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**Working Paper**

## Multi-criteria agency theory

Diskussionsbeiträge - Serie I, No. 280

**Provided in Cooperation with:**

Department of Economics, University of Konstanz

*Suggested Citation:* Vetschera, Rudolf (1996) : Multi-criteria agency theory, Diskussionsbeiträge - Serie I, No. 280, Universität Konstanz, Fakultät für Wirtschaftswissenschaften und Statistik, Konstanz

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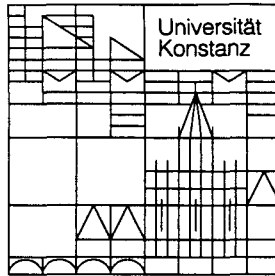
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## **Fakultät für Wirtschaftswissenschaften und Statistik**

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Rudolf Vetschera

### **Multi-Criteria Agency Theory**

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Diskussionsbeiträge

# **Multi-Criteria Agency Theory**

**Rudolf Vetschera**

**Serie I - Nr. 280**

66A249

**Februar 1996**

**W 284 (280)**



# MULTI-CRITERIA AGENCY THEORY

Rudolf Vetschera

## **Abstract**

This paper extends standard agency theoretic models to explicitly take into account the presence of multiple criteria at the agent level. Two possible approaches for this extension are discussed. In the first approach, we consider only the minimum information that the principal might possess about the agent's preferences towards different criteria. It can be shown that, in this context, the principal has only weak possibilities to influence the agent's behavior. In a second approach, a richer information set is considered. In this context, it is possible to determine probabilities of alternatives being selected by the agent. We also analyze how these probabilities can be influenced by the principal, and how this affects the principal's net profit.

## **Zusammenfassung**

In dieser Arbeit wird die Anwendung von Konzepten der Entscheidung bei mehrfacher Zielsetzung auf Agency-Beziehungen untersucht. Dabei werden zwei Szenarien bezüglich möglicher Informationsstände des Prinzipals über die Artenpräferenzen des Agenten betrachtet. Im ersten Szenario verfügt der Prinzipal nur über minimale Informationen bezüglich dieser Präferenzen und es kann gezeigt werden, daß unter diesen Annahmen der Prinzipal nur sehr geringe Einflußmöglichkeiten auf das Verhalten des Agenten hat. Im zweiten Szenario wird weitergehende Information unterstellt, die die Ableitung von Wahrscheinlichkeitsaussagen über die vom Agenten gewählte Alternative ermöglicht. Es wird untersucht, wie diese Wahrscheinlichkeitsverteilung von Aktionsparametern des Prinzipals abhängt und zur Gestaltung eines aus der Sicht des Prinzipals optimalen Anreizsystems benutzt werden kann.

# 1 Introduction

## 1.1 Motivation

Agency models have developed during the last two decades into a powerful framework for analyzing many problems in economics and management, especially concerning the analysis of organizational setups (Spremann, 1987). Since agency theory is rooted in traditional economic analysis, it also inherits many of the assumptions and limitations which characterize economic theory (Nilakant/Rao, 1994). Like most economic models, agency models are based on the assumption that the behavior of actors is driven by an underlying optimization model. In an agency context, this view is applied to both parties: the principal is often assumed to maximize profit, the agent maximizes his utility.

This focus on single-criterion optimization can be seen as rather unrealistic. For the principal, it can be argued that real goal systems of corporations are much more complex than just profit maximization (Schiemenz/Seiwert, 1979; Zeleny, 1982; Roy, 1988). For the agent, standard agency models already exhibit a more complex goal system underlying the utility function. The question here is whether the utility function is a valid and workable representation of the agent's preferences towards these goals.

## 1.2 Overview of the paper

In this paper, we will provide a first look into the "black box" of underlying goals. Our main focus will be on methods by which the agent's decision problem can be explicitly analyzed in more detail. The paper is the first in a planned series of working papers, in which we intend to develop a comprehensive methodology for studying multi-criteria agency problems.

The remainder of this paper is structured as follows: In section two, we provide further motivation for modeling agency problems as multi-criteria problems and analyze in which parts of the overall framework of agency theory multi-criteria models should be used. In sections three and four, we discuss two approaches to modeling the agent's problem as a multi-criteria problem in more detail. Section five draws some conclusions and outlines topics for further research.

## 2 The role of multiple criteria in agency problems

As we have just argued, using single-criteria optimization models to describe the behavior of economic actors might be seen as a very simplistic approach. In agency models, there are two actors: the agent, who carries out some activity, and the principal, on whose behalf that activity is carried out, who benefits from the activity and has to compensate the agent for his effort.

The model can be extended from the traditional economic viewpoint of single criterion maximization to a multi-criteria perspective for one or both of these actors. The resulting four cases are outlined in table 1:

		Principal	
		Criteria	
Agent	One	Case A	Case B
	Multiple	Case C	Case D

Table 1: Possibilities to introduce multiple criteria in agency models

Each of the four cases possesses unique characteristics, which can best be explained by formulating general optimization models (under rather simplistic assumptions) for them.

*Case A* corresponds to the standard way of formulating agency models: both actors are assumed to optimize a single criterion. The mathematical structure of such an agency model is that of a hierarchical optimization problem, i.e. an optimization problem in which at least one constraint involves the solution of another (lower level) optimization problem (Anandalingam/Friesz, 1992). A very simple single objective agency model can thus be formulated as follows (Spremann, 1987; Laux, 1990):

$$\begin{aligned}
 & \max_{c \in C} g(a^*) - c(g(a^*)) \\
 & s.t. \\
 & u(a^*, c(g(a^*))) = \max_{a \in A} u(a, c(g(a))) \\
 & u(a^*, c(g(a^*))) \geq u_{\min}
 \end{aligned} \tag{1}$$

In model (1),  $a$  represents the agent's level of activity. It should be noted that, while most agency models treat  $a$  as a scalar, it could as well be a vector if different types of

effort are considered. For example, for a salesperson, the number of customers visited and the persuasiveness of each visit might be considered two distinct types of effort. This extension does not change the structure of the problem at all. Activity  $a$  is assumed to lead to a result  $g(a)$ . Depending on this result, the principal pays a compensation  $c(g(a))$  to the agent. It should be noted that in this very simple model, both  $g(a)$  and  $c(g(a))$  are assumed to be known functions. While this assumption is not typical for agency problems and most models actually deal with situations in which  $g(a)$  is stochastic, this simple model is sufficient for our purposes. We also assume that compensation depends on the profit (and not, for example, directly on the effort). This formulation is common for many agency models.

The objective function of (1) represents the assumptions about the principal, who is assumed to maximize her<sup>1</sup> net profit. The net profit is the profit resulting from the agent's activity minus the compensation that has to be awarded to the agent. The principal has to decide upon a compensation function  $c$ . The first constraint represents the maximizing behavior of the agent, who is assumed to select a level of activity  $a^*$  at which his utility is maximized. The second constraint of (1), which is often called "participation constraint", guarantees the agent a certain minimum utility. It is assumed that an agent will participate in the relationship only if he can achieve at least the utility level  $u_{min}$ .

It should be noted that this formulation contains another implicit assumption about the agent. Whenever the value of  $a$ , which leads to maximum utility, is not unique, the agent will choose among those solutions the one that maximizes the principal's profit. While this assumption is sometimes mentioned in the literature, e.g. (Laux, 1990), it is not considered problematic since given the "right" type of functions, a scalar optimization model will not or only rarely fail to provide a unique solution. Thus, it is possible to link  $a$  more or less directly to  $c$ , which makes this model tractable by conventional solution methods for non-hierarchical optimization problems. For example, if  $a$  is scalar, then this constraint is replaced by the first order condition  $\partial u / \partial a = 0$ .

In case  $B$ , only the principal is assumed to pursue several criteria, while the behavior of the agent is modeled via single criterion optimization. Methodologically, this is the simplest way of introducing multiple criteria into the agency model. Model (1) now becomes

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<sup>1</sup> Throughout this paper, we will refer to the principal as "she" and the agent as "he" to allow for simple distinction

$$\begin{aligned}
& \text{vmax}_{c \in C} \mathbf{g}(a^*) - c(\mathbf{g}(a^*)) \\
& \text{s.t.} \\
& u(a^*, c(\mathbf{g}(a^*))) = \max_{a \in A} u(a, c(\mathbf{g}(a))) \\
& u(a^*, c(\mathbf{g}(a^*))) \geq u_{\min}
\end{aligned} \tag{2}$$

where "vmax" denotes vector maximization and  $\mathbf{g}$  is a vector of consequences from the activity. To consider different consequences separately is only meaningful if either the functions are stochastic or  $a$  is also a vector of different activities. Otherwise, the single scalar value of  $a$  would already imply all outcomes and thus provide a compact representation of the entire outcome vector. The compensation  $c$  can be either a scalar or a vector. In the first case, the value of  $c$  is subtracted from the corresponding component of  $\mathbf{g}$  in the objective function. If, for example, the only type of compensation is money and profit of the principal is one of the components of  $\mathbf{g}$ , the agent's pay is subtracted from the principal's profit. If compensations of different kinds are awarded to the agent (e.g. payment, extra days of vacation, privilege symbols etc.), the elements of  $c$  are subtracted from the corresponding elements of  $\mathbf{g}$ .

In (2), the agent's optimization problem is still a scalar problem. Therefore, similar arguments as above can be made concerning the replacement of this optimization problem by a standard constraint linking  $c$  and  $a^*$ . Problem (2) then becomes a standard multi-objective decision problem, which could be solved by the principal using any of the many methods which have been developed for this class of problems (Hwang/Masud, 1979; Shin/Ravindran, 1991).

In case C, problem (1) becomes

$$\begin{aligned}
& \max_{c \in C} \mathbf{g}(\mathbf{a}^*) - \mathbf{f}(\mathbf{c}(\mathbf{g}(\mathbf{a}^*))) \\
& \text{s.t.} \\
& (-\mathbf{a}^*, \mathbf{c}(\mathbf{g}(\mathbf{a}^*))) = \text{vmax}_{a \in A} (-\mathbf{a}, \mathbf{c}(\mathbf{g}(\mathbf{a}))) \\
& a_1^* \leq a_{1,\max} \\
& \vdots \\
& a_k^* \leq a_{k,\max} \\
& c_1(\mathbf{g}(\mathbf{a}^*)) \geq c_{1,\min} \\
& \vdots \\
& c_n(\mathbf{g}(\mathbf{a}^*)) \geq c_{n,\min}
\end{aligned} \tag{3}$$

There is now possibly a vector  $\mathbf{c}$  of compensations to be considered and the agent solves no longer a scalar optimization problem, but a vector problem in which those compensations (as well as possibly several kinds of effort) are simultaneously optimized. We assume that the agent is work-averse, so the effort is to be minimized. A vector



optimization problem would also arise if several types of effort are considered. Even if there is only one type of compensation and one type of effort, a multi-objective problem has to be solved if effort and compensation are considered separately.

Apart from the fact that the agent now has to solve an optimization problem with multiple criteria, two additional problems arise in this context: firstly, if there are several types of compensation, their impact on the principal's profit has to be evaluated. If the costs of different types of compensation (e.g. a more expensive office) are known, this can be done by a (possibly even linear) function, which we denote by  $f$  in (3). The second problem is more complex: if (several kinds of) effort and (several kinds of) compensation are treated separately, it is no longer possible to formulate a single participation condition. In (3), we therefore assumed that the agent has reservation levels for all his objectives and will participate only if all his aspiration levels are achieved or exceeded. While this might seem a straightforward and inconsequential extension of the participation condition in (1), it has dramatic effects as we will show in section 3.

Methodologically, this case is much more challenging than case B. It is no longer possible to establish in advance a direct mapping from  $c$  to  $a$ . The operator "vmax" in (3) can be interpreted in several ways. One possible interpretation is the classical vector optimization problem, in which the solution corresponds to the entire set of nondominated solutions. In this interpretation, it is possible that the set of alternatives remaining feasible due to the first constraint of (3) is rather large. This has significant implications for the entire optimization model. If we interpret "vmax" as classical vector optimization, we implicitly assume in (3) that the agent will be indifferent between all nondominated solutions. Among those nondominated solutions, the agent is further assumed to choose the level of activity  $a^*$  that maximizes the principal's profit. This assumption is much harder to justify for a large set of nondominated solutions to the agent's vector optimization problem than for a small set of equivalent solutions to a scalar optimization problem.

Another interpretation of "vmax" is that the agent will actually solve the multi-criteria optimization problem to find a unique "best" solution according to his preferences. While this might be a more realistic scenario, it leads to severe problems in solving the principal's optimization problem. It is no longer possible to replace the optimization constraint by a standard constraint, because this would require a closed analytical expression for the result of the multicriteria solution procedure, given the compensation function(s)  $c$ . This would clearly contradict most assumptions made in multi-criteria decision making, where it is usually assumed that not even the decision maker himself is able to provide a comprehensive representation of his preferences in advance.

It becomes evident that the amount of information on the agent's preferences which the principal possesses becomes crucial for the formulation and solution of models of type (3). In the next two sections, we will therefore discuss different scenarios about which information might be available and their consequences in more detail.

Case *D* is a combination of cases *B* and *C*. The model here becomes

$$\begin{aligned}
 & \text{vmax}_{c \in C} g(\mathbf{a}^*) - \mathbf{c}(g(\mathbf{a}^*)) \\
 & s.t. \\
 & (-\mathbf{a}^*, \mathbf{c}(g(\mathbf{a}^*))) = \text{vmax}_{a \in A} (-\mathbf{a}, \mathbf{c}(g(\mathbf{a}))) \\
 & a_1^* \leq a_{1,\max} \\
 & \vdots \\
 & a_k^* \leq a_{k,\max} \\
 & c_1(g(\mathbf{a}^*)) \geq c_{1,\min} \\
 & \vdots \\
 & c_n(g(\mathbf{a}^*)) \geq c_{n,\min}
 \end{aligned} \tag{4}$$

In this paper, we will not deal with this case, since most problems can also be illustrated using case *C*.

In comparing the four cases, it becomes evident that the main difference between single- and multi-objective models lies in the representation of preferences. Preference models are of even greater importance in agency theory than in other areas of economic analysis. In agency theory, the preference representation for the agent is not just a device which the modeling economist uses to represent that actor in the model. Implicitly, it is also assumed that the principal, in selecting an incentive system, uses the same model of the agent's preferences. To put it differently: if one were to design a compensation scheme based on a standard agency theoretic model like (1), one would have to know the agent's utility function. The standard model of case *A* (and also case *B*) therefore describes a situation in which the principal has full information about the agent's preferences.

This assumption of full information on the agent's preference has been relaxed in the literature (Laux, 1990). There are two main approaches to deal with uncertainty about the agent's preferences: In the first approach, the problem of the principal is modeled as a decision problem under uncertainty, e.g. by supposing a known probability distribution over different utility functions the agent might possess. The principal then designs the compensation system so that she maximizes her expected utility (or expected profit in the case of a risk neutral principal). The other approach is concerned with the design of so-called "self selection" contracts. Here, the principal offers different

compensation systems specifically designed for the different types of agents. One feature in the design of these contracts is that each kind of agent will be best off when actually choosing the type of contract designed for him (and at the same time maximizing the principal's profit).

Both approaches, while allowing for some uncertainty, presume that the principal has quite detailed information about the preferences of the possible types of agents. This information is still assumed to be rich enough to allow the representation of those preferences in the form of utility functions. In the following two sections, we will introduce alternative ways of modeling, which are based on less stringent information requirements. The first approach we will be discussing is concerned with the minimum set of information a principal might possess about potential agents. In the second approach, which is motivated by work on decision making under incomplete information and sensitivity analysis, we will consider a somewhat richer information set.

### **3 The minimum information approach**

#### **3.1 Assumptions**

What is the minimum information a principal will need in order to design an incentive system? One can plausibly argue that in order to design an incentive system for a given person (or class of persons), one has at least to know what the goals of that person are. One can hardly design a system of financial rewards for a guru who has vowed to live permanently in absolute poverty (and intends to keep that vow). One also needs to know the direction in which each goal is optimized (i.e., does that person prefer earning more money to earning less, working longer hours to shorter hours, etc.).

As soon as the set of criteria and their respective direction of improvement is established, it is a tempting next step to introduce the dominance criterion as an additional assumption about the agent's preferences. While this criterion introduces a certain requirement on the agent's level of rationality, this minimal level of rationality can plausibly be assumed for an economic actor. We therefore define the minimum level of information on the actor's preferences on which we will base the analysis of this section as follows:

1. The agent's goals are known.
2. The directions in which these goals are to be optimized are known.
3. It is known that the agent will not select a (weakly) dominated solution.

It should be noted that a third assumption besides assumptions 1. and 2. above is needed in order to exclude any solutions from further consideration. As long as we use only assumptions 1. and 2., the agent might still choose any feasible alternative under any system of incentives and so the preference model will not help to solve the principal's design problem.

Strictly spoken, assumption 3. above is not the minimum information about the agent's preferences. The minimum consists in knowing exactly one element of the preference relation (under a given incentive scheme). Knowing one element of the agent's preference relation would allow the principal to conclude that the less preferred alternative will not be chosen by the agent, thus eliminating it from further consideration. However, here we are concerned with defining a minimum level of preference information that can be defined in more general terms, and which is also plausible to assume. Plausibility also leads us to use the weak rather than the strong dominance relation as third assumption. While strong dominance will eliminate fewer alternatives (and can thus be regarded as the weaker assumption), it seems hardly imaginable that an agent would forego a possibility to improve on at least some of his goals without loss in the other goals.

### 3.2 Model Formulation

Using this set of assumptions, we can rewrite (3) more specifically as

$$\begin{aligned}
 & \max_{c \in C} g(\mathbf{a}^*) - f(\mathbf{c}(g(\mathbf{a}^*))) \\
 & s.t. \\
 & (-\mathbf{a}^*, \mathbf{c}(g(\mathbf{a}^*))) \in \underset{a \in A}{Eff}(-\mathbf{a}, \mathbf{c}(g(\mathbf{a}))) \quad (a) \\
 & a_1^* \leq a_{1,\max} \\
 & \vdots \\
 & a_k^* \leq a_{k,\max} \\
 & c_1(g(\mathbf{a}^*)) \geq c_{1,\min} \\
 & \vdots \\
 & c_n(g(\mathbf{a}^*)) \geq c_{n,\min} \quad (c)
 \end{aligned} \tag{5}$$

where

$$\underset{a \in A}{Eff}(-\mathbf{a}, \mathbf{c}(g(\mathbf{a}))) = \left\{ (-\mathbf{a}, \mathbf{c}(g(\mathbf{a}))) : \nexists \mathbf{a}' \in A : a'_i \leq a_i, c_j(g(\mathbf{a}')) \geq c_j(g(\mathbf{a})), \mathbf{a}' \neq \mathbf{a} \right\} \tag{6}$$

is the set of all nondominated alternatives. For simplicity, we will denote that set by  $E$ . In (6), we assume that all kinds of effort are to be minimized and all kinds of compensation are to be maximized. We also assume that all kinds of effort are indeed relevant

objectives for the agent (i.e. there is no component of  $\mathbf{a}$  about which the agent does not care).

As we have already stated before, this model also contains the implicit assumption that among those alternatives between which the agent is indifferent, he will chose the best one for the principal. As long as our preference model is restricted to dominance, this means that the agent will be indifferent between all nondominated alternatives.

### 3.3 Analysis of the model

The question now arises whether this weak set of information is actually sufficient for the principal to control the agent's behavior and to design an adequate incentive system. In model (5), control over the agent's behavior means changing the set  $E$ . Only if that set is changed can the principal expect a different outcome. Unfortunately, the principal's possibilities are rather restricted as the following theorem shows:

**Theorem 1:** Set  $E$  cannot be changed by strictly monotone changes in the compensation system.

**Proof:** The proof is straightforward. If alternative  $\mathbf{a}$  is dominated by alternative  $\mathbf{a}'$ , the relationships  $c_j(g(\mathbf{a})) \leq c_j(g(\mathbf{a}'))$  holds for all  $j$ , with at least one strict inequality. For any strictly monotone transformation of the  $c_j$ , this relationship is preserved and therefore alternative  $\mathbf{a}$  remains dominated by  $\mathbf{a}'$ . On the other hand, if  $\mathbf{a}$  is not dominated by  $\mathbf{a}'$ , at least one of two possible conditions must hold: The first possibility is that there is at least one inequality among the  $c_i$ 's in the other direction, which is also not changed by a strictly monotone transformation. The other possibility is that  $\mathbf{a}$  requires less effort than  $\mathbf{a}'$ . This property also does not change by a strictly monotone transformation of the compensation. Thus, no change in the dominance relation can be effected by a strictly monotone transformation of the compensation functions.

Therefore, if we assume that the compensation function is a strictly monotone function of the outcomes (i.e. the agent gets more compensation whenever the result of his activities is better), set  $E$  remains the same for all such strictly monotone compensation systems and therefore the agent will not change his behavior.

It is hard to imagine a compensation system which is not monotone at all in outcomes (i.e., where for some outcomes, higher outcomes will lead to a higher compensation but for other outcomes, higher outcomes will lead to lower compensation for the agent). The only remaining possibility to influence the agent's behavior therefore is a transformation of the compensation scheme which is monotone, but not strictly monotone:

**Theorem 2:** Set  $E$  can be reduced by a transformation of the compensation system, which is monotone but not strictly monotone.

**Proof:** Consider two alternatives  $\mathbf{a}$  and  $\mathbf{a}'$ , which possess the following characteristics:

1.  $a_i \leq a'_i$  for all  $i$ , with at least one strict inequality (i.e.,  $\mathbf{a}$  dominates  $\mathbf{a}'$  with respect to effort)
2.  $c_i(g(\mathbf{a})) \geq c_i(g(\mathbf{a}'))$  for all but one  $i$  (i.e.  $\mathbf{a}$  would also dominate  $\mathbf{a}'$  with respect to compensation, but is worse with respect to one kind of compensation).
3.  $c_j(g(\mathbf{a})) < c_j(g(\mathbf{a}'))$  (i.e., we denote the dimension in which  $\mathbf{a}$  is worse than  $\mathbf{a}'$  by  $j$ ).

When we apply a monotone (but not strictly monotone) transformation  $f$  to the  $j$ -th compensation, which has the property that  $f(c_j(g(\mathbf{a}))) = f(c_j(g(\mathbf{a}')))$ ,  $\mathbf{a}$  now dominates  $\mathbf{a}'$  and therefore  $\mathbf{a}'$  is eliminated from set  $E$ .

Two points should be noted with respect to theorem 2: First, this theorem only shows that the set of nondominated alternatives can be reduced by a transformation of the compensation functions, but it cannot be enlarged. The second part of the proof of theorem 1 still holds. It is not possible to make a dominated alternative nondominated by a monotone change in the compensation function. One can make two alternatives equal in terms of compensation. But in order to make a dominance relation vanish, these two would also have to be identical in the effort involved. This change would make them the same alternative. Furthermore, by a monotonous transformation of the compensation function, the principal can only eliminate alternatives which involve higher effort by the agent. It is intuitively clear that in the extreme case, where compensation is equal for all alternatives, a work-averse agent would choose to work as little as possible. Therefore, this possibility of influencing the agent's behavior is probably of little help to the principal.

Apart from this disappointing result, the minimum information approach has several other disadvantages. Since the different criteria are no longer aggregated into one utility function, there are several aspiration levels for the different criteria in model (5). The intersection of feasible points according to these constraints generates a cone in the agent's criteria space. Therefore, by selecting appropriate aspiration levels, the agent can restrict the set of feasible solutions to one element. It is thus possible for him to obtain any efficient solution, even if the set of feasible solutions is not convex. While this effect potentially poses a severe limit on the principal's ability to control the agency relationship, its actual impact will depend on various other factors. In order to actually reduce the feasible set to one element, a potential agent must be able to formulate arbitrary demands with respect to all aspects of the relationship. For example, a

potential employee must be able to formulate demands with respect to pay, hours worked, days off etc. and must credibly argue that he will accept employment only if all his conditions are met. If a potential agent's position is indeed that strong, he would probably be able obtain a comparable level of compensation even if his utility function were known to the potential principal. If the agent is in a weaker position, he will only be able to set his aspiration level at a dominated point, leading to a set of nondominated solutions that contains more than one element. Figure 1 illustrates this effect for the case of one kind of compensation and effort:

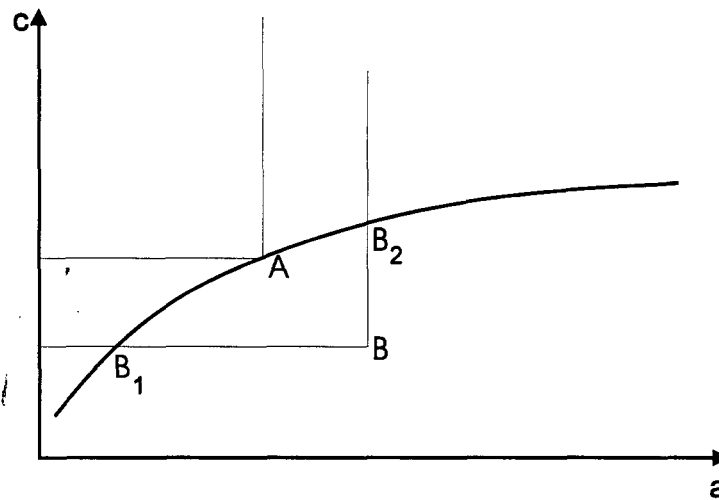


Figure 1: Aspiration levels and feasible efficient solutions

Figure 1 presents the agent's problem in effort/compensation-space. The solid curve represents the maximum compensation that can be awarded to the agent, given the corresponding level of effort. This maximum level can be equal to the principal's profit from the activity ( $g(a)$ ). If the principal has an aspiration level concerning her net profit, below which no agency relationship will be established, it is equal to  $g(a)$  minus that aspiration level.

If the agent's position is strong enough to plausibly argue that his true aspiration levels are at point A, this will be the only point remaining in the feasible efficient set. If, on the other hand, the agent has to fix his aspiration levels at the (dominated) point B, all efficient points which dominate point B will remain feasible. These are the points located on the curve between points  $B_1$  and  $B_2$ .

Another problem of the minimum information approach is caused by the potentially large size of set  $E$ . Model (5) is formulated as an optimization model, therefore the best solution (according to the principal's objective functions) which fulfills constraints (a) to (c) will be chosen. This means that among the efficient solutions, the agent will select

the solution which is best for the principal. This is probably not a realistic model of the agent's behavior.

This last problem could be overcome by a modification of model (5). For example, instead of always selecting the best efficient solution, it could be assumed that the agent randomly picks one efficient solution. However, since the other problems of this approach are rather severe, we will not elaborate such possible modifications.

## 4 A partial information approach

The preceding section has shown that simply assuming that the agent obeys the dominance criterion does not provide a preference model which is detailed enough to allow for effective control by the principal. On the other hand, full information of the principal on the agent's preferences is unrealistic. In this section, we will therefore develop an approach covering the middle ground between those two extremes. The general idea of our approach is outlined in figure 2:

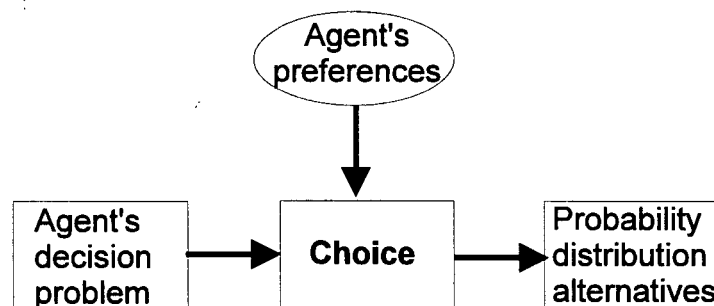


Figure 2: A partial information approach

Given the agent's decision problem about the level of inputs he wishes to apply, the choice the agent makes depends on his preferences. If the principal does not have complete information on the agent's preferences, she cannot make an exact prediction of the agent's choice. Our goal will therefore be to determine a probability distribution on the set of the agent's actions. If the principal knows the probability distribution of the agent's actions for a given system of incentives, her selection problem for the system of incentives becomes a decision problem under uncertainty. Depending on the principal's risk attitude, this problem then can be solved by determining the incentive system that has the highest expected payoff or highest expected utility.

To operationalize this general framework, two questions have to be answered:



1. How can incomplete information on preferences be specified?
2. How can this information on preferences be translated into a probability distribution of the agent's actions?

In the following subsection, we will develop a model for a specific choice situation, in which we show how these questions can be answered in that setting. Problems that might arise for other decision techniques will then be illustrated in the subsequent subsection.

## 4.1 A linear model

### 4.1.1 Assumptions

The model is based on the following situation:

1. The agent can only choose among a finite set  $A$  of possible actions. Each action is characterized by specific levels of effort(s) and compensation(s). To simplify the exposition, we will not distinguish any more between efforts and compensations. Rather, we will describe each action by a vector of  $K$  attributes:  $\mathbf{a}_i = (a_{i1}, \dots, a_{iK})$ . These values are known to the principal.
2. The agent's utility function is linear in the attributes and thus has the form

$$U(\mathbf{a}_i) = \sum_k w_k a_{ik} \quad (7)$$

where the  $w_k$ 's are unknown to the principal. The weights are assumed to be scaled so that  $\sum_k w_k = 1$ .

3. The principal might possess some additional information on the  $w_k$ 's.

The question now is how can the principal, given this information, determine the probability that the agent will select each alternative  $\mathbf{a}_i$ ?

To develop a model for this task, we first have to clarify in which form the principal's additional information on the weights is available. Two possibilities for encoding such information have been discussed in the literature (Arbel/Vargas, 1993): probability distributions and constraints.

Probability distributions can be formulated independently for single weights or jointly for the entire weight vector. Probability information about weights has been used in some papers on sensitivity analysis of linear models, especially the AHP (Saaty/Vargas, 1987; Paulson/Zahir, 1995) to determine the influence of parameter uncertainty on

decision outcomes. So far, no analytical methods have been developed to link probability distributions of parameters to the probabilities of choosing a certain alternative. In the papers published so far, simulation techniques have been used.

Constraints can either take the form of bounds on single weights or link several weights. This kind of information on weights has been considered in the fields of decision making under incomplete information (Franke, 1978; Hannan, 1981; Kirkwood/Sarin, 1985; Hazen, 1986; Weber, 1987; Arbel, 1989; Malakooti, 1989) and sensitivity analysis (Evans, 1984; Schneller/Sphicas, 1985; Vetschera, 1986; Barron/Schmidt, 1988; Rios Insua/French, 1991). The motivation for that kind of analysis was the problem of a decision maker who is uncertain about his preferences and consequently not able to specify the weights precisely, but prefers to provide only interval or ordinal information about the weights. In this context, analytical models have been developed, which will form the basis of our approach.

We therefore assume that the information the principal has about the agent's weights can be formulated as a set of constraints, which define a set  $W$  of feasible weight vectors. Apart from linearity, we do not make further assumptions on the types of constraints involved. For example, such constraints might take the form of lower and upper bounds on weights:

$$\underline{w}_k \leq w_k \leq \overline{w}_k \quad (8)$$

or they might be more complex constraints linking several weights. Constraints linking several weights might, for example, represent ordinal information on the importance of criteria.

The next step in the principal's problem can now be formulated as follows: Given that the agent's weights come from set  $W$ , what is the probability that the agent will chose a certain alternative  $a_i$ ? To answer this question, we also need information on the probability distribution of weight vectors within  $W$ . Using the principle of insufficient reason, we assume that weight vectors are uniformly distributed over  $W$ .

#### 4.1.2 Model formulation

Using these assumptions, we can now construct a formal model. We denote by  $O(a_i)$  the region in weight space in which alternative  $a_i$  is optimal from the agent's point of view. We will call this set  $a_i$ 's *region of optimality*. In order to be optimal,  $a_i$  has to be better than all the other alternatives according to the agent's linear utility function.  $O(a_i)$  is therefore defined by the following set of constraints:

$$\sum_{k=1}^K w_k \cdot a_{i,k} \geq \sum_{k=1}^K w_k \cdot a_{j,k} \quad \forall j \neq i \quad (9)$$

Furthermore, we have to take into account the scaling constraint

$$\sum_{k=1}^K w_k = 1 \quad (10)$$

and the information the principal has about the agent's preferences:

$$w \in W \quad (11)$$

(9), (10) and (11) together define  $O(a_i)$ . For the time being, we will assume that the principal has no additional information on the agent's preferences. In this case,  $W$  corresponds to the entire weight space and (11) need not be considered in the model.

(9) and (10) define a polyhedron in  $K-1$  - dimensional space. Since we assume uniform distribution of weights within the feasible region, we can apply Starr's domain criterion (Starr, 1962; Charnetski/Soland, 1978; Erkut/Tarimcilar, 1991; Eiselt/Laporte, 1992) to determine the probability that  $a_i$  is the optimal alternative for the agent. This probability is given as the ratio of the volume of the polyhedron  $O(a_i)$  to the volume of the  $K-1$  - dimensional unit simplex or to  $W$  in the general case. Algorithms to calculate the volumes of such polyhedra have been described in (Cohen/Hickey, 1979) and (Vetschera, 1996).

Since the volume of the unit simplex is constant, the probability depends only on (9) and (10). We can therefore use this system of linear inequalities to study the principal's possibilities to influence the agent's decision.

#### 4.1.3 Analysis of the model

For this purpose, we have to analyze how changes in the data value of one attribute will affect the probability distribution of alternatives. One might think of this attribute as the level of compensation that the principal pays to the agent. Since the compensation depends on the output of the decision (and we are dealing with certainty), the compensation which the agent will receive when selecting each alternative can directly be interpreted as an attribute of the alternatives.

Are there possibilities for the principal to change the probabilities of some alternatives being chosen by manipulating the attribute values? The answer to this question is trivially "yes" if arbitrary changes in attribute values are allowed, since the principal can then always make one alternative outstandingly attractive to the agent. More realistically, however, are transformations that affect the outcomes of all alternatives in

a similar manner. For simplicity, we will consider only additive and multiplicative transformations.

For additive transformations, we obtain the following

**Theorem 3:** The probability of choosing a certain alternative is not changed when outcomes of all alternatives in one attribute are changed by the same constant amount.

**Proof:** To prove theorem 3, we rewrite (9) as:

$$\sum_{k=1}^K w_k \cdot (a_{i,k} - a_{j,k}) \geq 0 \quad \forall j \neq i \quad (12)$$

If we add a constant  $z$  to the value of all alternatives in some attribute  $c$ , we obtain

$$(a_{i,c} + z) - (a_{j,c} + z) = a_{i,c} - a_{j,c} \quad (13)$$

and thus the constraints defining the polyhedron are not changed.

The case is more difficult for multiplicative transformations. The possibility to influence the agent's decision depends on how the agent interprets this change. The situation can be summarized as follows:

**Theorem 4:** When the outcomes of all alternatives in one attribute are multiplied by a constant factor, the probability of choosing a certain alternative changes only if the agent does not fully adjust for range effects.

To explain what we mean by range effects, we rewrite the model when outcomes in one attribute are multiplied by a factor  $\alpha$  as:

$$\begin{aligned} \sum_{k \neq c} w_k \cdot (a_{ik} - a_{jk}) + w_c \alpha \cdot (a_{ic} - a_{jc}) &\geq 0 \\ \sum_k w_k &= 1 \end{aligned} \quad (14)$$

The question now is whether the agent should (and will) adjust weight  $w_c$  to reflect the new scaling of outcomes. Whether such an adjustment is made depends on the nature of the change. If the change is perceived as a pure change in scale (e.g., if compensation to the agent is paid in Yen instead of Dollars), an adjustment is likely to take place. If the change is not perceived as a change of scale, but as a change in substance (e.g., the agent gets a raise of 10% on his compensation), than no adjustment will take place.

If weights are adjusted, the model does not become (14), but in fact becomes

$$\begin{aligned}
\sum_{k \neq c} w_k \cdot (a_{ik} - a_{jk}) + w_c \alpha \cdot (a_{ic} - a_{jc}) &\geq 0 \\
\sum_{k \neq c} w_k + \alpha \cdot w_c &= 1
\end{aligned}
\tag{15}$$

In model (15), one column is multiplied by a constant factor. Geometrically, this means that one dimension of the weight space is expanded (or contracted) by a factor of  $\alpha$ . Therefore, the volume of the polyhedron will change by the same factor. But so will the volume of the "unit" simplex (which then of course is no longer the unit), and the ratio will remain the same. Thus the probability of choosing alternative  $a_i$  will not be affected.

On the other hand, if no (or insufficient) weight adjustment takes place, the shape of the polyhedron will change according to (14) and thus the probability distribution of alternatives will be modified. Computational experiments have shown that the resulting change in probabilities is nonlinear in  $\alpha$ .

#### 4.1.4 Evaluation methods

So far, we have only discussed general properties of the model. In order to apply the model to a specific situation, methods are needed to compute the volume of the polyhedron defined by (9) and (10). Following (Arbel/Vargas, 1993), two approaches to this problem can be distinguished: exact algorithms based on mathematical programming techniques, and simulation techniques.

Algorithms to compute the volumes of an  $n$ -dimensional polyhedron have been presented by (Cohen/Hickey, 1979) and (Vetschera, 1996). Both algorithms provide exact (within numerical precision) solutions. They can be applied when  $W$  corresponds to the entire unit simplex and when linear constraints on the weights are given. Since these algorithms calculate the volume of a polyhedron, the probability that the agent selects a certain alternative can only be determined if the weights are uniformly distributed over set  $W$ . Simulation studies have shown that the running time grows exponentially with the number of attributes considered.

The probability that an alternative is optimal for the agent can also be determined via simulation. In simulation techniques, weight vectors are either systematically generated throughout the entire feasible parameter space or are randomly sampled (Charnetski/Soland, 1978; Saaty/Vargas, 1987; Steuer/Sun, 1995). The optimal alternative is then determined for each set of weights generated. After a sufficiently large number of experiments, the probability that an alternative is chosen can be determined as the ratio of the number of sample points in which that alternative was optimal to the total number of (feasible) points sampled.

In a strategy in which the parameter space is systematically scanned, the number of data points and thus the computational effort grows exponentially with the number of attributes. In random sampling strategies, the number of data points to be generated can be set by the experimenter, but the quality of results might deteriorate if an insufficient number of sample points is chosen. The algorithmic strategy and the simulation strategy will therefore probably not be very different with respect to the amount of computing time involved.

Unlike the algorithmic strategy, the simulation approach allows for the use of different distributions of the weights. In random sampling, weights can be directly drawn from their assumed joint distribution. Using the techniques of (Stewart, 1995), it is possible to generate vectors exhibiting a pre-specified correlation of weights. In a scanning approach, data points can be weighted according to the value of the density function. The simulation approach therefore offers more flexibility than the algorithmic approach.

#### 4.1.5 The principal's optimization model

Once the probability distribution of the agent's choice has been determined, it is possible to construct the principal's optimization problem. The basic structure of that model was defined in (3). Model (3) was formulated for the case of certainty, but now the principal has only imperfect information on the agent's preferences. For simplicity, we assume that the principal is risk neutral.

We only consider a simple compensation system, in which the agent receives a fixed percentage of the principal's gross profit as commission. The profit is modeled as one attribute of the alternatives, which will be denoted by the subscript  $g$ . The fixed fraction of the profit which is awarded to the agent is denoted by a factor  $c$  ( $0 \leq c \leq 1$ ). We also assume that the compensation is the only attribute which leads to costs for the principal and thus reduces her profit. Therefore, if the agent selects alternative  $\mathbf{a}_i$ , the principal obtains a gross profit of  $a_{ig}$ , from which the agent receives  $ca_{ig}$ . This leaves the principal a net profit of  $(1-c)a_{ig}$ .

The probability that the agent selects an alternative depends on  $W$  and the attribute values of the alternatives for the agent, which in turn are influenced by the compensation system. We will therefore denote these probabilities by  $p(\mathbf{a}_i|c, W)$ .

The principal's optimization problem can now be formulated as follows:

$$\begin{aligned}
 & \max_{c \in C} \sum_{i \in A} p(\mathbf{a}_i|c, W) \cdot (a_{ig} - c \cdot a_{ig}) \\
 & s.t. \\
 & A = \{i | c \cdot a_{ig} \geq \bar{a}_g; a_{ik} \geq \bar{a}_k \ \forall k \neq g\}
 \end{aligned} \tag{16}$$

where  $A$  is the set of alternatives satisfying the agent's aspiration levels for all attributes.

#### 4.1.6 Numerical example

In this subsection, we will illustrate the model by an numerical example. We consider the following decision problem involving five alternatives and three attributes:

Alternatives	Attributes		
	$Z1=ca_g$ for $c=0.1$	Z2	Z3
A1	0.30	0.86	0.80
A2	0.50	0.13	0.25
A3	1.00	0.00	0.40
A4	0.20	0.60	1.00
A5	0.00	1.00	0.00

Table 2: Data for example

The first attribute is the compensation the agent would receive at a compensation level of 10% of the principal's profit.

Since we are dealing with three attributes, the resulting problem after substitution of  $w_3$  will be two-dimensional and thus can be plotted in  $w_1/w_2$ -space. The resulting partition of the unit simplex is shown in figure 3:

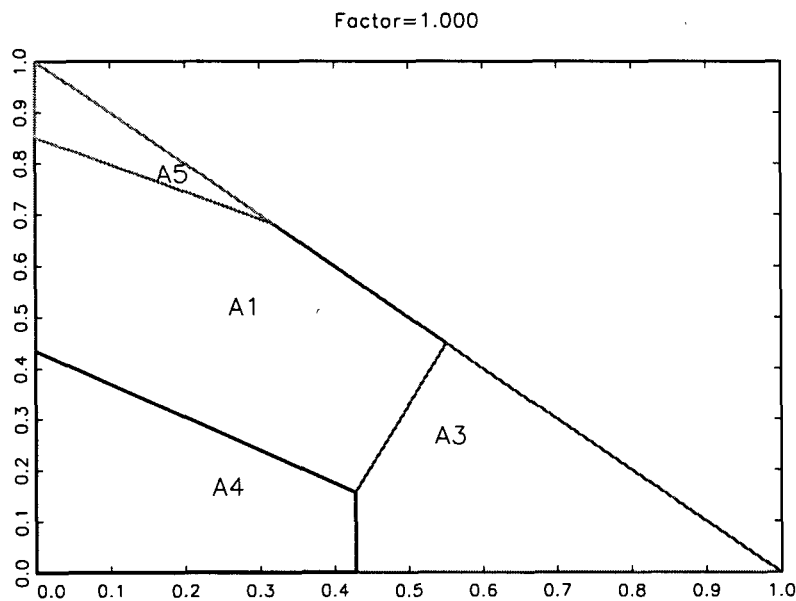


Figure 3: Partition of the unit simplex

The corresponding probabilities for choosing the alternatives are listed in table 3 for various levels of  $c$ .

Alternative	A1	A2	A3	A4	A5
$c=0.05$	0.474	0.000	0.128	0.326	0.072
$c=0.1$	0.424	0.000	0.275	0.253	0.047
$c=0.35$	0.238	0.000	0.628	0.116	0.018

Table 3: Probabilities of choosing the alternatives

As can be seen from figure 3 as well as table 3, there is no combination of weights that would make alternative A2 optimal. A1 is the most likely choice for  $c=0.1$ , the selection of A5 by the agent is possible, but unlikely. As a next step, we can analyze how the probabilities change if the compensation awarded to the agent changes. In performing this analysis, we assume that no adjustment for range effects takes place, i.e. the agent actually perceives these changes as changes in the level of compensation, not as changes in scale.

Figures 4 and 5 show the partition of  $w_1/w_2$ - space that would result if the compensation factor  $c$  is changed from 0.1 to 0.05 and 0.35, respectively. Since Alternative A3 is the best alternative with regard to compensation, its share of the simplex increases when the values of that attribute increase. The opposite holds for A5, which is the worst alternative with regard to compensation.

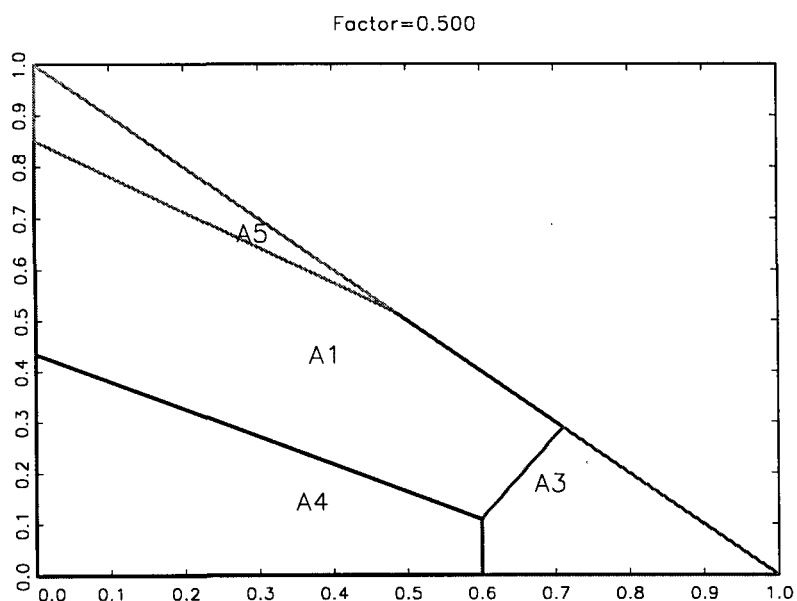


Figure 4: Partition of the unit simplex for  $c=0.05$



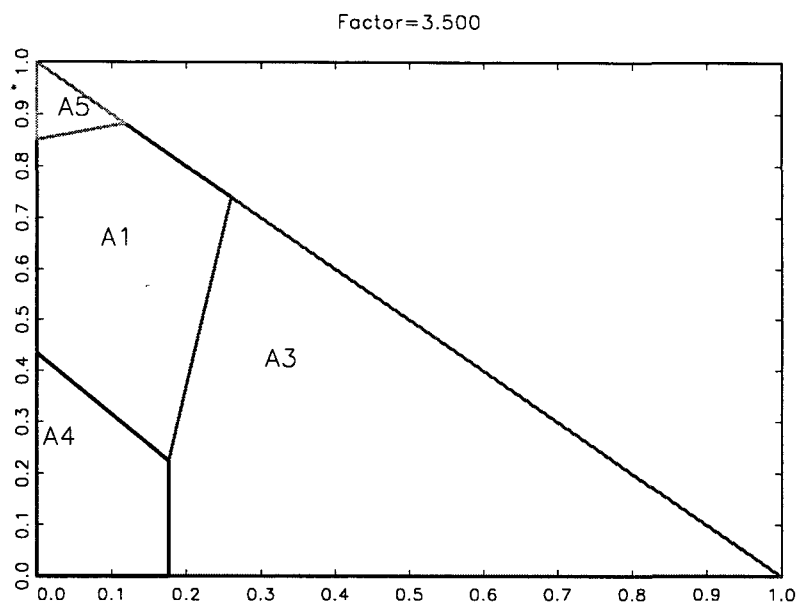
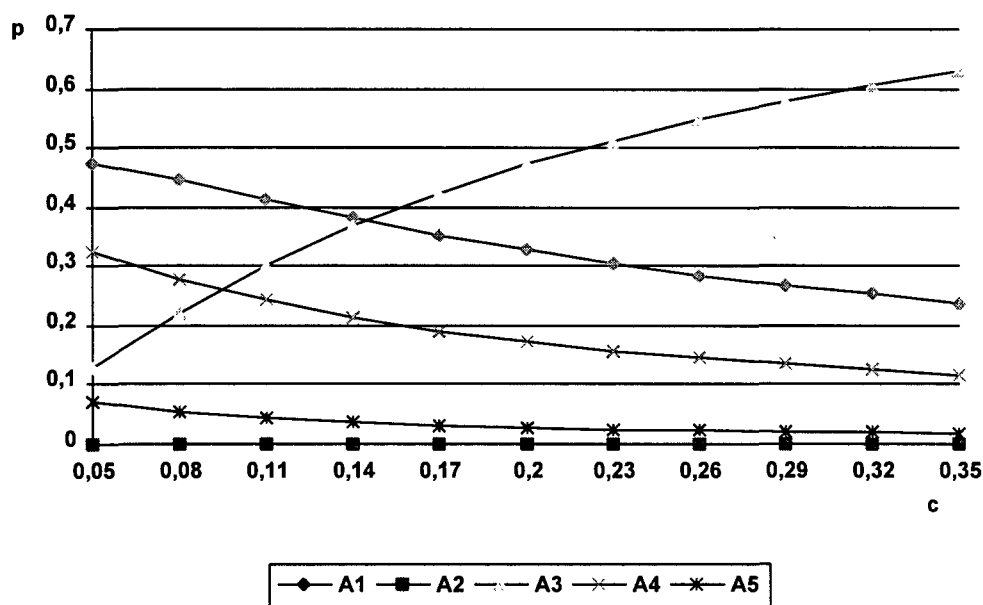
Figure 5: Partition of the unit simplex for  $c=0.35$ 

Figure 6 presents the development of the probabilities for all alternatives when factor  $c$  is varied from 0.05 to 0.35. As can clearly be seen from the figure, the changes in probabilities are nonlinear.

Figure 6: Development of choice probabilities for changing compensation factor  $c$ 

On the other hand, if the agent would fully adjust for range effects, no change in probabilities would take place. Figure 7 exhibits the partition of  $w_1/w_2$ -space for a factor of  $c = 2$  and full compensation. The region of feasible weights is compressed along the  $w_2$ -axis, but the ratio of areas remains the same.

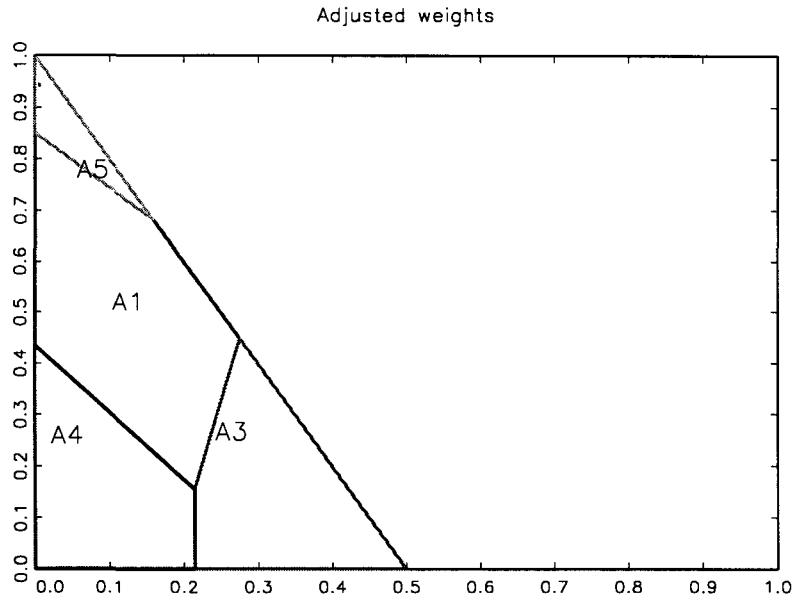


Figure 7: Partition of the unit simplex with adjustment for range effects

Using the probabilities shown in figure 6, we can compute the expected profit of the principal. Assuming that all alternatives are acceptable to the agent, i.e. aspiration levels for attributes Z2 and Z3 are zero, we obtain

$$\max_{c \in C} \sum_{i \in \{1,3,4,5\}} p(a_i | c, W) \cdot (a_{ig} - c \cdot a_{ig}) \quad (17)$$

Figure 8 shows the development of (17) when  $c$  is varied from 0.05 to 0.35.

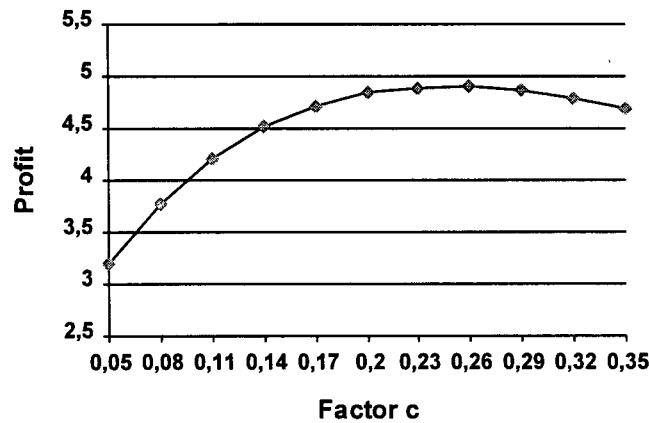


Figure 8: Expected profit for changing compensation factor  $c$

The maximum profit of the principal is obtained for a factor of about  $c=0.25$ . It should be noted that a unique, non-trivial optimum exists even in this simple model, where both the compensation function and the agent's utility function are linear.

## 4.2 Other decision techniques

The approach developed here can be used not only for linear utility functions. To illustrate its application to other decision criteria, and the problems that might arise in such applications, we consider a Tchebycheff criterion of the form

$$W(\mathbf{a}_i) = \min_k (a_{ik} - r_k) \quad (18)$$

where  $r_k$  is a reference or aspiration level for attribute  $k$ . This kind of criterion is widely used in multi-criteria decision making (Steuer, 1989) and has several advantages over additive functions (Wierzbicki, 1986). Most notably, it allows for the characterization of all (weakly) efficient solutions, while when using additive functions, alternatives that are dominated by linear combinations of other alternatives will never be chosen.

In the Tchebycheff criterion (18), the reference levels  $r_k$  play a similar role as the weights  $w_k$  in the additive model (7). They are the (unknown) parameters representing the agent's preferences towards the attributes. Similar to (9), the set of parameter combinations for which alternative  $\mathbf{a}_i$  is optimal can be described by a system of inequalities

$$\min_k (a_{ik} - r_k) \geq \min_k (a_{jk} - r_k) \quad \forall j \neq k \quad (19)$$

Unlike the additive model, there is no scaling constraint linking the reference levels of all attributes. In order to avoid system (19) being unbounded, the reference levels can be restricted to the same range as the data values. Assuming that the data values are all scaled to the unit interval, we obtain:

$$0 \leq r_k \leq 1 \quad \forall k \quad (20)$$

Due to the min operator, (19) is not a set of linear constraints. It is therefore possible that the region of optimality for an alternative is not a convex set. As an example, we consider the following two alternatives:

Alternatives	Attributes		
	Z1	Z2	Z3
A1	0.8	0.35	0.6
A2	0.6	0.55	0.4

Table 4: Example for the Tchebycheff criterion

For a vector of aspiration levels  $\mathbf{r}=(0.1, 0.4, 0.5)$ , we obtain the following evaluations:

$$A1: \min(0.8 - 0.1, 0.35 - 0.4, 0.6 - 0.5) = \min(0.7, -0.05, 0.1) = -0.05$$

$$A2: \min(0.6 - 0.1, 0.55 - 0.5, 0.4 - 0.5) = \min(0.5, 0.05, -0.1) = -0.1 < -0.05$$

and therefore A1 is preferred to A2. For another level of aspiration levels  $r'=(0.9, 0.6, 0.5)$ , we obtain:

$$A1: \min(0.8 - 0.9, 0.35 - 0.6, 0.6 - 0.5) = \min(-0.1, -0.25, 0.1) = -0.25$$

$$A2: \min(0.6 - 0.9, 0.55 - 0.6, 0.4 - 0.5) = \min(-0.3, -0.05, -0.1) = -0.3 < -0.25$$

and therefore A1 is also preferred. However, for the linear combination of aspiration levels  $0.5r + 0.5r' = (0.5, 0.5, 0.5)$ , A2 is preferred:

$$A1: \min(0.8 - 0.5, 0.35 - 0.5, 0.6 - 0.5) = \min(0.3, -0.15, 0.1) = -0.15$$

$$A2: \min(0.6 - 0.5, 0.55 - 0.5, 0.4 - 0.5) = \min(0.1, 0.05, -0.1) = -0.1 > -0.15$$

Figure 9 shows the regions of optimality for the two alternatives in  $r_1/r_2$  - space:

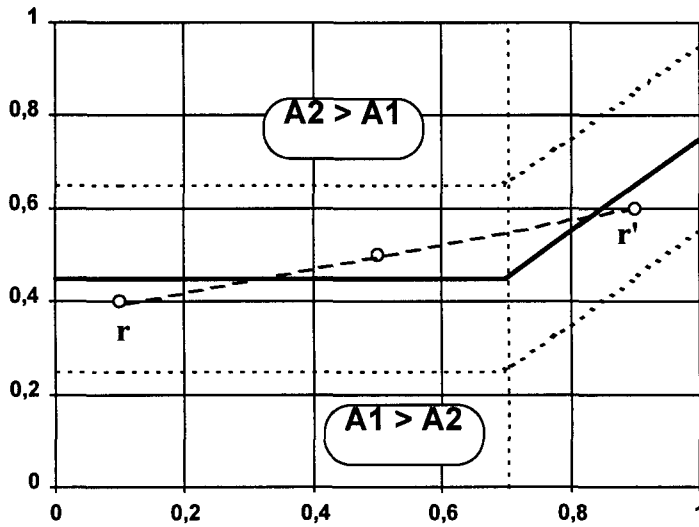


Figure 9: Non-convex regions of optimality for the Tchebycheff criterion

The solid line separates the two regions of optimality. Both points  $r$  and  $r'$  are in the region where A1 is better, but their linear combination (the middle of the dashed line) is in the region where A2 is preferred.

Although the region of optimality for an alternative as defined in (19) might be non-convex, it consists of convex parts. Let  $k_i^*$  denote the attribute in which the minimum is obtained in the Tchebycheff criterion (18) for alternative  $a_i$ . The region of optimality for the alternative can be determined as the union of subregions defined by the following systems of inequalities:

$$\begin{aligned}
a_{ik_i^*} - r_{k_i^*} &\geq a_{jk_j^*} - r_{k_j^*} \quad \forall j \neq i \\
a_{jk} - r_k &\geq a_{jk_j^*} - r_{k_j^*} \quad \forall j, \quad \forall k \neq k_j^*
\end{aligned} \tag{21}$$

The first set of inequalities in (21) ensures that  $a_i$  is the optimal alternative in that region, the second set ensures that the evaluation is indeed determined by  $k_j^*$ . By calculating the volume of the polyhedron (21) for all possible combinations of the  $k_j^*$ , the volume of the entire region of optimality can be determined. In figure 9, the boundaries of those subregions are indicated by the dotted lines.

It is therefore possible to use the techniques similar to those developed for the case of linear utility functions for other multicriteria decision methods.

## 5 Conclusions and topics for further research

In this paper, we have outlined some possibilities for considering multiple criteria in an agency context. Multi-criteria modeling can be seen as a more realistic way of representing preferences of actors in this setting.

One important result of the first group of models presented here is the great importance which the information available to the principal has on her possibilities to influence the agent's behavior. For the minimum information case, we have shown that the principal's possibilities are also very restricted. If more information is available, the principal's potential to influence the agent's decision also increases, although the relationships involved are rather complex. One could place standard, single-criteria agency models at the other end of the spectrum: in these models, the principal has even more information about the agent's preference structure and consequently, the problem becomes less complex. This result opens a wide field of research topics, in which various other information levels can be studied.

Another dimension, along which this research can be extended, is the decision situation considered. In this paper, we have dealt mainly with the choice among a finite set of well-defined alternatives. This situation is not typical for agency models, where decisions involving continuous variables are more frequently studied. In the field of multi-criteria problems, this field corresponds to multi-objective problems as opposed to the multi-attribute problem analyzed here. Both problems are relevant and occur in practical settings. Adapting the methodology developed here to multiobjective problems is, however, not a trivial task.

The analysis in this paper was also restricted to the case of certainty. In many agency problems, uncertainty is important and this aspect should also be taken into account in multi-criteria agency models.

Developing a comprehensive multi-criteria agency theory is therefore a considerable research agenda, from which, in the long run, both research areas will profit. On one hand, it should lead to more realistic agency models. On the other hand, problems arising out of agency settings should also lead to new developments in the field of multi-criteria decision making.

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