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Offshoring Domestic Jobs

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Offshoring Domestic Jobs

Abstract

We set up a two-country general equilibrium model, in which heterogeneous firms from one country (the source country) can offshore routine tasks to a low-wage host country. The most productive firms self-select into offshoring, and the impact on welfare in the source country can be positive or negative, depending on the share of firms engaged in offshoring. Each firm is run by an entrepreneur, and inequality between entrepreneurs and workers as well as intra-group inequality among entrepreneurs is higher with offshoring than in autarky. All results hold in a model extension with firm-level rent sharing, which results in aggregate unemployment. In this extended model, offshoring furthermore has non-monotonic effects on unemployment and intra-group inequality among workers. The paper also offers a calibration exercise to quantify the effects of offshoring.


Keywords: offshoring, heterogeneous firms, income inequality.

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1 Introduction

Fragmentation of production processes across country borders, leading to the offshoring of tasks that used to be performed domestically, is widely seen as a new paradigm in international trade. Public opinion in high-income countries has been very critical of this phenomenon, and much more so than of traditional forms of international trade, since it seems obvious that offshoring to low-wage countries destroys domestic jobs.\footnote{As pointed out by The Economist (2009), “Americans became almost hysterical” about the job destruction due to offshoring, when Forrester Research predicted a decade ago that 3.3 million American jobs will be offshore until 2015.} Academic research has drawn a more nuanced picture with respect to the effects of offshoring. Two major effects of offshoring have been identified. First, offshoring has indeed the obvious \textit{international relocation effect} emphasised in the public discussion, as tasks that were previously performed domestically are now performed offshore. In addition, however, there is a \textit{firm-level productivity effect}, as the ability to source tasks from a low-wage location abroad lowers a firm’s marginal cost.

We show in this paper that important additional insights into the effects of offshoring can be gained by adding firm differences to the picture, thereby acknowledging the empirical regularity that only a rather small share of firms makes actually use of the offshoring opportunity.\footnote{Based on information of the IAB Establishment Panel from the Institute for Employment Research in Nuremberg, Moser, Urban, and Weder di Mauro (2009) report that only 14.9 percent of the 8,466 plants in this data-set undertake some offshoring and that, on average, offshoring firms are larger, use better technology, and pay higher wages than their non-offshoring competitors. In a survey of 150 British firms, 30 percent of the producers stated that they conduct part of their production abroad (The Economist, 2004). By contrast, in a survey of 118,300 Japanese manufacturing firms only 3.3 percent of the producers declare to be involved in international outsourcing and/or FDI (Tomiura, 2007).} In particular, we show that both the international relocation effect and the firm-level productivity effect have new implications in the presence of firm heterogeneity, thereby jointly shaping welfare and inequality in the source country of offshoring. To conduct our analysis, we set up a two-country general equilibrium model that features monopolistic competition between heterogeneous firms, not all of which find it worthwhile to offshore in equilibrium. Firms need to be run by an entrepreneur, and in order to introduce a stark asymmetry between the two countries we assume that entrepreneurs exist in only one of them, making this country the source country...
of offshoring in equilibrium. In the source county, agents are symmetric in their productivity as production workers, but they differ in their entrepreneurial abilities. These abilities are instrumental for firm productivity and thus for the profit income the entrepreneur earns when becoming owner-manager of a firm. Agents in the source country are free to choose between occupations, and individual ability determines who becomes entrepreneur or production worker (see Lucas, 1978).

Similar to Grossman and Rossi-Hansberg (2008) and Acemoglu and Autor (2011) we model output of a firm as a composite of different tasks, and furthermore assume that only part of the tasks performed by a firm are offshorable. According to the taxonomy in Becker, Ekholm, and Muendler (2012), these are tasks that are routine (Levy and Murnane, 2004) and do not require face-to-face contact (Blinder, 2006). Since offshoring from the source country is the only employment opportunity for workers in the host country, wages there are generally lower than in the source country, which provides an incentive for offshoring from the perspective of firms in the source country. This incentive is not unmitigated, since firms relocating their routine tasks abroad need to hire a freelance offshoring agent, resulting in a fixed offshoring cost, and in addition shipping back to the source country the intermediate inputs produced in the host country is subject to iceberg trade costs. If these variable offshoring costs are sufficiently high, our model produces the well established result that only the most productive firms are active in international markets. In line with empirical evidence, this is the case we are focusing on in our analysis.

We show that offshoring reduces welfare in the source country if variable offshoring costs are so high that only a small share of firms choose to move their routine tasks abroad. Instrumental for the welfare loss is the well-known international relocation effect, which now interacts with firm-heterogeneity: The first firms to offshore are those with the highest productivity, and hence the highest employment. The absolute number of production jobs relocated abroad among the “early offshorers” is therefore large, not all workers losing their jobs as production workers can find employment as offshoring agents, and as a consequence some of the workers in the source country will necessarily turn to entrepreneurship themselves. The mass of firms in the source country therefore rises, which exacerbates the distortion of excessive firm entry already present
in the autarky equilibrium of our model. This *domestic reallocation effect*, moving workers into occupations that are less desirable from an economy-wide perspective, leads to a decrease in source-country welfare. By contrast, offshoring increases welfare of the source country relative to autarky if the variable offshoring costs are low. Two effects are responsible for the change in the sign of the welfare effect: On the one hand, the firm-level productivity effect (which is negligible at very high offshoring costs) now has a significant (positive) influence on aggregate welfare. On the other hand, the domestic reallocation effect becomes less unfavourable, and may even become welfare increasing by itself, with the least efficient firms leaving the market as in Melitz (2003), thereby reducing the distortion of excessive firm entry in our model.

Offshoring has a non-monotonic effect on inequality of entrepreneurial incomes according to the Gini criterion, with the reduction in variable offshoring costs leading to larger inequality of entrepreneurial incomes when the share of offshoring firms is low, and the reverse effect when the share of offshoring firms is high. The reasoning is straightforward: offshoring always boosts the profits of the newly offshoring firms, and these firms are those at the top of the productivity distribution in the early stages of offshoring, and those at the bottom of the productivity distribution later on. By contrast, the effect of offshoring on intergroup inequality between entrepreneurs and workers is monotonically increasing in the share of offshoring firms. Both types of inequality are higher in any offshoring equilibrium than in autarky, and hence offshoring generates a superstar effect favouring the incomes of the best entrepreneurs, similar to Gersbach and Schmutzler (2007).

In the main part of our paper, we assume that the market for production labour is perfectly competitive, in line with the key contributions to the offshoring literature, such as Jones and Kierzkowski (1990), Feenstra and Hanson (1996), Kohler (2004), Grossman and Rossi-Hansberg (2008), and Rodriguez-Clare (2010). While this version of our model serves the purpose well to isolate the role of firm heterogeneity in the offshoring process, we show that it is straightforward to extend the framework by using a more sophisticated model of the labour market. In this extended version of the model, there is rent-sharing at the firm level, leading to wage differentiation among production workers and to involuntary unemployment. Interestingly, all our results from the full-employment version of the model remain qualitatively unchanged. In addition, the
version with firm-level rent sharing generates new results regarding the effect of offshoring on aggregate unemployment, and on inequality within the group of production workers. In particular, we show that both the effect of offshoring on unemployment and the effect on intra-group inequality among production workers are non-monotonic in the share of offshoring firms, with unemployment and inequality being lower than in autarky when only few firms offshore, while the reverse is true when a large share of them does so. By considering homogeneous workers, our theory thereby offers an explanation for the large variation in wage effects that offshoring has on workers within the same skill group (cf. Hummels, Jørgensen, Munch, and Xiang, 2011).

Only few papers in the literature on offshoring consider firm heterogeneity. Antras, Garicano, and Rossi-Hansberg (2006) develop a model with team production, in which offshoring is synonymous to the formation of international teams. Individuals are heterogeneous in their skill level, and the highest-skill individuals self-select into becoming team managers. Since individuals with higher skills are more productive in the role of a production worker as well as in the role of a manager, offshoring – by providing access to a large, relatively low-skilled foreign labour force – not only increases the incentives of workers to become managers in the source country, but also reduces the average skill level of the domestic workforce. Due to positive assortative matching between managers and workers, the top managers therefore end up being matched with workers of a lower skill level in the open economy, and hence they lose relative to less able managers. This is a key difference to the superstar effect present in our model.

At a more general level, the model developed in this paper builds on the large literature that studies offshoring to low-wage countries in frameworks with either identical or atomistic firms. The literature distinguishes two possible forms of offshoring: The first one is vertical
multinational activity which associates offshoring with setting up a foreign affiliate and relocating production within the boundaries of a firm (see Helpman, 1984; Markusen, 2002). The second one is international outsourcing, which associates offshoring with imports of intermediates that are purchased from an external supplier at arm's length (see Jones and Kierzkowski, 1990; Feenstra and Hanson, 1996; Kohler, 2004; Rodriguez-Clare, 2010).\(^3\) Our paper is related in particular to Rodriguez-Clare (2010), whose Ricardian general equilibrium model shows a mechanism different from ours that leads to potential welfare losses for the source country of offshoring. In the paper by Rodriguez-Clare (2010), welfare losses for the source country can arise due to negative terms of trade effects at the final goods level, a channel that is not active in our model, which features only a single final good.\(^4\)\(^5\)

Since we also consider an extension to our basic framework, in which we model involuntary unemployment resulting from firm-level rent sharing, our paper is also related to contributions that consider offshoring in the presence of labour market imperfections. One of them is Egger and Kreickemeier (2008), who introduce a fair-wage effort mechanism into a multi-sector traditional trade model with high-skilled and low-skilled workers to investigate the consequences of offshoring on relative wages and unemployment. Grossman and Helpman (2008) explore in a model with fair wage preferences how offshoring alters workers’ fairness considerations and analyse to what extent this provides so far unexplored incentives for firms to shift production abroad. Keuschnigg and Ribi (2009) study the labour market implications of offshoring in a setting with search frictions and investigate the scope for government interventions to make

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\(^3\)The literature has also shed light on those factors that govern the decision to offshore as a vertical multinational or in the form of outsourcing. Thereby, contractual imperfections as well as the costs of searching a suitable input supplier are highlighted as important obstacles to a market solution (see McLaren, 2000; Grossman and Helpman, 2003; Antràs and Helpman, 2004).

\(^4\)In a broader interpretation, a country’s terms of trade also capture the price of intermediate goods. While the price of imported intermediates may indeed increase, thereby worsening the terms of trade for the source country, this is not instrumental for the potentially negative welfare effects of offshoring in our setting.

\(^5\)Acemoglu, Gancia, and Zilibotti (2012) consider a Ricardian model in which offshoring induces directed technical change. With technical change favoring high-skilled workers at low levels of offshoring, this model provides a rationale for the empirical observation of rising skill premia in developed as well as developing countries. Costinot, Vogel, and Wang (2012) use a Ricardian framework with many goods and countries to study vertical specialisation of countries along the international supply chain.
offshoring Pareto improving by introducing suitable instruments of redistribution. Mitra and Ranjan (2010) consider a two-sector traditional trade model with labour market imperfection due to search frictions and shed light on how the degree of inter-sectoral labour mobility influences the consequences of offshoring for employment and wages. While all of these studies highlight important channels through which offshoring can impact domestic labour markets, they do not shed light on the specific role of firm heterogeneity or the consequences of occupational choice.\(^6\)

The remainder of the paper is organised as follows. In Section 2, we set up the model and derive some preliminary results regarding the decision of firms to offshore and its implications for firm-level profits. We also characterize the factor allocation in the open economy equilibrium and show how the share of offshoring firms is linked to the variable cost of offshoring. In Section 3, we analyse how changes in the offshoring costs affect factor allocation, welfare and income distribution in our model. In Section 4 we present the extended version of our model that features firm-level rent sharing and involuntary unemployment. In Section 5, we use parameter estimates from existing empirical research to calibrate the extended version of our model, thereby quantifying the implications of offshoring on welfare, unemployment and income inequality in a model-consistent way. In Section 6 we analyse to what extent the link between offshoring and inequality in our setting depends on the specific measures of inequality considered in this paper. In addition, we introduce a comprehensive measure of inequality that allows us to rank the economy-wide distributions of income under offshoring and in autarky. Section 7 concludes our analysis with a brief summary of the most important results.

2 A model of offshoring and firm heterogeneity

We consider an economy with two sectors: a final goods industry that uses differentiated intermediates as the only inputs, and an intermediate goods industry that employs labour for

\(^6\)There is also a sizable literature that studies vertical multinational activity in models of union wage setting and product market imperfection. Existing contributions usually consider a partial equilibrium environment, and hence they are not equipped to identify economy-wide employment effects. Prominent examples include Mezzetti and Dinopulos (1991), Skaksen and Sørensen (2001), Skaksen (2004), and Lommerud, Meland, and Straume (2009).
performing two tasks, which differ in their offshorability. One task is non-routine and requires face-to-face communication, and it must therefore be produced at the firm’s headquarters location. The other task is routine and can be either produced at home or abroad. Each firm in the intermediates goods industry is run by an entrepreneur, who decides on hiring workers for both tasks. We embed the economy just described in a world economy with two countries, where the second country differs from the first in only one respect: the second country does not have any entrepreneurs. Given our production technology, the country without entrepreneurs cannot headquarter any firms, and therefore ends up being the host country of offshoring. The other country is the source country of offshoring. Trade is balanced in equilibrium, with the source country exporting the final good in exchange for the tasks offshored to the host country. In the remainder of this section, we discuss in detail the main building blocks of the model and derive some preliminary results.

2.1 The final goods industry

Final output is assumed to be a CES-aggregate of differentiated intermediate goods \( q(v) \):

\[
Y = \left[ M^{-(1-\rho)} \int_{v \in V} q(v)^\rho dv \right]^{1/\rho},
\]

where \( V \) is the set of available intermediate goods with Lebesgue measure \( M \), and \( \rho \in (0, 1) \) is a preference parameter that is directly linked to the elasticity of substitution between the different varieties in the production of \( Y \): \( \sigma \equiv (1 - \rho)^{-1} > 1 \). The production technology in Eq. (1) is a limiting case of the technology considered by Ethier (1982), featuring constant returns to scale in the economy-wide production of final output. Final output is sold under perfect competition. We choose \( Y \) as the numéraire and set its price equal to one. Profit maximization in the final goods industry determines demand for each variety \( v \) of the intermediate good:

\[
q(v) = \frac{Y}{M^\rho(v)^{-\sigma}}.
\]

2.2 The intermediates goods industry

In the intermediate goods sector, there is a mass \( M \) of firms that sell differentiated products \( q(v) \) under monopolistic competition. Each firm is run by a single entrepreneur who acts as owner-
manager and combines a non-routine task, which must be performed at the firm’s headquarters location in the source country, and a routine task, which can either be produced at home or abroad. We denote the non-routine task by superscript $n$ and the routine task by superscript $r$. In analogy to Antràs and Helpman (2004) and Acemoglu and Autor (2011), we assume that the two tasks are inputs in a Cobb-Douglas production function for intermediate goods. Assuming that one unit of labour is needed for one unit of each task, the production function for intermediates can be written as

$$q(v) = \varphi(v) \left[ \frac{l^n(v)}{\eta} \right]^{\eta} \left[ \frac{l^r(v)}{1-\eta} \right]^{1-\eta},$$  \hspace{1cm} (3)

where $\varphi(v)$ denotes firm-specific productivity, $l^n(v)$ and $l^r(v)$ are the labour inputs in firm $v$ for the production of the respective tasks, and $\eta \in (0,1)$ measures the relative weight (cost share) of the non-routine task in the production of the intermediate good.\(^7\) With a labour input coefficient of one, the unit production cost for the non-routine task is equal to the domestic wage rate $w$ for all firms. The unit production cost for the routine task $\omega^r(v)$ differs between firms, since it depends on whether a firm decides to perform the task domestically or abroad. We can write the marginal costs of producing output $q(v)$ as

$$c(v) = \frac{w}{\varphi(v)z(v)}, \quad \text{with} \quad z(v) \equiv \left[ \frac{w}{\omega^r(v)} \right]^{1-\eta}. \hspace{1cm} (4)$$

For purely domestic producers, which we denote by superscript $d$, the unit production cost for the routine task is given by $w$, which implies $z(v) = 1$ and $c^d(v) = w/\varphi(v)$. If a firm produces purely domestically, there is hence no difference between the two-task technology in (3) and the single-task technology usually considered in trade models with heterogeneous producers. By

\(^7\)Our production function can easily be extended to account for a continuum of tasks that differ in their offshorability as in Grossman and Rossi-Hansberg (2008). Firms would then not only choose their offshoring status, but also decide on the range of tasks they relocate abroad. In a supplement, available from the authors upon request, we show that all offshoring firms would choose to offshore the same range of tasks, irrespective of their own productivity $\varphi(v)$. As the only additional effect in this more sophisticated model variant, a decline in the cost of offshoring would not only be associated with more firms entering offshoring, but also with an increase in the range of tasks offshored by infra-marginal firms. Since the general equilibrium implications of the latter effect are well understood from Grossman and Rossi-Hansberg (2008), we focus here on the extensive margin of offshoring between rather than the intensive margin within firms.
contrast, the distinction between routine and non-routine tasks matters if we look at offshoring firms. An offshoring firm, denoted by superscript $o$, fragments the production process and hires workers in the host country to perform the routine task. The unit production cost for the routine task is therefore given by the effective foreign wage rate $\tau w^*$, where $\tau > 1$ represents the iceberg transport costs an offshoring firm has to incur when importing the output of the routine task from the offshore location.\(^8\) The marginal cost of an offshoring firm is therefore given by

$$c^o(v) = \frac{w}{\varphi(v) \kappa}, \quad \text{where} \quad \kappa \equiv \frac{c^d(v)}{c^o(v)} = \left[ \frac{w}{\tau w^*} \right]^{1-\eta}$$

measures the relative change in overall marginal cost that a firm achieves by moving its routine tasks abroad. Contrasting (4) and (5), it follows that $z(v) = \kappa$ must hold in the case of offshoring firms. Assuming that offshoring also requires the fixed input of one freelance offshoring agent, it is only attractive for source country producers to move routine tasks abroad if $\kappa > 1$, making $\kappa$ the \textit{marginal cost savings factor} that a firm can achieve by offshoring.\(^9\)

Firms in the intermediates sector set prices as a constant markup $1/\rho$ over marginal cost, and output and revenues follow as

$$q(v) = \frac{Y}{M} \left[ \frac{c(v)}{\rho} \right]^{-\sigma} \quad \text{and} \quad r(v) = \frac{Y}{M} \left[ \frac{c(v)}{\rho} \right]^{1-\sigma},$$

respectively. Using Eqs. (5) and (6), we can compute relative operating profits of two firms with the same productivity, but differing offshoring status. We get\(^10\)

$$\frac{\pi^o(\varphi)}{\pi^d(\varphi)} = \kappa^{\rho \sigma}. \quad (7)$$

With $\kappa > 1$, an offshoring firm makes higher operating profits than a purely domestic firm with identical productivity. Analogously, the relative operating profits by two firms with the same offshoring status but differing productivities $\varphi_1$ and $\varphi_2$ are given by

$$\frac{\pi^i(\varphi_1)}{\pi^i(\varphi_2)} = \left( \frac{\varphi_1}{\varphi_2} \right)^{\xi}, \quad i \in \{d, o\}, \quad (8)$$

\(^8\)We use an asterisk to denote variables pertaining to the host country of offshoring.
\(^9\)While $\kappa$ is endogenous, it is easy to see that at least some firms will offshore in equilibrium, provided that variable offshoring costs $\tau$ are finite. If no firm were to shift production of routine tasks abroad, $w^*$ would fall to zero and $\kappa$ would go to infinity, rendering offshoring an attractive choice for source country firms.
\(^10\)We suppress firm index $v$ from now on, because a firm’s performance is fully characterised by its position in the productivity distribution and its offshoring status.
where $\xi \equiv \sigma - 1$. Therefore, given their offshoring status, more productive firms make higher operating profits.

2.3 Equilibrium factor allocation

We assume that the source and the host country of offshoring are populated by $N$ and $N^*$ agents, respectively. While the population in the host country has only access to a single activity, namely the performance of routine tasks in the foreign affiliates of offshoring firms, agents in the source country can choose from a set of three possible occupations: entrepreneurship, employment as a production worker, and self-employment as an offshoring agent. An entrepreneur is owner-manager of the firm, and her ability determines firm productivity. To keep things simple, we assume that entrepreneurial ability maps one-to-one into firm productivity, and we can therefore use a single variable, $\phi$, to refer to ability as well as productivity. Being the residual claimant, the entrepreneur receives firm profits as individual income. Agents differ in their entrepreneurial abilities, and hence in the profits they can realize when running a firm. Following standard practice, we assume that abilities (and thus productivities) follow a Pareto distribution, for which the lower bound is normalised to 1: $G(\phi) = 1 - \phi^{-k}$, where both $k > 1$ and $k > \xi$ are assumed in order to guarantee that the mean of firm-level productivities and the mean of firm-level revenues, respectively, are positive and finite.

Entrepreneurial ability is irrelevant for the two alternative activities that can be performed in the source country of offshoring, so that agents are symmetric in this respect. If an individual becomes a freelance offshoring agent who provides the services required for setting up a foreign plant as a fixed input in the production process, she receives a fee $s$, which is determined in a perfectly competitive market in general equilibrium. Finally, agents in the source country can also apply for a job as production worker and perform the routine or non-routine task, receiving wage rate $w$. As shown below, our equilibrium features self-selection of the most productive firms into offshoring. This implies that the lowest-productivity firm is purely domestic. Denoting this firm’s productivity by $\phi^c$, we therefore arrive at indifference condition

$$\pi^d(\phi^c) = w = s.$$  

Eq. (9) gives the condition for indifference of the marginal entrepreneur between the three pos-
sible occupations in terms of variables that are all endogenous. A second indifference condition holds for the marginal offshoring firm with productivity $\varphi^o$. Since offshoring requires the fixed input of one freelance offshoring agent, we can characterize the entrepreneur of the marginal offshoring firm by

$$\pi^o(\varphi^o) - \pi^d(\varphi^o) = s,$$

i.e. for the indifferent entrepreneur the gain in operating profits achieved by offshoring equals the fixed offshoring cost. All variables in Eqs. (9) and (10) are endogenous, and both indifference conditions are linked via their dependence on $s$. Using Eqs. (7) to (10), it is straightforward to derive $\kappa = (\varphi^c/\varphi^o)^k$, and with Pareto distributed productivities, the fraction $\chi$ of offshoring firms follows directly as

$$\chi = 1 - G(\varphi^o) = \left(\frac{\varphi^c}{\varphi^o}\right)^k = (\kappa \rho \sigma - 1)^{\frac{1}{k}},$$

where we need to assume $\kappa < 2^{1/(\rho \sigma)}$ in order to ensure that $\chi < 1$. A value of $\kappa$ satisfying this inequality is the case we focus on in the following, and in Section 2.4 below we derive the condition on model parameters required for this outcome. In order to determine factor allocation in the source country, we now substitute for the endogenous variables in Eq. (9).

Due to constant markup pricing, the income of production workers is a constant fraction $\rho$ of aggregate income $Y$. In an equilibrium with offshoring, part of the labour income accrues to workers in the host country. Labour income per production worker in the source country, which is relevant for indifference condition (9), is given by

$$w = \frac{\gamma(\chi; \eta) \rho Y}{L},$$

where $L$ is the (endogenous) supply of production workers in the source country, and $\gamma(\chi; \eta) \equiv \alpha(\chi; \eta)/(1 + \chi)$, with

$$\alpha(\chi; \eta) \equiv 1 + \eta \chi - (1 - \eta) \chi^{k-\frac{k}{k+\frac{\rho}{\sigma}}},$$

is the share of the total wage bill that accrues to workers in the source country. It is easily confirmed that $\gamma(\chi; \eta)$ decreases monotonically in $\chi$, falling from the maximum value of 1 at $\chi = 0$ to the minimum value of $\eta$ at $\chi = 1$.

11Eq. (12) is derived from $wL = \rho N \left[ \int_{\varphi^c}^{\varphi^o} r^d(\varphi) dG(\varphi) + \eta \int_{\varphi^c}^{\varphi^o} r^o(\varphi) dG(\varphi) \right]$. 
With Pareto distributed productivities, average plant-level revenues $\bar{r} = Y/[M(1 + \chi)]$ are a constant multiple $k/(k - \xi)$ of revenues by the marginal firm, $\sigma \pi^d(\varphi^c)$.\footnote{This follows from solving $Y = N \left[ \int_{\varphi^c}^{\varphi^d} r^d(\varphi) dG(\varphi) + \int_{\varphi^c}^{\infty} r^o(\varphi) dG(\varphi) \right]$.} Rearranging terms, we can solve for the profits of the marginal firm

$$\pi^d(\varphi^c) = \left( \frac{k - \xi}{k} \right) \frac{Y}{\sigma M(1 + \chi)}. \quad (14)$$

According to Eqs. (12) and (14), both the wage rate of production workers and the profit income of the marginal entrepreneur are proportional to aggregate income $Y$. Using these equations to substitute for $w$ and $\pi^d(\varphi^c)$ in Eq. (9), we therefore get an indifference locus linking the mass of production workers $L$ and the mass of firms $M$ that is independent of aggregate income:

$$L = \gamma(\chi; \eta) (1 + \chi) \frac{k \rho \sigma}{k - \xi} M \quad (15)$$

It is immediate that $L$ and $M$ are positively linked via the indifference locus. The intuition is as follows: An increase in $L$ decreases labour income per worker, ceteris paribus, and therefore it needs to be accompanied by an increase in $M$ (thereby reducing average profits and also profit income of the marginal entrepreneur) to restore indifference. A second condition linking $L$ and $M$ is established by the resource constraint

$$L = N - (1 + \chi) M, \quad (16)$$

which illustrates that individuals can work as either entrepreneurs ($M$), offshoring agents ($\chi M$), or production workers ($L$). Together, Eqs. (15) and (16) pin down the equilibrium mass of intermediate goods producers $M$ and the equilibrium mass of production workers $L$ as a function of model parameters and a single endogenous variable, the share of exporting firms $\chi$:

$$M = \left\{ \frac{k - \xi}{(1 + \chi) [\gamma(\chi; \eta) k \rho \sigma + k - \xi]} \right\} N, \quad (17)$$

$$L = \left[ \frac{\gamma(\chi; \eta) k \rho \sigma}{\gamma(\chi; \eta) k \rho \sigma + k - \xi} \right] N. \quad (18)$$

The mass of firms is linked to the ability of the marginal entrepreneur by the condition $M = [1 - G(\varphi^c)]N$, and solving for $\varphi^c$ gives

$$\varphi^c = \left\{ \frac{(1 + \chi) [\gamma(\chi; \eta) k \rho \sigma + k - \xi]}{k - \xi} \right\}^{\frac{1}{1}}. \quad (19)$$

In the next subsection we show how $\chi$ is determined as a function of the cost of offshoring $\tau$. 
2.4 Determining the share of offshoring firms

In this subsection, we derive the formal condition in terms of model parameters for an interior offshoring equilibrium, i.e. a situation in which some but not all firms offshore, and we also show how the share of offshoring firms $\chi$ varies with the cost of offshoring $\tau$ in an interior equilibrium. For this purpose, we first look at Eq. (11), which combines all possible combinations of $\chi$ and $\kappa$ that are consistent with indifference of the marginal offshoring firm between domestic and foreign production of routine tasks. Accordingly, we use the term offshoring indifference condition (OC) to refer to the $\chi$-$\kappa$ relationship established by Eq. (11). A larger marginal cost savings factor $\kappa$ makes offshoring more attractive, and a larger share of firms chooses to move production of their routine tasks abroad. Hence, the offshoring indifference condition is represented by an upward sloping locus in Figure 1, and from our analysis above we know that $\chi \in (0, 1)$ requires $\kappa \in (1, 2^{1/(\rho \sigma)})$.

A second link between $\chi$ and $\kappa$ can be derived from Eq. (5). Using $wL = \gamma(\chi; \eta) \rho Y$ from Eq. (12), the equivalent condition for the host country given by $w^*N^* = [1 - \gamma(\chi; \eta)]\rho Y$, and replacing $L$ by Eq. (18) we obtain the labour market constraint (LC):

$$\kappa = \left\{ \frac{\gamma(\chi; \eta) k \rho \sigma + k - \xi}{\tau [1 - \gamma(\chi; \eta) k \rho \sigma]} \left( \frac{N^*}{N} \right) \right\}^{1 - \eta}. \tag{20}$$

Since $\gamma(\chi, \eta)$ decreases monotonically from 1 to $\eta$ as $\chi$ increases from 0 to 1, we know that the labour market constraint is monotonically decreasing in $\chi$, starting from infinity. This is intuitively plausible: At $\chi = 0$, there is no production in the host country, and wage rates there fall to zero, making the marginal cost savings factor $\kappa$ infinitely large. With more firms starting to offshore production, wages in the host country are bid up, thereby reducing $\kappa$. Combining Eqs. (11) and (20), we can conclude that an interior equilibrium with $\chi < 1$ is reached if the right-hand side of Eq. (20), evaluated at $\gamma(1, \eta) = \eta$, is smaller than $2^{1/(\rho \sigma)}$. This can obviously be achieved for sufficiently high values of $\tau$, and it is this parameter domain we consider in our analysis. The result is illustrated in Figure 1. Given our parameter constraints, the LC and OC loci intersect at some $\chi \in (0, 1)$.

Eq. (20) also illustrates that the marginal cost savings factor $\kappa$ is increasing in the relative population size $N^*/N$. This is quite intuitive: the larger the relative size of the host country population, ceteris paribus, the lower the endogenous relative wage $w^*/w$, and hence the larger the potential cost savings from offshoring.
While it is not possible to derive an explicit solution for $\chi$ in terms of $\tau$, we can determine the sign of partial derivative $\partial \chi / \partial \tau$ by combining Eqs. (11) and (20) to the implicit function $F(\chi, \tau) \equiv \{ \gamma(\chi; \eta) k \rho \sigma + k \xi \left( \frac{N^*}{N} \right) \}^{1-\eta} - \left( 1 + \chi \xi \right) \frac{1}{\rho \sigma} = 0$.

Implicit differentiation yields $d\chi / d\tau < 0$ for any interior equilibrium with $0 < \chi < 1$. In Figure 1, higher direct costs of shipping intermediate goods, i.e. a higher parameter $\tau$, shifts the LC locus downwards, but does not affect the OC locus. We therefore have the intuitive result that a higher $\tau$ reduces the marginal cost savings factor $\kappa$, and thus reduces $\chi$, the equilibrium share of firms that shift production of their routine task abroad. Due to the monotonic relationship between (endogenous) $\chi$ and (exogenous) $\tau$ we can equivalently derive comparative static results below in terms of either variable.\(^{14}\)

\(^{14}\)One can see in Eq. (20) that the limiting case $\chi \to 0$ is induced by $\tau \to \infty$. 

**Figure 1: Partitioning of firms by their offshoring status**
3 The effects of offshoring

The purpose of this section is to look at the effects of offshoring on key macroeconomic variables, namely aggregate welfare and income inequality within the group of entrepreneurs as well as between entrepreneurs and production workers. All effects are driven by the underlying reallocation of domestic factors between firms and occupations, and it is therefore the link between the level of offshoring and domestic factor allocation that we look at first. Throughout this section, we will derive comparative static results in terms of changes in $\chi$. As shown above, this is equivalent to considering exogenous changes in offshoring cost $\tau$, noting that $d\chi/d\tau < 0$. Also, we focus on the source country, since most effects for the host country are trivial due to our simplifying assumption that no firms are headquartered there.

3.1 Factor allocation

Changes in the extent of offshoring affect factor allocation in the source country via its effects on the labour indifference condition (15) and on the resource constraint (16). The curves $LI^a$ and $RC^a$ in the right quadrant of Figure 2 represent the labour indifference condition and the resource constraint in autarky (superscript $a$ is also used in the following to denote autarky values), while the $CA$ curve in the left quadrant shows how a given mass of firms translates into a value for the cutoff ability $\varphi^c$.

From Eq. (16), an increase in $\chi$, starting from $\chi = 0$, rotates the $RC$ locus counter-clockwise: for a given mass of production workers, the viable mass of firms is reduced, simply because offshoring firms now need to hire freelance offshoring agents. The effect on labour indifference locus $LI$ is less straightforward. Since the attractiveness of becoming a production worker depends on the mass of available jobs, it is useful to consider first what offshoring does to (domestic) firm-level employment. Using $l^d(\varphi) = q^d(\varphi)/\varphi$ and $l^o(\varphi) = \eta q^o(\varphi)/(\varphi \kappa)$, we obtain

$$\frac{l^o(\varphi)}{l^d(\varphi)} = \left[ \frac{l^o(\varphi)}{l^d(\varphi)/q^d(\varphi)} \right] \left[ \frac{q^o(\varphi)}{q^d(\varphi)} \right] = \eta \kappa^o = \eta \left( 1 + \chi \xi_k \right),$$

(21)

which can be greater or smaller than one. Eq. (21) shows how to split the effect of offshoring on firm-level employment into the two partial effects well-known from the literature: The in-
ternational relocation effect, showing that domestic employment per unit of output is affected negatively by offshoring, since on the one hand the routine task is now produced by foreign labour and on the other hand the input ratio changes in favour of the – now relatively cheaper – routine task, is equal to $\eta/\kappa < 1$. The firm-level productivity effect, showing the positive employment effect due to the increase in output that the offshoring firm achieves as a consequence of its decreased marginal cost, is equal to $\kappa^\sigma > 1$.\(^{15}\)

It is immediate that the effect of offshoring on firm-level employment is negative for small values $\chi$, since $\kappa$ is close to 1, and hence the firm-level productivity effect is small. What happens to the $LI$ locus in this case can be seen by looking at Eq. (15): For a given mass of firms $M$, the

\(^{15}\)Empirical evidence for the effect of offshoring on firm-level employment comes from Moser, Urban, and Weder di Mauro (2009) and Hummels, Jørgensen, Munch, and Xiang (2011), who sort out the firm-level productivity effect and the international relocation effect using matched employer-employee-data. While the former study finds that the firm-level productivity effect dominates for the case of Germany, the opposite seems to occur in Denmark as noted by Hummels, Jørgensen, Munch, and Xiang (2011).
mass of production workers $L$ decreases as a consequence of the reduced labour demand from offshoring firms, and the $LI$ locus rotates counter-clockwise. Formally, this result follows from $\lim_{\chi \to 0} (\partial \gamma / \partial \chi) = -\infty$. Due to the strong reduction in the mass of production workers, not all of them can find employment in the offshoring service sector, and, as shown by Eq. (17), the mass of firms increases unambiguously. Graphically, the low-$\chi$ scenario is captured by locus $LI^o$ in Figure 2, and the offshoring equilibrium is determined by the intersection of $LI^o$ with the new resource constraint $RC^o$. At higher levels of $\chi$, the firm-level productivity effect becomes more pronounced and it may eventually dominate if $\eta$ is sufficiently high, i.e. if the international relocation effect is not too strong. As a consequence, the counter-clockwise rotation of the $LI^o$ locus is less pronounced (or even reversed) at high levels of $\chi$, and hence the mass of firms may fall eventually below the autarky level. The mass of production workers on the other hand never returns to its autarky level, as can be verified by looking at Eq. (18).

### 3.2 Welfare

With just a single global consumption good, welfare for the source country is simply given by source country income per capita. Aggregate income $I$ in the source country is the sum of profit income and offshoring service income, totalling $(1 - \rho)Y$ due to markup-pricing, and domestic labour income, which is equal to $\gamma(\chi; \eta)\rho Y$. Using Eq. (6) for the marginal firm with productivity $\varphi^c$, as well as Eqs. (9) and (14), we get

$$\frac{I}{I^a} = \Phi(\chi), \quad (22)$$

where

$$\Phi(\chi) \equiv [1 - \rho + \gamma(\chi; \eta)\rho] (1 + \chi)^{\frac{1}{\rho}} \left\{ \frac{(1 + \chi)[\gamma(\chi; \eta)\rho \sigma + k - \xi]}{k\rho \sigma + k - \xi} \right\}^{\frac{1}{1-\rho}}. \quad (23)$$

By construction, $\Phi(0) = 1$. Income in the source country is therefore higher in an offshoring equilibrium than in autarky if for the specific share $\chi$ of offshoring firms in this equilibrium we have $\Phi(\chi) > 1$. In our model, an increase in the share of offshoring firms has two effects on $\Phi(\chi)$. First, there is the firm-level productivity effect, outlined in the previous subsection, which increases welfare in the source country, ceteris paribus. Second, the changes in firm-level domestic employment in newly offshoring firms induce a *domestic reallocation* of labour, either
towards the newly offshoring firms (if the firm-level employment effect derived in Eq. (21) is positive) or away from them (otherwise). In the process the wage rate changes, and since the pools of production workers and entrepreneurs are linked via indifference condition (9), the mass of entrepreneurs changes as well. It is this facet of the domestic reallocation effect that is directly welfare relevant, since the autarky equilibrium of our model features excessive firm entry relative to the social planner solution. Any offshoring-induced increase in the mass of firms reinforces this distortion and therefore has a negative effect on welfare in the source country, ceteris paribus, and vice versa.\textsuperscript{16}

At low levels $\chi$, the firm-level productivity effect is negligible (this is why only the most productive firms choose to offshore), while the domestic reallocation effect leads to an unambiguous increase in the mass of firms as offshoring firms are large and therefore dismiss a relatively large number of workers when shifting their production of routine tasks abroad. As a consequence, at low levels of $\chi$, welfare in the source country falls relative to autarky. Things are different if a large share of firms is engaged in offshoring. In this case, two effects work in favour of higher welfare: First, the positive firm-level productivity effect is now sizable, and it affects a larger share of firms. Second, the domestic reallocation effect is now more favourable since the newly offshoring firms are relatively small, so that a marginal increase in $\chi$ shifts fewer – if any – workers towards entrepreneurship. Under the empirically plausible sufficient condition $\eta \geq 1/2$ we can show that $\Phi'(1) > 0$ and $\Phi(1) > 1$, and hence that welfare increases at high levels of $\chi$, and that it is higher at high levels of offshoring than in autarky.\textsuperscript{17} The reason is that if $\chi$ is close to one and $\eta \geq 1/2$, fewer firms are active in the offshoring equilibrium than under autarky, so that offshoring provides a (partial) remedy for the distortion of firm entry in our model, and this is sufficient for a welfare stimulus in the source country.

\textsuperscript{16}Our production technology in Eq. (1) neutralises an externality that is present in the general CES production function used by Ethier (1982): increasing returns to scale in the economy-wide production of final goods. With increasing returns to scale, additional firm entry in the intermediates sector generates a positive externality, which we exclude here. In the more general Ethier case, the distortion resulting from excessive entry is therefore reduced relative to our case, and it disappears for the special case of the CES production function used, e.g., by Matusz (1996).

\textsuperscript{17}Blinder (2009) and Blinder and Krueger (2012) report that one quarter of all tasks in US production are offshorable, which suggests that $\eta$ is close to 0.75.
As outlined above, offshoring of just a few firms exerts negative welfare effects in the source country, because the domestic reallocation effect is relatively strong, while the firm-level productivity effect is relatively weak in this case. We now show that this result is a direct consequence of firm heterogeneity by comparing it to the outcome in a model where all firms have the same productivity, which is the special case of our framework with \( k \to \infty \). In the case of homogeneous producers, all firms have a finite productivity (equal to 1, the lower bound of the Pareto distribution). Consequently, the firm-level productivity effect of offshoring is of first order already at \( \chi = 0 \), since the marginal cost savings factor \( \kappa \) making firms indifferent between offshoring and not offshoring is now equal to \( 2^{1/(\rho \sigma)} \) independent of the level of \( \chi \). At the same time, for low levels of \( \chi \) the domestic reallocation effect is mitigated relative to the benchmark of heterogeneous firms because the first offshoring firms are now smaller, and hence the mass of workers losing their job in newly offshoring firms is reduced. One can show that as a result welfare is increasing monotonically in \( \chi \).

While the main focus of this paper is on the consequences of offshoring in the source country, it is also worth taking a closer look at its implications for global welfare. Noting that world-wide per-capita income is proportional to output \( Y \) and accounting for \( \frac{Y}{Y^a} = \Phi(\chi)/[1 - \rho + \gamma(\chi; \eta)\rho] \), it follows from Eq. (23) that offshoring unambiguously stimulates global welfare. This is intuitive, because in our model the only motive for offshoring is cost saving, and cheaper production provides a source of welfare gain, despite a negative impact of excessive firm entry at low levels of \( \chi \). We summarize our insights regarding the welfare implications of offshoring in the following proposition.

**Proposition 1** Welfare in the source country decreases with the share of offshoring firms at low levels of \( \chi \). Under the sufficient condition \( \eta \geq 1/2 \), the effect is reversed as more firms offshore, and welfare surpasses its autarky level if \( \chi \) is sufficiently large. Global welfare is higher in any offshoring equilibrium than in autarky.

**Proof** Analysis in the text and formal discussion in the Appendix.
3.3 Inequality among entrepreneurs and between groups

Intra-group inequality of entrepreneurial income is measured by the Gini coefficient for profit income, which, as formally shown in the Appendix, is given by

\[ A_M(\chi) = \frac{\xi}{2k - \xi} \left[ 1 + \frac{(k - \xi)(2 - \chi)}{k + \xi \chi} \right]. \] (24)

The relationship between Gini coefficient \( A_M(\chi) \) and the share of offshoring firms \( \chi \) is non-monotonic. An increase in \( \chi \) generates a profit gain for newly offshoring firms. If the share of offshoring firms is small, an increase in \( \chi \) implies that newly offshoring firms are run by entrepreneurs with high ability, and these are firms that already ranked high in the profit distribution prior to offshoring. Hence, an increase in \( \chi \) raises the dispersion of profit income in this case. Things are different at high levels of \( \chi \), because newly offshoring firms are now firms with a low rank in the distribution of profit income and an increase in \( \chi \) therefore lowers the dispersion of profit income. Furthermore, comparing \( A_M(\chi) \) for \( \chi > 0 \) with \( A_M(0) \), we find that offshoring increases the dispersion of profit income relative to the benchmark of no offshoring, irrespective of the prevailing level of \( \chi \). This result is due to the fact that the common fixed cost of offshoring disproportionately affects the profits of less productive firms, thereby contributing to an increase in the dispersion of profit incomes.

Inter-group inequality is measured by the ratio of average entrepreneurial income and average labour income, where the latter is simply given by wage rate \( w \). According to (14), average entrepreneurial income, \( \bar{\psi} \), is equal to \( \pi^d(\varphi^e)(1 + \chi)k/(k - \xi) - \chi s \). Applying indifference condition (9), the ratio of average entrepreneurial income and average income of production workers is therefore given by

\[ \frac{\bar{\psi}}{w} = \frac{k + \chi \xi}{k - \xi}. \] (25)

It follows immediately that inter-group inequality rises monotonically in the share of offshoring firms \( \chi \). The intuition is as follows. A higher value of \( \chi \) indicates that the marginal cost saving factor \( \kappa \) must be higher, which in turn implies that profits of all offshoring firms increase, both in absolute terms and relative to the profits of the marginal firm in the market. Since the marginal firm’s profits are equal to \( w \), it is clear that inter-group inequality has to go up in response to an increase in \( \chi \).
The following proposition summarizes the results.

**Proposition 2** The inequality of entrepreneurial income, measured by the Gini coefficient, rises with the share of offshoring firms at low levels of $\chi$, and decreases at high levels of $\chi$, while always staying higher than in the benchmark situation without offshoring. Increasing the share of offshoring firms $\chi$ leads to a monotonic increase in inter-group inequality between entrepreneurs and workers.

4 Offshoring in the presence of firm-level rent-sharing

In this section, we extend our framework by a more sophisticated model of the labour market in order to address the widespread concern that offshoring has a negative effect on aggregate employment in a country that shifts production of routine tasks to a low-wage location. More specifically, we develop a model of firm-level rent sharing in an imperfectly competitive labour market to get a framework that features involuntary unemployment and at the same time captures the stylised fact that more profitable firms pay higher wages (cf. Blanchflower, Oswald, and Sanfey, 1996). As we show in the following, all the results derived so far are robust with respect to this extension.

The labour market model proposed in this section is a fair-wage effort model which builds upon the idea of gift exchange, and whose main assumptions are rooted in insights from psychological research (see Akerlof, 1982; Akerlof and Yellen, 1990). The model postulates a positive link between a firm’s wage payment and a worker’s effort provision, and workers exert full effort, normalised to equal 1, if and only if they are paid at least the wage they consider fair. As in Egger and Kreickemeier (2012) we assume that the fair wage $\hat{w}$ is a weighted average of firm-level operating profits $\pi(\varphi)$ and the average wage of production workers $(1 - U)\bar{w}$, where $U$ is the unemployment rate of production workers and $\bar{w}$ is the average wage of those production workers who are employed:

$$\hat{w}(\varphi) = [\pi(\varphi)]^\theta[(1 - U)\bar{w}]^{1-\theta}$$

An analogous condition holds in the host country of offshoring. Following Akerlof and Yellen (1990), we assume that effort decreases proportionally with the wage, and hence firms have
no incentive to pay less than \( \hat{w} \). On the other hand, one can show that the model features involuntary unemployment in equilibrium, and therefore even low-productivity firms do not need to pay more than \( \hat{w} \) to attract workers. Hence, \( w(\varphi) = \hat{w}(\varphi) \), and Eq. (26) describes the distribution of wages across firms as a function of firm-level operating profits.\textsuperscript{18} In contrast to the full employment version of our model the decision to become a production worker now carries an income risk, since wages are firm-specific. We make the standard assumption that workers have to make their career choice before they know the outcome of the job allocation process among applicants (cf. Helpman and Itskhoki, 2010).\textsuperscript{19} With risk neutrality of individuals, the indifference condition for the marginal entrepreneur now becomes

\[
\pi^d(\varphi^c) = (1 - U) \bar{w} = s. \tag{9'}
\]

In comparison to the full employment version of our model, the relative operating profits of more productive firms are lower with rent-sharing, since part of the advantage stemming from higher productivity is compensated by having to pay a higher wage rate. Formally, the elasticity of firm-level relative operating profits with respect to relative firm productivity (cf. Eq. (8)) is no longer given by \( \xi \equiv \sigma - 1 \), but by \( \bar{\xi} \equiv (\sigma - 1) / [1 + \theta(\sigma - 1)] \), which is smaller than \( \xi \) if \( \theta \) is strictly positive.\textsuperscript{20} It then follows from Eq. (26) that the elasticity of firm-level relative wages with respect to relative productivities is given by \( \theta \bar{\xi} \). Notably, this holds true not only in the source country, but also in the host country: highly productive firms pay higher wages in the host country than their less productive competitors.\textsuperscript{21}

\textsuperscript{18} Even though firms set wages unilaterally, their profit maximization problem does not differ from the one in Section 2.2. As pointed out by Amiti and Davis (2012), wages depend positively on profits due to fair wage constraint (26), and hence the firm has no incentive to manipulate the wage, but instead treats it parametrically at the equilibrium level \( w(\varphi) = \hat{w}(\varphi) \).

\textsuperscript{19} Production workers would of course prefer to work for a firm that offers higher wages and, in the absence of unemployment compensation, those who do not have a job would clearly benefit from working for any positive wage rate. However, since due to contractual imperfections it is impossible to fix effort of workers ex ante, firms are not willing to accept underbidding by outsiders: Once employed, the new workers would adopt the reference wage of insiders and thus reduce their effort when the wage paid by the firm falls short of the wage considered to be fair (see Fehr and Falk, 1999).

\textsuperscript{20} In the borderline case \( \theta = 0 \), firm-level operating profits have zero weight in the determination of the fair wage, Eq. (26) simplifies to \( \hat{w} = w \), and the model collapses to the full employment version.

\textsuperscript{21} Evidence on international rent sharing within a firm but across country borders has been documented e.g.
There is one additional important mechanism in our model that arises due to firm-level rent sharing: for an offshoring firm, there is a feedback effect on firm-level marginal costs in the source country, since higher operating profits lead to higher firm-level wage rates via fair wage constraint (26). This implies that the input ratio changes more strongly in favour of the imported routine task, and the international relocation effect identified in Section 3.1 is therefore more strongly negative than in the full employment model, now equaling $\eta/\kappa(\xi/\bar{\xi})$ instead of $\eta/\kappa$. However, the functional relationships between $\chi$, welfare and the two inequality measures in Section 3 remain to be given by Eqs. (22) to (25), with the mere difference that $\bar{\xi}$ replaces $\xi$. As a consequence, the comparative static effects of offshoring on aggregate welfare and on income inequality among entrepreneurs as well as between entrepreneurs and workers remains qualitatively the same in the more sophisticated model variant considered here, and Propositions 1 and 2 continue to hold. This allows us to focus on the labour market side of our model in the subsequent analysis.

In the presence of firm-level rent sharing, $L$ is the mass of individuals looking for employment as production workers in the source country, while the mass of employed production workers is now given by $(1-U)L$. Neither entrepreneurs nor offshoring agents can be unemployed, and therefore the economy-wide unemployment rate in the source country is given by $u \equiv UL/N$. When looking at $u/u^a$, it is helpful to consider separately the effect of offshoring on the unemployment rate of production workers, measured by $U/U^a$, and the effect on the supply of production labour due to adjustments in the occupational choice, measured by $L/L^a$. As shown in the Appendix, the unemployment rate of production workers is given by

$$U = \frac{\theta \bar{\xi} + \left[1 - \Delta(\chi; \eta\right)(k - \bar{\xi})}{k - (1 - \theta)\bar{\xi}},$$

where

$$\Delta(\chi; \eta) \equiv \frac{\beta(\chi; \eta)}{\alpha(\chi; \eta)}$$

and

$$\beta(\chi; \eta) \equiv 1 + \chi \left[\eta \left(1 + \chi^{\frac{k-1}{k}}\right)^{(1-\theta)} - 1\right],$$

by Budd, Konings, and Slaughter (2005).

\textsuperscript{22}A detailed discussion on how firm-level rent-sharing alters the equations in Section 2 is deferred to a supplement, which is available upon request.

\textsuperscript{23}The importance of occupational choice for understanding how a country’s labour market absorbs the consequences of trade and offshoring has recently been pointed out by Liu and Trefler (2011) and Artuç and McLaren (2012).
and \( \alpha(\chi; \eta) \) has been defined in Eq. (13). It is easily checked that \( \Delta(0, \eta) = 1 \), and therefore \( U \) is lower in an equilibrium with offshoring than in autarky if \( \Delta(\chi; \eta) > 1 \) and higher than in autarky if \( \Delta(\chi; \eta) < 1 \). The effect of offshoring on \( L \) follows directly from Eq. (18), and as discussed in Subsection 3.1, the supply of production labour is smaller in an offshoring equilibrium than in autarky. By reducing \( L \), this effect reduces aggregate unemployment \( u \), ceteris paribus. Putting together these partial effects leads to

\[
\frac{u}{u^a} = \Lambda(\chi; \eta), \quad \text{with} \quad \Lambda(\chi; \eta) \equiv \frac{\theta \bar{\xi} + [1 - \Delta(\chi; \eta)](k - \bar{\xi})}{\theta \xi} \frac{\gamma(\chi; \eta)(k \rho \sigma + k - \bar{\xi})}{\gamma(\chi; \eta) k \rho \sigma + k - \xi},
\]

(29)

where \( u^a \) can be computed from Eqs. (18) and (27). The first fraction of \( \Lambda \) is equal to \( U/U^a \) and the second fraction is equal to \( L/L^a \). Unemployment rate \( u \) is lower with \( \chi > 0 \) than with \( \chi = 0 \) if \( \Lambda(\chi; \eta) < 1 \), while the opposite is true if \( \Lambda(\chi; \eta) > 1 \). We show the following result:

**Proposition 3** Unemployment in the source country decreases with the share of offshoring firms at low levels of \( \chi \). Under the sufficient condition \( \eta \geq 1/2 \) the effect is reversed as more firms offshore, and unemployment surpasses its autarky level if \( \chi \) is sufficiently large.

**Proof** See the Appendix.

The intuition for this result is straightforward. Since the labour supply effect works unambiguously in favour of a reduction in overall unemployment, all potentially harmful employment effects must work via an increase in the unemployment rate of production workers \( U \). This effect is analysed most easily by noting that due to the indifference condition for the marginal entrepreneur, Eq. (9), and the fair wage constraint, Eq. (26), there is a direct link in any equilibrium with \( \chi < 1 \) between \( U \), the average wage for production labour \( \bar{w} \) and the wage paid by the marginal firm, \( w(\varphi^c) \), given by

\[
U = \frac{\bar{w} - w(\varphi^c)}{\bar{w}},
\]

and hence if \( U \) changes \( \bar{w}/w(\varphi^c) \) has to change in the same direction: an increase in \( \bar{w} \), which makes entrepreneurship less attractive, has to be compensated by an increase in \( U \) in order to restore indifference. Now consider an increase in \( \chi \), starting from zero. The newly offshoring firms in this scenario are the high-productivity firms paying high wages, and their domestic employment levels fall unambiguously due to the international relocation effect (as noted before,
\( \kappa \) is close to 1 if \( \chi \) is close to zero, and therefore the productivity effect is small), resulting in a decrease in the domestic average wage \( \bar{w} \). Hence, for low levels of offshoring \( U \) decreases with an increase in \( \chi \), and so does the aggregate unemployment rate \( u \).

The effect of a marginal increase in offshoring on \( U \) is reversed at high levels of \( \chi \), since now the newly offshoring firms have low productivity and pay low wages, and since the international relocation effect reduces the weight their wage has in the domestic average wage, \( \bar{w} \) increases.\(^{24}\) Hence, the unemployment rate of production workers \( U \) increases, and overall unemployment is driven by two opposing effects: the supply of production workers decreases, but a larger share of them is without a job. If \( \eta \) is large, and hence the international relocation effect is small, the negative impact of offshoring on \( U \) dominates the decline in \( L \) at high levels of \( \chi \).\(^ {25}\)

The ratio \( \bar{w}/w(\varphi^c) \) provides one measure of income inequality among production workers, but not a very informative one, since it ignores information on individual wage rates by everybody but the workers in the marginal firm. Hence, in analogy to the measurement of entrepreneurial income inequality we now look at the Gini coefficient as a more sophisticated measure of wage dispersion. As formally shown in the Appendix, this Gini coefficient is given by

\[
A_L(\chi) = \frac{\theta \xi}{2(k - \xi) + \theta \xi} \left\{ 1 + \frac{2(k - \xi) \left[ 1 - \chi \frac{k - (1 - \theta)\xi}{k - \bar{\xi}} \right] [\alpha(\chi; \eta) - 1]}{\alpha(\chi; \eta) \beta(\chi; \eta) \theta \xi} \right. \\
- \frac{2 \left[ k - (1 - \theta)\xi \right] \left[ 1 - \chi \frac{k - \xi}{k - \bar{\xi}} \right] \beta(\chi; \eta) - 1]}{\alpha(\chi; \eta) \beta(\chi; \eta) \theta \xi} \left. \right\}. \tag{30}
\]

Inequality of wage income is the same in the polar cases where either no firms or all firms offshore: \( A_L(0) = A_L(1) = \frac{\theta \xi}{2[k - (1 - \theta)\xi] - \theta \xi} \).

\(^{24}\)This is different only in the limiting case of \( \eta = 0 \). In this case, all tasks are routine, and therefore if a firm moves tasks abroad, all domestic production jobs of this firm are lost. As a consequence, with each newly offshoring firm the source country loses its highest-paying domestic production jobs, and therefore \( \bar{w} \) decreases monotonically with an increase in \( \chi \), leading to a monotonic decrease in \( U \).

\(^{25}\)In the limiting case of \( \eta \to 1 \), \( \chi \) loses its impact on the supply of production labour \( L/L^a \). At the same time, a larger share of offshoring firms increases the unemployment rate of production workers, i.e. \( dU/d\chi > 0 \), because offshoring firms have to pay higher wages in the source country and thus reduce employment in the non-routine task.

\(^{26}\)An analogous result holds for the trade models of Egger and Kreickemeier (2009, 2012) and Helpman, Itskhoki,
lower than the autarky level at low levels of offshoring, and higher than the autarky level at high levels of offshoring. Figure 3 illustrates the resulting S-shape of the $A_L$ locus, alongside the Gini-coefficient for entrepreneurial income $A_M$ that we computed in the previous section, with the only modification that now $\tilde{\xi}$ replaces $\xi$.

$$A_M, A_L$$

$$A_M(0)$$

$$A_L(0)$$

$$\chi$$

Figure 3: Gini coefficients for entrepreneurial income and wage income

The intuition is analogous to the one for the effect of offshoring on $\bar{w}/w(\varphi^c)$. In a situation where the offshoring strategy is only chosen by the most productive firms, the international relocation effect shifts high-wage jobs abroad, and this effect is responsible for the reduction of wage inequality at low levels of $\chi$. The influence of the relocation effect is reversed at high levels of $\chi$, since now the low-productivity firms shift low-wage jobs abroad, thereby contributing and Redding (2010), where wage inequality is the same in the cases of autarky and exporting by all firms.
to an increase in wage inequality in the source country. There is also a firm-level wage effect due to the rent-sharing mechanism in our model: it increases wage dispersion at low levels of $\chi$ (wage-boosting increase in profits by high-wage firms) and reduces wage dispersion at high levels of $\chi$ (wage-boosting increase in profits by low-wage firms). The firm-level wage effect thereby influences wage inequality in the opposite direction to the international relocation effect, and it dominates the overall effect when many firms offshore.

The following proposition summarizes the main insights regarding the distributional effects of offshoring within the group of (employed) production workers.

**Proposition 4** The impact of offshoring on the dispersion of wage income, measured by the Gini coefficient, is non-monotonic. Wage income inequality falls relative to the benchmark without offshoring if $\chi$ is small, while it rises relative to this benchmark if $\chi$ is sufficiently large.

**Proof** Analysis in the text and formal discussion in the Appendix.

## 5 A calibration exercise

In this section, we use the extended version of our model from Section 4 to quantify the implications of offshoring for welfare, employment, and income distribution in the source country. For this purpose, we calibrate the model, using parameter estimates from the empirical trade literature. A useful starting point are the findings in Egger, Egger, and Kreickemeier (2011), who conduct a structural empirical estimation of the Egger and Kreickemeier (2012) model. Employing information from the Amadeus data-set, Egger, Egger, and Kreickemeier (2011) report the following parameter estimates for the average country in their data-set, which covers five European economies: $\theta = 0.102, \sigma = 6.698, k = 4.306$. While, to the best of our knowledge, there are no other directly comparable estimates for the rent-sharing parameter available, the estimate of $\sigma$ lies in the range of parameter estimates reported by Broda and Weinstein (2006) and is well in line with the parameter value considered by Arkolakis (2010) in his calibration exercise. The parameter estimate of $k$ is higher than the estimate of 2 reported by Del Gatto, Mion, and Ottaviano (2006). However, it is consistent with findings by Arkolakis and Muendler (2010) and – together with the estimates for $\theta$ and $\sigma$ – guarantees that the parameter constraint

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\( k > \bar{\xi} \) is fulfilled. Regarding the share of offshorable tasks, we consider the findings by Blinder (2009) and Blinder and Krueger (2012) and set \( \eta = 0.75 \).

We then compute how a given exposure to offshoring alters our variables of interest relative to a benchmark without offshoring. The results from this exercise are summarised in Table 1. We see that using our parameter estimates the decline in \( \chi \) at low levels of \( \chi \) is only small, while for larger values of \( \chi \) offshoring exerts a sizable positive welfare effect in the source country. Furthermore, offshoring widens the income gap between entrepreneurs and workers significantly, and it generates ‘managerial superstars’ by increasing income inequality within the group of entrepreneurs (see Manasse and Turrini, 2001; Gersbach and Schmutzler, 2007). As outlined in Section 4, offshoring exerts a non-monotonic effect on the distribution of wage income. Evaluated at our parameter estimates, the model suggests that offshoring has only moderate effects on unemployment and intra-group inequality among production workers.

As guidance for what constitutes an empirically plausible share of exporting firms we use the results from Moser, Urban, and Weder di Mauro (2009). Using a large sample of 8,466 German plants from the IAB Establishment Panel, they find that the share of offshoring firms is 14.9 percent. This share is somewhat lower than the share of offshoring firms reported by The Economist (2004) from a small survey of 150 British firms, while it is significantly higher.

### Table 1: Impact of offshoring on welfare, unemployment, inequality

<table>
<thead>
<tr>
<th>( \chi )</th>
<th>( I ) in pct.</th>
<th>( u ) in ppt.</th>
<th>( \tilde{\omega} ) in pct.</th>
<th>( A_M ) in pct.</th>
<th>( A_L ) in pct.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>−0.827</td>
<td>−2.654</td>
<td>0.837</td>
<td>0.322</td>
<td>−6.910</td>
</tr>
<tr>
<td>0.10</td>
<td>2.067</td>
<td>−0.944</td>
<td>8.369</td>
<td>2.860</td>
<td>−0.488</td>
</tr>
<tr>
<td>0.25</td>
<td>7.202</td>
<td>0.898</td>
<td>20.922</td>
<td>5.902</td>
<td>2.270</td>
</tr>
<tr>
<td>0.50</td>
<td>15.222</td>
<td>2.560</td>
<td>41.844</td>
<td>8.626</td>
<td>2.362</td>
</tr>
<tr>
<td>0.75</td>
<td>22.564</td>
<td>3.502</td>
<td>62.766</td>
<td>9.395</td>
<td>1.224</td>
</tr>
<tr>
<td>0.90</td>
<td>26.692</td>
<td>3.839</td>
<td>75.319</td>
<td>9.211</td>
<td>0.477</td>
</tr>
</tbody>
</table>

*Notes:* All reported figures, except of the one for unemployment, refer to percentage changes relative to autarky. In the case of unemployment, figures refer to changes in percentage points.
than the share of firms conducting international outsourcing and/or FDI in Japan as reported by Tomiura (2007). Evaluated at $\chi = 0.149$, our model predicts that the observed pattern of offshoring has increased welfare in Germany by 3.8 percent and at the same time has widened the gap between entrepreneurial income and wages by 12.5 percent. Furthermore, offshoring has increased inequality within the group of entrepreneurs by 4 percent and inequality within the group of production workers by 0.9 percent. In contrast to the widespread perception of large negative employment effects, our model predicts that offshoring has lowered unemployment in Germany by 0.2 percentage points.

6 Alternative measures of inequality

In this section, we extend the analysis of the link between offshoring and inequality in three directions. First, we include the two groups left out of the analysis of intra-group inequality so far – namely the freelance offshoring agents and the unemployed production workers, where the latter group only exists in the version of our model with firm-level rent sharing. Second, we analyse the effect of offshoring on the Lorenz curves for group-specific incomes to assess the robustness of our earlier analysis using Gini coefficients. Third, we look at the effect of offshoring on economy-wide inequality.\(^{27}\)

For the mentioned more comprehensive view of intra-group inequality, we combine entrepreneurs with freelance offshoring agents, thereby creating the group of self-employed agents, leaving everybody else – employed and unemployed production workers – for the second broadly defined income group. The Gini coefficient for income of self-employed agents can be expressed as

$$A_S(\chi) = \frac{\xi}{2k - \xi} \left[ 1 + \frac{2(k - \xi)}{k} \frac{\chi}{(1 + \chi)^2} \right].$$

(31)

In analogy to Eq. (24), the inequality measure in Eq. (31) applies in the basic model variant without rent sharing, and it can be adapted to the extended model with rent sharing by simply replacing $\xi$ by $\bar{\xi}$. Differentiating (31), we find $A'_S(\chi) > 0$, and hence inequality in the broadly

\(^{27}\)In the interest of readability, we keep the analysis in the section informal and refer readers who are interested in a more formal discussion to a supplement, which is available upon request.
defined group of all self-employed agents increases monotonically with $\chi$. From a comparison between (24) and (31), we can furthermore conclude that $A_M(\chi) > A_S(\chi)$ holds for all $\chi > 0$. This implies that inequality within the group of all self-employed agents is less pronounced than inequality within the subgroup of entrepreneurs.

In the extended model with firm-level rent sharing, the Gini coefficient for income of all production workers, including those who are unemployed, is given by

$$A_U(\chi) = U + (1 - U) A_L(\chi)$$

(32)

and thus larger than $A_L(\chi)$. Since $U$ is smaller than $U^a$ at low levels of $\chi$, while the reverse is true at high levels of $\chi$, the non-monotonic effect of $\chi$ on $A_L(\chi)$ extends to $A_U(\chi)$. However, there is one important difference between the two indices. The Gini coefficient for the income of all production workers does not fall back to its autarky level if all firms offshore. The reason is that the unemployment rate of production workers is higher at high levels of $\chi$ than under autarky, and this increases $A_U(\chi)$ ceteris paribus. As a consequence, $A_U(1) > A_U(0)$, whereas $A_L(1) = A_L(0)$.

In order to assess the robustness of our results with respect to the use of alternative measures of inequality, we analyse the effect of offshoring on the Lorenz curves for group-specific incomes. The main insight from this analysis is that the income distribution for self-employed agents in autarky Lorenz dominates the respective income distribution under offshoring. Since Lorenz dominance is equivalent to mean-preserving second-order stochastic dominance, our previous insight regarding the impact of offshoring on income inequality within the group of self-employed agents extends to all inequality measures that respect second-order stochastic dominance – including, for instance, the Gini coefficient or the Theil index. Regarding the relationship between offshoring and inequality within the broadly defined group of production workers, we find that for high levels of $\chi$ the distribution of labour income with offshoring Lorenz dominates the respective distribution under autarky, while the opposite is true if $\chi$ is close to zero. This implies that the non-monotonicity in the impact of $\chi$ on the distribution of labour income extends to all other measures of inequality that respect second-order stochastic dominance.28

28For intermediate levels of $\chi$, Lorenz curves for the scenarios with and without offshoring can intersect, implying
In a final step of our analysis, we are interested in the impact of offshoring on economy-wide inequality. For this purpose, we have to find a comprehensive measure of inequality. A particularly useful metric in this respect is the Theil index, which is decomposable in the sense that it can be written as a weighted average of inequality within subgroups, plus inequality between these subgroups (cf. Shorrocks, 1980). More specifically, in our model the Theil index for economy-wide income distribution can be written as

\[
T = a_S \left[ T_S + \ln \left( \frac{k}{k - \xi} \right) \right] + a_U T_U + \ln \left( a_S \frac{k - \xi}{k} + a_U \right),
\]

where \( T_S, T_U \) are the Theil indices for the income distributions of self-employed agents and all production workers, respectively, while

\[
a_S = \frac{1 - \rho}{\rho \gamma(\chi; \eta) + 1 - \rho}, \quad a_U = \frac{\rho \gamma(\chi; \eta)}{\rho \gamma(\chi; \eta) + 1 - \rho},
\]

are the income shares of the two subgroups of population. While we can show that offshoring raises Theil index \( T \) and hence economy-wide income inequality in the benchmark case without firm-level rent sharing (\( \theta = 0 \)), this result does not extend in general to the more sophisticated model variant with positive levels of \( \theta \), as the following simulation exercise shows by way of a counter-example.

Using the parameter estimates from the previous section, we can quantify the impact of offshoring on the different measures of inequality analysed above. The main insights from this calibration exercise are summarised in Table 2. This table reports changes (relative to autarky) in the Gini coefficients \( A_S(\chi), A_U(\chi) \) (columns 2 and 3), the group-specific Theil indices \( T_S, T_U \) (columns 4 and 5), and the economy-wide Theil index \( T \) (column 6) for different levels of \( \chi \). Thereby, columns 2 and 4 confirm our previous insight that offshoring makes the distribution of income of self-employed agents more unequal, while the figures in columns 3 and 5 depict the non-monotonicity in the effect of offshoring on labour income inequality. Finally, column 6 indicates a positive impact of offshoring on economy-wide income inequality. However, evaluated that the distributions of labour income for \( \chi = 0 \) and \( \chi > 0 \) cannot be ranked according to the criterion of second-order stochastic dominance. In this case, it cannot be ruled out that different metrics of inequality – such as the Gini coefficient or the Theil index – lead to different results regarding the impact of offshoring on labour income inequality.
at our parameter estimates, offshoring lowers economy-wide income inequality if $\chi$ is sufficiently close to zero. For instance, the economy-wide Theil index $T$ falls by 0.2 percent, if the source country of offshoring moves from autarky to $\chi = 0.001$.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.320</td>
<td>-6.354</td>
<td>0.018</td>
<td>-2.729</td>
<td>2.599</td>
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<td>0.10</td>
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<td>-0.682</td>
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<td>0.25</td>
<td>5.220</td>
<td>2.588</td>
<td>0.525</td>
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</tr>
<tr>
<td>0.50</td>
<td>7.250</td>
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<td>1.434</td>
<td>2.785</td>
<td>24.405</td>
</tr>
<tr>
<td>0.75</td>
<td>7.990</td>
<td>6.840</td>
<td>2.626</td>
<td>3.593</td>
<td>27.078</td>
</tr>
<tr>
<td>0.90</td>
<td>8.133</td>
<td>7.233</td>
<td>3.432</td>
<td>3.883</td>
<td>27.982</td>
</tr>
</tbody>
</table>

Notes: All reported figures refer to percentage changes relative to autarky.

We complete the discussion in this section, by quantifying the impact of observed levels of offshoring for income inequality in Germany. As outlined in Section 5, about 14.9 percent of German firms undertake offshoring, with moderate effects on income inequality. To be more specific, according to our calibration exercise, the observed pattern of offshoring has increased the Gini coefficient for the income of self-employed agents by 3.7 percent, while the Theil index for this income group has been raised by just 0.3 percent. The impact of the observed pattern of offshoring on income inequality of production workers is even less pronounced and amounts to 0.2 according to the Gini criterion, while it is almost zero when applying the metric of the Theil index. Despite the small impact of offshoring on measures of intra-group inequality, our calibration results point to a considerable increase in economy-wide income inequality, with the respective Theil index increasing by 14.6 percent. This is in line with our previous insight that offshoring exerts relatively strong inter-group effects.
7 Concluding remarks

In this paper, we have developed an analytically tractable general equilibrium framework for analysing offshoring to low-wage countries. It is a key feature of our framework that firms differ from each other in terms of their productivity. As a consequence, the costly option to offshore routine tasks to the low-wage country, while available to all firms, is chosen only by a subset of them in equilibrium. The effects that offshoring has on welfare and the income distribution depends on the share of firms that offshore tasks in equilibrium, and we are therefore able to show that considering firm heterogeneity adds a relevant dimension to the established offshoring literature that has mainly focused on the heterogeneity of tasks.

The model is analytically tractable, although the wages in both countries are determined endogenously. We buy analytical tractability by making the strong assumption that the option of becoming an entrepreneur is open only to individuals in one of the countries. This country is therefore the headquarters location of all firms, making it the source country of offshoring, while the other country is the host country of offshoring, with all employment in the performance of routine tasks for firms in the source country. The host country emerges endogenously as the low-wage country, since offshoring involves both a fixed and a variable cost, and hence for offshoring to be profitable firm-level wage rates have to be lower than in the source country. With this deliberately simple modelling of the host country, our interest is focused on the effects of offshoring in the source country.

Whereas income inequality between entrepreneurs and workers increases monotonically with the share of offshoring firms, we find that offshoring has a non-monotonic effect on income inequality among entrepreneurs: income inequality within this group increases if only a few firms shift the production of routine tasks abroad and decreases (while always staying above the autarky level) if offshoring becomes common practice among high and low productivity firms. Furthermore, offshoring may exert a positive or negative effect on aggregate welfare in the source country: Welfare decreases relative to autarky if only a small share of firms offshore their routine tasks to the low-wage country, while it is higher than in autarky if this share is large. Low offshoring shares are the result of high offshoring costs, and in this situation only the high-productivity firms offshore their routine tasks. This leads to the loss of jobs in the
most productive firms in the source country, and the workers are forced to seek employment in other firms, while some opt for opening new firms. The latter effect is welfare decreasing, since already the autarky equilibrium has too many firms from a social planner’s point of view. The effect is reversed if offshoring costs are low, and therefore many firms move their routine tasks offshore.

In an extension with firm-level rent sharing, which preserves all results derived in the benchmark model with perfectly competitive labour markets, our model allows us to address the public concern that offshoring invariably destroys domestic jobs. Our analysis shows that it is important to distinguish between what happens at the level of offshoring firms (firm-level effect) and what happens in the aggregate, after taking into account general equilibrium effects. We find that firm-level employment of production workers and aggregate employment tend to move in opposite directions: Aggregate employment increases unambiguously at low levels of $\chi$, where the negative firm-level effects on source country employment are largest. The reverse is true at high levels of $\chi$: Firm level employment of production workers goes up, while aggregate employment falls. The key to understanding this result lies in the occupational choice mechanism at the heart of our model: Individuals losing their jobs in newly-offshored tasks either find employment as offshoring agents, or they decide to become entrepreneurs, for whom the reservoir of available production workers makes it now worthwhile to run a firm themselves. The offshoring of domestic jobs may therefore result in a decrease of domestic unemployment under precisely those circumstances where one would expect it least if guided by firm-level effects. The model extension with rent sharing also leads to richer distributional effects, and we show that intra-group inequality within the group of production workers is affected by offshoring in a non-monotonic way, being lower than in autarky if only the most productive firms find it attractive to shift the production of routine tasks abroad, and higher than in autarky if offshoring becomes common practice among high and low productivity firms.

To round off the analysis in this paper, we have conducted a calibration exercise for our model variant with firm-level rent sharing, in order to quantify the implications of offshoring. Relying on parameter estimates from the empirical trade literature, our model predicts strong positive welfare effects and sizable distributional consequences between entrepreneurs and workers as well
as within the group of entrepreneurs, whereas the consequences of offshoring for unemployment and inequality within the group of workers are moderate. We have also aggregated the different income groups to arrive at a comprehensive measure of economy-wide income inequality and have evaluated this comprehensive measure at the parameter estimates at hand. The insights from this exercise indicate that the movement from autarky to observed patterns of offshoring has significantly increased economy-wide income inequality in the source country.

A Appendix

A.1 Offshoring and welfare in the source country

In the main text, we claim that raising $\chi$ from zero to a small positive level exerts a negative impact on $\Phi(\chi)$. To show this effect, we can differentiate $\Phi(\chi)$. According to (23), this gives

$$\Phi'(\chi) = \frac{\sigma \Phi(\chi)}{1 + (\sigma - 1)\gamma(\chi; \eta)} \left\{ \frac{\sigma - 1}{\sigma} \frac{1 - \xi + (\sigma - 1)\gamma(\chi; \eta)}{\gamma(\chi; \eta)k(\sigma - 1) + k - \xi} \frac{\partial \gamma(\chi; \eta)}{\partial \chi} + \left[ \frac{1}{\sigma} + \frac{\sigma - 1}{\sigma} \gamma(\chi; \eta) \right] \frac{k + \sigma - 1}{k(\sigma - 1) + 1} \right\}. \tag{A.1}$$

Evaluating this derivative at $\chi = 0$ and noting that $\gamma(0; \eta) = \Phi(0) = 1$, further implies

$$\Phi'(0) = \frac{\sigma - 1}{\sigma} \frac{\sigma - \xi}{k(\sigma - 1) + k - \xi} \left[ \lim_{\chi \to 0} \frac{\partial \gamma(\chi; \eta)}{\partial \chi} \right] + \frac{k + \sigma - 1}{k(\sigma - 1)} \tag{A.2}$$

Noting that $\sigma > \xi$ and that $\lim_{\chi \to 0} \frac{\partial \gamma(\chi; \eta)}{\partial \chi} = -\infty$, it follows that $\Phi'(0) < 0$.

In the main text, we also claim that $\Phi'(\chi) > 0$ and $\Phi(\chi) > 1$ hold for sufficiently high levels of $\chi$ if $\eta \geq 1/2$. To show that $\eta \geq 1/2$ implies $\Phi(1) > 1$, it is useful to rewrite (23) as follows

$$\Phi(\chi) = \frac{\left\{ \frac{1 + (\sigma - 1)\gamma(\chi; \eta)}{\sigma [1 + (\sigma - 1)\gamma(\chi; \eta) - \xi / k]} \right\} (1 + \chi)^{-\frac{1}{2}}} {T_1(\chi)} \left[ \frac{(1 + \chi) \frac{\gamma(\chi; \eta)k(\sigma - 1) + k - \xi}{k(\sigma - 1) + k - \xi}} {T_2(\chi)} \right] \left[ \frac{(1 + \chi) \frac{\gamma(\chi; \eta)k(\sigma - 1) + k - \xi}{k(\sigma - 1) + k - \xi}} {T_3(\chi)} \right] \tag{A.3}$$

It is easily confirmed that $T_1(\chi) > 1$, $T_2(\chi) > 1$ hold for any $\chi > 0$. Accordingly, $T_3(1) \geq 1$ is sufficient for $\Phi(1) > 1$. Noting that $T_3(1) >, =, < 1$ is equivalent to $k - \xi >, =, < k(\sigma - 1)(1 - 2\eta)$, therefore implies that $\eta \geq 1/2$ is sufficient for $\Phi(1) > 1$. 

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Finally, to show that $\Phi'(1) > 0$ if $\eta \geq 1/2$, it is useful to consider

$$\Phi'(1) = \frac{(\sigma - 1) (k - \xi) (1 - \eta)}{2k [1 + (\sigma - 1) \eta]} \Phi(1) B, \quad (A.4)$$

with

$$B \equiv \frac{[1 + (\sigma - 1) \eta] (k + \sigma - 1)}{(\sigma - 1)^2 (k - \xi) (1 - \eta) - \frac{1 - \xi + (\sigma - 1) \eta}{\eta k (\sigma - 1) + k - \xi}}$$

$$> \frac{1}{(\sigma - 1) (k - \xi) (1 - \eta)} - \frac{1}{\eta k (\sigma - 1) + k - \xi}, \quad (A.5)$$

according to (A.1). Noting that $(\sigma - 1) [\eta k - (1 - \eta) (k - \xi)] + k - \xi > 0 \ \forall \ \eta \geq 1/2$, therefore implies $B > 0$, and thus $\Phi'(1) > 0$. This completes the proof. $Q.E.D.$

### A.2 Derivation of the Gini coefficient in equation (24)

For characterizing the Gini coefficient in (24), we must distinguish between firms which offshore and those that produce only domestically. Cumulative profits of purely domestic firms with productivity $\bar{\varphi} \in [\varphi^c, \varphi^o]$ are given by $\Psi(\bar{\varphi}) \equiv N \int_{\varphi^c}^{\bar{\varphi}} \pi^d(\varphi)dG(\varphi)$. Considering $\pi^d(\varphi)/\pi^d(\varphi^c) = (\varphi/\varphi^c)^\xi$ from (8), we can solve for

$$\Psi(\bar{\varphi}) = M \pi^d(\varphi^c) \frac{k}{k - \xi} \left[ 1 - \left( \frac{\bar{\varphi}}{\varphi^c} \right)^{\xi - k} \right]. \quad (A.6)$$

Economy-wide profit income is given by $\Psi = M(1 + \chi) [k/(k - \xi)] \pi^d(\varphi^c) - M \chi s$. Accounting for $s = \pi^d(\varphi^c)$ from (9), gives $\Psi = M \pi^d(\varphi^c)(k + \chi \xi)/(k - \xi)$. The share of cumulative profits realised by firms with a productivity level up to $\bar{\varphi} \in [\varphi^c, \varphi^o]$ is therefore given by

$$\frac{\Psi(\bar{\varphi})}{\Psi} = \frac{k}{k + \chi \xi} \left[ 1 - \left( \frac{\bar{\varphi}}{\varphi^c} \right)^{\xi - k} \right]. \quad (A.7)$$

Denoting the fraction of firms with a productivity level $\varphi \leq \bar{\varphi}$ by $\mu \equiv 1 - (\bar{\varphi}/\varphi^c)^{-k}$, equation (A.7) can be rewritten as the first segment of the Lorenz curve for the distribution of profit income:

$$Q^1_M(\mu) = \frac{k}{k + \chi \xi} \left[ 1 - (1 - \mu)^{k - \xi} \right], \quad (A.8)$$

which is relevant for parameter domain $\mu \in [0, 1 - \chi)$.

We now follow the same steps as above to calculate the second segment of the Lorenz curve for the distribution of profit income. We can first note that cumulative profits of all firms with
productivities up to $\bar{\varphi} \in [\varphi^o, \infty)$ are given by $\Psi(\varphi) = \Psi(\varphi^o) + N \int_{\varphi^o}^{\bar{\varphi}} \pi^o(\varphi) dG(\varphi) - N \int_{\varphi^o}^{\bar{\varphi}} s dG(\varphi)$. Accounting for $\pi^d(\varphi)/\pi^d(\varphi^c) = (\varphi/\varphi^c)^\xi$ from (8) and $\pi^o(\varphi)/\pi^d(\varphi) = 1 + \chi^{\xi/k}$, according to (7) and (11), we can calculate

$$\Psi(\bar{\varphi}) = \Psi(\varphi^o) + M \pi^d(\varphi^c) \left\{ \frac{k}{k - \xi} \left[ \chi^{\frac{k - \xi}{k - \xi}} - \left( \frac{\bar{\varphi}}{\varphi^c} \right)^{\xi-k} \right] - \chi - \left( \frac{\bar{\varphi}}{\varphi^c} \right)^{-k} \right\}. \quad (A.9)$$

Dividing the latter by economy-wide profit income $\Psi$ gives the share of profit income accruing to entrepreneurs with an ability up to $\bar{\varphi} \in [\varphi^o, \infty)$:

$$\frac{\Psi(\bar{\varphi})}{\Psi} = Q^1_M (1 - \chi) + \frac{k - \xi}{k + \chi^\xi} \left\{ \frac{k}{k - \xi} \left[ \chi^{\frac{k - \xi}{k - \xi}} - \left( \frac{\bar{\varphi}}{\varphi^c} \right)^{\xi-k} \right] - \chi - \left( \frac{\bar{\varphi}}{\varphi^c} \right)^{-k} \right\}. \quad (A.10)$$

Substituting $\mu$ from above, (A.10) can be reformulated to the second segment of the Lorenz curve, which is relevant for $\mu \in (1 - \chi, 1)$

$$Q^2_M(\mu) = Q^1_M (1 - \chi) + \frac{k}{k + \chi^\xi} \left[ \chi \left( \frac{k + \xi}{k - \xi} \right) - \chi + \frac{k - \xi}{k - \xi} \left( \frac{\bar{\varphi}}{\varphi^c} \right)^{\xi-k} \right], \quad (A.11)$$

with $Q^2_M(1 - \chi) = Q^1_M (1 - \chi)$. Together Eqs. (A.8) and (A.11) form the Lorenz curve.\footnote{The Lorenz curve in (A.12) has the usual properties: $Q_M(0) = 0$, $Q_M(1) = 1$ and $Q'_M(\mu) > 0 \forall \mu \in (0, 1)$.}

$$Q_M(\mu) \equiv \begin{cases} Q^1_M(\mu) & \text{if } \mu \in [0, 1 - \chi) \\ Q^2_M(\mu) & \text{if } \mu \in [1 - \chi, 1] \end{cases}. \quad (A.12)$$

The Gini coefficient for the distribution of profit income in (24) can then be calculated according to $A_M(\chi) \equiv 1 - 2 \int_0^1 Q_M(\mu) d\mu$. QED

**A.3 Derivation of Eq. (27)**

Adding up domestic employment over all purely domestic and offshoring firms in the source country gives $(1 - U) L = N \left[ \int_{\varphi^c}^{\varphi^o} t^d(\varphi) dG(\varphi) + \int_{\varphi^c}^{\infty} t^o(\varphi) dG(\varphi) \right]$. Using $t^d(\varphi)/t^d(\varphi^c) = (\varphi/\varphi^c)^{(1-\theta)\xi}$ and $t^o(\varphi)/t^d(\varphi^c) = \eta^{\kappa(\sigma-1)(1-\theta)} (\varphi/\varphi^c)^{(1-\theta)\xi}$, according to (6), (8), the equivalent of (21) for the scenario with $\theta > 0$, and (26), and accounting for the definition of $\beta(\chi; \eta)$ in (28), we can calculate

$$(1 - U) L = t^d(\varphi^c) \beta(\chi; \eta) \frac{k}{k - (1 - \theta)\xi}. \quad (A.13)$$
Furthermore, combining (9'), (12), (14) and noting that constant markup pricing implies \((\sigma - 1)\pi(\varphi^c) = l^d(\varphi^c)w^d(\varphi^c)\), we can express the total wage bill in the source country as follows:

\[(1 - U)L\tilde{w} = l^d(\varphi^c)w^d(\varphi^c)\frac{\alpha(\chi; \eta)}{k - \xi}. \tag{A.14}\]

Together (A.13) and (A.14) determine the wage ratio \(w(\varphi^c)/\tilde{w} = \Delta(\chi; \eta)(k - \xi)/(k - (1 - \theta)\xi)\), where \(\Delta(\chi; \eta) = \beta(\chi; \eta)/\alpha(\chi; \eta)\) has been considered. Applying the fair-wage constraint (26) for the marginal firm and considering indifference condition (9), we can compute \(U = 1 - w(\varphi^c)/\tilde{w}\). Substituting for \(w(\varphi^c)/\tilde{w}\), then gives (27). \(QED\)

### A.4 Offshoring and unemployment rate \(u\)

Let us first consider the impact of offshoring on \(U\). From Eqs. (13) and (28), we can conclude that, for all \(\chi \in (0, 1]\), \(\Delta(\chi; \eta) > , =, < 1\) is equivalent to \(\Omega(\zeta) \equiv (\eta\zeta^{1-\theta} - 1)(\zeta - 1)^\theta - (\eta\zeta - 1) > , =, < 0\), with \(\zeta \equiv 1 + \chi^{\xi/k} \in (1, 2]\). Differentiating \(\Omega(\zeta)\) gives \(\Omega'(\zeta) = -\eta[1 - (1 - \theta)\zeta^{-\theta}(\zeta - 1)\theta] + \theta(\zeta - 1)\theta - 1 (\eta\zeta^{1-\theta} - 1)\) and \(\Omega''(\zeta) = \theta(1 - \theta)(\zeta - 1)\theta - 2[1 - \eta/\zeta^{1+\theta}]\). Accounting for \(\Omega''(\zeta) > 0\) and \(\Omega'(2) = -\eta(1 - 2^{-\theta}) - \theta(1 - 2^{2-\theta}) < 0\), it follows that \(\Omega'(\zeta) < 0\) must hold for all \(\zeta \in (1, 2]\).

Noting finally that \(\Omega(1) = 1 - \eta > 0\) and \(\Omega(2) = -2\eta[1 - (1/2)^\theta] < 0\), we can define a unique \(\hat{\chi} \in (0, 1)\), such that offshoring lowers \(U\) if \(\chi < \hat{\chi}\), while it raises \(U\) if \(\chi > \hat{\chi}\).

From inspection of (29) we can note that \(\Lambda > 1\) requires \(\Delta < 1\) and thus a positive effect of offshoring on \(U\). This implies that \(\Lambda(\chi; \eta) > 1\) can only materialize if \(\chi > \hat{\chi}\). Furthermore, it is worth noting that partially differentiating \(\Delta(\chi; \eta)\) with respect to \(\eta\) gives

\[
\frac{\partial \Delta(\chi; \eta)}{\partial \eta} = -\frac{\chi - \tilde{\xi}}{\alpha(\chi; \eta)^2} \left[ \left( 1 - \chi \frac{\chi - (1 - \theta)\xi}{k} \right) - \left( 1 - \chi \frac{\chi - (1 - \theta)\xi}{k} \right) \frac{\theta \tilde{\xi}^2}{\alpha(\chi; \eta)^2} \right] < 0. \tag{A.15}\]

Additionally accounting for \(\partial \gamma(\chi; \eta)/\partial \eta < 0\) it follows from (29) that \(\partial \Lambda(\chi; \eta)/\partial \eta > 0\). We finally show that \(\eta \geq 1/2\) is sufficient for \(\Lambda(1; \eta) > 1\). For this purpose, we can note that \(\lambda(1; 1/2) > , =, < 0\) if \(\hat{\Lambda}(\theta) \equiv (1 - 2^{-\theta})(k\sigma - \tilde{\xi}) - \theta\tilde{\xi} > , =, < 0\). In view of \(\partial \lambda(1, \eta)/\partial \eta > 0\), this implies that \(\hat{\Lambda}(\theta) \geq 0\) is sufficient for \(\Lambda(1; \eta) > 0\) if \(\eta \geq 1/2\). Differentiating \(\hat{\Lambda}(\theta)\) gives \(\hat{\Lambda}'(\theta) = 2^{-\theta} \ln(2(k\sigma - \tilde{\xi}) + \tilde{\xi}^2[1 - 2^{-\theta} - \theta + 1 - 1/(\sigma - 1)]\). It is noteworthy that \(\tilde{\xi} \geq 1\) is equivalent to \(1 \geq \theta + 1/(\sigma - 1)\), and hence implies \(\hat{\Lambda}'(\theta) > 0\). Accounting for \(\hat{\Lambda}(0) = 0\), we can thus conclude that \(\hat{\Lambda}(\theta)\) holds for all \(\theta\)-levels that are consistent with \(\tilde{\xi} \geq 1\). But what happens if \(\tilde{\xi} < 1\)? In
this case, $\hat{\Lambda}(\theta)$ might have an extremum at some $\hat{\theta} \in (0,1)$. Evaluating the second derivative of $\hat{\Lambda}(\theta)$ at $\hat{\theta}$, we obtain $\hat{\Lambda}''(\hat{\theta}) = -2^{1-\hat{\theta}} \ln 2(\sigma - \bar{\xi}) - \bar{\xi}^2 - 2^{-\hat{\theta}} \ln 2(\sigma - \bar{\xi})$. Accounting for $\sigma > 1 + \bar{\xi}$, we get $k\sigma - \bar{\xi} > k(1 - \bar{\xi})\bar{\xi}$ and thus $\hat{\Lambda}''(\hat{\theta}) < 0$. Hence, if $\hat{\Lambda}(\theta)$ has an extremum, this extremum must be a maximum. Noting finally that $\hat{\Lambda}(0) = 0$ and $\hat{\Lambda}'(0) > 0$, it follows that $\hat{\Lambda}(1) = (2\sigma)^{-1} \left[ k\sigma^2 - 3(\sigma - 1) \right] > 0$ is sufficient for $\hat{\Lambda}(\theta) \geq 0$. Recollecting from above that $\Lambda(\chi; \eta) < 1$ holds if $\chi \leq \hat{\chi}$, $u/u^a$ must be non-monotonic in $\chi$ if $\eta \geq 1/2$, because in this case, we have $\Lambda(1; \eta) \geq 1$. This completes the proof. QED

A.5 Derivation of the Gini coefficient in equation (30)

To characterize the Gini coefficient for the distribution of wage income we must distinguish workers employed in purely domestic firms from those employed in offshoring firms. Cumulative labour income of workers employed in purely domestic firms with a productivity level up to $\bar{\phi} \in [\phi^c, \phi^o]$ is given by $W(\bar{\phi}) \equiv N \int_{\phi^c}^{\phi^o} w(\phi)l(\phi)dG(\phi)$. Since constant markup pricing implies that a firm’s wage bill is proportional to its revenues, we can make use of $w^d(\phi)l^d(\phi) = (\sigma - 1)\pi^d(\phi)$. Considering $\pi^d(\phi)/\pi^d(\phi^c) = (\phi/\phi^c)^{\bar{\xi}}$ from (8), then implies

$$W(\bar{\phi}) = (\sigma - 1)M\pi^d(\phi) \left[ 1 - \left( \frac{\phi}{\phi^c} \right)^{\bar{\xi}} \right].$$

(A.16)

Total economy-wide labour income equals $W = \rho\gamma(\chi; \eta)Y$. Using the definition of $\gamma(\chi; \eta)$, (13) and (14), we obtain $W = (\sigma - 1)M\pi^d(\phi^c)\left[ k/(k - \bar{\xi}) \right]\alpha(\chi; \eta)$. Hence, the share of wage income accruing to workers, who are employed in firms with a productivity level up to $\bar{\phi} \in [\phi^c, \phi^o]$, can be expressed as

$$\frac{W(\bar{\phi})}{W} = \frac{1}{\alpha(\chi; \eta)} \left[ 1 - \left( \frac{\phi}{\phi^c} \right)^{\bar{\xi}} \right].$$

(A.17)

To calculate the Lorenz curve for the distribution of labour income, we must link the income ratio in (A.17) to the respective employment ratio. For this purpose, we first note that total employment in all firms with a productivity level up to $\bar{\phi} \in [\phi^c, \phi^o]$ is given by $L(\bar{\phi}) \equiv N \int_{\phi^c}^{\phi^o} l^d(\phi)dG(\phi)$.

Noting that expression $k\sigma^2 - 3(\sigma - 1)$ has an unique minimum at $\sigma = (3/2)k^{-1}$, we can conclude that $\hat{\Lambda}(1) \geq \left[ 3 - (3/2)k^{-1} \right]/(2\sigma) > 0$. 

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Substituting \( l^d(\varphi)/l^d(\varphi^c) = (\varphi/\varphi^c)^{(1-\theta)}\xi \), we can calculate
\[
L(\bar{\varphi}) = Ml^d(\varphi^c) \frac{k}{k - (1 - \theta)\xi} \left[ 1 - \left( \frac{\varphi}{\varphi^c} \right)^{(1-\theta)\xi-k} \right].
\] (A.18)

In a similar vein, we can show that economy-wide employment of production workers in the source country equals \((1-U)L = Ml^d(\varphi^c)\beta(\chi;\eta)k/[k-(1-\theta)\xi]\). Hence, the share of production workers that are employed in firms with a productivity level up to \(\bar{\varphi} \in [\varphi^c, \varphi^o]\) is given by \(\lambda = \beta(\chi;\eta)^{-1}[1-(\bar{\varphi}/\varphi^c)^{(1-\theta)\xi-k}]\). Combining the latter with (A.17), we obtain the first segment of the Lorenz curve for the distribution of labour income
\[
Q^1_L(\lambda) = \frac{1 - [1 - \beta(\chi;\eta)\lambda]^{k-\xi}}{\alpha(\chi;\eta)},
\] (A.19)

which is relevant if \(\lambda \in [0, b_L]\), with \(b_L \equiv \beta(\chi;\eta)^{-1}(1 - \chi^{1-(1-\theta)\xi/k})\).

We now follow the same steps as above to calculate the second segment of the Lorenz curve. We first compute the total domestic wage bill of firms with a productivity level up to \(\bar{\varphi} \in [\varphi^o, \infty)\). This gives \(W(\bar{\varphi}) \equiv W(\varphi^o) + N \int_{\bar{\varphi}}^{\infty} w^o(\varphi)l^o(\varphi)dG(\varphi)\). Accounting for \(w^o(\varphi)l^o(\varphi) = \eta(\sigma-1)\pi^o(\varphi)\) and considering \(\pi^d(\varphi)/\pi^d(\varphi^c) = (\varphi/\varphi^c)^\xi\) from (8) as well as \(\pi^o(\varphi)/\pi^d(\varphi) = 1 + \chi^{\xi/k}\), according to (7) and (11), we can calculate
\[
W(\bar{\varphi}) = W(\varphi^o) + M\pi^d(\varphi^c)\frac{k(\sigma - 1)}{k-\xi} \eta \left( 1 + \chi^{\xi/k} \right) \left[ \frac{k-\xi}{k-\xi} - \left( \frac{\varphi}{\varphi^c} \right)^{\xi-k} \right].
\] (A.20)

Dividing (A.20) by economy-wide labour income \(W\), yields
\[
\frac{W(\bar{\varphi})}{W} = 1 - \frac{\eta \left( 1 + \chi^{\xi/k} \right)}{\alpha(\chi;\eta)} \left( \frac{\varphi}{\varphi^c} \right)^{\xi-k}.
\] (A.21)

The mass of domestic workers employed by firms with a productivity level up to \(\bar{\varphi} \in [\varphi^o, \infty)\) is given by \(L(\bar{\varphi}) = L(\varphi^o) + N \int_{\bar{\varphi}}^{\infty} l^o(\varphi)dG(\varphi)\). Accounting for \(l^o(\varphi)/l^d(\varphi) = \eta\kappa(\sigma-1)(1-\theta) = \eta(1 + \chi^{\xi/k})^{1-\theta}\) and \(l^d(\varphi)/l^d(\varphi^c) = (\varphi/\varphi^c)^{(1-\theta)\xi}\), we can further write
\[
L(\bar{\varphi}) = L(\varphi^o) + Ml^d(\varphi^c) \frac{k\eta \left( 1 + \chi^{\xi/k} \right)^{1-\theta}}{k - (1 - \theta)\xi} \left[ \frac{k-(1-\theta)\xi}{k-\xi} - \left( \frac{\varphi}{\varphi^c} \right)^{(1-\theta)\xi-k} \right].
\] (A.22)

Dividing \(L(\bar{\varphi})\) by economy-wide employment \((1-U)L\), then gives \(\lambda = 1 - \eta\beta(\chi;\eta)^{-1}(1 + \chi^{\xi/k})^{1-\theta}(\bar{\varphi}/\varphi^c)^{(1-\theta)\xi-k}\). Solving the latter for \(\bar{\varphi}/\varphi^c\) and substituting the resulting expression
into (A.21), we obtain the second segment of the Lorenz curve

\[
Q_L^2(\lambda) = 1 - \frac{\eta \left(1 + \chi \tilde{\xi}\right)}{\alpha(\chi; \eta)} \left[\frac{(1 - \lambda)\beta(\chi; \eta)}{\eta \left(1 + \chi \tilde{\xi}\right)^{1-\theta}}\right]^\frac{k - \tilde{\xi}}{k - (1 - \theta)\xi},
\]

(A.23)

which is relevant if \( \varphi \in [\varphi^o, \infty) \). Together (A.19) and (A.23) form the Lorenz curve

\[
Q_L(\lambda) \equiv \begin{cases} 
Q_L^1(\lambda) & \text{if } \lambda \in [0, b_L) \\
Q_L^2(\lambda) & \text{if } \lambda \in [b_L, 1]
\end{cases}.
\]

(A.24)

The Gini coefficient for the distribution of labour income in (30) can then be calculated according to

\[
A_L(\chi) \equiv 1 - 2 \int_0^1 Q_L(\lambda) d\lambda.
\]

QED

A.6 The properties of Gini coefficient \( A_L \)

From the definitions of \( \alpha(\chi; \eta) \), \( \beta(\chi; \eta) \) and inspection of (30), it follows that \( A_L(1) = A_L(0) \). Furthermore, if \( \chi \in (0, 1) \), the sign of \( A_L(\chi) - A_L(0) \) is equivalent to the sign of

\[
\delta(\chi; \eta) \equiv \frac{k - \tilde{\xi}}{k - (1 - \theta)\xi} \left(1 - \chi \frac{k - (1 - \theta)\xi}{k}\right) \left[\eta \left(1 + \chi \tilde{\xi}\right) - 1\right]
\]

\[
- \left(1 - \chi \frac{k - \tilde{\xi}}{k}\right) \chi^{\frac{\theta}{\tilde{\xi}}} \left[\eta \left(1 + \chi \tilde{\xi}\right)^{1-\theta} - 1\right].
\]

(A.25)

Noting further that

\[
\frac{k - \tilde{\xi}}{k - (1 - \theta)\xi} \left(1 - \chi \frac{k - (1 - \theta)\xi}{k}\right) > \left(1 - \chi \frac{k - \tilde{\xi}}{k}\right) \chi^{\frac{\theta}{\tilde{\xi}}}.
\]

(A.26)

holds for any possible \( \chi \in (0, 1) \), it is straightforward to show that \( \delta(\chi; \eta) > 0 \) must hold if \( \eta(1 + \chi \tilde{\xi}/k) \geq 1 \), or, equivalently, if \( \chi \geq [(1 - \eta)/\eta]^{k/\tilde{\xi}} \).

But what is the sign of \( \delta(\chi; \eta) \) if \( \chi < [(1 - \eta)/\eta]^{k/\tilde{\xi}} \)? Or, more specifically, since we know that \( \chi \) must be smaller than one: What is the sign of \( \delta(\chi; \eta) \) if \( \chi < \bar{\chi} \), where \( \bar{\chi} \equiv \min\{[(1 - \eta)/\eta]^{k/\tilde{\xi}}, 1\} \) and thus \( \bar{\chi} = 1 \) if \( \eta \leq 1/2 \), while \( \bar{\chi} = [(1 - \eta)/\eta]^{k/\tilde{\xi}} \) if \( \eta > 1/2 \)? To answer this question, we can first note that if \( \chi < \bar{\chi} \), condition \( \delta(\chi; \eta) > =, < 0 \) is equivalent to

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31The Lorenz curve in (A.24) has the usual properties: \( Q_L(0) = 0, Q_L(1) = 1 \) and \( Q_L^\prime(\lambda) > 0 \forall \lambda \in (0, 1) \)
condition $\delta_0(\chi; \eta) >, =, < \delta_1(\chi)$, with

\[
\delta_0(\chi; \eta) \equiv \frac{1 - \eta \left(1 + \chi^{\frac{k}{k}}\right)^{1-\theta}}{1 - \eta \left(1 + \chi^{\frac{k}{k}}\right)}, \quad \delta_1(\chi) \equiv \frac{k - \bar{\xi}}{k - (1 - \theta)\bar{\xi}} \left(1 - \frac{k - \bar{\xi}}{1 - \chi^{\frac{k}{k}}}\right) \chi^{\frac{\alpha}{\theta}}. \tag{A.27}
\]

It is easily confirmed $\delta_0(\chi; \eta)$ increases in $\chi$ over the relevant interval, reaching a minimum function value of $\delta_0(0; \eta) = 1$ at $\chi = 0$. Accordingly, $\delta_0(\chi; \eta)$ reaches a maximum function value at $\tilde{\chi}$. This maximum function value is given by $(1 - 2^{1-\theta}\eta)/(1 - 2\eta)$ if $\eta \leq 1/2$, while it equals $\infty$ if $\eta > 1/2$. In a similar way, we can show that $\delta_1(\chi)$ is decreasing in $\chi$, reaching a maximum function value of $\infty$ at $\chi = 0$ and a minimum function value of $1$ at $\chi = 1$. Putting together, this implies that there exists a unique $\tilde{\chi}(\eta)$ such that $\delta_0(\chi; \eta) >, =, < \delta_1(\chi)$ and thus $\delta(\chi; \eta) >, =, < 0$ if $\chi >, =, < \tilde{\chi}(\eta)$. Finally, accounting for $\partial \delta_0(\chi; \eta)/\partial \eta > 0$, it follows that $\tilde{\chi}(\eta)$ shrinks in $\eta$ and reaches a minimum value of $0$ at $\eta = 1$. In this case $\delta(\chi; 0) > 0$ holds for any $\chi \in (0, 1)$. Furthermore, $\tilde{\chi}(\eta)$ reaches a maximum value of $1$ at $\eta = 0$, implying that in this case $\delta(\chi; 0) < 0$ must hold for any $\chi \in (0, 1)$. This completes the formal discussion of the properties of $A_L$. QED

References


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Supplement
(Not intended for publication)

If not stated differently, the formal analysis in this supplement refers to the sophisticated model variant with $\theta > 0$.

Derivation details for the model variant with $\theta > 0$

In this subsection, we show in detail how the equations in Section 2 must be modified, when allowing for rent sharing between workers and firms. The first equation that has to be modified is Eq. (4). With rent sharing wages are firm-specific, and hence we can rewrite unit production costs as follows:

$$c(v) = \frac{\omega^n(v)}{\varphi(v)z(v)}$$

with

$$z(v) \equiv \left[ \frac{\omega^r(v)}{\omega^o(v)} \right]^{1-\eta}, \quad (S.1)$$

where $\omega^r(v)$ is the domestic wage rate paid by firm $v$ to workers conducting non-routine tasks. Thereby, we have $\omega^r(v) = w^d(v)$ if the firm produces all tasks at home, while we have $\omega^r(v) = w^o(v)$ if routine tasks are produced offshore. As in Section 2, we have $z(v) = 1$ and thus $c^d(v) = w^d(v)/\varphi$ if the firm produces purely domestically. For an offshoring firm, we obtain $z(v) = z^o(v)$ and, instead of (5),

$$c^o(v) = \frac{w^o(v)}{\varphi(v)z^o(v)},$$

where

$$z^o(v) \equiv \left[ \frac{w^o(v)}{\tau w^*(v)} \right]^{1-\eta} = \left[ \frac{(1-U) \bar{w}}{(1-U^*)\bar{w}^*} \right]^{(1-\eta)(1-\theta) - 1}. \quad (S.2)$$

Thereby, we have made use of the fair-wage constraint in (26) in order to substitute for $w^o(v)/w^*(v)$.

Combining (6) and (26), we can furthermore compute

$$\frac{\pi^o(v)}{\pi^d(v)} = \kappa \bar{\xi} \quad \text{and} \quad \frac{w^o(v)}{w^d(v)} = \kappa^\theta \bar{\xi}, \quad (S.3)$$

where

$$\kappa \equiv \frac{c^o(v)}{c^d(v)} = \left\{ \frac{(1-U) \bar{w}}{(1-U^*)\bar{w}^*} \right\}^{(1-\eta)(1-\theta) - 1}. \quad (S.4)$$

Using (8) and (S.3) in indifference condition (10), and accounting for $\pi^d(\varphi^r) = s$ from (9'), we can easily verify that the link between $\chi$ and $\kappa$ remains to be given by (11), with $\bar{\xi}$ assuming
the role of $\xi$ if $\theta > 0$. Labour income per capita in the source and host country are given by

$$
(1 - U)\bar{w} = \frac{\gamma(\chi; \eta)\rho Y}{L} \quad \text{and} \quad (1 - U^*)\bar{w}^* = \frac{[1 - \gamma(\chi; \eta)]\rho Y}{N^*},
$$

(S.5)

respectively. Substituting (S.5) into (S.4) allows us to compute

$$
\kappa = \left[ \frac{\gamma(\chi; \eta) k\rho \sigma + k - \bar{\xi}}{1 - \gamma(\chi; \eta)} \left( \frac{N^*}{N} \right) \right]^{(1-\eta)(1-\theta)} \eta - 1
$$

(S.6)

And combining (11) and (S.6) we can conclude that the relationship between $\kappa$ and $\chi$ in the model variant with $\theta > 0$ is characterized by the implicit function

$$
F(\chi, \tau) \equiv \left[ \left( \frac{\gamma(\chi; \eta) k\rho \sigma + k - \bar{\xi}}{1 - \gamma(\chi; \eta)} \left( \frac{N^*}{N} \right) \right) \right]^{(1-\eta)(1-\theta)} \eta - 1 - \left( 1 + \frac{\bar{\xi}}{k} \right) = 0.
$$

This completes our discussion on how rent sharing affects the equations reported in Section 2.

QED

Existence of involuntary unemployment in an offshoring equilibrium

Involuntary unemployment exists in our model, if $U > 0$. To show that this is the case in any offshoring equilibrium with $\chi \in (0, 1)$, we must prove that $\Delta(\chi; \eta) = [k - (1 - \theta)\bar{\xi}]/(k - \bar{\xi})$ holds in the relevant parameter domain (see (27)). Partially differentiating $\Delta(\cdot)$ with respect to $\eta$, gives $\partial\Delta(\cdot)/\partial\eta < 0$, according to (A.15), and we can therefore safely conclude that $\Delta(\chi; \eta) \leq \Delta(\chi; 0)$ holds for any $\eta > 0$. Accounting for

$$
\Delta(\chi; 0) = \frac{1 - \chi}{1 - \chi \frac{k - (1 - \theta)\bar{\xi}}{k}} \equiv \hat{\Delta}(\chi),
$$

(S.7)

it then follows that $\hat{\Delta}(\chi) < [k - (1 - \theta)\bar{\xi}]/(k - \bar{\xi})$ is sufficient for involuntary unemployment to exist in equilibrium. Differentiating $\hat{\Delta}(\chi)$ yields $\hat{\Delta}'(\chi) = \chi^{-\frac{\bar{\xi}}{k}}(1 - \chi^{1-\xi/k})^2$, with

$$
\hat{\iota}(\chi) = \frac{k - (1 - \theta)\bar{\xi}}{k} \chi^{-\frac{\bar{\xi}}{k}}(1 - \chi^{1-\xi/k}) + \frac{k - \bar{\xi}}{k} \chi^{-\frac{k - (1 - \theta)\bar{\xi}}{k}}.
$$

(S.8)

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32Rent sharing affects Eqs. (13)-(19) because $\bar{\xi}$ assumes the role of $\xi$ if $\theta > 0$, but there are not additional effects.
Noting that \( i(0) > 0, i(1) = 0 \), and \( i'(\chi) < 0 \), we obtain \( i'(\chi) > 0 \), implying that \( i(\chi) < i(1) \) must hold for any \( \chi \in [0,1) \). Noting finally that \( \lim_{\chi \to 1} i(\chi) = |k - (1 - \theta)\xi|/(k - \xi) \), we can safely conclude that there must be involuntary unemployment in the offshoring equilibrium. 

\[ \text{QED} \]

**Further details for the derivation of (30)**

Using the insights from the Appendix, we can note that

\[
\int_{0}^{b_{L}} Q_{L}^{1}(\lambda) d\lambda = \frac{1}{\alpha(\chi; \eta)} \left[ \lambda + (1 - \beta(\chi; \eta)\lambda)^{\frac{2(1 - \xi + \theta\xi)}{2(k - \xi + \theta\xi)\beta(\chi; \eta)}} \right]_{0}^{b_{L}} = \frac{b_{L}}{\alpha(\chi; \eta)} + \frac{k - (1 - \theta)\xi}{2(k - \xi + \theta\xi)} \left[ \frac{\chi^{\frac{2(1 - \xi + \theta\xi)}{k}}}{\alpha(\chi; \eta)\beta(\chi; \eta)} - \frac{1}{\alpha(\chi; \eta)\beta(\chi; \eta)} \right], \tag{S.9}
\]

while

\[
\int_{b_{L}}^{1} Q_{L}^{2}(\lambda) d\lambda = \left. \lambda \right|_{b_{L}}^{1} + \frac{\eta}{\alpha(\chi; \eta)} \left( \frac{1 + \chi^{\frac{k}{k - (1 - \theta)\xi}}}{{\beta(\chi; \eta)}^{\chi^{\frac{k}{k - (1 - \theta)\xi}}} \right) \left. \frac{\chi^{\frac{2(1 - \xi + \theta\xi)}{k}}}{\alpha(\chi; \eta)\beta(\chi; \eta)} \right|_{b_{L}}^{1} = 1 - b_{L} - \frac{\eta^{2}}{\alpha(\chi; \eta)\beta(\chi; \eta)} \left. \frac{k - (1 - \theta)\xi^{2 - \theta}}{2(k - \xi + \theta\xi)} \frac{\chi^{\frac{2(1 - \xi + \theta\xi)}{k}}}{\alpha(\chi; \eta)\beta(\chi; \eta)} \right|_{b_{L}}^{1}. \tag{S.10}
\]

Substituting (S.9) and (S.10) into

\[ A_{L} = 1 - 2 \int_{0}^{b_{L}} Q_{L}^{1}(\lambda) d\lambda - 2 \int_{b_{L}}^{1} Q_{L}^{2}(\lambda) d\lambda, \tag{S.11} \]

we obtain

\[ A_{L} = -1 + 2b_{L} \left. \frac{\alpha(\chi; \eta) - 1}{\alpha(\chi; \eta)} \right|_{b_{L}}^{1} + \frac{2}{\alpha(\chi; \eta)\beta(\chi; \eta)} \left. \frac{k - (1 - \theta)\xi^{2 - \theta}}{2(k - \xi + \theta\xi)} \right|_{b_{L}}^{1} + 2Z(\chi; \eta), \tag{S.12} \]

with

\[ Z(\chi; \eta) \equiv \left. \left( \frac{\eta^{2}}{\alpha(\chi; \eta)\beta(\chi; \eta)} - \frac{1}{\alpha(\chi; \eta)\beta(\chi; \eta)} \right) \frac{k - (1 - \theta)\xi^{2 - \theta}}{2(k - \xi + \theta\xi)} \frac{\chi^{\frac{2(1 - \xi + \theta\xi)}{k}}}{\alpha(\chi; \eta)\beta(\chi; \eta)} \right|_{b_{L}}^{1}. \tag{S.13} \]
Using the definition of $b_L$, we can rewrite $A_L$ in the following way

$$A_L = \frac{\theta \xi}{2(k - \xi) + \theta \xi} + \frac{2 \left[ k - (1 - \theta) \xi \right] [1 - \alpha(\chi; \eta) \beta(\chi; \eta)]}{\alpha(\chi; \eta) \beta(\chi; \eta) \left[ 2(k - \xi) + \theta \xi \right]} + \frac{2 \left( 1 - \frac{k - (1 - \theta) \xi}{k} \right) [\alpha(\chi; \eta) - 1]}{\alpha(\chi; \eta) \beta(\chi; \eta)} + 2Z(\chi; \eta). \quad (S.14)$$

Accounting for (13) and (28), we can show that

$$1 - \alpha(\chi; \eta) \beta(\chi; \eta) = - \left[ \alpha(\chi; \eta) - 1 \right] - \left[ \beta(\chi; \eta) - 1 \right] - \left[ \alpha(\chi; \eta) - 1 \right] \left[ \beta(\chi; \eta) - 1 \right]$$

$$= - \left[ \alpha(\chi; \eta) - 1 \right] \left( 1 - \chi \frac{k - (1 - \theta) \xi}{k} \right) - \left[ \beta(\chi; \eta) - 1 \right] \left( 1 - \chi \frac{k - \xi}{k} \right)$$

$$- \left[ \eta^2 \left( 1 + \chi \frac{\xi}{k} \right)^{2 - \theta} - 1 \right] \chi \frac{2(k - \xi) + \theta \xi}{\xi^2}. \quad (S.15)$$

Substituting (S.15) into (S.14), it is straightforward to calculate (30). QED

A continuum of tasks that differ in offshorability

In this extension, we shed light on the firm-internal margin of offshoring, by considering a continuum of tasks that differ in offshorability, as suggested by Acemoglu and Autor (2011). For this purpose, we replace production function (3) by

$$q(v) = \varphi(v) \exp \int_0^1 \ln \ell(v, \eta) d\eta, \quad (S.16)$$

where $\ell(v, \eta)$ is the input of task $\eta \in [0, 1]$ in the production of $q(v)$. Tasks are symmetric in the labour input they require to be performed and, as in the main text, we impose the additional assumption that one unit of labour must be employed to produce one unit of task $\eta$. However, as in Grossman and Ross-Hansberg (2008), tasks differ in their offshorability and this is captured by an iceberg cost parameter $t$ that is task specific: $t(\eta)$. An intuitive way to interpret parameter $t$ is to think of it as task-specific trade cost parameter, implying that total costs of shipping the output of a task $\eta$, whose production has been moved offshore, back to the source country amounts to $t(\eta) \tau > 1$. To facilitate the analysis, we impose the additional assumption that $t(1) = 1$, $t(0) = \infty$ and $t'(\eta) < 0$. This implies that tasks are ranked according to their offshorability and it allows us to identify a unique firm-specific $\hat{\eta}(v)$, which separates the tasks performed at home, $\eta < \hat{\eta}(v)$, from the tasks performed abroad $\eta \geq \hat{\eta}(v)$. 

S4
Once a firm has decided to engage in offshoring, it is left with two further decisions on how to organize its production, which are taken in two consecutive stages. In stage one, the firm chooses how many tasks to move offshore and sets \( \eta(v) \) accordingly, while in stage two, the firm chooses optimal employment in domestic and offshored tasks. As it is common practice, we solve this two stage problem through backward induction and first determine the profit-maximizing employment levels for a given \( \eta(v) \). For this purpose, we can recollect from the main text that wages paid to domestic and foreign workers are given \( w^o(v) \) and \( w^*(v) \), respectively. We can write labour demand for domestic and foreign task production as follows:

\[
l_n(v) = \int_0^{\eta(v)} \ell_n(v, \eta) \, d\eta = \eta(v) \ell_n(v) \quad \text{and} \quad l_r(v) = \int_{\eta(v)}^1 \ell_r(v, \eta) \, d\eta = \int_{\eta(v)}^1 \ell_r(v) \, d\eta.
\]

Therefore, firm \( v \)'s cost minimisation problem can be expressed as follows:

\[
\min_{l_n(v), l_r(v)} \omega_n(v) l_n(v) + \omega_r(v) l_r(v) \quad \text{s.t.} \quad 1 = \varphi \left[ \frac{l_n(v)}{\eta(v)} \right]^{1-\eta(v)} \left[ \frac{l_r(v)}{1-\eta(v)} \right]^{1-\eta(v)}, \tag{S.17}
\]

where \( \omega_n(v) = w^o(v) \), \( \omega_r(v) = \tau w^*(v) \) hold according to the main text and

\[
\epsilon[\eta(v)] \equiv \frac{1 - \eta(v)}{\int_{\eta(v)}^1 t(\eta) \, d\eta}, \tag{S.18}
\]

reflects the average productivity loss arising from the extra labour costs \( t(\eta) \), when producing a task abroad. Solving maximisation problem (S.17) gives unit production costs \( c(v) = w^o(v) / \left[ \varphi(v) \tilde{z}(v) \right] \), where

\[
\tilde{z}(v) \equiv \left\{ \frac{w^o(v)}{w^*(v)} \epsilon[\eta(v)] \right\}^{1-\eta(v)} \tag{S.19}
\]

At stage one, the firm sets \( \eta(v) \) to minimise its unit costs \( c(v) \). Thus, for the optimal \( \eta(v) \)-level the following first-order condition must hold: \( \partial c(v) / \partial \eta(v) = 0 \). In view of (S.18) and

\[\text{As in the main text, we define } l'(v) \text{ such that foreign labour demand of offshoring firm } v \text{ is given by } \tau l'(v). \text{ While this definition of } l'(v) \text{ might seem awkward at a first glance, it is useful for our purpose because it allows us to directly compare the production technology in (S.17) with the respective technology in (3).}

\[\text{It is notable that } \tilde{z}(v) \text{ degenerates to } z(v), \text{ when considering a discrete offshoring technology, with}
\]

\[
t(\eta) = \begin{cases} \infty & \forall \eta \in [0, \tilde{\eta}) \\ 1 & \forall \eta \in [\tilde{\eta}, 1] \end{cases}.
\]
\[\frac{\partial \ln \tilde{z}(v)}{\partial \tilde{\eta}(v)} = -\ln \left( \frac{\omega_0(v)}{\omega^*(v)\tau} \frac{\epsilon[\tilde{\eta}(v)]}{\tau} \right) + t[\tilde{\eta}(v)]\epsilon[\tilde{\eta}(v)] - 1 \equiv 0. \quad (S.20)\]

Acknowledging Eq. (26) in the main text, we know that \(\omega_0(v)/\omega^*(v)\) is the same for all producers, and hence Eq. (S.20) determines the same cost-minimising \(\tilde{\eta}\) for all firms, which we denote by \(\tilde{\eta}_0\). Since the second-order condition of the stage one cost-minimisation problem requires \(\partial^2 \ln \tilde{z}(v)/\partial \tilde{\eta}(v)^2 < 0\), while \(\partial^2 \ln \tilde{z}(v)/\partial \tilde{\eta}(v)\partial \tau > 0\) follows from inspection of (S.20), we can finally conclude that \(d\tilde{\eta}_0/d\tau > 0\), and hence firms offshore a lower share of tasks if the costs of shipping foreign output back to the source country increase. This completes our formal discussion. \textit{QED}

\textbf{Alternative measures of income inequality}

While in the main text, we focus on the distribution of wage and entrepreneurial income, we now take a somewhat broader perspective and add those who do not have a job and those who become self-employed as service agents to our picture of inequality. For this purpose, we extend the Gini coefficients from Sections 3 and 4, and define a Gini coefficient for the income distribution of \textit{all} production workers (including those who do not have a job) as well as a Gini coefficient for the income distribution of self-employed agents (including entrepreneurs as well as freelance agents).

\textbf{Income inequality among self-employed agents}

To characterize income inequality among \textit{all} self-employed agents, we rely on the Lorenz curve for this income group, which now has three segments.\(^{35}\) The first segment captures the share of income attributed to freelance offshoring service providers. It is given by \(Q^0_\xi(\mu) = [(k - \xi)/k]\mu\) and relevant for all \(\mu \in [0, \chi/(1 + \chi)]\). The second segment of the Lorenz curve captures income of freelance offshoring agents plus cumulative profits of purely domestic firms with a productivity

\(^{35}\)In this subsection, we consider the basic model variant without rent sharing. The respective results for the model variant with rent sharing are obtained when replacing \(\xi\) by \(\tilde{\xi}\).
level up to $\bar{\varphi} \in [\varphi^c, \varphi^o)$. Following the derivation steps in Appendix A.2, this gives

$$\hat{\Psi}(\bar{\varphi}) = M\pi^d(\varphi^c) \left\{ \chi + \frac{k}{k - \xi} \left[ 1 - \left( \frac{\bar{\varphi}}{\varphi^c} \right)^{\xi-k} \right] \right \}. \quad (S.21)$$

Economy-wide profits plus service fees add up to total operating profits $\hat{\Psi} = M\pi^d(\varphi^c)(1 + \chi) \left[ k/(k - \xi) \right]$. Hence, the cumulative share of (profit income) realised by offshoring agents and firms with a productivity level up to $\bar{\varphi} \in [\varphi^c, \varphi^o)$ is given by

$$\hat{\Psi}(\bar{\varphi}) = M\pi^d(\varphi^c) \left\{ \chi + \frac{k}{k - \xi} \left[ 1 - \left( \frac{\bar{\varphi}}{\varphi^c} \right)^{\xi-k} \right] \right \}. \quad (S.22)$$

We have to link (S.22) with the ratio of self-employed agents receiving the respective income share. Denoting the fraction of these agents by $\mu \equiv (1 + \chi)^{-1} \left[ \chi + 1 - (\bar{\varphi}/\varphi^c)^{-k} \right]$, Eq. (S.22) can be reformulated to the second segment of the Lorenz curve

$$Q^1_S(\mu) = \frac{k - \xi}{k} \frac{\chi}{1 + \chi} + \frac{1}{1 + \chi} \left\{ 1 - \left[ (1 + \chi)(1 - \mu) \right] \frac{k - \xi}{k} \right\}, \quad (S.23)$$

which is relevant for parameter domain $\mu \in [\chi/(1 + \chi), 1/(1 + \chi))$.

In a final step, we compute the cumulative income of all freelance offshoring agents and entrepreneurs with an ability up to $\bar{\varphi} \in [\varphi^o, \infty)$ as a share of the total income of self-employed agents, $\hat{\Psi}$. Substituting $\mu$ from above, this gives the third segment of the Lorenz curve

$$Q^2_S(\mu) = Q^1_S \left( \frac{1}{1 + \chi} \right) + \frac{1}{1 + \chi} \left\{ (1 + \chi)^{\xi/k} \left[ \frac{k - \xi}{k} - [(1 + \chi)(1 - \mu)] \frac{k - \xi}{k} \right] \right. - \frac{k - \xi}{k} \left[ \chi - (1 + \chi)(1 - \mu) \right] \right\}. \quad (S.24)$$

Putting the three segments together, we obtain the new Lorenz curve

$$Q^S_S(\mu) \equiv \begin{cases} Q^0_S(\mu) & \text{if } \mu \in [0, \frac{\chi}{1 + \chi}) \\ Q^1_S(\mu) & \text{if } \mu \in \left[ \frac{\chi}{1 + \chi}, \frac{1}{1 + \chi} \right) \\ Q^2_S(\mu) & \text{if } \mu \in \left[ \frac{1}{1 + \chi}, 1 \right] \end{cases}. \quad (S.25)$$

The Gini coefficient for the distribution of income among self-employed agents can then be calculated according to $A_S(\chi) \equiv 1 - 2 \int_0^1 Q^S_S(\mu) d\mu$. Substituting (S.25), we can compute (31). QED
Income inequality among employed and unemployed production workers

To characterize income inequality among all production workers, we rely on the Lorenz curve for labour income. Since this Lorenz curve now also captures unemployed individuals, it consists of three segments. The first segment represents the share of income attributed to those who do not have a job. Abstracting from unemployment compensation, it is clear that the income share of this group is zero, and we can thus note that the respective Lorenz curve segment is given by \( Q_U^0(\lambda) = 0 \) and relevant for all \( \lambda \in [0, U) \).

To calculate the second segment of the Lorenz curve, we follow the steps in Appendix A.5 and combine the labour income share of workers employed in purely domestic firms with a productivity level up to \( \bar{\varphi} \in [\varphi^c, \varphi^o] \) – as determined by Eq. (A.17) – with the share of all production workers who are either unemployed or employed in firms up to productivity \( \bar{\varphi} \):

\[
\lambda = U + \frac{1 - U}{\beta(\chi; \eta)} \left[ 1 - \frac{1 - (\bar{\varphi}/\varphi^c)^{(1-\theta)\xi-k}}{1 - U} \right].
\]  

(S.26)

This gives the second segment of the Lorenz curve for the distribution of labour income

\[
Q_U^1(\lambda) = 1 - \frac{1 - \beta(\chi; \eta)}{\alpha(\chi; \eta)} \frac{1 - (\bar{\varphi}/\varphi^c)^{(1-\theta)\xi-k}}{1 - U}
\]  

(S.27)

which is relevant for \( \lambda \in [U, b_U) \), with \( b_U = \frac{1 - U}{\beta(\chi; \eta)} \left( 1 - \chi^{1-(1-\theta)\xi/k} \right) \).

To determine the third segment of the Lorenz curve, we compute the share of total domestic labour income accruing to workers who are either unemployed or employed in firms with a productivity level up to \( \bar{\varphi} \in [\varphi^o, \infty) \) – as represented by (A.21) – with the share of production workers who are either unemployed or employed by these firms:

\[
\lambda = 1 - \eta \left( 1 + \chi^{\xi} \right)^{1-\theta} \frac{1 - U}{\beta(\chi; \eta)} \left( \bar{\varphi}/\varphi^c \right)^{(1-\theta)\xi-k}.
\]  

(S.28)

This allows us to calculate the third segment of the Lorenz curve

\[
Q_U^2(\lambda) = \frac{\eta \left( 1 + \chi^{\xi} \right)^{1-\theta}}{\alpha(\chi; \eta)} \left[ \frac{1 - \lambda}{1 - U} \frac{\beta(\chi; \eta)}{\eta \left( 1 + \chi^{\xi} \right)^{1-\theta}} \right]^{\frac{k - k}{\xi - (1-\theta)\xi}}.
\]  

(S.29)
which is relevant if $\lambda > b_U$. Putting the three segments together, gives the (extended) Lorenz curve for labour income distribution

$$Q_U(\lambda) = \begin{cases} 0 & \text{if } \lambda \in [0, U) \\ Q^1_U(\lambda) & \text{if } \lambda \in [U, b_U) \\ Q^2_U(\lambda) & \text{if } \lambda \in [b_U, 1] \end{cases} \quad \text{(S.30)}$$

The Gini coefficient for the distribution of labour income can then be calculated according to $A_U(\chi) \equiv 1 - 2 \int_0^1 Q_U(\lambda) d\lambda$, and it is given by (32). QED

The concept of Lorenz dominance

We now consider a second criterion for ranking distributions and look at the criterion of Lorenz dominance. Thereby, we say that distribution $A$ Lorenz dominates distribution $B$ if the Lorenz curve of $A$ lies above the Lorenz curve of $B$ for any cumulative share of population $p$. Since the Lorenz dominance is equivalent to mean-preserving second-order stochastic dominance, all measures of inequality that respect this criterion – such as the Gini coefficient or the Theil index – rank $A$ as a more equal distribution than $B$ if $A$ Lorenz dominates $B$.

Self-employed agents

The Lorenz curve for the income distribution of self-employed agents under autarky is given by

$$Q^S_S(\mu) = 1 - (1 - \mu)^{k - \xi}. \quad \text{(S.31)}$$

Hence, the income distribution of self-employed agents under autarky Lorenz dominates the respective income distribution under partial offshoring if $Q_S(\mu) < Q^S_S(\mu)$ holds for any $\mu \in (0, 1)$. We have to check this inequality separately for the three segments of $Q_S(\mu)$. Let us first look at domain $\mu \in (0, \chi/(1 + \chi))$. In this case, $Q_S(\mu) < Q^S_S(\mu)$, is equivalent to $D^0_S(\mu, b) \equiv b\mu - 1 + (1 - \mu)b < 0$, with $b \equiv (k - \xi)/k$. Twice differentiating $D^0_S(\mu, b)$ with respect to $b$ gives

$$\frac{\partial D^0_S(\mu, b)}{\partial b} = \mu + \ln(1 - \mu)(1 - \mu)^b, \quad \frac{\partial^2 D^0_S(\mu, b)}{\partial b^2} = \left[\ln(1 - \mu)\right]^2 (1 - \mu)^b. \quad \text{(S.32)}$$
with \( \partial D^0_S(\mu, 0)/\partial b = \mu + \ln(1 - \mu) < 0 \), \( \partial D^0_S(\mu, 1)/\partial b = \mu + \ln(1 - \mu)(1 - \mu) > 0 \), and \( \partial^2 D^0_S(\mu, b)/\partial b^2 > 0 \). Accounting for \( D^0_S(\mu, 0) = D^0_S(\mu, 1) = 0 \), we can therefore conclude that 
\( D^0_S(\mu, b) < 0 \) and thus \( Q_S(\mu) < Q^c_S(\mu) \) must hold in the relevant parameter range.

For domain \( \mu \in [\chi/(1+\chi), 1/(1+\chi)] \), it follows from (S.23) and (S.31) that \( Q_S(\mu) < Q^c_S(\mu) \) is equivalent to \( D^1_S(\mu, b) \equiv (b-1)\chi + [1-(1+\chi)^{b-1}](1+\chi)(1-\mu)^b < 0 \). Therefore, \( \partial D^1_S(\mu, b)/\partial \mu < 0 \) implies that \( D^1_S(\chi/(1+\chi), b) \equiv \hat{D}^1_S(b) = (b-1)\chi + (1+\chi)^{1-b} - 1 < 0 \) is sufficient for \( Q^1_S(\mu) < Q^c_S(\mu) \) to hold in the relevant parameter domain. Twice differentiating \( \hat{D}^1_S(b) \) yields 
\[
\frac{d\hat{D}^1_S(b)}{db} = \chi - \ln(1 + \chi)(1 + \chi)^{1-b}, \quad \frac{d^2\hat{D}^1_S(b)}{db^2} = [\ln(1 + \chi)]^2(1 + \chi)^{1-b}.
\]
Accounting for \( d\hat{D}^1_S(0)/db = \chi - \ln(1 + \chi) > 0 \), \( d\hat{D}^1_S(1)/db = \chi - \ln(1 + \chi) > 0 \), and \( d^2\hat{D}^1_S(b)/db^2 > 0 \), it follows from \( \hat{D}^1_S(0) = \hat{D}^1_S(1) = 0 \) that \( \hat{D}^1_S(b) < 0 \) and thus \( Q_S(\mu) < Q^c_S(\mu) \) must hold for all \( \mu \in [\chi/(1+\chi), 1/(1+\chi)] \).

Finally, we look at domain \( \mu \in [1/(1+\chi), 1] \). In this case, \( Q_S(\mu) < Q^c_S(\mu) \) is equivalent to \( D^2_S(\mu, b) \equiv -(1 + \chi)^b(1 - \mu)^b + b(1 + \chi)(1 - \mu) < 0 \), according to (S.24) and (S.31). Twice differentiating \( D^2_S(\mu, b) \) with respect to \( \mu \) gives \( \partial D^2_S(\mu, b)/\partial \mu = b(1 + \chi)^b(1 - \mu)^{b-1}[1 - (1 + \chi)^{1-b}(1 - \mu)^{1-b}], \partial^2 D^2_S(\mu, b)/\partial \mu^2 = b(1 - b)(1 + \chi)^b(1 - \mu)^{b-2} > 0 \). Accounting for \( \partial D^2_S(1/(1+\chi), b)/\partial \mu = b(1 + \chi)(\chi^{b-1} - 1) > 0 \), it is thus immediate that \( D^2_S(1, b) = 0 \) is sufficient for \( Q_S(\mu) < Q^c_S(\mu) \) to hold in the relevant parameter domain.

Putting together, we can thus conclude that \( Q_S(\mu) < Q^c_S(\mu) \) holds for any \( \mu \in (0, 1) \), which proves that the income distribution of self-employed agents under autarky Lorenz dominates the respective income distribution in an offshoring equilibrium for arbitrary values of \( \chi \in (0, 1) \).

QED

Production workers

The Lorenz curve for the distribution of labour income under autarky has two segments and is given by

\[
Q^0_U(\lambda) = \begin{cases} 
0 & \text{if } \lambda \in [0, U^a) \\
1 - \left(\frac{1 - \lambda}{1 - U^a}\right)^{1/(1-\theta)} & \text{if } \lambda \in [U^a, 1]
\end{cases},
\]

where \( U^a = \theta \xi / [k - (1 - \theta)\xi] \), according to (27). The ranking of \( Q^0_U(\lambda) \) and \( Q_U(\lambda) \) depends on the unemployment rate of production workers in the offshoring scenario relative to autarky.
Furthermore, as outlined in the main text, the ranking of $U$, $\beta(\chi; \eta)$ is equivalent to the ranking of $1$, $\alpha(\chi; \eta)$ and thus equivalent to the ranking of $\alpha(\chi; \eta) >, =, < \beta(\chi; \eta)$. From Appendix A.4 we know that there exists a unique $\hat{\chi} \in (0, 1)$, such that $\alpha(\chi; \eta) >, =, < \beta(\chi; \eta)$ if $\chi >, =, < \hat{\chi}$.

Let us first consider $\chi \geq \hat{\chi}$, which corresponds to an offshore equilibrium with $U \geq U^a$. In this case, we have $Q^a_U(\lambda) = Q_U(\lambda) = 0$ for all $\lambda \in [0, U^a)$ and $Q^a_U(\lambda) > Q_U(\lambda) = 0$ for all $\lambda \in [U^a, U)$. Furthermore, combining (S.27) and (S.33), it follows that, for domain $\lambda \in [U, b_U)$, the ranking of $Q_U(\lambda) >, =, < Q^a_U(\lambda)$ is equivalent to the ranking of

$$D^1_U(\hat{\lambda}) \equiv 1 - \alpha(\chi; \eta) + \alpha(\chi; \eta) \left(1 - \hat{\lambda}\right)^b \Delta(\chi; \eta)^b - \left[1 - \beta(\chi; \eta)\hat{\lambda}\right]^b >, =, < 0,$$  \hspace{1cm}  \text{(S.34)}

where $\hat{b} \equiv (k - \xi)/(k - (1 - \theta)\xi)$ and $\hat{\lambda} \equiv (\lambda - U)/(1 - U)$. Differentiating $D^1_U(\hat{\lambda})$ gives

$$\frac{dD^1_U(\hat{\lambda})}{d\hat{\lambda}} = \hat{b} \alpha(\chi; \eta) \Delta(\chi; \eta)^b \left[\left(1 - \hat{\lambda}\right)^{1-b} \left(\frac{1 - \hat{\lambda}}{1 - \beta(\chi; \eta)\hat{\lambda}}\right)^{1-b} - 1\right].$$  \hspace{1cm}  \text{(S.35)}

Consider first the case of $\beta(\chi; \eta) \leq 1$. Since $\chi \geq \hat{\chi}$ implies $\beta(\chi; \eta) \leq \alpha(\chi; \eta)$ and thus $\Delta(\chi; \eta) \leq 1$, it is immediate that $\beta(\chi; \eta) \leq 1$ is sufficient for $dD^1_U(\hat{\lambda})/d\hat{\lambda} < 0$. Noting further that $\lambda = U$ implies $\hat{\lambda} = 0$ and thus $D^1_U(0) = \alpha(\chi; \eta)[\Delta(\chi; \eta)^b - 1] < 0$, we can therefore safely conclude that $Q_U(\lambda) < Q^a_U(\lambda)$ holds for all $\lambda \in [U, b_U)$ in this case.

But what happens if $\beta(\chi; \eta) > 1$? In this case, we cannot rule out that $dD^1_U(\hat{\lambda})/d\hat{\lambda} < 0$. However, computing the second derivative of $D^1_U(\hat{\lambda})$, we obtain

$$\frac{d^2D^1_U(\hat{\lambda})}{d\hat{\lambda}^2} = \frac{1 - \hat{b}}{1 - \hat{\lambda}} \left\{ \frac{dD^1_U(\hat{\lambda})}{d\hat{\lambda}} - \hat{b} \alpha(\chi; \eta) \Delta(\chi; \eta)^b \left(\frac{1 - \hat{\lambda}}{1 - \beta(\chi; \eta)\hat{\lambda}}\right)^{1-b} \left(1 - \frac{1 - \hat{\lambda}}{1 - \beta(\chi; \eta)\hat{\lambda}}\right)\right\}. \hspace{1cm} \text{(S.36)}$$

From inspection of (S.35) and (S.36) we can therefore conclude that $dD^1_U(\hat{\lambda})/d\hat{\lambda} \geq 0$ is sufficient for $d^2D^1_U(\hat{\lambda})/d\hat{\lambda}^2 > 0$ if $\beta(\chi; \eta) > 1$. To see this, note that $\beta(\chi; \eta)\hat{\lambda} < \beta(\chi; \eta)\hat{\lambda}_U$, with $\hat{\lambda}_U \equiv (b_U - U)/(1 - \lambda)$ must hold on the relevant parameter domain. Substituting for $b_U$, we obtain

$$\beta(\chi; \eta)\hat{\lambda} < 1 - \chi^{1-(1-\theta)\xi/k} < 1.$$  \hspace{1cm}  \text{(S.37)}

From inspection of (S.35) and (S.36) it therefore follows that if $dD^1_U(\hat{\lambda})/d\hat{\lambda} \geq 0$ holds for some $\hat{\lambda}_0 \in (0, \hat{\lambda}_U)$, then $dD^1_U(\hat{\lambda})/d\hat{\lambda} > 0$ must hold for all $\hat{\lambda} \in (\hat{\lambda}_0, \hat{\lambda}_U)$. Furthermore, recollecting from above that $D^1_U(0) < 0$, this implies that if $D^1_U(\hat{\lambda}) \geq 0$ holds for
some $\hat{\lambda} \in (0, \hat{\lambda}_U)$, then $D^1_U(\hat{\lambda}_U) > 0$ must hold as well. Accordingly, we can infer insights on the sign of $D^1_U(\hat{\lambda})$ by evaluating (S.34) at $\hat{\lambda} = \hat{\lambda}_U$. This gives

$$D^1_U(\hat{\lambda}_U) = \alpha(\chi; \eta) (1 - \chi \hat{\xi}k) \left[ \eta \left( 1 + \chi \hat{\xi}k \right) \right] \left[ \left( 1 + \chi \hat{\xi}k \right)^{-\theta \beta} - \left( \frac{\eta \left( 1 + \chi \hat{\xi}k \right)}{\alpha(\chi; \eta)} \right)^{1-b} \right].$$

(S.37)

However, since $\beta(\chi; \eta) > 1$ implies $\alpha(\chi; \eta) > 1$ if $\chi \geq \hat{\chi}$, it is immediate that $\alpha(\chi; \eta) < \eta(1 + \chi \hat{\xi}k)$, and this implies $D^1_U(\hat{\lambda}_U) < 0$. Putting together, we can therefore safely conclude that $Q_U(\lambda) < Q^a_U(\lambda)$ holds for all $\lambda \in [U, b_U]$ irrespective of the ranking of $\beta(\chi; \eta) >, =, < 1$.

In a final step, we have to look at domain $\lambda \in [b_U, 1]$. According to (S.29) and (S.33), for this parameter domain the ranking of $Q_U(\lambda) >, =, < Q^a_U(\lambda)$ is equivalent to the ranking of

$$D^2_U(\lambda) \equiv \left[ 1 - \left( 1 + \chi \hat{\xi}k \right)^{\theta \beta} \left( \frac{\eta \left( 1 + \chi \hat{\xi}k \right)}{\alpha(\chi; \eta)} \right)^{1-b} \right] \left( 1 - \hat{\lambda} \right)^b \Delta(\chi; \eta)^b >, =, < 0.$$  

(S.38)

Notably, the sign of $D^2_U(\lambda)$ does not depend on the specific level of $\hat{\lambda}$, so that $\text{sgn}[D^2_U(\lambda)] = \text{sgn}[D^2_U(\hat{\lambda}_U)]$. However, since $D^1_U(\hat{\lambda}_U) = D^1_U(\hat{\lambda}_U)$ holds by definition, it follows that $Q_U(\lambda) < Q^a_U(\lambda)$ extends to interval $\lambda \in [b_U, 1]$. Summing up, we can thus conclude that the income distribution of production workers under autarky Lorenz dominates the income distribution of production workers in the offshoring equilibrium if the share of offshoring firms is sufficiently high, i.e. if $\chi \geq \hat{\chi}$.

Let us now consider $\chi < \hat{\chi}$, which implies $\Delta(\chi; \eta) > 1$ and thus $U < U^a$. In this case, we have $Q_U(\lambda) = Q^a_U(\lambda) = 0$ for all $\lambda \in [0, U]$ and $Q_U(\lambda) > Q^a_U(\lambda) = 0$ for all $\lambda \in (U, U^a)$. For domain $\lambda \in (U, b_U)$, the ranking of $Q_U(\lambda) >, =, < Q^a_U(\lambda)$ depends on the sign of $D^1_U(\lambda)$, where $D^1_U(0) = \alpha(\chi; \eta) |\Delta(\chi; \eta)^b - 1| > 0$ holds if $\chi < \hat{\chi}$. But what can we say about the sign of $D^1_U(\lambda)$ if $\hat{\lambda} > U$? To answer this question, it is worth looking at (S.36). From the formal discussion in Appendix A.4, we know that $\Delta(\chi; \eta) > 1$ requires $\hat{\Omega}(\hat{\kappa}) \equiv (\hat{\kappa} \hat{\xi} - \theta - 1)(\zeta - 1) > 1$, where $\zeta \equiv 1 + \chi \hat{\xi}k$ and $\hat{\kappa} \equiv \eta \zeta$. In view of $\hat{\Omega}'(\hat{\kappa}) = (1 - 1/\zeta) / (\zeta - 1) > 0$ and $\hat{\Omega}(1) = (\zeta - \theta - 1)(\zeta - 1) < 0$, we can conclude that $\eta \zeta < 1$ is necessary for $\Delta(\chi; \eta) > 1$. This implies that $\beta(\chi; \eta) < 1$ must hold for all $\chi < \hat{\chi}$. Hence, if $D^1_U(\lambda)$ has an extremum at $\hat{\lambda} \in (0, \hat{\lambda}_U)$, this extremum must be a maximum. In view of $D^1_U(0) > 0$, we can therefore conclude that $D^1_U(\lambda)$ is positive for all
\( \lambda \in [U, b_U) \) if \( D^1_U(\hat{\lambda}_U) \geq 0 \), while \( D^1_U(\hat{\lambda}_U) < 0 \) implies that there exists a unique \( \lambda_0 \in [U, b_U) \) such that \( D^1_U(\hat{\lambda}) >, =, < 0 \) if \( \lambda_0 >, =, < \lambda \). Noting finally that \( \text{sgn}[D^1_U(\hat{\lambda}_U)] = \text{sgn}[D^2_U(\hat{\lambda})] \) holds for all \( \lambda \in [b_U, 1) \) and accounting for \( \lim_{\chi \to 0} D^2_U(\hat{\lambda}_U; \beta) = (1 - \eta^{1-b})(1 - \hat{\lambda})^b \Delta(\chi; \eta)^b > 0 \), \( \lim_{\chi \to \hat{\chi}} D^2_U(\hat{\lambda}_U; \beta) < 0 \) (see our extensive discussion for domain \( \chi \geq \hat{\chi} \)), the following conclusion is immediate: For sufficiently small \( \chi \), the distribution of labour income with offshoring Lorenz dominates the respective distribution without offshoring. For \( \chi \) smaller than but close to \( \hat{\chi} \), Lorenz curves \( Q^0_U \) and \( Q_U \) intersect and it is therefore not possible to rank the distributions of labour income with and without offshoring according to the criterion of Lorenz dominance. This completes our discussion on Lorenz curve dominance. \( QED \)

**Economy-wide income distribution**

For computing a comprehensive measure of economy-wide income inequality, we have to solve the problem that distributions of profit and labour income overlap if \( \theta > 0 \). Due to this overlap, we cannot simply calculate Gini coefficients for ranking the economy-wide income distributions with and without offshoring, but instead look at the Theil index as an alternative measure of income inequality. In discrete notation the Theil index for the income distribution in a group of agents with population size \( n \) can be calculated according to

\[
T = \frac{1}{n} \sum_{i=1}^{n} \frac{y_i}{\bar{y}} \ln \left( \frac{y_i}{\bar{y}} \right), \tag{S.39}
\]

where \( y_i \) is income of agent \( i \), while \( \bar{y} \) is the average income. One of the main advantages of the Theil index as compared to other measures of inequality is its decomposability. For instance, if there are \( m \) subgroups of population, Theil index \( T \) can be decomposed according to

\[
T = \sum_{j=1}^{m} \frac{n_j \bar{y}_j}{n \bar{y}} T_j + \sum_{j=1}^{m} \frac{n_j \bar{y}_j}{n \bar{y}} \ln \left( \frac{\bar{y}_j}{\bar{y}} \right). \tag{S.40}
\]

In our model, we can distinguish two main income groups and hence can write the Theil index for the economy-wide income distribution as follows

\[
T = a_S T_S + a_L T_L + a_S \ln \left( a_S \frac{N}{N - L} \right) + a_L \ln \left( a_L \frac{N}{L(1 - U)} \right), \tag{S.41}
\]

where \( T_S, T_L \) are the Theil indices for income distributions of self-employed agents and employed production workers, respectively, while \( a_S, a_L \) represent these groups income shares, as
determined by (34). Accounting for (18), we can furthermore compute

\[
\frac{a_S}{N} - \frac{N}{N - L} = \frac{\rho \gamma(\chi; \eta) + (1 - \rho)(1 - \bar{\xi}/k)}{\rho \gamma(\chi; \eta) + 1 - \rho} \quad \frac{k}{k - \xi}, \quad \frac{a_L}{L} = \frac{\rho \gamma(\chi; \eta) + (1 - \rho)(1 - \bar{\xi}/k)}{\rho \gamma(\chi; \eta) + 1 - \rho}, \quad (S.42)
\]

and thus obtain\(^{36}\)

\[
T = \frac{1 - \rho}{\rho \gamma(\chi; \eta) + 1 - \rho} T_S + \frac{\rho \gamma(\chi; \eta)}{\rho \gamma(\chi; \eta) + 1 - \rho} T_L + \ln \left( \frac{\rho \gamma(\chi; \eta) + (1 - \rho)(1 - \bar{\xi}/k)}{\rho \gamma(\chi; \eta) + 1 - \rho} \right) \\
+ \frac{1 - \rho}{\rho \gamma(\chi; \eta) + 1 - \rho} \ln \left( \frac{k}{k - \xi} \right) - \frac{\rho \gamma(\chi; \eta)}{\rho \gamma(\chi; \eta) + 1 - \rho} \ln \left( \frac{(k - \bar{\xi})\Delta(\chi; \eta)}{k - (1 - \theta)\bar{\xi}} \right). \quad (S.43)
\]

In autarky, we can explicitly compute the Theil indices for the income distribution of self-employed agents and production workers, respectively:

\[
T_S^a = \frac{k - \bar{\xi}}{\xi} \int_1^\infty x^{-\frac{k}{\xi}} \left[ \ln x - \ln \left( \frac{k}{k - \xi} \right) \right] dx = \frac{\bar{\xi}}{k - \xi} - \ln \left( \frac{k}{k - \xi} \right), \quad (S.44)
\]

and

\[
T_L^a = \frac{k - \bar{\xi}}{k - (1 - \theta)\xi} \int_1^\infty y^{-\frac{k-(1-\theta)\xi}{\theta \xi}} \left[ \ln y - \ln \left( \frac{k - (1 - \theta)\bar{\xi}}{k - \xi} \right) \right] dy \\
= \frac{\theta \bar{\xi}}{k - \xi} - \ln \left( \frac{k - (1 - \theta)\bar{\xi}}{k - \xi} \right). \quad (S.45)
\]

Substituting for \(T_S, T_L\) and setting \(\chi = 0\) then yields

\[
T^a = \frac{(1 - \rho)\bar{\xi}}{k - \xi} + \frac{\rho \theta \bar{\xi}}{k - \xi} + \ln \left( 1 - \frac{(1 - \rho)\bar{\xi}}{k} \right). \quad (S.46)
\]

To determine the impact of offshoring on Theil index \(T\) we first consider the benchmark case of \(\theta = 0\). In this case, (S.43) reduces to

\[
T = \frac{1 - \rho}{\rho \gamma(\chi; \eta) + 1 - \rho} \left[ T_S + \ln \left( \frac{k}{k - \xi} \right) \right] + \ln \left( \frac{\rho \gamma(\chi; \eta) + (1 - \rho)(1 - \bar{\xi}/k)}{\rho \gamma(\chi; \eta) + 1 - \rho} \right). \quad (S.47)
\]

Since we know from our extensive discussion on Lorenz curves that the income distribution within the group of self-employed agents in autarky Lorenz dominates the respective income distribution under offshoring and since the Theil index, although not relying on the Lorenz curve, is invariant under monotonic transformations of the income distribution, we can straightforwardly show that (33) and (S.41) are equivalent, when setting \(T_U = T_L - \ln(1 - U), a_L = a_U\) and accounting for (34).
curve, respects (mean-preserving) second-order stochastic dominance – which is equivalent to Lorenz dominance – we can safely conclude that $T_S > T_S^a$ holds for all $\chi > 0$. Using (S.44) in (S.47), this implies that $T - T^a > \hat{\Delta}_T(\chi)$, with

$$\hat{\Delta}_T(\chi) \equiv \left(\frac{\xi/k}{1 - \xi/k}\right) \frac{\rho (1 - \rho)(1 - \gamma(\chi; \eta))}{\rho \gamma(\chi; \eta) + 1 - \rho} + \ln \left(\frac{\rho \gamma(\chi; \eta) + (1 - \rho)(1 - \xi/k)}{\rho \gamma(\chi; \eta) + 1 - \rho}\right) - \ln \left(1 - \frac{(1 - \rho)\xi}{k}\right).$$

(S.48)

Differentiating $\hat{\Delta}_T(\chi)$ gives

$$\frac{\hat{\Delta}_T(\chi)}{d\chi} = -\frac{\rho^2 (1 - \rho)\gamma(\chi; \eta)(\xi/k)^2}{(1 - \xi/k) [\rho \gamma(\chi; \eta) + (1 - \rho)]^2 [\rho \gamma(\chi; \eta) + (1 - \xi/k)]} \frac{d\gamma(\chi; \eta)}{d\chi} > 0.$$  

(S.49)

Noting further that $\hat{\Delta}_T(0) = 0$, it follows that $\hat{\Delta}_T(\chi) > 0$ holds for all $\chi > 0$. For $\theta = 0$, we can therefore conclude that an increase of $\chi$ from zero to any positive level increases Theil index $T$, and hence renders the economy-wide distribution of income less equal.

Let us now consider the more sophisticated scenario with $\theta > 0$ (and thus $\tilde{\xi}$ instead of $\xi$). While in the model variant with labour market imperfection, we are not able to rank $T$ and $T^a$ for arbitrary levels of $\chi$, we can at least compare Theil indices for the two limiting cases $\chi = 0$ and $\chi = 1$. Since $T_L = T_L^a$, while $T_S > T_S^a$ hold if $\chi = 1$, we can safely conclude that $T - T^a > \Delta_T(\theta; \hat{a})$, with

$$\Delta_T(\theta; \hat{a}) \equiv \frac{\rho (1 - \rho)(1 - \eta)}{\rho \eta + 1 - \rho} \frac{\hat{a}}{1 - \hat{a}} + \ln \left(\frac{\rho \eta + (1 - \rho)(1 - \hat{a})}{\rho \eta + 1 - \rho} \frac{[1 - (1 - \rho)\hat{a}]}{[1 - (1 - \rho)\hat{a}]}\right)$$

$$\frac{\Delta_T(\hat{a})}{\hat{a}} + \frac{\rho \theta}{\rho \eta + 1 - \rho} \left[\eta \ln 2 - \frac{(1 - \rho)(1 - \eta)\hat{a} \theta}{1 - \hat{a}}\right]$$

(S.50)

and $\hat{a} \equiv \tilde{\xi}/k$. Differentiating $\Delta_T(\hat{a})$ gives

$$\frac{d\Delta_T(\hat{a})}{d\hat{a}} = \frac{\rho^2 (1 - \rho)(1 - \eta)\hat{a} [\rho \eta + (1 - \rho)(1 - \hat{a}) + \eta(1 - \hat{a})]}{[\rho \eta + 1 - \rho] [\rho \eta + (1 - \rho)(1 - \hat{a})] [1 - (1 - \rho)\hat{a}] (1 - \hat{a})^2} > 0.$$  

(S.51)

In view of $\Delta_T(0) = 0$, this implies that $\Delta_T(\hat{a}) > 0$ holds for all $\hat{a} \in (0, 1)$. While the sign of $\Delta_T(\theta; \hat{a})$ is not clear in general, it is immediate that $\Delta_T(\theta; \hat{a}) > 0$ holds for sufficiently small levels of $\theta$. Hence, we can conclude that a movement from autarky to high levels of offshoring will unambiguously increase economy-wide income inequality if the motive for rent-sharing is not too strong. This completes our discussion on the Theil index. QED
The calibration exercise has been executed in *Mathematica.* We offer here the source code as well as the parameter estimates used in our calibration. At first, we set parameter values: \(k = 4.306\), \(\sigma = 6.698\) and \(\theta = 0.102\), based on the results in Egger, Egger, and Kreickemeier (2011), and \(\eta = 0.75\), based on Blinder (2009) and Blinder and Krueger (2012).

```mathematica
k=4.306;
\[\sigma=6.698;\]
\[\theta=0.102;\]
\[\eta=0.75;\]
```

As all variables of interest can be expressed in terms of the share of offshoring firms, \(\chi\), we define

\[
\chi^*; \\
\chi_G=0.149;
\]

where \(\chi_G = 0.149\) is the share of offshoring firms in Germany as computed from Moser, Urban, and Weder di Mauro (2009). We then define \(\xi\) and check that \(k > \xi\) holds.

```mathematica
ξ=(σ−1)/(1+θ(σ−1));
If[k<=ξ, Print["Error:k<=ξ"]];
```

We also define \(\alpha(\chi; \eta)\) from (13) and \(\beta(\chi; \eta)\) from (28) as well as \(\gamma(\chi; \eta) = \alpha(\chi; \eta)/(1 + \chi)\) and \(\Delta(\chi, \eta) = \beta(\chi; \eta)/\alpha(\chi; \eta)\).

```mathematica
α=1+\chi^*((k-ξ)/k)(η(1+\chi^*(ξ/k))−1);
β=1+\chi^*((k−(1−θ)ξ)/k)(η(1+\chi^*(ξ/k))^(1−θ)-1);
γ=α/(1+\chi);
Δ=β/α;
```

Now we can turn to aggregate income in the source country relative to autarky, see Eq. (22):
\[ \Phi = \left\{ \frac{1}{\sigma} + \frac{(\sigma-1)/\sigma}{\gamma} \right\} \left( 1 + \chi \right)^{\gamma} \left( \frac{(k+\sigma-1)/(k(\sigma-1))}{(k^*\sigma-\xi)/(\gamma\*k(\sigma-1)+k-\xi)} \right)^{((k-1)/k)} \]

Equation (29) translates into

\[ \Lambda = \left( 1 + \left( \frac{(k-\xi)/(\theta\*\xi))}{(1-\Delta)} \right) \right) \times \left( \frac{\gamma(k^*\sigma-\xi)/(\gamma\*k^*\sigma-\xi))}{(k(\sigma-1))/(k^*\sigma-\xi)} \right) ; \]

\[ u = \left( \frac{(\theta\*\xi)/k}{(k(\sigma-1))/(k^*\sigma-\xi)} \right) \times \Lambda ; \]

\[ u_a = u / \{ \chi \rightarrow 0 \} ; \]

where \( u_a \) refers to the autarky unemployment rate. Finally, inter-group inequality between entrepreneurs and workers as well as intra-group inequality within both groups, each normalised to one for its respective autarky level, follow from (25), (24) and (30).

\[ \omega = \frac{(k+\chi\*\xi)}{k} ; \]

\[ A_M = 1 + \left( \frac{(k-\xi)*\chi*(2-\chi)}{(k^*\sigma+\chi)} \right) ; \]

\[ A_L = 1 + \left( \frac{(2(k-\xi)/(1-\chi)((k-(1-\theta)\xi)/k)) ((\alpha-1)) - (2(k-(1-\theta)\xi)/(1-\chi)((k-\xi)/k)) (\beta-1))}{(\alpha\*\beta\*\theta\*\xi)} \right) ; \]

In a last step we evaluate the above defined functions at \( \chi = 0.01, 0.1, 0.25, 0.5, 0.75, 0.9 \) and \( \chi = \chi_G \) to produce the results in Table 1.

\[ \text{Do}\{z=z0; \ n\Phi=\Phi/. \{\chi \rightarrow z\} ; \}
\]

\[ \text{nu}=u / . \{\chi \rightarrow z\} ; \]

\[ \text{n}\omega=\omega / . \{\chi \rightarrow z\} ; \]

\[ \text{nA}_M=\text{A}_M / . \{\chi \rightarrow z\} ; \]

\[ \text{nA}_L=\text{A}_L / . \{\chi \rightarrow z\} ; \]

\[ \text{Print}["\chi = ", z, " \text{ I = } ", 100*(n\Phi-1), " \text{ u = } ", 100*(nu-u_a), " \text{ \omega = } ", 100*(n\omega-1), " \text{ A}_M = ", 100*(nA_M-1), " \text{ A}_L = ", 100 \text{ (nA}_L-1) ; \]

\[ ,\{z0, \{0.01, 0.1, 0.25, 0.5, 0.75, 0.9, 1, \chi_G\}] \]

In a next step, we determine the Gini coefficient for income of self-employed agents as given by (31):

\[ \text{AS1} = \xi/(2k-\xi) * (1 + (2*(k-\xi))/k*k/(1+\chi)^2) ; \]

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To determine the Gini coefficient for income of all production workers in (32) we first need to specify the Gini coefficient for income of employed production workers in (30). Considering AL from above, we obtain

\[ AL1 = \frac{(\theta \xi)}{(2(k-\xi) + \theta \xi) \cdot (AL)}; \]

Combining AL1 with the unemployment rate of production workers (see (27))

\[ U1 = \frac{(\theta \xi + (1-\Delta)(k-\xi))}{(k-(1-\theta)\xi)}; \]

we can compute

\[ AU1 = (1-U1) \cdot AL + U1; \]

For the definition of Theil indices we first need to specify the income share of entrepreneurs, freelance offshoring workers and employed production workers. This gives

\[ aM = \frac{(1-\rho)}{(\rho \gamma + (1-\rho) \cdot (k+\chi \xi)/(k+\chi))/\rho \gamma + (1-\rho) \cdot (k-\xi)/(k+\chi)}; \]

\[ aF = (1-\rho)/(\rho \gamma + (1-\rho) \cdot ((k-\xi) \cdot \chi)/(k+\chi)); \]

\[ aL = (\rho \gamma)/(\rho \gamma + 1-\rho); \]

respectively. Furthermore, we also determine the income share of self-employed agents, as defined in (34):

\[ aS = (1-\rho)/(\rho \gamma + 1-\rho); \]

Average income of the three subgroups – entrepreneurs, freelance offshoring agents, and employed production workers – relative to the economy-wide income average is given by

\[ vM = ((\rho \gamma + (1-\rho) \cdot (1-\xi/k)) / (\rho \gamma + 1-\rho)) \cdot (k+\chi \xi)/(k-\xi); \]

\[ vF = ((\rho \gamma + (1-\rho) \cdot (1-\xi/k)) / (\rho \gamma + 1-\rho)) \cdot (k-(1-\theta)\xi)/(k-\xi); \]

\[ vL = ((\rho \gamma + (1-\rho) \cdot (1-\xi/k)) / (\rho \gamma + 1-\rho)) \cdot (k-(1-\theta)\xi)/(k-\xi); \]

while for the self-employed we obtain

\[ vS = ((\rho \gamma + (1-\rho) \cdot (1-\xi/k)) / (\rho \gamma + 1-\rho)) \cdot k/(k-\xi); \]
We now determine the product of income ratios and log income ratios for entrepreneurial income multiplied by the relative frequency the respective income ratios are realized. For purely domestic firms, this gives

\[ Alt1 = \frac{((k-\xi)/(k+\chi*\xi))}{k} \times (\xi-k-1) \times \log \left[ \frac{((k-\xi)/(k+\chi*\xi))}{x^{\xi}} \right]; \]

while for offshoring firms, we obtain

\[ Alt2 = \frac{((k-\xi)/(k+\chi*\xi))}{k} \times (1+\chi^{\xi/k}) \times (-k-1) \times \log \left[ \frac{((k-\xi)/(k+\chi*\xi))}{x^{\xi}} \right]; \]

We can compute similar expressions for production workers and obtain

\[ Alt3 = \frac{((k-\xi)*\Delta)/(\theta*\xi*\beta)}{y} \times \left[ \log \left( \frac{((k-\xi)*\Delta)/(\theta*\xi)}{((1-\theta)*\xi-k)/(\theta*\xi)} \right) \right]; \]

for workers employed in purely domestic firms and

\[ Alt4 = \frac{((k-\xi)*\Delta)/(\theta*\xi*\beta)}{y} \times \left[ \log \left( \frac{((k-\xi)*\Delta)/(\theta*\xi)}{((1-\theta)*\xi-k)/(\theta*\xi)} \right) \right]; \]

for workers employed in exporting firms.

In a last step we evaluate the new inequality measures at \( \chi = 0.01, 0.1, 0.25, 0.5, 0.75, 0.9 \) and \( \chi = \chi_G \) to produce the results in Table 2.

We first evaluate the Gini coefficient for income of self-employed agents at \( \chi = 0 \) and \( \chi = z \), respectively. This gives

\[ ASa = AS1/.\{$\chi$->0} \]

\[ AS = AS1/.\{$\chi$->z} \]

In a second step, we evaluate the Gini coefficient for income of all production workers at \( \chi = 0 \) and \( \chi = z \), respectively. This gives

\[ AUa = AU1/.\{$\chi$->0} \]

\[ AU = AU1/.\{$\chi$->z} \]
We now turn to the Theil index for entrepreneurial income, which in autarky can be computed according to

\[ Alt_{00} = Alt_{1} / \chi \to 0; \]

\[ T_{Ma} = \text{NIntegrate}[Alt_{00}, \{x, 1, \infty\}]; \]

The respective Theil index in the case of offshoring can be determined according to

\[ Alt_{11} = Alt_{1} / \chi \to z; \]
\[ Alt_{22} = Alt_{2} / \chi \to z; \]
\[ T_{M} = \text{NIntegrate}[Alt_{11}, \{x, 1, z^{(-1/k)}\}] + \text{NIntegrate}[Alt_{22}, \{x, z^{(-1/k)}, \infty\}]; \]

In a similar vein, we can calculate the Theil index for income of employed production workers under autarky and in the scenario with offshoring. This gives

\[ Alt_{55} = Alt_{3} / \chi \to 0; \]
\[ T_{La} = \text{NIntegrate}[Alt_{55}, \{y, 1, \infty\}]; \]

and

\[ Alt_{33} = Alt_{3} / \chi \to z; \]
\[ Alt_{44} = Alt_{4} / \chi \to z; \]
\[ T_{L} = \text{NIntegrate}[Alt_{33}, \{y, 1, z^{(-\theta*\xi/k)}\}] + \text{NIntegrate}[Alt_{44}, \{y, z^{(-\theta*\xi/k)}(1+z^{\xi/k})^{\theta}, \infty\}]; \]

respectively. Thereby, it is notable that in the scenario with offshoring, firms which shift production abroad pay a wage premium to their domestic workers, and this wage premium is captured by an upward shift of the lower bound of the second integral in the equation for \( T_{L} \).

The economy-wide Theil index under autarky is then given by

\[ T_{a1} = a_{M}*(T_{Ma}) + a_{L}*(T_{La}) + a_{M}*(\text{Log}[v_{M}]) + a_{F}*(\text{Log}[v_{F}]) + a_{L}*(\text{Log}[v_{L}]); \]
\[ T_{a} = T_{a1} / \chi \to 0; \]

while the economy-wide Theil index in the scenario with offshoring is given by
\[ T_1 = aM \cdot TM + aL \cdot TL + aM \cdot \log[vM] + aF \cdot \log[vF] + aL \cdot \log[vL]; \]

\[ T = T_1 / \{x \rightarrow z\}; \]

To avoid rounding errors, we can manipulate the result in the following way

\[ \text{If}[TL < TLa + 0.1 \cdot (10) \&\& TLa > 0.1 \cdot (10), TL = TLa]; \]

Finally, we can compute \( T_U \), considering the calibrated values of \( T_L \).

\[ \Delta a = \Delta / \{x \rightarrow 0\}; \]

\[ \Delta 1 = \Delta / \{x \rightarrow z\}; \]

we can compute

\[ T_{Ua} = TLa - \log[((k - \xi) \ast \Delta a)/(k - (1 - \theta \ast \xi))]; \]

\[ T_{U} = TL - \log[((k - \xi) \ast \Delta 1)/(k - (1 - \theta \ast \xi))]); \]

In a similar vein, we can compute \( T_S \), relying on the calibrated values of \( T_M \):

\[ T_{S1} = (aM \cdot TM + aM \cdot \log[vM] + aF \cdot \log[vF] + aS \cdot \log[vS]) / aS; \]

\[ T_{Sa} = T_{S1} / \{x \rightarrow 0\}; \]

\[ T_{S} = T_{S1} / \{x \rightarrow z\}; \]

To complete the calibration exercise, we finally add

\[ \text{Print["x="}, z, " \Delta AS="}, 100 \ast (AS - ASa) / ASa, " \Delta AU="}, 100 \ast (AU - AUa) / AUa, " \Delta TS="}, 100 \ast (TS - TSa) / TSa, " \Delta TU="}, 100 \ast (TU - T_{Ua}) / T_{Ua}, " \Delta T="}, 100 \ast (T - Ta) / Ta]; \]

\[ \{z0, \{0.01, 0.1, 0.25, 0.5, 0.75, 0.9, 1, \chi_G\}\} \]

References


