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## Transition from a pay-as-you-go to a fully-funded pension system: The case of differing individuals and intragenerational fairness

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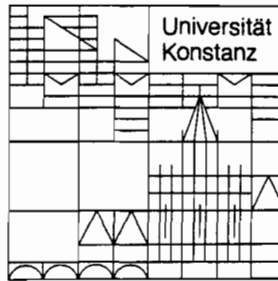
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**Fakultät für  
Wirtschaftswissenschaften  
und Statistik**

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Johann K. Brunner

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a Pay-as-You-Go to  
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Diskussionsbeiträge

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**Johann K. Brunner**

**Serie I - Nr. 266**

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## Transition from a pay-as-you-go to a fully-funded pension system: The case of differing individuals and intragenerational fairness\*

### *Abstract*

In recent contributions to the theory of public pension systems it was argued that a Pareto-improving transition from an established unfunded pension system to a funded one is possible. This result is derived in an overlapping-generations model with identical individuals. In the present study an extended model, with differing individuals, of an intragenerationally fair unfunded pension scheme is introduced. Within this more realistic framework it is shown that, in general, a Pareto-improving transition to a funded system is not possible, because any instrument applied for the financing of pensions in the phase of transition involves intragenerational redistribution.

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\* I am grateful to Josef Falkinger for helpful comments.

## I Introduction

Since the end of the nineteenth century, and especially since world war II, most industrialized countries have established a public pension system following the pay-as-you-go method, where contributions of the active are immediately given as transfers to the retired. It is well-known that the rentability of this system is equal to the growth rate of the economy (Aaron 1966), while the return on contributions to a funded system clearly equals the interest rate, that is, in a competitive capital market, the marginal productivity of capital. As both the growth rate of population and that of labor productivity, which determine the growth rate of the economy, have declined in the last decades, the question arises whether a transition from the established unfunded to a funded system can be performed. The problem is that in the phase of transition the active would have to bear a two-fold burden: paying for the pensions of the retired and accumulating a sufficient stock of capital, from which their own pensions could be financed. Hence, the usual opinion says that such a transition would make at least one generation worse off than it would be with the given system.

However, in recent studies some authors have argued that a Pareto-improving transition from an unfunded to a funded system is in fact possible (Homburg 1990, Homburg and Richter 1990, Breyer and Straub 1993). Their argument rests on the observation that contributions to the unfunded system are usually levied in the form of a tax on labor income and distort, thus, the labor-leisure decision of the individuals, causing a deadweight-loss. A change of the system would mean that this deadweight-loss is removed, and the corresponding surplus could be used to design a Pareto-improving transition.

More precisely, the idea of these authors is to substitute, in the phase of transition, the income tax by a lump-sum tax, which could then gradually be reduced to zero, because of

the gains in efficiency. The intention of the present study is to show that this argument is based on an incomplete consideration of the reason why contributions are imposed as a tax on labor income in the existing system. As is obvious from taxation theory, the introduction of an income tax instead of a lump-sum tax only makes sense if the economy consists of different individuals, who should be treated in a different way, but who cannot be identified by their primary characteristic (say, ability). Income is then used as a surrogate variable for distinguishing between individuals.

In view of this, an appropriate analysis requires the formulation of an overlapping-generations model with differing individuals. In such a model, two motives for a differentiated treatment of individuals in the public pension system can be investigated: (i) intragenerational redistribution or (ii) intragenerational fairness. I have shown in another paper (Brunner 1993) that if a pay-as-you-go pension system performs intragenerational redistribution (in the form that contributions are proportional to income but benefits are uniform), then, in general, no Pareto-improving transition to a funded system exists. In this study I argue that the same is true if the unfunded pension system follows intragenerational fairness, i.e., if contributions as well as pensions of an individual are proportional to his income in the active period. Such a system was studied in Breyer and Straub (1993); their positive result on the existence of a Pareto-improving transition is based on a comparison of the conditions for a first-best system with the properties of a distorted system, without any account of institutional restrictions (as they acknowledge themselves, p. 89). If the question, which instruments are available for a transition, is considered explicitly, the positive result turns out to be valid only under rather restrictive assumptions, such as nearly identical individuals.

The condition for a Pareto-improving transition requires that no individual of any generation may be made worse off, which obviously represents a more severe restriction in

the case of differing individuals. The essential point of this study is to show that any change of the system, intended to reduce the deadweight-loss, normally involves intragenerational redistribution and conflicts, therefore, with the Pareto criterion. The main argument goes as follows: If the growth rate of the economy is lower than the interest rate, any contribution to the unfunded pension system reduces life-time income. Clearly, the loss is the higher, the higher the contribution. Thus, given that individuals have different incomes, even an intragenerationally fair system (with contributions and benefits proportional to income) imposes differing losses of life-time income on them (as does, obviously, a redistributive system). As a consequence, if this system is replaced by one with lump-sum contributions (and benefits), the condition for a Pareto-improvement requires that no individual has to pay "considerably" more than before. That is, in a Pareto-improving first-best system, lump-sum contributions would have to be in some way differentiated between individuals (as in the proportional system). However, as was mentioned above, this instrument is not available, because the government cannot be assumed to have precise knowledge necessary to identify individuals by their primary characteristic.

As an alternative method (instead of using lump-sum taxation) of initiating a transition to a funded system, one could think of replacing the proportionality rule by a more sophisticated method of determining income-related contributions and benefits. Such a change might also reduce the excess burden caused by the unfunded system, and this gain might allow a Pareto-improving transition. In this study we consider an affine (or "linear") relation between contributions and gross income. But again, the introduction of such a rule would, in general, represent an act of redistribution, making at least one individual worse off. Altogether, we find that, except in special cases, no implementable instrument exists which could be used for a Pareto-improving transition from an unfunded to a funded pension system.

In Section II an overlapping-generations model with differing individuals will be formulated, which represents the framework for the detailed presentation of the argument. Section III contains the main analysis of instruments for a transition, where III.1 deals with lump-sum taxation and III.2 with linear income taxation; III.3 contains remarks on debt-financed transition. In Section IV the results are summarized and some concluding comments are added.

## II The model

As was mentioned in the introduction, a necessary prerequisite for an adequate formulation of the problem is to consider a model with different individuals. For simplicity we assume that only two types of individuals exist (the conclusion would be the same with more types), who are characterized by their abilities  $a^1 < a^2$ . As is usual in the simple overlapping-generations model, every individual lives for two periods, he is active in the first and retired in the second. The ability parameter influences the wage rate  $w_t^i = a^i w_t$  of the individuals, where  $w_t$  denotes the wage rate per efficiency unit of labor,  $t$  is a period index. The preferences of a type- $i$  individual are described by a strictly quasi-concave utility function  $u^i(c_t^i, z_{t+1}^i, l_t^i)$ , where  $c_t^i$ ,  $z_{t+1}^i$  are consumption in the active and retirement period, resp., and  $l_t^i$  denotes labor supply.  $u^i$  is taken as twice differentiable with partial derivatives  $u_c^i$ ,  $u_z^i > 0$ ,  $u_l^i < 0$ . The groups are assumed to be of equal size, denoted by  $N_t$ , with exogenously given growth factor  $G_t$ , i.e.,  $N_t = N_{t-1} G_t$ . For simplicity, we assume that the abilities  $a^i$  remain constant over time.

In this paper we consider an unfunded pension system where contributions depend on gross income with a constant contribution rate  $b$ , and pension payments  $p_{t+1}^i$  to the retired are



differentiated according to contributions in the period before. This property is formulated as follows. Let  $\omega_t \equiv (w_t^1 l_t^1 + w_t^2 l_t^2)/2 = w_t(a_t^1 l_t^1 + a_t^2 l_t^2)/2$  denote average gross income in period  $t$  and  $\rho_t^i \equiv w_t^i l_t^i / \omega_t$  the ratio of individual gross income to  $\omega_t$ . Total available revenue for pension payments in period  $t + 1$  is  $2b\omega_{t+1}N_{t+1}$ , which gives  $b\omega_{t+1}N_{t+1}/N_t$  as average benefit of the retired. We define intragenerational fairness by the property:  $p_{t+1}^i = \rho_t^i b\omega_{t+1}G_{t+1}$ . (By definition,  $\rho_t^1 + \rho_t^2 = 2$ , hence  $N_t(p_{t+1}^1 + p_{t+1}^2) = 2b\omega_{t+1}N_{t+1}$ .)

With these preparations, the following individual decision problem results:

- $$\begin{aligned}
 (1) \quad & \max u^i(c_t^i, z_{t+1}^i, l_t^i) \\
 \text{s.t.} \quad & \\
 (2) \quad & c_t^i = (1-b)w_t^i l_t^i - s_t^i, \\
 (3) \quad & z_{t+1}^i = s_t^i R_{t+1} + \rho_t^i b G_{t+1} \omega_{t+1}, \\
 (4) \quad & c_t^i, z_{t+1}^i, l_t^i \geq 0,
 \end{aligned}$$

where  $R_{t+1}$  denotes the return on saving, i.e., one plus the interest rate. Eliminating  $s_t^i$ , substituting for  $\rho_t^i$ , and introducing the growth factor of wage income  $\Omega_{t+1} \equiv \omega_{t+1}/\omega_t$ , one gets from (2) and (3)

$$(5) \quad c_t^i + z_{t+1}^i / R_{t+1} = w_t^i l_t^i - b w_t^i l_t^i (1 - G_{t+1} \Omega_{t+1} / R_{t+1}).$$

In this framework, a fully-funded system is characterized by  $b = 0$ , which means that it is equivalent to private saving for old-age consumption; the relevant issue is that its rentability is determined by the interest factor  $R_{t+1}$ . The rentability of the contributions to the unfunded system is  $p_{t+1}^i / (b w_t^i l_t^i) = G_{t+1} \Omega_{t+1}$ . Hence we have the familiar condition

$G_{t+1}\Omega_{t+1} \gtrless R_{t+1}$ , which determines whether the unfunded ( $>$ ) or the funded ( $<$ ) system is preferable from the point of view of individual rentability. In the latter case,  $D_{t+1} \equiv 1 - G_{t+1}\Omega_{t+1}/R_{t+1}$  is positive, and, as (5) shows,  $bw_t^i l_t^i D_{t+1}$  describes the loss in life-time income caused by the contribution  $bw_t^i l_t^i$  to the unfunded system.

Taking  $w_t^i = a^i w_t$  and  $R_{t+1}$  as given, and forming expectations concerning  $G_{t+1}$ ,  $l_{t+1}^i$ ,  $w_{t+1}^i$  and, thus,  $\omega_{t+1}$ ,  $\Omega_{t+1}$ , each individual maximizes (1) under the constraints (4) and (5). Optimal values  $\hat{c}_t^i(b; w_t^i, R_{t+1})$ ,  $\hat{z}_{t+1}^i(b; w_t^i, R_{t+1})$ ,  $\hat{l}_t^i(b; w_t^i, R_{t+1})$  are determined by the first-order conditions

$$(6) \quad \frac{u_c^i(\hat{c}_t^i, \hat{z}_{t+1}^i, \hat{l}_t^i)}{u_z^i(\hat{c}_t^i, \hat{z}_{t+1}^i, \hat{l}_t^i)} = R_{t+1},$$

$$(7) \quad -\frac{u_l^i(\hat{c}_t^i, \hat{z}_{t+1}^i, \hat{l}_t^i)}{u_c^i(\hat{c}_t^i, \hat{z}_{t+1}^i, \hat{l}_t^i)} = w_t^i - bw_t^i D_{t+1},$$

together with the budget constraint (5). (For the sake of shortness we neglect the possibility of corner solutions, they would not change the conclusions.)

Next we model the production sector in a conventional way: We assume a linear-homogeneous production function  $F(K_t, L_t)$ , defined on the stock of capital  $K_t$  and on total labor supply in efficiency units, given by

$$(8) \quad L_t = N_t(a_t^1 \hat{l}_t^1 + a_t^2 \hat{l}_t^2).$$

Total goods supply in period  $t$  is  $K_t + F(K_t, L_t)$ <sup>1</sup>, which can be used for consumption and saving. The stock of capital is equal to savings one period ago, that is ( $K_t^i$  denotes capital possessed by group  $i$ )

$$(9) \quad K_{t+1}^i = s_t^i N_t, \quad i=1,2,$$

$$(10) \quad K_{t+1} = K_{t+1}^1 + K_{t+1}^2.$$

Competitive factor markets will ensure that

$$(11) \quad R_t = 1 + F_K(K_t, L_t),$$

$$(12) \quad w_t = F_L(K_t, L_t).$$

Additionally, we have

$$(13) \quad w_t^i = a^i w_t, \quad i = 1,2.$$

We generally assume that individuals have perfect foresight concerning future labor supply as well as growth of population and wages, thus they know  $G_{t+1}$ ,  $\Omega_{t+1}$ . Then, in period  $t = t_0$ , where capital equipments  $K_{t_0}^i = S_{t_0-1}^i$ ,  $i = 1,2$  and, hence,  $K_{t_0}$  are given, equilibrium values for the sixteen variables  $\hat{c}_{t_0}^i(b)$ ,  $\hat{z}_{t_0+1}^i(b)$ ,  $\hat{l}_{t_0}^i(b)$ ,  $\hat{s}_{t_0}^i$ ,  $L_{t_0}$ ,  $w_{t_0}$ ,  $w_{t_0}^i$ ,  $K_{t_0+1}^i$ ,  $K_{t_0+1}$ ,  $R_{t_0+1}$ ,  $i = 1,2$ , are determined by the equations (2), (5) - (13), where it should be noted that all but (8), (10), (11), (12) hold for  $i = 1,2$ .

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<sup>1</sup> $F(K_t, L_t)$  is production net of depreciation.

### III Transition

The question, we want to answer in this study, is whether a change of the pension system is possible without making any individual worse off than with the former system. Of course, this question is relevant only if the rentability of the unfunded system is lower than that of the funded system. Thus, we assume in the following generally, for any pension system we consider, that the growth rate of the economy is lower than the interest rate, formally

$$(16) \quad G_{t+1} \Omega_{t+1} < R_{t+1} \text{ (or } D_{t+1} > 0), \text{ for all } t.$$

Let an unfunded pension system with contribution rate  $\bar{b}$  be given in period  $t_0$  and let  $v_t^i(\bar{b})$ ,  $t \geq t_0$  denote indirect utility of a type- $i$  individual, born in period  $t$ . Clearly,  $v_t^i(\bar{b}) = u^i(\hat{c}_t^i(\bar{b}), \hat{z}_{t+1}^i(\bar{b}), \hat{l}_t^i(\bar{b}))$ , and the optimal values are determined as in the process described at the end of the previous section. (To be precise,  $v_t^i$  also depends on the initial values  $K_{t_0}^i$ , for shortness of notation we neglect this dependency in the following and take initial values as fixed.)

As is well-known, an existing unfunded pension system cannot simply be abolished, because this would mean that in some period the retired get no benefits, though they paid contributions when they were active. Thus, if the system is to be substituted by a funded one, contributions of the active, given as transfers to the retired, are still needed during some phase of transition. This in turn means that the only way to initiate a change of the system is to begin with an alternative method of raising the revenue from the active. We analyze in this section whether such an alternative method of raising contributions exists, where no individual is made worse off. If the answer is negative, we can conclude that no transition to an unfunded pension system can be performed.

Formally, we say that a **Pareto-improving transition path** to a funded system exists, if we can find  $t_1 > t_0$  and feasible values  $\tilde{c}_t^i, \tilde{z}_t^i, \tilde{l}_t^i, \tilde{K}_t^i, t \geq t_0$  such that  $\tilde{z}_{t_0}^i = \hat{z}_{t_0}^i(b)$ ,  $u^i(\tilde{c}_t^i, \tilde{z}_{t+1}^i, \tilde{l}_t^i) \geq v_t^i(b)$  for any  $t \geq t_0$ , and  $(\tilde{c}_t^i, \tilde{z}_{t+1}^i, \tilde{l}_t^i) = (\hat{c}_t^i(0), \hat{z}_{t+1}^i(0), \hat{l}_t^i(0))$  for  $t > t_1$ ,  $i = 1, 2$ .

In this definition the attribute feasible has to be given a more precise meaning. On the one hand it requires, of course, that the goods market is in equilibrium in every period. On the other hand, feasibility means that  $\tilde{c}_t^i, \tilde{z}_{t+1}^i, \tilde{l}_t^i, \tilde{K}_t^i$  result as decentralized decisions of the individuals, influenced by measures of the government. Thus, feasibility can only be defined relative to the available instruments of the government, that is, to alternative ways of raising contributions, as discussed above.

Therefore we consider, in the following Subsections III.1 and III.2, resp., two instruments, which are familiar from taxation theory, namely lump-sum contributions and contributions being a linear function of gross income. In III.3 some comments on the possibility of debt-financed transition are provided.

### III.1 Transition by means of lump-sum contributions

#### III.1.1 A lump-sum pension system

In a system of lump-sum contributions and benefits, the revenue condition of the state implies that average benefits in period  $t$  equal  $G_t(\beta_t^1 + \beta_t^2)/2$ , where  $\beta_t^i$  denotes contributions per worker. In order that the rentability and fairness of the system is comparable with that of proportional contributions, we assume that the contributions grow

with factor  $\Omega_{t+1}$ , and that benefits are distributed to the retired in accordance with their contributions in the previous period. This gives us the following constraints for an individual:

$$(17) \quad c_t^i = w_t^i l_t^i - s_t^i - \beta_t^i,$$

$$(18) \quad z_{t+1}^i = s_t^i R_{t+1} + \beta_t^i G_{t+1} \Omega_{t+1},$$

which can be combined to

$$(19) \quad c_t^i + z_{t+1}^i / R_{t+1} = w_t^i l_t^i - \beta_t^i D_{t+1}.$$

As before,  $\beta_t^i D_{t+1}$  is the loss in life-time income caused by the contribution  $\beta_t^i$  to the unfunded system. The first-order conditions for the maximization of (1) under constraints (4) and (19) are (6) and

$$(20) \quad - \frac{u_l^i(\hat{c}_t^i, \hat{z}_{t+1}^i, \hat{l}_t^i)}{u_c^i(\hat{c}_t^i, \hat{z}_{t+1}^i, \hat{l}_t^i)} = w_t^i.$$

Using (19) and (20) instead of (5) and (7), the dynamics of the economy, when the unfunded pensions are financed by lump-sum contributions, can be described analogous to that at the end of Section II. We denote by  $v_t^i(\beta_t^i)$  indirect utility of a type-i individual in this process.

Of course, the way contributions have to be paid by the individuals influences their labor supply and savings, thus the growth rate of total income and the interest rate. However, one can expect that both the growth rate and the interest rate do not change too much, if

proportional contributions are substituted by lump-sum ones, when total revenue is fixed.

We formulate this as an explicit assumption:

**GRL:** Let  $R_{t+1}(\bar{b})$ ,  $\Omega_{t+1}(\bar{b})$  and  $R_{t+1}(\beta)$ ,  $\Omega_{t+1}(\beta)$  be the interest factor and the growth factor of total income, given proportional contributions and lump-sum contributions, resp., where  $\bar{b}(w_t^1 \hat{1}_t^1(\bar{b}) + w_t^2 \hat{1}_t^2(\bar{b})) = \beta_t^1 + \beta_t^2$ . Then the difference between  $R_{t+1}(\bar{b})$  and  $R_{t+1}(\beta)$  as well as between  $\Omega_{t+1}(\bar{b})$  and  $\Omega_{t+1}(\beta)$  is "small".<sup>2</sup>

If we write  $D_{t+1}(\bar{b})$ ,  $D_{t+1}(\beta)$ , resp., for the factors determining the loss in life-time income through the respective unfunded system, GRL implies that the difference between them is negligible, compared to the direct welfare effects of a change of the contribution system. Note that the individual losses  $\bar{b}w_t^i \hat{1}_t^i(\bar{b})D_{t+1}(\bar{b})$ ,  $\beta_t^i D_{t+1}(\beta)$ , with both methods of collecting contributions, are nearly the same (due to GRL) if  $\beta_t^i = \bar{b}w_t^i \hat{1}_t^i(\bar{b})$ , but still, the first system imposes an excess burden.

### III.1.2 The impossibility of transition

Now suppose that in some period  $t$  the government, initiating a transition from the unfunded to a funded system, substitutes the proportional (to gross income) contributions by lump-sum ones. In order that the retired generation does not lose in this process, the constraint  $\beta_t^1 + \beta_t^2 = \bar{b}(w_t^1 \hat{1}_t^1(\bar{b}) + w_t^2 \hat{1}_t^2(\bar{b}))$  must be fulfilled. The question is, how to choose  $\beta_t^1$  and  $\beta_t^2$ .

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<sup>2</sup>It should be noted that this condition does not refer to a comparison between a funded and an unfunded system, but only to the method by which a fixed revenue for the unfunded system is raised. The reason for the introduction of GRL is to avoid complications, which are inessential for the main message, when we prove the result of this section.

Obviously, the first-best choice would be to set  $\beta_t^i = \bar{w}_t^i \hat{l}_t^i(\bar{b})$ , which would mean that lump-sum contributions differentiated according to the type of individual are imposed. However, to implement this instrument, government needs appropriate information (in our model: knowledge of the individual abilities) which, as was mentioned in the introduction, it usually does not possess. We formulate this assumption explicitly:

IN: The authority knows that two types of individuals exist, and knows the utility function  $u^i$  and ability  $a^i$  of each type, but cannot identify individuals.

As a consequence of IN, the government can impose lump-sum taxes only in an arbitrary way (that is, such that they are not associated to a specific type of individual). We ask whether it can happen that an individual is made worse off by a change to this method of raising contributions.

**Lemma 1:** Assume GRL and let an unfunded pension system with contribution rate  $\bar{b}$  be given and let, in some period  $t$ ,  $\bar{w}_t^i \hat{l}_t^i(\bar{b})$  be the contribution of a type- $i$  individual and  $\bar{w}_t^i \hat{l}_t^i(\bar{b}) G_{t+1} \Omega_{t+1}$  his correctly anticipated pension. For  $i = 1, 2$  there exists  $\beta_t^{i, \max}$  such that, for any lump-sum contribution  $\beta_t^i$ , if  $\beta_t^i > \beta_t^{i, \max}$ , then  $v_t^i(\beta_t^i) < v_t^i(\bar{b})$ .

Proof: Let, for  $i = 1, 2$ ,  $\bar{\delta}^i \equiv \max \{ \delta^i \mid \delta^i = w_t^i \hat{l}_t^i - c_t^i - z_{t+1}^i / R_{t+1} \text{ and } u^i(c_t^i, z_{t+1}^i, l_t^i) \geq v_t^i(\bar{b}) \}$ , which is the maximum income loss an individual can be imposed on through a lump-sum system without making him worse off than with the existing system. Because of the excess burden and GRL, we have  $\bar{\delta}^i > \bar{w}_t^i \hat{l}_t^i(\bar{b}) D_{t+1}(\bar{b})$ . The maximum lump-sum contribution corresponding to  $\bar{\delta}^i$  is  $\beta_t^{i, \max} \equiv \bar{\delta}^i / D_{t+1}(\bar{b})$ . QED.



In order to develop some intuition for  $\beta_t^{i,\max}$ , it is useful to think of the money value of the excess burden as some fraction  $\lambda^i$  of the loss  $\bar{b}w_t^1\hat{l}_t^i(\bar{b})D_{t+1}(\bar{b})$ . It follows that, by means of a lump-sum tax, a loss of  $(1+\lambda^i)\bar{b}w_t^1\hat{l}_t^i(\bar{b})D_{t+1}(\bar{b})$  can be imposed on a type- $i$  individual without reducing his welfare. From this, in turn, and GRL, one can conclude that the corresponding maximum lump-sum contribution is  $\beta_t^{i,\max} = (1+\lambda^i)\bar{b}w_t^1\hat{l}_t^i(\bar{b})$ .

This lemma shows that indeed, if the government substitutes proportional contributions by lump-sum ones, it does matter which amount is imposed on each individual. As a consequence, we get the result that a Pareto-improving transition path exists only if the difference between the individuals is small.

Result 1: Given IN and GRL. With the use of lump-sum contributions a Pareto-improving transition path to a fully-funded system exists if and only if  $\beta_t^{1,\max} \geq \bar{b}(w_t^1\hat{l}_t^1(\bar{b}) + w_t^2\hat{l}_t^2(\bar{b}))/2$ .

Proof: Suppose, government wants to change the existing unfunded system in some period  $t$  by the introduction of a lump-sum tax. As it cannot identify the individuals, a possibility is to impose a head tax, then  $\beta_t^1 = \beta_t^2 = \bar{b}(w_t^1\hat{l}_t^1(\bar{b}) + w_t^2\hat{l}_t^2(\bar{b}))/2$ , so that the pensions of the retired generation remain unchanged. By this, the contributions of the more qualified are reduced, therefore a Pareto-improvement occurs if and only if we have for the less qualified:  $\beta_t^{1,\max} \geq \bar{b}(w_t^1\hat{l}_t^1(\bar{b}) + w_t^2\hat{l}_t^2(\bar{b}))/2$ , due to Lemma 1.

If, instead of a head tax, the government chooses arbitrary  $\beta_t^1 \neq \beta_t^2$ , such that its budget condition is fulfilled, then obviously  $\max \{\beta_t^1, \beta_t^2\} > \bar{b}(w_t^1\hat{l}_t^1(\bar{b}) + w_t^2\hat{l}_t^2(\bar{b}))/2$ . As it cannot identify individuals, it may happen that the less qualified have to pay the larger amount, which certainly makes them worse off if  $\beta_t^{1,\max} < \bar{b}(w_t^1\hat{l}_t^1(\bar{b}) + w_t^2\hat{l}_t^2(\bar{b}))/2$ . QED.

As the proof of Theorem 2 in Breyer and Straub (1993) makes clear, their positive result on a Pareto-improving transition path from an unfunded to a funded pension system is completely based on the possibility of lump-sum taxation; they start with a comparison of optimality conditions for the distorted pay-as-you-go system and an undistorted system. Essentially, however, this procedure is valid only if identical individuals are assumed, as Breyer and Straub (1993, p.80) do. In the more realistic case of differing individuals, Result 1 shows that the institutional and informational framework, in which a change should take place, must not be neglected. Indeed, its consideration reveals the real problem.

### III.2 Transition by means of income-dependent contributions

Having dealt with the problematic character of a transition by use of lump-sum taxes, we turn now to a discussion of the possibilities opened up by income-dependent contributions. We assume again that an intragenerationally fair unfunded pension system, with contributions and benefits being proportional to income, is given, and that a change to a funded system is intended. The idea is that a better second-best instrument than the proportionality rule, i.e., a less distorting income-related tax could be used to produce a surplus through a reduction of the excess burden, and to finance a Pareto-improving transition by this surplus. Of course, the excess burden can never be removed completely with an income-dependent tax, but it may be reduced, and this may create a sufficiently large surplus for our purpose.

### III.2.1 A linear contribution rule

It is immediately clear that the system would have to be more complicated than that of proportional contributions, because in general no other rate than  $\bar{b}$  would raise just the revenue required from the active generation for the benefits of the retired. As the most general approach we could think of a function  $\tau_t: \mathbb{R} \rightarrow \mathbb{R}$ , where  $\tau_t(y_t^i)$  is the required contribution of an individual having gross income  $y_t^i = w_t^i l_t^i$ . However, we do not consider the general case but limit our analysis to a specific, simple type of function, namely when  $\tau_t$  is an affine function,  $\tau_t(y) = \alpha_t + b_t y$ . This type is widely used in taxation theory, frequently under the heading of "linear" taxation.

In order that in period  $t$  a change of the pension system can start, contributions are collected according to the function  $\alpha_t + b_t y$ . As the retired generation must not lose by the change, the revenue condition  $2\alpha_t + b_t(\hat{y}_t^1(b_t, \alpha_t) + \hat{y}_t^2(b_t, \alpha_t)) = \bar{b}(\hat{y}_t^1(\bar{b}) + \hat{y}_t^2(\bar{b}))$  has to be fulfilled. For comparability of the systems, we assume again that the return on contributions is  $G_{t+1}\Omega_{t+1}$ , which also reflects intragenerational fairness. Moreover, it is clear that the same limits for the individual contributions as in case of lump-sum taxation (see III.1) must hold, that is,  $\alpha_t + b_t \hat{y}_t^i \leq \beta_t^{i, \max}$  must be fulfilled. Otherwise an individual would certainly be made worse off.

As in III.1, we want to avoid unessential complications and assume that the influence of the contribution rule on the growth rate of total income and the interest rate can be neglected.

**GRI:** Let  $R_{t+1}(b_t, \alpha_t)$  and  $\Omega_{t+1}(b_t, \alpha_t)$  be the interest factor and the growth factor of total income, given that contributions are imposed according to the function  $\alpha_t + b_t y$ , where  $2\alpha_t + b_t(\hat{y}_t^1(b_t, \alpha_t) + \hat{y}_t^2(b_t, \alpha_t)) = \bar{b}(\hat{y}_t^1(\bar{b}) + \hat{y}_t^2(\bar{b}))$ . Then the difference

between  $R_{t+1}(\bar{b})$  and  $R_{t+1}(b_t, \alpha_t)$  as well as between  $\Omega_{t+1}(\bar{b})$  and  $\Omega_{t+1}(b_t, \alpha_t)$  is "small".

The life-time budget constraint in case of a linear tax rule is

$$(21) \quad c_t^i + z_{t+1}^i / R_{t+1} = y_t^i - \alpha_t D_{t+1}(b_t, \alpha_t) - b_t y_t^i D_{t+1}(b_t, \alpha_t),$$

thus the distortion of the wage rate, caused by any  $b_t$ , is  $b_t D_{t+1}(b_t, \alpha_t)$ , while the total reduction of life-time income, due to the contribution, is  $(\alpha_t + b_t y_t^i) D_{t+1}(b_t, \alpha_t)$ , similar to the corresponding expressions above. The most important consequence for our further analysis is that any variation of the contribution  $\alpha_t + b_t y_t^i$  means at the same time a variation in life-time income in the same direction, where the respective values are related by the factor  $D_{t+1}(b_t, \alpha_t) > 0$ .

### III.2.2 The impossibility of transition

We ask, whether a substitution of proportional contributions with rate  $\bar{b}$  by a linear rule can create a surplus, which allows a Pareto-improving transition to a funded pension system. For a detailed analysis we have to introduce some additional definitions: Let  $v_t^i(b_{t+1}, \alpha_{t+1}) = u^i(\hat{c}_t^i(b_{t+1}, \alpha_{t+1}), \hat{z}_{t+1}^i(b_{t+1}, \alpha_{t+1}), \hat{l}_{t+1}^i(b_{t+1}, \alpha_{t+1}))$  be the indirect utility function, given a linear contribution rule, that is,  $\hat{c}_t^i$ ,  $\hat{z}_{t+1}^i$ ,  $\hat{l}_{t+1}^i$  result from the maximization of (1) subject to (21) and (4), given equilibrium wage and interest rate and with perfect foresight of future values.

Furthermore, we define the transformed utility function  $U^i(c_t, z_{t+1}, y_t) \equiv u^i(c_t, z_{t+1}, y_t / w_t^i)$ . As in Brunner (1993), we consider the expenditures on  $c_t$  and  $z_{t+1}$  as a Hicksian composite

commodity  $x_t$  with corresponding utility function  $\Psi^i(x_t, y_t) \equiv \max \{U^i(c_t, z_{t+1}, y_t) \mid c_t + z_{t+1}/R = x_t\}$ . Finally, we define the slope of an indifference curve of  $\Psi^i$ ,  $\sigma^i(x_t, y_t) \equiv -\Psi_y^i(x_t, y_t)/\Psi_x^i(x_t, y_t)$ . With these preparations we can formulate the condition of "Agent Monotonicity" (Seade 1982):

**AM.**  $\sigma^1(x_t, y_t) > \sigma^2(x_t, y_t)$ , for any  $x_t$  and  $y_t$ .

This condition, which is usual in optimum income taxation theory, requires that, for any bundle  $(x_t, y_t)$ , the marginal rate of substitution between gross income and composite consumption is lower for the more able individual. As an important consequence, AM implies  $\hat{y}_t^1(b_t, \alpha_t) < \hat{y}_t^2(b_t, \alpha_t)$  for any parameters  $b_t, \alpha_t$  of a linear tax function. That is, labor supply of the more able individual is at least as large to earn more gross (and net) income than the less able, irrespective of the tax function. (In fact, this also holds for any nonlinear tax  $\tau_t(y)$ . For a discussion see Brunner 1989, p.26). In other words, the ordering of abilities extends to that of the individuals' incomes.

With these preparations we can analyze the effect of a marginal change from  $(\bar{b}, 0)$  to some  $(b_t, \alpha_t)$ . Clearly, the marginal contribution rate can either be increased or decreased. We consider the former possibility first, i.e.,  $b_t > \bar{b}$ . As this leads to an increase of the excess burden, we have a straightforward result:

**Lemma 2:** Assume AM and GRI and let an unfunded pension system with contribution rate  $\bar{b}$  be given. If leisure is a normal good for the less qualified individuals, then by a linear tax rule  $(b_t, \alpha_t)$ , with  $b_t > \bar{b}$  and where  $2\alpha_t + b_t(\hat{y}_t^1(b_t, \alpha_t) + \hat{y}_t^2(b_t, \alpha_t)) = \bar{b}(\hat{y}_t^1(\bar{b}) + \hat{y}_t^2(\bar{b}))$ , the more qualified are made worse off.

Proof: Assume, as a first step, that the increase of  $\bar{b}$  is accompanied by the introduction of some appropriate  $\alpha_t$  in such a way that utility of the more qualified individuals remains constant. We ask whether this leads to a reduction of total tax revenue. If so, then in a second step additional revenue must be raised by an increase of  $\alpha_t$ , which means that welfare of the type-2 individuals is decreased.

The proof is given diagrammatically, by use of figure 1, where the horizontal axis describes gross income, while the vertical axis describes expenditures on the composite commodity. The 45°-line shows the budget line of both individuals in the absence of any contribution (i.e.,  $\bar{b} = 0$ ).  $B(\bar{b}, 0)$ , defined by the equation  $x_t = (1 - \bar{b}D_{t+1})y_t$  is the budget line, given a proportional contribution with rate  $\bar{b}$  (and  $\alpha_t = 0$ ). The points  $A^1$  and  $A^2$ , where  $B(\bar{b}, 0)$  is tangent to the indifference curves  $I^1$ ,  $I^2$ , resp., describe the decisions of the two individuals. (Due to AM,  $A^2$  is to the right of  $A^1$ .) The distance  $H^i A^i$ , divided by  $D_{t+1}$ , shows the respective contributions.

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Figure 1

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The move from  $A^2$  to  $C^2$  is caused by an increase of  $\bar{b}$  to  $b_t$  with associated decrease of  $\alpha_t$ , such that the more qualified individual remains on the same utility level (indifference-curve  $I^2$ ). The equation for  $B(b_t, \alpha_t)$  is  $x_t = -\alpha_t D_{t+1} + (1 - b_t D_{t+1})y_t$ , where  $b_t > \bar{b}$  and  $\alpha_t < 0$ . Obviously, due to the increase of the excess burden, the contribution of the more qualified individual is reduced (observe the distance  $F^2 C^2$ ). Now consider the low-qualified individual, whose new choice is  $C^1$ . The relevant part of the new budget line  $B(b_t, \alpha_t)$  lies above that of  $B(\bar{b}, 0)$ , hence utility is higher at  $C^1$  than at  $A^1$ . Moreover, normality of leisure and  $b_t > \bar{b}$  imply that  $C^1$  is to the left of  $A^1$ , where the vertical distance between

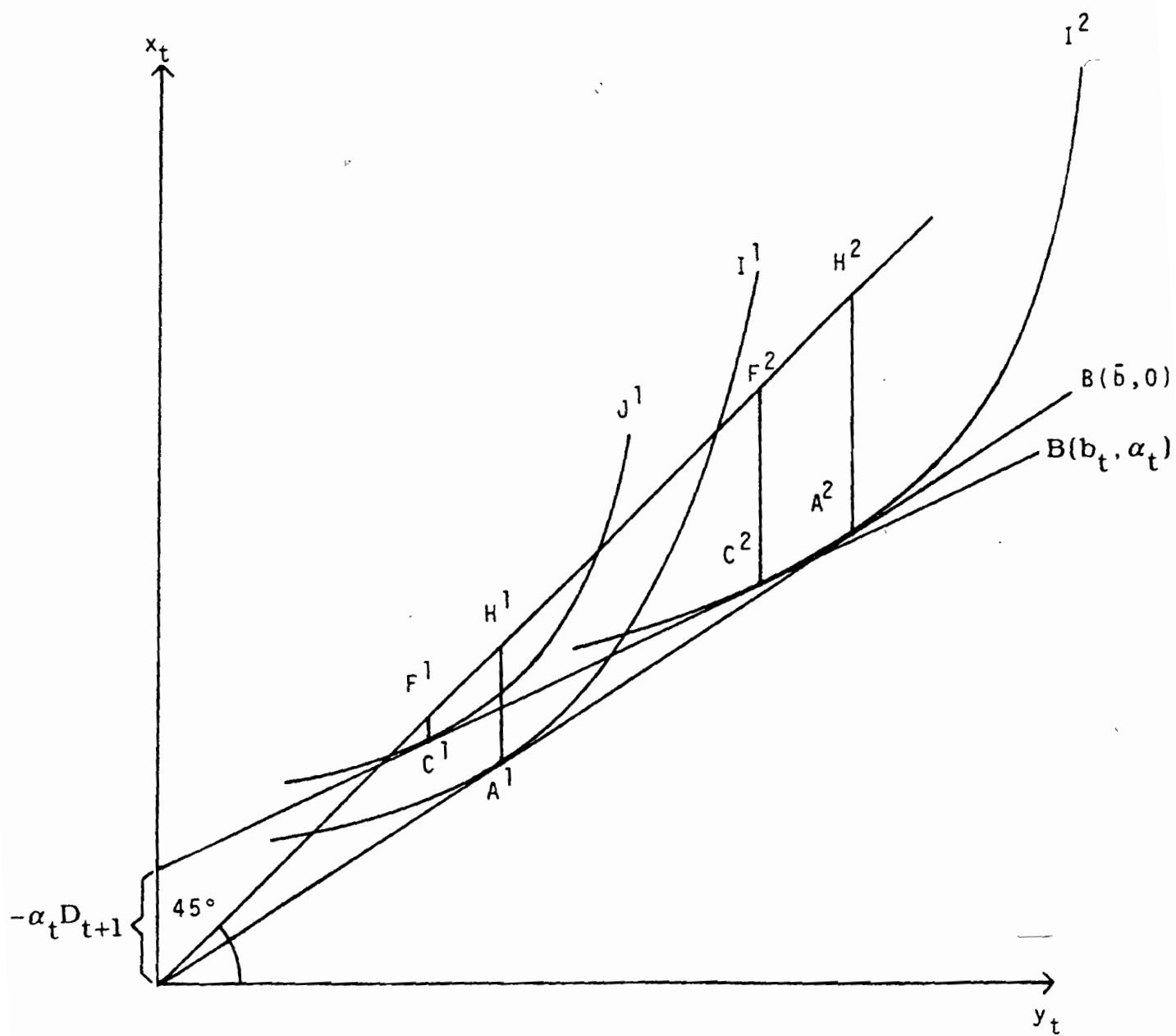


Fig. 1

the 45°-line and  $B(b_t, \alpha_t)$  and, thus, the contribution  $C^1 F^1$  is lower than  $A^1 H^1$ .

Hence, if utility of the more qualified is held constant, an increase of the marginal contribution rate violates the budget constraint of the pension system. Therefore,  $\alpha_t$  must be increased, which means that at least the more qualified are made worse off than with  $\bar{b}$ . QED.

Usually, an increase of the marginal contribution rate (with the adaptation of  $\alpha_t$  such that revenue remains constant) represents an instrument of redistribution from the top to the bottom: A higher marginal rate hits the type-2 individuals (who earn more income, due to AM) more than the others, while the associated reduction of  $\alpha_t$  (financed by the increase of the marginal rate) is the same for both types. Due to the increase of the excess burden, this reduction of  $\alpha_t$  cannot compensate both, but only the type-1 individuals; they usually win in this procedure. However, it may happen that even these cannot be compensated, because the additional revenue, raised by the increase of  $\bar{b}$ , is too small, then both types lose.

Of course, if we turn to the effect of lowering  $\bar{b}$ , we simply find the opposite of that of an increase: Usually a decrease of the marginal contribution rate (with the adaptation of  $\alpha_t$  such that revenue remains constant) means redistribution from the bottom to the top, by the reversed procedure as described above. But now, corresponding to the case that all *lose* by an *increase*, it can happen that all *win* by a *decrease* of the marginal contribution rate, because the excess burden is now reduced.

Unfortunately, not clear-cut condition can be given of when a decrease of  $\bar{b}$  represents a Pareto-improving measure. It certainly depends on the size of  $\bar{b}$ , for if  $\bar{b}D_{t+1}$  is close to 1, nobody would work any more, which means that the positive substitution effect of a



decrease of  $\bar{b}$  would be large enough to outweigh its negative direct revenue effect. The opposite limiting case is described by the following lemma:

**Lemma 3:** Assume AM and GRI and let an unfunded pension system with contribution rate  $\bar{b}$  be given. If  $\bar{b}D_{t+1}$  is sufficiently small, then by a linear tax rule  $(b_t, \alpha_t)$ , with  $b_t < \bar{b}$  and where  $2\alpha_t + b_t(\hat{y}_t^1(b_t, \alpha_t) + \hat{y}_t^2(b_t, \alpha_t)) = \bar{b}(\hat{y}_t^1(\bar{b}) + \hat{y}_t^2(\bar{b}))$ , the less qualified are made worse off.

Proof: Consider fig. 2, which is similar to fig. 1., but  $\bar{b} = 0$  (i.e., the case of zero contributions) is assumed. The budget line  $B(b_t, \alpha_t)$  depicts a decrease of  $\bar{b}$  to  $b_t$  (and associated  $\alpha_t > 0$ ), where utility of the type-1 individuals is held constant. It is immediately seen that this change makes tax revenue definitely negative, ( $C^1$  and  $C^2$  lie above the 45°-line).<sup>3</sup> Hence, in order that the budget constraint remains fulfilled,  $\alpha_t$  must be increased further, which makes the first individual worse off than with  $\bar{b} = 0$ . By continuity, we can conclude that the same effect occurs for sufficiently small values of  $\bar{b} > 0$  as well. QED.

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Figure 2

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**Remark:** Concerning the range of  $\bar{b}$ , for which this result holds, we can conclude from the above considerations that it depends on the elasticity of labor supply. (Clearly, it holds for

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<sup>3</sup>This corresponds to a well-known result in taxation theory that the deadweight loss is of second order with respect to the tax rate (see, e.g., Atkinson and Stiglitz (1980, p.368)), while the revenue effect is of first order. For the same reason, the marginal tax rate of the optimal linear income tax is positive, if some redistribution is desired (Hellwig 1986, Brunner 1989, p.184).

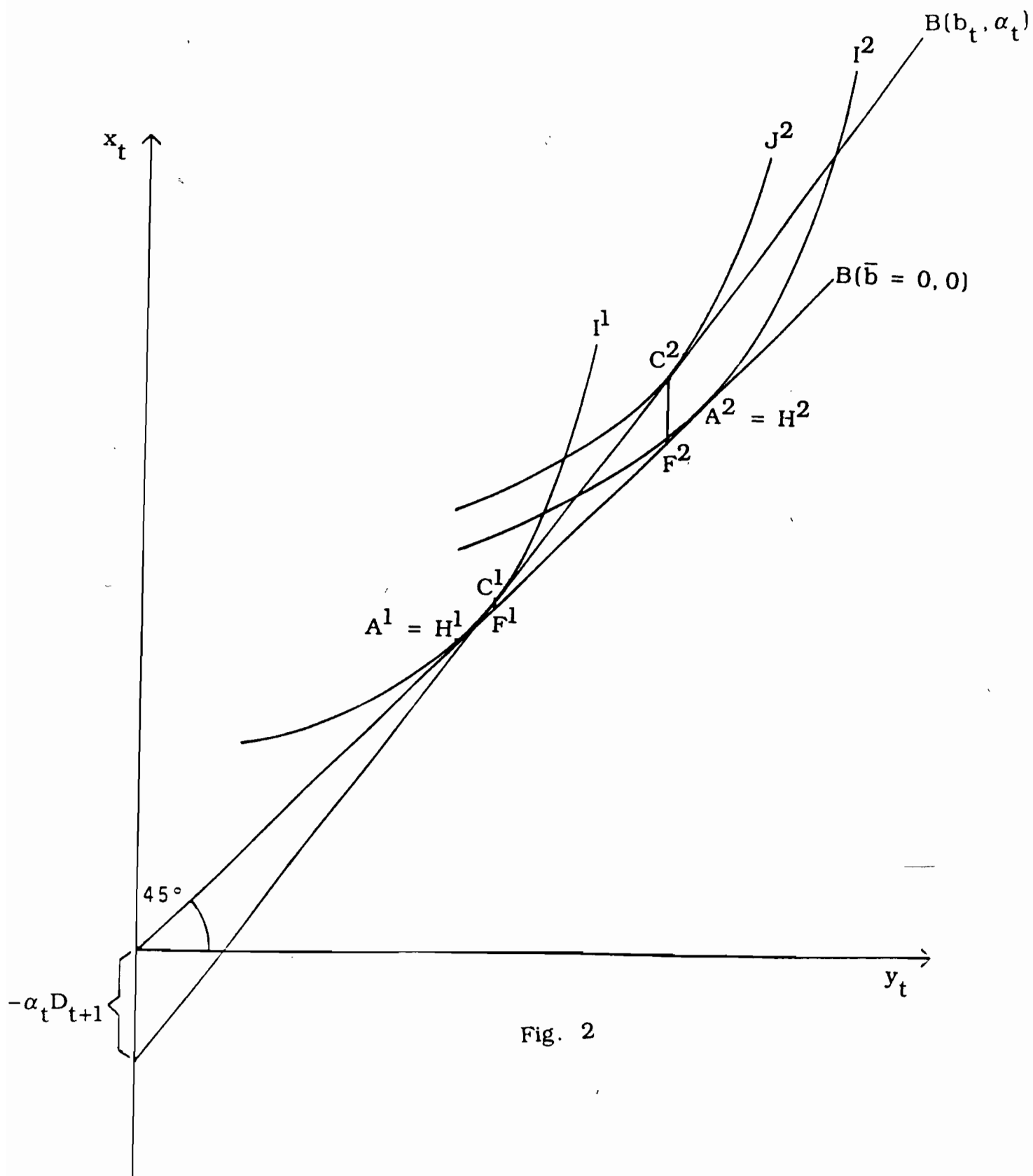


Fig. 2

any  $\bar{b}$ , if the elasticity is zero.) Moreover, analogous to the situation in III.1, also the extent, to which individuals differ, matters. If income of the type-2 individuals is high above that of the others, a decrease of the marginal contribution rate with appropriate adaptation of  $\alpha_t$  is less likely to represent a Pareto-improvement, because it entails redistribution from the bottom to the top. In any case, it should be noticed that the relevant quantity determining the distortion of the labor supply decision is not  $\bar{b}$  but  $\bar{b}D_{t+1}$ , where  $D_{t+1} < 1$ .<sup>4</sup>

Altogether, we can summarize our findings as follows:

Result 2: Assume AM and GRI and let an unfunded pension system with contribution rate  $\bar{b}$  be given. If leisure is a normal good and  $\bar{b}$  is not too large, no Pareto-improving transition to a funded system by means of a linear contribution rule is possible.

Proof: Follows from the Lemmas 2 and 3. QED.

### III.3 Debt-financed transition

For the sake of completeness the possibility of using public debt as an instrument for a transition from the unfunded to a funded pension system must be taken into account. This means that in some period  $t$  the pensions are financed by a credit instead of the contributions of the active. Of course, in a closed economy<sup>5</sup>, the only creditors can be the

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<sup>4</sup>To give a numerical illustration: If, on a yearly basis,  $R_{t+1} = 1.03$  and the economy grows with  $G_{t+1}\Omega_{t+1} = 1.01$ , then  $D_{t+1} = .019$ . Taking a period to consist of 30 years, we get  $D_{t+1} = 1 - (1.01/1.03)^{30} = .445$ .

<sup>5</sup>For an open economy it was shown in Brunner (1993) that debt financing does not provide

active population, they now offer a credit instead of their former contribution. The difference is that the credit is given voluntarily, because its return is  $R_{t+1}$ , which is larger than  $G_{t+1}\Omega_{t+1}$ , by assumption (16).

One detects immediately that this procedure cannot work, because to repay the credit plus interest, the government would have to collect - as a credit or as obligatory contributions - a higher amount than with the former system from the active generation in the next period. If it uses a credit again, it ends up with a Ponzi scheme, where public debt grows permanently faster than the economy, which must lead to a collapse. Otherwise we are back to the former system, but with higher contributions. It follows from these short considerations that public debt represents no method, which could make a transition to a funded system possible.

#### IV Concluding remarks

The intention of this paper was to model an intragenerationally fair pay-as-you-go pension system, when individuals differ in earning ability, and to investigate the possibility of a Pareto-improving transition to a fully-funded system. In this framework, the Pareto criterion requires that no individual of any generation is made worse off, which turns out to be a severe restriction on the construction of measures for a transition.

With a fair rule, contributions and benefits are proportional to gross income, with a fixed contribution rate  $b$ . At the same time, the choice of  $b$  determines the loss in life-time

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a method for a Pareto-improving transition from an intragenerationally redistributive unfunded pension system to a funded one. It is not difficult to see that the same is true, if the existing unfunded system is intragenerationally fair.

income, by  $\hat{y}_t^i D_{t+1}^i$ , for any individual, caused by the existence of an obligatory unfunded pension system. Any attempt to alter the system must start with a change of the contribution rule, without reducing total revenue. Only ability-specific lump-sum contributions would represent an improvement for all, but these must be excluded, due to a lack of information on the side of the authority. As was shown in Section III, if individuals are sufficiently different, the use of arbitrary lump-sum contributions or of a linear rule in general represents an act of redistribution within a generation and makes therefore at least one individual worse off.

Probably, in the model of this paper, a more complicated rule than the linear would allow a Pareto improvement. However, one has to observe that in reality there exist many different types of individuals, and that such a rule would presumably be characterized by a number of parameters which is larger than that of types. Therefore, the implementation of such a measure seems hardly realistic.

A problematic aspect of our analysis may lie in the fact that throughout the paper we concentrated on the immediate effect of the contribution rule on the distortion of the labor-leisure decisions of the individuals, but spent little attention to changes in capital formation. (Remember the assumptions GRL and GRI.) This is justified, firstly, by the fact that Breyer and Straub (1993) considered the same problem, and our intention was to analyze their result in a more detailed model. But also, it seems to be a rather plausible assumption that a change of the contribution rule, with constant tax revenue, does not per se influence capital formation and future labor supply to such an extent that the distributive effects, analyzed in section III, are outweighed. Still, this question would be interesting to be settled by further research.

## References

- Aaron, H. (1966), The social insurance paradox. *Canadian Journal of Economics and Political Science* 32, 371–374.
- Atkinson, A. B and Joseph E. Stiglitz (1980), *Lectures on Public Economics*. New York et al.: McGraw–Hill.
- Breyer, F. and M. Straub (1993), Welfare effects of unfunded pension systems when labor supply is endogenous. *Journal of Public Economics* 50, 77–91.
- Brunner, J.K. (1993), Redistribution and the efficiency of the pay-as-you-go pension scheme. *Diskussionsbeiträge der Fakultät für Wirtschaftswissenschaften und Statistik, Serie I – Nr. 265*, Universität Konstanz.
- Brunner, J.K. (1989), *Theory of equitable taxation*. Berlin et al.: Springer
- Hellwig, M. F. (1986), The optimal linear income tax revisited. *Journal of Public Economics* 31, 163–179.
- Homburg, S. (1990), The efficiency of unfunded pension schemes. *Journal of Institutional and Theoretical Economics* 146, 640–647.
- Homburg, S. and W.F. Richter (1990), Eine effizienzorientierte Reform der GRV. In: B. Felderer (ed), *Bevölkerung und Wirtschaft*. Berlin: Duncker und Humblot, 183–191.