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Welfare effects of unfunded pension systems when labor supply is endogenous

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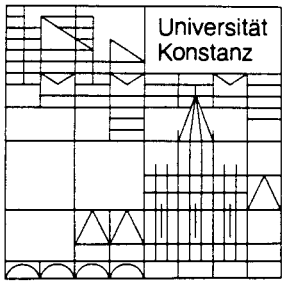

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Welfare Effects of Unfunded Pension Systems when Labor Supply is Endogenous

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Welfare Effects of Unfunded Pension Systems when Labor Supply is Endogenous

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1. Introduction

Since the seminal articles by SAMUELSON (1958) and AARON (1966) it is well-known that in unfunded pension systems the contributions of the working-age members "earn" a return which is composed of the rates of growth of population ("biological rate of interest") and of wages, whereas for funded systems as well as for private savings the market rate of interest and thus the marginal productivity of capital is relevant. From this perspective, it comes as no surprise that many industrial countries introduced or expanded unfunded public pension schemes in the years following the post-war baby boom. Considering the recent decline in birth rates, however, economists as well as politicians in these countries are contemplating on a reverse transition.[1]

Moreover, since FELDSTEIN's (1974) analysis it is well-known that in a world of life-cycle savers without bequest motive, the introduction of an unfunded social pension system reduces private savings and thus, if it is a closed economy, capital accumulation and future per-capita production.[2] Unless the economy is on a dynamically inefficient "over-capitalized" time path, where the growth rate exceeds the rate of interest, this means that steady-state welfare is reduced by the presence of the unfunded pension system.

However, as was shown in BREYER (1989), this does not imply that the transition to a funded system - or, equivalently, the abolition of the mandatory pension system - would lead to an intergenerational Pareto improvement since it is impossible to compensate the losers of the transition (i.e. the first generation which no longer receives payments from the unfunded system) without making at least one of the later generations strictly worse off. This impossibility was proved both for a (small) open

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[1] For Germany, see e.g. NEUMANN (1986).

[2] This statement is true at least for short-run equilibria which are stable in the Walrasian sense. If Marshallian stability is assumed, the effect on equilibrium saving may be reversed (see JAEGER 1990).

and for a closed economy, but labor supply was assumed to be exogenous.

Now HOMBURG (1990) has argued that this result was an artefact of a model with exogenous labor supply. If instead labor supply was assumed to react to net wages and - as is generally true in reality - contributions to the pension system are levied in the form of payroll taxes with fixed rates, then unfunded social security is no longer Pareto efficient. More precisely, replacing contributions in one period by external government debt would reduce distortions in the labor supply decision and thus raise the welfare level of the generation that was active in that period without hurting any other generation.

Although this consideration is correct, HOMBURG's analysis is confined to small open economies, and obviously the recourse to external debt is not available to closed economies. So it remains an unresolved question whether a similar result is true for economies that are either closed or so large that their domestic savings rate has an impact on the rate of interest on world capital markets. Scenarios of the transition from an unfunded to a funded pension system have been analyzed in several simulation studies,[3] but in none of them has the existence of a transition path been demonstrated which would improve the welfare of each generation. Similarly, HOMBURG/RICHTER (1990) have calculated the (static) deadweight loss for the German economy that is inherent in the labor-supply distortions due to the payroll-tax nature of social security contributions. But again, it was not shown that the avoidance of these losses would suffice to build up a capital stock big enough to "fund" the existing level of pensions.

As an instantaneous abolition of the unfunded pension system would definitely hurt the generation that is retired when the transition is performed, the key to a Pareto improvement must lie here in a change of the manner in which contributions to the

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[3] See, e.g. AUERBACH/KOTLIKOFF (1987) and SEIDMAN (1986) for the United States and RAFFELHÜSCHEN (1989) and KITTERER/RAFFELHÜSCHEN (1990) for the Federal Republic of Germany.

pension system are levied. Obviously, the labor supply decision is no longer distorted if

- either contributions and benefits are tied together according to the principle of actuarial fairness
- or contributions take the form of lump-sum taxes.

Both possibilities shall be explored in this paper.

The paper is organized as follows. In Section 2, we present the assumptions of the model and introduce three pure kinds of unfunded pension systems. Subsequently, in Section 3, we characterize steady-state equilibria associated with each of these types of pension systems. In Section 4, a condition for Pareto optimal paths is derived, and in Section 5 we analyze the possibility of a Pareto-improving transition from a pension system as described in Section 3. Finally, in Section 6 we draw some conclusions.

2. The Model

2.1 Basic Assumptions

Following the tradition of SAMUELSON (1958) and DIAMOND (1965), we consider a one-good economy with overlapping generations, where each individual lives for exactly two periods (working age and retirement age). N_t denotes the number of workers in period t and G_t the corresponding growth factor, i.e.

$$(2.1) \quad N_t = G_t \cdot N_{t-1}.$$

If the average worker of the t-generation provides l_t units of labor, then total labor supply in period t is

$$(2.2) \quad L_t = l_t \cdot N_t.$$

There is only one good, which is produced in each period t according to the (twice differentiable) constant-returns-to-scale production function

(2.3) $Y_t = F(K_t, L_t)$ with

$$F_L(K_t, L_t) \to \infty \text{ as } L_t \to 0 \text{ and } F_K(K_t, L_t) \to \infty \text{ as } K_t \to 0,$$

where K_t denotes the stock of the only good at the beginning of period t, which completely merges in the product Y_t.[4]

Perfect competition in the factor markets ensures that in equilibrium the interest factor and the income per worker are equal to the respective marginal products, i.e.

(2.4) $R_t = 1 + r_t = F_K(K_t, L_t)$,

(2.5) $w_t = F_L(K_t, L_t)$.

As we assumed constant returns to scale, we can replace F by the per-worker production function f where

$$(2.6) \quad y_t = \frac{Y_t}{L_t} = F\left(\frac{K_t}{L_t}, 1\right) = f(k_t)$$

and k_t denotes capital intensity in period t.

In this model of two overlapping generations and no bequest motive, the only reason for saving is providing for retirement age so that in each period the capital stock (plus interest) is transformed into consumption by the old, and a new capital stock is formed by the savings of the young. Therefore, if S_t denotes total savings and s_t savings per individual of working age, the latter determines the capital intensity of the following period via

$$(2.7) \quad k_{t+1} = \frac{K_{t+1}}{L_{t+1}} = \frac{S_t}{l_{t+1} \cdot N_{t+1}} = \frac{s_t \cdot N_t}{l_{t+1} \cdot N_t \cdot G_{t+1}} = \frac{s_t}{l_{t+1} \cdot G_{t+1}}$$

If we use the symbols c_t^i and z_{t+1}^i for consumption of the i-th member of the generation t in the two periods of his (economic) life, the values of these variables are determined by

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[4] Note that in one-good models it makes no difference if capital is assumed to be perfectly durable as in DIAMOND (1965) or circular or anything in-between (depreciation rate positive but less than one).

$$(2.8) \qquad c_t^i = w_t \cdot l_t^i - s_t^i - B_t(l_t^i),$$

$$(2.9) \qquad z_{t+1}^i = R_{t+1} \cdot s_t^i + P_{t+1}(l_t^i, l_t, l_{t+1}, w_{t+1}, R_{t+1}, G_{t+1})$$

where B_t denotes his contributions to and P_{t+1} his future benefits from an unfunded public pension system. Clearly, both equal zero if no such system exists. The superscript i is introduced to emphasize that contributions and benefits may depend upon the individual labor supply decision.[5] Nevertheless, all individuals are assumed to be identical so that it suffices to analyze optimal behavior of a representative person.

2.2 Individual Utility Maximization

The individual working-age person in period t chooses his labor supply l_t^i and savings s_t^i so as to maximize - under the constraints (2.8) and (2.9) - the twice differentiable utility function

$$(2.10) \qquad U_t^i = U(c_t^i, z_{t+1}^i, l_t^i) \qquad \text{with}$$

$$U_c, U_z > 0, \; U_l \leq 0 \quad \text{and} \quad U_c \; (U_z) \to \infty \quad \text{as} \quad c \; (z) \to 0,$$

where U is assumed quasiconcave in (c,z,l) and strictly quasiconcave in (c,z) for given l.

At the time when he plans his life-cycle behavior, he does not yet know l_{t+1}, the average labor supply of the subsequent period, so he is forced to form a (point) expectation l_{t+1}^e. From the necessary first-order conditions for an interior optimum with respect to s_t^i and l_t^i we obtain the equations

$$(2.11) \qquad \frac{\partial U / \partial c_t^i}{\partial U / \partial z_{t+1}^i} = R_{t+1} \cdot$$

$$(2.12) \qquad - \frac{\partial U / \partial l_t^i}{\partial U / \partial c_t^i} = w_t + \frac{\partial P_{t+1} / \partial l_t^i}{R_{t+1}} - \frac{\partial B_t}{\partial l_t^i}$$

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[5] Variables without superscript denote average values.

from which the functions

$$(2.13) \quad s_t^i = s_t(w_t, B_t(\cdot), R_{t+1}, P_{t+1}(\cdot))$$

$$(2.14) \quad l_t^i = l_t(w_t, B_t(\cdot), R_{t+1}, P_{t+1}(\cdot))$$

can be obtained. Given the expectation on future labor supply, l_{t+1}^e, the variables R_{t+1} and w_{t+1} appearing in (2.13) and (2.14) can be endogenized via (2.4) and (2.5), which completes the characterization of the short-run equilibrium of the economy from the perspective of a period-t decision-maker:

$$(2.15) \quad R_{t+1} = R_{t+1}(s_t, l_{t+1}^e)$$

$$(2.16) \quad w_{t+1} = w_{t+1}(s_t, l_{t+1}^e).$$

2.3 Funded and unfunded pension systems

A pension system is characterized by a contribution function $B_t(\cdot)$ and a benefit function $P_{t+1}(\cdot)$. We shall distinguish the following three types of unfunded systems:

a) constant contribution rate b with intragenerational fairness:[6]

$$(2.17) \quad B_t = b \cdot w_t \cdot l_t^i, \qquad P_{t+1} = b \cdot w_{t+1} \cdot l_{t+1} \cdot G_{t+1} \cdot l_t^i / l_t$$

b) full actuarial fairness:

$$(2.18) \quad B_t = b_t \cdot w_t \cdot l_t^i, \qquad P_{t+1} = b_t \cdot w_t \cdot l_t^i \cdot R_{t+1},$$

so that the contribution rate b_t changes over time according to

$$(2.19) \quad b_{t+1} = b_t \cdot (w_t \cdot l_t \cdot R_{t+1}) / (w_{t+1} \cdot l_{t+1} \cdot G_{t+1})$$

c) lump-sum-contributions and benefits:

$$(2.20) \quad B_t = \underline{B}_t, \qquad P_{t+1} = \underline{B}_{t+1} \cdot G_{t+1}$$

Clearly, a funded system can be expressed as a special case of type a) by setting b=0.

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[6] Analogously, the benefit level could be fixed as a percentage of current wages. We omit this variant for reasons of space.

For each of these types of pension systems we shall in the following specify the value of the variable

$$(2.21) \qquad \pi := \frac{1}{R_{t+1}} \cdot \frac{\partial P_{t+1}}{\partial l_t^{\,i}} - \frac{\partial B_t}{\partial l_t^{\,i}},$$

which, according to (2.12) drives a wedge between the marginal rate of substitution of consumption for leisure and the market wage and thus leads to a distortion of the labor supply decision:

$$(2.21a) \qquad \pi = b \cdot \left[\frac{w_{t+1} \cdot G_{t+1} \cdot l_{t+1}}{R_{t+1} \cdot l_t} - w_t\right].$$

$$(2.21b) \qquad \pi = b_t \cdot w_t - b_t \cdot w_t \cdot R_{t+1}/R_{t+1} = 0$$

$$(2.21c) \qquad \pi = 0.$$

3. Perfect-foresight equilibria

3.1 General remarks

A logically consistent way to specify the expectations of each generation t on their successors' labor supply, $l_{t+1}^{\,e}$, is to assume perfect foresight, i.e.

$$(3.1) \qquad l_{t+1}^{\,e} = l_{t+1}.$$

However, since l_{t+1} itself is a choice variable, which depends (among other things) upon R_{t+2}, this introduces an infinite regress into the future, which can be resolved for instance by confining the analysis to steady states, i.e. time paths in which all per-capita variables including labor supply l are constant over time.

3.2 Equilibrium with constant contribution rate

Assuming identical individuals and using (2.21a), the optimality conditions (2.11) and (2.12) become

$$(3.2)\qquad \frac{U_c(c_t, z_{t+1}, l_t)}{U_z(c_t, z_{t+1}, l_t)} = R_{t+1}$$

$$(3.3)\qquad -\frac{U_l(c_t, z_{t+1}, l_t)}{U_c(c_t, z_{t+1}, l_t)} = w_t \cdot [1 + b(\frac{G_{t+1}}{R_{t+1}} - 1)].$$

Using (2.4) and (2.5) for period t+1 and (2.7) through (2.9), (3.2) and (3.3), we obtain a system of 7 equations for the 8 endogenous variables c_t, z_{t+1}, s_t, l_t, l_{t+1}, R_{t+1}, w_{t+1} and k_{t+1} (given G_{t+1}, w_t and b). If then R_{t+1}, s_t and w_{t+1} are eliminated, one obtains the reduced system

$$(3.4a)\quad c_t = (1-b)\cdot w_t \cdot l_t - k_{t+1}\cdot G_{t+1}\cdot l_{t+1}$$

$$(3.4b)\quad z_{t+1} = G_{t+1}\cdot l_{t+1}\{f'(k_{t+1})\cdot k_{t+1} + b[f(k_{t+1}) - f'(k_{t+1})\cdot k_{t+1}]\}$$

$$(3.4c)\quad U_c(c_t, z_{t+1}, l_t) = f'(k_{t+1})\cdot U_z(c_t, z_{t+1}, l_t)$$

$$(3.4d)\quad -f'(k_{t+1})\cdot U_l(c_t, z_{t+1}, l_t) = U_c(c_t, z_{t+1}, l_t)\cdot w_t \cdot [(1-b)\cdot f'(k_{t+1}) + b\cdot G_{t+1}]$$

Assuming constant population growth $G_t = G$ for all t, one can remove the indeterminacy of the system (3.4) by considering a steady-state-equilibrium in which the endogenous variables l_t and l_{t+1} coincide and all other time indexes can be dropped as well:

$$(3.5a)\quad c = (1-b)\cdot l\cdot [f(k) - k\cdot f'(k)] - k\cdot G\cdot l$$

$$(3.5b)\quad z = G\cdot l\cdot \{k\cdot f'(k) + b\cdot [f(k) - k\cdot f'(k)]\}.$$

$$(3.5c)\quad U_c(c,z,l) = f'(k)\cdot U_z(c,z,l)$$

$$(3.5d)\quad -f'(k)\cdot U_l(c,z,l) = U_c(c,z,l)\cdot [f(k) - k\cdot f'(k)]\cdot \{(1-b)\cdot f'(k) + b\cdot G\}$$

3.3 Equilibrium with actuarially fair benefits

In the type b) pension system, the contribution rate b receives a time index, and thus the equilibrium system (3.4) is replaced by

(3.6a) $c_t = (1-b_t)\cdot l_t\cdot w_t - k_{t+1}\cdot G_{t+1}\cdot l_{t+1}$

(3.6b) $z_{t+1} = f'(k_{t+1})\cdot\{b_t\cdot w_t + k_{t+1}\cdot G_{t+1}\}.$

(3.6c) $U_c(c_t,z_{t+1},l_t) = f'(k_{t+1})\cdot U_z(c_t,z_{t+1},l_t)$

(3.6d) $-U_l(c_t,z_{t+1},l_t) = U_c(c_t,z_{t+1},l_t)\cdot w_t$

(3.6e) $G_{t+1}\cdot b_{t+1}\cdot l_{t+1}[f(k_{t+1})-k_{t+1}\cdot f'(k_{t+1})] = b_t\cdot w_t\cdot l_t\cdot f'(k_{t+1}).$

and the steady-state system (3.5) by

(3.7a) $c = (1-b)\cdot l\cdot[f(k)-k\cdot f'(k)] - k\cdot G\cdot l$

(3.7b) $z = f'(k)\cdot\{b\cdot[f(k)-k\cdot f'(k)] + k\cdot G\}.$

(3.7c) $U_c(c,z,l) = f'(k)\cdot U_z(c,z,l)$

(3.7d) $-U_l(c,z,l) = U_c(c,z,l)\cdot[f(k)-k\cdot f'(k)]$

(3.7e) $G = f'(k).$

Equation (3.7e) states that in a steady-state equilibrium with an unfunded, but actuarially fair pension system, the rate of interest must coincide with the (population) growth rate, so that a golden-rule steady-state is attained. This is compatible with a positive contribution rate b only if in the absence of an unfunded pension system the economy would be on a dynamically inefficient "over-capitalized" growth path with $R < G$. Since it was assumed that at the outset an unfunded system with positive contribution rate exists and according to (2.19) b_t has to remain positive forever, this rules out the existence of a steady state with a type b) pension system as long as the initial type a) steady state is dynamically efficient.

3.4 Equilibrium with lump-sum contributions and benefits

Using (2.20), the equilibrium system (3.4) is modified to

(3.8a) $c_t = l_t\cdot w_t - B_t - k_{t+1}\cdot l_{t+1}$

(3.8b) $z_{t+1} = f'(k_{t+1})\cdot k_{t+1}\cdot l_{t+1}\cdot G_{t+1} + B_{t+1}\cdot G_{t+1}$

(3.8c) $U_c(c_t,z_{t+1},l_t) = f'(k_{t+1})\cdot U_z(c_t,z_{t+1},l_t)$

(3.8d) $-U_l(c_t,z_{t+1},l_t) = w_t\cdot U_c(c_t,z_{t+1},l_t)$,

and in the steady-state case to

(3.9a) $c = l\cdot[f(k)-k\cdot f'(k)] - B - k\cdot l$

(3.9b) $z = f'(k)\cdot k\cdot l\cdot G + B\cdot G$

(3.9c) $U_c(c,z,l) = f'(k)\cdot U_z(c,z,l)$

(3.9d) $-U_l(c,z,l) = [f(k)-k\cdot f'(k)]\cdot U_c(c,z,l)$.

4. Pareto Optimal Time Paths of Consumption, Labor, and Capital Accumulation

4.1 Properties

We now consider infinite sequences S of labor-when-young, savings-when-young and consumption-when-old, $(l_t,s_t,z_{t+1})_{t=0,\ldots}$, where the remaining variables c_t, k_t, w_t and R_t are determined by the respective definitions or equilibrium conditions (2.4)-(2.9), and introduce the concept of short-run Pareto efficiency:

Definition 4.1: A sequence of consumption, savings and labor, $S = (l_t,s_t,z_{t+1})_{t=0,\ldots}$ is called short-run Pareto efficient in the interval [T,V] if there is no other feasible sequence $S' = (l_t',s_t',z_{t+1}')_{t=0,\ldots}$ with

a) $U_t(c_t',z_{t+1}',l_t') \geq U_t(c_t,z_{t+1},l_t)$ for $t=T,\ldots,V$,

b) $U_t(c_t',z_{t+1}',l_t') > U_t(c_t,z_{t+1},l_t)$ for at least one t

c) $(s_{T-1},z_T) = (s_{T-1}',z_T')$

d) $(s_V,z_{V+1}) = (s_V',z_{V+1}')$

S is called short-run Pareto efficient if it is short-run Pareto-efficient in every finite interval.[7]

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[7] Here the term "interval" is used somewhat loosely for an uninterrupted sequence of discrete points of time.

Then we have the following theorem:

Theorem 1: The sequence $S = (l_t, s_t, z_{t+1})_{t=0,\ldots}$ with $l_t, c_t, z_{t+1} > 0$ $(t=0,\ldots,\infty)$ is short-run Pareto efficient in the interval $[T,V]$ if and only if:

$$(4.1) \quad \frac{-U_l{}^t}{U_c{}^t} := \frac{-U_l}{U_c}(c_t, z_{t+1}, l_t) = w\left(\frac{s_{t-1}}{l_t}\right) =: w^t \qquad \text{or}$$

$$\frac{-U_l{}^t}{U_c{}^t} < w^t \quad \text{and} \quad l_t = 1 \qquad \text{for } t=T,\ldots,V$$

$$(4.2) \quad \frac{U_c{}^t}{U_z{}^t} = R^{t+1} \qquad \text{for } t=T,\ldots,V-1$$

Proof: Without loss of generality it suffices to show that the theorem is true for T=1, V=2. (The structure of the proof for the general case is exactly identical, only the notation required is a bit more involved.)

a) "only if": According to Definition 4.1 we take the values of (s_0, z_1) and (s_2, z_3) as given and consider a sequence S which is short-run Pareto efficient in the interval $[1,2]$. This means that the variables l_1, l_2, s_1, z_2 are such that they solve the following optimization problem:

$$(4.3) \quad \max_{l_1, l_2, s_1, z_2} U[F(s_1, l_2) - z_2 - s_2, z_3, l_2]$$

$$\text{s.t.} \quad U[F(s_0, l_1) - z_1 - s_1, z_2, l_1] = \underline{U}_1 = \text{const.}$$

For an interior optimum of (4.3), i.e. $l_1, l_2 < 1$, the following necessary first-order conditions have to be fulfilled:[8]

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[8] Note that border solutions with $l_t=0$ or $s_t=0$ or $s_t=F(s_{t-1}, l_t)-z_t$ are excluded by the assumptions on the production and utility functions, (2.3) and (2.10).

$$(4.4) \quad 0 = \lambda \cdot \{U_c^1 \cdot F_L(s_0, l_1) + U_l^1\} = \lambda \cdot \{U_c^1 \cdot w^1 + U_l^1\}$$

$$(4.5) \quad 0 = U_c^2 \cdot F_L(s_1, l_2) + U_l^2 = U_c^2 \cdot w^2 + U_l^2$$

$$(4.6) \quad 0 = U_c^2 \cdot F_K(s_1, l_2) - \lambda \cdot U_c^1 = U_c^2 \cdot R^2 - \lambda \cdot U_c^1$$

$$(4.7) \quad 0 = - U_c^2 + \lambda \cdot U_z^1.$$

Equation (4.5) directly implies (4.1) for t=2. From (4.7) and $U_c>0$ we have $\lambda>0$. Thus (4.4) implies (4.1) for t=1, and (4.6) and (4.7) together imply (4.2) for t=1.

Next we consider boundary solutions and apply the corresponding Kuhn-Tucker conditions: If $l_1=1$, the first equality sign in (4.4) is replaced by ">", and thus the second line of (4.1) is true for t=1. Similarly, $l_2=1$ changes the first equality sign in (4.5) to ">" and implies the second line of (4.1) for t=2.

b) "if": The maximand of the problem (4.3) is quasi-concave, the set of feasible solutions is obviously convex, and the partial derivative of U_2 with respect to z_2 is not zero (due to $U_c>0$) so that the first-order conditions (4.1) and (4.2) are also sufficient for a short-run Pareto efficient allocation,[9] q.e.d.[10]

Remark: The system of equations (4.1), (4.2) does not uniquely determine the values of a Pareto efficient path since for every period t and for given values of the variables s_{t-1}, l_t, z_t it contains only two equations for the three unknowns s_t, l_{t+1} and z_{t+1}. Now suppose that the economic institutions (e.g. factor markets and the pension system) imply that in all periods t a functional relationship between savings-when-young and consumption-when-old holds,

$$(4.8) \quad z_t = g_t(s_{t-1}),$$

---

[9] See TAKAYAMA (1975), p.110f., Theorem 1.E.2 (Arrow-Enthoven).

[10] The Inada properties assumed in (2.3) and (2.10) that marginal utility and marginal products go to infinity when the respective arguments go to zero can be dispensed with easily without losing the assertion of Theorem 1. Only the proof would have to be changed by allowing for additional boundary optima.

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where, of course, the intergenerational budget constraint requires that

(4.9) $g_t(s_{t-1}) + c_t + s_t = F(G_t \cdot s_{t-1}, l_t)$.

Then (4.8) can be used to eliminate z_{t+1}, and assuming that the solution to (4.1) and (4.2) is unique, we obtain a system of two difference equations, which must be filfilled for all Pareto efficient sequences (s_t) and (l_t) - and only for those:

(4.10) $s_t = m(s_{t-1}, l_t)$

(4.11) $l_{t+1} = h(s_{t-1}, l_t)$.

4.2 Applications to Unfunded Pension Systems

From the general result in Theorem 1 we can derive two corollaries which prove the Pareto efficiency of unfunded pension systems in certain special cases:

Corollary 1.1: Suppose that the assumptions of the model stated in Section 2 are fulfilled and labor supply is not an argument in the utility function (i.e. $U_l \equiv 0$). Then - in the absence of any further distortions - an allocation resulting from the existence of an unfunded pension system is short-run Pareto efficient.

Proof: In the equilibrium, $U_l=0$ implies $l_t=1$ and $-U_l/U_c = 0 < w^t$ for every period t so that (4.1) is fulfilled. In the absence of any other distortions (4.2) holds as well. Then we get from Theorem 1b) the assertion.

The result stated in Corollary 1.1 was first proved by BREYER (1989), using the assumption that consumption in both periods of life is a normal good. Since this assumption was not needed here, Corollary 1.1 is a generalization of BREYER's earlier result.

Corollary 1.2: Suppose that the assumptions of the model stated in Section 2 are fulfilled. Then - in the absence of any further distortions - an allocation resulting from the existence of an unfunded pension system with lump-sum contributions is short-run Pareto efficient.

Proof: With lump-sum contributions to the pension system, (4.1) and (4.2) are fulfilled for every period t. So the assertion follows immediately from Theorem 1.

5. Pareto-improving Transitions to a Funded Pension System

The main message from the two corollaries proved in the preceding section is the following: If the way in which the contributions to an unfunded pension system are levied does not distort the labor/leisure choice, then it is impossible to improve intergenerational welfare, as measured by the Pareto criterion, by changing the pension system in a finite number of periods. In other words, if there are any negative welfare consequences of the pension system in the Pareto sense, they must come from static misallocations. These are clearly absent if either labor supply is not "taxed" or the individuals have no preference for leisure.

Is the inverse inference also true? Are static distortions in the labor market sufficient for the existence of a Pareto-improving transition from an unfunded to a funded pension system? Before we can answer this question in general, we formulate the following lemma on the set of steady states corresponding to an unfunded pension system with varying levels of payroll-tax contributions:

Lemma 1: Let $l(G,b)$, $c(G,b)$, $z(G,b)$, $k(G,b)$ denote the steady-state values of the variables when population grows by the constant factor G and there is an unfunded pension system with contributions at the constant payroll-tax rate b ($0 \leq b \leq 1$), i.e. the solution to the system (3.5). Assume further that $R(G,b) > G$ for all feasible b and that $k(G,b)$

is monotone nonincreasing in b. Then U^b = $U[c(G,b),z(G,b),l(G,b)]$ is monotone decreasing in b.[11]

Proof: We write the equations in the system (3.5) as functions of b and differentiate (3.5a) and (3.5b) with respect to b, using

$$(5.1) \quad w'(b) = w'(k)\cdot k'(b) = -k\cdot f''(k)\cdot k'(b),$$

thus we get

$$\begin{aligned}(5.2)\ c'(b) &= (1-b)\cdot l\cdot w'(b) + (1-b)\cdot w\cdot l'(b) - w\cdot l \\ &\quad - l\cdot k'(b)\cdot G - k\cdot l'(b)\cdot G \\ &= -(1-b)\cdot l\cdot k\cdot f''(k)\cdot k'(b) - l\cdot k'(b)\cdot G \\ &\quad + \{(1-b)\cdot[f(k)-k\cdot f'(k)]\}\cdot l'(b) - k\cdot l'(b)\cdot G - w\cdot l \\ &= \{-(1-b)\cdot l\cdot k\cdot f''(k) - G\cdot l\}\cdot k'(b) \\ &\quad + \{(1-b)\cdot f(k) - (1-b)\cdot k\cdot f'(k) - G\cdot k\}\cdot l'(b) - w\cdot l\end{aligned}$$

$$\begin{aligned}(5.3)\ z'(b) &= G\cdot l\cdot k\cdot f''(k)\cdot k'(b) + G\cdot l\cdot f'(k)\cdot k'(b) \\ &\quad + G\cdot k\cdot f'(k)\cdot l'(b) + G\cdot b\cdot w\cdot l'(b) + G\cdot b\cdot l\cdot w'(b) + G\cdot l\cdot w \\ &= G\cdot l\cdot k\cdot f''(k)\cdot k'(b) + G\cdot l\cdot f'(k)\cdot k'(b) \\ &\quad - G\cdot b\cdot l\cdot k\cdot f''(k)\cdot k'(b) + G\cdot k\cdot f'(k)\cdot l'(b) \\ &\quad + G\cdot b\cdot[f(k) - k\cdot f'(k)]\cdot l'(b) + G\cdot w\cdot l \\ &= G\cdot\{(1-b)\cdot l\cdot k\cdot f''(k) + f'(k)\cdot l\}\cdot k'(b) \\ &\quad + G\cdot\{b\cdot f(k) + (1-b)\cdot k\cdot f'(k)\}\cdot l'(b) + G\cdot w\cdot l\end{aligned}$$

Differentiating steady-state utility $U[c(b),z(b),l(b)]$ with respect to b yields

$$(5.4) \quad \frac{dU}{db} = U_c\cdot c'(b) + U_z\cdot z'(b) + U_l\cdot l'(b)$$

and thus, using (3.5c) and (3.5d):

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[11] To avoid notational clutter, the argument G will be supressed from the functions c(.) etc. since G will not be varied.

$$(5.5)\quad \frac{f'(k)}{U_c}\cdot\frac{dU}{db} = f'(k)\cdot c'(b) + z'(b)$$
$$- \{f(k) - k\cdot f'(k)\}\cdot[G\cdot b + (1-b)\cdot f'(k)]\cdot l'(b)$$
$$= k'(b)\cdot\{-(1-b)\cdot l\cdot k\cdot f'(k)\cdot f''(k) - G\cdot l\cdot f'(k)$$
$$+ G\cdot(1-b)\cdot l\cdot k\cdot f''(k) + G\cdot l\cdot f'(k)\}$$
$$+ l'(b)\cdot\{(1-b)\cdot f'(k)\cdot f(k) - (1-b)\cdot k\cdot[f'(k)]^2$$
$$- G\cdot k\cdot f'(k) + G\cdot b\cdot f(k) + G\cdot(1-b)\cdot k\cdot f'(k) - G\cdot b\cdot f(k)$$
$$+ G\cdot b\cdot k\cdot f'(k) - (1-b)\cdot f(k)\cdot f'(k) + (1-b)\cdot k\cdot[f'(k)]^2\}$$
$$+ l\cdot w\cdot[G-f'(k)]$$
$$= \{l\cdot k\cdot(1-b)\cdot f''(k)\cdot[G-f'(k)]\}\cdot k'(b) + l\cdot w\cdot[G-f'(k)].$$

As we assumed that $R = f'(k) > G$ and k is a decreasing function of b, the first term on the RHS of (5.5) is clearly negative, and so is the second term. This concludes the proof that steady-state utility is a decreasing function of b.

The assumptions involved in Lemma 1 are not very strong. That the interest factor exceeds the population growth factor guarantees the dynamic efficiency of the time paths under consideration, given that there are no static inefficiencies. Capital intensity k is the ratio of savings s and labor supply l. Ignoring income effects for a moment, both s and l will fall when b is raised. Our assumption now states that savings fall by a larger percentage than labor supply so that the ratio k falls as well, which is true for most utility and production functions.[12]

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[12] Suppose, e.g., that the production function (2.4) is of the Cobb-Douglas type $Y_t = K_t^{\beta}\cdot L_t^{1-\beta}$ and that the utility function (2.11) is log-linear: $U(c_t, z_{t+1}, l_t) = \alpha\cdot\ln c_t + \gamma\cdot\ln z_{t+1} + \ln(1-l_t)$. Then the solution of (3.5) yields a steady-state value of k of

$$k(b) = \left\{\frac{\gamma\beta(1-\beta)(1-b)}{(\alpha+\gamma)\beta G + \alpha(1-\beta)bG}\right\}^{1/(1-\beta)}.$$

Equipped with this lemma, we can now state the main theorem of this section:

Theorem 2: Let $S(b^\circ)$ be the steady state resulting from constant population growth with growth factor G and an unfunded pension system with constant payroll tax $b^\circ>0$. Assume that for all steady states corresponding to such systems with tax rate b ($0\leq b\leq b^\circ$)

1. $R(b) > G$ and
2. $k(b)$ is monotone nonincreasing in b.

Assume further that $U_1<0$ for all feasible vectors (c,z,l). Then there is a time path S' which constitutes a Pareto-improving transition from the unfunded to a funded system in finite time, i.e. an interval $[0,T]$ such that

- at time t=0 all variables have the same values as in $S(b^\circ)$,
- at time t=T all variables have the same values as in $S(0)$,
- for all intermediate periods t: $U_t \geq U(b^\circ)$.

Proof: From Theorem 1 we know that $S(b^\circ)$ is not short-run Pareto efficient. Then this is true for the interval [1,2], and for given values of (s_0,z_1) and (s_2,z_3) equal to $[s(b^\circ),z(b^\circ)]$ like in $S(b^\circ)$ there exist feasible values of (l_1,l_2,s_1,z_2) such that $U_1 = U(b^\circ)$ and $U_2 > U(b^\circ)$. From the continuity of U there is then a $\delta_1>0$ and a vector (l_1,l_2,s_1,z_2) such that $U_2(c_2,l_2,z_3) = U(b^\circ)$ and (s_2,z_3) as in the steady state $S(b^\circ-\delta_1)$ corresponding to a payroll tax with rate $b = b^\circ-\delta_1$. From Lemma 1 we know that $U(b^\circ-\delta_1) > U(b^\circ)$.

Suppose that $b^\circ-\delta_1 > 0$. Then we know from Theorem 1 that the steady state $S(b^\circ-\delta_1)$ is not short-run Pareto-efficient. Hence this is true for the interval [3,4], and

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and clearly $k(b)$ is decreasing in b because when b rises, the numerator shrinks and the denominator rises.

for given values of (s_2,z_3) and (s_4,z_5) equal to $[s(b^\circ-\delta_1), z(b^\circ-\delta_1)]$ there exist feasible values of (l_3,l_4,s_3,z_4) such that $U_3 = U(b^\circ-\delta_1)$ and $U_4 > U(b^\circ-\delta_1)$. From the continuity of U there is then a $\delta_2>0$ and a vector (l_3,l_4,s_3,z_4) such that $U_3(c_3,l_3,z_4) = U_4(c_4,l_4,z_5) = U(b^\circ)$ and (s_4,z_5) as in the steady state $S(b^\circ-\delta_1-\delta_2)$ corresponding to a payroll tax with rate $b = b^\circ-\delta_1-\delta_2$. From Lemma 1 we know that $U(b^\circ-\delta_1-\delta_2) > U(b^\circ-\delta_1)$.

For every $b\in[0,b^\circ]$, the maximum δ_j determined in this way can be expressed as a function $\delta(b)$. From $U(b) > U(b^\circ)$ for each $b<b^\circ$ we know that $\delta(b)>0$ for all $b\in[0,b^\circ-\delta_1]$, so the values of $\delta(b)$ are bounded away from zero for all $b\in[0,b^\circ]$. Thus after a finite number J of steps each involving two periods we have

$$(5.6) \qquad b_J = b^\circ - \sum_{j=1}^{J} \delta_j \leq \delta(b_J),$$

so that δ_{J+1} can be chosen equal to b_J. But this concludes the proof that the pay-as-you-go pension system can be abolished within a finite number of periods in a Pareto-improving way.

Theorem 2 confirms for the closed-economy case the conjecture made by HOMBURG/RICHTER (1990) that the avoidance of the deadweight loss implied by an unfunded pension system would by itself suffice to build up the fund required to replace the unfunded by a funded system.

6. Concluding remarks

In this paper we considered a standard overlapping-generations model to study the Pareto efficiency of unfunded pension systems. Our first result was that under quite general circumstances, intergenerational Pareto efficiency is violated unless

- either there is no utility attached to leisure
- or the contributions are levied in a lump-sum fashion.

In the real world contributions are typically levied as payroll taxes with fixed rates over time. Economic intuition suggests that in order to overcome the adverse effects of this arrangement on labor supply, it should be changed either to a system with actuarially fair benefits or with lump-sum contributions. However, it turns out that the first suggestion is not a viable strategy if the economy is on a dynamically efficient steady state growth path because the contribution rate would have to rise beyond limit to maintain actuarial fairness.

On the other hand, we were able to prove that under certain conditions the conversion to lump-sum contributions is in fact sufficient not only to achieve an intergenerational Pareto improvement but also to reduce the contribution rates in finite time to zero and thus to replace the unfunded by a funded pension system. This is true not only for the small open economy as established by HOMBURG (1990) but for the closed economy as well. Thus our analysis not only generalizes an earlier result due to BREYER (1989) but also proves a conjecture made by HOMBURG/RICHTER (1990). It also contradicts earlier results derived from simulation studies which seemed to suggest that <u>any</u> conversion from an unfunded to a funded pension system even with variable labor supply was bound to hurt at least one of the transition generations.

A further question, which was not definitely answered in this paper, refers to the implementation of such a Pareto-improving transition. So far we only proved that under certain circumstances such a path exists, but it remains open what institutional design can ascertain that the voluntary transactions of utility-maximizing agents in a market economy with overlapping generations can bring it about. Thus there is room for important further research.

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