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## Human-capital investment and the wage gap

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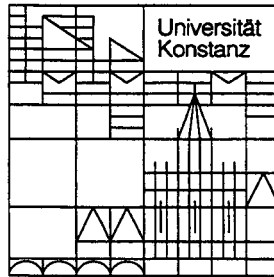
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Diskussionsbeiträge

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# **Human-Capital Investment and the Wage Gap**

**Jürgen Meckl  
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**Serie I – Nr. 307**

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# Human–Capital Investment and the Wage Gap

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## **Abstract**

According to empirical studies, the relation between the relative wage of skilled workers and their relative supply is U-shaped. This finding is explained by the effect of technological change on the incentives for human–capital investments made by heterogeneous individuals.

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# 1 Introduction

There has been considerable debate about explaining the behavior of wage inequality and relative supply of skilled labor. According to empirical studies (cf. Katz and Autor 1999 for an overview), the relative wage of skilled labor fell during the 1970s, but increased during the 1980s and 1990s. Relative supply of skilled labor rose in both periods, though more in the 1970s. These observations imply a U-shaped relation between the relative wage of skilled workers and their relative supply. Most of the literature to the so called wage-gap debate concentrates on explaining the labor-market developments of the 1980s and 1990s. This literature focuses on changes in relative labor demand—either generated by global market integration (cf. Wood 1994) or by factor-biased technological change (cf. Aghion and Howitt 1998)—as the principal causes for the increase in wage inequality. The increase in the skill premium may then induce changes in the relative labor supply, thus generating a positive correlation between the relative wage and supply of skilled labor. In order to account for the observations in the 1970s, however, this demand-side literature typically has to refer to additional exogenous labor-supply shocks (Acemoglu (1998), e.g., postulates a sudden increase of skilled labor that moreover explains the factor bias in technological change).

We offer a complete supply-side explanation for the observed non-monotonic evolution of the relative wage which is based on technological change. People who differ with respect to their abilities have to incur some fixed cost when they want to become skilled (i.e. make a human-capital investment). Technological progress

affects both the unskilled and the skilled wage rate and changes the individual investment incentives. Specifically, as the technology improves more individuals are inclined to become educated, thus increasing the relative supply of skilled labor. We show that this rise can be accompanied by non-monotonic behavior of the relative wage under fairly general conditions. The proposed mechanism should be seen as complementary to the widely discussed demand-side models, as it explains the U-shaped relation between the skill premium and the relative labor supply without referring to exogenous shocks in labor supplies.<sup>1</sup>

## 2 The model

Our static economy is populated by a continuum of agents with the mass normalized to 1. Individuals have heterogeneous inherent abilities  $a$ . The distribution of these abilities can be characterized by some probability measure with support  $I := [\underline{a}, \bar{a}]$ . We assume it to have a density function  $f$  which is continuously differentiable and strictly positive in the interior of  $I$  (let  $F$  denote the respective distribution function).

People can either work as unskilled supplying  $a$  units of labor on the competitive labor market or they can upgrade their abilities according to some strictly increasing continuously differentiable function  $g$  which satisfies  $g' > 1$  and  $g(\underline{a}) > \underline{a}$ .

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<sup>1</sup>Galor and Moav (2000) also develop a supply-side explanation of the change in relative wages and labor supply that is based on human-capital-investment decisions of individuals with heterogeneous abilities. Their explanation of the non-monotonicity, however, crucially depends on the existence of exogenous capital-market imperfections.

Doing so they can supply  $g(a)$  units of labor, but they have to incur some fixed cost  $C$ . The wage rate for an effective unit of labor is supposed to be linear in the state of technology  $A$  (i.e. technological progress improves individual productivities). So people can either receive  $aAw$  when refraining from investment or  $g(a)Aw - C$  as skilled workers. In our simple framework, the amount of labor supplied does not affect the productivity-adjusted wage rate  $w$ ,<sup>2</sup> so we normalize  $w$  to unity in the sequel. In order to avoid corner solutions we make the following assumption

(A1) A pair  $(A, C)$  is admissible, iff there is an  $a_0 \in (\underline{a}, \bar{a})$  such that  $g(a_0) - x = a_0$  holds, with  $x := C/A$ .

Due to  $g' > 1$ , this  $a_0 \in (\underline{a}, \bar{a})$  is unique and the function  $\phi : x \mapsto (g - \text{id})^{-1}(x)$  is well defined for all  $x \in (g(\underline{a}) - \underline{a}, g(\bar{a}) - \bar{a})$  ( $\text{id}$  is the identity mapping). Given  $x$ , an individual chooses education iff his ability  $a$  is not smaller than  $a_0 = \phi(x)$  (implicit in this is that an individual that is indifferent between skilled and unskilled work chooses to become educated). So the supply of unskilled labor is given by  $F(a_0)$  and the supply of skilled workers by  $1 - F(a_0)$ .

The following proposition is immediate:

**Proposition 1** *The function  $\phi$  is differentiable and increasing in  $x$ :  $\phi'(x) > 0$ . Hence, technological progress or a decline of the costs of education raises the relative supply of skilled labor.*

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<sup>2</sup>This assumption holds, e.g., in the case of a small open economy with unrestricted access to international capital markets and neoclassical production possibilities.

To study the behavior of the relative wage we have to choose a wage index for each group of labor. For ease of exposition, we choose the median income, but our results remain valid for the mean income as well.

Let  $a_l$  ( $a_h$ ) denote the median of the ability distribution conditional on the fact that the ability is smaller (not smaller) than  $a_0$ , i.e.  $a_l := F^{-1}(F(a_0)/2)$  and  $a_h := F^{-1}((1 + F(a_0))/2)$ . Due to the strict monotonicity between ability and wage the median income of the unskilled  $w_l(a_0)$  and the skilled  $w_h(a_0)$  are

$$w_l(a_0) = F^{-1}\left(\frac{F(a_0)}{2}\right) A \quad (1)$$

$$w_h(a_0) = g\left[F^{-1}\left(\frac{1 + F(a_0)}{2}\right)\right] A \quad (2)$$

Reducing  $x$  obviously shifts down the threshold level of human capital investment  $a_0$ , so the most talented unskilled workers become educated. This drives down the productivity-adjusted wage rates of the median of both skilled and unskilled workers ( $w_h/A$  resp.  $w_l/A$ ). Whether the relative wage  $\omega := w_h(a_0)/w_l(a_0) = g(a_h)/a_l$  is falling or rising depends on the relative size of those changes.

The sign of  $d\omega/dx$  depends on the shape of the function  $g$  and the shape of the density function  $f$ . This can be seen by calculating the effect of an exogenous change in  $x$  on  $\omega$ :

$$\frac{d\omega}{dx} = \left[ \frac{g'(a_h) f(a_0)}{g(a_h) f(a_h)} - \frac{1 f(a_0)}{a_l f(a_l)} \right] \frac{g(a_h)}{2a_l} \phi'(x). \quad (3)$$

In (3),  $g'(a_h)/g(a_h)$  and  $1/a_l$  are the growth rates of  $g(a_h)$  and  $a_l$  with respective weights  $f(a_0)/f(a_h)$  and  $f(a_0)/f(a_l)$ . This is because for a small change  $da_0 <$

$0$  the medians of both groups have to decrease so that  $\int_{a_k+da_k}^{a_k} f(a) da = 1/2 \int_{a_0+da_0}^{a_0} f(a) da$



( $k = l, h$ ) holds. For a marginal change of  $a_0$  the integral can be approximated by  $f(a_k)|da_k| = 1/2f(a_0)|da_0|$ . The larger  $f(a_k)$  for given  $f(a_0)|da_0|$ , i.e. the more mass there is at the respective median position, the smaller the size of the change of  $a_k$ .

To demonstrate that a non-monotone relation between  $\omega$  and  $x$ —and hence between the skill premium and relative labor supply—is indeed possible without referring to unrealistic specifications of  $g$  or  $f$  we consider the following examples.

### 3 Examples

Our first example isolates the effect of the shape of the  $g$  function on non-monotonicity. Therefore, we assume inherent abilities to be distributed uniformly within the interval  $[\underline{a}, \bar{a}]$ ,<sup>3</sup> so that both of the weights  $f(a_0)/f(a_h)$  and  $f(a_0)/f(a_l)$  are equal to 1.

$$(A2) \quad g(a) = a^z \text{ and } f(a) = 1/(\bar{a} - \underline{a})\mathbb{1}_{[\underline{a}, \bar{a}]}(a) \text{ with } \underline{a} > 1, z > 1.^4$$

Under (A2), (3) reads

$$\frac{d\omega}{dx} = \left[ \frac{z}{a_h} - \frac{1}{a_l} \right] \frac{a_h^z}{2a_l} \phi'(x). \quad (4)$$

Hence,  $\text{sgn}(d\omega/dx) = \text{sgn}(z - a_h/a_l)$ , and we get

**Lemma 1** *Under (A2), a sufficient condition for a U-shaped relation between  $\omega$*

<sup>3</sup>This assumption is quite common in the literature; cf. Galor and Moav (2000).

<sup>4</sup>As usual,  $\mathbb{1}$  denotes the indicator function of the set  $I$ , i.e.  $\mathbb{1}_I(a)$  is 1 if  $a \in I$ , and 0 otherwise.

and  $x$  is given by  $2\bar{a}/(\bar{a} + \underline{a}) < z < (\bar{a} + \underline{a})/2\underline{a}$ .<sup>5</sup>

*Proof.* Under (A2),  $d(a_h/a_l)/dx = (1 - a_h/a_l)\phi'(x)/2a_l < 0$ . Hence,  $z - a_h/a_l$  must be strictly increasing in  $x$ . Due to the postulated condition we have  $\lim_{x \rightarrow g(\underline{a}) - \underline{a}} (z - a_h/a_l) = z - (\bar{a} + \underline{a})/2\underline{a} < 0 < z - 2\bar{a}/(\bar{a} + \underline{a}) = \lim_{x \rightarrow g(\bar{a}) - \bar{a}} (z - a_h/a_l)$  which proves the result.  $\square$

Our second example isolates the effect of the shape of the distribution of inherent abilities on non-monotonicity by assuming linear  $g$ :

$$(A3) \quad g(a) = ba \text{ with } b > 1, \text{ and } f(\underline{a}) = f(\bar{a}) = 0$$

Under (A3),  $g'(a_h)/g(a_h) = 1/a_h$ , and (3) reads

$$\frac{d\omega}{dx} = \left[ \frac{1}{a_h f(a_h)} - \frac{1}{a_l f(a_l)} \right] \frac{ba_h}{2a_l} f(a_0) \phi'(x). \quad (5)$$

Defining  $h(x) := (f(a_l)/f(a_h) - a_h/a_l)(x)$ , we get  $\text{sgn}(d\omega/dx) = \text{sgn}(h)$ . The following result is quite evident

**Lemma 2** *Under (A3) there is a non-monotone relation between  $\omega$  and  $x$ .*

*Proof.* From  $\lim_{x \rightarrow g(\underline{a}) - \underline{a}} h(x) = -F^{-1}(1/2)/\underline{a} < 0$  and  $\lim_{x \rightarrow g(\bar{a}) - \bar{a}} h(x) = +\infty$ , it is immediate that the sign of  $d\omega/dx$  is non-unique.  $\square$

If we additionally assume

$$(A4) \quad f \text{ is symmetric and single peaked on } [\underline{a}, \bar{a}]$$

<sup>5</sup>The condition  $2\bar{a}/(\bar{a} + \underline{a}) < z < (\bar{a} + \underline{a})/2\underline{a}$  requires that  $\bar{a} - \underline{a}$  is sufficiently great if  $z \geq 2$ , and that  $\bar{a} - \underline{a}$  lies within certain boundaries if  $1 < z < 2$ .

we arrive at

**Lemma 3** *Under (A3) and (A4), the relation between  $\omega$  and  $x$  is U-shaped.*

*Proof.* Since  $\text{sgn}(d\omega/dx) = \text{sgn}(h)$  it is sufficient to consider the properties of  $h$ . Symmetry and single peakedness of  $f$  guarantee that  $\frac{df(a_l)/f(a_h)}{dx} > 0$ . Furthermore, because of  $\frac{da_h/a_l}{dx} = 1/b \frac{d\omega}{dx}$  we have  $\text{sgn}\left(\frac{da_h/a_l}{dx}\right) = \text{sgn}(h)$ . Combining yields that for all  $x$  with  $h(x) \leq 0$  we must have  $h'(x) > 0$ . This in turn implies that  $h$  can have at most one root. From the proof of the preceding lemma we have  $\lim_{x \rightarrow g(\underline{a}) - \underline{a}} h(x) < 0 < \lim_{x \rightarrow g(\bar{a}) - \bar{a}} h(x)$  so that the intermediate value theorem guarantees the existence of  $\tilde{x}$  such that  $h(\tilde{x}) = 0$ . So we have  $h(x) < 0$  for  $x < \tilde{x}$ ,  $h(x) = 0$  for  $x = \tilde{x}$  and  $h(x) > 0$  for  $x > \tilde{x}$  which proves the result.  $\square$

Lemmata 1 and 3 can be summarized in the following proposition

**Proposition 2** *Under (A2) and differences in abilities of a certain size or (A3) and (A4), permanent technological change or a permanent reduction in the cost of education first lower and eventually raise the relative wage of skilled workers.*

## 4 Conclusions

This paper has shown that some simple human–capital–investment considerations in conjunction with individuals having heterogeneous inherent abilities are sufficient to cause a non–monotonic evolution of the relative wage when there is technological change. An integration of the proposed mechanism into an intertemporal general–equilibrium model seems to be a promising line of future research

offering a more comprehensive explanation of the dynamics of wage inequality and labor supply.

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