Prettner, Klaus; Trimborn, Timo

Working Paper
Demographic change and R&D-based economic growth: Reconciling theory and evidence

cege Discussion Papers, No. 139

Provided in Cooperation with:
Georg August University of Göttingen, cege - Center for European, Governance and Economic Development Research

Suggested Citation: Prettner, Klaus; Trimborn, Timo (2012) : Demographic change and R&D-based economic growth: Reconciling theory and evidence, cege Discussion Papers, No. 139, University of Göttingen, Center for European, Governance and Economic Development Research (cege), Göttingen

This Version is available at:
http://hdl.handle.net/10419/70221

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
DEMOGRAPHIC CHANGE AND R&D-BASED ECONOMIC GROWTH: RECONCILING THEORY AND EVIDENCE

Klaus Prettner, Timo Trimborn
Demographic change and R&D-based economic growth: reconciling theory and evidence

Klaus Prettner\textsuperscript{a} and Timo Trimborn\textsuperscript{b}

\textsuperscript{a) University of Göttingen
Chair of International Economics
Platz der Göttinger Sieben 3
37073 Göttingen, Germany

\textsuperscript{b) University of Göttingen
Chair of International Economics
Platz der Göttinger Sieben 3
37073 Göttingen, Germany

Abstract

In recent decades, most industrialized countries experienced declining population growth rates caused by declining fertility and associated with rising life expectancy. We analyze the effect of continuing demographic change on medium- and long-run economic growth by setting forth an R&D-based growth model including an analytically tractable demographic structure. Our results show that, in response to demographic change, technological progress and economic growth accelerate in the medium run but slow down in the long run. Numerical investigation reveals that the time period during which technological progress and economic growth are faster than without demographic change can be very long. Since the theoretical predictions for the medium run are consistent with the negative association between population growth and economic growth found in the empirical literature, the present framework can reconcile R&D-based growth theory with the available empirical evidence.

\textbf{JEL classification:} J11, O30, O41
\textbf{Keywords:} demographic change, technological progress, economic growth, semi-endogenous growth theory, transitional dynamics
1 Introduction

Over the last decades substantial demographic changes have been taking place in industrialized countries. While increasing life expectancy allowed individuals to gain additional years of life, declining fertility has slowed down population growth. These developments have been the main driving forces behind population aging. For the G-7 countries, the situation is depicted in Table 1 which lists the crude birth rate (CBR) and life expectancy (LE) in 1950 and 2010. Without exception, each of these countries experienced substantial decreases in birth rates and increases in life expectancy. The US, for example, experienced a decline of birth rates from 24 to 14 per thousand inhabitants and a rise of life expectancy at birth from 69 to 78 years. Consequently, population growth declined from 1.62% to 0.89% during this period.

While the literature discusses several channels through which aging may affect economic prosperity (e.g. pensions, health-care), the sheer fact that population growth declines may also impact economic growth. Since most of the industrialized countries experienced considerable population growth during the last century, it is unclear whether economic growth continues at the same pace if population growth declines. In this paper we investigate how this particular aspect of demographic change impacts medium- and long-run economic growth perspectives in the context of modern economic growth theory. We do this by setting forth an R&D-based economic growth model with a tractable demographic structure. Consistent with the post World War II experience of industrialized countries, we analyze declining population growth caused by declining fertility and accompanied by rising life expectancy. We then trace the consequences of demographic change on economic growth during the transition phase toward the long-run equilibrium as well as in the long-run equilibrium itself.

Table 1: CBR and LE for the G-7 countries in 1950 and 2010 according to United Nations (2011)

<table>
<thead>
<tr>
<th>Country</th>
<th>CBR 1950</th>
<th>CBR 2010</th>
<th>LE 1950</th>
<th>LE 2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>28</td>
<td>11</td>
<td>69</td>
<td>81</td>
</tr>
<tr>
<td>France</td>
<td>19</td>
<td>13</td>
<td>67</td>
<td>81</td>
</tr>
<tr>
<td>Germany</td>
<td>16</td>
<td>8</td>
<td>68</td>
<td>80</td>
</tr>
<tr>
<td>Italy</td>
<td>18</td>
<td>9</td>
<td>66</td>
<td>81</td>
</tr>
<tr>
<td>Japan</td>
<td>24</td>
<td>9</td>
<td>62</td>
<td>83</td>
</tr>
<tr>
<td>U.K.</td>
<td>15</td>
<td>12</td>
<td>69</td>
<td>80</td>
</tr>
<tr>
<td>USA</td>
<td>24</td>
<td>14</td>
<td>69</td>
<td>78</td>
</tr>
</tbody>
</table>

At first sight, the prediction of R&D based growth theory about how declining population growth affects economic growth is gloomy. Considering semi-endogenous growth models, the balanced growth rate of output is proportional to population growth and,
hence, declining population growth slows down economic growth in the long-run. The reason for the connection lies in the research sector, where the existing stock of technology has a positive external effect on the accumulation of new technology (“standing on shoulders effect”). However, the positive effect is diminishing with respect to the amount of already accumulated technology. Hence, for sustained economic growth a rising supply of researchers is needed. On a balanced growth path, this perpetual increase of researchers can only be achieved by population growth. Declining population growth impedes the supply and, therefore, hampers economic development.

Empirical evidence, on the other hand, has not confirmed this prediction. Studies on the connection between population growth and output growth have rather found a negative association (see for example Brander and Dowrick, 1994; Kelley and Schmidt, 1995; Ahituv, 2001; Bernanke and Gürkaynak, 2001). This implies that demographic change toward slower population growth could have a positive impact on economic growth. In order to reconcile theory and evidence, Dalgaard and Kreiner (2001), Strulik (2005) and Strulik et al. (2011) argue that it is not the sheer size of the labor force but a country’s aggregate human capital stock that counts in producing new technology. Hence, countries could escape the curse of declining population growth by higher investment into human capital. Dalgaard and Kreiner (2001) and Strulik (2005) point out that the average human capital of a society decreases in the population growth rate because newborns enter the economy with a lower than average human capital level. If this force is strong enough to offset the positive impact of population growth on human capital accumulation, the mechanism could indeed be responsible for a negative association between population growth and technological progress. By contrast, Strulik et al. (2011) implement a child quality-quantity trade-off following Becker (1993) into R&D-based growth models such that parents can either invest in the number of their children (quantity) or in their education (quality). They show that demographic change leads to a shift toward having fewer but better educated children, which can essentially lead to more available aggregate human capital and therefore to faster technological progress and economic growth.

We propose an alternative way to reconcile theory and evidence that is already rooted in the standard semi-endogenous growth literature. Demographic change can have a completely different effect during the transition period as compared to its effect on the long-run balanced growth path. To show this, we integrate an overlapping generations structure into a Jones (1995) model along the lines of Buiter (1988) and Prettner (2012). We show

---

1Early endogenous growth models in the vein of Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) imply that a country’s population size is proportional to its long-run growth rate (strong scale effect). Since important predictions from this strand of the literature seem to be inconsistent with the corresponding empirical evidence, we follow the semi-endogenous growth literature.

2This idea has already been expressed in Romer (1990) but it was not explicitly modeled. For other contributions with human capital see e.g. Eicher (1996), Arnold (1998) and Arnold (2002). However, in these models, there is no interaction between population growth and investments in education.

3There remains some doubt whether the quality-quantity trade-off mechanism is strong enough to generate an increase in aggregate human capital in response to fertility declines (see Lee and Mason, 2010; Prettner et al., 2012, for different views). Therefore, it is not clear whether the quantity-quality trade-off alone can account for the negative association between population and output growth.
that households adjust to higher life expectancy by increasing savings which leads to faster accumulation of capital and technology during transition. This boosts output above the level that would prevail without demographic change. The extended semi-endogenous growth model, therefore, predicts a positive association between population and output growth only for the long run. In the medium run the relation is negative, in line with the empirical evidence and without the need to rely on a mechanism describing human capital accumulation.

We parameterize the model with US data and simulate transitional dynamics following a stylized demographic change. In so doing we rely on a numerical method developed by Trimborn et al. (2008) that allows to obtain impulse responses up to an arbitrarily small error. We show that the boom in output following demographic change is pronounced and lasts for more than 30 years. This observation confirms that the impact of demographic change on growth is qualitatively different during transition and on the balanced growth path which supports Solow’s critique: “The steady state is not a bad place for the theory of growth to start, but may be a dangerous place for it to end” (Solow, 2000, p. 7).

The contribution of our paper is, therefore, twofold. First, we propose an alternative (and probably simpler) way to reconcile empirical evidence and theoretical considerations about the negative association between population growth and economic growth. Second, we are able to analyze the effects of demographic change not only along the balanced growth path as in Prettner (2012) but also during the transition period. Thereby we ensure that fertility and mortality mimic the stylized facts of demographic change in industrialized countries over the last decades. Altogether we expect that this framework allows us to gain a better understanding of the economic consequences of demographic change toward longer life spans and lower fertility.

The paper proceeds as follows: section 2 contains the model, which we analyze theoretically in section 3. In section 4 we obtain numerical simulations and section 5 concludes.

## 2 The model

This section describes the overlapping generations version of the semi-endogenous growth framework of Jones (1995). We follow Buiter (1988) and Prettner (2012) by augmenting the model with a demographic structure.

### 2.1 Basic assumptions

The production technology follows Jones (1995). There are three sectors in our model economy: final goods production, intermediate goods production and R&D. The final goods sector uses labor (in the form of workers) and machines to produce for a perfectly competitive market. The Dixit and Stiglitz (1977) monopolistically competitive intermediate goods sector produces differentiated capital goods using foregone consumption. Each firm in this sector has to buy one blueprint as a fixed input to become operative. Finally,
the R&D sector produces these blueprints by using labor (in the form of scientists) for which it competes with the final goods sector in a perfectly competitive labor market.

Households consist of overlapping generations in the spirit of Buiter (1988). The different cohorts are distinguishable by their date of birth \( t_0 \) and each individual has to face a constant risk of death each instant which we denote by \( \mu. \) It follows that life expectancy of an individual is \( 1/\mu. \) The law of large numbers then implies that a fraction \( \mu \) of the population dies at each moment in time. Consequently, the population grows at rate \( n = \beta - \mu, \) with \( \beta \) referring to the constant birth rate which is equal to the period fertility rate in this demographic setting (cf. Preston et al., 2001). It follows that each cohort consists of a measure \( N(t_0, t) \) of individuals at a certain point in time \( t > t_0. \) The size of each cohort can be traced as \( N(t_0, t) = \beta N(0) e^{\beta t_0} e^{-\mu t}, \) where \( N(0) \) refers to the initial population size.

### 2.2 Households

For expositional reasons, we suppress time arguments except where a distinction between time and age is necessary. Individuals maximize their discounted stream of lifetime utility

\[
U = \int_0^\infty \ln(c) e^{-(\rho + \mu)t} dt
\]  

(1)

by choosing optimal consumption \( c, \) while \( \rho > 0 \) represents the subjective time discount rate which has to be augmented by the mortality rate \( \mu > 0 \) because individuals who face the risk of death are less likely to postpone consumption. We take the consumption good as numéraire and assume that there exists a perfect life insurance company at which individuals insure themselves against the risk of dying with positive assets (cf. Yaari, 1965). In this case, individual assets \( a \) yield an annuity premium \( \mu \) in addition to the interest rate \( r. \) They evolve according to

\[
\dot{a} = (r + \mu) a + w - c,
\]  

(2)

where \( r \) denotes the interest rate and \( w \) refers to the wage rate. The reason for the mortality rate showing up in the individual asset accumulation equation is that the wealth of dying individuals is redistributed by the life insurance company among those individuals who survived. The result of this optimization problem is the familiar consumption Euler

---

4Allowing for age-dependent mortality would not change our qualitative results. However, it would seriously complicate the derivation of aggregate capital and consumption. Adapting the framework for age-dependent mortality is therefore left for further research.

5Note that all individuals have the same level of fertility. For an alternative specification with differential fertility see e.g. de la Croix and Doepke (2003).

6Formally, individuals maximize the expected stream of instantaneous utility discounted at rate \( \rho. \) This is equivalent to equation (1) because the time until death is exponentially distributed.
equation
\[ \frac{\dot{c}}{c} = r - \rho \]  
(3)
stating that optimal individual consumption expenditures grow as long as the interest rate exceeds the rate of pure time preference.

\section*{2.3 Household aggregation}

Agents are heterogeneous with respect to age and consequently also with respect to wealth. The reason is that older agents have had more time to build up assets in the past. In order to get expressions for the dynamic behavior of aggregate consumption expenditures — denoted by \( C \) — and aggregate assets — denoted by \( \Omega \) — we have to apply the following rules to integrate over all cohorts alive at time \( t \) (cf. Buiter, 1988)

\[ C(t) \equiv \int_{-\infty}^{t} c(t_0, t)N(t_0, t)dt_0 = \beta N(0)e^{-\mu t} \int_{-\infty}^{t} c(t_0, t)e^{\beta t_0}dt_0, \]  
(4)
\[ \Omega(t) \equiv \int_{-\infty}^{t} a(t_0, t)N(t_0, t)dt_0 = \beta N(0)e^{-\mu t} \int_{-\infty}^{t} a(t_0, t)e^{\beta t_0}dt_0. \]  
(5)
Note that we apply our demographic assumptions in the second step. The corresponding calculations are carried out in appendix A.2. Altogether, we derive the following expressions

\[ \dot{C} = (r - \rho + n)C - \beta(\rho + \mu)\Omega, \]  
(6)
\[ \dot{\Omega} = r\Omega - C + W, \]  
(7)
which show that aggregate consumption expenditure growth differs from individual consumption expenditure growth in two ways. First, population growth raises consumption expenditure growth at the economy-wide level which is captured by the term \( nC \). Second, at each instant a fraction of older and wealthier individuals who can afford more consumption die, and they are replaced by poorer newborns who start with a lower level of consumption. This continually ongoing process of generational turnover slows down aggregate consumption expenditure growth as compared to individual consumption expenditure growth and is captured by the term \( \beta(\rho + \mu)\Omega \). In contrast to individual assets, the evolution of aggregate assets does not depend on mortality. The reason is that the life insurance company only redistributes wealth between cohorts such that the mortality effect drops out in the aggregation procedure.
2.4 Firms

The production side of our model economy closely follows Jones (1995). Consumption goods are produced in the final goods sector according to

\[ Y = L_Y^{1-\alpha} \int_0^A x_i^{\alpha} di, \]  

(8)

where \( Y \) represents total production of final output and therefore also represents a country’s gross domestic product (GDP), \( L_Y \) denotes the amount of labor used in the final goods sector, \( x_i \) is the amount of an intermediate input, that is, a specific machine \( i \), used in final goods production, \( A \) represents the technological frontier, that is, the spectrum of differentiated machines available, and \( \alpha \in (0, 1) \) is the output elasticity of an intermediate product. Profit maximization and perfect competition imply the following factor rewards

\[ w_Y = (1 - \alpha) \frac{Y}{L_Y}, \]  

(9)

\[ p_i = \alpha L_Y^{1-\alpha} x_i^{\alpha-1}, \]  

(10)

where \( w_Y \) refers to the wage rate paid to workers in the final goods sector, and \( p_i \) denotes prices for machines.

Firms in the intermediate goods sector have to purchase one blueprint from the R&D sector as fixed input and can afterwards employ capital as variable input to optimally adjust the volume of production. We assume that after an intermediate goods producer has incurred the fixed costs, it can transform one unit of capital, denoted by \( k \), into one unit of the intermediate good such that \( k_i = x_i \). In this case operating profits can be written as

\[ \pi_i = p_i k_i - r k_i = \alpha L_Y^{1-\alpha} k_i^{\alpha} - r k_i. \]  

(11)

Profit maximization yields optimal prices of machines

\[ p_i = \frac{r}{\alpha}, \]  

(12)

with \( 1/\alpha \) being the markup over marginal cost. We drop the index \( i \) from now on because equation (12) holds for all firms. Furthermore, we rewrite the interest rate as \( r = \alpha^2 Y/K \). The aggregate capital stock, denoted by \( K \), is then equal to the amount of all intermediates produced, that is, \( K = Ax \). Therefore equation (8) becomes

\[ Y = (AL_Y)^{1-\alpha} K^\alpha \]  

(13)

and technological progress appears as labor augmenting.

Firms in the R&D sector employ scientists to discover new blueprints. Denoting the productivity of scientists by \( \nu > 0 \) and the size of technology spillovers by \( 0 < \phi < 1 \) (cf.
where $L_A$ refers to the amount of scientists employed. The technological frontier expands faster the more scientists are employed and the more productive these scientists work. Furthermore, the existing stock of knowledge has a positive intertemporal spill-over effect on researchers’ productivity (“standing on shoulders effect”). The size of this effect is captured by $\phi$. R&D firms maximize their profits $\pi_A$ according to

$$\max_{L_A} \pi_A = p_A \nu A^\phi L_A - w_A L_A,$$

(15)

where $p_A$ represents the price of a blueprint. The first order condition pins down wages in the R&D sector to

$$w_A = p_A \nu A^\phi.$$

(16)

From this equation it is evident that wages of scientists increase in the prices of blueprints, the productivity of scientists and the stock of knowledge combined with the extent of knowledge spillovers.

### 2.5 Equilibrium

There is perfect labor mobility between sectors, therefore wages of workers in final good production and wages of scientists equalize and we can insert equation (9) into equation (16) to get the equilibrium condition

$$p_A \nu A^\phi = (1 - \alpha) \frac{Y}{L_Y}.$$

(17)

Firms in the R&D sector charge prices for blueprints that are equal to the present value of operating profits in the intermediate goods sector because there is always a potential entrant who is willing to pay that price due to free entry. Therefore we have

$$p_A = \int_{t_0}^{\infty} e^{-\int_{t_0}^{t} r(s) \, ds} \pi_A \, d\tau,$$

(18)

where the discount rate is the market interest rate. Using Leibniz’ rule, we obtain the following relationship

$$r = \frac{\pi}{p_A} + \frac{\dot{p}_A}{p_A}.$$

(19)

This is a no-arbitrage rule equating the return of investing in capital and investing in intermediate firms. Therefore, the return of investing one dollar in capital ($r$) has to
be equal to dividend payments from the intermediate firm \((\pi/p_A)\) and the value gain of intermediate firm shares \((\dot{p}_A/p_A)\). Next, we obtain operating profits by using equation (11) as \(\pi_A = (1 - \alpha)\alpha Y/A\) such that the evolution of prices for blueprints can be written as

\[
\dot{p}_A = rp_A - (1 - \alpha)\alpha Y/A.
\]

Finally we assume that the markets for capital and labor clear, that is,

\[
\begin{align*}
L &= L_A + L_Y, \\
\dot{K} &= Y - C,
\end{align*}
\]

stating that all available labor \(L\) is either employed in the final goods sector or in the R&D sector and that everything that is not consumed adds to the capital stock in our closed economy. At this stage we can then formally define an equilibrium.

**Definition 1.** An equilibrium of the economy is defined by the system of equations (6), (7), (14), (20) and (22) for which the conditions (13), (17), (21) and \(r = \alpha^2 Y/K\) are fulfilled.

### 2.6 The balanced growth path

Along a balanced growth path \(\dot{A}/A = \dot{Y}/Y - n = \dot{C}/C - n = \dot{K}/K - n = \dot{y}/y\) holds, where \(y\) denotes per capita GDP. Furthermore, the division of labor between the final goods sector and the R&D sector does not change and consequently both grow at rate \(n\). Hence, the growth rate of the economy along the balanced growth path \(g^*\) can be calculated by taking the time derivative of the logarithm of equation (14) and imposing that the long-run growth rate itself is constant. In appendix A.3 we show that this yields

\[
g^* = \frac{n}{1 - \phi} = \frac{\beta - \mu}{1 - \phi}.
\]

Note that the balanced growth rate is proportional to population growth and that changes in fertility or mortality affect the balanced growth rate through population growth only.

### 3 The impact of demographic change on economic growth: theory

Between 1970 and 2010 life expectancy in the US increased substantially, while birth rates have fallen to the extent that the total fertility rate reached a value close to the replacement rate of 2.1. To analyze the effect of demographic change on long-run economic growth and to reveal transitory effects we focus on a stylized scenario. We assume that, initially, the economy grows along a balanced growth path with constant mortality and fertility rates.
Then, the economy is hit by a demographic shock represented by a permanently lower mortality rate and an even stronger permanent decline of fertility such that population growth declines. This induces a transition during which the economy converges towards the new balanced growth path implied by the new mortality and fertility rates. This stylized scenario allows us to derive theoretically how demographic change affects the balanced growth rate of the economy. In a second step we analyze how households react by adjusting savings and consumption, and how this affects the economy during transition. In the next section we confirm the theoretical results by numerical simulation.

The effect of demographic change on long-run economic growth is unambiguously grim because the per-capita long-run growth rate of output, $g^*$, is proportional to the population growth rate, that is,

$$\frac{dg^*}{dn} > 0.$$  \hspace{1cm} (24)

The reason for declining per-capita growth rates in the long run is routed in the research sector because long-run output growth is ultimately driven by accumulation of knowledge/technology. The existing stock of knowledge has a positive external effect on the accumulation of new knowledge (“standing on shoulders effect”). This positive effect, however, is diminishing with respect to the existing stock of knowledge. To see this, inspect $\dot{A}/A = A^{\phi-1} L_A$ and notice that $\phi - 1 < 0$. This causes the growth rate of knowledge to decline, the higher the stock of already accumulated knowledge is, given the hypothetical case of a constant number of researchers. Hence, a perpetually rising number of researchers is needed to keep the growth rate of knowledge constant.

Consider now population growth and a constant share of population working in the research sector instead of a constant number of researchers. In this case, perpetual population growth means that the number of researchers grows with the same rate as the population. The constant flow of new researchers, hence, enables the economy to overcome the diminishing standing on shoulders effect and to grow at a constant balanced growth rate. If population growth declines and the share of researchers remains constant, the number of researchers grows at a lower rate, which drives the growth rate of knowledge down. In fact, the population share working in the research sector is bounded between zero and one and, hence, cannot grow indefinitely. Therefore, a shift of the population share working in research can only have a transitory effect on the growth rate of knowledge. Lower population growth inevitably slows down technological progress in the long run, which in turn leads to a lower economic growth rate along the balanced growth path.

If an economy initially following a balanced growth path experiences demographic change, the new balanced growth rate will be lower. However, the economy will not immediately grow with this rate. During transition there are several mechanisms at work that can potentially move the economy’s growth rate in opposite direction — even above the level prevalent before the demographic shock. Thus, output can be higher during transition compared to an economy that did not experience a demographic shock.
To understand how the economy’s growth rate is affected during transition, it is crucial to investigate the households’ saving decision. We do this by assuming that the economy rests in a steady state initially. We then investigate how households would change their saving and consumption decision if demographic change occurs and leaves prices unaffected. Declining death rates have an immediate effect on households’ consumption and saving decision. Note first that both fertility and mortality changes do not affect individual consumption growth, which depends only on the interest rate and the households’ discount rate. However, lower mortality rates do impact households’ initial consumption and, hence, households’ savings. To see this, consider the households’ lifetime budget constraint

\[
\int_0^\infty c(0, t)e^{-(r+\mu)t}dt = \int_0^\infty w(t)e^{-(r+\mu)t}dt.
\] (25)

Inserting the growth rates of individual consumption \(\dot{c}(0, t)/c(0, t) = r - \rho\) and wages in the steady state \(\dot{w}/w = g\) and integrating we obtain

\[
c(0, 0) = w(0)\frac{\rho + \mu}{r + \mu - g}.
\] (26)

We can derive the effect of mortality on initial consumption as

\[
\frac{\partial c(0, 0)}{\partial \mu} = w(0)\frac{r - \rho - g}{(r - g + \mu)^2}.
\]

To assess the sign of the effect, we have to investigate if in steady state individual consumption grows faster or slower compared to the aggregate economy, that is, if \(r - \rho \gtrless g\). This can be derived from the growth rate of aggregate consumption \(\dot{C}\), which is \(g + n\) in steady state and, hence,

\[
g + n = \frac{\dot{C}}{C} = r - \rho + n - \beta(\rho + \mu)\frac{\Omega}{C}.
\]

Since the last summand is always positive, \(r - \rho > g\) holds and consequently, the derivative \(\partial c(0, 0)/\partial \mu\) is unambiguously positive. Therefore the direct effect of declining mortality on initial consumption is negative and households increase savings.

To gain intuition for this effect note first that mortality enters a household’s optimization problem twice: first via the discount factor, \(\rho + \mu\) (lower mortality increases the expected lifespan and, hence, leads to lower discounting of future utility), and second via the annuity interest rate, \(r + \mu\) (lower mortality decreases the annuity premium on interest rates). While both effects cancel each other out with respect to the growth rate of consumption, the annuity interest rate affects the level of consumption through the budget constraint. The reason is that households choose a consumption growth rate higher than the growth rate of wages. Households start with a level of consumption lower than wage income, but, due to the higher growth rate, eventually consume more than their contem-
poraneous wage income. To put it differently, households delay consumption to the future as compared to what they could consume with their current wage income, that is, they save. Inspection of the household’s lifetime budget constraint (equation (25)) shows that the net present value of consumption and wages is affected differently by a change of the annuity interest rate \( r + \mu \). If mortality decreases, future consumption and future wage income receive more weight. Since consumption accumulates more at later points in time, the net present value of consumption increases more than the net present value of wage income. To clear the budget constraint, households have to decrease initial consumption. Therefore, given that the growth rate of consumption is not affected by declining mortality, the effect of declining mortality on initial consumption is a pure budgetary effect.

To summarize, the instantaneous effect of declining mortality on households’ savings is positive, and it is negative on households’ consumption. On the other hand, fertility does not affect households’ decisions directly. This can be seen by inspecting the first order condition (equation 3) and the budget constraint (equation 25) and noticing that neither the fertility rate nor the population growth rate enter these conditions directly. However, lower fertility affects aggregate savings via the number of households, because declining fertility decreases population growth and, hence, the number of households that save. But different to any direct effect on households’ saving rate, the indirect effect via population growth phases in only gradually. Therefore, declining population growth rates lead to higher aggregate savings in the short and medium run, and reduced aggregate savings in the long run because, eventually, the positive level effect will be dominated by the negative growth effect.

Higher aggregate savings in the medium run can, nonetheless, lead to a prolonged period of higher growth compared to an economy that is not exposed to a demographic shock. Higher savings increase the amount of accumulated assets, which are composed of physical capital and blueprints \((K + p_A A)\). The mechanism by which physical capital increases is straightforward: the fraction of output goods converted to capital goods increases. The mechanism by which the number of blueprints increases is more involved: higher aggregate savings yield a lower interest rate; therefore, future profits of intermediate firms are discounted at a lower rate which causes the price for blueprints to increase. This lifts the marginal product of researchers and causes labor to shift from final output production to R&D which, finally, increases the accumulation rate of blueprints. Note that this mechanism is just a different way for households to delay consumption to the future: Factor reallocation towards R&D decreases consumption and increases factor accumulation for the initial periods.

Besides the negative effect of demographic change on output in the long run, higher \( K \) and higher \( A \) shift output above the level that would prevail without demographic change in the medium run. In the short run, an additional effect can lead to a decline in output namely the reallocation of labor from final output production to R&D (recall that output is produced from capital, \( K \), knowledge, \( A \), and labor, \( L_Y \)). Conceptually, we know that accumulation of \( A \) or \( K \) affects output only in the medium run but has no effect at impact
because it needs time to accumulate these production factors. A decline of labor in the output producing sector can, on the other hand, affect output on impact because labor can be reallocated immediately and without any costs. This is the mechanism driving output down at impact.

4 The impact of demographic change on economic growth: quantitative results

In this section we explore the quantitative effects of demographic change on factor accumulation, output, and consumption. We stick to the stylized scenario in which the economy grows along a balanced growth path and experiences a permanent and nonrecurring increase in life expectancy and an even more pronounced decrease of fertility. This allows us to compare the numerical results with theoretical predictions from Section 3. We calibrate the model with US data and show that the transitory effects of demographic change on factor accumulation and output are large and long-lasting.

4.1 Parametrization

The calibrated pre-shock economy matches US data in 1970 and the post-shock economy matches US data in 2010. We average the observed variables for several years to reduce the influence of fluctuations on the calibrated values. The instantaneous mortality rate \( \mu = 0.014 \) resembles an expected lifespan of \( 1/\mu = 70 \) years, the five-year average for men and women observed in the US between 1968 and 1972. We set \( \beta \) to the average crude birth rate between 1968 and 1972 of 17 births per thousand inhabitants, that is, \( \beta = 0.017 \). Finally, we calibrate \( \phi = 0.78 \) to match the US per capita GDP growth rate of 1.5% between 1960 and 1970. The remaining parameters are set in accordance with the literature: \( \rho = 0.015 \) guarantees a steady state interest rate of 3.5% and \( \alpha = 0.38 \) and \( \delta = 0.07 \) are used in various calibration studies for the US (cf. Trabandt and Uhlig, 2011).

The demographic shock is represented by a simultaneous decrease of mortality and fertility such that population growth declines. Life expectancy increases from 70 to 78 years (\( \mu \) decreases from 0.014 to 0.0128), and fertility decreases from \( \beta = 0.017 \) to \( \beta = 0.014 \). Both final values are observed in the US in 2010. Consequently, population growth decrease from 0.32% to 0.12% and the balanced growth rate decrease from 1.45% to 0.5%.

4.2 Quantitative assessment

The quantitative effect of demographic change on output growth can be seen in Figure 1. The left panel shows the growth rate of per capita output for a long time horizon of 200 years. The right panel zooms in for a shorter time horizon of 30 years. Immediately

\footnote{Birth and death rate combined lead to a population growth rate of 0.32%, which is below the rate of 1.3% observed for this period in the US. This deviation is mainly attributed to migration that is not part of our analysis.}
<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0.015</td>
<td>interest rate of 3.5%</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.38</td>
<td>Trabandt and Uhlig (2011)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.07</td>
<td>Trabandt and Uhlig (2011)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.78</td>
<td>balanced growth rate of 1.5%</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.014</td>
<td>average 1968-1972</td>
</tr>
<tr>
<td>$\mu$ (new)</td>
<td>0.0128</td>
<td>2010</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.017</td>
<td>average 1968-1972</td>
</tr>
<tr>
<td>$\beta$ (new)</td>
<td>0.014</td>
<td>2010</td>
</tr>
<tr>
<td>$n$</td>
<td>0.0032</td>
<td>implied</td>
</tr>
<tr>
<td>$n$ (new)</td>
<td>0.0012</td>
<td>implied</td>
</tr>
<tr>
<td>$g$</td>
<td>0.0145</td>
<td>implied</td>
</tr>
<tr>
<td>$g$ (new)</td>
<td>0.005</td>
<td>implied</td>
</tr>
</tbody>
</table>

after the shock, output growth increases from the old balanced growth rate of 1.45% (indicated by a dotted line) to 1.55%. For about 21 years output grows faster compared to the old balanced growth rate. After this period, output growth declines below 1.45% and converges slowly toward its new long-run balanced growth rate of 0.5%. Although the demographic shock has a considerable negative impact on long-run growth, there is a prolonged period in which growth is higher compared to the pre-shock balanced growth path, and adjustment speed towards the new balanced growth rate is very low with a half-life of about 130 years.

The reason why a demographic shock has a long-lasting positive impact on per capita output growth can be seen by inspecting the engine of long-run growth: R&D displayed in Figure 2. The left panel shows the growth rate of R&D for a long time horizon of 200 years and the right panel zooms in for a shorter time horizon of 30 years. The rate of technological progress increases from 1.45% to 1.5% at impact and remains above the original level for almost 9 years. After this period R&D growth declines below the old balanced growth rate (indicated by a dotted line) and converges slowly toward the new balanced growth rate of 0.5% (indicated by a dashed line).

We can also visualize the impact of the demographic shock by comparing the level of output of an economy hit by the shock and an economy continuing to grow along the old balanced growth path. Figure 3, panel (i) shows output of an economy in transition compared to steady state output, $y/y^*$. The ratio starts below one, that is, output jumps down after the demographic shock has hit the economy. It then increases well above its old balanced growth path level and eventually declines. The peak of the inverse U-shaped curve is at year 21 — exactly when per capita GDP growth falls below 1.45%. However, output is above the value it would have been without demographic shock for a much longer period. Only in year 35 an economy without demographic shock would catch up with an economy hit by a demographic shock. Eventually, the output ratio converges to zero because the long-run growth rate of output after the shock is below the pre-shock level.
Adjustment dynamics following a demographic shock: \( g_y \) denotes the growth rate of output. The old balanced growth rate is indicated by a dotted line, the new balanced growth rate is indicated by a dashed line.

Adjustment dynamics following a demographic shock: \( g_A \) denotes the growth rate of blueprints. The old balanced growth rate is indicated by a dotted line, the new balanced growth rate is indicated by a dashed line.

Why output decreases at impact and follows a humped-shape pattern afterwards can be seen by conducting a growth accounting exercise. Panels (ii), (iii), and (iv) of Figure 3 show the level of R&D scaled by steady-state R&D \((A/A^*)\), capital compared to steady-state capital \((K/K^*)\), and the labor share working in final output production \((l_Y = L_Y/L)\). Since output is produced from these three factors only, changes in output can be traced back to the producing factors. Two variables, blueprints and capital, are state variables and, thus, cannot change at impact. Therefore, the decline of output right after the shock can fully be attributed to workers moving from final output production to R&D, that is, \( l_Y \) falls at impact. These additional scientists increase the accumulation of blueprints for
Adjustment dynamics following a demographic shock. Per capita output, $y$, technology, $A$, and per-capita capital $k$ are divided by their pre-shock steady state values indicated by a star. $l_Y$ denotes the share of workers in final output production. The old balanced growth values are indicated by a dotted line.

a prolonged period of time. The peak of $A/A^*$ in year 9 again corresponds to the point in time when the growth rate of R&D falls below its pre-shock value, and the level of R&D is higher compared to the old balanced growth path until year 18. Accumulation of capital, in principle, follows the pattern of R&D only that the point of time at which the steady-state economy catches up with respect to capital accumulation is much later (year 55).

The decomposition with respect to production factors shows which of them are responsible for higher output in the medium run but not why they change. In Section 3 we explain that a demographic shock causes households to increase the saving rate and argue that this leads to a higher level of accumulable factors $K$ and $A$ in the medium run. Figure 4 confirms this mechanism: panel (ii) shows the interest rate which falls at impact from the pre-shock steady-state value of 3.5% (indicated by a dotted line) to 3.4%. The reason is a general equilibrium effect: labor moves from final output production to the research sector. Since capital cannot change at impact, the marginal product of capital
decreases. In the subsequent periods, capital accumulation accelerates faster than accumulation of blueprints, which decreases the interest rate further down toward the new steady-state value of 2.5% (indicated by a dashed line).

The declining interest rate has an effect on the price of patents. Since future profits are now discounted at a lower interest rate, the price of blueprints increases, as can be seen in panel (iv) of Figure 4. However, after around 64 years, \( p_A \) falls below the pre-shock steady-state value. This can be explained by a drop in profits, which are displayed in panel (iii). They decrease at impact and with respect to time due to a market size effect. Lower \( L_Y \) (panel (v)) at impact (due to factor reallocation) and decreasing \( L_Y \) over time (due to lower population growth) lowers the marginal product of each intermediate good in final output production and, consequently, decreases the demand for intermediate goods. Since the negative effect of profits on patent prices phases in only gradually (in contrast to the discount rate effect), patent prices decline below their pre-shock values only after a prolonged period of time. Higher prices of blueprints increase the marginal product of researchers measured in units of final goods. The resulting higher wage induces workers to move from final output production into the research sector. However, panel (vi) shows that declining population growth rates eventually result in declining absolute numbers of researchers. The economy only escapes lower growth in the medium run but not in the long run.

Finally, we can assess the impact of a demographic shock on consumption. Section 3 has revealed two opposite effects: Higher saving rates of households have a negative impact on consumption in the short and medium run, while higher output due to faster factor accumulation increases consumption. Panel (i) of Figure 4 shows that the second effect dominates for a substantial period of time. Consumption follows the pattern of output and declines on impact, increases above its pre-shock value before year 4 and peaks at year 21. After that it declines and passes the pre-shock level at year 37. Note that consumption in relation to its pre-shock value shrinks to zero in the long-run, because the demographic shock diminishes the balanced growth rate of consumption.

### 5 Conclusions

We set up a semi-endogenous economic growth model incorporating demography. Demographic change is represented by declining mortality and fertility rates such that population growth slows down. We analytically show that the long-run rates of technological change and per capita output growth decrease in response to demographic change but that the economy accumulates more physical capital and knowledge along the transition path such that there is a temporal boost of technological progress and per capita output growth. We then numerically assess the reaction of an economy to an exogenous reduction in mortality and fertility driving population growth down. In response investments in physical capital and knowledge creation accelerate and labor shifts from the production sector to the R&D sector such that economic growth speeds up. Eventually, however, the negative
Adjustment dynamics following a demographic shock. Consumption, $c$, profits, $\pi$, prices of blueprints, $p_A$, labor in final output production, $L_Y$, and the number of researchers, $L_A$, are divided by their pre-shock steady state values indicated by a star. The old balanced growth values are indicated by a dotted line, the new balanced growth values are indicated by a dashed line.

effect of declining fertility on the flow of new scientists into the R&D sector dominates and knowledge creation slows down. Nevertheless, the time frame during which economic growth is faster than without a demographic shock lasts for around 20 years.

Consequently, our model is able to reconcile the theoretical interrelations between population growth and economic growth with the empirical findings in the vein of Brander and Dowrick (1994), Kelley and Schmidt (1995), Ahituv (2001) and Bernanke and Gürkan (2001). This has been achieved without relying on the quality-quantity trade-off effect emphasized by Lee and Mason (2010) and Strulik et al. (2011). However, in reality, both effects are likely to be present and both might contribute to mitigating the negative impact of declining fertility on economic prosperity. Furthermore, other mechanisms associated with declining fertility could also play an important role like increasing female labor supply (cf. Bloom et al., 2009), or the standard demographic dividend of declining youth dependency ratios (cf. Bloom et al., 2003, 2010).

Altogether, our result is also interesting with regards to the critiques expressed by Solow (2000) and mentioned in the introduction: we showed that demographic change can
have qualitatively different effects during transition toward the long-run balanced growth path and along the long-run balanced growth path itself. Consequently, focusing only on long-run steady-state outcomes might be very misleading.

We hope that our contribution is able to shed some additional light on the interrelations between demographic change and economic development. Nevertheless, we acknowledge that — for expositional reasons — our modeling set-up has been very stylized. It could prove useful to introduce more realistic age-dependent mortality structures and/or to model endogenous fertility decisions. However, for the questions at hand, we are convinced that our set-up is a reasonable compromise between realism and tractability.

Acknowledgments

We would like to thank Holger Strulik for valuable comments. Furthermore, we are very grateful for the financial support granted by the Max Kade foundation regarding the post-doctoral fellowship 30393 “Demography and Long-run Economic Growth Perspectives”.

Appendix

A Derivations

A.1 The individual optimization problem

The current value Hamiltonian reads

\[ H = \ln(c) + \lambda [(r + \mu) a + w - c]. \]  \hspace{1cm} (A.1)

The associated first order conditions are

\[ \frac{\partial H}{\partial c} = 0 \quad \Rightarrow \quad \frac{1}{c} = \lambda, \]
\[ \frac{\partial H}{\partial a} = (\rho + \mu) \lambda - \dot{\lambda} \quad \Rightarrow \quad \frac{\dot{\lambda}}{\lambda} = \rho - r. \]

Differentiating the first equation and using the second equation yields the familiar Euler equation

\[ \frac{\dot{c}}{c} = r - \rho. \]  \hspace{1cm} (A.2)
A.2 Aggregation

Differentiating equations (4) and (5) with respect to time yields:

\[
\dot{C}(t) = \left[ \int_{-\infty}^{t} \beta N(0) e^{-\mu t} c(t_0, t) e^{\beta t_0} dt_0 \right. \\
+ \beta N(0) e^{-\mu t} c(t, t) e^{\beta t} - 0 \\
\left. = \beta N(0) e^{-\mu t} c(t, t) e^{\beta t} - \mu C(t) + \beta N(0) e^{-\mu t} \int_{-\infty}^{t} \dot{c}(t_0, t) e^{\beta t_0} dt_0 \right]
\]

\[
\dot{\Omega}(t) = \left[ \int_{-\infty}^{t} \beta N(0) e^{-\mu t} \dot{a}(t_0, t) e^{\beta t_0} - \mu \beta N(0) e^{-\mu t} a(t_0, t) e^{\beta t_0} dt_0 \right]
\]

\[
+ \beta N(0) e^{-\mu t} a(t, t) e^{\beta t} - 0 \\
\left. = \beta N(0) e^{-\mu t} a(t, t) e^{\beta t} - \mu \Omega(t) + \beta N(0) e^{-\mu t} \int_{-\infty}^{t} \dot{a}(t_0, t) e^{\beta t_0} dt_0. \right]
\]

From equation (2) it follows that

\[
\dot{\Omega}(t) = -\mu K(t) \\
+ \beta N(0) e^{-\mu t} \int_{-\infty}^{t} [(r + \mu) a(t_0, t) + w(t) - c(t_0, t)] e^{\beta t_0} dt_0 \\
= -\mu K(t) + (r + \mu) \beta N(0) e^{-\mu t} \int_{-\infty}^{t} a(t_0, t) e^{\beta t_0} dt_0 \\
- \beta N(0) e^{-\mu t} \int_{-\infty}^{t} c(t_0, t) e^{\beta t_0} dt_0 + N(0) e^{-\mu t} \left( \frac{\beta w(t) e^{\beta t_0}}{\beta} \right)_{-\infty}^{t} \\
= r\Omega(t) - C(t) + W(t)
\]

which is the aggregate law of motion for assets. Note that the definition of aggregate wages is $W(t) = N(0) w(t) e^{\beta - \mu}$. We reformulate an agent’s optimization problem

\[
\max_{c(t_0, \tau)} U = \int_{t}^{\infty} e^{(\rho + \mu)(t-\tau)} \left[ \ln(c(t_0, \tau)) \right] d\tau 
\]

subject to its lifetime budget restriction, stating that the present value of lifetime consumption expenditures have to be equal to the present value of lifetime wage income plus initial assets

\[
a(t_0, t) + \int_{t}^{\infty} w(\tau) e^{-R^\Omega(t, \tau)} d\tau = \int_{t}^{\infty} c(t_0, \tau) e^{-R^\Omega(t, \tau)} d\tau,
\]

where we used the definition $R^\Omega(t, \tau) = \int_{t}^{\tau} (r(s) + \mu) ds$. The first order condition for optimal consumption is

\[
\frac{1}{c(t_0, \tau)} e^{(\rho + \mu)(t-\tau)} = \lambda(t) e^{-R^\Omega(t, \tau)}.
\]
In period \((\tau = t)\) we have
\[
c(t_0, t) = \frac{1}{\lambda(t)}.
\]

Therefore we can write
\[
\frac{1}{c(t_0, \tau)} e^{(\rho + \mu)(t-\tau)} = \frac{1}{c(t_0, t)} e^{-R\Omega(t, \tau)}
\]
\[
c(t_0, t)e^{(\rho + \mu)(t-\tau)} = c(t_0, \tau)e^{-R\Omega(t, \tau)}.
\]

Integrating and using equation (A.4) yields
\[
\int_0^\infty c(t_0, t)e^{(\rho + \mu)(t-\tau)} d\tau = \int_0^\infty c(t_0, \tau)e^{-R\Omega(t, \tau)} d\tau
\]
\[
\frac{c(t_0, t)}{\rho + \mu} \left[ -e^{(\rho + \mu)(t-\tau)} \right]_t^\infty = \left[ a(t_0, t) + \int_t^\infty w(\tau)e^{-R\Omega(t, \tau)} d\tau \right]
\]
\[
\Rightarrow c(t_0, t) = (\rho + \mu) \left[ a(t_0, t) + h(t) \right],
\]
(A.5)

where \(h(t)\) refers to human wealth of individuals which does not depend on the date of birth because productivity is age independent. The above calculations show that optimal consumption in the planning period is proportional to total wealth with a marginal propensity to consume of \(\rho + \mu\). By making use of equation (A.5), we can write aggregate consumption as
\[
C(t) \equiv \beta N(0)e^{-\mu t} \int_{-\infty}^t c(t_0, t)e^{\beta t_0} dt_0
\]
\[
= \beta N(0)e^{-\mu t} \int_{-\infty}^t (\rho + \mu) \left[ a(t_0, t) + h(t) \right] e^{\beta t_0} dt_0
\]
\[
= \beta N(0)e^{-\mu t}(\rho + \mu) \int_{-\infty}^t e^{\beta t_0} h(t) dt_0
\]
\[
= (\rho + \mu) [\Omega(t) + H(t)].
\]
(A.6)

Note that the definition \(H(t) = N(0)e^{(\beta - \mu)t} h(t)\) applies. Newborns do not have financial wealth because there are no bequests, therefore
\[
c(t, t) = (\rho + \mu) h(t)
\]
(A.7)
holds for each newborn individual. Putting equations (3), (A.3), (A.6) and (A.7) together yields

\[ \dot{C}(t) = \beta N(0) e^{-\mu t} c(t, t) e^{\beta t} - \mu C(t) + \beta N(0) e^{-\mu t} \int_{-\infty}^{t} \dot{c}(t_0, t) e^{\beta t_0} dt_0 \]

\[ = \beta N(0) e^{(\beta - \mu) t} (\rho + \mu) h(t) - \mu (\rho + \mu) [\Omega(t) + H(t)] \]

\[ + \beta N(0) e^{-\mu t} \int_{-\infty}^{t} \dot{c}(t_0, t) e^{\beta t_0} dt_0 \]

\[ = \beta (\rho + \mu) H(t) - \mu (\rho + \mu) [\Omega(t) + H(t)] \]

\[ + \beta N(0) e^{-\mu t} \int_{-\infty}^{t} (r - \rho) c(t_0, t) e^{\beta t_0} dt_0 \]

\[ = \beta (\rho + \mu) H(t) - \mu (\rho + \mu) [\Omega(t) + H(t)] + (r - \rho) C(t) \]

\[ = (r - \rho + n) C(t) - \beta (\rho + \mu) \Omega(t). \]  

(A.8)

A.3 The balanced growth path

Along a balanced growth path we have

\[ g = \frac{\dot{A}}{A} = \frac{\nu L_A}{A^{1-\phi}}. \]

Taking logarithms yields

\[ \log g = \log(\nu) + \log(L_A) - (1 - \phi) \log(A). \]

Taking the derivative of this expression with respect to time and noting that along the balanced growth path the economic growth rate is constant yields

\[ \frac{\partial g}{\partial t} = n - (1 - \phi) g = 0 \]

\[ \Rightarrow g = \frac{n}{1 - \phi} = \frac{\beta - \mu}{1 - \phi}. \]
A.4 The system describing the model economy

To summarize, the economy is governed by the following dynamic system:

\[ \dot{A} = \nu A^\phi L_A, \]
\[ \dot{p}_A = r p_A - (1 - \alpha)\alpha \frac{Y}{A}, \]
\[ \dot{K} = Y - C, \]
\[ \dot{\Omega} = r \dot{\Omega} + W - C, \]
\[ \dot{H} = (r + \beta) H - W, \]
\[ \dot{C} = (r - \rho + n) C - (\rho + \mu) \beta \Omega, \]
\[ r = \alpha^2 \frac{Y}{K}, \]
\[ p_A \nu A^{\phi - 1} = (1 - \alpha) \left( \frac{K}{AL_Y} \right)^\alpha, \]
\[ L = L_Y + L_A, \]
\[ Y = K^\alpha (AL_Y)^{1-\alpha}. \] (A.9)

We scale each variable \( x \) by its respective growth rate \( \gamma_x \) according to \( \tilde{x}(t) = x(t) e^{-\gamma_x t} \).

Then, the scale-adjusted system of equations reads

\[ \dot{\tilde{A}} = \nu \tilde{A}^\phi \tilde{L}_A - g \tilde{A}, \]
\[ \dot{\tilde{p}}_A = (r - n) \tilde{p}_A - (1 - \alpha)\alpha \frac{\tilde{Y}}{\tilde{A}}, \]
\[ \dot{\tilde{K}} = \tilde{Y} - \tilde{C} - (\delta + n + g) \tilde{K}, \]
\[ \dot{\tilde{\Omega}} = r \tilde{\Omega} + \tilde{W} - \tilde{C} - (\delta + n + g) \tilde{\Omega}, \]
\[ \dot{\tilde{H}} = (r + \beta) \tilde{H} - \tilde{W} - (g + n) \tilde{H}, \]
\[ \dot{\tilde{C}} = (r - \rho - g) \tilde{C} - (\rho + \mu) \beta \tilde{\Omega}, \]
\[ r = \alpha^2 \left( \frac{\tilde{Y}}{\tilde{K}} \right), \]
\[ \tilde{p}_A \nu \tilde{A}^{\phi - 1} = (1 - \alpha) \left( \frac{\tilde{K}}{\tilde{A} \tilde{L}_Y} \right)^\alpha, \]
\[ 1 = l_Y + l_A, \]
\[ \tilde{Y} = \tilde{K}^\alpha (\tilde{A} \tilde{L}_Y)^{1-\alpha}, \]
\[ \tilde{W} = (1 - \alpha) \frac{\tilde{Y}}{l_Y}. \] (A.10)
References


Bisher erschienene Diskussionspapiere

Nr. 139: Prettner, Klaus;Trimborn, Timo: Demographic change and R&D-based economic growth: reconciling theory and evidence, September 2012

Nr. 138: König, Jörg; Ohr, Renate: Homogeneous groups within a heterogeneous community - Evidence from an index measuring European economic integration, August 2012

Nr. 137: Schwager, Robert: Student Loans in a Tiebout Model of Higher, July 2012

Nr. 136: Martínez-Zarzoso, Inmaculada: Exporting and Productivity: Evidence for Egypt and Morocco, April 2012

Nr. 135: König, Jörg; Ohr, Renate: Messung ökonomischer Integration in der Europäischen Union – Entwicklung eines EU-Integrationsindexes -, April 2012

Nr. 134: Gehringer, Agnieszka: Financial liberalization, growth, productivity and capital accumulation: The case of European integration, März 2012

Nr. 133: Berner, Eike; Birg, Laura: Retailers and Consumers. The pass-through of import price changes, März 2012

Nr. 132: Gehringer, Angnieszka: Current accounts in Europe: implications of the external imbalances for the future of the common monetary policy, März 2012

Nr. 131: Ohr, Renate; Özalbayrak, Mehmet: The Euro – A „MUST“ for Small European States?, Januar 2012

Nr. 130: Zeddies, Götz: Der Euro als Triebfeder des deutschen Exports?, November 2011

Nr. 129: Geishecker, Ingo; Siedler, Thomas: Job Loss Fears and (Extreme) Party Identification: First Evidence from Panel Data, Oktober 2011

Nr. 128: König, Jörg; Ohr, Renate: Small but Beautiful? Economic Impacts of the Size of Nations in the European Union, August 2011

Nr. 127: Schüder, Stefan: Monetary Policy Trade-Offs in a Portfolio Model with Endogenous Asset Supply, Juni 2011

Nr. 126: Hiller, Sanne: The Export Promoting Effect of Emigration: Evidence from Denmark, Juni 2011

Nr. 125: Martínez-Zarzoso, Inmaculada; Voicu, Anca M.; Vidovic, Martina: CEECs Integration into Regional and Global Production Networks, Mai 2011


Nr. 123: Dreher, Axel; Vreeland, James Raymond: Buying Votes and International Organizations, Mai 2011

Nr. 122: Schürenberg-Frosch, Hannah: One Model fits all? Determinants of Transport Costs across Sectors and Country Groups, April 2011

Nr. 121: Verheyen, Florian: Bilateral Exports from Euro Zone Countries to the US - Does Exchange Rate Variability Play a Role?, April 2011

Nr. 120: Ehlers, Tim: University Graduation Dependent on Family’s Wealth, Ability and Social Status, April 2011

Nr. 119: Cho, Seo-Young; Dreher, Axel; Neumayer, Eric: The Spread of Anti-trafficking Policies – Evidence from a New Index, März 2011

Nr. 94: Bernauer, Thomas; Kalbhem, Anna; Koubi, Vally; Ruoff, Gabi: On Commitment Levels and Compliance Mechanisms – Determinants of Participation in Global Environmental Agreements, Januar 2010


Nr. 92: Dreher, Axel; Nunnenkamp, Peter; Thiel, Susann; Thiele, Rainer: Aid Allocation by German NGOs: Does the Degree of Public Refinancing Matter?, Januar 2010

Nr. 91: Bjørnskov, Christian; Dreher, Axel; Fischer, Justina A. V.; Schnellenbach, Jan: On the relation between income inequality and happiness: Do fairness perceptions matter?, Dezember 2009

Nr. 90: Geishecker, Ingo: Perceived Job Insecurity and Well-Being Revisited: Towards Conceptual Clarity, Dezember 2009

Nr. 89: Kühl, Michael: Excess Comovements between the Euro/US dollar and British pound/US dollar exchange rates, November 2009

Nr. 88: Mourmouras, Alex, Russel, Steven H.: Financial Crises, Capital Liquidation and the Demand for International Reserves, November 2009

Nr. 87: Goerke, Laszlo, Pannenberg, Markus: An Analysis of Dismissal Legislation: Determinants of Severance Pay in West Germany, November 2009

Nr. 86: Marchesi, Silvia, Sabani, Laura, Dreher, Axel: Read my lips: the role of information transmission in multilateral reform design, Juni 2009

Nr. 85: Heinig, Hans Michael: Sind Referenden eine Antwort auf das Demokratiedilemma der EU?, Juni 2009

Nr. 84: El-Shagi, Makram: The Impact of Fixed Exchange Rates on Fiscal Discipline, Juni 2009


Nr. 81: Williamson, Jeffrey G.: History without Evidence: Latin American Inequality since 1491, Mai 2009

Nr. 80: Erdogan, Burcu: How does the European Integration affect the European Stock Markets?, April 2009

Nr. 79: Oelgemöller, Jens; Westermeier, Andreas: RCAs within Western Europe, März 2009

Nr. 78: Blonski, Matthias; Lilienfeld-Toal, Ulf von: Excess Returns and the Distinguished Player Paradox, Oktober 2008

Nr. 77: Lechner, Susanne; Ohr, Renate: The Right of Withdrawal in the Treaty of Lisbon: A game theoretic reflection on different decision processes in the EU, Oktober 2008

Nr. 76: Kühl, Michael: Strong comovements of exchange rates: Theoretical and empirical cases when currencies become the same asset, Juli 2008

Nr. 75: Höhenberger, Nicole; Schmiedeberg, Claudia: Structural Convergence of European Countries, Juli 2008

Nr. 74: Nowak-Lehmann D., Felicitas; Vollmer, Sebastian; Martinez-Zarzoso, Inmaculada: Does Comparative Advantage Make Countries Competitive? A Comparison of China and Mexico, Juli 2008

Nr. 73: Fendel, Ralf; Lis, Eliza M.; Rülke, Jan-Christoph: Does the Financial Market Believe in the Phillips Curve? – Evidence from the G7 countries, Mai 2008

Nr. 72: Hafner, Kurt A.: Agglomeration Economies and Clustering – Evidence from German Firms, Mai 2008
Nr. 71: Pegels, Anna: Die Rolle des Humankapitals bei der Technologieübertragung in Entwicklungsländer, April 2008
Nr. 70: Grimm, Michael; Klasen, Stephan: Geography vs. Institutions at the Village Level, Februar 2008
Nr. 69: Van der Berg, Servaas: How effective are poor schools? Poverty and educational outcomes in South Africa, Januar 2008
Nr. 66: Eckel, Carsten: International Trade and Retailing: Diversity versus Accessibility and the Creation of “Retail Deserts”, August 2007
Nr. 65: Stoschek, Barbara: The Political Economy of Environmental Regulations and Industry Compensation, Juni 2007
Nr. 64: Martinez-Zarzoso, Inmaculada; Nowak-Lehmann D., Felicitas; Vollmer, Sebastian: The Log of Gravity Revisited, Juni 2007
Nr. 63: Gundel, Sebastian: Declining Export Prices due to Increased Competition from NIC – Evidence from Germany and the CEEC, April 2007
Nr. 62: Wilekens, Sebastian: Should WTO Dispute Settlement Be Subsidized?, April 2007
Nr. 60: Janebra, Eckhard: Exports, Unemployment and the Welfare State, März 2007
Nr. 59: Lambsdoff, Johann Graf; Nell, Mathias: Fighting Corruption with Asymmetric Penalties and Leniency, Februar 2007
Nr. 58: Köller, Mareike: Unterschiedliche Direktinvestitionen in Irland – Eine theoriegestützte Analyse, August 2006
Nr. 56: Görlich, Dennis; Trebesch, Christoph: Mass Migration and Seasonality Evidence on Moldova’s Labour Exodus, Mai 2006
Nr. 55: Brandmeier, Michael: Reasons for Real Appreciation in Central Europe, Mai 2006
Nr. 54: Martínez-Zarzoso, Inmaculada; Nowak-Lehmann D., Felicitas: Is Distance a Good Proxy for Transport Costs? The Case of Competing Transport Modes, Mai 2006
Nr. 53: Ahrens, Joachim; Ohr, Renate; Zeddies, Götz: Enhanced Cooperation in an Enlarged EU, April 2006
Nr. 52: Stöwhase, Sven: Discrete Investment and Tax Competition when Firms shift Profits, April 2006
Nr. 51: Pelzer, Gesa: Darstellung der Beschäftigungseffekte von Exporten anhand einer Input-Output-Analyse, April 2006
Nr. 50: Elschner, Christina; Schwager, Robert: A Simulation Method to Measure the Tax Burden on Highly Skilled Manpower, März 2006
Nr. 49: Gaertner, Wulf; Xu, Yongsheng: A New Measure of the Standard of Living Based on Functionings, Oktober 2005
Nr. 48: Rincke, Johannes; Schwager, Robert: Skills, Social Mobility, and the Support for the Welfare State, September 2005
<table>
<thead>
<tr>
<th>Nr. 47:</th>
<th>Bose, Niloy; Neumann, Rebecca: Explaining the Trend and the Diversity in the Evolution of the Stock Market, Juli 2005</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nr. 46:</td>
<td>Kleine, Jörn; Toubal, Farid: Gravity for FDI, Juni 2005</td>
</tr>
<tr>
<td>Nr. 45:</td>
<td>Eckel, Carsten: International Trade, Flexible Manufacturing and Outsourcing, Mai 2005</td>
</tr>
<tr>
<td>Nr. 44:</td>
<td>Hafner, Kurt A.: International Patent Pattern and Technology Diffusion, Mai 2005</td>
</tr>
<tr>
<td>Nr. 43:</td>
<td>Nowak-Lehmann D., Felicitas; Herzer, Dierk; Martínez-Zarzoso, Inmaculada; Vollmer, Sebastian: Turkey and the Ankara Treaty of 1963: What can Trade Integration Do for Turkish Exports, Mai 2005</td>
</tr>
<tr>
<td>Nr. 41:</td>
<td>Carlberg, Michael: International Monetary Policy Coordination, April 2005</td>
</tr>
<tr>
<td>Nr. 40:</td>
<td>Herzog, Bodo: Why do bigger countries have more problems with the Stability and Growth Pact?, April 2005</td>
</tr>
<tr>
<td>Nr. 39:</td>
<td>Marouani, Mohamed A.: The Impact of the Multifiber Agreement Phaseout on Unemployment in Tunisia: a Prospective Dynamic Analysis, Januar 2005</td>
</tr>
<tr>
<td>Nr. 38:</td>
<td>Bauer, Philipp; Riphahn, Regina T.: Heterogeneity in the Intergenerational Transmission of Educational Attainment: Evidence from Switzerland on Natives and Second Generation Immigrants, Januar 2005</td>
</tr>
<tr>
<td>Nr. 37:</td>
<td>Böttner, Thiess: The Incentive Effect of Fiscal Equalization Transfers on Tax Policy, Januar 2005</td>
</tr>
<tr>
<td>Nr. 36:</td>
<td>Feuerstein, Switgard; Grimm, Oliver: On the Credibility of Currency Boards, Oktober 2004</td>
</tr>
<tr>
<td>Nr. 35:</td>
<td>Michaelis, Jochen; Minich, Heike: Inflationsdifferenzen im Euroraum – eine Bestandsaufnahme, Oktober 2004</td>
</tr>
<tr>
<td>Nr. 34:</td>
<td>Neary, J. Peter: Cross-Border Mergers as Instruments of Comparative Advantage, Juli 2004</td>
</tr>
<tr>
<td>Nr. 33:</td>
<td>Bjorvatn, Kjetil; Cappelen, Alexander W.: Globalisation, inequality and redistribution, Juli 2004</td>
</tr>
<tr>
<td>Nr. 32:</td>
<td>Stremmel, Dennis: Geistige Eigentumsrechte im Welthandel: Stellt das TRIPs-Abkommen ein Protektionsinstrument der Industrieländer dar?, Juli 2004</td>
</tr>
<tr>
<td>Nr. 31:</td>
<td>Hafner, Kurt: Industrial Agglomeration and Economic Development, Juni 2004</td>
</tr>
<tr>
<td>Nr. 30:</td>
<td>Martinez-Zarzoso, Inmaculada; Nowak-Lehmann D., Felicitas: MERCOSUR-European Union Trade: How Important is EU Trade Liberalisation for MERCOSUR’s Exports?, Juni 2004</td>
</tr>
<tr>
<td>Nr. 29:</td>
<td>Birk, Angela; Michaelis, Jochen: Employment- and Growth Effects of Tax Reforms, Juni 2004</td>
</tr>
<tr>
<td>Nr. 28:</td>
<td>Broll, Udo; Hansen, Sabine: Labour Demand and Exchange Rate Volatility, Juni 2004</td>
</tr>
<tr>
<td>Nr. 27:</td>
<td>Bofinger, Peter; Mayer, Eric: Monetary and Fiscal Policy Interaction in the Euro Area with different assumptions on the Phillips curve, Juni 2004</td>
</tr>
<tr>
<td>Nr. 26:</td>
<td>Torlak, Elvisa: Foreign Direct Investment, Technology Transfer and Productivity Growth in Transition Countries, Juni 2004</td>
</tr>
<tr>
<td>Nr. 25:</td>
<td>Lorz, Oliver; Willmann, Gerald: On the Endogenous Allocation of Decision Powers in Federal Structures, Juni 2004</td>
</tr>
<tr>
<td>Nr. 24:</td>
<td>Felbermayr, Gabriel J.: Specialization on a Technologically Stagnant Sector Need Not Be Bad for Growth, Juni 2004</td>
</tr>
<tr>
<td>Nr. 23:</td>
<td>Carlberg, Michael: Monetary and Fiscal Policy Interactions in the Euro Area, Juni 2004</td>
</tr>
<tr>
<td>Nr. 22:</td>
<td>Stähler, Frank: Market Entry and Foreign Direct Investment, Januar 2004</td>
</tr>
</tbody>
</table>
Nr. 20: Eckel, Carsten: Does globalization lead to specialization, November 2003

Nr. 19: Ohr, Reneate; Schmidt, Andre: Der Stabilitäts- und Wachstumspakt im Zielkonflikt zwischen fiskalischer Flexibilität und Glaubwürdigkeit: Ein Reform-ansatz unter Berücksichtigung konstitutionen- und institutionenökonomischer Aspekte, August 2003

Nr. 18: Ruehmann, Peter: Der deutsche Arbeitsmarkt: Fehlentwicklungen, Ursachen und Reformansätze, August 2003


Nr. 16: Graf Lambsdorff, Johann; Schinke, Michael: Non-Benevolent Central Banks, Dezember 2002

Nr. 15: Ziltener, Patrick: Wirtschaftliche Effekte des EU-Binnenmarktprogramms, November 2002

Nr. 14: Haufler, Andreas; Wooton, Ian: Regional Tax Coordination and Foreign Direct Investment, November 2001


Nr. 12: Lewis, Mervyn K.: Risk Management in Public Private Partnerships, Juni 2001

Nr. 11: Haaland, Jan I.; Wooton, Ian: Multinational Firms: Easy Come, Easy Go?, Mai 2001


Nr. 8: Angermuller, Niels-Olaf: Währungskrisenmodelle aus neuerer Sicht, Oktober 2000

Nr. 7: Nowak-Lehmann, Felicitas: Was there Endogenous Growth in Chile (1960-1998)? A Test of the AK model, Oktober 2000

Nr. 6: Lunn, John; Steen, Todd P.: The Heterogeneity of Self-Employment: The Example of Asians in the United States, Juli 2000

Nr. 5: Gußfeldt, Jörg; Streit, Clemens: Disparitäten regionalwirtschaftlicher Entwicklung in der EU, Mai 2000

Nr. 4: Haufler, Andreas: Corporate Taxation, Profit Shifting, and the Efficiency of Public Input Provision, 1999

Nr. 3: Rühmann, Peter: European Monetary Union and National Labour Markets, September 1999

Nr. 2: Jarchow, Hans-Joachim: Eine offene Volkswirtschaft unter Berücksichtigung des Aktienmarktes, 1999

Nr. 1: Padoa-Schioppa, Tommaso: Reflections on the Globalization and the Europeanization of the Economy, Juni 1999

Alle bisher erschienenen Diskussionspapiere zum Download finden Sie im Internet unter: http://www.uni-goettingen.de/de/60920.html.