bernoulli distribution over outcomes  $\alpha, \beta$  with probabilities  $\mu_i, 1 - \mu_i$ ;
2. If outcome  $\alpha$  has realized, receiver  $i$  informs the sender privately of  $x = (x_1, \dots, x_{N_j-1})$ , otherwise he informs the sender of  $y = (y_1, \dots, y_{N_j-1})$ , but in either case he does not tell the sender whether the reported vector is  $x$  or  $y$ . Receiver  $i$  also informs receiver  $j$  of  $x$  privately;
3. Sender sends private messages to both receivers.

It is straightforward to verify that this protocol has the following equilibrium. Receiver  $i$  randomizes according to the description above. Sender's message to receiver  $j$  is a uniform draw from  $[0, 1]$  if  $\theta \in [0, \theta_{j,1})$  and the  $k$ th element of the vector reported to her by receiver  $i$  if  $\theta \in [\theta_{j,k}, \theta_{j,k+1})$ ,  $k = 1, \dots, N_j - 1$ . After receiving a message from the sender, receiver  $j$  takes action  $a_j = 0$  if the number he gets from the sender does not coincide with any of the  $N_j - 1$  numbers reported to him by receiver  $i$  at stage 2; receiver  $j$  takes action  $a_{j,k} = \frac{1}{2}(\theta_{j,k} + \theta_{j,k+1}) + b_j$  if the number he gets from the sender coincides with the  $k$ th element of the vector reported to him by receiver  $i$  at stage 2. ■

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