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Redistribution and the efficiency of the pay-as-you-go pension system

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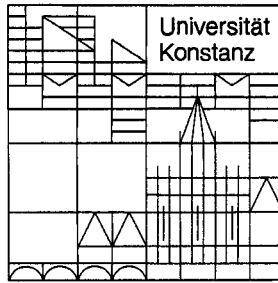
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**Fakultät für
Wirtschaftswissenschaften
und Statistik**

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**Redistribution and the
Efficiency of the Pay-as-You-Go
Pension System**

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**Redistribution and the efficiency
of the pay-as-you-go pension system**

Johann K. Brunner

Serie I - Nr. 265

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J.K. Brunner, Redistribution and the efficiency of the pay-as-you-go pension system.*

Abstract:

The paper considers the efficiency of the pay-as-you-go pension system with endogenous labor supply, when contributions are collected as an income tax. Results from the literature state that in this case a Pareto-improving transition path to a fully-funded system exists, if the interest rate is permanently higher than the growth rate of the economy. It is shown that these results depend crucially on the possibility of replacing the income tax by a lump-sum tax. If one adopts the realistic assumption that such a change is impossible, no Pareto-improving transition path from an unfunded to a funded pension system exists.

Keywords: Social Security, Income Taxation, Redistribution.

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I Introduction

In recent years there has been an intensive discussion of the merits of a fully-funded versus a pay-as-you-go pension system (for an overview see Breyer 1990). A specific issue in this discussion has been the negative effect of the latter on private saving and capital formation, which occurs if one considers life-cycle saving in a simple overlapping-generations model. Another aspect is that declining birth rates tend to lower the rentability of the pay-as-you-go method.¹ Problems of this kind have induced an analysis of the possibility of a transition from the pay-as-you-go system to a fully-funded system. While Breyer (1989) showed in a model with lump-sum contributions and payments that the former is intergenerationally efficient (as is the latter), so that a change of the system would make at least one generation worse off, Homburg (1990) questioned this result, because in reality contributions are not lump-sum. He employed a model with variable labor supply, income-dependent contributions and lump-sum pension payments to show that in this framework a Pareto improving transition path from an unfunded to a funded system exists.

In this study I argue that the last result, though formally correct, is based on an incomplete consideration of the reason for the tax character of the contributions. To be more precise: In the representative-consumer model used by Homburg it indeed makes no sense to collect contributions from the active in form of a distorting tax, and it is no wonder that a Pareto-improving abolition is possible. For an appropriate investigation, however, one has to ask why such a tax is introduced at all.

¹It should be mentioned that these are not the only criteria for an evaluation of the systems. Other could be, e.g., insurance against collective risks, such as inflation, or the availability of appropriate institutional arrangements.

As is known from the theory of income taxation, the choice between a lump-sum and a distorting tax becomes relevant only in an economy with differing individuals, who are to be treated differently. In this situation a problem of information must be solved, namely how to identify individuals. If the authority has enough information, given exogenously, to know, say, who is more and who is less able, it can impose a differentiated lump-sum tax, and this is of course a first-best solution. Normally, however, this information does not exist and is too costly to be collected. In this case the usual procedure is to identify individuals by their incomes, which means to impose a (distorting) income tax.

The existence of an income-related tax and a lump-sum transfer, as in Homburg's model, typically describes a situation where redistribution is desired: the individuals contribute different amounts but all get the same benefit out of the system. Accordingly, in the following the model will be extended to account for this aspect explicitly, and it will be shown that in the extended framework the availability of a Pareto-improving transition path from a pay-as-you-go system to a funded system indeed depends on the information of the authority, whether a differentiated lump-sum tax can be imposed or not. If not, then no transition path which is at the same time inter- and intragenerationally Pareto improving, exists.

II The Model

We extend the standard overlapping-generations model in order to incorporate the issue of redistribution. That is, we assume that every generation consists of n groups of individuals (for simplicity we will assume $n = 2$ in the following, and that every group consists of only one individual; without loss of generality) with different abilities $h^1 < h^2$, which influence their wage rate $w_t^i = h^i w_t$, $i = 1, 2$, where w_t is the wage per efficiency unit of labor and t is

a period index. Every generation lives for two periods, it is active (providing labor time l_t^i , i.e., $h^i l_t^i$ units of labor) in the first and is retired in the second. With c_t^i, z_{t+1}^i denoting consumption in the respective periods, and s_t^i denoting saving in period t , the decision problem of an individual with ability h^i is

$$\max U^i(c_t^i, z_{t+1}^i, l_t^i)$$

$$\text{s.t.} \quad c_t^i + s_t^i = w_t^i l_t^i (1-\tau) + \alpha_t + \beta_t^i,$$

$$z_{t+1}^i = s_t^i R + p_{t+1},$$

where R is the interest factor $1 + r$ (which we assume as given, describing the case of a small open economy, as in Homburg's model; furthermore it is constant over time), τ is the tax rate for the contribution to the pay-as-you-go pension system, also taken as constant, for simplicity. α_t, β_t^i and p_{t+1} denote lump-sum payments to the individuals in the active and retirement period, resp., where β_t^i is differentiated according to the types. The series of w_t^i are taken as exogenously fixed in the following, with $w_t^1 < w_t^2$. U^i is assumed to be strictly quasiconcave and twice differentiable, with partial derivatives $U_c^i, U_z^i > 0, U_l^i < 0$. Solving this maximization problem we derive the individually optimal values $\hat{c}_t^i, \hat{z}_{t+1}^i, \hat{l}_t^i$, depending on w_t^i, R , and $\tau, p_{t+1}, \alpha_t, \beta_t^i$. For simplicity we take the population as being constant.

For a reasonable formulation of the case of redistribution we suppose that preferences fulfill the property of "Agent-Monotonicity", AM (this mild condition is usual in optimum income taxation theory, for a definition see the Appendix), which guarantees that gross income increases with ability, i.e., $w_t^1 \hat{l}_t^1 < w_t^2 \hat{l}_t^2$. From this assumption we obviously get that a pension system with $\tau > 0, p_{t+1} > 0$, (and $\alpha_t = 0, \beta_t^i = 0$) is redistributive in the

sense that contributions fulfill $\tau w_t^1 \hat{1}_t^1 < \tau w_t^2 \hat{1}_t^2$, while payments out of the system are equal for both groups.

It is clear that in this framework a fully-funded pension system is equivalent to the accumulation of private saving for the period of retirement: The rentability is determined by the interest factor R , no details of the institutional setting are considered.

We are now in the position to define three different systems of old age income provision:

(i) If $\alpha_t = 0$, $\beta_t^i = 0$, and $\tau > 0$, $p_{t+1} > 0$ fulfill the equation

$$\tau(w_t^1 \hat{1}_t^1 + w_t^2 \hat{1}_t^2) = 2p_t,$$

then a *pay-as-you-go system with linear intragenerational redistribution (PLIR)* is established.

(ii) If $\tau = 0$, $p_{t+1} = 0$, $\alpha_t = 0$, and $\beta_{t+1}^1 > 0$, $\beta_{t+1}^2 < 0$ fulfill the equation

$$\beta_t^1 = -\beta_t^2,$$

then a *fully-funded system with intragenerational lump-sum redistribution (FLUR)* is established.

(iii) If $\beta_t^i = 0$, $p_{t+1} = 0$, and $\tau > 0$, $\alpha_t > 0$ fulfill the equation

$$\tau(w_t^1 \hat{1}_t^1 + w_t^2 \hat{1}_t^2) = 2\alpha_t,$$

then a *fully-funded system with linear intragenerational redistribution (FLIR)* is established.

In PLIR the contributions of the active generation are used to finance the benefits of the retired. Obviously, for constant τ , the overall rentability of the system is

$$(w_{t+1}^1 \hat{l}_{t+1}^1 + w_{t+1}^2 \hat{l}_{t+1}^2) / (w_t^1 \hat{l}_t^1 + w_t^2 \hat{l}_t^2),$$

while the redistributive effect of the system can be seen from differing individual rentabilities

$$(w_{t+1}^1 \hat{l}_{t+1}^1 + w_{t+1}^2 \hat{l}_{t+1}^2) / (2w_t^i \hat{l}_t^i), \quad i=1,2.$$

Thus, given AM, rentability is higher for the less able and lower for the more able individuals.

PLIR can be regarded as an integrated system of both inter- and intragenerational redistribution. In contrast, FLUR and FLIR are systems where only redistribution within a generation is performed, FLUR uses a lump-sum tax, FLIR a linear one.

For the definition of FLUR it obviously makes no difference whether lump-sum redistribution takes place in the first (as is assumed above) or in the second (using $\beta_{t+1}^i R$) period of the life-cycle, given rational individuals and a perfect capital market.

III Transition

Now we turn to the main question, namely whether it is possible to find a Pareto-improving transition path from an established pay-as-you-go system to a funded system. Let $V^i(\tau, p_{t+1}, \alpha_t, \beta_t^i)$, $i = 1, 2$, be the indirect utility function for individual i of the generation born in t , depending on the parameters of the pension system.

Let PLIR with parameters $\bar{\tau}$, \bar{p}_{t+1} , $\bar{\alpha}_t (= 0)$, $\bar{\beta}_t^i (= 0)$ at $t = t_0 - 1$ be given. A Pareto-improving transition path to another of the systems defined above exists, if there are $t_1 > t_0$ and parameters $\tau_t, p_{t+1}, \alpha_t, \beta_t^i$, for $t \geq t_0$, such that for all $t > t_1$ the second system is established and $V^i(\tau_t, p_{t+1}, \alpha_t, \beta_t^i) \geq V^i(\bar{\tau}, \bar{p}_{t+1}, \bar{\alpha}_t, \bar{\beta}_t^i)$, for $i = 1, 2$ and all $t \geq t_0$.

Before we investigate the question in detail, let us recall the argument applied by Homburg for the case of one representative individual: As contributions are levied in the way of a distorting tax, abolition of this distortion produces a surplus which can be used for a stepwise reduction of pension payments out of the unfunded system (and, thus, also for a reduction of the contributions). To be more precise, the suggested procedure is to substitute, in some period t_0 , the distortionary tax $\tau w_t \hat{l}_t^i$ by a lump-sum contribution of the same amount, whereby, due to the removal of the excess burden, welfare of the concerned active generation increases. Part of this increase is collected by means of a "small" tax with revenue ϵ . Obviously, accumulated revenues (plus interest) grow to infinity, moreover, for the whole problem to be relevant, it is assumed that the growth rate of the economy (thus, in particular, of the pensions) is permanently lower than the interest rate. Thus, after finitely many periods the accumulated revenues are large enough to finance completely the pensions, that is, the contributions of the active generation. From that moment all future generations are free of obligations to contribute to the

pay-as-you-go system and can care for their old age by private saving, which is equivalent to a fully-funded system.

In fact, Homburg constructs a little more complicated system by introducing in every period t a public debt D_t which substitutes (part of) the income tax of the active in the transition phase. When retired, they have to pay back $D_t R$, and this repayment is financed by \bar{p}_{t+1} , their lump-sum pension. Formally, the original constraints (a) $c_t + s_t = (1 - \bar{\tau})w_t \hat{l}_t$, (b) $z_{t+1} = s_t R + \bar{p}_{t+1}$ are replaced by (a') $c_t + s_t = (1 - \tau')w_t \hat{l}_t$, (b') $z_{t+1} = s_t R + \bar{p}_{t+1} - D_t R$. Moreover, one has $D_t = \bar{\tau}w_t \hat{l}_t - \tau'w_t \hat{l}_t > 0$ and, in the relevant case, $D_t R = \bar{p}_{t+1}$. As $\bar{\tau} > \tau'$, the excess burden is reduced and, because life-time income remains constant, welfare of the individuals is increased by this procedure.

One observes, now, that (a'), (b') are completely equivalent to the constraints (a'') $c_t + s_t = (1 - \tau')w_t \hat{l}_t - \bar{p}_{t+1}/R$, (b'') $z_{t+1} = s_t R + \bar{p}_{t+1}$. This shows that in fact no public debt needs to be introduced, the transition path can equally well be defined without it. As described above, part of the gains from the removal of the excess burden can be collected and accumulated, in order to decrease the contributions of the active gradually to zero. Homburg argues that by the suggested procedure (a'), (b') no lump-sum taxation is required for the possibility of a Pareto-improving transition. However, it is clear that not receiving a lump-sum pension \bar{p}_{t+1} (which, by (b'), is used to pay back the debt $D_t R$) is equivalent to paying a lump-sum tax.

As was stated in the introduction, the viable strategy in case of different individuals depends on the information of the authority. Two possible assumptions are:

II. The government can observe individual preferences and abilities.

I2. The government knows that two types of individuals exist, where each type has preferences U^i and ability h^i (i.e., wage rate w_t^i), $i = 1, 2$, but cannot identify individuals.

Denote by $G_t \equiv \frac{\sum_{i=1}^2 w_t^i \hat{l}_t^i(\tau, p_{t+1}, \alpha_t, \beta_t^i)}{\sum_{i=1}^2 w_{t-1}^i \hat{l}_{t-1}^i(\tau, p_t, \alpha_{t-1}, \beta_{t-1}^i)}$ the growth factor of the economy. Obviously, G_t depends on the parameters of the tax - transfer system. In the following, the assumption $G_t < R$ should be interpreted as $G_t(\tau, p_{t+1}, \alpha_t, \beta_t^i) < R$ for all "reasonable" values of $(\tau, p_{t+1}, \alpha_t, \beta_t^i)$, occurring in the transition from one system to the other. In particular, it is not possible to increase the rentability of the pay-as-you-go system above that of the funded system by changing the parameters.

Result 1: Assume that $R > G_t$, for all $t \geq t_0$. With information I1 a Pareto-improving path from PLIR to FLUR exists.

Proof: The proof is similar to the above consideration of the case of one individual: Let PLIR with parameters $\bar{\tau} > 0$, $\bar{p}_{t+1} > 0$, $\bar{\alpha}_t = 0$, $\bar{\beta}_t^i = 0$ be given and let $\hat{l}_t^i(\bar{\tau}, \bar{p}_{t+1}, 0, 0)$ be labor supply of a type- i individual. Replace in period t_0 the income-dependent contributions $\tau w_{t_0}^i \hat{l}_{t_0}^i(\bar{\tau}, \bar{p}_{t_0+1}, 0, 0)$ by lump-sum contributions $\tilde{\beta}_{t_0}^i$ of the same amount, $i = 1, 2$, to finance the pensions in that period. As the utility level of all members of the active generation is increased by the removal of the excess burden, it is possible to reduce their benefit in retirement to some amount $\tilde{p}_{t_0+1}^i$, defined implicitly by the condition

$$V^i(0, \tilde{p}_{t_0+1}^i, 0, \tilde{\beta}_{t_0}^i) = V^i(\bar{\tau}, \bar{p}_{t_0+1}, 0, 0),$$

where $\tilde{p}_{t_0+1}^1 + \tilde{p}_{t_0+1}^2 < 2\bar{p}_{t_0+1}$. Hence, in period $t_0 + 1$, contributions $\tilde{\beta}_{t_0+1}^i$, which are again collected in a lump-sum manner, can for both types of active individuals be lower

than with the former system. We define, for any $t \geq t_0 + 1$, $\tilde{\beta}_t^i, \tilde{p}_{t+1}^i$, $i = 1, 2$, recursively as the solutions of the following minimization problems (MP_t):

$$\min_{\beta_t^i, p_{t+1}^i} p_{t+1}^1 + p_{t+1}^2$$

s.t.

$$(1) \quad \beta_t^1 + \beta_t^2 = \bar{p}_t^1 + \bar{p}_t^2,$$

$$(2) \quad V^i(0, p_{t+1}^i, 0, \beta_t^i) \geq V^i(\bar{\tau}, \bar{p}_{t+1}^i, 0, 0), \quad i = 1, 2,$$

$$(3) \quad p_{t+1}^i \geq 0, \quad i = 1, 2.$$

It is easily seen that as long as for the minimal values $\bar{p}_{t+1}^1 + \bar{p}_{t+1}^2 > 0$ holds, this problem has a unique solution, where (2) is fulfilled with equality. We want to show that there exists t_1 such that $\bar{p}_{t+1}^i = 0$, $i = 1, 2$, for all $t \geq t_1$. For this, we define $\epsilon_t \equiv 2\bar{p}_t - (\bar{p}_t^1 + \bar{p}_t^2)$, for any $t \geq t_0 + 1$. We have $\epsilon_{t_0+1} > 0$ from above.

Now assume that $\bar{p}_{t+1}^1 + \bar{p}_{t+1}^2 > 0$ for all $t \geq t_0$, which means $\epsilon_t < 2\bar{p}_t$. As ϵ_t denotes the reduction of the contributions of the active in period t , compared to the former system PLIR, it follows that their pension benefits, when retired, can be reduced at least by $\epsilon_t R$ without reducing their welfare (in fact, by more, due to the removal of the excess burden). Hence, as long as $\bar{p}_{t+1}^1 + \bar{p}_{t+1}^2 > 0$, we have $\epsilon_{t+1} > \epsilon_t R$, or, equivalently, $2\bar{p}_{t+1} - \epsilon_{t+1} < 2\bar{p}_{t+1} - \epsilon_t R$. On the other hand, \bar{p}_t grows with factor G_t , thus $2\bar{p}_{t+1} - \epsilon_{t+1} < 2\bar{p}_t G_{t+1} - \epsilon_t R$ follows. Repeated application of this argument gives

$$2\bar{p}_{t+1} - \epsilon_{t+1} < 2\bar{p}_{t_0+1} \prod_{m=t_0+2}^{t+1} G_m - \epsilon_{t_0+1} R^{t-t_0}, \quad \text{for any } t \geq t_0 + 1.$$

As $G_m < R$, for all $m \geq t_0$, the right-hand side becomes negative at some $t = t'$, thus $2\bar{p}_{t'+1} < \epsilon_{t'+1}$, which contradicts the above assumption. Hence, t_1 must exist, such that $\bar{p}_{t_1+1}^1 = 0$, $\bar{p}_{t_1+1}^2 = 0$ are the solution of (MP_{t_1}) . Moreover, if the contributions of the active are zero once, i.e., $\epsilon_{t_1+1} = 2\bar{p}_{t_1+1}$, this generation obviously needs no pension benefits to preserve welfare at the same level as with PLIR. Hence $\bar{p}_{t_1+2}^1 = 0$, $\bar{p}_{t_1+2}^2 = 0$, and the same holds for all further $t \geq t_1$.

It follows from (1) that $\tilde{\beta}_t^1 = -\tilde{\beta}_t^2$, i.e., a FLUR system is indeed established, and the transition is Pareto improving, due to (2). (It remains open how precisely the gains are distributed, for $t \geq t_1$ the $\tilde{\beta}_t^i$ can be chosen arbitrarily, as long as (1) and (2) are satisfied.)

QED.

This result confirms what one expects: If the excess burden of a distorting tax can be removed, the profit can be used to finance a transition from PLIR to FLUR, even in the case of redistribution between individuals. However, it is clear that this possibility depends crucially on the assumption of a completely informed authority. Usually, this assumption is considered unrealistic, and with the more realistic version I2 it is obviously impossible to introduce an individually differentiated lump-sum tax.² We have

Result 2: With information I2 no Pareto-improving transition path from PLIR to FLUR exists.

Even if the use of a differentiated lump-sum tax is precluded under a realistic modeling of

²To mention the obvious, it is clear that the introduction of government debt would in no way change this conclusion.

government information, one has to ask whether an intra- and intergenerational system like PLIR could be substituted by a pure system of intragenerational redistribution, via a linear income tax, which was denoted by FLIR in Section II. Obviously, future generations would profit from such a change, if $R > G_t$, because with PLIR every individual gets the uniform payment p_{t+1} , whose discounted value is $p_{t+1}/R = \tau(w_{t+1}^1 \hat{l}_{t+1}^1 + w_{t+1}^2 \hat{l}_{t+1}^2)/(2R) = \tau(w_t^1 \hat{l}_t^1 + w_t^2 \hat{l}_t^2)G_{t+1}/(2R)$, while with FLIR he gets $\alpha_t = \tau(w_t^1 \hat{l}_t^1 + w_t^2 \hat{l}_t^2)/2$. The question is, however, whether a transition from one system to the other is possible, without making any one worse off than he would have been with PLIR.

In order to formulate this problem meaningfully, we have to assume that PLIR is established in an efficient way, which is expressed by the following condition:³

EF. Let PLIR with parameters $\bar{\tau} > 0$, $\bar{p}_{t+1} > 0$, $\bar{\alpha}_t = 0$ be given. For any t , there exist no $\tau \neq \bar{\tau}$ and $\alpha_t \neq 0$ such that $\tau(w_t^1 \hat{l}_t^1 + w_t^2 \hat{l}_t^2) - \alpha_t = \bar{p}_t$ and $V^i(\tau, \bar{p}_{t+1}, \alpha_t) \geq V^i(\bar{\tau}, \bar{p}_{t+1}, 0)$ for $i = 1, 2$, with strict inequality for at least one i .

This condition guarantees that the proportional tax $\bar{\tau}$ to finance pension payments p_t is Pareto efficient, in every period t . As is well-known, an increase of the marginal tax rate in general increases tax revenue and allows the introduction of a uniform negative tax α_t . This procedure normally represents an act of redistribution from the more to the less qualified individual (i.e., by assumption AM, from the high- to the low-income individual), because the higher marginal tax rate hits the former more than the latter. However, if the increase of τ does not raise sufficient additional revenue, it may happen that the procedure makes both individuals worse off. Then a reduction of τ and of α_t would

³For shortness of notation we neglect in the following the parameters β_t^i , which are zero with both PLIR and FLIR.

be a Pareto improvement. Condition EF excludes that such an extreme situation occurs with the parameters of PLIR. What we are interested in are potential gains in dynamic efficiency due to a change from PLIR to FLIR, not in gains caused by the abolition of static inefficiencies. As an immediate consequence we have

Lemma: Let PLIR with parameters $\bar{\tau} > 0$, $\bar{p}_{t+1} > 0$, $\bar{\alpha}_t = 0$, fulfilling EF, be given. There exist constants $\delta_t^i > 0$, $\kappa_t > 0$, for any t and $i = 1, 2$, such that, for any t ,

$$(4) \quad \sum_{i=1}^2 \delta_t^i \frac{\partial V^i}{\partial \tau} + \kappa_t \sum_{i=1}^2 (w_t^i \hat{l}_t^i + \bar{\tau} w_t^i \frac{\partial \hat{l}_t^i}{\partial \tau}) = 0,$$

$$(5) \quad \sum_{i=1}^2 \delta_t^i \frac{\partial V^i}{\partial \alpha_t} + \kappa_t \sum_{i=1}^2 (\bar{\tau} w_t^i \frac{\partial \hat{l}_t^i}{\partial \alpha_t} - 1) = 0.$$

Proof: The efficiency condition EF entails that $\tau = \bar{\tau}$, $\alpha_t = 0$ solve, for any t , the maximization problem

$$(6) \quad \max_{\tau, \alpha_t} V^2(\tau, \bar{p}_{t+1}, \alpha_t)$$

s.t.

$$(7) \quad \tau(w_t^1 \hat{l}_t^1 + w_t^2 \hat{l}_t^2) - 2\alpha_t \geq 2\bar{p}_t,$$

$$(8) \quad V^1(\tau, \bar{p}_{t+1}, \alpha_t) \geq V^1(\bar{\tau}, \bar{p}_{t+1}, 0).$$

We write the corresponding Lagrangean as $V^2(\tau, \bar{p}_{t+1}, \alpha_t) + \delta_t^1 (V^1(\tau, \bar{p}_{t+1}, \alpha_t) - V^1(\bar{\tau}, \bar{p}_{t+1}, 0)) + \kappa_t (\tau(w_t^1 \hat{l}_t^1 + w_t^2 \hat{l}_t^2) - 2\alpha_t - 2\bar{p}_t)$, with $\delta_t^1 \geq 0$, $\kappa_t \geq 0$ as Lagrange variables, and define $\delta_t^2 \equiv 1$. One observes immediately, then, that (4) and (5) are the necessary

conditions for a solution of the maximization problem. Moreover, $\delta_t^2 > 0$ and $\frac{\partial V^i}{\partial \alpha_t} > 0$ imply $\kappa_t > 0$, due to (5). Finally, for $\tau = \bar{\tau}$ and $\alpha_t = 0$, (8) is trivially fulfilled with equality, which means that, in general, $\delta_t^1 > 0$.⁴ QED.

With this preparation we are now in a position to investigate the possibility of a Pareto-improving transition from PLIR to FLIR:

Result 3: Let PLIR with parameters $\bar{\tau} > 0$, $\bar{p}_{t+1} > 0$, $\bar{\alpha}_t = 0$, fulfilling EF, be given. No Pareto-improving transition path to FLIR exists.

Proof: We have to show that among all possible systems characterized by the parameters $\tau_t, p_{t+1}, \alpha_t, b_t$, where b_t denotes a public debt in period t , the one with $\tau = \bar{\tau}$, $p_{t+1} = \bar{p}_{t+1}$, $\alpha_t = 0$, $b_t = 0$ is Pareto-efficient in every finite sequence of periods $t = t_0, \dots, t_1$. We do this by showing that for every $t_1 > t_0$ weights $\gamma_t^i > 0$ exist such that $\bar{\tau}, \bar{p}_{t+1}, 0, 0$ solve the following problem (\bar{p}_{t_0} as well as some arbitrary expectation $p_{t_1+1}^e$ are taken as given):

$$\max_{\tau_t, p_{t+1}, \alpha_t, b_t} \sum_{t=t_0}^{t_1} \sum_{i=1}^2 \gamma_t^i V^i(\tau_t, p_{t+1}, \alpha_t)$$

$$\text{s.t.} \quad \sum_{i=1}^2 (\tau_t w_t^i \hat{l}_t^i(\tau_t, p_{t+1}, \alpha_t) - \alpha_t - p_t) + b_t \geq 0, \quad t = t_0, \dots, t_1$$

$$\sum_{t=t_0}^{t_1} b_t R^{t_1-t} \leq 0.$$

⁴We assume, thus, a slightly stronger version of EF, namely that $\tau = \bar{\tau}$ and $\alpha_t = 0$ represent a solution of (6) - (8) where (8) is in fact binding.

The last constraint requires that total public debt is repaid at the end of the time horizon.

The corresponding Lagrangean L with additional variables $\mu_t \geq 0$, $\nu \geq 0$, $t = t_0, \dots, t_1$, is

$$L = \sum_{t=t_0}^{t_1} \sum_{i=1}^2 \gamma_t^i V^i(\tau_t, p_{t+1}, \alpha_t) + \sum_{t=t_0}^{t_1} \mu_t \left(\sum_{i=1}^2 (\tau_t w_t^i \hat{l}_t^i(\tau_t, p_{t+1}, \alpha_t) - \alpha_t - p_t) + b_t \right) - \nu \sum_{t=t_0}^{t_1} b_t R^{t_1-t}.$$

The first-order conditions are derived as

$$(9) \quad \frac{\partial L}{\partial \tau_t} = \sum_{i=1}^2 \gamma_t^i \frac{\partial V^i}{\partial \tau_t} + \mu_t \sum_{i=1}^2 (w_t^i \hat{l}_t^i + \tau_t w_t^i \frac{\partial \hat{l}_t^i}{\partial \tau_t}) = 0, \quad t = t_0, \dots, t_1$$

$$(10) \quad \frac{\partial L}{\partial \alpha_t} = \sum_{i=1}^2 \gamma_t^i \frac{\partial V^i}{\partial \alpha_t} + \mu_t \sum_{i=1}^2 (\tau_t w_t^i \frac{\partial \hat{l}_t^i}{\partial \alpha_t} - 1) = 0, \quad t = t_0, \dots, t_1,$$

$$(11) \quad \frac{\partial L}{\partial p_{t+1}} = \sum_{i=1}^2 \gamma_t^i \frac{\partial V^i}{\partial p_{t+1}} + \mu_{t+1} \sum_{i=1}^2 (-1) + \mu_t \sum_{i=1}^2 \tau_t w_t^i \frac{\partial \hat{l}_t^i}{\partial p_{t+1}} = 0, \quad t = t_0, \dots, t_1-1,$$

$$(12) \quad \frac{\partial L}{\partial b_t} = \mu_t - \nu R^{t_1-t} = 0, \quad t = t_0, \dots, t_1.$$

Our aim is to find $\bar{\gamma}_t^i > 0$, which together with $\bar{\tau}$, \bar{p}_{t+1} , $\bar{\alpha}_t = 0$, $b_t = 0$ and with appropriate $\bar{\mu}_t$, $\bar{\nu}$ solve (9) - (12). We define $\bar{\gamma}_{t_0}^i \equiv \delta_{t_0}^i > 0$, $\bar{\mu}_{t_0} \equiv \kappa_{t_0} > 0$, whose existence is guaranteed

by the foregoing lemma. Similarly, for $t > t_0$, we define $k_t \equiv \bar{\mu}_{t_0} R^{t_0-t} / \kappa_t$ and set $\bar{\gamma}_t^i \equiv$

$$k_t \delta_t^i > 0, \quad \bar{\mu}_t \equiv k_t \kappa_t (= \bar{\mu}_{t_0} R^{t_0-t}) > 0.$$

As $(\bar{\mu}_t, \bar{\gamma}_t^i)$ are proportional to (κ_t, δ_t^i) , they fulfill (assumption EF), together with $\tau_t = \bar{\tau}$, $p_{t+1} = \bar{p}_{t+1}$, $\bar{\alpha}_t = 0$ the equations (4) and (5), which are clearly identical with (9) and (10).

Next, one observes that (12) can be transformed to $\nu = \mu_t R^{t-t_1}$, which is fulfilled, by the definition of $\bar{\mu}_t$, $t = t_0, \dots, t_1$, if we set $\bar{\nu} \equiv \bar{\mu}_{t_0} R^{t_0-t_1}$.

Finally, to see that (11) holds for the parameters of PLIR and with $b_t = 0$, one notes that α_t and p_{t+1} are both lump-sum benefits, occurring in different periods, hence $\partial V^i / \partial \alpha_t = R \partial V^i / \partial p_{t+1}$, $\partial \hat{l}_t^i / \partial \alpha_t = R \partial \hat{l}_t^i / \partial p_{t+1}$. Therefore, (11) can be written as

$$(11') \quad \sum_{i=1}^2 \gamma_t^i \frac{\partial V^i}{\partial \alpha_t} \frac{1}{R} + \mu_{t+1} \sum_{i=1}^2 (-1) + \mu_t \sum_{i=1}^2 \tau_t w_t^i \frac{\partial \hat{l}_t^i}{\partial \alpha_t} \frac{1}{R} = 0, \quad t = t_0, \dots, t.$$

As we have $\bar{\mu}_{t+1} = \bar{\mu}_t / R$, by definition, it follows that with $\bar{\tau}$, \bar{p}_{t+1} , $\bar{\alpha}_t = 0$, $b_t = 0$, with the weights $\bar{\gamma}_t^i$ and with the Lagrange variables $\bar{\mu}_t$, $\bar{\nu}$, (11') is satisfied, because it is equivalent to (10). Thus, we have shown, altogether, that PLIR is a Pareto-efficient system. QED.

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IV Concluding Remarks

The intention of this study was to analyze in detail the question of whether a *Pareto-improving* transition path from a *pay-as-you-go* pension system to a *fully-funded* system is possible, given that contributions to the former are collected by a (distorting) proportional income tax. As such a tax clearly makes no sense in an economy with identical

individuals, the problem was formulated in a model with two different types of individuals. In this framework it becomes apparent that with a system, where contributions are proportional to gross income while benefits are uniform, intragenerational redistribution comes along with old-age security.

Once this issue is recognized appropriately, the results proved in this study conform with what one expects: If the authority has enough information to substitute the income tax by a (differentiated, that is, redistributive) lump-sum tax, this change creates a surplus through the removal of the excess burden. If the interest rate is assumed to be permanently higher than the growth rate of the economy, it is clear that any surplus, whatever small, will after sufficiently many periods be large enough to finance pensions of one generation. At this time, no contributions of the active are required, and a funded system can start.

However, the existence of the excessive information required for differentiated lump-sum taxation is usually considered unrealistic. As a consequence, the procedure described above appears illusionary. If the redistributive (within generations) character of the system is to be preserved, a realistic alternative might be to substitute the pay-as-you-go system by a linear redistributive tax within the active. But Result 3 showed that such a change cannot be performed in a Pareto-improving way.

In theory, there seems to exist another way to improve the working of the system without destroying its redistributive aspect, and this would mean to replace the proportional tax by a nonlinear system, with differing marginal tax rates. Obviously, a tax with more parameters than just one marginal tax rate τ would allow to lower (but not remove!) the excess burden of the system, and it might again be possible to use the gains for a transition

to a funded system. The disadvantage of a nonlinear tax is, of course, its more complex nature.⁵

Appendix *Alky*

In order that income taxation performs redistribution in the "right" direction, i.e., from the more to the less qualified individuals, one has to assume that gross income of the former is indeed higher than that of the latter, given any linear tax. The appropriate condition on preferences, for the case of one good, was called "Agent Monotonicity" by Seade (1982), for a discussion see also Brunner (1989, p. 26). We extend this condition for the case of two goods in a natural way:

Let $y_t = w_t^i l_t$ denote gross income earned by an individual of type i in period t . We define the transformed utility function $\bar{U}^i(c_t, z_{t+1}, y_t) \equiv U^i(c_t, z_{t+1}, y_t/w_t^i)$. Moreover, we consider the expenditures on c_t and z_{t+1} as a Hicksian composite commodity x_t and introduce the utility function $\Psi^i(x_t, y_t) \equiv \max \{ \bar{U}^i(c_t, z_{t+1}, y_t) / c_t + z_{t+1}/R = x_t \}$. Finally, we define the slope of an indifference curve of Ψ^i , $\sigma^i(x_t, y_t) \equiv - \Psi_y^i(x_t, y_t) / \Psi_x^i(x_t, y_t)$.

AM. $\sigma^1(x_t, y_t) > \sigma^2(x_t, y_t)$, for any x_t and y_t .

This condition requires that, for any bundle (x_t, y_t) , the marginal rate of substitution between gross income and composite consumption is lower for the more able individual. It is completely analogous to the case of one good to show that AM implies $y^1(\tau, p_{t+1}, \alpha_t, \beta_t) < y^2(\tau, p_{t+1}, \alpha_t, \beta_t)$ for any values of the parameters of the pension system.

⁵The optimum nonlinear income tax was first discussed by Mirrlees 1971. See also Ebert 1992, Brunner 1989, 1993.

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