

Schweinberger, Albert G.

**Working Paper**

## Capital, heterogeneous labour, global goods markets and unemployment

Diskussionsbeiträge - Serie I, No. 309

**Provided in Cooperation with:**

Department of Economics, University of Konstanz

*Suggested Citation:* Schweinberger, Albert G. (2001) : Capital, heterogeneous labour, global goods markets and unemployment, Diskussionsbeiträge - Serie I, No. 309, Universität Konstanz, Fachbereich Wirtschaftswissenschaften, Konstanz

This Version is available at:

<https://hdl.handle.net/10419/68932>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

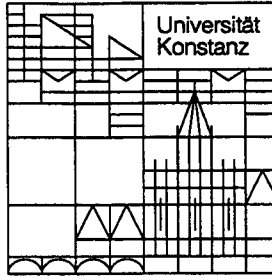
Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*



475 977 696

**Rechts-, Wirtschafts- und  
Verwaltungswissenschaftliche Sektion**  
Fachbereich  
Wirtschaftswissenschaften

---

Albert G. Schweinberger

**Capital, Heterogeneous Labour,  
Global Goods Markets  
and Unemployment**

W 284 (309)

Diskussionsbeiträge

---

**Capital, Heterogeneous Labour, Global Goods Markets  
and Unemployment**

**Albert G. Schweinberger**

**Serie I – Nr. 309**

**Januar 2001**

# Capital, Heterogeneous Labour, Global Goods Markets and Unemployment

by

Albert G. Schweinberger  
Faculty of Economics and Statistics  
University of Konstanz  
Januar 2001

**Keywords:** Minimum Wages, Global Markets, Unemployment  
**JEL classification index:** F 11, J31, E24

Address for correspondence:  
Prof. Albert G. Schweinberger  
Faculty of Economics and Statistics  
University of Konstanz  
Fach D 150  
78457 Konstanz  
Germany  
Phone: +49/7531/88-2632  
Telefax-No.: +49/7531/88-4119  
E-mail: [Albert.Schweinberger@uni-konstanz.de](mailto:Albert.Schweinberger@uni-konstanz.de)

# Capital, Heterogeneous Labour, Global Goods Markets and Unemployment

## Abstract

A two country model of trade between a flexi wage and a minimum wage economy or two minimum wage economies is developed. The main novelty is that there are three factors of production: capital, skilled and unskilled labour. This unlocks the terms of trade. Unskilled labour is subject to the same or different minimum wages in one or both countries. The trading pattern of the two countries is explained in terms of differences in various factor intensities, endowments with the two fully employed factors (capital and skilled labour) and the binding minimum wage rates. Free trade may reduce the wage of the unskilled workers in the flexi wage economy and lower the unemployment in the minimum wage economy. This follows because unemployment may be a source of competitive advantage due to the income effect. If the two countries differ only in terms of factor endowments (but operate the same minimum wage) free trade moves the two countries towards an equalisation of unemployment.

# 1 Introduction

Many economists would argue that one of the main obstacles to further trade liberalisation is the regulation of national labour markets in many (especially European) countries. Most often this regulation assumes the form of setting minimum wage rates for unskilled workers. As is well known, binding minimum wage rates for unskilled workers are widely regarded as one of the main "causes" of involuntary unemployment in closed and open economies.

Will international trade exacerbate unemployment of unskilled workers? This is surely one of the key issues in any discussions of further trade liberalisation. A very popular view is that it is obvious that by setting too high minimum wage rates the economy becomes uncompetitive. The unskilled workers price themselves out of world markets. Furthermore the too high minimum wage rates in the home country benefit foreign unskilled workers because of a diversion of demand from domestic towards foreign unskilled labour. The wage rates of the foreign unskilled workers are bound to rise and so will unemployment in the home country.

The preceding popular argument about the effects of minimum wage rates in open economies was made precise for the first time in two elegant papers by Davis [see Davis (1998a) and (1998b)].

Davis proved in a benchmark model that if two countries which are identical (except that one does and the other does not impose minimum wage rates for unskilled workers) start trading there is not only rising unemployment of the unskilled workers in the country with binding minimum wage rates but also a rise in the wage of the foreign unskilled workers.

Davis (op. cit.) also showed that whilst the former result (the rising unemployment) may not hold in economies which also differ in regard to factor endowments the latter result (the rise in the foreign wage) is very robust.

Making use of the classical work by Brecher [see Brecher (1974a) and (1974b)] Davis has unearthed an important effect of free international trade on unemployment and wage rates. The fact that this effect captures effectively popular opinion makes it all the more topical and relevant.

My first aim in this paper is to develop a novel Heckscher-Ohlin modelling framework taking into account the existence of binding minimum wage rates in at least one of the trading countries allowing for heterogeneous labour as well as capital. There are three factors of production: capital, skilled and unskilled labour. Only the latter is subject to a binding minimum wage. The model is based upon extensions of the classical work by Brecher (op. cit.) in Schweinberger (1978) and Neary (1985). The other standard assumptions of traditional Heckscher-Ohlin model such as identical production functions and preferences are retained.

Allowing for capital as well as heterogeneous labour seems important for two main reasons. Firstly, it should make the model and the results more acceptable to economists who are not trade theorists and secondly (and more importantly) it leads to several rather drastic changes in the results. The latter are mainly due to the fact that allowing for three factors of production (one of which is subject to a minimum wage) and two produced goods "unlocks" the terms of trade. In a model without capital [as in Davis (op.cit.)], a binding minimum wage and diversification in production imply that both countries trade (in the free trade equilibrium) at the autarkic prices of the minimum wage economy.

Unlocking the terms of trade opens up many possibilities but, as will be shown, one can still derive a number of sharp results. Let, for example, trading patterns be determined more by minimum wage rates. Then it follows that the commencement of international trade between the minimum wage and the flexi wage economies may well imply an increase in the employment of the unskilled in the minimum wage economy and a fall in the wage of the unskilled workers in the flexi wage economy (if the minimum wage economy exports the good which is intensive in the use of unskilled labour).

This surprising result highlights one of the main contributions of the paper: the importance of income effects brought about by the existence of unemployment. The very existence of unemployment may be a source of competitive advantage due to lower income and therefore lower demand for the good which is intensive in the use of the unemployed factor (unskilled labour).

All the results are also relevant to the wage-gap debate<sup>1</sup>. In my view it would appear imperative to take into account trade liberalisation not only between advanced and low wage countries but also between countries which do and others which do not impose minimum wage rates for unskilled workers.

The trade liberalisation between Europe and the USA may well have contributed to lowering unemployment in Europe and falling wages of the unskilled workers in the USA. On the other hand, the increased trade between Europe and the Asian tiger countries may have increased unemployment in Europe and raised the wages of the unskilled workers in Asia. The model put forward in this paper can explain both these possible scenarios in terms of differences in national labour market policies, various patterns of

---

<sup>1</sup>There is a huge literature on the wage gap debate. It has recently been reviewed in Davis (op. cit.) and Aghion P. et alii (1999). One of the best known articles is Freeman (1995), see also Atkinson (2000) and Wood (1994).



factor intensities of the two goods (taking into account the three factors) and factor endowment differences.

Introducing capital as a third factor also opens up a new perspective on the relationship between skilled and unskilled labour. Both skilled and unskilled labour may gain or both may lose from free international trade. If, for example, the minimum wage economy has a price advantage in the good which makes intensive use of unskilled labour and this good is intensive in skilled labour relative to capital both unskilled and skilled labour gain from free international trade.

Another major result concerns the effect of free international trade on unemployment if both countries differ in terms of the endowments with capital and skilled labour but impose the same (binding) minimum wage for unskilled labour. Contrary to popular opinion, the unemployment will rise (fall) in the country with the lower (higher) unemployment in the autarkic equilibrium as a result of free international trade. This result seems to be particularly relevant to trade between European countries belonging to the European union.

The paper is structured as follows: The basic analytical techniques emphasizing the antiprotective effect of binding minimum wage rates on industries and firms are set out in the following part two. In part three a benchmark model is analysed. In this case the two countries are identical except that one imposes a binding minimum wage on unskilled labour. The effect of differences in the endowments with the fully employed factors on trading patterns in minimum wage economies is the main focus of part four. Part five combines the analyses of parts three and four. Precise conditions under which factor endowment differences reinforce or counteract the trading patterns implied by different national labour market policies are derived and interpreted.

## 2 The antiprotective effect of minimum wage rates and the relationship between fix and flexi wage economies

The following key assumptions are taken to hold throughout parts two and three unless stated otherwise.

1. Two goods are produced by means of three factors of production: capital, skilled and unskilled labour. There exists a binding minimum wage for unskilled workers in (at least) one of the two trading countries. The binding minimum wage is exogenous.
2. There is perfect competition in all goods and factor markets. Production functions are constant returns to scale and strictly quasiconcave in all inputs.
3. Preferences are homothetic or quasihomothetic and identical for all households.
4. Good two is chosen as the numéraire. The minimum wage is therefore fixed in terms of good two.
5. Both goods are internationally tradable. There are no nontraded goods.

I shall make use of the following notation:

|                             |   |  |
|-----------------------------|---|--|
| $x_j$ and $x_j^*(j = 1, 2)$ | = | outputs of goods one and two in the home and foreign countries respectively          |
| $L, L^S, K$                 | = | the endowments with unskilled labour, skilled labour and capital of the home country |
| $N$                         | = | the employment of unskilled labour in the home country                               |
| $p_1$ and $p_2$             | = | the prices of the two goods  |

|                       |   |  |
|-----------------------|---|--|
| $w, w^s, r$           | = | the wage of unskilled and skilled labour respectively<br>and the rental of capital             |
| $c_1$ and $c_2$       | = | the consumption of goods one and two   |
| $a_{N1}$ and $a_{N2}$ | = | the amount of unskilled labour required per unit out-<br>put of goods one and two respectively |

As is well known from the received literature, see Schweinberger (1978) and Neary (1985), the distortion brought about by the existence of a binding minimum wage for unskilled labour may be defined as follows. The production agents choose outputs so that only the value added by the fully employed factors (instead of all factors) is maximised. Algebraically, outputs and the employment of unskilled labour is determined from:

**Program A:**

$$\begin{aligned}
 \max_{x_1, x_2, N} : & \quad p_1 x_1 + p_2 x_2 - wN \\
 \text{subject to: } & \quad \tilde{g}(x_1, x_2; N; L^s, K) \leq 0 \\
 & \quad x_1, x_2, N \geq 0
 \end{aligned}$$

This entails that the standard production possibility locus in the output space alone is no longer relevant for the determination of the equilibrium outputs. Instead an appropriately defined locus in the space of outputs and employment of unskilled labour becomes relevant [see again Schweinberger (op. cit.) and Neary (op. cit.)].

To make the role of the antiprotective effect of minimum wage rates as transparent as possible it is convenient to assume that the input coefficients  $a_{N1}$  and  $a_{N2}$  are fixed. As we shall see in a moment, this assumption enables us to make use of concepts of the

theory of effective protection [see e. g. Corden (1971)]<sup>2</sup>.

**Assumption 6:**

The input coefficients of unskilled labour  $a_{N1}$  and  $a_{N2}$  are fixed.

If  $a_{N1}$  and  $a_{N2}$  are fixed it is straightforward to show that program A may be rewritten as the following program B.

**Program B**

$$\begin{aligned} \max_{x_1, x_2} : & (p_1 - a_{N1}w)x_1 + (p_2 - a_{N2}w)x_2 \\ \text{subject to : } & g(x_1, x_2; L^s, K) \leq 0 \\ & x_1, x_2 \geq 0 \\ \text{where : } & g(x_1, x_2; L^s, K) \equiv \tilde{g}(x_1, x_2; N; L^s, K) \\ \text{and } & N = a_{N1}x_1 + a_{N2}x_2 \end{aligned}$$

It is easy to show that the convexity of the reduced form transformation set  $g(\cdot) \leq 0$  in the space of  $x_1$  and  $x_2$  is implied by the convexity of the unreduced transformation set  $\tilde{g}(\cdot) \leq 0$  in  $x_1$ ,  $x_2$  and  $N$ .

The symbols  $v_1$  and  $v_2$  stand for the values added by the fully employed factors in the production of goods one and two respectively; the symbol  $\Pi$  for the total value added by the fully employed factors. Clearly,  $v_1$  and  $v_2$  could be interpreted as effective prices, see again Corden (op. cit.), to highlight the isomorphism with the theory of effective protection. The percentage changes in  $v_1$  and  $v_2$  brought about by changes in the prices

<sup>2</sup>This assumption is not as unrealistic as it may appear. Skilled labour is generally considered to be much more flexible than unskilled labour, see e.g. Aghion P. et alii (1999).

of the two goods and/or the minimum wage ( $w$ ) could be referred to as effective rates of protection. *Ceteris paribus* an increase in  $w$  implies an antiprotective effect.

My next aim is to prove that in an economy which does not impose a binding minimum wage for unskilled workers Program B may still be used to determine the output (and employment) effects of changes in the goods prices  $p_1$  and  $p_2$  and the wage of the unskilled workers. In such an economy all three factors are fully employed and the wage of the unskilled workers is endogenous. To this end consider the market equilibrium conditions for the fully employed factors:

$$a_{K1}(\cdot)x_1 + a_{K2}(\cdot)x_2 = K \quad (1)$$

$$a_{L1}(\cdot)x_1 + a_{L2}(\cdot)x_2 = L^S \quad (2)$$

$$\text{where:} \quad a_{Kj} = a_{Kj}(r, w^S), \quad j = 1, 2$$

$$a_{Lj} = a_{Lj}(r, w^S), \quad j = 1, 2$$

denote the input coefficients of capital and skilled labour per unit output of goods one and two respectively<sup>3</sup>.

Differentiating totally (1) and (2) and multiplying the two equations by  $r$  and  $w^S$  respectively, one obtains after addition (having made use of the conditions of cost minimisation:  $rda_{K1} + w^S da_{L1} = 0$  and  $rda_{K2} + w^S da_{L2} = 0$ ) the following expression:

$$(a_{K1}r + a_{L1}w^S)dx_1 + (a_{K2}r + a_{L2}w^S)dx_2 = 0 \quad (3)$$

Considering the zero profitability conditions:

<sup>3</sup>Note that the input coefficients whilst flexible do not depend upon the wage of unskilled workers. This follows because of the assumption that there is no substitution between unskilled and skilled labour or capital.

$$a_{K1}r + a_{L1}w^S + a_{N1}w = p_1 \quad (4)$$

$$a_{K2}r + a_{L2}w^S + a_{N2}w = p_2 \quad (5)$$

one arrives at:

$$v_1 dx_1 + v_2 dx_2 = 0 \quad (6)$$

where:

$$v_1 = p_1 - a_{N1}w \quad (7)$$

$$v_2 = p_2 - a_{N2}w \quad (8)$$

Expression (6) shows that the outputs are again chosen by production agents so that the total value added by capital and skilled labour is maximised subject to the transformation locus defined by equations (1) and (2).

The wage of unskilled labour is now determined from the following market equilibrium condition for unskilled labour:

$$a_{N1}x_1(p_1 - a_{N1}w, p_2 - a_{N2}w) + a_{N2}x_2(p_1 - a_{N1}w, p_2 - a_{N2}w) = L \quad (9)$$

The functions  $x_1(\cdot)$  and  $x_2(\cdot)$  denote the supply functions of goods one and two respectively. They exist if the reduced form production possibility set  $g(\cdot) \leq 0$  is weakly bowed out. This is the case if:

$$a_{K1}a_{L2} - a_{K2}a_{L1} \neq 0.$$

Having solved expression (9) for  $w$ , it is straightforward to determine the equilibrium outputs of the two goods.

To prove the isomorphism between an economy subject to a binding minimum wage (given the stated assumptions) and a flexi-wage (full employment) economy, I totally differentiate equation (9) with respect to  $w$ . This yields:

$$a_{N1} \frac{\partial x_1}{\partial w} dw + a_{N2} \frac{\partial x_2}{\partial w} dw = 0 \quad (10)$$

Equation (10) may be rewritten as:

$$a_{N1} dx_1 + a_{N2} dx_2 = 0 \quad (11)$$

Substituting (11) into:

$$(p_1 - a_{N1}w)dx_1 + (p_2 - a_{N2}w)dx_2 = 0 \quad (12)$$

(see Program B) one readily obtains:

$$p_1 dx_1 + p_2 dx_2 = 0 \quad (13)$$

Equation (13) is the standard result of a full employment economy: outputs are chosen so that the value added by all factors of production is maximised (at the given goods prices). The results are now formalised in the following LEMMA.

**LEMMA:**

Let assumptions (1) to (6) be satisfied. Then the equilibrium outputs of goods one and two are determined from the constrained optimisation program in an economy with a binding minimum wage, see Program B. Now let all three factors be fully employed; i.e.: the wage of the unskilled workers is fully flexible. It is determined by equation (9). Then the equilibrium outputs of goods one and two are still determined from Program B having solved the market equilibrium condition for unskilled labour [see expression (9)] for the full employment wage of unskilled labour.

It will be shown that this LEMMA is extremely useful for our purposes. It enables us to relate directly economies which are subject to binding minimum wage rates (as many countries in Europe) to countries with flexible wages and (almost) full employment such as the USA<sup>4</sup>.

### 3 The determination of trading patterns and unemployment

There seems to be general agreement among trade theorists that the regulation in national labour markets could play as important a role in the determination of trading patterns as differences in factor endowments and differences in technology. As mentioned in the Introduction the same applies to popular arguments in the media. To ensure tractability of the analysis I now make the following Assumption 7.

#### **Assumption 7:**

The production functions for all goods are identical in both countries.

Assumption 7 may or may not imply factor price equalisation. It definitely does if there are only two factors of production: skilled and unskilled labour and there are no factor intensity reversals. This is not the case in a model with two goods and three factors of production (as developed in part two) unless by coincidence.

To make the analysis and its results as transparent as possible, I assume first that the two trading economies are completely identical except that one does and the other does not regulate the market for unskilled labour by enforcing a minimum wage. The main question which I wish to put to this model is whether a commencement of trade

---

<sup>4</sup>It should be noted that the results stated in the Lemma may be regarded as applications of fundamental optimisation theorems in perfectly competitive economies, see e. g. Woodland (1982).



between the two countries can explain the rising unemployment in the minimum wage economy (Europe) and/or the (almost) simultaneous immiserisation of unskilled workers in the USA. Needless to emphasize the observation of rising unemployment in Europe (possibly due to increased trade between Europe and the USA) and the decline in the relative real wages of the unskilled workers in the USA is right at the heart of the wage gap debate. Furthermore it should be noted that the model of the USA may readily be reinterpreted as referring to any country with flexible wages and full employment (such as until recently the Asian tiger countries).

In order to determine the autarkic equilibrium values of  $p_1$  and  $\bar{w}$  one needs two equations: (14) and (15).

$$\begin{aligned} x_1(\cdot) &= c_1[p_1, 1; p_1 x_1(\cdot) + p_2 x_2(\cdot)] \quad \text{or} \\ e_1(\cdot) &= x_1(\cdot) - c_1(\cdot) = 0 \end{aligned} \quad (14)$$

Equation (14) represent the autarkic market equilibrium condition for good 1 in Europe and the USA. Equation (9) is restated here for convenience:

$$a_{N1}x_1(\cdot) + a_{N2}x_2(\cdot) = L \quad (15)$$

where:  $x_1 = x_1(p_1 - a_{N1}w, p_2 - a_{N2}w; K, L^S)$

$$x_2 = x_2(p_1 - a_{N1}w, p_2 - a_{N2}w; K, L^S)$$

$$c_1 = c_1(\cdot) = \text{the uncompensated demand function for good 1 in the home and foreign countries}$$

Equations (14) and (15) stand for the autarkic equilibrium conditions for Europe and the USA for good 1 and unskilled labour. Equations (14) and (15) can be solved

for  $p_1$  and  $w$  keeping  $K$  and  $L^S$  fixed but for a lower value of  $L$  in the minimum wage economy. The latter is taken to be the home country (Europe).

Many of the following results hinge on the following **factor intensity condition**. Good 1(2) is said to intensive in the use of unskilled labour relative to good 2(1) if and only if:

$$\alpha_{N1} > \alpha_{N2} \quad (\alpha_{N2} > \alpha_{N1})$$

where:  $\alpha_{N1} = \frac{a_{N1}w}{p_1}$       and       $\alpha_{N2} = \frac{a_{N2}w}{p_2}$

It is very straightforward to show that if  $dw > 0$  and  $dp_1 = dp_2 = 0$  then:

$$\begin{aligned} dv_1/v_1 < 0 \quad \text{and} \quad dv_2/v_2 < 0 \\ \text{and} \quad \alpha_{N1} \geq \alpha_{N2} \iff dv_2/v_2 \geq dv_1/v_1 \end{aligned} \tag{16}$$

I first assume that good two (the numéraire good) is intensive in the use of unskilled labour. As will be shown, in this case it is relatively straightforward to generalize the key result obtained by Davis[in Davis (op. cit.)]; namely that the commencement of international trade between the minimum wage economy (Europe) and the flexi wage economy (USA) increases the wage of the unskilled in the USA and also raises unemployment in Europe. Consider Figure 1 on page 15.

Since good two is intensive in the use of unskilled labour it can be shown that the loci determined by equations (14) and (15) must (both) be downward sloping. Taking the schedule  $e_1$  first, note that a fall in  $p_1$  (keeping  $w$  fixed), entails an excess demand for good one. To reestablish the equilibrium in the market for good one the wage of unskilled labour has to rise (for given  $p_1$ ) because this implies a fall in the output of good two and an increase in the output of good one [see expression (16)].

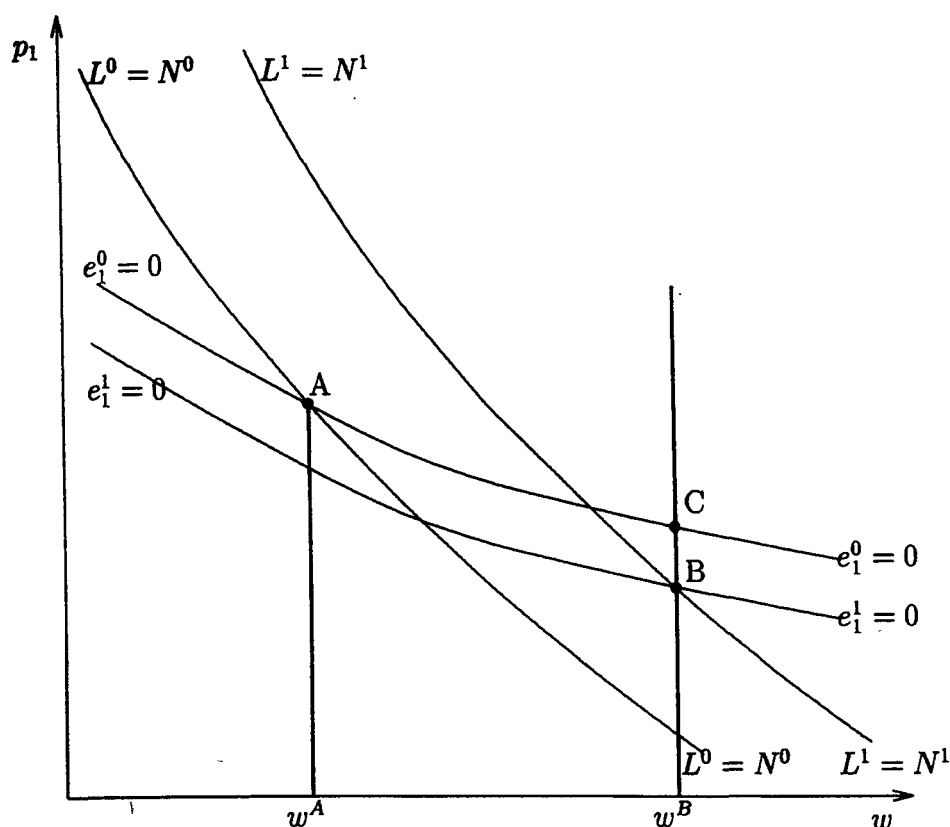


Figure 1:

The locus  $L = N$  is negatively sloped because a fall in  $p_1$  (given  $w$ ) increases the demand for unskilled labour if good two is intensive in the use of unskilled labour. A ceteris paribus fall in  $p_1$  entails a fall in the output of good one and a rise in the output of good two. To reestablish the equilibrium in the market for unskilled labour the wage  $w$  has to rise.

In drawing Figures 1 and 2 I have also assumed that **both markets are stable**. To be precise, I have assumed that "own effects always dominate cross effects". This means that a change in  $p_1$  (for given  $w$ ) always has a bigger effect on the market for good one than on the market for unskilled labour. Something completely analogous holds for the effects of changes in  $w$  on the market for unskilled labour and the market for good one.

The schedules shown on Figure 1 correspond to the two countries. The foreign country (the flexible wage economy) is denoted by the superscript 0, the home country by the superscript 1.

The autarkic equilibrium values of  $w$  and  $p_1$  in the foreign country are shown by the point  $A$ . The corresponding values for the home country by  $B$ . The two countries differ in terms of the employment of unskilled labour. The binding minimum wage in the home country brings about a lower level of employment of unskilled labour in the home country in the autarkic equilibrium. As shown in part two this employment effect may be analysed in terms of a lower endowment with unskilled labour in the home country.

This lower endowment with unskilled labour in the home country implies that the  $L = N$  schedule for the home country must lie to the right of  $L = N$  schedule for the foreign country.

The lower value for  $L$  in the home country also implies that the  $e_1 = 0$  schedule for the home country must lie below the corresponding schedule for the foreign country. This follows because a lower value of  $L$  entails, for given  $p_1$  and  $w$ , that there is an excess supply of good one. Therefore  $p_1$  must fall to reestablish the equilibrium (keeping  $w$  fixed). Note that the changes in  $w$  on the schedule  $e_1^0 = 0$  must be such that equilibrium in the market for unskilled labour is preserved, i.e.: for small changes in  $w$ , one must have:  $p_1 dx_1 + p_2 dx_2 = 0$ . The following Proposition I follows from Figure 1.

**PROPOSITION I:**

Assume that two countries are identical except that one operates a binding minimum wage for unskilled labour and the other does not and that good two (the numéraire good) is intensive in the use unskilled labour (see expression (18)) and that the markets

for good one and unskilled labour are stable (in the sense described above). Then:

- (a) the minimum wage economy has a lower price for good one in the autarkic equilibrium;
- (b) the commencement of international trade implies an improvement of the wage of the unskilled workers in the flexi wage economy (the USA) and a rise in unemployment in the home country (Europe), the minimum wage economy.

**Proof:** Part (a) of the proposition follows directly from the Figure. Part (b) follows because a rise in  $p_1$  in the home country implies for the given minimum wage  $w^B$  that the schedule  $L = N$  must shift to the right even further. Note that the equilibrium schedule  $e_1^1 = 0$  is no longer relevant once trade occurs. The wage of unskilled labour in the USA (the flexi wage economy) clearly must rise after the commencement of trade because  $p_1$  must fall and the economy moves along the schedule  $L^0 = N^0$ . Q.E.D.

Proposition I generalises one of the main results obtained by Davis in op. cit. to minimum wage economies with heterogeneous labour and capital. Actually as is obvious from Figure 1 Proposition I can readily be reinterpreted in terms of two minimum wage economies trading with each other (the one with the higher minimum wage experiences more unemployment than the other).

I now turn to the case where the nonnuméraire good, namely good one, is intensive in the use of unskilled labour.

A reasoning analogous to the one underlying Figure 1 leads to Figure 2

Figure 2 has been drawn so that the home country (Europe) has a lower autarkic price of good one. But this is not necessarily the case. Assume first that the home

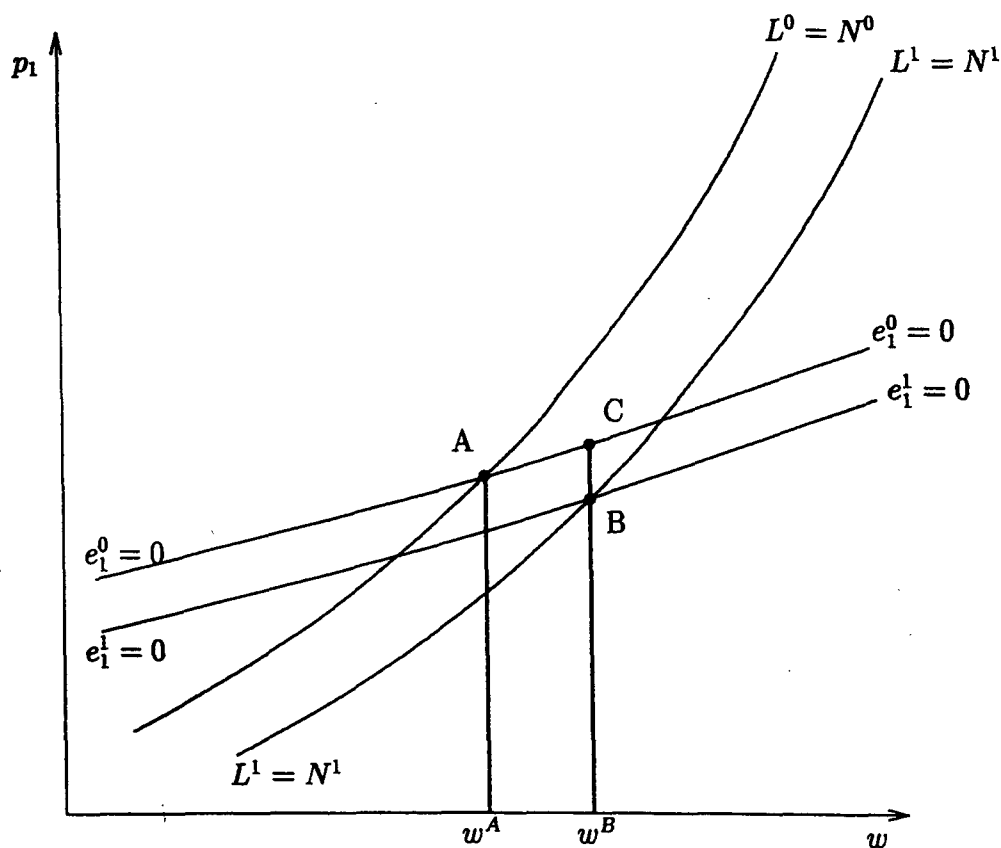


Figure 2:

country actually has a lower price of good one in the autarkic equilibrium. Then it follows from Figure 2 that the commencement of trade between the two countries lowers the wage of unskilled labour in the flexiwave economy (USA) and increases employment of the unskilled workers in Europe. This is a very interesting result because it is counterintuitive. Note especially it does not depend upon assumed differences in the factor endowments of the two countries

If, on the other hand, the autarkic equilibrium price in the home country of good one is higher in the minimum wage economy (Europe) it follows at once that even though good one is intensive in the use of unskilled labour (and not good two as on Figure 1) the effect of international trade is the same as on Figure 1: the wage of the unskilled

workers rises in the flexi wage economy and employment in the minimum wage economy falls (and unemployment rises). These results are now formalised as Proposition II.

**PROPOSITION II:**

Assume that the two countries are identical except that one operates a binding minimum wage for unskilled labour and the other does not. Further assume that good one (the nonnuméraire good) is intensive in the use of unskilled labour and that the markets for good one and unskilled labour are stable (in the sense explained above). Then the commencement of trade implies that:

- (a) the wage of the unskilled workers rises in the flexi wage economy and employment of the unskilled workers falls in the minimum wage economy if the minimum wage economy has a higher autarkic equilibrium price for good one.
- (b) the wage of the unskilled workers falls in the flexi wage economy and employment of the unskilled workers rises in the minimum wage economy if the minimum wage economy has a lower autarkic equilibrium price for good one.
- (c) the autarkic price of good one is higher in the minimum wage economy if the wage and employment differences between the two economies are "very small" and there are no inferior goods.
- (d) the autarkic price of good one may be lower in the minimum wage economy if the differences between the two economies with regard to employment and the wage (of the unskilled) are large.

**Proof:** See Appendix A.

A useful corollary follows from Propositions I and II.

**COROLLARY:**

Assume two identical economies as described in Propositions I and II. Then the commencement of international trade is necessarily conflict generating between the owners of unskilled labour in the flexi wage and minimum wage economies.

The Corollary follows directly from Propositions I and II. It is noteworthy because it formalises the intuitive conclusion that in the economies described above the owners of unskilled labour in one of the two countries must lose from international trade and therefore will oppose it. Most importantly, the analysis shows that the losers may not be the unskilled workers in the country which operates the minimum wage policy.

This apparently paradoxical result may arise because the demand for good 1 in the country with the minimum wage is lower because income is lower in this country. The latter is due to the possibly high level of unemployment amongst the unskilled workers. What is interesting and novel about this result is that a binding minimum wage and the resulting unemployment of the unskilled workers may give a country a competitive advantage. The probability of the said income effect more than offsetting the substitution effect is higher if the difference between the full employment wage and the binding minimum wage is bigger, see parts (c) and (d) of Proposition II and Appendix A.



## 4 The Heckscher-Ohlin theorem with heterogeneous labour, capital and unemployment

In part three I analysed in some detail the effect of international trade on factor prices and unemployment if trade occurs between two countries which are identical except for the existence of a binding minimum wage for unskilled labour in one of the two countries. I now turn to the analysis of the trading patterns and the effects of trade on factor prices and unemployment if the two trading countries operate the same binding minimum wage for unskilled labour and are in fact identical except for differences in the endowments with the two fully employed factors, capital and skilled labour. In a nutshell I now generalise the standard Heckscher-Ohlin model and theorem to three factors of production, where one of the three factors is unemployed due to a binding minimum wage.

To generalise the Heckscher-Ohlin theorem I differentiate totally

$$e_1 = x_1(\cdot) - c_1[p_1, 1; p_1 x_1(\cdot) + x_2(\cdot)] \quad (17)$$

with respect to  $K$  and  $L^S$ , keeping  $p_1$ , fixed and assuming that in the initial equilibrium  $x_1(\cdot) = c_1(\cdot)$ . This yields:

$$de_1 = \left( \frac{\partial x_1}{\partial K} K - \frac{\partial c_1}{\partial Y} \frac{\partial Y}{\partial K} K \right) \frac{dK}{K} + \left( \frac{\partial x_1}{\partial L^S} L^S - \frac{\partial c_1}{\partial Y} \frac{\partial Y}{\partial L} L \right) \frac{dL}{L} \quad (18)$$

$$\text{where: } Y = p_1 x_1(\cdot) + x_2(\cdot) .$$

From the fact that  $x_1(\cdot)$  and  $Y(\cdot)$  are homogeneous of degree one in  $K$  and  $L^S$  we know that:

$$L^S \frac{\partial x_1}{\partial L^S} = x_1 - \frac{\partial x_1}{\partial K} K \quad (19)$$

$$L^S \frac{\partial Y}{\partial L^S} = Y - \frac{\partial Y}{\partial K} K \quad (20)$$

Substituting (19) and (20) into (18) I obtain:

$$de_1 = \frac{\partial x_1}{\partial K} K \left( \frac{dK}{K} - \frac{dL^S}{L^S} \right) - \frac{\partial c_1}{\partial Y} \frac{\partial Y}{\partial K} K \left( \frac{dK}{K} - \frac{dL^S}{L^S} \right) + x_1 \left( 1 - \frac{\partial c_1}{\partial Y} \frac{Y}{c_1} \right) \frac{dL^S}{L^S} \quad (21)$$

Noting that:

$$\frac{\partial Y}{\partial K} = p_1 \frac{\partial x_1}{\partial K} + \frac{\partial x_2}{\partial K}$$

I finally arrive at:

$$de_1 = \left( \frac{\partial c_2}{\partial Y} \frac{\partial x_1}{\partial K} - \frac{\partial c_1}{\partial Y} \frac{\partial x_2}{\partial K} \right) K \left( \frac{dK}{K} - \frac{dL^S}{L^S} \right) + x_1 \left( 1 - \frac{\partial c_1}{\partial Y} \frac{Y}{c_1} \right) \frac{dL^S}{L^S} \quad (22)$$

To generalise the Heckscher-Ohlin theorem I now make three additional assumptions.

**Assumption 8:**

The market for good one is Walras stable.

**Assumption 9:**

The preferences of the households in the two countries are not only identical, but also homothetic.

**Assumption 10:**

There are no factor intensity reversals, i.e.: if one good is intensive in capital (relative to skilled labour) at one set of factor prices it is intensive in capital at all factor prices.

Assumptions 8, 9 and 10 and equation (22) give rise to the following Proposition III.

**PROPOSITION III:**

Assume two countries. Each produces two goods with three factors of production: capital, skilled labour and unskilled labour. The two countries are identical except for the endowments with capital and skilled labour. Both countries operate the same binding

minimum wage for unskilled labour. The autarkic equilibrium price of the capital (skilled labour) intensive good is lower (higher) in the country which is relatively abundantly endowed with capital (skilled labour).

**Proof:** Proposition III follows directly from expression (22) and the stated assumptions. The latter imply that:

$$\frac{\partial c_1}{\partial Y} \quad \text{and} \quad \frac{\partial c_2}{\partial Y} > 0$$

If good one is capital intensive, then:

$$\frac{\partial x_1}{\partial K} > 0 \quad \text{and} \quad \frac{\partial x_2}{\partial K} < 0 \quad (23)$$

If good one is intensive in skilled labour then:

$$\frac{\partial x_1}{\partial K} < 0 \quad \text{and} \quad \frac{\partial x_2}{\partial K} > 0 \quad (24)$$

Expressions (23) and (24) follow from total differentiation of the equilibrium conditions for the fully employed factors, see equations (1) and (2).

Proposition III in conjunction with the results of part 3, especially Propositions I and II now enable me to derive results concerning the determination of the trading pattern and the effect of trade on factor prices and unemployment if the two trading countries differ first only in regard to the endowments with the two fully employment factors, capital and skilled labour (but operate the same minimum wage) and then differ in regard to both: the endowment differences with fully employment factors and the binding minimum wage rates. It hardly needs emphasizing that the following section 5 seems of special relevance to the analysis of the effect of free trade on employment levels of European countries belonging to the European Union.

## **5 Endowment Differences, Trading Patterns and the Effect of Trade on Unemployment**

Section 5 is divided into two subsections. The key assumptions of subsection 5.1 are that the two trading countries differ only with regard to their endowments with the fully employed factors, capital and skilled labour but that the minimum wage of unskilled labour is the same. First it is also assumed that the minimum wage is binding in both countries. Then this assumption is relaxed. The minimum wage may be binding in the autarkic equilibrium but not binding in the free trade equilibrium or the converse.

In the final subsection 5.2 the two countries are allowed to differ with regard to factor endowments and binding minimum wage rates.

### **5.1**

One of the most interesting and important issues concerning the effects of the rising trade between European countries on unemployment is whether international trade between two minimum wage economies increases or reduces unemployment in one or both of the countries. Unemployment rates differ considerably between European countries. The view expressed in discussions in the media generally is tantamount to the following: the strong (competitive) economies (i.e.: the economies with the low unemployment rates) benefit from free international trade and the weak (uncompetitive) economies (with the high unemployment rates) are hurt by free international trade. The conclusion generally is that free international trade lowers the already low unemployment in the competitive countries and exacerbates the unemployment in the countries which are already ridden with high unemployment.

The first task in this subsection is to show that the view expressed in discussions in

the media as stated above is wrong (subject to reasonable assumptions). To this end the following Proposition IV is put forward.

**PROPOSITION IV:**

- (a) Assume two countries with identical production functions and homothetic preferences. There are two fully employed factors, capital and skilled labour. There is one unemployed factor: unskilled labour. The unemployment is due to a binding minimum wage which is the same in both countries.
- (b) The effect of international trade is to lower unemployment (increase employment) in the country with high unemployment (in the autarkic equilibrium) and increase unemployment (lower employment) in the country with low unemployment (in the autarkic equilibrium).
- (c) The result stated under (b) holds whatever the endowment differences between the two countries and for any pattern of trade and factor intensities.

**Proof:** A formal proof can be found in the Appendix B, Tables 1 to 4.

Proposition IV can be demonstrated by means of an inspection of Figures 1 and 2. Let  $w^B$  be the common minimum wage of unskilled labour which is binding in both countries. Assume further that:

$$\frac{a_{K1}}{a_{L1}} > \frac{a_{K2}}{a_{L2}} \quad \text{and} \quad d \log K > d \log L^s$$

Good 1 is capital intensive (relative to skilled labour) and the home country (country one) is relatively abundant in capital (again relative to skilled labour).

If we make these assumptions it follows that the autarkic equilibrium point for the foreign country (country zero) may be represented by point  $C$  and the autarkic equilibrium point of the home country (country one) by point  $B$ . The home country has a competitive advantage in the production of good one and exports it; the foreign country therefore imports good one. The price of good one rises in the home country and falls in the foreign country.

The reader's attention is now drawn to Figures 1 and 2. An inspection of Figure 1 shows that the level of employment of unskilled labour in the autarkic equilibrium is lower in the foreign country than in the home country. Since  $p_1$  rises in the home country employment of unskilled labour falls in the home country. Clearly employment rises in the foreign country as  $p_1$  falls.

Turning to Figure 2 and making use of exactly the same reasoning it is easy to see that employment of unskilled labour is lower in the home country in the autarkic equilibrium than in the foreign country. The opening up of the two countries to trade results in a rise of  $p_1$  in the home country and a fall of  $p_1$  in the foreign country. Employment rises in the home country and falls in the foreign country.

## 5.2

In this subsection two of the key assumptions of Proposition IV are relaxed. Firstly, endowment differences between the two countries may be such that the minimum wage rates are not binding in one or both countries. Secondly, the two countries may differ not only with regard to factor endowments but also minimum wage rates. A minimum wage is binding in a country if and only if the following inequality is satisfied:

$$a_{N1}x_1(v_1, v_2; K, L^S) + a_{N2}x_2(v_1, v_2; K, L^S) = N < L \quad (25)$$

**where:**  $v_1 = p_1 - a_{N1}w$  and  $v_2 = 1 - a_{N2}w$

The assumption that the minimum wage rate for unskilled labour is binding implies that  $p_1$ ,  $K$  and  $L^S$  are such that expression (25) is satisfied. As shown by Brecher in Brecher (1980), a country may be endowed with more capital and/or skilled labour but this does not necessarily imply that  $N$  (employment of unskilled labour) is higher.

To shed more light on the relationship between the endowments  $K$  and  $L^S$  on the one hand and  $N$  on the other it is convenient to regard the employment of unskilled labour as being produced by the outputs of the two final goods,  $x_1$ ,  $x_2$ . Solving equations (1) and (2) for  $x_1$  and  $x_2$  in terms of  $K$  and  $L^S$  and substituting into expression (25) one can readily derive the following reduced form:

$$A_{KN}K + A_{LN}L^S = N \quad (26)$$

**where:**  $A_{KN}$  and  $A_{LN}$  denote the employment of unskilled labour generated by one unit of capital and skilled labour respectively.

The coefficients  $A_{KN}$  and  $A_{LN}$  may both be positive but one of them may be negative. Both coefficients are positive if the following condition is satisfied: the vector of input coefficients  $(a_{N1}, a_{N2})$  lies strictly within the cone spanned by the input coefficient vectors  $(a_{K1}, a_{L1})$  and  $(a_{K2}, a_{L2})$ . I refer to this condition as **Assumption 11**.

Assumption 11 and Proposition III now enable us to generalize Propositions I and II to countries which differ not only in terms of minimum wage rates but also in endowments with the fully employed factor, skilled labour and capital.

Because of lack of space I concentrate on the case where good 2 is intensive in the use of unskilled labour, see Figure 1.

**PROPOSITION V:**

Let the two countries differ in terms of minimum wage rates and endowments with the two fully employed factors. The home country (country 1) operates at a higher minimum wage than the foreign country (country 0). The home country is more abundantly endowed with skilled labour and capital than the foreign country. Assumption 11 holds.

Then:

The home country experiences a fall in the wage of the unskilled workers and possibly an increase in unemployment and the foreign country a rise in the wage of the low skill workers if:

either the capital skilled labour ratio in the home country is higher than in the foreign country and good 1 is capital intensive

or the capital skilled labour ratio in the home country is lower than in the foreign country and good 1 is intensive in skilled labour.

**Proof:** It can be shown, see Figure 1, that subject to the assumptions of Proposition V the trading pattern implied by the higher minimum wage in the home country is reinforced by the stated endowment differences between the two countries. Note first of all that it follows from Assumption 2 that the curve  $L^1 = N^1$  must shift to the right because of  $dK$  and  $dL^s$  are positive by assumption. Furthermore from Proposition III the assumed pattern of differences in relative factor intensities of the two fully employed factors imply that the schedule  $e_1^1 = 0$  must shift downward. Hence the minimum wage



given by  $w^B$ , see point B in Figure 1, is no longer binding. The home country a fortiori exports good 1 and the foreign country imports it. As the price of good 1 rises in the home country the wage of the unskilled workers falls until  $w^B$  is reached. If  $p_1$  rises more there will be unemployment. Q.E.D.

An interesting possibility arises if the home country is endowed with less capital and skilled labour than the foreign country and the capital skilled labour ratio is higher in the home country than in the foreign country. Assume that good 1 is intensive in skilled labour. In this case it follows that the  $L^1 = N^1$  schedule shifts to the left and the goods market equilibrium condition  $e_1^1 = 0$  upward. The new intersection point of the two schedules lies to the left of the minimum wage  $w^B$ . Factor endowment differences have made the minimum wage even more binding. As a matter of fact it is easy to see that in this case the trading pattern implied by the existence of a minimum wage in the home country may be reversed by the endowment differences between the two countries. The home country may be importing good 1 rather than exporting it. Employment of the unskilled workers rises in the home country and the wage of the unskilled workers in the foreign country falls as a result of free international trade. This analysis shows again that the argument that binding minimum wage rates in Europe prop up the wages of the unskilled in the USA is subject to a number of qualifications if we allow for capital as well as skilled and unskilled workers.

Finally note that it is straightforward to prove that if the endowment differences and factor intensity pattern are as postulated in Proposition V but good 1 is intensive in the use of unskilled labour, see Figure 2, the case for the home country exporting good 1 and thereby reducing its unemployment at the expense of the foreign unskilled workers

(see Proposition II) is considerably strengthened.

## 6 Conclusions and Extensions

The tension between globalisation and the national regulation of labour markets is the driving force behind a very fast growing literature. Its importance can hardly be overemphasized. This literature has led to many important insights however most contributions are based upon the premise that the effects of globalisation on national labour markets can be analysed without formal two country models of trade. Furthermore many of the contributions (though featuring heterogeneous labour in terms of skilled and unskilled workers) neglect capital. The main tenet of this paper is that in order to analyse the effects of globalisation on national labour markets a two country model with heterogeneous labour and capital is indispensable. The said modelling approach is crucial if the aim is to achieve a unifying framework of market interaction between the national labour market and the global goods markets.

Adopting a model which unlocks the terms of trade a number of novel and substantive results have been derived, see Propositions I-V. These results generalise many of the conclusions of the received literature but also open up new perspectives. Many worthwhile extensions could not be pursued due to lack of space.

Firstly, it would be interesting to allow for a binding minimum wage for skilled as well as unskilled labour. This is relevant for many European countries. It locks, of course, the terms of trade and therefore the flexi wage economy has to trade in the free trade equilibrium at the autarkic relative prices of the minimum wage economy (if there is nonspecialisation in production) Skilled and unskilled labour workers may be complementary or competitive. How the latter effects the pattern of competitive advantage and

trading patterns has to the best of my knowledge not received any attention in formal two country model of international trade.

Secondly, a major shortcoming of a considerable part of the literature and this paper is that an important fact of international trade is ignored: most trade takes place in intermediate and not in final goods. Allowing trade in intermediate goods is bound to affect many of the results because imported intermediate goods may be substitutes or complements for skilled or unskilled labour<sup>5</sup>.

Finally and most importantly, if the aim of the model is to explain the growing wage gap or the long term relationship between the minimum wage rates in some and the level of the flexible wage rates in other trading countries it seems imperative to adopt a dynamic growth oriented approach. This approach must allow for endogenous accumulation of capital as well as production of skilled labour from capital and unskilled labour.

From recent empirical research it appears that vertical intraindustry trade (trade in quality goods) plays an increasingly important part in international trade. It may be conjectured that the increase in vertical intraindustry trade and its effect on national markets for skilled und unskilled workers can only be understood within a formal dynamic framework<sup>6</sup>.

---

<sup>5</sup>For a recent contribution to the wage gap debate which focusses on trade in intermediate goods, see Falk M. and B. Koebel (1997) and Aghion P. et alii (1999).

<sup>6</sup>Interesting evidence on the growing importance of vertical intraindustry trade can be found in Greenaway et alii (1994).

## References

- [1] **Aghion, P., E. Caroli and C. García-Penãlosa** (1999). *Inequality and economic growth: the perspective of the new growth theories*. *Journal of Economic Literature* 37: 1615-1660.
- [2] **Atkinson, A.** (2000). *The changing distribution of income: evidence and explanations*. *German Economic Review* 1(1): 3-18.
- [3] **Brecher, R. A.** (1974a). *Minimum Wage Rates and the Pure Theorie of International Trade*. *Quarterly Journal of Economics*, February, 88(1): 98-116.
- [4] **Brecher, R. A.** (1974b). *Optimal Commercial Policies for an Minimum Wage-Economie*. *Journal of International Economics*, 4(2): 139-149.
- [5] **Brecher, Richard A.** (1980). *Increased Unemployment from Capital Accumulation in a Minimum-Wage Model of an Open Economy*. *Canadian Journal of Economics* 13(1): 152-158.
- [6] **Corden, W. M.** (1974). *The Theory of Protection* Oxford, Clarendon Press.
- [7] **Davis, D.** (1998A). *Does European unemployment prop up American wages? National labor markets and global trade*. *American Economic Review* 88(3): 478-494.
- [8] **Davis, D.** (1998B). *Technology, unemployment, and relative wages in a global economy*. *European Economic Review* 42: 1613-1633.
- [9] **Falk, M. and B. Koebel** (1997). *The Demand for Heterogeneous Labour in Germany* Centre for European Research in Economics, Mannheim, Germany.
- [10] **Freeman, R. B.** (1995). *Are your Wages Set in Beijing?* *Journal of Economic Perspectives*, Summer, 9(3): 15-32.
- [11] **Greenaway, D. et alii** (1994) *Country Specific Factors and the Pattern of Horizontal and Vertical Intra-industry Trade in the UK*. *Weltwirtschaftliches Archiv*, Vol. 130: 77-100.
- [12] **Neary, S. P.** (1985). *International Factor Mobility, Minimum Wage Rates and Factor Price Equalisation: A Synthesis*. *Quarterly Journal of Economics*, Vol. 100: 551-570.
- [13] **Schweinberger, A.** (1978). *Employment Subsidies and the Theory of Minimum Wage Rates in General Equilibrium*. *Quarterly Journal of Economics* 92(3): 361-374.
- [14] **Wood, A.** (1994) *North South Trade, Employment and Inequality: Changing Fortunes in a Skill-Driver World*. Clarendon Press, Oxford.
- [15] **Woodland, A. D.** (1982). *International trade and Resource Allocation*. North-Holland.

## A

The proof of Proposition II parts (a) and (b) follows directly from an inspection of Figure 1. I now proceed to prove parts (c) and (d). To prove parts (c) and (d) it follows from Figure 2 that the autarkic price of good 1 is lower in the minimum wage economy if and only if, keeping  $p_1$  constant,  $\frac{dw}{dL}$  derived from  $e_1 = 0$  exceeds in absolute terms  $\frac{dw}{dL}$  derived from  $L = N$ , i.e.:

$$\left| \left( \frac{dw}{dL} \right)_{e_1=0} \right| > \left| \left( \frac{dw}{dL} \right)_{L=N} \right|$$

Differentiating totally  $x_1(\cdot) = c_1(\cdot)$  with respect to  $w$  and  $L$  we obtain (making use of the cost minimisation conditions:  $Kdr + L^s dw^s + Ldw = 0$ ).

$$\begin{aligned} \frac{\partial x_1}{\partial w} dw &= \frac{\partial c_1}{\partial Y} w dL \quad \text{and therefore} \\ \left( \frac{dw}{dL} \right)_{e_1=0} &= \frac{w \frac{\partial c_1}{\partial Y}}{\frac{\partial x_1}{\partial w}} \end{aligned} \tag{27}$$

where:  $Y = p_1 x_1 + p_2 x_2 = Kr + L^s w^s + Lw$ .

Differentiating totally  $L = N$  we have:

$$\left( \frac{dw}{dL} \right)_{L=N} = \frac{1}{a_{n1} \frac{\partial x_1}{\partial w} + a_{n2} \frac{\partial x_2}{\partial w}} < 0 \tag{28}$$

Since good 1 is intensive in unskilled labour we know that

$$\left( \frac{dw}{dL} \right)_{e_1=0} < 0$$

if there are no inferior goods. Now let:

$$\left( \frac{dw}{dL} \right)_{e_1=0} < \left( \frac{dw}{dL} \right)_{L=N} \tag{29}$$

It is straightforward to show that expression (29) cannot be satisfied if all goods are normal and good one is intensive in the use of unskilled labour. Substituting (27) and (28) into (29) we have:

$$w \frac{\partial c_1}{\partial Y} \left( a_{N1} \frac{\partial x_1}{\partial w} + a_{N2} \frac{\partial x_2}{\partial w} \right) < \frac{\partial x_1}{\partial w} \quad (30)$$

or

$$\frac{\partial x_1}{\partial w} \left( w a_{N1} \frac{\partial c_1}{\partial Y} - 1 \right) + w \frac{\partial c_1}{\partial Y} a_{N2} \frac{\partial x_2}{\partial w} < 0 \quad (31)$$

For normal goods  $p_1 \frac{\partial c_1}{\partial Y}$  and  $p_2 \frac{\partial c_2}{\partial Y}$  are positive and less than one. Clearly,

$$1 > p_1 \frac{\partial c_1}{\partial Y} > w a_{N1} \frac{\partial c_1}{\partial Y} > 0$$

Since  $\frac{\partial x_1}{\partial w} < 0$  and  $\frac{\partial x_2}{\partial w} > 0$  it follows that the left hand side of expression (31) is positive.

This proves part (c) of Proposition II. The proof and result stated above is undoubtedly useful because it shows that if the differences between the minimum and the flexi wage economy are small it does not matter whether good one or good two is intensive in the use of unskilled labour. However the proof and analysis have one major shortcoming: they are based upon a local analysis only.

It is straightforward to show that in a global analysis the result stated under (c) may not hold. The reason for this is that in the local analysis one obtains (differentiating totally  $Y = rK + w^s L^s + wL$ ):

$$dY = wdL$$

but in the global analysis:

$$\Delta Y = k\Delta r + L^s \Delta w^s + L\Delta w + w\Delta L + \Delta w\Delta L$$

Taking total differences of  $e_1 = 0$  and assuming for simplicity that  $\frac{\partial c_1}{\partial Y}$  is fixed and equal

to  $\alpha$  we readily obtain

$$\left(\frac{\Delta w}{\Delta L}\right)_{e_1=0} = \frac{\alpha \left(K \frac{\Delta r}{\Delta L} + L^s \frac{\Delta w^s}{\Delta L}\right)}{\frac{\Delta x_1}{\Delta w} - \alpha L} + \frac{(w + \Delta w)}{\frac{\Delta x_1}{\Delta w} - \alpha L} \quad (32)$$

and

$$\left(\frac{\Delta w}{\Delta L}\right)_{L=N} = (a_{N1} \frac{\Delta x_1}{\Delta w} + a_{N2} \frac{\Delta x_2}{\Delta w})^{-1} \quad (33)$$

Proceeding as in the local analysis we set

$$\left(\frac{\Delta w}{\Delta L}\right)_{e_1=0} < \left(\frac{\Delta w}{\Delta L}\right)_{L=N} \quad (34)$$

Rearranging this it is straightforward to show that (34) implies (35):

$$\begin{aligned} & \alpha \left(K \frac{\Delta r}{\Delta L} + L^s \frac{\Delta w^s}{\Delta L}\right) \left(a_{N1} \frac{\Delta x_1}{\Delta w} + a_{N2} \frac{\Delta x_2}{\Delta w}\right) + \dots \\ & \dots + \frac{\Delta x_1}{\Delta w} [\alpha(w + \Delta w)a_{N1} - 1] + \alpha(w + \Delta w)a_{N2} \frac{\Delta x_2}{\Delta w} < -\alpha L^0 \end{aligned} \quad (35)$$

Expression (35), the global result, should be compared with expression (31), the local result. Clearly, there is now an additional term. The second and third term on the left hand side are positive (as in the local analysis). However the first term must be negative.

Note that:

$$\alpha \left(K \frac{\Delta r}{\Delta L} + L^s \frac{\Delta w^s}{\Delta L}\right)$$

must be positive. A fall in  $L$  (given the goods prices) raises  $w$  and therefore either  $r$  and  $w^s$  or at least one of  $r$  and  $w^s$  must fall. Therefore  $K \frac{\Delta r}{\Delta L} + L^s \frac{\Delta w^s}{\Delta L}$  must be positive. In the local analysis we have:

$$K dr + L^s dw^s + L dw = 0$$

and therefore a rise in  $w$  always implies:

$$K dr + L^s dw^s < 0$$

$$\text{and} \quad K \frac{dr}{dL} + L^s \frac{dw^s}{dL} > 0$$

In the global analysis we know from cost minimization that:

$$K\Delta r + L^s\Delta w^s + L\Delta w > 0$$

Since we also know that:

$$\frac{\Delta w}{\Delta L} < 0$$

it follows that:

$$K\frac{\Delta r}{\Delta L} + L^s\frac{\Delta w^s}{\Delta L} + L\frac{\Delta w}{\Delta L} > 0$$

implies:

$$K\frac{\Delta r}{\Delta L} + L^s\frac{\Delta w^s}{\Delta L} > 0.$$

Also note that  $a_{N1}\frac{\Delta x_1}{\Delta w} + a_{N2}\frac{\Delta x_2}{\Delta w}$  must be negative. This follows from the fact that a *ceteris paribus* increase in  $w$  must lower employment.

The inequality stated as expression (35) can therefore always be satisfied by appropriate choices of  $\alpha$  and  $L^0$ . This proves part (d) of Proposition III. The global proof highlights the importance of income effects which drop out in the local analysis.



## B

|   |                         |                         |
|---|-------------------------|-------------------------|
| $\frac{\alpha_{K1}}{\alpha_{L1}} > \frac{\alpha_{K2}}{\alpha_{L2}}$ | $N_A^1 > N_A^0$         | $N_A^1 < N_A^0$         |
| $\frac{\alpha_{K2}}{\alpha_{L2}} > \frac{\alpha_{K1}}{\alpha_{L1}}$ | $N_A^1 < N_A^0$         | $N_A^1 > N_A^0$         |
|   | $d \log K > d \log L^s$ | $d \log L^s > d \log K$ |

Table 1: Employment differences in autarkic equilibria, good 2 intensive in unskilled labour. It follows from an inspection of Figure 1 and the main text, see section 5.1

|   |                         |                         |
|---|-------------------------|-------------------------|
| $\frac{\alpha_{K1}}{\alpha_{L1}} > \frac{\alpha_{K2}}{\alpha_{L2}}$ | $N_A^0 > N_A^1$         | $N_A^1 > N_A^0$         |
| $\frac{\alpha_{K2}}{\alpha_{L2}} > \frac{\alpha_{K1}}{\alpha_{L1}}$ | $N_A^1 > N_A^0$         | $N_A^0 > N_A^1$         |
|   | $d \log K > d \log L^s$ | $d \log L^s > d \log K$ |

Table 2: Employment differences in autarkic equilibria, good 1 intensive in unskilled labour.

|   | Country 0                    |                              | Country 1                    |                              |
|---|------------------------------|------------------------------|------------------------------|------------------------------|
| $\frac{\alpha_{K1}}{\alpha_{L1}} > \frac{\alpha_{K2}}{\alpha_{L2}}$ | $N \uparrow$                 | $N \downarrow$               | $N \downarrow$               | $N \uparrow$                 |
| $\frac{\alpha_{K2}}{\alpha_{L2}} > \frac{\alpha_{K1}}{\alpha_{L1}}$ | $N \uparrow$                 | $N \downarrow$               | $N \downarrow$               | $N \uparrow$                 |
|   | $d \log K$<br>$> d \log L^s$ | $d \log L^s$<br>$> d \log K$ | $d \log K$<br>$> d \log L^s$ | $d \log L^s$<br>$> d \log K$ |

Table 3: Changes in employment, good 2 intensive in unskilled labour.

|   | Country 0                    |                              | Country 1                    |                              |
|---|------------------------------|------------------------------|------------------------------|------------------------------|
| $\frac{\alpha_{K1}}{\alpha_{L1}} > \frac{\alpha_{K2}}{\alpha_{L2}}$ | $N \downarrow$               | $N \uparrow$                 | $N \uparrow$                 | $N \downarrow$               |
| $\frac{\alpha_{K2}}{\alpha_{L2}} > \frac{\alpha_{K1}}{\alpha_{L1}}$ | $N \uparrow$                 | $N \downarrow$               | $N \downarrow$               | $N \uparrow$                 |
|   | $d \log K$<br>$> d \log L^s$ | $d \log L^s$<br>$> d \log K$ | $d \log K$<br>$> d \log L^s$ | $d \log L^s$<br>$> d \log K$ |

Table 4: Changes in employment, good 1 intensive in unskilled labour. Tables 3 and 4 follow from an inspection of Figures 1 and 2 respectively and the main text, see section 5.1.