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ABSTRACT

In-Work Benefits and the Nordic Model*

Welfare benefits in the Nordic countries are often tied to employment. We argue that this is one of the factors behind the success of the Nordic model, where a comprehensive welfare state is associated with high employment. In a general equilibrium setting, the underlining mechanism works through wage moderation and job creation. The benefits make it more important to hold a job, thus lower wages will be accepted, and more jobs created. Moreover, we show that the incentive to acquire higher education improves, further boosting employment in the long run. These positive effects help counteracting the negative impact of taxation.

JEL Classification: H24, J21, J24

Keywords: Nordic model, in-work benefits, wage adjustment, unemployment, education, skill formation, earnings

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1 Introduction

A prominent feature of the so-called Nordic model is a comprehensive welfare state financed by taxes on labor. In fact, the public sector in many of the Nordic countries is responsible for the distribution and allocation of resources amounting to more than half of their country’s GDP (Eurostat, 2012). With an emphasis on redistributional transfers and service provision financed by taxes on labor, a concern with the model is, of course, that it induces weak incentives to work. In a more long term perspective, such a system may also reduce incentives to acquire skills, with a negative impact on future productivity and labor market outcomes. However, external observers are often surprised that the Nordic countries manage to combine low unemployment and high labor force participation with high taxes and generous welfare arrangements. So, how is this possible?

One answer to this question is that many of the welfare arrangements in the Nordic countries are closely tied to market work. The generosity of the benefits are, in general, related to earnings. In addition, eligibility to a number of benefits and social services is conditional on employment. Subsidized childcare, for example, is, in principle, only available to employed workers. Also, some generous elements of the paid parental leave schemes are only accessible to employed workers. In addition, the more recently introduced earned income tax credit is by definition exclusively targeted to employed workers. The idea is that these benefits, by increasing the returns from working, increase the supply of labor.

The observation that the Nordic countries have sustained high economic activity because benefits are closely tied to market work is not new. In fact this was noted as a contributing factor to the high participation rate observed in Sweden when a group of NBER economists studied the Swedish welfare state in the mid 1990s (see Freeman et al., 1997). This was also an important message in the discussion on the prospects and challenges of the Scandinavian model in Andersen (2008).

The starting point for this paper is that entitlement to many of the benefits available in the Nordic countries is conditional on employment. As discussed above, this tends to increase the gains from working, which encourages labor supply. However, we argue that this is not the end of the story. To investigate the full impact of welfare state arrangements of this type, one needs to account for the general equilibrium effects. This is particularly relevant because many benefits have been available to the whole population for a long
period of time. Clearly, to investigate the effects of these benefits on employment, which is an equilibrium outcome, both supply-side and demand-side factors must be included in the analysis. Moreover, beside considering the equilibrium outcome for the existing workforce, it is important to account for the impact of these benefits on incentives to acquire skills. The equilibrium composition of the workforce in terms of educational attainment is a crucial variable for the sustainability of the Nordic model, both in terms of its growth potential and international competitiveness (Andersen, 2008) and in terms of the political support for the welfare state (Hassler et al., 2003).

To carry out such an analysis, we develop a simple model of a non-clearing labor market featuring involuntary unemployment as an equilibrium outcome. Labor force participation is also endogenously determined. Moreover, individuals differ in their ability to acquire education and choose educational attainments based on a cost-benefit analysis. In particular, we focus on the choice between proceeding to higher, i.e. tertiary, education or not. The aim is to investigate the implications of benefits that are conditional on work on unemployment and labor force participation, accounting for their long term impact on educational attainments.

We show that benefits available only to employed workers moderate wages, reduce unemployment rates, and increase labor force participation and employment. Moreover, one could expect that welfare benefits, even if conditional on work, could induce an outright reduction in education as they represent an important subsidy for low skilled workers. What we find instead is that the incentives to proceed to higher education are actually strengthened. This is a consequence of the relatively stronger increase in labor market opportunities for highly educated workers that follow when wages are moderated. Wages, in turn, fall because workers are more willing to accept lower wages when benefits are conditional on work and thus the value of having a job is higher. Lower wages increase job creation and lower the unemployment rate. Thus, total employment increases for three sets of reasons. First, the benefits reduce the unemployment rate for workers at all educational levels. Second, more workers choose to proceed to higher education where expected unemployment spells are shorter. Third, as labor force participation increases with the benefits, a larger share of the population will be employed.

We also look at the impact of benefits when they are financed through a proportional tax on wages. Taxation actually reinforces wage moderation and, as such, does not overrule that benefits reduce wages, increase job creation, and reduce unemployment rates. However, it weakens the incentives to
acquire higher education and participate in the labor force, thus inducing a counteracting effect on educational attainment and labor force participation.

The element of the Nordic model that this paper underlines is the wage moderation stemming from benefits conditional on work. Also, we find this mechanism to be very robust to the choice of model. Moreover, looking at benefits through this channel highlights how they have a positive impact on educational attainment and participation, thus counteracting, at least partly, the negative effect that taxation has on skill acquisition and labor force participation.

The analytical results are followed up with a numerical example illustrating the effects of the benefits on labor market performance and educational attainment. The simulations indicate that benefits can have an important impact on unemployment for both low- and high-skilled. Without distortiary taxation, benefits also have a positive impact on skill acquisition, thus further reducing overall unemployment in the long run. When financing through proportional taxation on wages is included in the model, the negative effect of taxation on educational attainment dominates the positive effect of benefits, thus resulting in a decrease in the share of the workforce acquiring tertiary education. Nonetheless, benefits still have a positive overall impact on unemployment.

Considering the previous literature, there are a number of studies that have tried to explain why the Nordic countries have performed so well despite high taxes and generous welfare arrangements. As mentioned, some of these studies have emphasized the importance of that benefits are tied to market work for the successful outcome in terms of employment and participation (see Aronsson and Walker, 1997). A related view is provided by Rogerson (2007). He argues that the governments’ spending pattern in the Scandinavian countries, compared to other high tax countries, can potentially explain the large number of aggregate work hours observed in these countries. He shows, holding tax rates constant, that it matters if the revenue is spent on disability payments which may only be received when an individual does not work or subsidies for day care for working mothers. The reason is that childcare subsidies create jobs. Our study also finds that how the government choose to spend tax revenues matters for labor market performance, although for a different reason. In contrast to Rogerson (2007), our results materialize through general equilibrium effects working through wage moderation.

There is also a large number of studies focusing on particular features of the welfare state in the Nordic countries, looking for instance at the impact
of childcare subsidies and paid parental leave schemes on labor supply and a number of other outcome variables.\footnote{See Datta Gupta et al (2008) and Kolm and Lazear (2010). For a recent empirical study on the impact of childcare subsidies on female labor supply in Norway see Havnes and Mogstad (2011).} In contrast to our study, this literature has, in general, only been concerned with the short run impact of these policies on labor supply, thus disregarding the impact on wages and job creation in the long run.

This study is also related to the large literature on earned income tax credits (EITCs) as such a tax credit is available only to workers with income from work.\footnote{Theoretical papers, usually based on standard neoclassical labor supply models, investigate the effects of the EITC on work hours (Eissa and Hoynes, 2006) or on the extensive margin (Saez, 2002). For empirical papers, see Eissa and Liebman, 1996, Meyer and Rosenbaum, 2001, Chetty, Friedman and Saez, 2012.} This literature, however, has only recently been concerned with the implications of EITC policies when wages respond to the policy. Rothstein (2010) investigates the impact of the US EITC in a model featuring a perfectly competitive labor market, accounting for the behavioral responses in labor force participation and work hours. He finds that the increased labor supply following the EITC leads to lower wages in equilibrium. This, in turn, dampens the equilibrium impact on labor supply. Kolm and Tonin (2011) contrast the impact of an EITC when wages are fixed and when equilibrium wage adjustments are accounted for using a search and matching model. They also find that wages fall with the tax credit in equilibrium, but this actually amplifies the positive impact of the EITC on search intensity, participation, employment, and unemployment. The theoretical predictions of wage restraints of the EITC are supported by Leigh (2010) who uses variation in state EITC supplements in the US to investigate how hourly wages are affected by the tax credit. He finds that there is a significant reduction in the wages following the tax credits at the lower end of the income distribution.\footnote{Using microdata from Sweden, Bennmarker et al. (2011) investigate the combined impact of an earned income tax credit, reduced generosity in the unemployment benefits, and reduce payroll taxes, on wages. They show that wages were restrained, although they are not able to disentangle the effects from the different policy changes.}

Research on the educational impact of an EITC is rather limited. While not looking at education, the paper by Heckman, Lochner and Cossa (2003) is related as it studies the impact of wage subsidies on on-the-job skill formation, distinguishing between a model with learning-by-doing and a model with training on the job. They show that on-the-job training models predict...
that wage subsidies reduce skill formation, while learning-by-doing models predict the opposite. A recent paper by Malul and Luski (2009) contrasts the effects of a minimum wage and an EITC on incentives to acquire human capital. They find that a minimum wage policy increases the professional level, as individuals need to "defend" themselves against unemployment, while the EITC reduces the incentive to invest in human capital because of the implicit tax created by the "phase out" of the EITC subsidy. In contrast to the existing literature, our paper highlight the impact of in-work benefits on educational attainment going through the general equilibrium effects, in particular through the impact on wages and job creation.

The rest of the paper is organized as follows. Section 2 presents a simple model of the labor market and analyses the implications of benefits conditional on work on labor market outcomes and educational choice. Section 3 introduces proportional taxation to finance benefits. Section 4 provides a numerical example to illustrate the mechanisms at work. In section 5 we show that the impact of conditioning benefits to work on labor market outcomes is robust to alternative models of wage settings. The last section concludes.

2 The Model

This section develops a simple model of a non-clearing labor market with unemployment featuring as an equilibrium outcome. More specifically, the labor market is characterized by trading frictions due to the costly and time-consuming matching of workers and firms.\footnote{The Nordic model is, in addition to its comprehensive welfare state and high taxes, associated with strong unions. This suggests that a union-firm wage bargaining model is very relevant. In section 5.1 we show that the results derived in this paper are qualitatively the same using a model where unions that represent workers bargain with firms over wages.}

The policy in consideration is benefits conditional on work. As mentioned, a crucial feature of many welfare policies in the Nordic countries is that benefits, in different ways, are conditional on employment. For example, highly subsidized childcare has, until recently, only been available to employed workers.\footnote{In 2002 unemployed workers got limited access to subsidized childcare in Sweden.} Moreover, some of the generous features of the paid parental leave schemes are only accessible to employed workers (Kolm and Lazear, 2010). For example, employed workers have the legal right to return to their job after a more than 12 month period of parental leave in Sweden.
In addition, we have the more recently introduced earned income tax credits, which by definition are exclusively targeted to employed workers. In the model, we will let one parameter, denoted $IWB$, capture the in-work benefits. To highlight the impact of these benefits, we will initially abstract from their financing (or equivalently, consider financing through lump-sum taxes).

The population is heterogeneous in terms of ability to acquire education. Abilities, $a$, are for simplicity distributed according to a standard uniform distribution, and individuals decide on the educational level they wish to pursue based on their individual ability. For simplicity the educational choice is between acquiring higher education, such as a college education, or not. Allowing for more educational levels in this setting would produce the same results.

Also labor force participation is endogenously determined. The population is heterogeneous in terms of how leisure when out of the labor force, $l$, is valued, which for simplicity is also distributed according to a standard uniform distribution and, for analytical tractability, is assumed to be independent from the distribution of ability.

### 2.1 Matching

Unemployed workers with a higher level of education will only search for jobs targeted to workers with higher educational level, and vice versa for workers with a lower level of education. Along the lines of Mortensen and Pissarides (1999), we can allow workers to look for jobs where they are 'over qualified', and thus allow firms to employ workers with an educational level above what is required for the job. In equilibrium, however, workers will not find it optimal to search for jobs where they are over qualified, and firms will not find it optimal to hire overqualified workers, leading to the endogenous outcome of a segmented equilibrium as modelled here.

The matching process of vacancies and unemployed job searchers within an educational category is captured by a concave and constant-returns-to-scale matching function of the Cobb-Douglas form, $X_j = v_j^{1-\eta}u_j^\eta$, where $X_j$ is the matching rate, $v_j$ is the vacancy rate, and $u_j$ is the unemployment

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6 Also benefits that are accessible when not in employment, like unemployment benefits, are strongly tied to market work due to the fact that their generosity, as well as the entitlement to them, often is based on earnings in previous periods. Although these benefits may weaken the incentives to quickly find a new job, the fact that entitlement and generosity are tied to earnings increases the returns to work.
rate. Index $j = L, H$ refers to the educational categories: low educated (L), and high educated (H). The matching, unemployment, and vacancy rates are defined relative to the labor force of the educational category.

The transition rate into employment for a worker with a given level of education is $X_j/u_j = \lambda (\theta_j) = \theta_j^{1-\eta}$, where $\theta_j = v_j/u_j$ denotes labor market tightness. Firms fill vacancies at the rate $X_j/v_j = q (\theta_j) = \theta_j^{-\eta}$. Higher labor market tightness, $\theta_j$, increases workers’ probability of finding a job, but reduces the probability of a firm finding a worker, i.e., $\lambda' (\theta_j) > 0$ and $q' (\theta_j) < 0$, where $\eta = -q' (\theta_j)/q (\theta_j)$ is the elasticity of the expected duration of a vacancy with respect to tightness.

### 2.2 Workers and Firms

Let $E_j$ and $U_j$ denote the expected present values of employment and unemployment of workers with a given educational level. The flow value functions for a worker $i$ with education $j$ can then be written:

\[
\begin{align*}
    rE_{ji} &= w_{ji} + IWB - s (E_{ji} - U_{ji}) - C_j (a_i), j = L, H, \quad (1) \\
    rU_{ji} &= \lambda (\theta_j) (E_{ji} - U_{ji}) - C_j (a_i), j = L, H, \quad (2)
\end{align*}
\]

where $r$ and $s$ are the exogenous discount and separation rates and $w$ is the wage. The term $IWB$ represents the in-work benefit which, by definition, is a benefit accessible only when employed.

To acquire higher education is costly in terms of effort to the individual, and potentially also in terms of pecuniary means. The cost of acquiring the low level of education is, for simplicity, normalized to zero, whereas the cost of attaining higher education is $C_i = c (a_i)$, where $c' (a_i) < 0$ captures that workers with high ability face lower effort costs of education.

There is a large number of small firms searching for workers with a particular education. Each firm employs one worker only. Let $J_j$ and $V_j$ denote the expected present values of an occupied and vacant job for a given level of educational requirements. The asset equations of a specific occupied job and a vacant job can then be written as:

---

7 We model the educational cost as a cost to acquire and maintain skill. This is a simplifying assumption and is not important for the results. The assumption enables us to use a model without having workers continuously being born and dying. Such a model would, however, generate the same qualitative expressions.
where \( k \) denotes vacancy costs and \( y_j \) denotes productivity. Firms that search for highly educated workers adopt a more advanced technology, which implies that the productivity will be higher in those firms once production starts. For the same reason will firms that search for less educated workers adopt a less advanced technology with the implication that productivity is lower once production gets started. Thus, we have \( y_H > y_L \).

### 2.3 Wage Formation and Tightness

Matching frictions create quasi-rents for any matched pair providing a scope for Nash bargaining. In symmetric equilibrium with free entry, i.e. with \( V_j = 0 \), the bargaining solution satisfies \( \beta J_j = (1 - \beta) (E_j - U_j) \), where \( \beta \) is the worker’s bargaining power. This condition and the flow value functions in (1)-(4) yield the wage rule:

\[
 w_j = \beta (y_j + k \theta_j) - (1 - \beta) IW B, \ j = L, H. \tag{5}
\]

From the free entry assumption facing firms, \( V_j = 0 \), and equations (3)-(4), tightness in equilibrium is determined by:

\[
 k (r + s) \\
 q (\theta_j) = (1 - \beta) (y_j + IW B) - \beta k \theta_j, \ j = L, H, \tag{6}
\]

where the equilibrium wage follows recursively from (5) once tightness is pinned down by (6).

In equilibrium, the flow into unemployment equals the flow out of unemployment for each category of educated workers.\(^8\) The equilibrium unemployment rate facing workers with a given level of education is:

\[
 u_j = \frac{s}{s + \theta_j^{1-\eta}}, \ j = L, H, \tag{7}
\]

\(^8\)Thus, \( s (1 - u_L) LFP_L = \lambda (\theta_L) u_L LFP_L \), and \( s (1 - u_H) LFP_H = \lambda (\theta_H) u_H LFP_H \), where \( LFP_j, j = L, H \), denotes the labor force for each educational category. The size of the labor force for each educational level is endogenous and will be determined in the next section. However, as the unemployment rates are independent of the size of the labor force it is of no importance how we note them here.
which depends positively on the separation rate and negatively on tightness.

We can now derive the following results:

**Proposition 1** An IWB will reduce wages, increase tightness, and reduce the unemployment rate for workers in all educational categories.

**Proof.** See appendix. ■

The in-work benefit, which by definition is conditional on work, simply increases the attractiveness of having a job. When holding a job becomes more attractive, wage demands will be moderated. This makes it more profitable for firms to open vacancies, which in turn, induces tightness to increase and the equilibrium rate of unemployment to fall.

Considering the effect on take-home pay, the following proposition summarizes the result:

**Proposition 2** An IWB will increase the take-home pay, $w_j + IWB$, $j = L, H$, although not by the full amount of the IWB.

**Proof.** See appendix. ■

The IWB will thus restrain wage demands leading to a smaller increase in take-home pay compared to the value of the benefit. As wage restraint stimulates job creation which, in turn, reduces the expected unemployment spells, more workers will transit from unemployment into jobs, thus leading to higher expected lifetime earnings.

### 2.4 Education, Labor Force Participation, and Employment

We assume that educational attainment only gives a payoff to workers in jobs.\(^9\) Thus, only workers that will participate in the labor market will consider whether they should acquire higher education or not. As workers enter the labor market into the state of unemployment, in their decision they compare the value of unemployment at different educational attainments. This comparison reveals that the educational gain in terms of a higher expected income needs to exceed the individual cost of acquiring education, in order

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\(^9\)Education could, of course, also have some consumption value. Accounting for this would not change the results and one could consider the cost of education as modelled here to be net of any benefit enjoyed regardless of labor market status.
for the individual to attain additional education. Thus, workers with very low ability will not find it worthwhile to proceed to higher education, whereas very high ability workers will find it more than worthwhile to do so.

Using (1) and (2), we can write the condition determining the ability level of the marginal worker as:

$$r_U = r_H (\hat{a}) ,$$

where \(\hat{a}\) is the ability level of the worker which is indifferent between acquiring higher education or not. Thus, all workers that participate in the labor market and have an ability level equal to or higher than \(\hat{a}\) will proceed to higher education, whereas workers with an ability level below \(\hat{a}\) (and high ability workers who will not participate in the labor market) will not.

The equation in (8) can be rewritten using (1), (2), together with the first order conditions for wages, and the equations in (4) under the assumption of free entry. This yields:

$$c (\hat{a}) = \frac{\beta k}{(1 - \beta)} (\theta_H - \theta_L) .$$

The right hand side of equation (9) is the expected income gain of getting a college education. In order to guarantee that at least some workers acquire additional education, expected income must increase with education. Ignoring the IWB, this can be shown to hold formally by use of the equations in (6) where \(\theta_H > \theta_L\) if \(y_H > y_L\). The IWB may affect the individual incentives to acquire education by affecting tightness, and thus the expected income, in a different way at the two levels of education. This is the particular issue up for investigation here.

By assuming that the cost function fulfills \(\lim_{a \to 1} c(a) = 0\) and \(\lim_{a \to 0} c(a) = +\infty\), we can focus on the non-trivial case where at least some workers find it worthwhile to acquire higher education while others don’t. Although equation (9) is used to pin down whom in the labor force will proceed to higher education and whom will not, to get an expression for the number of workers in the population with higher education, we also need to know whom will participate in the labor market.

---

By use of (1) and (2), the value of unemployment is written as 

$$rU_j = (1 - \phi(u_j)) [w_j + IWB_j] - C_j (a_j),$$

where \(1 - \phi(u_j)\) can be interpreted as the expected time in employment. The weight, \(1 - \phi(u) = \frac{\lambda(\theta_j)}{\lambda(\theta_j) + \lambda(\theta_j)}\), reduces down to the employment rate, \(1 - u_j\), when the discount rate approaches zero.
A worker enters the labor force into the state of unemployment by becoming available to the labor market. It will be worthwhile to enter the labor market if the returns of entering exceed the returns from not entering. Let \( N \) denote the expected present value of non-participation. The flow value of not participating in the labor force is given by the per period real value of leisure, \( l \), which differs across workers.

The flow value function for non-participation, \( rN_i = l_i \), is then added to the flow value functions for employment and unemployment in (1)-(2). The assumption is that it is not important if the worker has a higher education or not for the workers evaluation of leisure when out of the labor force, and thus subindex \( j \) is absent. The function determining the valuation of leisure which makes the worker indifferent between participating and not participating in the labor market is given by the following continuous function:

\[
\hat{l} = \begin{cases} 
  rU_L & \text{if } a < \hat{a}, \\
  rU_H(a) & \text{if } a \geq \hat{a}.
\end{cases}
\]

This function can be rewritten by use of the flow equation in (2) in symmetric equilibrium, the Nash bargaining solutions, \( \beta J_j = (1 - \beta) (E_j - U_j) \), and the free entry condition, \( V_j = 0 \), together with (4), \( j = L, H \), as\(^{11}\):

\[
\hat{l} = \begin{cases} 
  \frac{\theta_L \beta k}{(1 - \beta)} & \text{if } a < \hat{a}, \\
  \frac{\theta_H \beta k}{(1 - \beta)} - c(a) & \text{if } a \geq \hat{a}.
\end{cases}
\]

A worker that would not proceed to higher education when participating, i.e. a worker with \( a < \hat{a} \), will not find it worthwhile to participate in the labor market if his or her valuation of leisure exceeds \( \hat{l} = \frac{\theta_L \beta k}{(1 - \beta)} \). Workers with very high ability, on the other hand, may choose to participate in the labor market even if they have a high valuation of leisure. This follows as their pay-off on the labor market is very high accounting for that they fairly effortlessly can acquire higher education and reap a higher expected income.

\(^{11}\)As we use the standard uniform distribution for \( \hat{l} \), the value of the function should not exceed unity. For simplicity we assume that this is not binding, that is, this threshold level is lower than unity. In what follows we will assume to be in an interior solution.
Figure 1: Labor force participation and educational attainment

Figure 1 illustrates the choice of participation and education in the population. Areas $A$, $B$ and $C$ in the left panel give the stock of workers participating in the labor market. Areas $B$ and $C$ give the stock of workers that will acquire higher education. Area $D$ captures workers that would, in case of participation, acquire higher education. However, as they will not participate in the labor market due to their high valuation of leisure, they will not acquire higher education as education is costly and only gives a payoff when working. This implies that the labor force participation rate is larger for workers with high ability. More specifically, the labor force participation rate is given by the area $(B + C) / (B + C + D)$ for workers with high ability, i.e. $a \geq \hat{a}$, and by the area $A / (A + E)$ for workers with lower ability, i.e. $a < \hat{a}$. The labor force participation rate for workers with low education is $A / (A + D + E)$, while by assumption workers acquire high education only if they intend to participate in the labor market.

It is then straightforward to derive labor force participation, $LFP$, the stock of educated workers, $Edu$, and total employment, $Emp$, in the economy as:
\[ LFP = \int_0^1 \hat{\alpha} k \theta_L \frac{(1 - \hat{\alpha}) \beta k \theta_H}{(1 - \beta)} \, da + (1 - \hat{\alpha}) \beta k \theta_H - \int_\hat{\alpha}^1 c(a) \, da, \quad (10) \]

\[ Edu = \frac{(1 - \hat{\alpha}) \beta k \theta_H}{(1 - \beta)} - \int_\hat{\alpha}^1 c(a) \, da, \quad (11) \]

\[ Emp = (1 - u_L) \hat{\alpha} \beta k \theta_L + (1 - u_H) \left[ \frac{(1 - \hat{\alpha}) \beta k \theta_H}{(1 - \beta)} - \int_\hat{\alpha}^1 c(a) \, da \right]. \quad (12) \]

The effect of in-work benefits on labor force participation, education, and employment are summarized in the following proposition.

**Proposition 3** An IWB will increase labor force participation, the stock of workers with higher education, and aggregate employment.

**Proof.** See appendix. ■

Labor force participation increases when benefits are conditional on work because job creation is stimulated. This tends to increase the expected income of participation both by reducing the expected unemployment spells and by increasing the take home pay for workers at all educational levels. Perhaps more surprisingly is that benefits conditional on work increase the incentives for workers to acquire higher education. There are two reasons for this. First, an IWB will increase the relative demand for workers with a higher level of education. The improved labor market opportunities for

\[ ^{12} \text{Labor market opportunities improves for all workers. However, the increased demand for workers will be dampened by the fact that it will take a longer time to fill a vacancy when the competition for workers becomes higher. More fierce competition for workers will hit firms hiring low educated more severely, leading to that labor market conditions particularly improve for workers with higher education. This is because tightness is lower for low educated workers and, thus, one more firm entering with a vacancy will increase the expected time to fill a vacancy relatively more for these workers compared to workers with higher education. To put it in another way, there is more competition for workers with a college education relative to workers with no higher education. Then, an additional vacancy has a relatively smaller negative externality on other firms with an open vacancy for college educated. This increases the relative demand for college educated workers, and therefore increases the incentives for workers to proceed to higher education.} \]
highly educated workers relative to less educated workers increase the attractiveness of acquiring higher education. Second, as an IWB increases the return to participation, some workers with very high ability will find it worthwhile to leave their state of non-participation and join the labor force. As these workers have a very high ability, they will fairly effortlessly acquire higher education first. The right panel in Figure 1 illustrates how labor force participation and the stock of workers with higher education are affected by an IWB. What can be noticed is that there is a leftward shift in the line denoting the ability level where workers are indifferent between acquiring higher education or not. This corresponds to the first of the two reasons provided above for the increase in higher education. The second reason is represented by the upward shift in the line denoting, for each ability level, the value of leisure leaving a worker indifferent between participating or not in the labor market.

Aggregate employment increases for three sets of reasons. First, employment increases because benefits conditional on work increase job creation which reduces the unemployment rate for all educational categories. Second, as the benefits improve the incentives to acquire education, employment increases as the expected unemployment spells are shorter among highly educated workers. Third, as labor force participation increases, employment increases because some of the workers entering the labor market will become employed.

3 IWB Financed by Proportional Tax

In this section, we study the effects of an IWB when it is financed through distortionary income taxation. There are a number of ways to finance the benefit. Below we formally consider the case when taxation is proportional.

The flow value function for employment in (1) is now written:

$$rE_{ji} = w_{ji}(1 - t) + IWB_{ji} - s(E_{ji} - U_{ji}) - C_j(a_i), \quad j = L, H,$$

where $t$ is a proportional income tax rate. The rest of the equations in (1)-(4) and the flow value of participation remain unchanged. To derive the equilibrium equations determining wages and tightness, we follow the

13 The IWB being financed by payroll taxation would yield the same results.
same procedure as in the basic setting, taking into account that the Nash bargaining solution now satisfies \( \beta (1 - t) J_j = (1 - \beta) (E_j - U_j) \) for \( j = L, H \). This yields the following equations:

\[
    w_j = \beta (y_j + k\theta_j) - (1 - \beta) \frac{IWB}{1 - t}, \quad j = L, H, 
\]

\[
    \frac{k(r + s)}{q(\theta_j)} = (1 - \beta) \left( y_j + \frac{IWB}{1 - t} \right) - \beta k\theta_j, \quad j = L, H. 
\]

The tax rate will now have a direct effect on the wage, \( w_j \), and tightness, \( \theta_j \). In fact, as the tax rate, \( t \), increases, the wage demands are reduced and tightness will increase.\(^{14}\) This follows as a higher tax rate reduces the value of the earned income, but it will not reduce the value of the \( IWB \). Thus, an increase in the tax rate increases the importance of the \( IWB \) as a source of income when employed and will thus work in a similar way as an increase in the \( IWB \).

Taxes will also have a direct impact on the incentives to acquire education. The ability level of a worker on the labor market that is indifferent between acquiring or not acquiring higher education when participating in the labor force is given by equation

\[
    c(\tilde{a}) = \frac{\beta k(1 - t)}{1 - \beta} (\theta_H - \theta_L). \tag{16}
\]

The equation determining the valuation of leisure which makes the worker indifferent between participating and not participating in the labor market is now given by:

\[
    \hat{l} = \frac{\theta_L \beta k (1 - t)}{(1 - \beta)} \text{ if } a < \tilde{a}, 
\]

\[
    \hat{l} = \frac{\theta_H \beta k (1 - t)}{(1 - \beta)} - c(a) \text{ if } a \geq \tilde{a}. \tag{17}
\]

It is then straightforward to derive labor force participation, \( LFP \), the stock of educated workers, \( Edu \), and total employment, \( Emp \), in the economy as:

\(^{14}\) This is a standard result in models of the equilibrium rate of unemployment. The tax rates will have an impact on producer costs, tightness and unemployment if there is a fixed compensation or cost on the employed or unemployed workers’ side. See Pissarides (1998).
The total wage bill in the economy is given by
\[
WageBill = w_L (1 - u_L) \frac{\bar{\alpha} k \theta_L (1 - t)}{(1 - \beta)} + w_H (1 - u_H) \left[ \frac{(1 - \bar{\alpha}) \beta k \theta_H (1 - t)}{(1 - \beta)} - \int_{\bar{a}}^1 c(a) da \right],
\]
where the expressions for wages are given by (14). Finally, the government budget constraint for an IWB financed by proportional taxation is given by
\[
t \ast WageBill = IWB \ast Emp.
\]
workers increases with the IWB, more workers tend to proceed to higher education. However, the fact that the taxation directly reduces the payoff from education will reduce the incentives for workers to attain higher education. Taxes will also have a direct negative effect on labor force participation and employment. Next we turn to numerical simulations to illustrate these effects.

4 A Numerical Example

In this section we calibrate the model and simulate the impact of an in-work benefit on the main variables of interest. Given the simplicity of the model we consider these calculations as illustrative of how the mechanisms highlighted in the previous sections work, without aiming to provide specific guidance in terms of the empirical impact of having benefits conditioned on work.

4.1 Calibration

The month is the basic time unit. To ensure that the labor force participation rate for low skilled workers is always less than 1, productivity for low skilled workers, $y_L$, is fixed at 0.75. Worker bargaining power, $\beta$, is set to 0.6, while the real interest rate $r$ is 0.0025. $\eta$ equals 0.5, while parameters $k$, $s$, and $y_H$ are set to replicate an average duration of unemployment for the low educated of 6 months and an unemployment rate of 0.08 for low skilled, and 0.06 for high skilled in absence of an in-work benefit. For analytical convenience, we assume that the cost of acquiring high school education is given by

$$c(a_i) = 2\delta (1 - a_i).$$

Then, the expression $\int_{\hat{a}}^{1} c(a) \, da$ is given by

$$\int_{\hat{a}}^{1} c(a) \, da = \delta (1 - \hat{a})^2.$$
\[
\hat{a} = 1 - \frac{\beta k (\theta_H - \theta_L)}{2\delta (1 - \beta)}.
\]  

The share of people with high education in the population, \( Edu \), is given by (11). The parameter \( \delta \) is set to replicate a skill distribution in absence of an in-work benefit with \( Edu = 0.35 \). The table below summarizes the parametrization, with further details about the calibration provided in the Appendix.

<table>
<thead>
<tr>
<th>( y_L )</th>
<th>( y_H )</th>
<th>( \beta )</th>
<th>( k )</th>
<th>( r )</th>
<th>( s )</th>
<th>( \eta )</th>
<th>( \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>1.34</td>
<td>0.6</td>
<td>15.4</td>
<td>0.0025</td>
<td>0.014</td>
<td>0.5</td>
<td>0.718</td>
</tr>
</tbody>
</table>

### 4.2 Numerical Results

We first analyze the effect of benefits conditional on work without distortionary taxation. Figure 2 plots the main variables of interest as a function of an in-work benefit going from 0 to 0.25, equivalent to one third of the productivity of people with low education. As expected, tightness increases as wages fall. The share of the population with higher education increases, going from 35% of the population with no benefits to 44% when benefits are at 0.25. Unemployment rates also fall, both for the highly educated (from 6% to 5.5%) and for people with low education (from 8% to 6.9%). Beside the unemployment rate for each of the two educational categories, the overall unemployment rate is also influenced by the composition of the workforce, and it decreases from 7% to 6.3%. Also as expected, the labor force participation rate increases and, as a result of a higher participation and lower unemployment, the total employment rate in the population increases.

When we consider in-work benefits financed by proportional taxation, the picture is rather different (see Figure 3). The tax rate corresponding to the maximum in-work benefit of 0.25 is 28%. The most striking difference is that higher benefits now reduce the incentives to acquire higher education, as they imply a higher fiscal pressure. As showed by Proposition 4, the unemployment rate falls, both for the highly educated (from 6% to 5.3%) and for people with low education (from 8% to 6.6%). Overall, unemployment falls from 7% to 6.2%. The overall labor force participation rate and employment rate are rather stable. This is due to the combination of a positive effect for low-educated and a negative composition effect as less people acquire higher education.
Figure 2: IWB - No Distortionary Tax

Figure 3: IWB - Distortionary Tax
To appreciate the difference made by benefits conditional on work even when they are financed by distortionary taxation, in Figure 4 we show the impact when distortionary taxes are instead used to finance a lump-sum transfer available to all. To facilitate the comparison, the x-axis in each graph is still $IWB$. This should be interpreted as the tax rate resulting in the corresponding level of in-work benefits when taxes are used to finance them, so that $IWB = 0.25$ corresponds to a tax rate of 28%. As expected from (14) and (15), when taxes are used to finance a lump-sum transfer, they have no effect on tightness, wages, and unemployment rates. They do have a very strong negative effect on the incentive to acquire education. At $t = 28\%$, the share of the population with higher education falls from 35\% to 18\% when benefits are not conditional on work, as opposed to 24.7\% when benefits are conditional on work. Because of this composition effect, the overall unemployment rate increases with taxation, even if the unemployment rate for the two educational categories is unaffected. Labor force participation decreases, both because of the composition effect and because of the negative incentive to participate for the low-educated. As a result, employment also falls. Instead, with benefits conditional on work, both participation and employment remained substantially stable, while unemployment declined.

5 Alternative Models of Wage Setting

In this paper we underline the impact of benefits conditional on work stemming from wage moderation. This mechanism is very robust to various possible assumptions about the way wages are set. The induced wage moderation, increased hires, and the lower unemployment rate follow in all basic standard models featuring unemployment as an equilibrium outcome. In an efficiency wage model, for instance, benefits conditional on work would provide firms with an instrument to discipline workers. Thus, the firm would not need to pay workers as high wages in order to prevent them from shirking, since the threat of losing the benefit when fired will do the job. Also in a static or dynamic union-firm wage bargaining model of the Right-to-manage type, or a Monopoly union model, would the same result materialize.\footnote{In fact, also a model with a perfectly competitive labor market will do the job of explaining the fall in wages and the job creation. However, such model can not explain the observed low unemployment rate in the Nordic countries as involuntary unemployment is absent.} As a union-
firm wage bargaining model is particularly relevant for the Nordic countries, in this section we set up such a model to demonstrate the effects of in-work benefits on the main labor market outcomes.

5.1 Union-Firm Wage Bargaining

One feature of the Nordic countries is the strong presence of unions. Here we assume that unions, representing workers at the firm level, bargain with firms over the wage. However, once the wage is set, the firms will decide on how many workers to hire. The problem is solved through backward induction. Thus, at the second stage, firms decide on the number of workers to hire, \( N \), so as to maximize their profit, \( \Pi \), taking the wage, \( w \), as predetermined. Firms then solve \( Max_N \ \Pi = N^\alpha - wN \), where, for simplicity, the production technology is captured by a Cobb-Douglas function, and the individual index is dropped. The firm’s demand for labor is then given by

\[
N = \left( \frac{w}{\alpha} \right)^{-\frac{1}{1-\alpha}}, \quad \text{with} \quad \varepsilon = \frac{dN}{dw} \frac{w}{N} = \frac{1}{1-\alpha}.
\]
Wage are determined through decentralized union-firm Nash bargaining. The union's utilitarian objective function is captured by \( \Omega = N (w + IW B) + (\bar{N} - N) B \), where \( \bar{N} \) is the number of members, and \( B \) capture the unemployment benefit which is received by all unemployed workers. The union face a trade-off in that a higher wage improves the well-being of their employed members, but a higher wage will, at the same time, render more members unemployed which reduces the well-being of those members. As all workers receive unemployment benefits in case the bargain breaks apart, \( \bar{N}B \) captures the union fallback position, leaving \( N (w + IW B - B) \) as the union "rent" contribution in the bargain. As the firm makes no profit in case the bargain breaks apart, the Nash product is given by

\[
\Lambda = [N (w + IW B - B)]^\lambda [N^\alpha - w N]^{1-\lambda},
\]

where \( \lambda \) is the relative bargaining strength of the union compared to the firm, \( \lambda \in (0, 1] \). The Nash product is maximized by choosing \( w \), accounting for that \( N = N (w) \) through (28). From the first order condition, the following wage setting curve can be derived:

\[
w = \frac{\alpha + \lambda (1 - \alpha)}{\alpha} [B - IW B],
\]

where it is clear that also in a union-firm wage bargaining model will wage moderation follow due to in-work benefits. The intuition is analogous to the one found in the basic matching model. As an in-work benefit increases the value of having a job, the union wage demand is restrained because they want more of their member to be in jobs. Employment increases as more workers are hired when wages are lower (see (28) which determines employment in the economy with the number of firms normalized to unity). The model collapses to a Monopoly union model when \( \lambda = 1 \) is imposed, i.e. all bargaining power is given to the union.

Note that using flow value functions in this union-firm wage bargaining set-up would yield the same steady state result as this simple static model of decentralized bargains or union wage setting.

6 Conclusions

As mentioned in the introduction, the Nordic countries are characterized by a comprehensive welfare state financed through taxes and social security contribution, with public expenditures amounting in 2011 to more than half of
GDP. However, one could argue that high social spending financed through high taxes characterizes many other countries, especially in continental Europe. Indeed, in 2011, general government total expenditure amounted to 49.1% of GDP in the EU-27 (the average for the period 2002-2011 is 47.7%), with the largest share, 43.4%, of this expenditure devoted to the redistribution of income through social transfers in cash or in kind (Eurostat, 2012). What this paper has emphasized is that one feature of the Nordic model, namely the fact that many of the welfare arrangements are strongly tied to work, makes a difference. In particular, we have underlined how benefits structured in such a way induce job creation and lower the unemployment rate through their wage moderating effect. Moreover, they do provide incentives to pursue further education, and increase labor force participation and employment. For a given amount of tax revenue, the labor market outcome would be very different if spending would instead be directed towards programs that are not conditional on work, or, even more, towards programs conditional on not working.

We have also emphasized one crucial aspect behind the long-term sustainability of the Nordic model, namely its effect on incentives to pursue higher education. As Andersen (2008) noticed in his discussion on the prospects and challenges of the Nordic model, "a compressed wage structure and high taxation have a negative effect on the return to education". This paper shows how benefits conditional on work mitigate this negative incentive and may contribute, together with other policies like the public financing of education, to maintain the educational attainment in the Nordic countries at high levels.16

References


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16 According to OECD data (OECD, 2011), the percentage of the population with at least upper secondary education or tertiary education in the Nordic countries is above the OECD and the EU-21 averages both for the working age population (25-64) and for youth (25-34).


7 Appendix

A1 Proofs of propositions

Propositions 1-4. Differentiation of the equations in (6) yields: \( \frac{d\theta_j}{dIWB} = \frac{(1-\beta)}{\beta k} \left( 1 + (r + s) \frac{\eta}{\beta \theta_j^{\eta - 1}} \right)^{-1} > 0 \). Then differentiation of (7) yields \( \frac{du_j}{dIWB} = \frac{(1-\theta_j\eta)}{(s+\theta_j^{\eta - 1})^2} \frac{d\theta_j}{dIWB} < 0 \). Differentiation of (5) making use of the expressions for \( \frac{d\theta_j}{dIWB} \) yields \( dIWB \) \( dw_j \), and employment is considered through differentiation of (9)-(12) and using the previous proposition. Differentiation of (10) yields \( \frac{dLFP}{dIWB} = \hat{\alpha} \beta k \frac{d\theta_j}{dIWB} + \frac{(1-\hat{\alpha})\beta k}{(1-\beta)} \frac{d\theta_H}{dIWB} > 0 \) as changes in \( \hat{\alpha} \) will have no impact on \( LFP \). Differentiation of (9) using the expression for \( \frac{d\theta_j}{dIWB} \) and the facts that \( c'(\cdot) < 0 \) and \( \theta_H > \theta_L \) yields \( \frac{da}{dIWB} = \frac{1}{c'(\hat{\alpha})} \frac{(1-\beta)}{\beta_k} \left( \left( 1 + \frac{(r+s)\eta}{\beta \theta_H^{\eta - 1}} \right)^{-1} - \left( 1 + \frac{(r+s)\eta}{\beta \theta_L^{\eta - 1}} \right)^{-1} \right) < 0 \).

Differentiation of (11) and (12) yield \( \frac{dEdu}{dIWB} = \frac{d\alpha}{dIWB} \frac{(1-\beta)\beta_k \theta_H}{(1-\beta)} + \frac{(1-\hat{\alpha})\beta k}{(1-\beta)} \frac{d\theta_H}{dIWB} > 0 \) and \( \frac{dEmp}{dIWB} = -\frac{du_L}{dIWB} \frac{(1-\beta)\beta_k \theta_L}{(1-\beta)} - \frac{du_H}{dIWB} \left[ \frac{(1-\hat{\alpha})\beta_k \theta_H}{(1-\beta)} - \int_{\hat{\alpha}}^{1} c(a) \, da \right] - \frac{d\alpha}{dIWB} \frac{(1-\beta)\beta_k \theta_L}{(1-\beta)} \left( u_L - u_H \right) + (1 - u_L) \frac{(1-\beta)\beta_k \theta_H}{(1-\beta)} > 0 \) using that \( \frac{(1-\hat{\alpha})\beta_k \theta_H}{(1-\beta)} - \int_{\hat{\alpha}}^{1} c(a) \, da > 0 \) as that is the labor force participation of highly educated workers, and that \( \frac{da}{dIWB} < 0 \) and \( \frac{d\theta_H}{dIWB} > 0 \). This proves proposition 3.

Differentiation of (15) yields \( \frac{d\theta_j}{dIWB} = \frac{(1-\beta)}{(1-t)^2} \left[ 1 + (r + s) \frac{\eta}{\beta \theta_j^{\eta - 1} + \beta k} \right] > 0 \) if \( \frac{dt}{dIWB} > 0 \). From (7) it then follows that \( \frac{du_j}{dIWB} = -\frac{(1-\eta)\theta_j^{-\eta}}{(s+\theta_j^{\eta - 1})^2} \frac{d\theta_j}{dIWB} < 0 \) if \( \frac{dt}{dIWB} > 0 \). Also, differentiation of (14) and by use of the expression for \( \frac{d\theta_j}{dIWB} \) we have

27
\[ \frac{dw_j}{dIWB} = -\frac{(1-\beta)}{(1-\epsilon)} \frac{k(r+s)\theta^{\eta-1}}{\beta k + k(r+s)\theta^\eta} [1 - t + IWB \cdot \frac{dt}{dIWB}] < 0 \text{ if } \frac{dt}{dIWB} > 0. \]

It is clear that \( \frac{dt}{dIWB} > 0 \) from the direct effect in (24) (ignoring the indirect effects working through the tax bases of employment and the wage bill). Accounting for the dynamic effects implies that the government revenue can both increase and fall with higher taxes. By assuming \( \frac{dt}{dIWB} > 0 \), we assume that the dynamic effects working through the tax bases are not dominating the direct effects. This proves proposition 4.

**A2 Calibration - Derivation**

The parameters \( k \) and \( s \) are set to replicate, in absence of an \( IWB \), an unemployment rate for the low skilled of \( \bar{u}_L \) and an average duration of unemployment for the low skilled of \( d_{uL} \) months. Given (7) and the fact that \( d_{uL} = \frac{1}{\lambda(\bar{u}_L)} = \frac{1}{\theta_L^{-1}} \), we get

\[ s = \frac{\bar{u}_L}{d_{uL}(1-\bar{u}_L)}, \]  

and

\[ \theta_L = \left(s \frac{1-\bar{u}_L}{\bar{u}_L}\right)^{\frac{1}{\eta}}. \]  

Then, using (6) and the fact that \( q(\theta_j) = \theta_j^{-\eta} \), we can get the following expression for \( k \)

\[ k = \frac{(1-\beta) y_L}{(r+s) \theta_L^\eta + \beta \theta_L}. \]  

Given \( \bar{u}_H \) and using the expression for \( \theta_H \) corresponding to (30), it is possible to derive the implied \( \theta_H \). Using (6) again and the value for \( \theta_H \) derived above, it is then possible to calculate \( y_H \)

\[ y_H = \frac{k(r+s)\theta_H^{\eta} + \beta k \theta_H}{1-\beta}. \]  

Finally, using (27) and the fact that the share of people with higher education, \( Edu \), is given by (11), we get, after using (26),

\[ \delta = \frac{(\beta k)^2(\theta_H-\theta_L)(\theta_H+\theta_L)}{4(1-\beta)^2 Edu}. \]
A3 Simulations with Financing

Using equations (15), (16), and (24), when financing is taken into account and for a given level of \( IWB \), we have four unknowns \((\theta_H, \theta_L, \bar{a}, t)\) solving the system of equations

\[
\begin{align*}
  k (r + s) \theta_L^\eta - (1 - \beta) \left( y_L + \frac{IWB}{1 - t} \right) + \beta k \theta_L &= 0 \quad (33) \\
  k (r + s) \theta_H^\eta - (1 - \beta) \left( y_H + \frac{IWB}{1 - t} \right) + \beta k \theta_H &= 0 \quad (34) \\
  1 - \bar{a} - \frac{\beta k (1 - t)}{2 \delta (1 - \beta)} (\theta_H - \theta_L) &= 0 \quad (35) \\
  (1 - \bar{a}) \left( \frac{\theta_H^{1 - \eta}}{s + \theta_H^{1 - \eta}} \right) \left[ \beta (y_H + k \theta_H) - (1 - \beta) \frac{IWB}{1 - t} - \frac{IWB}{t} \right] + \\
  + \bar{a} \left( \frac{\theta_L^{1 - \eta}}{s + \theta_L^{1 - \eta}} \right) \left[ \beta (y_L + k \theta_L) - (1 - \beta) \frac{IWB}{1 - t} - \frac{IWB}{t} \right] &= 0 \quad (36)
\end{align*}
\]

We can use equation (33) to solve for \( IWB \) as a function of \( \theta_H \) and \( t \),

\[
IWB = \frac{k (r + s) \theta_H^\eta + \beta k \theta_H - y_H (1 - \beta)}{(1 - \beta)} (1 - t). \quad (37)
\]

We can then use this expression to replace \( IWB \) in (34) to get

\[
(1 - \beta) (y_H - y_L) - k (r + s) (\theta_H^\eta - \theta_L^\eta) - \beta k (\theta_H - \theta_L) = 0. \quad (38)
\]

We can use expression (35) to replace \( \bar{a} \) in (36), expression (37) to replace \( IWB \) in the first line of (36) and a similar expression using (34) instead of (33) to replace \( IWB \) in the second line of (36). This gives the following equation

\[
\begin{align*}
  &\left( \frac{\theta_L^{1 - \eta}}{s + \theta_L^{1 - \eta}} \right) \theta_L \left[ 2 \delta (1 - \beta) - \beta k (1 - t) (\theta_H - \theta_L) \right] * \\
  &\cdot [(1 - \beta) y_L - (1 - \beta t) k (r + s) \theta_L^\eta - (1 - t) \beta k \theta_L] + \\
  &+ \left( \frac{\theta_H^{1 - \eta}}{s + \theta_H^{1 - \eta}} \right) \frac{1}{2} \beta k (1 - t) (\theta_H - \theta_L) (\theta_H + \theta_L) * \\
  &\cdot [(1 - \beta) y_H - (1 - \beta t) k (r + s) \theta_H^\eta - (1 - t) \beta k \theta_H] = 0 \quad (41)
\end{align*}
\]
Using (38) and (39), we have a system of two equations in two unknowns $(\theta_H, \theta_L)$ that we solve numerically. We can then get $\tilde{a}$ from (35) and $IWB$ from (37).