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## Lay-off restraints, employment subsidies, and the demand for labour

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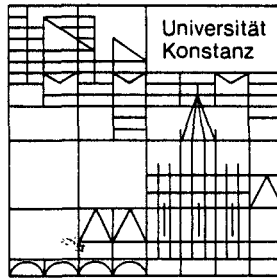
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**Fakultät für  
Wirtschaftswissenschaften  
und Statistik**

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Horst Siebert und  
Ngo Van Long

**Lay-Off Restraints,  
Employment Subsidies,  
and the Demand for Labour**

Diskussionsbeiträge

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LAY-OFF RESTRAINTS, EMPLOYMENT SUBSIDIES,  
AND THE DEMAND FOR LABOUR

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Serie A: Volkswirtschaftliche Beiträge

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Serie C: Betriebswirtschaftliche Beiträge

## LAY-OFF RESTRAINTS, EMPLOYMENT SUBSIDIES, AND THE DEMAND FOR LABOUR

The paper introduces the concept of a firm's normal employment level as a weighted average of past employment levels and it analyzes the impact of an incentive scheme in which a firm receives a reward (or pays a penalty) when it deviates above (below) its normal employment level. The result is that such an institutional setting may imply a cyclical demand in labour. Thus, institutional arrangements may be responsible for business cycles.

### 1. INTRODUCTION

In an earlier paper, we investigated the firm's demand for labour when there is a quantity constraint on the rate of lay-off and showed that the observed pattern of labour demand may be cyclical with a leading indicator of the underlying cycle being the demand for the final product. (See Long and Siebert, 1983.) The present paper is intended as a comparison to our earlier paper: We will show that financial penalties (or rewards) imposed by the government on the firm for its deviation from its "normal employment level" may cause the firm to change its demand for labour even in a steady state and, in some cases, to generate a business cycle even if there is no underlying cycle in the demand for the final product.

### 2. THE MODEL AND THE NECESSARY CONDITIONS

#### 2.1 The Economic Setting

Let  $L(t)$  denote the number of workers actually employed by the firm at time  $t$ . The government defines the firm's normal employment level as a weighted average of past levels of employment:

$$(1) \quad A(t) = m \int_{-\infty}^t L(s) e^{-m(t-s)} ds \quad m > 0 \quad (1)$$

$$\text{Hence: } \dot{A}(t) = \frac{dA}{dt} = m [L(t) - A(t)] \quad (1')$$

The parameter  $m$  reflects the relative weight given to more recent employment levels. (It is easy to verify that if  $L(s)$  is a constant, say  $\bar{L}$ , then  $A(t) = \bar{L}$ ).

If  $L(t) > A(t)$ , the firm receives a financial reward:

$$R(t) = f(E(t)) \quad (E(t) \geq 0) \quad (2)$$

where  $E(t)$  is the excess of  $L(t)$  over  $A(t)$ . It is assumed that  $f(0) = 0$ ,  $f'(E) \geq 0$  for  $E \geq 0$ ,  $f'(0) = \alpha \geq 0$ .

If  $L(t) < A(t)$ , the firm must pay a penalty (or tax)  $T$ :

$$T(t) = g(D(t)) \quad (D(t) \geq 0) \quad (3)$$

where  $D(t)$  is the short fall (deficiency) of  $L(t)$  below the normal level  $A(t)$ . It is assumed that  $g(0) = 0$ ,  $g'(D) \geq 0$  for  $D \geq 0$ , and  $g'(0) = \beta \geq 0$ .

The firm's objective is to maximize the discounted value of the stream of cash flow:

$$\text{Max}_{L(t)} \int_0^{\infty} e^{-rt} [P(t)F(L(t)) - W(t)L(t) + R(t) - T(t)] dt \quad (4)$$

where  $F(L)$  is a strictly concave production function, and  $P(t) > 0$ ,  $W(t) > 0$  represent the price of the product and the wage rate.

In (4), not both  $R(t)$  and  $T(t)$  can be positive at any given time  $t$ , because by definition either  $E(t) \geq 0$  or  $D(t) \geq 0$ . In this problem it is assumed that the firm can choose its  $L(t)$  freely. Thus  $L(t)$  is the control variable and  $A(t)$  is the state variable. The price of the product  $P(t)$ , and the wage rate  $W(t)$ , are both exogenous.

## 2.2 Derivation of the necessary Conditions

The problem we posed above, has some asymmetric properties. For example, if  $f(E) = \alpha E$  and  $g(D) = \beta D$ , then the "net reward" function has a "kink" at  $L - A = 0$ , if  $\beta \neq \alpha$ ; see Figure 1.

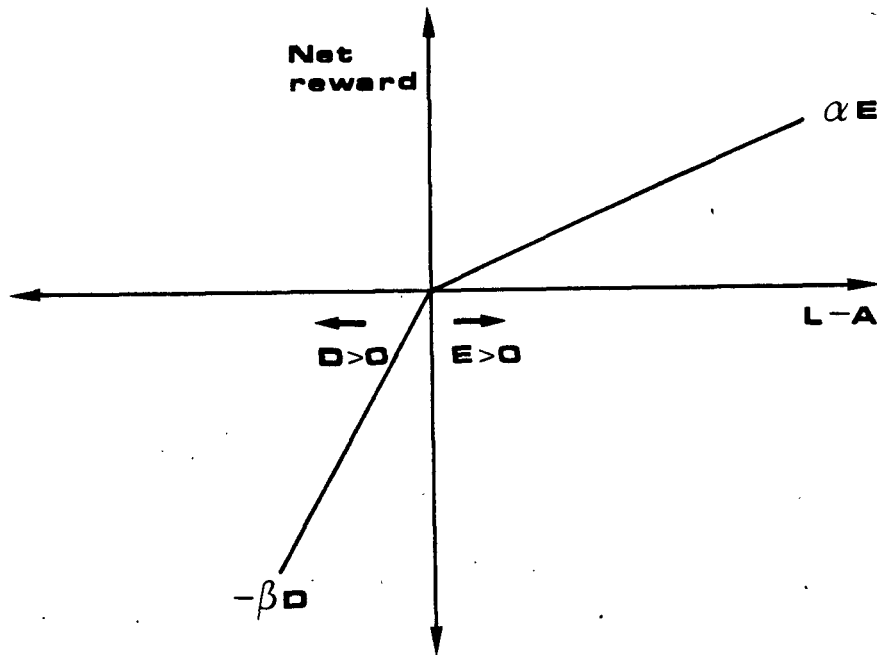


Figure 1

In order to take into account the "kink" in the net reward function, we use a transformation of variables. We keep  $A(t)$  as the state variable, and treat  $D(t)$  and  $E(t)$  as control variables, thus eliminating  $L(t)$ , because

$$L(t) = A(t) + E(t), \quad \text{if } E \geq 0 \quad (5)$$

or

$$L(t) = A(t) - D(t), \quad \text{if } D \geq 0 \quad (6)$$

We can express (5) and (6) compactly as:

$$L(t) = A(t) + E(t) - D(t); \quad \begin{array}{l} E(t) D(t) = 0, \\ D(t) \geq 0, \\ E(t) \geq 0 \end{array} \quad (7)$$

(In other words, we let the firm choose  $D(t)$  and  $E(t)$ , subject to the constraint that at any point of time either  $D(t) \geq 0$  or  $E(t) \geq 0$  and that not both can be positive at the same time (technically,  $D(t) E(t) = 0$ ,  $E(t) \geq 0$ ,  $D(t) \geq 0$ ), so that the transformed problem is identical to the original problem.)

Formally, the transformed problem is:

$$\begin{aligned} \text{Max}_{D(t), E(t)} \int_0^{\infty} e^{-rt} \{ & P(t)F[A(t)+E(t)-D(t)] - W(t)[A(t)+E(t)-D(t)] - \\ & + f(E(t)) - g(D(t)) \} dt \end{aligned} \quad (8)$$

subject to:

$$D(t) \geq 0 \quad (9)$$

$$E(t) \geq 0 \quad (10)$$

$$D(t)E(t) = 0 \quad (11)$$

and, from (1') and (7)

$$\dot{A}(t) = m [E(t) - D(t)] \quad (12)$$

In addition, we must add the condition

$$A(t) + E(t) - D(t) \geq 0 \quad (13)$$

to reflect the fact that  $L(t) \geq 0$ . However, since  $E(t)$  is non-negative by (10), we may simplify (13) as:

$$A(t) - D(t) \geq 0 \quad (13')$$

Technically, the constraint (11) may create certain problems<sup>1</sup> because it gives rise to an irregular constraint set. However, it will be seen that if  $f(E) - g(D)$  is concave (possibly linear) and if  $f'(0) \leq g'(0)$  then the constraint (11) is redundant.

The Lagrangian for problem (8) is

$$\begin{aligned} \mathcal{L} = & PF(A+E-D) - W(A+E-D) + f(E) - g(D) + \psi m[E-D] + \\ & + \lambda_1 D + \lambda_2 E + \lambda_3 DE + \lambda_4 (A-D+E) \end{aligned} \quad (14)$$

The variable  $\psi(t)$  is the shadow price of the normal level of employment. It will be seen that this shadow price is negative because for any given  $L(t)$ , the higher is  $A(t)$ , the lower will be the subsidy received (the subsidy being an increasing function of  $L-A$ ); or the higher will be the penalty (in the case  $A-L > 0$ ). The multipliers  $\lambda_1$ ,  $\lambda_2$ ,  $\lambda_3$  and  $\lambda_4$  are associated with constraints (9), (10), (11) and (13'). The multipliers  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_4$  are all non-negative, but the multiplier  $\lambda_3$  is indeterminate in sign, because (11) is an equality constraint.

The necessary conditions are:



$$\frac{\partial \mathcal{L}}{\partial E} = PF' - W + f'(E) + \psi m + \lambda_2 + D\lambda_3 + \lambda_4 = 0 \quad (15)$$

$$\frac{\partial \mathcal{L}}{\partial D} = -PF' + W - g'(D) - \psi m + \lambda_1 + E\lambda_3 - \lambda_4 = 0 \quad (16)$$

$$\dot{\psi} = r\psi - \frac{\partial \mathcal{L}}{\partial A} = r\psi - PF' + W - \lambda_4 \quad (17)$$

The interpretations of conditions (15) and (16) will be facilitated if we assume for the time being that  $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0$ . Under this assumption, condition (15) says that any increase in the employment level above the average level,  $A$ , should be made until the marginal value product of labour plus the additional employment subsidy just equals the direct wage cost ( $W$ ) plus the cost of raising the average level of employment ( $A$ ), as the increase in  $A$  will reduce future subsidies (or increase future penalties):

$$PF' + f'(E) = W + |\psi m| \quad (18)$$

(Recall that  $\psi$  is negative.)

Equation (16) has a similar interpretation: if the firm finds it optimal to reduce the labour force below the normal employment level,  $A$ , then this reduction should be carried out until the marginal gain ( $W + |\psi m|$ , i.e. wage saving plus the desirable effect of a fall in  $A$ ) just equals the marginal cost, namely the loss of output and the penalty for the shortfall in employment below its average normal level, i.e.  $PF' + g'(D)$ .

### 3. ANALYSIS: THE CASE OF HIGH PENALTY AND LOW SUBSIDY

In this section, we analyse the case where the penalty for employing less than the normal level is higher than the subsidy for employing more than normal level. For example, if  $f(E) = \alpha E$  and  $g(D) = \beta D$ , we assume that

$$\beta > \alpha$$

More generally, we assume that  $f(E)$  is concave or linear in  $E$ , with  $f'(0) = \alpha < W$ , and  $g(D)$  is convex or linear in  $D$ , with  $g'(0) = \beta > \alpha$ ; see Figure 2.

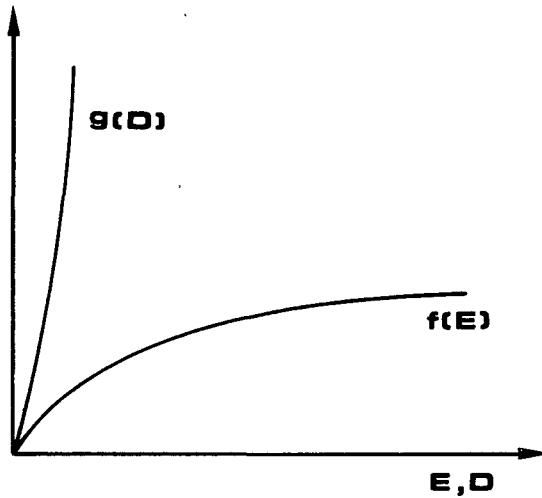


Figure 2

We will concentrate on the case where  $P$  and  $W$  are positive constants, so that a phase diagram can be drawn.

Let us investigate the region where  $E > 0$  and  $D = 0$ . Since  $E > 0$ ,  $\lambda_2 = 0$ . Also  $D = 0$  implies that (13) holds with strict inequality, hence  $\lambda_4 = 0$ . Equation (15) thus becomes:

$$PF'(A+E) - W + f'(E) = -\psi m \quad (19)$$

From (19),  $E$  is a function of  $\psi$  and  $A$ , with derivatives

$$\frac{\partial E}{\partial \psi} = \frac{-m}{PF'' + f''} > 0 \quad (20)$$

$$\frac{\partial E}{\partial A} = \frac{-PF''}{PF'' + f''} < 0 \quad (21)$$

Therefore the curve along which  $E = \text{constant}$  has the slope:

$$\left. \frac{d\psi}{dA} \right|_{E \text{ const.}} = - \frac{\partial E / \partial A}{\partial E / \partial \psi} = \frac{-PF''}{m} > 0 \quad (22)$$

In particular, the region where  $E > 0$  is separated from the region where  $E = 0$  by the curve  $\psi_1(A)$  which satisfies the equation

$$\psi = [W - PF'(A) - f'(0)] m^{-1} \quad (23)$$

This curve is depicted in Figure 3.

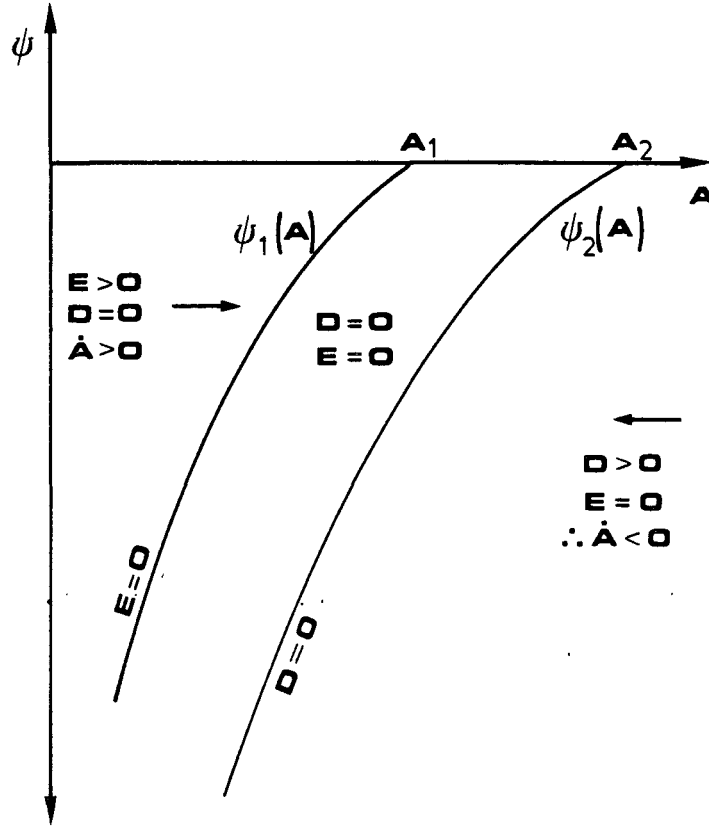


Figure 3

When  $\psi = 0$ , the curve  $\psi_1(A)$  cuts the horizontal axis at  $A_1$ , where

$$PF'(A_1) = W - f'(0) > 0 \quad (24)$$

To the left of this curve,  $E$  is positive implying that  $\dot{A} > 0$ .

Next, we investigate the region where  $D > 0$  and  $E = 0$ . Since  $D > 0$ ,  $\lambda_1 = 0$ . For a sufficiently small  $D$ , (13) will hold with strict inequality and equation (16) becomes

$$PF'(A-D) - w + g'(D) = -\psi m \quad (25)$$

From (25),  $D$  is a function of  $\psi$  and  $A$ , with derivatives

$$\frac{\partial D}{\partial \psi} = \frac{-m}{-PF'' + g''(D)} < 0 \quad (26)$$

$$\frac{\partial D}{\partial A} = \frac{-PF''}{-PF'' + g''(D)} > 0 \quad (27)$$

Therefore the curve along which  $D = \text{constant}$  has the slope

$$\left. \frac{d\psi}{dA} \right|_{D \text{ const.}} = \frac{-\partial D / \partial A}{\partial D / \partial \psi} = \frac{-PF''}{m} > 0 \quad (28)$$

In particular, the region where  $D > 0$  is separated from the region where  $D = 0$  by the curve  $\psi_2(A)$  which satisfies the equation

$$\psi = [W - PF'(A) - g'(0)] m^{-1} \quad (29)$$

This curve is also depicted in Figure 3. It lies below the  $\psi_1(A)$  curve because, by assumption,  $g'(0) > f'(0)$ . Given any  $A$ , the vertical distance between the two curves is  $[g'(0) - f'(0)] m^{-1}$ . When  $\psi = 0$ , the curve  $\psi_2(A)$  cuts the horizontal axis at  $A_2$ , where  $A_2$  is the solution of

$$PF'(A_2) = W - g'(0) > 0 \quad (30)$$

In the region between the two curves  $\psi_1(A)$  and  $\psi_2(A)$ , both  $E$  and  $D$  are zero.

Let us turn our attention to the locus of points where  $\dot{\psi} = 0$ . As we remarked earlier, for  $D = 0$  or  $D$  sufficiently small, the constraint (13') holds with strict inequality, implying  $\lambda_4 = 0$ . Equation (17) becomes

$$\dot{\psi} = r\psi - PF'(A+E-D) + W \quad (31)$$

In the region where  $E > 0$  and  $D = 0$ , equation (31) becomes

$$\dot{\psi} = r\psi - PF'[A+E(\psi, A)] + W \equiv M_1(A, \psi) \quad (32)$$

where from (32) and (21)

$$\frac{\partial M_1}{\partial A} \equiv \frac{\partial \dot{\psi}}{\partial A} = (-PF'') \left[ 1 + \frac{\partial E}{\partial A} \right] = (-PF'') \frac{f''}{(PF'' + f'')} \geq 0 \quad (33)$$

and from (32) and (20)

$$\frac{\partial M_1}{\partial \psi} \equiv \frac{\partial \dot{\psi}}{\partial \psi} = r - (PF'') \frac{\partial E}{\partial \psi} > 0 \quad (34)$$

Therefore the curve  $M_1(A, \psi) = 0$  has the slope

$$\left. \frac{d\psi}{dA} \right|_{M_1=0} = \frac{-\partial M_1 / \partial A}{\partial M_1 / \partial \psi} = \frac{-f''}{(m+r) + (rf''/PF'')} \leq 0 \quad (35)$$

In the region situated between the two curves  $\psi_1(A)$  and  $\psi_2(A)$ , both D and E are zero, therefore

$$\dot{\psi} = r\psi - PF'(A) - W \equiv M_3(A, \psi) \quad (36)$$

The locus of  $\dot{\psi} = 0$  in this region has the slope

$$\left. \frac{d\psi}{dA} \right|_{M_3=0} = - \frac{\partial M_3 / \partial A}{\partial M_3 / \partial \psi} = \frac{PF''}{r} < 0 \quad (37)$$

From (32) and (36), the curve  $M_1 = 0$  meets the curve  $M_3 = 0$  at the point  $(A^*, \psi^*)$  which is the solution of the two equations

$$PF'(A^*) - W + f'(0) = -m\psi^* \quad (38)$$

$$r\psi^* - PF'(A^*) + W = 0 \quad (39)$$

i.e.

$$\psi^* = - \frac{f'(0)}{r+m} < 0 \quad (40)$$

$$F'(A^*) = \left\{ \frac{W - [rf'(0)/(r+m)]}{p} \right\} > 0 \quad (41)$$

Finally, in the region where  $D > 0$  and  $E = 0$ , equation (31) takes the form

$$\dot{\psi} = r\psi - PF'[A-D(\psi, A)] + W \equiv M_2(A, \psi) \quad (42)$$

with derivatives

$$\frac{\partial M_2}{\partial A} = \frac{\partial \dot{\psi}}{\partial A} = (-PF'') \left( 1 - \frac{\partial D}{\partial A} \right) = (-PF'') \left[ \frac{g''(D)}{-PF'' + g''} \right] \geq 0 \quad (43)$$

$$\frac{\partial M_2}{\partial \psi} = \frac{\partial \dot{\psi}}{\partial \psi} = r + (PF'') \frac{\partial D}{\partial \psi} > 0 \quad (44)$$

Therefore the curve  $M_2(A, \psi) = 0$  has the slope

$$\left. \frac{d\psi}{dA} \right|_{M_2=0} = \frac{-\partial M_2 / \partial A}{\partial M_2 / \partial \psi} \leq 0 \quad (45)$$

The curve  $M_2 = 0$  meets the curve  $M_3 = 0$  at the point  $(A^{**}, \psi^{**})$  which is the solution of the pair of equations:

$$PF'(A^{**}) - W + g'(0) = -m\psi^{**} \quad (46)$$

$$r\psi^{**} - PF'(A^{**}) + W = 0 \quad (47)$$

i.e.

$$\psi^{**} = \frac{-g'(0)}{r+m} < 0 \quad (48)$$

$$F'(A^{**}) = \frac{W - [rg'(0)/(r+m)]}{p} > 0 \quad (49)$$

Figures 4A and 4B are the phase diagrams.

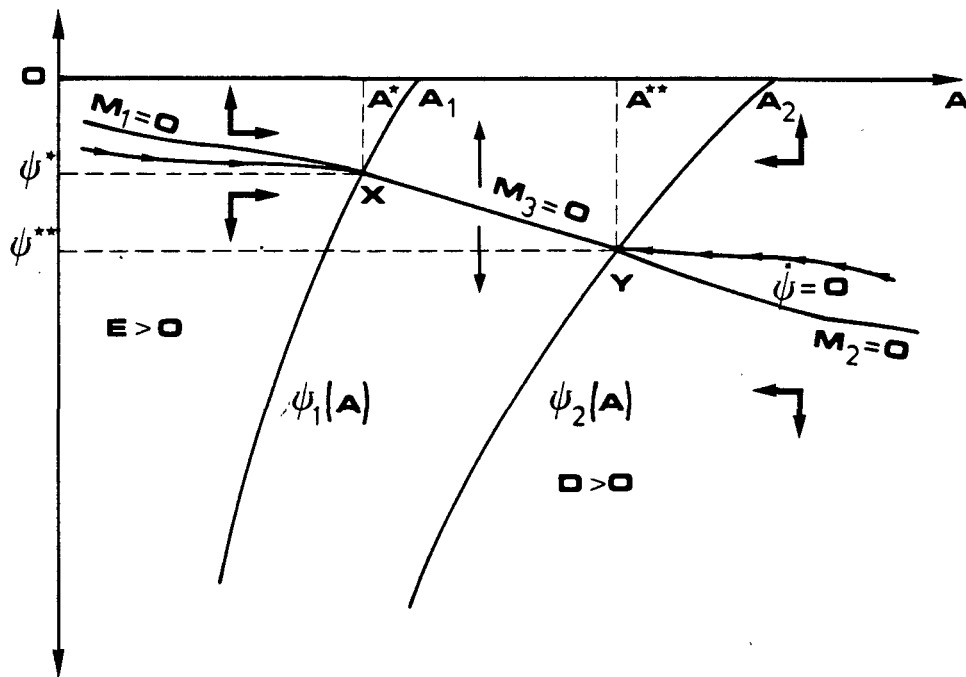


Figure 4A

In Figure 4A, the stable branches of the saddle points are negatively sloped, because (35) and (45) hold with strict inequality. In Figure 4B, the linearity of  $g(D)$  and  $f(E)$  implies that (35) and (45) hold with equality, so that the curves  $M_1 = 0$  and  $M_2 = 0$  are both horizontal.

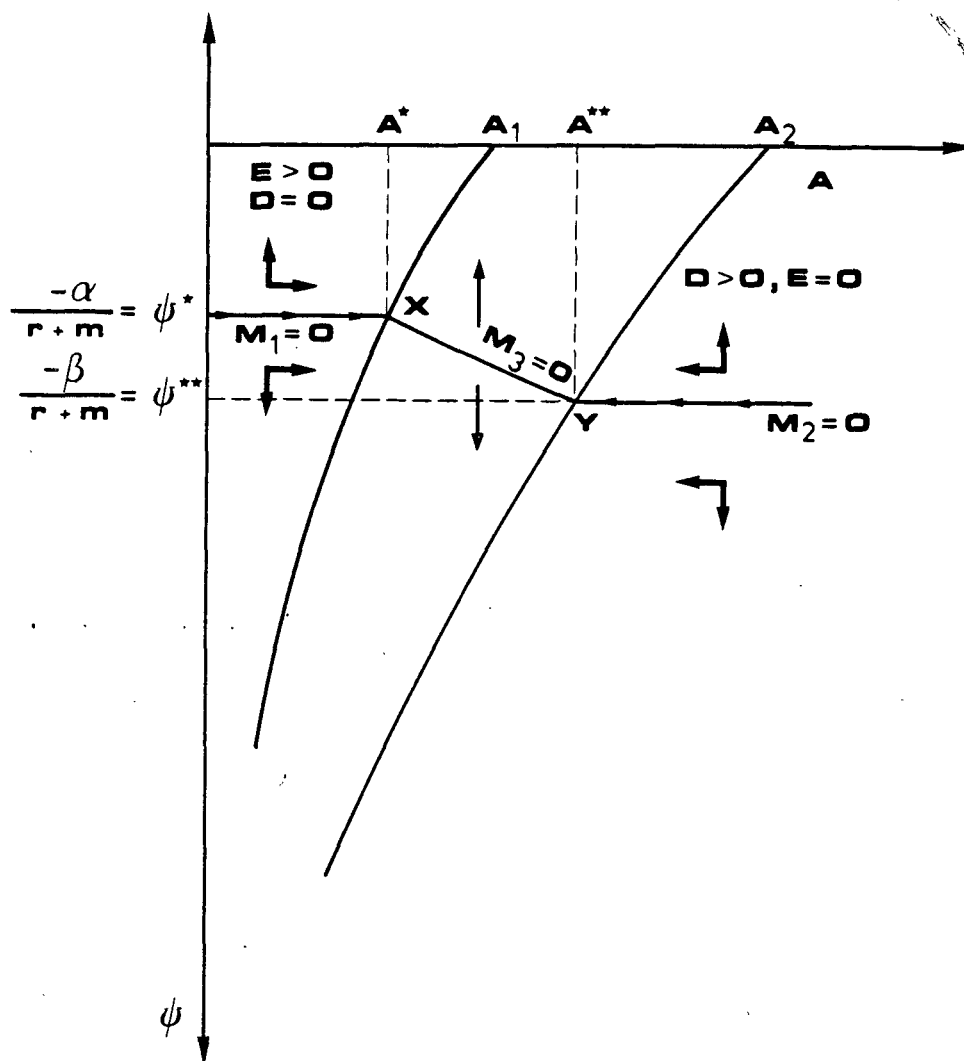


Figure 4B

From Figure 4, if the initial value of  $A$ ,  $A(0)$ , is less than  $A^*$ , then the firm will increase its employment level until it reaches the level  $A^*$ . The associated shadow price converges to  $\psi^*$  (negative). The movement of the system is the stable branch of a saddle, converging to the point  $X = (A^*, \psi^*)$ .

If the initial value of  $A$  is greater than  $A^{**}$ , then the firm will reduce its employment level until it reaches the level  $A^{**}$ . The shadow price converges to  $\psi^{**}$ . The system converges to the stationary

equilibrium at  $Y = (A^{**}, \psi^{**})$ .

If the initial value of  $A$  is less than  $A^{**}$  and greater than  $A^*$ , then the firm will maintain its present employment level. Any point along the curve  $XY$  is an equilibrium point.

Let  $\hat{A}$  be the employment level under laissez-faire, that is

$$PF'(\hat{A}) = W$$

From (39) and (41),

$$\hat{A} < A^* < A^{**}$$

Thus the effect of the tax subsidy scheme is to increase the level of employment in the steady state, provided  $g'(0) > f'(0)$ .

It is interesting to note that both  $A^*$  and  $A^{**}$  are increasing functions of  $r$  and decreasing functions of  $m$ . Intuitively, this makes sense in view of the following points:

(i)  $m$  may be interpreted as the rate of depreciation of  $A$ . Since  $A$  is a "bad stock" (its shadow price being negative), the higher is the rate of depreciation  $m$ , the less concern the firm will have about the effect of its current employment decision on  $A$ . As  $m$  tends to infinity,  $A^*$  and  $A^{**}$  tend to  $\hat{A}$ : "only the present matters" in this polar case.

(ii)  $r$  is the rate of interest (the rate of discounting future cash flow). If "capital" is a "good stock", a higher  $r$  will lead to a smaller steady state stock of capital. But  $A$  is a "bad stock". Therefore a higher  $r$  will cause the firm to be less concerned about the effects of its current employment decision on the future stock of  $A$ . This leads to a higher steady state level of  $A$ . (This is quite similar to the "pollution problem": the higher is the rate of discount, the higher is the steady state level of pollution).

Proposition 1: If the subsidy rate,  $f'(0)$ , is less than the penalty rate,  $g'(0)$ , then the firm's employment level in the long run depends on its initial normal employment level,  $A(0)$ .

A special case:  $g'(0) > 0$  and  $f(E) = 0$  (positive penalty for reducing the work force, and zero subsidy for increasing the work force).



In this case, from (40) and (41),

$$\psi^* = 0 \quad (50)$$

$$F'(A^*) = W/P \quad (51)$$

so that  $A^*$  coincide with  $A_1$ . The locus of  $M_1(\psi, A) = 0$  is horizontal because  $f'' = f' = 0$ . The phase diagram is Figure 5. For  $A(0) < A_1$ , the saddle path is along the horizontal axis, and  $\psi(t) = 0$  identically.

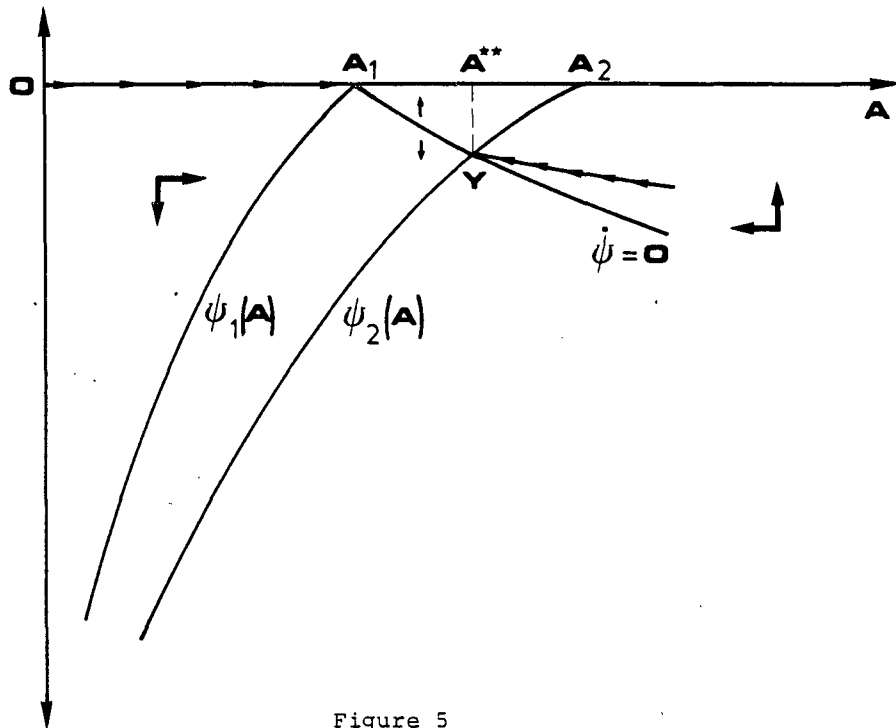


Figure 5

Proposition 2: If there is a positive penalty from employing at a level lower than the normal (average) employment level, and no reward for increasing employment above its normal level, then

(a) If the initial average level of employment is low ( $A(0) < A_1$ ), then the firm will approach the laissez-faire employment level  $A^*$ , characterized by

$$PF'(A^*) = W.$$

b) If the initial average level of employment is higher than  $A^{**}$ , where

$$PF'(A^{**}) = W - [rg'(0)/(r+m)]$$

then the firm will eventually reduce its employment level to  $A^{**}$ , which is greater than  $A^*$ .

c) If  $A(0)$  is between  $A^*$  and  $A^{**}$ , then the firm will maintain that employment level.

#### 4. THE CASE OF HIGH SUBSIDY AND LOW PENALTY

We now reverse our assumption, so that

$$0 \leq g'(0) < f'(0) \quad (52)$$

(We still require that  $f(E)$  be linear or concave, and  $g(D)$  be linear or convex.) For the time being let us temporarily ignore constraint (11). As before, let  $\psi_2(A)$  be the curve which separates the region  $D > 0$  from the region  $D = 0$ , and let  $\psi_1(A)$  be the curve separating the region  $E > 0$  from the region  $E = 0$ . For any given  $A$ , the  $\psi_2(A)$  curve now lies above the  $\psi_1(A)$  curve, because by (52),  $f'(0) > g'(0)$ . Figure 6 illustrates the phase diagram. Clearly, in the region between  $\psi_1(A)$  and  $\psi_2(A)$ , both  $D$  and  $E$  may be positive. But, because of constraint (11), they cannot be positive at the same time. Let us concentrate on the case where  $g(D) = \beta D$  and  $f(E) = \alpha E$ , with  $\alpha > \beta > 0$ . In this case, the locus  $M_1 = 0$  is horizontal (in the region where  $E > 0$ ), the locus  $M_2 = 0$  is also horizontal (in the region where  $D > 0$ ).

In the region YVXZ (see Figure 6) the direction of the movement may be North-East (if  $E > 0$  and  $D = 0$ ) or South-West (if the firm chooses to have  $E = 0$  and  $D > 0$ ). Thus we have the possibility of cyclical behaviour, as illustrated by the closed curve (with arrows) inside this region. The economic explanation for this possibility lies in the fact that the firm finds it optimal to exploit the asymmetric incentive scheme: at first, it increases its level of employment (to get the subsidy), but as the subsidy vanishes if there is no change in employment level, the firm will reduce its employment level (thus incurring a penalty) in order to increase it again (and reaping the subsidy again). Thus  $\psi$  will oscillate between  $\psi^*$  and  $\psi^{**}$ , implying that the marginal revenue product will also oscillate. When  $E > 0$  and  $D = 0$ , the marginal revenue product varies between

$$PF' = W - \alpha - m\psi^* - \lambda_4 = W - \frac{r\alpha}{(r+m)} - \lambda_4 \equiv W_1 \quad (53)$$

and

$$PF' = W - \alpha - m\psi^{**} - \lambda_4 = W - \frac{[r\alpha + m(\alpha - \beta)]}{(r+m)} - \lambda_4 \equiv W_2 \quad (54)$$

When  $D > 0$  and  $E = 0$ , the marginal revenue product will oscillate between

$$PF' = W - \beta - m\psi^{**} + \lambda_4 = W - \frac{r\beta}{r+m} \equiv W_3 \quad (55)$$

and

$$PF' = W - \beta - m\psi^* = W - \frac{[r\beta + m(\beta - \alpha)]}{r+m} \equiv W_4 \quad (56)$$

where  $W_1, W_2, W_3$  and  $W_4$  are "shadow wage rates" at switching points.

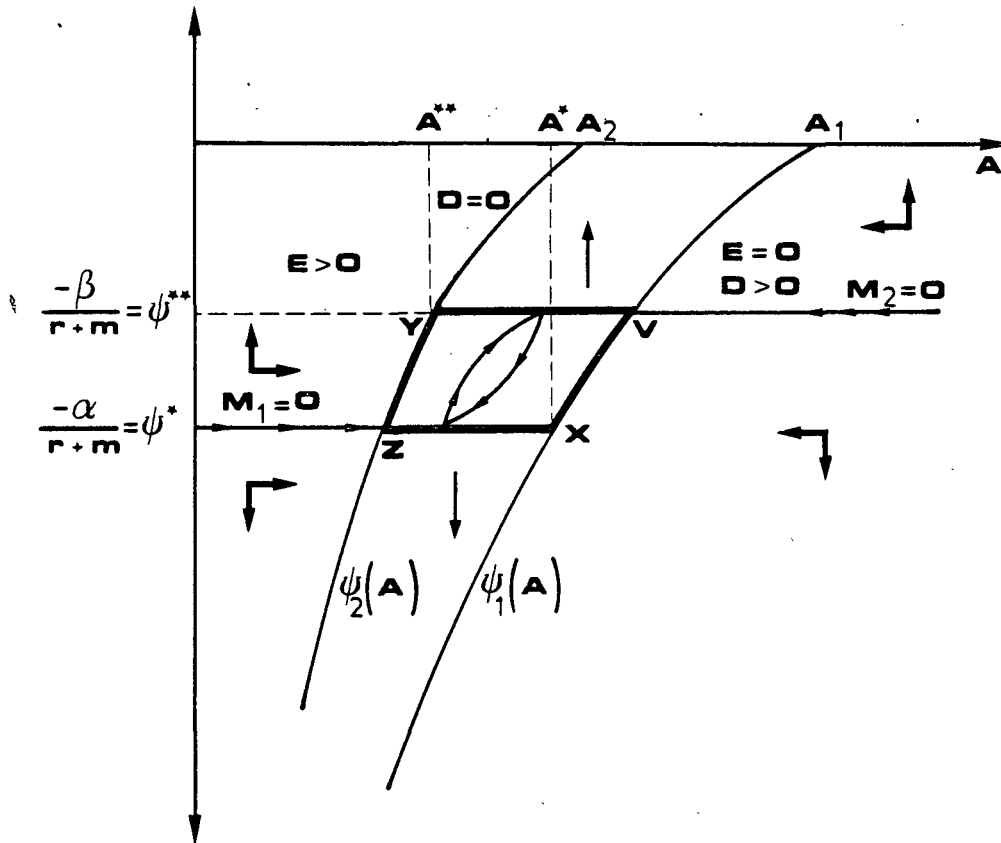


Figure 6

From (53) and (56) [or (54) and (55)], at any switch point

$$PF'(A-D) - PF'(A+E) = \alpha - \beta > 0 \quad (57)$$

implying a jump in the marginal value product - when a switch from  $D > 0$  to  $E > 0$  (or vice versa) takes place.

While it is difficult to establish a formal proof that oscillatory behaviour is optimal (from the point of view of the firm) because the constraint set is irregular, the following intuitive economic argument based on a simplified model<sup>2</sup> might perhaps convince the reader of the plausibility of our result.

Assume that time is discrete and that there are only three more periods to go. Assume further that the normal employment level,  $A(t)$ , is not defined as the average of all past employment levels, but is defined as the actual employment level in the previous period:

$$A(t) = L(t-1)$$

Let  $\hat{L}$  be defined by

$$PF'(\hat{L}) = W$$

Assume that the firm has always been employing  $\hat{L}$  units of workers, and that suddenly the government announces a tax-subsidy scheme, whereby the firm receives a subsidy at the rate  $\alpha > 0$  per additional worker in excess of the previous employment level. The total subsidy to be received in period  $t$  is

$$R(t) = \alpha[L(t) - L(t-1)] \quad \alpha > 0$$

provided  $L(t) > L(t-1)$ . The tax (penalty) is

$$T(t) = \beta[L(t) - L(t-1)] \quad \beta \geq 0$$

if  $L(t) < L(t-1)$ .

Assume that the rate of discount is zero. The firm has to make the decision concerning  $L(1)$ ,  $L(2)$  and  $L(3)$ . (By assumption,  $L(0) = \hat{L}$ .) For simplicity, assume

$$\beta = 0$$

Then there is nothing to lose, and everything to gain, by choosing  $L(1) = L^*$  such that

$$PF'(L^*) = W - \alpha$$

In doing so, the firm makes a net gain equal to the area ABC (Figure 7): the value of the additional output is the area ABB'C', while the additional wage cost (net of subsidies on the incremental workers) is only BB'CC'.

In period 2, the firm receives no subsidy if  $L(2) = L(1)$ . There is no gain in choosing  $L(2) > L(1)$ . It might seem that the optimal policy is to return to the level  $\hat{L}$  (set  $L(2) = \hat{L}$ ), so that in the final period, another subsidy is received if  $L(3) = L(1)$ . Upon reflection, however, this policy is not the best, for the firm can gain more subsidy in period 3 by having a lower employment level in period 2. By setting  $L(2) < \hat{L}$ , the marginal loss in period 2's profit is  $PF'(L(2)) - W$ , while the marginal gain (in period 3) is the additional subsidy. The truly optimal policy is to set  $L(2)$  at  $L^{**}$ , where

$$PF'(L^{**}) - W = \alpha$$

(so that the marginal gain just equals the marginal loss).

If the number of periods exceed 3, there will be a regular oscillation in employment between  $L^{**}$  and  $L^*$ .

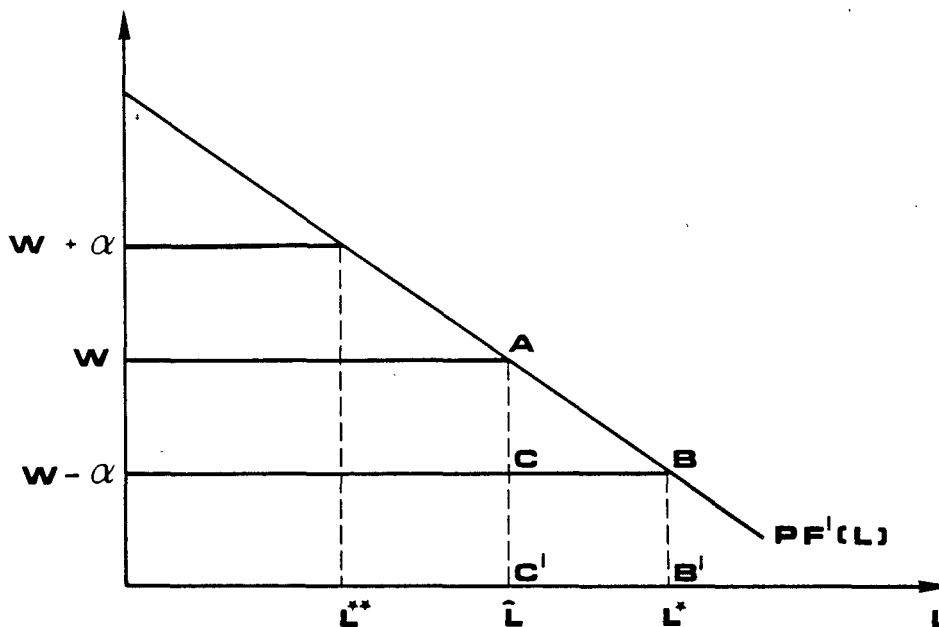


Figure 7

Our model is more complicated, because

- (i)  $A(t)$  is not defined as the actual level of employment in the previous period. Instead, it is a weighted average of all past levels of employment,
- (ii) there is a positive penalty for reducing employment,
- (iii) the discount rate is positive, and
- (iv) the time horizon is infinite.

However, the same basic force drives both models. It is not surprising that both models are characterized by endogenous employment cycles when the reward is greater than the penalty.

Our results in this section can be summarized in the following proposition:

Proposition 3: If the reward for employing additional labour above the normal employment level is higher than the penalty for sacking workers, the firm may find it optimal to behave cyclically, so that its average and actual employment levels will display a regular pattern of oscillations.

The pattern of switches that arises from our model is illustrated in Figure 8 for the case  $\alpha > \beta > 0$  and Figure 9 for the case  $\alpha > \beta = 0$ .

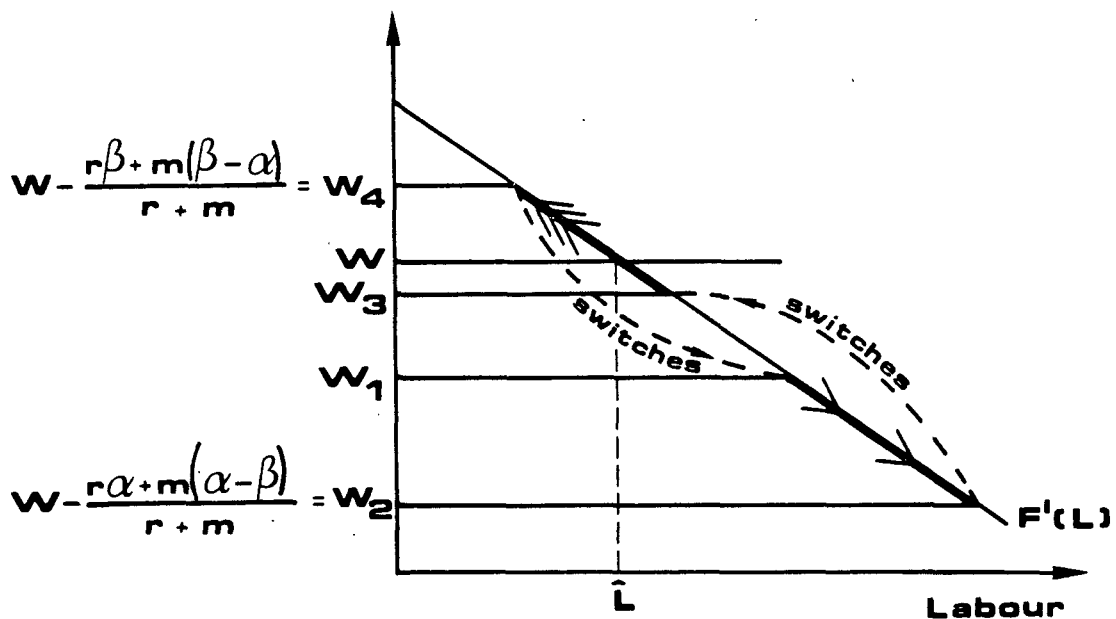


Figure 8

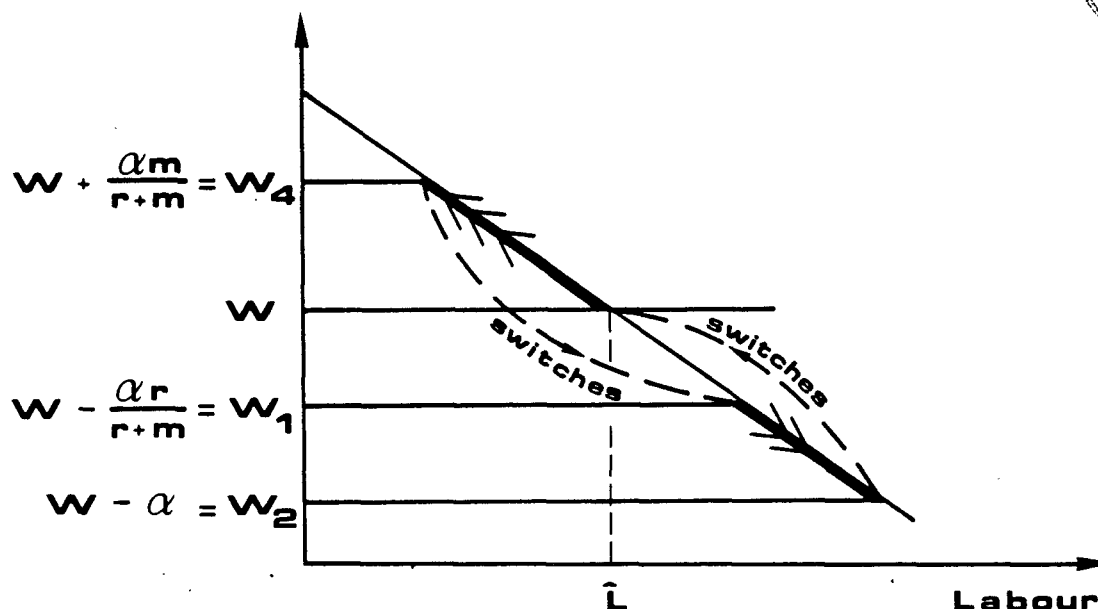


Figure 9

## 5. CONCLUSION

One of our major results is that business cycles may be caused by optimizing behaviour of firms in an environment ridden by government regulations or incentive schemes.

It is well known that cycles may exist in an economy where everyone optimizes individually; see, for example, Gale (1973), Grandmont (1983), Ryder and Heal (1973). These models, however, differ from our present model in that we highlight the possibility that incentive schemes created by government may cause cycles. It is apt to repeat what we said in our earlier paper: the effect of the institutional device which aims at the protection of the workers may run counter to its good intention. As Samuelson ((1980), p. 671) puts it, "the problem is to move beyond our good intentions".

FOOTNOTES:

- 1 In terms of the Kuhn-Tucker theory, this is known as the "constraint qualifications" problem.
- 2 An earlier version of this finite horizon, discrete time model (without discounting) was developed by S. Wilkie (1984) in a study of the Australian "incremental export subsidy" scheme.

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