Kind, Hans Jarle; Sørgard, Lars

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Market Segmentation in Two-Sided Markets:
TV Rights for Premier League

Hans Jarle Kind
Lars Sørgard

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Market Segmentation in Two-Sided Markets: TV Rights for Premier League

Abstract

This paper analyzes market segmentation in a two-sided market that consists of media consumers and advertisers. The analysis is motivated by a European Court of Justice Decision in October 2011, which allowed viewers to take advantage of international price differences and buy access to Premier League TV matches from whichever country they like. We compare complete market segmentation with the new situation where consumers can purchase from abroad (allowing for passive sales). Clearly, such a change is likely to harm Premier League, which at present is sold at different prices to viewers in different countries. More surprisingly, we find that all viewers – including those that switch to purchasing from abroad - might also be harmed. We further show that the two-sidedness of the market may break down in the country that attracts foreign viewers.

JEL-Code: D430, K210, L130.

Keywords: market segmentation, two-sided market, price discrimination, media market, passive sales.

Hans Jarle Kind
Norwegian School of Economics (NHH)
Helleveien 30
5045 Bergen
Norway
hans.kind@nhh.no

Lars Sørgard
Norwegian School of Economics (NHH)
Helleveien 30
5045 Bergen
Norway
lars.sorgard@nhh.no

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1 Introduction

Karen Murphy - a pub landlady in England - switched her Premier League subscription from an English satellite distributor to the Greek distributor Nova, and the price dropped from £ 7000 to £ 800.1 The Premier League brought the case to the European Court of Justice, but Karen Murphy won and was allowed to continue purchasing from the foreign distributor.2 The decision reflects that market segmentation in the form of denying consumers to buy from abroad may be in conflict with the idea behind EU’s Common Market and with EU’s competition law. In this paper we show that a ban on market segmentation in a two-sided market (of which a TV program that is financed by both viewer payments and advertising revenues is a prime example) might be detrimental to welfare, and actually harm all consumers, including those who would like to purchase from abroad.

Market segmentation, for example televised transmission of Premier League sold to viewers at different prices in different countries, is an example of third degree price discrimination. The consequences of such price discrimination in one-sided markets are analyzed in detail in the existing literature.3 For example, Varian (1985) has shown that a necessary condition for third degree price discrimination to increase total welfare is that output increases. Unfortunately, this insight cannot guide us in two-sided markets, where it is crucial to capture the interaction between the price setting on the two sides of the market. 4 In the Premier League case, for instance, we need to take into

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1See, for example, The Telegraph: 'Premier League braced for European Court of Justice ruling on selling of lucrative television rights’, online October 3 2011 on http://www.telegraph.co.uk/sport/football/competitions/premier-league/8805350/Premier-League-braced-for-European-Court-of-Justice-ruling-on-selling-of-lucrative-television-rights.html

2See judgements in case C-403/08 (Football Association Premier League and Others v QC Leisures and Others) and case C-429/08 (Karen Murphy v Media Protection Services Ltd. A later decision by a UK court was interpreted by Premier League as if their business practice is legal. See the UK court ruling of February 3 2012, that delivered its judgment on an ECJ ruling relating to a company called QC Leisure, a provider of Greek and Arabic decoder cards to publicans in the UK. On February 24 2012 the high court ruled in Karen Murphy’s favour, although they stated that many issues had to be settled at a later date. See http://www.bbc.co.uk/news/business-17150054.


4The importance of the interaction between the two sides of the market is clearly shown in the existing literature. See for example Evans (2003a, 2003b), Rochet and Tirole (2003) and Weyl (2010) for a general analysis, and for example Anderson and Coate (2005),
account the interrelationship between the consumer side and the advertiser side of the market.

The starting point for our analysis is a situation with third degree price discrimination between two countries. We call this regime, where we assume that consumers cannot purchase the product we consider from abroad, complete segmentation. To make it interesting, we set up the model such that there is a price difference between the countries. Formally, we assume that in the more expensive country there is both a consumer group with low price elasticity and a consumer group with a high price elasticity, while in the other country there is only a consumer group with high price elasticity.

The seller can earn revenue only from consumers (one-sided market), only from advertisers, or from both consumers and advertisers (two-sided market). If there is advertising, we assume that it is tailored to domestic consumers. This implies that if a consumer watches a foreign TV channel, then he will be of no value to the firms that advertise on that channel.\(^5\)

We compare the complete segmentation regime with what we call a no segmentation regime, where we allow consumers to purchase the product from abroad if that is cheaper. This is in line with the EU court decision we have referred to. It is also in line with the EU competition law guidelines on vertical restraints concerning passive sales, where any restrictions imposed by an upstream firm on the downstream firm concerning consumers’ purchasing from a neighboring territory will be regarded as a violation of a hardcore restriction.\(^6\) On the other hand, EU competition law allows the upstream firm to place restrictions on the downstream firms’ active sales. This means that the upstream firm can force its distributors not to sell actively in other places than their own exclusive territory, and thus not compete for consumers

\(^5\)This is a simplification; what matters for our results is that domestic consumers generally are more valuable than foreign consumers for the advertisers.

\(^6\)Passive sales is defined as ‘responding to unsolicited requests from individual customers’ coming from another territory (see Guidelines for Vertical Restraints, 2010/C 130/01, par. 51). See Commission Regulation 330/2010, art. 4(b), where it is stated that if an upstream firm imposes an exclusive territory on a downstream firm with restrictions on passive sales then this is outside the block exemption and must therefore be regarded as a hardcore restriction. It indicates that it is likely that it would be a violation of the competition law (article 101 in the Treaty of the Functioning of the European Union) if an upstream firm put restrictions on a downstream firm’s passive sales.
We take seriously the EU competition law rules by (i) allowing individual consumers to switch to a foreign distributor (allowing passive sales), and (ii) by prohibiting the distributors in the two countries to compete for each others’ consumers (prohibiting active sales).

As a benchmark, we consider the consequences of shifting from complete to no segmentation in a one-sided market. We do so by assuming that the TV channels only rely on consumer payment (no advertising revenue). Since it is the most price sensitive consumers who switch to the low-price country, we arrive at the standard result that the price will increase in the country with the highest initial price. Assuming that those who switch are of the same type as the consumers in the low-price country, the price there will not change.

We contrast these predictions with those in a corresponding model with a two-sided market. The consumers will then pay a direct price for viewing, as in a one-sided market, and, in addition, an indirect price because they are interrupted by ads (it is well documented that viewers generally consider TV ads as a nuisance). The consumers’ generalised price is the sum of the direct and the indirect price. In the no segmentation regime we assume that a non-loyal consumer will switch if he observes a lower generalised price in the other country.

We find that the direct prices increase in both countries following a shift to no segmentation. It is straightforward to understand why this happens in the high-price country; its most price sensitive viewers now buy from abroad. These viewers are clearly valuable to the foreign TV channel, but they do not generate any advertising revenue. Once we open up for international trade, direct consumer payment thus becomes relatively more important for the TV channel in the low-price country. The initial motive for setting a relatively low direct price - attracting consumers and thereby increasing revenues from advertisers - is therefore dampened in the low-price country. It will consequently increase the direct price and reduce the advertising volume.

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7This is a restriction on 'active sales', which means 'actively approaching individual customers', in a neighboring territory. The upstream firm can impose restrictions on the downstream firm's active sales and still be part of the block exemption regulation for vertical restraints (see Guidelines for Vertical Restraints, 2010/C 130/01, par. 51, as well as Commission Regulation 330/2010, art. 4b). This indicates that an upstream firm that imposes restrictions on a downstream firm’s active sales is probably not violating Article 101 of the Treaty of the Functioning of the European Union.

8See for example Danahar (1995) and Moriarty and Everett (1994).
The advertising level in the country that loses viewers will in contrast increase. This might seem surprising, since demand for advertising is higher the greater is the size of the audience, other things equal. The explanation for this result hinges on the fact that consumers tend to dislike ads on TV. However, when only the most loyal viewers remain, the channel can increase the advertising level without risking that too many viewers stop watching TV.

Since both the direct and the indirect price increase in the high-price country, the generalised price in this country must necessarily increase too. In addition, we show that the generalised price increases in the low-price country as well (the increase in the direct price outweighs the reduction in the indirect price). This is in contrast to a corresponding model with a one-sided market, where prices increase only in the high-price country.

The inflow of viewers to the low-price country means that a share of its audience will be of no value to the advertisers in this country. This mismatch in the advertising market implies that a shift from complete to no segmentation may lead to a shift from a two-sided to a one-sided market in the low-price country. This is true if the gain from advertising is sufficiently low. The TV channel in the low-price country will then switch to being financed only by consumer payment, which is obviously detrimental to the advertisers in the low-price country.

An interesting finding, not least from a policy point of view, is that a shift from complete to no segmentation can be detrimental to total welfare - the sum of consumer surplus and profits in the two countries. Even more interestingly, prices may increase so much that those who would like to switch will be worse off if they are allowed to buy from the cheaper country. As long as each non-loyal consumer reacts to any price difference by switching, and there is no coordination among the consumers, this will emerge as a prisoners’ dilemma outcome. This is in sharp contrast to standard predictions. It reflects the fact that market segmentation may be efficiency-enhancing in two-sided markets.

There are articles discussing price discrimination in two-sided markets, see for example Callauid and Jullien (2003), Armstrong (2006) and Liu and Serfes (2010). But none of these consider third degree price discrimination or allow for a shift from complete to no market segmentation. This implies that they do not capture the potential mismatch between consumers and advertisers, on which we focus. Moreover, some of their results are qualitatively different from ours. For example, Liu and Serfes (2010) find that price discrimination
leads to higher prices while we find the opposite.

Our main issue is how international trade might lead to a mismatch in two-sided markets. Jeon, Jullien and Klimenko (2012) study a similar issue. However, their focus is very different from ours, since their main concern is how the dominance of English web content may affect the production of web content in a non-English home language. Another important difference is that while they analyze potential mismatch between consumers and the content providers, we concentrate on potential mismatch between consumers and the advertising market.

The article is organized as follows. In the next section we present the model, and we report the results of a shift from complete to partial segmentation in a one-sided market. In section 3 we present a two-sided market model and derive the effects of a shift from complete to partial segmentation. In section 4 we present a model where demand is specified explicitly, and in section 5 we offer some concluding remarks and comment on the optimal public policy concerning market integration.

2 Some preliminaries

Let us consider a model where a media product is sold in two countries, Country 1 and Country 2. The product could for instance be a televised transmission of Premier League soccer games; hereafter labelled TV program. The program is aired by TV1 in Country 1 and TV2 in Country 2.\(^9\)

Consumers are heterogeneous concerning their willingness to pay for watching the TV program, and each consumer buys either one or zero units. It is not possible to charge different prices from different consumers within a country (no domestic price discrimination). We assume that there are two types of consumers, group A and group B. The countries have the same number of group A consumers. Abstracting from other factors that influence demand, we thus have \(C_1^A(p) = C_2^A(p)\) and \(\partial C_1^A / \partial p = \partial C_2^A / \partial p < 0\), where \(C_i^A\) denotes the number of consumers in group A who watch TV channel \(i\), and \(p\) denotes the viewer price. These consumers may switch to purchasing the product from the other country if they are allowed to do so. We call

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\(^9\)In reality, there might be an upstream agent that produces the content and sells this to the consumers through a TV channel. We do not model the relationship between the content provider and the TV channel. One interpretation of our model is that the content provider and the TV channel in each country write efficient contracts.
these consumers the \textit{non-loyals}.

Consumer group \textit{B} exists only in Country 1. These consumers are not willing to switch to purchasing from abroad, and we call them the \textit{loyals}. Let \( \varepsilon_i^k \) denote (the negative of) the price elasticity of demand for group \( k \) in country \( i \), where \( i = 1, 2 \) and \( k = A, B \), defined as \( \varepsilon_i^k \equiv - (\partial C_i^k / \partial p_i) \cdot (p_i / C_i^k) \). We assume that \( \varepsilon_1^A > \varepsilon_1^B \), which means that consumer demand in group \( A \) is more price elastic than in group \( B \). We further define the price elasticity for the total market in each of the countries as \( \varepsilon_1 \) and \( \varepsilon_2 \) (hereafter labelled average elasticities). Since Country 2 has only type \( A \) consumers, we have \( \varepsilon_2 = \varepsilon_A \).

In addition to the revenue they raise from the consumer charges, the TV channels can also earn revenue from the advertising market. If both advertisers and viewers are on board, this is a two-sided market. Consistent with empirical evidence, we assume that advertising is a nuisance to the viewers (and for simplicity we let the nuisance costs be identical for all consumers). The number of viewers at TV channel \( i \) thus depends negatively on the viewer price and on the advertising level, \( A_i \). Letting \( C_i \) denote the total number of viewers at TV channel \( i \), we thus have that \( C_i(p_i, A_i) \) is decreasing in both its arguments. Note that if no-one watches the foreign TV channel, we have \( C_1 = C_1^A + C_1^B \) and \( C_2 = C_2^A \).

The commercials are tailored towards the countries’ domestic consumers, implying that foreign consumers have no value on the advertising market (this is not crucial; what is important for our analysis is that foreign viewers are worth less than domestic viewers for the advertisers). With a downward-sloping demand curve, the advertisers’ demand for commercial time thus depends negatively on the price \( r_i \) they pay per slot and positively on the number of domestic viewers, \( C_{iL} ; \partial A_i(r_i, C_{iL}) / \partial r_i < 0 \) and \( \partial A_i(r_i, C_{iL}) / \partial C_{iL} > 0 \). We assume that all domestic consumers are equally valuable for the advertisers.

We may now write profits for the TV channel in Country \( i \) as

\[ \pi_i = p_i C_i(p_i, A_i) + r_i A_i(r_i, C_{iL}) - \phi(\circ), \]  

where the cost function \( \phi(\circ) \geq 0 \) may depend both on the ad level and on the number of viewers. The TV channel maximizes profits with respect to the advertising price \( r_i \) and the viewer price \( p_i \).

We compare two regimes. In the first regime the TV channels segment the markets completely by not allowing a consumer in one country to purchase
from the other country (complete segmentation). In the second regime we allow the non-loyals to buy from the country with the lowest total viewer price (free trade).

Since we assume homogeneity concerning each viewer’s nuisance cost, we can easily see that - all else equal - the price sensitivity on the viewer side is decisive for the viewer price:

**Remark 1:** All else equal, \( p_i > p_j \) if \( \varepsilon_j > \varepsilon_i \).

Furthermore, assuming that \( A_i(r_i, C_{iL}) \) is invertible, it follows that because \( \partial C_{iL}/\partial p_i < 0 \) we also have\(^{10}\):

**Remark 2:** \( \partial r_i/\partial p_i < 0 \).

The intuition for Remark 2 is simply that a higher viewer price lowers the number of viewers, which in turn reduces the willingness to pay for ads.

## 3 General results

### 3.1 Benchmark: One-sided markets (pure pay-TV)

For comparison, let us first consider the case with \( A_1 = A_2 = 0 \). This is the case with revenues only from the consumer side of the market, and therefore no two-sidedness. Solving \( \partial \pi_i/\partial p_i = 0 \) yields the standard first-order condition that marginal revenue is equal to marginal costs:

\[
\frac{\partial \pi_i}{\partial p_i} = \left[ C_i + p_i \frac{\partial C_i}{\partial p_i} - \frac{\partial \phi}{\partial p_i} \right] = 0
\] (2)

With complete segmentation, the consumers are forced to purchase their product in their own country. Since the consumers in Country 1 on average have a lower price elasticity of demand than the consumers in Country 2, it is optimal to set \( p^{A=0}_1 > p^{A=0}_2 \).

We compare complete segmentation with a regime where the distributors still sell actively only in their own country, but where the consumers are allowed to switch to the distributor in the neighboring country. The latter, called a no segmentation regime, is in line with the EU court decision on

\(^{10}\)This assumption is met in many two-sided models for the media market, see for example Kind et al. (2009).
Premier League and the existing competition law rules (allowing passive sales and at the same time imposing restrictions on active sales). By assumption, only the non-loyals will consider doing this, and since \( p_{1A}^{A=0} > p_{2A}^{A=0} \) it follows that non-loyals in Country 1 switch to Country 2.

Since the consumers switching from Country 1 to Country 2 are identical to those in Country 2, there will be no change in the price elasticity of demand and thus no change in the viewer price in Country 2. In Country 1 the remaining consumers, the loyals, are by definition less price elastic (\( \varepsilon_{B1} < \varepsilon_{A1} \)). This leads to a higher viewer price in Country 1.

Summing up, we have the following result:

**Proposition 1** Assume revenues only from the consumer side. Then a shift from complete segmentation to no segmentation leads to a higher viewer price in Country 1 and has no effect on the viewer price in Country 2.

A direct implication of this proposition is that

**Corollary 1** Assume that there is a shift from complete segmentation to no segmentation, and revenues only from the consumer side. The loyal consumers in Country 1 then lose, while the non-loyals in Country 1 gain. Consumers in Country 2 are not affected (no price change).

### 3.2 Two-sided markets

Let us now assume that the TV channels are financed by revenues from both viewers and advertisers. The subjective cost of watching TV then depends on the monetary viewer price \( p_i \) as well as on the non-monetary (indirect) costs of being interrupted by ads. Denoting the latter by \( f(A_i) > 0 \), with \( f'(A_i) > 0 \), we follow the convention in the media economics literature and define the generalized viewer price in Country \( i \) as

\[
g_i = p_i + f(A_i).
\]

The total (subjective) viewer cost per program is thus equal to the sum of the monetary and the non-monetary price.

The monetary viewer price will be lower in a two-sided market compared to the benchmark case. To show this, let us consider the first-order conditions for the optimal advertising and viewer prices in Country \( i \):
\[
\frac{\partial \pi_i}{\partial p_i} = \left[ C_i + p_i \frac{\partial C_i}{\partial p_i} - \frac{\partial \phi}{\partial p_i} \right] + r_i \frac{\partial A_i}{\partial C_{iL}} \frac{\partial C_{iL}}{\partial p_i} = 0 \quad (3)
\]

\[
\frac{\partial \pi_i}{\partial r_i} = \left[ A_i + r_i \frac{\partial A_i}{\partial r_i} - \frac{\partial \phi}{\partial r_i} \right] + p_i \frac{\partial C_i}{\partial A_i} \frac{\partial A_i}{\partial r_i} = 0 \quad (4)
\]

The term in the square bracket of (3) is equal to zero in a one-sided market, c.f. equation (2). The term outside the bracket captures the fact that a higher viewer price reduces the number of viewers and thus also the demand for ads. This term is therefore negative, making it clear that the viewer price will be relatively low in a two-sided market.

The interpretation of (4) is similar; if the viewers were indifferent to the ad-level, the advertising price should be set such that the term in the square bracket is equal to zero. However, the last term in equation (4) implies that the optimal advertising price is higher than this, because a higher advertising price reduces the advertising level such that the non-monetary viewer price falls. This in turn increases the number of viewers.\footnote{This is a robust finding in the literature on two-sided media markets. See Anderson and Gabszewicz (2006), which is a survey of the literature on models for the media market.}

Let us now consider how the viewer price \( p_i \) changes when we have a two-sided market and we shift from complete segmentation to a situation where consumers can switch (no segmentation). Let \( p^*_i \) denote viewer price in Country \( i \) with complete segmentation, and \( p^{**}_i \) the corresponding price under no segmentation. We use similar notations for the other variables. Note that due to the low price elasticity for the loyals, the generalized price under complete segmentation will be higher in Country 1 than in Country 2; \( g^1_1 > g^2_2 \).

Let us now assume the following:

**Assumption 1**: \( g^{**}_1 \geq g^{**}_2 \).

Given this assumption, it is profitable for a non-loyal to switch. As we shall see below, the generalized price in Country 1 with no segmentation will typically be strictly higher than that in Country 2 if all non-loyals switch, but will hold with equality if only a share of them do so.

If Assumption 1 is satisfied, we have:
Proposition 2 Assume revenues from both the advertising and the consumer side. Then a shift from complete to no segmentation increases the viewer prices in both countries; $p_i^{**} > p_i^{*}$.

Proof: Since the viewer price will be lower in a two-sided market than in a one-sided market, we have $p_2^{A=0} > p_2^{*}$ under complete segmentation. Suppose that TV2 could price discriminate between foreign and domestic consumers. It would then charge $p_2^{A=0}$ from the foreign viewers and $p_2^{*}$ from the domestic ones. As this is not possible, it follows straightforwardly that $p_2^{A=0} > p_2^{**} > p_2^{*}$. In Country 1, at least a share of the non-loyals buy from abroad. The price elasticity of demand in the relevant market in Country 1 has thus decreased, so with profit maximizing prices we have $p_1^{**} > p_1^{*}$.

We further have:

Proposition 3 Assume revenues from both the advertising and the consumer side. Then a shift from complete to no segmentation increases the generalized price in Country 1 ($g_1^{**} > g_1^{*}$) and reduces the advertising volume in Country 2 ($A_2^{**} < A_2^{*}$).

The proof and intuition for why $g_1^{**} > g_1^{*}$ are the same as for why $p_1^{**} > p_1^{*}$; since the most price sensitive consumers in Country 1 buy from abroad in a no segmentation regime, it is optimal for the TV channel in this country to increase both the monetary and the generalized viewer price. The intuition for why $A_2^{**} < A_2^{*}$ follows from the discussion above; by reducing the advertising volume, the TV channel in Country 2 can increase the monetary viewer price. This is optimal, since the viewers from Country 1 do not generate any advertising revenue.

To see formally that $A_2^{**} < A_2^{*}$, note that since only domestic viewers have any value on the ad market, the term in the square bracket of (4) for Country 2 is independent of whether TV2 has any foreign viewers. However, for each unit increase in the advertising level, the number of viewers falls more when the channel also serves foreign non-loyals: $\left| \frac{\partial (C_1^A + C_2^A)}{\partial A_2} \right| > \left| \frac{\partial C_2^A}{\partial A_2} \right|$. Since moreover we have $p_2^{**} > p_2^{*}$, the term outside the bracket in (4) is unambiguously larger under no segmentation than under complete segmentation. Thus, the advertising price in Country 2 will be higher ($r_2^{**} > r_2^{*}$) and the advertising level lower ($A_2^{**} < A_2^{*}$) in the no segmentation regime.

We also set up the following conjecture:
Conjecture 1 Assume revenues from both the advertising and the consumer side. Then a shift from complete to no segmentation might increase the generalized price in Country 2 ($g_{2}^{*} > g_{2}^{*}$) and the advertising volume in Country 1 ($A_{1}^{*} > A_{1}^{*}$).

The claim that $g_{2}^{*} > g_{2}^{*}$ is intuitive; since the switchers from Country 1 are valueless on the advertising market in Country 2, there is less reason for the TV channel in that country to attract a large audience in order to increase advertising revenue. Thus, it is optimal to set a higher generalized price, which means that the increase in $p_{2}$ dominates over the reduction in the ad level (i.e. $\Delta g_{2} = \Delta p_{2} + \Delta A_{2} > 0$).

At first sight the claim that $A_{1}^{*} > A_{1}^{*}$ might seem counter-intuitive; when the size of the audience falls, it becomes less attractive to advertise. Other things equal, this results in a lower advertising volume. However, since the remaining viewers in Country 1 - the loyals - are less price sensitive than those who have left, we might nonetheless expect that it is optimal to increase both the monetary ($p_{1}$) and the non-monetary ($A_{1}$) viewer price in that country.

4 A specific model

The relationships stated in Conjecture 1 cannot be proved to hold for any demand system, but below we provide an illustration of the mechanisms through a specific model. We also analyze in some detail the welfare implications of a shift from market segmentation to free trade, and in particular whether the non-loyals might be worse off if they are allowed to purchase from the foreign country. This possibility cannot be disregarded if the generalized viewer prices increase in both countries.

4.1 Market segmentation

Suppose for the moment that the consumers are prohibited from making their purchases abroad, such that the markets are completely segmented. Let demand from consumer group $A$ in Country $i = 1, 2$ be given by $C_{i}^{A} = 1 - p_{i} - \eta A_{i}$, where the positive parameter $\eta$ measures the viewers’ disutility of ads. We normalize by setting $\eta = 1$ :

\[ C_{i}^{A} = 1 - p_{i} - A_{i}. \]
Demand from consumer group $B$, which exists only in Country 1, is likewise given by

$$C_1^B = \beta - p_1 - A_1.$$  \hfill (6)

To capture the assumption that these consumers on average are less price elastic than those in Group A, we assume that $\beta > 1$ (the price elasticity is strictly decreasing in $\beta$).

Since the platforms are unable to price discriminate between the groups, it is clear that TV1 will choose to serve only group $B$ if $\beta$ is sufficiently high. However, for our purpose the most interesting case is the one where no consumer group is excluded, so we assume that $\beta$ is sufficiently low to ensure that this holds. The conditions for this to be true, and some consequences of allowing higher values on $\beta$, are discussed in the Appendix.

Without loss of generality, we assume that there is only one advertiser in each country. Let the profit function for the advertiser in Country 1 and Country 2, respectively, be given by

$$\Pi_1 = [\gamma (C_1^A + C_1^B) - r_1] A_1$$  and $$\Pi_2 = (\gamma C_2^A - r_2) A_2,$$

where $r_i$ is the advertising price in Country $i = 1, 2$. The parameter $\gamma > 0$ in (7) scales the gain from advertising; the higher is $\gamma$, the more valuable it is to expose the viewers to ads.

Solving $\partial \Pi_i / \partial A_i = 0$, subject to (5) and (6), we find that the profit maximizing advertising levels are given by

$$A_1 = \frac{(\beta + 1) - 2p_1}{4} - \frac{r_1}{4\gamma}$$  and $$A_2 = \frac{1-p_2}{2} - \frac{r_2}{2\gamma}. \hfill (8)$$

For simplicity we set the TV channels’ costs equal to zero, such that their profit levels equal

$$\pi_1 = p_1(C_1^A + C_1^B) + r_1A_1$$  and $$\pi_2 = p_2C_2^A + r_2A_2. \hfill (9)$$

Each TV channel maximizes profits with respect to the prices it charges in the viewer and advertising markets, $p_i$ and $r_i$. It can be shown that the second-order conditions require that $X_1 \equiv 6\gamma - \gamma^2 - 1 > 0$, which means that we must have $\gamma \in (3 - 2\sqrt{2}, 3 + 2\sqrt{2}) \approx (0.17, 5.82)$. Assuming that this is true, we find the first-order conditions

$$p_1^* = \frac{(\beta + 1) \gamma (3-\gamma)}{2X_1}; \quad p_2^* = \frac{p_1^*}{\beta + 1}$$  and

$$r_1^* = \frac{(\beta + 1) \gamma (\gamma + 1)}{X_1}; \quad r_2^* = \frac{r_1^*}{\beta + 1}. \hfill (10)$$
Using equation (8) and that \( g_i = A_i + p_i \) we further have

\[
A_1^* = \frac{(\beta + 1)(\gamma - 1)}{2X_1}; \quad A_2^* = \frac{2A_1^*}{\beta + 1}
\]

and

\[
g_1^* = \frac{\beta + 1}{4} \left( 1 - \frac{(\gamma - 1)^2}{X_1} \right); \quad g_2^* = \frac{2g_1^*}{\beta + 1}.
\]

Not surprisingly, we see that the generalized price in Country 1 is increasing in group B’s willingness to pay for watching TV; \( dg_1/d\beta > 0 \). Equation (11) also reveals that the ad level is positive if and only if \( \gamma > 1 \); for smaller values of \( \gamma \) the willingness to pay for ads is so low that the TV channels prefer to be advertising-free and only charge the viewers. If \( \gamma \geq 3 \), on the other hand, the willingness to pay for ads is so high that the TV channels maximize profits by not charging the viewers at all. Thereby they attract large audiences, and make high profits from the advertising side of the market.

We can now state:

**Lemma 1** The TV channels are financed by a combination of advertising and viewer payments if the markets are completely segmented and \( \gamma \in (1, 3) \).

From equations (10) and (11) we further note:

**Lemma 2** Both the direct and the indirect price are higher in Country 1 than in Country 2 \((p_1 > p_2, A_1 > A_2)\) if the markets are completely segmented and \( \gamma \in (1, 3) \).

Since the generalized price \( g_i = p_i + A_i \) is unambiguously higher in Country 1 than in Country 2, group A consumers in Country 1 are worse off than their identical counterparts in Country 2. This indicates that these consumers would benefit from being allowed to make their purchases from Country 2.

### 4.2 No market segmentation

The analysis above presupposed that consumers are not allowed to purchase from the foreign country. Suppose now that they are allowed to do so. Since \( g_1^* > g_2^* \), a share \( s \in (0, 1] \) of the non-loyals in Country 1 will find it beneficial
to buy from Country 2. Total demand for watching TV in Country 1 will then be equal to \( D_1 = (1 - s) C_1^A + C_1^B \), while in Country 2 we have \( D_2 = s C_1^A + C_2^A \). The profit functions for the TV channels, \( \pi_1 = p_1 D_1 + r_1 A_1 \) and \( \pi_2 = p_2 D_2 + r_2 A_2 \), are consequently given by

\[
\begin{align*}
\pi_1 &= p_1 [(1 - s)(1 - p_1 - A_1) + (\beta - p_1 - A_1)] + r_1 A_1 \quad \text{and} \\
\pi_2 &= p_2 [s (1 - p_2 - A_2) + (1 - p_2 - A_2)] + r_2 A_2.
\end{align*}
\]

Since only domestic viewers matter for the advertisers we further have

\[
\begin{align*}
\Pi_1 &= \{ \gamma [(1 - s)(1 - p_1 - A_1) + (\beta - p_1 - A_1)] - r_1 \} A_1 \quad \text{and} \\
\Pi_2 &= [ \gamma (1 - p_2 - A_2) - r_2 ] A_2.
\end{align*}
\]

The TV channel and the advertiser in Country 1 lose revenue if (some of) the non-loyals buy from Country 2 rather than from their home country. As noted above, this loss is not likely to be fully counterweighted by higher revenues for the firms in Country 2, since the switchers by assumption are of no value in the foreign advertising market.

In the Appendix we show that the first-order conditions for profit maximization in the no segmentation regime are given by:

\[
\begin{align*}
p_1^{**} &= \frac{[\beta + (1 - s)] \gamma (3 - \gamma)}{X_1 (2 - s)}; \\
p_2^{**} &= \frac{3 (1 + s) - \gamma}{X_2} \\
A_1^{**} &= \frac{[\beta + (1 - s)] (\gamma - 1)}{X_1 (2 - s)}; \\
A_2^{**} &= \frac{(1 + s) (\gamma - (1 + s))}{X_2},
\end{align*}
\]

where \( X_2 \equiv X_1 + s [2 (3 \gamma - 1) - s] \). Whenever the TV channel’s second-order conditions in Country 2 hold, we have \( X_2 > 0 \).

Adding \( p_i \) and \( A_i \) we find that the generalized prices equal

\[
\begin{align*}
g_1^{**} &= \frac{[\beta + (1 - s)] (4 \gamma - \gamma^2 - 1)}{X_1 (2 - s)} \quad \text{and} \\
g_2^{**} &= \frac{X_2 - 2 \gamma (1 + s))}{X_2}.
\end{align*}
\]

If the consumers are not allowed to buy from abroad, we know from equation (11) that the generalized price is higher in Country 1 than in Country 2 \( g_1^* > g_2^* \), such that we will have \( s > 0 \) in a no segmentation equilibrium. If all non-loyals buy from Country 2 \( (s = 1) \), we have \( g_1^{**} \geq g_2^{**} \). However, if only a share of them do so, the generalized prices in the two countries must be the same, \( g_1^{**} = g_2^{**} \). Only then can we have \( 0 < s < 1 \) in equilibrium.
Let us first characterize equilibrium in the special case where \( s = 1 \).

**All non-loyals in Country 1 buy from abroad (s = 1)**

A necessary requirement for having an equilibrium where all non-loyals in Country 1 shift to Country 2 is that \( g_1^{**} \geq g_2^{**} \). Inserting for \( s = 1 \) into equation (16) we find that this holds if and only if

\[
\beta \geq \tilde{\beta} \equiv \frac{X_1 (4 - 8\gamma + \gamma^2)}{X_3 (1 - 4\gamma + \gamma^2)}, \tag{17}
\]

where \( X_3 \equiv 12\gamma - \gamma^2 - 4 \).

With \( s = 1 \), it follows from equations (14) and (15) that \( A_2^{**} > 0 \) require \( \gamma > 2 \), while \( p_1^{**} > 0 \) requires \( \gamma < 3 \). Equations (14) - (16) thus hold only when \( \gamma \in [2, 3] \).

For \( \gamma \in [1, 2] \), where \( A_2^{**} = 0 \), we have

\[
p_2^{**} = g_2^{**} = \frac{1}{2}, \tag{18}
\]

in which case \( g_1^{**} \geq g_2^{**} \) is true if and only if

\[
\beta \geq \tilde{\beta} \equiv \frac{X_1}{2(4\gamma - 1 - \gamma^2)}. \tag{19}
\]

Note that the equilibrium in Country 1 is independent of whether the market in Country 2 is one-sided or two-sided.

It is straightforward to show that \( \tilde{\beta} = \tilde{\beta} \) at \( \gamma = 2 \), and it is useful to define \( \beta^P \equiv \tilde{\beta} \) for \( \gamma \equiv [1, 2] \) and \( \beta^P \equiv \tilde{\beta} \) for \( \gamma \equiv [2, 3] \). This allows us to state:

**Proposition 4** Assume no segmentation and that \( \beta > \beta^P \). Then \( g_1^{**} > g_2^{**} \), and all non-loyals buy from Country 2 (\( s = 1 \)). Suppose that

a) \( \gamma \in [1, 2] \). Then the platform in Country 2 is one-sided (\( p_2^{**} > 0, A_2^{**} = 0 \)), while the platform in Country 1 is two-sided (\( p_1^{**} > 0, A_1^{**} > 0 \))

b) \( \gamma \in (2, 3) \). Then the platforms in both countries are two-sided (\( p_i^{**} > 0, A_i^{**} > 0 \)).

Proposition 4 is illustrated in Figure 1, where the upward-sloping curve shows \( \beta^P(\gamma) \). Beneath this curve we have \( s \in (0, 1) \), so that no segmentation generates international price equalization (see further discussion below).
Both platforms will then be two-sided. However, above the curve $\beta^{P}(\gamma)$ the generalized price is higher in Country 1 than in Country 2, and only for $\gamma > 2$ will both countries have a two-sided market structure.

![Figure 1: Market structures with no segmentation](image)

To see why the curve $\beta^{P}$ is upward-sloping, consider an arbitrary point $(\gamma', \beta^{P}(\gamma'))$. By definition we have $g_1 = g_2$ at any point along this curve, so that the marginal switcher is indifferent as to which country he buys from. Assume that he buys from Country 2. Now, suppose that $\gamma$ increases. Then the advertising market becomes more profitable, and both TV channels will have incentives to reduce the generalized viewer price in order to attract a

\[\text{Along the boundary TV2 has equally large populations of foreign and domestic viewers, and finds it optimal set } A = 0. \text{ If } s < 1, \text{ there is a larger number of domestic than foreign viewers, and it is then optimal to set } A > 0. \text{ Thus, it is only for } s = 1 \text{ that there might exist an equilibrium where the market is one-sided in Country 2. However, if the number of foreign viewers is greater than the number of domestic viewers (e.g. because Country 1 is "large"), then we might have a one-sided market also for } s < 1.\]
larger audience. However, since only domestic viewers have any value on the advertising market, the relative importance of the advertising market is smaller in Country 2 than in Country 1. But this means that we must have \( g_1 < g_2 \) subsequent to an increase in \( \gamma \). Thus, it cannot be rational for all non-loyals to buy from Country 2 any more, unless also \( \beta \) increases (such that Platform 1 uses a higher mark-up than it did initially).

Some non-loyals in Country 1 may buy at home \((s \leq 1)\)

Let us now open up for the possibility that not all non-loyals buy from Country 2; \( s \leq 1 \). We then find that no segmentation has the following consequences (independent of whether the market in Country 2 is one-sided or two-sided and independent of the value of \( s \)):

**Proposition 5** Assume a shift from complete to no segmentation. Then

a) the monetary as well as the generalized price increase in both countries \( (p_i^{**} > p_i^* \) and \( g_i^{**} > g_i^* \)).

b) the advertising level increases in Country 1 and decreases in Country 2 \( (A_1^{**} > A_1^* \) and \( A_2^{**} < A_2^* \)).

**Proof:** Consider first the consequences of a shift from complete market segmentation \((s = 0)\) to a no segmentation equilibrium where not all non-loyals buy from Country 2 \((s < 1)\). Using equations (14) - (16) we have\(^{13}\):

\[
\frac{dp_i^{**}}{ds} = \frac{\gamma (3 - \gamma) (\beta - 1)}{(2 - s)^2 X_1} > 0 \quad \text{and} \quad \frac{dp_i^*}{ds} = \frac{2s (3 - \gamma) + 3 (1 + s^2) + \gamma (3\gamma - 2)}{X^2_2} > 0
\]

\[
\frac{dg_i^{**}}{ds} = \frac{(\beta - 1) g_1}{(2 - s) (1 - s + \beta)} > 0 \quad \text{and} \quad \frac{dg_i^*}{ds} = \frac{2\gamma^2 - (1 + s)^2}{X_2^2} > 0
\]

\[
\frac{dA_1^{**}}{ds} = \frac{2\gamma (\gamma - 1) (\gamma^2 - (1 + s)^2)}{(4\gamma - \gamma^2 - 1) X_2^2} > 0 \quad \text{and} \quad \frac{dA_2^{**}}{ds} = \frac{-\gamma 5s^2 + 2 (5 - \gamma) s + (\gamma^2 - 2\gamma + 5)}{X_2^2} < 0.
\]  

This proves Proposition 5 for \( s < 1 \). By setting \( s = 1 \) into equations (14) - (16) and comparing with equations (10) - (11) it follows straightforwardly that the Proposition holds also for the case where all non-loyals buy from Country 2. Q.E.D.

The results that \( p_i^{**} > p_i^* \), \( g_i^{**} > g_i^* \) and \( A_2^{**} < A_2^* \) are in accordance with Propositions 2 and 3 for the general model, while the results that \( g_2^{**} > g_2^* \) and \( A_1^{**} > A_1^* \) confirm Conjecture 1.

\(^{13}\)Note that \( \gamma^2 - (1 + s)^2 > 0 \) as long as \( A_2 > 0 \), which always holds if \( s < 1 \).
Equation (20) shows that the generalized price in both countries is always increasing in the share of non-loyals in Country 1 who buy in Country 2. Figure 2, which measures the generalized prices as functions of $s$, provides an illustration for a specific set of parameter values ($\beta = 1.2, \gamma = 2.5$).

![Figure 2: Price consequences of no segmentation](image)

No segmentation unambiguously increases prices in both countries, and more so in the country which initially has the lower price level. An important implication of this is that no segmentation - where consumers are allowed to switch - harms the consumers if $s < 1$ in equilibrium. Clearly, the higher generalized prices in the two countries harm those who buy at home both before and after we open up for switching and thereby for international trade. However, also those who switch must be harmed. The reason is that we cannot have an equilibrium where $s < 1$ unless $g_1 = g_2$. But it then follows that the loyals as well as the non-loyals pay a higher generalized price once we allow for international trade.

Actually, we can say more than this. By definition we have $g_1 = g_2$ and $s = 1$ along the curve $\beta^p$ in Figure 1. A shift to a no segmentation regime which brings the economy to this boundary must therefore involve a discrete positive jump in prices for all the consumers. By continuity, the same must be true also in some neighborhood above the curve. The only agent that
gains from international trade in this area, is TV2. Thus, since the higher generalized prices increase the deadweight loss on the consumer side of the market in both countries, it is not surprising that aggregate welfare (i.e., the sum of consumer surplus and profits in the two countries) falls once we open up for international trade if we are in the neighborhood of \((\gamma, \beta) = (\gamma, \beta(\gamma))\). This is formally proved in the Appendix:

Proposition 6 Below, and in the neighborhood above, the curve \(\beta^P(\gamma)\), the non-loyals lose when they are allowed to buy from abroad, and aggregate welfare falls.

Note that if non-loyals in Country 1 could coordinate their actions, they would not buy from abroad to such an extent that they must pay a higher generalized price than in absence of trade. In this sense the negative outcome is a result of a prisoner’s dilemma situation.

Proposition 6 does not imply that international trade is necessarily negative in our context. On the contrary, it improves welfare if \(\beta\) is sufficiently high (and in particular so high that \(s = 1\) and \(g_1^{**} > g_2^{**}\)). This is because the price the switchers have to pay in the completely segmented market is then very high; the non-loyals would even be excluded if \(\beta\) were high enough. In the Appendix we therefore derive a boundary \(\beta^W(\gamma)\) above which no segmentation is preferable from a welfare point of view. The result is shown in Figure 3; below the curve \(\beta^W\) international trade is detrimental to welfare.
Some concluding remarks

In this article we have shown that restrictions on firms’ abilities to segment an integrated market into national markets can be more detrimental to welfare in a two-sided market than in a corresponding one-sided market. When consumers are allowed to switch to a neighboring country it creates a mismatch between the two sides of the market and a less efficient market structure. Even those switching by purchasing from the neighboring country might be worse off.

Although our modelling is motivated by the organization of the TV rights for Premier League, it raises questions about other similar types of markets. For example, many types of TV programs are sold at different prices in different countries and are partly financed by commercials. Our analysis suggests that one should be very careful when intervening in that type of market, since it can violate the two-sidedness and thereby be detrimental both for viewers and advertisers.

We have taken seriously the present competition law rules in EU concerning vertical restraints, which are negative towards restrictions on passive sales but lenient towards restrictions on active sales. This is modelled by
allowing a downstream firm to serve a consumer that has switched from a neighboring country (passive sales), but prohibiting the downstream firms from competing for each others’ consumers (active sales). In this respect our analysis is an illustration that the existing competition law rules on vertical restraints are not well suited for two-sided markets. By allowing for passive but not active sales, one prevents downstream competition and at the same time allows for a mismatch between the two sides of the market.

The upstream firm could change the wholesale price to the downstream firms and thereby trigger changes in end-user prices and the volume of advertising that would prevent any switching of consumers from one country to another. But this could be interpreted as de facto the same as violating the consumers’ option to switch. If so, it might be a violation of competition rules, as well as of the idea behind the common market. But despite this, the upstream firm might pursue such a practice. Alternatively, the upstream firm might decide to sell to only one instead of all EU countries and thereby prevent passive sales. We leave these options for future research.

6 Appendix

6.1 Complete segmentation

6.1.1 No exclusion

Assume complete segmentation and that no consumer group is excluded. Using equations (5) - (10) we then find that consumption levels are given by

\[
C_1^{A*} = \frac{4\gamma - (\beta - 1)(4\gamma - 1 - \gamma^2)}{2X_1}; \quad C_1^B = C_1^{A*} + (\beta - 1); \quad C_1^* = (\beta + 1) \frac{2\gamma}{X_1} \\
C_2^* = C_2^{A*} = \frac{2\gamma}{X_1},
\]

(21)

where \(X_1 = 6\gamma - \gamma^2 - 1\).

The profit levels are equal to

\[
\pi_1^* = \frac{(\beta + 1)^2 \gamma}{2X_1}; \quad \Pi_1^* = \frac{(\beta + 1)^2 \gamma (\gamma - 1)^2}{2X_1^2} \\
\pi_2^* = \frac{\gamma}{6\gamma - \gamma^2 - 1}; \quad \Pi_2^* = \gamma \frac{(\gamma - 1)^2}{X_1^2}
\]

(22)
Using that consumer surplus for consumer group $A$ in Country $i = 1, 2$ equals $CS^A_i = \int_0^{C_i^A} (1 - C_i^A) dC_i^A = C_i^A \left( 1 - \frac{C_i^A}{2} \right)$, with an analogue expression for consumer group $B$ we further have

\[
CS^A_1^* = \frac{((\beta - 1)(1 + \gamma^2 - 4\gamma) + 4\gamma)(4\gamma - (\beta + 3)(1 + \gamma^2 - 4\gamma))}{8X_1^2};
\]

\[
CS^B_1^* = CS^A_1^* + \frac{\beta^2 - 1}{2};
\]

\[
CS^A_2^* = \frac{2\gamma(5\gamma^2 - 1 - \gamma^2)}{X_1^2}.
\]

Defining welfare as the sum of consumer surplus and profits yields

\[
W_1^* = \frac{2 \beta + 9\beta^2 + 9}{16} + \frac{(\beta + 1)^2(\gamma - 1)^2(34\gamma - 5(\gamma^2 + 1))}{16X_1^2};
\]

\[
W_2^* = \frac{2\gamma(7\gamma - (\gamma^2 + 1))}{X_1^2}.
\]

### 6.1.2 Exclusion of low-demand consumers in Country 1

It might be the case that it is optimal for the TV channel in Country 1 to serve only consumer group $B$ under complete segmentation. A possible exclusion of group $A$ in Country 1 does not affect the market outcome in Country 2 when there is no trade, so the expressions derived in Section 4.1 for Country 2 still hold. We shall consequently focus solely on Country 1 below.

If the low-demand consumer group in Country 1 is excluded, we must modify the advertiser’s profit function in that country to $\Pi_1 = (\gamma C_1^B - r_1) A_1$. Maximization with respect to $A_1$ yields the following demand for ads

\[
A_1 = \frac{\beta - p_1}{2} - \frac{r_1}{2\gamma}.
\]

Solving $\frac{\partial \pi_1}{\partial p_1} = \frac{\partial \pi_1}{\partial r_1} = 0$ subject to (25) implies that the prices and the advertising volume are equal to (with superscript $E$ for exclusion):

\[
p_1^E = \frac{\beta \gamma (3 - \gamma)}{X_1}, \quad r_1^E = \frac{\beta \gamma (\gamma + 1)}{X_1} \quad \text{and} \quad A_1^E = \frac{\beta \gamma - 1}{X_1}.
\]
We now find that
\begin{align}
C_1^E &= \beta \frac{2\gamma}{X_1}; \\
CS_1^E &= \beta^2 \frac{2\gamma (X_1 - \gamma)}{X_1^2} \\
\pi_1^E &= \beta^2 \frac{\gamma}{X_1}; \\
\Pi_1^E &= \beta^2 \frac{(\gamma - 1)^2}{X_1^2}; \\
W_1^E &= \beta^2 \frac{2\gamma ((7 - \gamma) \gamma - 1)}{X_1^2}.
\end{align}

The outcome in Country 2 is still given by equations (21) - (24).

Using equation (10) we find \( \pi_1^E = \beta^2 \frac{\gamma}{X_1} \). Since \( \pi_1^E > \pi_1^* \) for \( \beta > \beta^E \equiv 1 + \sqrt{2} \approx 2.41 \), we thus have an indication that above this critical value of \( \beta \) the equilibrium prices are given by equation (26). A necessary condition for this to be true is that the low-demand consumers will not watch TV at these prices. However, inserting for (26) into (6) we find
\begin{align}
C_A^1 &= \max \left\{ 0, (\beta^L - \beta) / \beta^L \right\},
\end{align}
where \( \beta^L \equiv \frac{X_1}{\gamma - X_1 - 1} \). It is now straight forward to show that \( \beta^L \) is monotonically increasing in \( \gamma \), with \( \beta^L = 1 \) at \( \gamma = 1 \) and \( \beta^L = 4 \) at \( \gamma = 3 \). Thereby equation ((26)) does not describe an equilibrium for sufficiently high values of \( \gamma \) (specifically, at \( \beta = \beta^E \) we have \( C_A^1 > 0 \) if \( \gamma > \gamma^L \equiv \left( \sqrt{14 - 8\sqrt{2}} - \sqrt{2} + 4 \right) / 2 \approx 2.1124 \)). In order to derive the candidate equilibrium in this case, it is useful to note that
\begin{align}
A_1^E - A_1^* &= \frac{(\gamma - 1)(\beta - 1)}{2X_1} > 0.
\end{align}

Equation (29) has the interesting implication that the advertising volume is highest in the case where the number of viewers is lowest, i.e. when only the \( B \)-group watches TV (though the advertising price falls; \( r_1^E - r_1^* = -\gamma \frac{\gamma + 1}{X_1} < 0 \)). This is in accordance with Conjecture 1.

The greater the advertiser’s benefit of advertising, the more profitable it is for the TV channel to increase the advertising volume. This indicates that if \( \gamma \) is so high that \( C_A^1 \) in equation (28) is positive, then it might be profitable for the TV channel to find a way of convincing the advertising market that only the \( B \)-group will watch TV. This it can do by setting the generalized viewing price \( (g_1 = p_1 + A_1) \) so high that the low-demand consumers do not buy access to the TV programs. Then the advertisers will be willing to buy more advertising space. In effect, this implies that the TV channel might

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find it profitable to employ a limit-pricing strategy (superscript $L$) where the prices are chosen such that $C_A^1$ is exactly equal to zero. Inserting for this into equation (25) we find $A_1 = \frac{(\beta - 1)\gamma - r_1}{\gamma}$ and $p_1 = \frac{r_1 - (\beta - 2)\gamma}{\gamma}$. Solving $\partial \pi_1 / \partial r = 0$ then yields

$$p_1^L = 1 - \frac{(\beta - 1)(\gamma - 1)}{2\gamma}, \quad r_1^L = \frac{(\beta - 1)(\gamma + 1)}{2} \quad \text{and} \quad A_1^L = \frac{(\beta - 1)(\gamma - 1)}{2\gamma},$$

which further implies that

$$C_1^{BL} = \beta - 1; \quad CS_1^{BL} = \frac{\beta^2 - 1}{2};$$
$$\pi_1^L = \frac{\beta - 1}{4} \frac{\beta(\gamma - 1)^2 + X_1}{\gamma}; \quad \Pi_1^L = \frac{(\beta - 1)^2(\gamma - 1)^2}{4\gamma};$$
$$W_1^L = \frac{(\beta - 1)(\beta(1 + \gamma(\gamma - 1) + X_1 - \gamma))}{2\gamma}. $$

**Figure A1:** Complete segmentation. Market outcome Country 1.

Figure A1 summarizes the discussion above, assuming that there exists an equilibrium with complete segmentation. Below the (weakly) upward-sloping curve in the Figure the difference between the two groups in the willingness to
pay for watching TV is so small that both groups will be served. The viewer prices and the ad volume are then given by $p^*_1$ and $A^*_1$ from equations (10) and (11). Above the curve, on the other hand, the $B-$group has such a high willingness to pay for watching TV that the low-demand group is excluded from the market. For $\gamma < \gamma^L$ Platform 1 chooses its unconstrained profit-maximizing prices $p^E_1$ and $r^E_1$ as given by equation (26), while for $\gamma > \gamma^L$ it engages in limit-pricing in order to exclude the low-demand group ($p^L_1$ and $r^L_1$). It is straightforward to show that if the market we consider had been one-sided (i.e. pure pay-TV), then group $A$ would be excluded from the market if and only if $\beta \geq \beta^E$. This is the same outcome as we have in our context if $\gamma < \gamma^L$. However, if $\gamma > \gamma^L$ then it would be unprofitable to exclude group $A$ from watching TV if $\beta = \beta^E$; this is due to the revenue they generate in the advertising market. A higher value of $\beta$ (greater difference between the groups) would then be required to make it profitable for TV channel to exclude group $A$. In this sense exclusion is less likely in a two-sided than in a one-sided market.\textsuperscript{14}

We can state:

**Lemma 3** Assume complete market segmentation. Consumer group $A$ in Country 1 is excluded from the market if $\gamma \leq \gamma^L$ and $\beta \geq \beta^E$. If $\gamma > \gamma^L$, then group $A$ is excluded only if $\beta > \beta^L > \beta^E$.

In the main body of the paper we restricted attention to the case where no consumer group is excluded and where the platforms receive revenue from both the consumer and advertiser side of the market if the markets are completely segmented. Letting $\beta^U \equiv \beta^E$ for $\gamma \leq \gamma^L$ and $\beta^U \equiv \beta^L$ for $\gamma > \gamma^L$, this means that we implicitly made the assumption that $\beta < \beta^U$ and $\gamma \in (1, 3)$.

### 6.2 Proof of equations (14) and (15)

From equation (13) we find that the solution to $A_i = \arg \max \Pi_i$ is given by

\[
A_1 = \frac{[\beta + (1 - s)] - p_1 (2 - s)}{2 (2 - s)} - \frac{r_1}{2 \gamma (2 - s)} \quad \text{and} \quad A_2 = \frac{1 - p_2}{2} - \frac{r_2}{2 \gamma}. \quad (31)
\]

\textsuperscript{14}It can be shown that $\pi^E > \pi^L > \pi^*$ in the relevant area. The result that $\pi^E > \pi^L$ is quite natural, since the firm has one degree of freedom less with limit-pricing compared to unconstrained profit-maximization. The firm will thus employ limit-pricing in the relevant area only when $C^{A}_1$ otherwise would be positive.
Inserting for (31) into (12) and solving $\partial \pi_i / \partial p_i = \partial \pi_i / \partial r_i = 0$ yields equation (14) and (15).

6.3 No segmentation equilibrium with $A_2 = 0; \; \gamma \in [1, 2]$.

Equation (15) does not hold for $\gamma < 2$, since that would imply a negative advertising level in Country 2. Maximizing profits for the platform in Country 2 under the restriction that $A_2 = 0$ and $s = 1$, we find

$$p_2^{**} = \frac{1}{2},$$

which further implies that

$$C_i^{A**} = \frac{1}{2}; \; CS_i^{A**} = \frac{3}{8}; \; \pi_2^{**} = \frac{1}{2}.$$ (33)

Profits in Country 1 and consumer surplus for consumer group $B$ are independent of whether the market in Country 2 is one-sided or two-sided, and using equations (13), (14) and (15) we have

$$C_1^{B**} = \frac{2\beta \gamma}{X_1}; \; CS_1^{B**} = \frac{2\beta^2 (5\gamma - 1 - \gamma^2) \gamma}{X_1^2};$$ (34)

$$\pi_1^{**} = \frac{\beta^2 \gamma}{X_1}; \; \Pi_1^{**} = \frac{\beta^2 \gamma (\gamma - 1)^2}{X_1^2}.$$ (34)

Equations (33) and (34) imply that welfare equals

$$W_1^{**} = \frac{2\beta^2 \gamma (7\gamma - \gamma^2 - 1)}{X_1^2} + \frac{3}{8} \text{ and } W_2^{**} = \frac{7}{8}.$$ (35)

There are two conditions that must be fulfilled for this to be an equilibrium. First, all non-loyals must prefer to purchase in Country 2, and the platform in this country must make a higher profit by being advertising-free than by setting the prices that apply in an equilibrium with complete segmentation (in which case we have a two-sided market in both countries). Using that $g_2^{**} = p_2^{**} = 1/2$ for $\gamma \in [1, 2]$ while $g_1^{**} = \beta \frac{4\gamma - 1 - \gamma^2}{X_1}$ for $\gamma \in [1, 3]$, c.f. equation (16), we find that $g_2^{**} < g_1^{**}$ in the relevant area if $a > \beta$, where $\beta$ is given by equation (19). For $\gamma \in [1, 2]$ we further have $\pi_2^{**} - \pi_2^* = \frac{4\gamma - 1 - \gamma^2}{2X_1} > 0$, so that both conditions are fulfilled whenever $a > \beta$. 27
6.4 No segmentation equilibrium with $A_2 > 0$; $\gamma \in (2, 3)$.

In the no segmentation equilibrium where the market is two-sided in both countries and $s = 1$ we have $r_1 = \frac{2\gamma (\gamma + 1)}{X_1}$ and $r_2 = \frac{2\gamma (\gamma + 2)}{X_3}$, with viewer prices given by equation (14). Using this we find $C_A^i = 4\gamma / X_3$ and $C_B^i = 2\gamma / X_1$. This implies that consumer surplus and profits are equal to

$$CS_A^i = \frac{4\gamma (10\gamma - \gamma^2 - 4)}{X_3}; \quad CS_B^i = \frac{2\beta^2 (5\gamma - \gamma^2 - 1) \gamma}{X_1^2}$$

$$\pi_1 = \frac{\gamma \beta^2}{X_1}; \quad \pi_2 = \frac{4\gamma}{X_3}$$

$$\Pi_1 = \frac{\beta^2 \gamma (\gamma - 1)^2}{X_1^2}; \quad \Pi_2 = \frac{4\gamma (\gamma - 2)^2}{X_3^2}.$$ 

Summing consumer surplus and profits we arrive at

$$W_1 = \frac{2\beta^2 \gamma (7\gamma - \gamma^2 - 1)}{X_1^2} + \frac{4\gamma (10\gamma - \gamma^2 - 4)}{X_3^2} \quad (36)$$

$$W_2 = \frac{4\gamma (18\gamma - \gamma^2 - 4)}{X_3^2}$$

6.5 Proof of Proposition 6

The boundary $\beta^P(\gamma)$ is given by $\hat{\beta}(\gamma)$ and $\tilde{\beta}(\gamma)$ from equations (17) and (19) for $\gamma \geq 2$ and $\gamma \leq 2$, respectively.

For $\gamma > 2$ we have

$$\Delta(\gamma \geq 2) = g^{**} - g^* = \frac{4\gamma - \gamma^2 - 1}{2X_1} \left( \frac{X_1X_3 - 6\gamma^2 (4 - \gamma)}{X_3 (4\gamma - \gamma^2 - 1) - \beta} \right).$$

If this difference is positive, the switchers face a higher generalized price if they purchase in Country 2 than if they are not allowed to do so. Inserting for the boundary value, $\beta = \hat{\beta}(\gamma)$, we find

$$(\Delta(\gamma \geq 2)|_{\beta = \hat{\beta}(\gamma)} = \frac{\gamma (\gamma^2 - 2)}{X_1X_3} > 0.$$ 

For $\gamma \leq 2$ we likewise have
\begin{equation}
(\Delta(\gamma \geq 2)|_{\beta=\beta(\gamma)} = \frac{(\gamma - 1)^2}{4X_1} > 0. Q.E.D.
\end{equation}

6.6 Derivation of the boundary $\beta^W$

Welfare in the two countries under complete segmentation is given by equation (24). Under no segmentation welfare is given by equation (35) for $\gamma < 2$. Let $\Delta(A_2 = 0)$ measure welfare under no segmentation minus welfare under complete segmentation. Using equations (24) and (35) we find:

$$\Delta(A_2 = 0) > 0 \text{ if } \beta > \beta_1(\gamma) \equiv 1 + (\gamma - 1) \sqrt{\frac{34\gamma - 5(\gamma^2 + 1)}{8\gamma (\gamma^2 + 1) - (\gamma^4 + 1) - 10\gamma^2}}.$$  

Using equations (24) and (36) we similarly find

$$\Delta(A_2 > 0) > 0 \text{ if } \beta > \beta_2(\gamma) \equiv 1 + 4\sqrt{\frac{\gamma (\gamma^2 - 2)(3\gamma (2\gamma - 1) + (3\gamma + X_3) X_1)}{X_3^2 (8\gamma (\gamma - 1)^2 + 6\gamma^2 - \gamma^4 - 1)}}.$$  

The curve $\beta^W(\gamma)$ is given by $\beta_1(\gamma)$ for $\gamma \leq 2$ and by $\beta_2(\gamma)$ for $\gamma \geq 2$.

References


