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by

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Approximate Truth of Perfectness

- An Experimental Test -

Siegfried Berninghaus, Werner Güth, King King Li

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Abstract

“Approximate truth” refers to the principle that border cases should be analyzed by solving generic cases and solving border cases as limits of generic ones (Brennan et al., 2008). Our study experimentally explores whether this conceptual principle is also behaviorally appealing. To do so, we focus on perfectness (Selten, 1975) and use his example game with (no) multiplicity of (perfect) equilibria. Distinguishing three uniform perturbation levels, we check for monotonicity (all players react monotonically to the perturbation level) and then explore the behavioral relevance of “approximate truth.”

Keywords: experimental games, trembling hand perfectness, perturbed strategies

JEL classification: C70, C72, C91

1. Introduction

In economics but also in its neighboring disciplines, there is a long tradition of solving ideal cases directly rather than by first solving generic cases and then solving the ideal or border cases by determining the limits of solutions of generic ones (Brennan, Güth, and Kliemt, 2008). This principle is more or less explicitly propagated by Harsanyi (1973), Selten (1975), and Harsanyi and Selten (1988). Examples of studies violating this principle are discussed in Brennan et al. (2008), whose list is, of course far from being exhaustive. Here we focus on the ideal case of unperturbed games and perfectness as a requirement of approximate truth (Selten, 1975).

In an unperturbed game, players can perfectly realize their intended activities, i.e., an unintended action is impossible. The consequence of such ideal power of command are unreached information sets in extensive form games or, analogously, impossible strategy vectors in normal form games. In a generic, i.e., perturbed game, the power to command one's actions is limited: for all players any action has possibly a very small, but strictly positive probability of being used unintentionally.

This obviously avoids unreached information sets in extensive form games and ensures that Bayes' rule (see Bayes, 1763) is always applicable. Similarly, all possible strategy vectors are realized with positive probability in normal form games. A specific consequence of the latter phenomenon is that whatever strategy is weakly dominated in the unperturbed game will be strictly dominated in a perturbed game. This obviously eliminates equilibria in weakly dominated strategies as candidates for perfect or "approximately true" equilibria.¹

We experimentally implement Selten's (1975) example of a three-person game with multiple equilibria only one of which is perfect, however. In line with an abstract solution requirement² (see Harsanyi and Selten, 1988), and to avoid demand effects of asymmetric minimum choice probabilities we induce constant minimum choice probability for all choices of all players. We thus explore the "approximate truth" of uniformly perfect equilibria by exogenously imposing constant minimum choice probabilities which vary between treatments.

In section 2 we describe the experimental protocol. Section 3 analyzes the experimental data. Section 4 concludes.

¹ In Industrial Organization such equilibria are nevertheless propagated, e.g., as the solution for price competition in asymmetric homogeneous markets with constant unit costs as originally suggested by Bertrand (1883). Here the market is extreme not only in the sense of perfect product homogeneity but also by letting prices vary continuously.

² By denying any rationality in making mistakes (see, e.g., Myerson, 1978, who postulates such rationality), all actions should have the same positive minimum probability of being used.

2. Experimental Protocol

Our experimental workhorse is the three-person game in Figure 1 (see Selten, 1975) with players $i = 1, 2, 3$ who have to decide between R_i and L_i . Since R_3 is weakly dominated, the equilibrium $R = (R_1, R_2, R_3)$ is not perfect in spite of its attractivity (R is, e.g., payoff undominated). Thus the “approximately true” solution is the perfect equilibrium $L = (L_1, L_2, L_3)$.

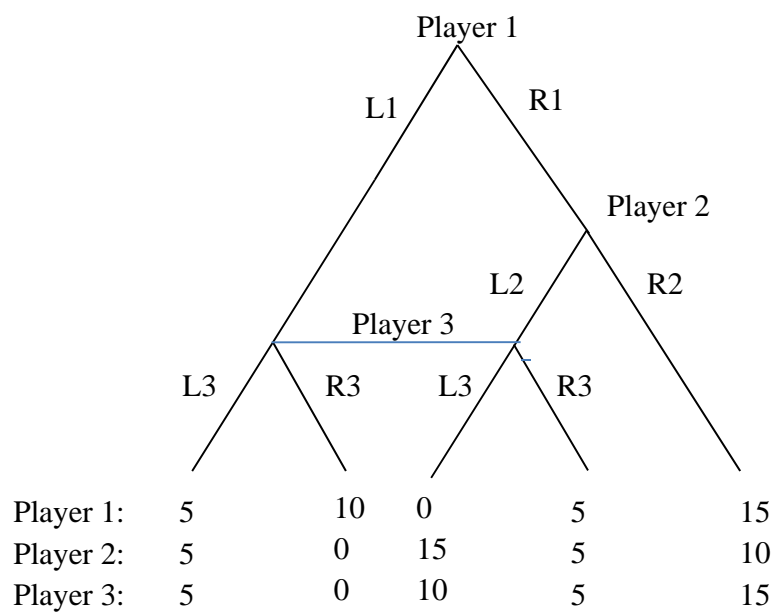


Figure 1: The extensive form game with players $i = 1, 2, 3$ and choices L_i and R_i . In the experimental instruction for $i = 1, 2, 3$, the choices are color labeled. By connecting the two decision nodes of player 3, it is indicated that they lie in one information set.

Epistemic prerequisites: To provide a best-case scenario for confirming the selection of a perfect equilibrium which usually requires common knowledge of rationality we intentionally implemented experimentally a game which does not require uncontrolled beliefs. More specifically, the equilibrium $L=(L_1, L_2, L_3)$ can be justified by repeated elimination of (in the unperturbed game weakly and in the perturbed games strictly) dominated strategies as follows:

- i) Player 3 must eliminate his dominated strategy R_3 ,
 - ii) what, if player 2 knows i), induces player 2 to eliminate his dominated strategy R_2 in the (by elimination of R_3) truncated game,
 - iii) so that player 3, knowing i) and ii) finally would eliminate his dominated strategy R_1 in the (by elimination of R_3 and R_2) truncated game. Thus
- (1) all three players $i=1,2,3$ must eliminate their dominated strategy,
 - (2) additionally, player 2 must be aware that (1) holds for $i=3$,
 - (3) whereas player 1 has to know (1) for $i=3$ and $i=2$ as well as to be aware of (2) to justify L as the only solution candidate without requiring common knowledge of rationality in the sense of the infinite sequence that all three players $i=1,2,3$ know that they all know that (1) holds.

What does it mean to test the broad principle of approximate truth in its specific application for justifying perfect equilibria (Selten, 1975) via implementing a best-case scenario? If the principle is already rejected when it relies on rather weak assumptions concerning not only decision rationality (according to (1) players only have to eliminate dominated strategies) but also knowledge conditions about the rationality of others (the requirements (2) and (3) above) one obviously can reject the behavioral appeal of approximate truth in equilibrium selection quite generally. If, on the other hand, we would confirm the principle, applied to equilibrium selection, for the example above, this should have induced us to test its behavioral reliability also for games whose perfect equilibria require stronger rationality and knowledge assumptions concerning others' rationality than (1), (2), and (3). This could be done by increasing the number of steps of eliminating dominated strategies or by example games whose equilibria rely on common(ly known) rationality.

Experimental setup: Participants were randomly and anonymously matched into groups of three participants to play the game in Figure 1. Player 1 (2, 3) determines the probability that the play will proceed with move L_1 (L_2 , L_3) respectively R_1 (R_2 , R_3). For $i = 1, 2, 3$, the probability for L_i , respectively R_i is determined by the following urn: in the 25 free balls game with altogether 27 balls, only one (of them) is predetermined as red (for L_i) and one as black (for R_i). As for the remaining 25 balls, player i can freely determine how many of them are red or black. Once the urn is composed by determining the color of all 27 balls, a ball will be randomly drawn from it. If a red ball is drawn, the game will proceed with route L_i . If a black ball is drawn, the game will

proceed with route R_i . Similarly, in the 21 free balls game, there are 3 red balls, and 3 predetermined black balls, and player i determines the color of the remaining 21 free balls. In the 9 free balls game, 9 red balls and 9 black balls have a predetermined color, meaning that player i determines the color of only 9 balls.

Note that even in the 9 free ball game, the avoidance of R_3 as far as possible, i.e., player 3 plays L_3 with probability $2/3$ and R_3 with probability $1/3$, suffices to render L_2 uniquely optimal for player 2 (since L_2 yields $35/3$ and R_2 only 10). Anticipating that both, player 2 and player 3, use their L -strategy with maximal probability $2/3$ in the 9 free balls game, finally implies that for player 1 the choice of L_1 , yielding $60/9$, is better than R_1 , yielding only $55/3$. Thus even the largest uniform perturbation level allows for no ambiguity of (sequential) rationality. For all perturbation levels all three players $i = 1, 2, 3$ should therefore use the perfect equilibrium strategy L_i with maximal probability.

For the sake of more informative data, and to encourage more thorough deliberations on what to do in the various roles (1, 2, and 3), choices were elicited using the strategy vector method. That is, participants specified their choice s_1, s_2 , and s_3 via determining the urns with red and black balls to govern the choice of player 1, 2, and 3. Given these choices, the computer randomly determined the roles, implemented the decisions, and computed the payoffs accordingly. For example, if L_1 and L_3 are realized, each player receives 5 euros.

To investigate within-subject differences across treatments, subjects participated in all three games, differing only in their uniform trembles where the order was varied between subjects. The three treatments can be distinguished by the sequence of the numbers of “free balls”:

<i>Treatments</i>		
25-21- 9	21-21-21	9-21-25

For example, in the 25-21-9 treatment, subjects first played the 25 free balls game, then the 21 free balls game, and finally the 9 free balls game. Subjects were not informed that they would play the second (third) game when they played the first (second) game in the sequence. They learned about payoffs only after playing all three games and additionally reported their beliefs concerning the choice behavior of others.

A total of 135 subjects participated in the computerized (via z-tree, Fischbacher, 2007) experiment. They were randomly recruited from a pool of approximately 2,500 subjects using an e-mail recruitment system (ORSEE, see Greiner, 2004). All subjects were university students from Jena, Germany. The experiments were conducted in German. In the beginning, subjects were given paper instructions. They were informed that their decisions would be anonymous and that they would receive a show-up fee of 2.5 euros in addition to what they earned by playing the games. Then each subject was asked to key in her decisions.

3. Experimental Results

Let us begin by analyzing first-play behavior, i.e., we use the combined first choices

- of “25 free balls” from treatment 25-21-9
- of “21 free balls” from treatment 21-21-21
- of “9 free balls” from treatment 9-21-25

where we concentrate on the proportion of “free balls” allocated to R_i and L_i , respectively.

Since only the equilibrium $L = (L_1, L_2, L_3)$ is perfect, we check monotonicity by testing

Hypothesis M: Monotonic convergence to L .

For $i = 1, 2, 3$ the tendency to voluntarily use L_i increases with more “free balls.”

Table 1. Average probability allocated

Treatment	Average Number of Allocated Balls (Std.)		
	L1	L2	L3
25 Free Balls	5.63 (9.01)	5.30 (8.29)	20.81 (6.27)
21 Free Balls	5.80 (8.43)	11.47 (9.79)	19.03 (4.73)
9 Free Balls	3.00 (3.34)	2.47 (3.25)	7.87 (2.11)
	Average Probability Allocated (Std.)		
25 Free Balls	0.25 (0.33)	0.23 (0.31)	0.81 (0.23)
21 Free Balls	0.33 (0.31)	0.54 (0.36)	0.82 (0.18)
9 Free Balls	0.44 (0.12)	0.42 (0.12)	0.62 (0.08)

Notes: 25 Free Balls refers to the first round in the 25-21-9 treatment. 21 Free Balls refers to the first round in the 21-21-21 treatment. 9 Free Balls refers to the first round in the 9-21-25 treatment.

Table 1 shows voluntary probability of L_i as depending on the number of “free balls,” separately for each player $i = 1, 2, 3$.

Clearly, the weakly dominating move L_3 is realized with higher probability than L_1 and L_2 , which could be expected as the non-rationality of L_3 is more obvious. Concerning the effect of variation in the number of free balls, the change from 25 to 21 “free balls” is minor, compared to the one

from 25 or 21 to 9 “free balls.” The intermediate treatment of “21 free balls” was mainly introduced to check only minor “perturbance differences.” As can be seen from Table 1, due to noisy choice behavior, one cannot expect monotone reactions to minor changes in the perturbation level. However, the differences between 25 or 21 “free balls” and 9 “free balls” are statistically significant for all three player roles, see Table 2.³

Table 2. Comparisons of Probability Allocated for L

Average Probability Allocated for L	Mean Differences	t-Test
L1		
25 vs. 9	0.20	3.04***
21 vs. 9	0.12	1.93*
L2		
25 vs. 9	0.19	3.16***
21 vs. 9	-0.11	-1.59
L3		
25 vs. 21	-0.18	-4.08***
21 vs. 9	-0.19	-5.46***

Notes: *, **, and *** represent significance at 10, 5, and 1 percent levels, respectively.

The average probabilities observed in 25 “free balls” for L_1 and L_2 are significantly lower than in the 9 “free balls,” whereas the reverse is observed with L_3 . We thus can conclude the following:

Observation 1: When the number of “free balls” decreases, the voluntary probabilities of relying on the perfect equilibrium strategies L_1 and L_2 increases where the effects when comparing 25 or 21 “free balls” with 9 “free balls” are statistically significant. For L_3 an opposite effect is observed.

The observation for L_1 and L_2 contradicts hypothesis M, whereas the observation for L_3 is in line with hypothesis M. As in every perturbed game R_3 is strictly dominated, one might expect the strongest avoidance of R_3 , whose non-optimality is more obvious. The probability allocated for L_3 is significantly higher than L_1 and L_2 (p-value = 0.00). There is no significant difference between L_1 and L_2 .

Observation 2: For all numbers of “free balls” player 3 avoids the dominated choice R_3 significantly more often than the two other players, 1 and 2, for whom the incentive to avoid R_i anticipates that player 3 avoids R_3 to such an extent that L_2 and then also L_1 become optimal.

³ Except the case of 21 vs. 9 in L2.

Let us finally investigate how earlier experience with another “perturbance” level affects behavior. Table 3 below tabulates the probability allocated by players in respective treatments. Contrary to the expectation that any earlier experience should strengthen the tendency of voluntarily using L_i rather than R_i , $i = 1, 2, 3$, there is no significant difference.

Observation 3: When for $i = 1, 2, 3$ comparing the first and third play

- of 25 “free balls” (i.e., using the 1st (3rd) play data of treatments 25-21-9 (9-21-25), the use of L_i is not significantly stronger,
- of 9 “free balls” (i.e., using the 1st (3rd) play data of treatments 9-21-25 (25-21-9), the use of L_i is not significantly stronger.

Table 3. The Effect of Sequence of Play on Probability Allocated

	Probability Allocated			
	25 Free Balls		9 Free Balls	
	25-21-9	9-21-25	25-21-9	9-21-25
Player 1	0.25 (0.06)	0.27 (0.06)	0.43 (0.03)	0.44 (0.02)
Player 2	0.23 (0.06)	0.39 (0.07)	0.40 (0.02)	0.42 (0.02)
Player 3	0.81 (0.04)	0.85 (0.05)	0.62 (0.02)	0.62 (0.01)

Observation 3 that former experiences do not strengthen the use of perfect equilibrium strategies is additionally supported by the 21-21-21-treatment.

Observation 4: The voluntary probabilities of using L_i rather than R_i does not increase significantly with the round of play of the 21-21-21-treatment, even when the game, captured by the noise level, remains constant.

Table 4. Probability Allocated to L_i in Treatment 21-21-21

	First Round	Second Round	Third Round
Player 1	0.33 (0.06)	0.41 (0.06)	0.44 (0.06)
Player 2	0.54 (0.07)	0.46 (0.06)	0.52 (0.06)
Player 3	0.82 (0.03)	0.80 (0.03)	0.78 (0.03)

4. Conclusion

Since the principle of approximate truth applied to equilibrium selection already fails for one best-case scenario its test for games whose perfect equilibria require stronger assumptions seems superfluous what justifies our approach to begin with a best-case scenario. This, of course, renders the principle of approximate truth questionable also in other applications than equilibrium selection (see the other examples, discussed by Brennan et al.).

Probably, it would have been naïve to expect that the unique perfect equilibrium prediction could be experimentally confirmed. One might have hoped that less perturbation would shift behavior toward its benchmark. But this could be confirmed only for player 3 whose behavior is much closer to the predicted choice (of L_3) but not for player 1 and player 2. In our view, the non-optimality of L_3 is much more obvious than that of L_1 and L_2 rendering the choices by players 1 and 2 much more noisy. This explains that minor changes in perturbation (21 versus 25 “free balls”) have little effect and that the use of L_1 respectively L_2 is far from being close to what was predicted. Altogether, we have not consistently proved that the principle of approximate truth is in line with the experimental evidence collected. Equilibrium refinements relying on this principle are therefore only philosophically but not psychologically appealing.

One may object to our analysis that we assume common(ly known) material opportunism in the sense that each player only cares for his own monetary payoff expectation. Since the imperfect equilibrium R payoff dominates the perfect equilibrium L , assuming efficiency concerns would allow to justify R rather than L as solution. However, this would require some common knowledge of such “social preferences,” which seems highly unrealistic. But the fact that R payoff dominates L means that our experimental game is a worst-case scenario for confirming the principle of approximate truth. Across the board, our results indicate that the principle obviously does not have behavioral appeal. If at all, it needs more favorable conditions than those provided by our experimental game.

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