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Abstract

The paper integrates human–capital investments of heterogeneous individuals into a neoclassical growth framework. The accumulation of physical capital changes relative factor prices and thus incentives to acquire skills, thereby altering the composition of the labor force. This interplay between the accumulation of human and physical capital provides a theoretical foundation for several important observations on wage inequality (e.g., the non–monotonic evolution of the skill premium) without having to hinge on specific exogenous shocks. Additional incorporation of wage rigidities emphasizes the trade off between residual wage inequality and employment opportunities for unskilled labor that is consistent with country–specific evidence.

JEL Classification I21, J31, O15

Keywords wage inequality, human–capital investment, transitional adjustment dynamics

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1 Introduction

The development of wage inequality by skills has been the subject of a lot of theoretical and empirical research in recent years. The great interest in this question is mainly due to two reasons. First, labor earnings constitute the major part of most households' incomes, and hence a changing wage structure has important implications for overall income inequality. Second, the behavior of some variables measuring wage dispersion was quite surprising and hardly to reconcile with conventional explanations.

Some of the facts about the evolution of wage inequality can be summarized as follows:

a) Although the fraction of skilled workers increased during the past decades, wage differentials by skill did not behave monotonically; while many countries shared a narrowing of the skill differentials in the 70s, the trend reversed in the 80s and 90s: the skill premium has been steadily increasing since then.[1]

b) The empirical studies unanimously display increasing inequality within the group of skilled workers.[2] The picture is less homogeneous for wage dispersion in the unskilled group. While many countries (esp. the US and the UK) experienced more unequal wages for the unskilled workers, other economies (among them Belgium and Germany[3]) had a period of low–skilled wage compression.

Most theoretical contributions concerned themselves with the evolution of the skill premium. Moreover, most authors focused on the increasing wage gap in the 80s and 90s, whereas only few articles addressed its non–monotonic behavior over the past 30 years. Without claiming to be comprehensive, we can distinguish three major lines of research dealing with fact a).

First, an increasing skill premium is argued to be a consequence of skill–biased technological change which causes larger demand for skilled labor (cf., e.g., Mincer, 1991, Autor et al., 1998). This argument seems to be suitable only for the last

[1] See the summary of stylized facts in Katz and Autor (1999), and the references therein. Particularly helpful and illustrative is Table 2.1 in Freeman and Katz (1994).

[2] Cf. Katz and Autor (1999) for the United States and, e.g., OECD (1993) for other OECD countries.

[3] Cf. Steiner and Wagner (1998), Fitzenberger (1999), or Möller (1999).

20 years. The basic mechanism, however, has been adapted by Acemoglu (1998) and Kiley (1999) to account for the development since the 70s. They emphasize that whether technological change is complementary to skilled or unskilled labor is subject to firms' decisions. The chosen design depends on the relative availability of the two types of labor. Acemoglu and Kiley assert that while in the early 70s unskilled labor was relatively abundant (thus making technological change biased towards unskilled labor), an exogenous shock (educational reforms, attempts to avoid participation in the Vietnam war) raised the relative supply of skilled workers. This labor–supply shock caused an immediate drop of the relative skilled wage in the 70s. However, it also made technological progress switch to skill complementarity, thus generating the increasing relative wage in the 80s and 90s. Necessary for the U–shaped evolution of the skill premium is an exogenous shift in labor supply.

Second, increasing wage differentials by skill are attributed to changing patterns of international trade (globalization). There are two channels through which increased integration of world goods markets affects wage inequality. On the one hand, globalization causes changes in the relative goods prices, thus affecting relative factor prices and demands (Wood, 1994, Leamer, 1998). According to standard reasoning, these price changes generate an increase in the relative wage of skilled labor in industrialized countries (Stolper–Samuelson effects). On the other hand, globalization also affects the size of markets and thus changes the incentives for product development (Dinopolous and Segerstrom, 1999). As a result, relative demand for skilled labor increases driving up their wage. All these trade–based explanations of wage inequality, however, only address the behavior of between–group wage inequality in the 80s and 90s.

Third, several contributions (Card, 1996, DiNardo et al., 1996, or Freeman, 1996) explain the altering wage structure by changes in labor–market institutions. Specifically, erosion of the real and relative value of the minimum wage, the decline in unionization, and changes in the distribution of bargaining power in the wage–setting process cause a relative decline of the unskilled wages. Again, these contributions address only the development of the relative wage since the 80s.

A major drawback of all these theoretical explanations focusing on the development of between–group wage inequality is that they treat skilled and unskilled labor as homogeneous groups of workers. For a more complete analysis of wage inequality that also explains the stylized facts about within–group inequality mentioned in b), it is necessary to introduce some heterogeneity with respect to the different groups of labor. This can be achieved by introducing differences in the inherent abilities of individuals that generate differences in their productivities and thus in their incentives to acquire skills.[4] Galor and Moav (2000a), which is closest to our approach, combine such a differentiated analysis of supply–side effects with skill–biased technological change. In their framework, technological progress is biased towards skilled labor and raises incentives to acquire skills. Moreover, they presume technological change to reduce total working time, as agents have to spend time on getting acquainted with new technologies. The faster technical progress is, the more demanding is the acquisition of skills, i.e. schooling time is increasing in the growth rate. Given that the more able individuals can better deal with technical change (i.e., the effort needed to handle with new machines is negatively correlated with individual abilities), their model provides an explanation of the facts about within group inequality and rising skill differentials. Combining this setup with a changing degree of capital–market imperfection for financing the investment costs provides an answer to the question of non–monotonicity of the skill premium.

Although there are several mechanisms which can account for the increasing skill premium in the 80s and 90s, only few contributions tackle the question of non–monotonicity of the skill differential. Acemoglu (1998), Kiley (1999), and Galor and Moav (2000a) build on the fact that the complementarity of technological change to specific labor raises its relative wage. But these models have a serious flaw, as they have to rely on exogenous shocks in explaining the decrease of the skill premium in the 70s: the non–monotonicity either rests on an exogenous change in the extent of capital–market imperfection (Galor and Moav) or on exogenous labor–supply shocks (Acemoglu, Kiley). It is at least subject to some doubt whether shocks which are to some degree country specific can explain the co–movement of the relative wage since the 70s in many countries.

The present paper argues that none of these special exogenous shocks is essential for an explanation of the observations about between–group wage inequality.

[4]Acemoglu (1998, 2000) suggests an imperfect correlation of education and productivities as an alternative way to account for residual wage inequality.

There is an endogenous tendency for the wage differential by skills to behave non–monotonically in a neoclassical growth model (Solow model) when augmented by endogenous human–capital investments. As people have different innate productivities, they earn different wage incomes and they have heterogeneous incentives to qualify for skilled work (which is equivalent to making a human–capital investment in our setting). In general, the most talented individuals choose to acquire skills, while the less able work as unskilled. Since both the group of skilled and of unskilled labor are made up of individuals that are heterogeneous with respect to their productivities, we can analyze the development of both between–group wage inequality and within–group wage inequality (residual wage inequality). Of course, the wage differential by skills has to be measured by some index for both types of labor. Following the empirical literature, we choose the average wage income as such an index.

In our model, the returns to and the opportunity costs of education depend on relative factor prices. In the course of economic growth, the accumulation of physical capital alters factor prices, thus implying larger returns to and less opportunity costs of higher education. As successively less talented individuals are inclined to upgrade their inherent abilities, the fraction of skilled workers rises. At the same time, average wages decline for both types of labor. Combined with a convex education technology, a non–monotone development of the wage differential by skills follows. Additionally, we obtain increasing wage dispersion for the high–skilled as more and more people with inferior productivity become skilled. On the other hand, while the most talented among the less able individuals gradually choose to become skilled, wage dispersion within this groups declines. Such a compression has been observed for some countries (e.g., Germany). In order to also account for contrary developments (as in the US or UK), we extend our basic model following the above mentioned argument about institutional change in the labor markets. We show that the observation of increasing inequality within the low–skilled group can be explained by the introduction of a wage floor that does not keep up with the increase in labor productivity over time (cf. the literature cited above). This extended version of our model, which stresses the trade off between wage inequality and employment opportunities, also offers some explanation for country–specific differences in the evolution of unemployment.

The paper proceeds as follows. Section 2 introduces our basic neoclassical growth model with heterogeneous agents. Section 3 considers the static equilibrium, and Section 4 analyzes the steady–state equilibrium of the model and its transitional dynamics. Section 5 presents the development of wage inequality implied by the basic model. Section 6 discusses extensions of the basic model necessary for a comprehensive explanation of the empirical findings. Section 7 concludes.

2 The Model

2.1 Factor Supplies

Our economy is populated by a continuum of individuals of mass one. Each generation lives for one period having one off–spring at the end of their lives implying that population is constant over time. Agents are heterogeneous with respect to their inherent abilities, a, and their wealth endowment, x, inherited from the parent generation.

Assumption 1 *Inherent abilities are distributed uniformly on* $[\underline{a}, \overline{a}]$ *(*$\underline{a} > 1$*). They are independent across generations and across individuals.*

Using uniform abilities is technically convenient and enables us to obtain explicit solutions but it is not crucial for our results. Let Φ denote the distribution function of inherent abilities. Individuals are supposed to have identical homothetic utility functions with consumption and bequest as arguments. As usual, utility is strictly increasing and concave in both arguments. This assumption brings about a constant fraction $1 - s$ of life time income is spent on consumption.[5] The agents maximize utility, iff they maximize end–of–life wealth.

At the beginning of their lives individuals realize the amount of wealth inherited from their parents and their inherent ability. Individuals can either refrain from education (i.e., work as unskilled) thereby supplying a units of effective labor. In this case, they earn a wage income aw and a capital income $x(1+r)$. Alternatively, they can upgrade their abilities at some fixed cost I, and thus supply a^2

[5] Assuming a constant bequest share simplifies the analysis considerably, as otherwise aggregate wealth bequeathed to the next generation would depend on the distribution of parents' wealth and not only on aggregate wealth.

units of effective labor.[6] Total income of a skilled individual is $a^2w+(x-I)(1+r)$. Individuals that don't have sufficient wealth to cover upfront schooling costs can borrow at a perfect capital market.

An agent chooses education, iff end–of–life wealth resulting from human–capital accumulation is larger than that for unskilled work, i.e. whenever

$$a(a-1) \geq I\frac{1+r}{w}. \tag{1}$$

As there are no capital–market imperfections, the investment decision solely depends on ability for given values of w, r and I. Obviously, the following result holds:

Lemma 1 *The function $a_0 : \mathbb{R}_+^2 \to \mathbb{R}_+$ which maps every pair $(w, r) \in \mathbb{R}_+^2$ to*

$$a_0(w,r) := \frac{1+\sqrt{1+4I\frac{1+r}{w}}}{2}$$

describes minimum ability for which investment in education is profitable, i.e. every individual with ability of at least a_0 finds it attractive to become skilled.[7] We have: $\partial a_0/\partial w < 0$ and $\partial a_0/\partial r > 0$.

The sign of the partial derivatives of a_0 reflects the fact that for increasing w and decreasing r investment becomes attractive for less talented individuals due to increasing returns to and decreasing costs of human–capital investment, respectively. Using the properties of a_0, we can easily calculate the fraction of skilled workers in the economy.

Lemma 2 *The fraction of skilled (unskilled) workers n is $1-\Phi(a_0)$ ($\Phi(a_0)$). As a result, n is non–increasing in a_0.[8]*

An individual works as skilled whenever the productivity gain from education is able to compensate for the fixed cost. For $a_0 \geq \bar{a}$ nobody wants to become

[6] The assumption on increasing returns to education may be interpreted as a shortcut for the fact that the more able individuals typically learn faster or better. Suppose, for instance, that agents need not only incur I, but they also have to spend time on education where this time input diminishes working time. Combining larger productivity with less schooling requirements can yield increasing returns to ability.

[7] Note that we do not presume the image of a_0 to be a subset of $[\underline{a}, \bar{a}]$.

[8] We cannot claim n to be strictly decreasing in a_0, since $a_0(\mathbb{R}_+) \not\subset [\underline{a}, \bar{a}]$.

educated, for $a_0 \leq \underline{a}$ everybody chooses to invest in education. For a given pair (w, r), demand for capital resulting from investment in education is nI. The amount of effective labor supplied on the labor market is determined by

Lemma 3 *For any pair (w, r), effective labor supply can be characterized by the continuous function $H : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ with*

$$H(a_0(w,r)) := \begin{cases} \dfrac{\underline{a}+\overline{a}}{2} & \textit{for} \quad a_0(w,r) > \overline{a} \\ \dfrac{1}{2}\dfrac{a_0^2-\underline{a}^2}{\overline{a}-\underline{a}} + \dfrac{1}{3}\dfrac{\overline{a}^3-a_0^3}{\overline{a}-\underline{a}} & \textit{for} \quad a_0(w,r) \in [\underline{a}, \overline{a}] \\ \dfrac{\underline{a}^2+\underline{a}\overline{a}+\overline{a}^2}{3} & \textit{for} \quad a_0(w,r) < \underline{a}\,. \end{cases} \tag{2}$$

Proof. If the pair (w, r) induces a threshold ability larger than $\overline{a}$, nobody wants to become skilled. Effective labor supply then is given by

$$H = \int_{\underline{a}}^{\overline{a}} a \, d\Phi(a)\,.$$

With inherent abilities uniformly distributed within $[\underline{a}, \overline{a}]$, we get

$$H = \frac{1}{\overline{a}-\underline{a}} \int_{\underline{a}}^{\overline{a}} a \, da = \frac{\underline{a}+\overline{a}}{2}\,.$$

For $a_0(w, r) < \underline{a}$ everybody becomes skilled implying an effective labor supply of

$$H = \frac{1}{\overline{a}-\underline{a}} \int_{\underline{a}}^{\overline{a}} a^2 \, da = \frac{\underline{a}^2+\underline{a}\overline{a}+\overline{a}^2}{3}\,.$$

For $\underline{a} \leq a_0(w, r) \leq \overline{a}$, individuals with ability greater than a_0 choose to become skilled so that

$$H = \frac{1}{\overline{a}-\underline{a}} \left(\int_{\underline{a}}^{a_0} a \, da + \int_{a_0}^{\overline{a}} a^2 \, da \right) = \frac{1}{2}\frac{a_0^2-\underline{a}^2}{\overline{a}-\underline{a}} + \frac{1}{3}\frac{\overline{a}^3-a_0^3}{\overline{a}-\underline{a}}\,.$$

Continuity is obvious. □

The aggregate amount of effective labor must lie in a compact interval for all pairs (w, r)

$$H \in [\underline{H}, \overline{H}] \quad \text{with} \quad \underline{H} := \frac{\underline{a}+\overline{a}}{2}\,, \overline{H} := \frac{\underline{a}^2+\underline{a}\overline{a}+\overline{a}^2}{3}\,,$$

with $\underline{H}$ being strictly larger than 0.

2.2 Production Decisions

There is a large number of firms acting competitively on goods and factor markets. The aggregate technology can be described by some neoclassical production function $F(K_P, H)$ with physical capital used in the production of physical output K_P and effective labor H as arguments. More specifically, we impose

Assumption 2 *The production function $F : \mathbb{R}^2_+ \to \mathbb{R}_+$ is twice continuously differentiable and has constant returns to scale: $F(\lambda K_P, \lambda H) = \lambda F(K_P, L)$. We have strictly positive, but diminishing returns to both factors: $F_i > 0$ and $F_{ii} < 0$ for $i = K_P, H$. The first derivatives satisfy the Inada–conditions (i.e. $\lim_{i\to 0} F_i = +\infty$ and $\lim_{i\to +\infty} F_i = 0$ for $i = K_P, H$), and the second derivatives are bounded.*

For any capital input $K_P > 0$, the marginal product of labor is bounded from above, as $F_H(K_P, H) \leq F_H(K_P, \underline{H})$ due to diminishing returns to H and $H \geq \underline{H}$.

Making use of the homogeneity property, we obtain $f : \mathbb{R}_+ \to \mathbb{R}_+$ with $f(k) := F(k, 1)$ (where $k := K_P/H$ is the capital intensity of production) so that $F(K_P, H) = Hf(k)$.

Assuming a depreciation rate of 1, profit maximization of the firms implies the demand for capital and effective labor as

$$
\begin{aligned}
1 + r &= f'(k) && (3)\\
w &= f(k) - kf'(k). && (4)
\end{aligned}
$$

3 The Static Equilibrium

For the economy to be in equilibrium, all markets must clear simultaneously. Due to Walras' law, we can concentrate on factor markets. At any instant, aggregate capital supply, K, is given by aggregate bequests. K is used for accumulating skills and for production of final goods, i.e. firms as well as households demand capital. As derived in the last section, capital demand for acquiring skills at factor prices (w, r) is equal to $nI = [1 - \Phi(a_0(w, r))]\, I$. Effective labor supply is given by $H(a_0(w, r))$ (cf. (2)).

Equilibrium factor prices must satisfy (3) and (4). This yields factor prices as a function of k: $w = w(k)$, $r = r(k)$, with $w'(k) > 0$, $r'(k) < 0$. The static equilibrium is fully characterized by the equilibrium capital intensity. Given aggregate capital K, a capital–labor ratio k employed in production is compatible with static equilibrium iff the corresponding factor rates $w(k)$ and $r(k)$ bring about investment incentives that induce k. This means that dividing capital not used for production of human capital, $K - [1 - \Phi(a_0(w,r))]\, I$, by the amount of physical labor, $H(a_0(w,r))$, must imply k. To be able to characterize the equilibrium condition for k in a more transparent way, we introduce the function $h : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$ with

$$h(k,K) := \frac{K - I[1 - \Phi(a_0(w(k), r(k)))]}{H(a_0(w(k), r(k)))} .$$

For arbitrary positive K, the capital intensity k constitutes a static equilibrium iff

$$k = h(k,K) \tag{5}$$

holds.

The following proposition describes the set of equilibrium capital–labor ratios for given aggregate capital.

Proposition 1 *For any $K > 0$, there is one and only one $k > 0$ which is compatible with static equilibrium.*

Proof. It suffices to prove that, for given K, there exists one and only one solution of the equation $k = h(k,K)$. To show this we examine the function h for fixed K. With factor prices being a function of k, we can calculate critical values $\underline{k}$ and $\overline{k}$ from (1) such that $a_0(w(\underline{k}), r(\underline{k})) = \overline{a}$ and $a_0(w(\overline{k}), r(\overline{k})) = \underline{a}$. This means that no individual becomes skilled if $k \leq \underline{k}$, and all individuals become skilled if $k \geq \overline{k}$.

From (1), we derive $\underline{k}$ as the solution of

$$\overline{a}(\overline{a} - 1) = I\psi(k) ,$$

where the function $\psi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is defined by $\psi(k) := f'(k)/[f(k) - kf'(k)]$. Equivalently, $\overline{k}$ solves

$$\underline{a}(\underline{a} - 1) = I\psi(k) .$$

Assumption 2 guarantees that $0 < \psi(k) < \infty$ and $\psi'(k) < 0$. With $a(a-1) > 0$ from Assumption 1, there exist unique solutions $0 < \underline{k} < \overline{k} < \infty$. Note that neither $\underline{k}$ nor $\overline{k}$ depends on K. The preceding considerations show that h as a function of k alone is constant for $k \leq \underline{k}$ and $k \geq \overline{k}$

$$h(k,K) = \begin{cases} \frac{K}{\underline{H}} & \text{for } k \leq \underline{k} \\ \frac{K-I}{\overline{H}} & \text{for } k \geq \overline{k}. \end{cases} \tag{6}$$

For $\underline{k} < k < \overline{k}$, h is strictly decreasing in k: increasing k, raises w and lowers r which induces larger investment incentives. The identical function which maps k to k is strictly increasing while h as a function of k is non–increasing. This guarantees uniqueness of a solution of $k = h(k,K)$. Existence follows from the intermediate value theorem as the identical function and h are continuous, and $\lim_{k\to 0} k < \lim_{k\to 0} h(k,K)$ and $\lim_{k\to +\infty} k > \lim_{k\to +\infty} h(k,K)$ hold. □

Figure 1 illustrates the determination of the equilibrium value k for a given capital stock K by the intersection of the graph of $h(k,K)$ and the 45–degree line. Specifically, it depicts the case of an interior solution $k \in (\underline{k}, \overline{k})$, where both skilled and unskilled labor exist.

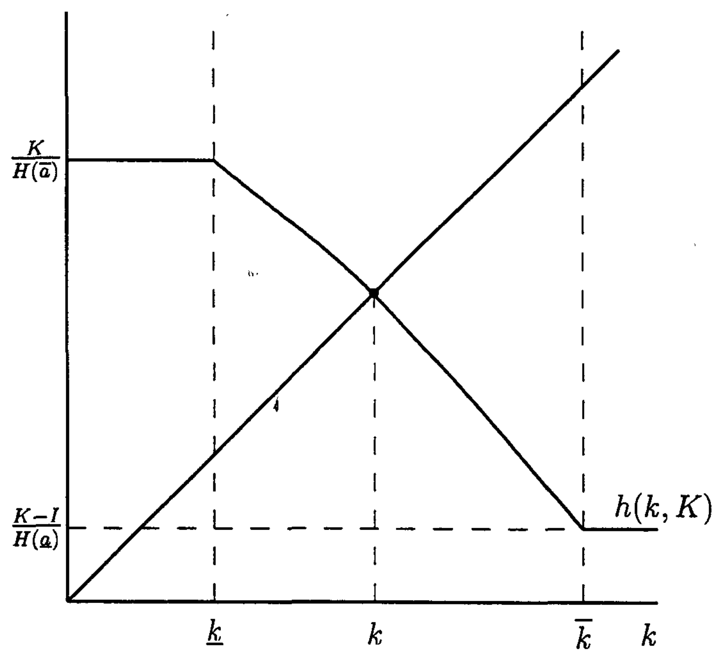

Figure 1: Determination of the equilibrium capital intensity

Proposition 1 guarantees that the function $g : \mathbb{R}_+ \to \mathbb{R}_+$ which assigns the unique equilibrium capital–labor ratio k to every aggregate capital stock K is well–defined. In the above proof, $\underline{k}$ and $\overline{k}$ were defined so that the corresponding factor prices provide only the most able ($\overline{a}$) or even the least able ($\underline{a}$) with human capital investment incentives. From our characterization of the static equilibrium, the following corollaries are straightforward.

Corollary 1 *The function g which maps every initial K to the equilibrium k is continuous and strictly increasing in K.*

Proof. Continuity follows from the corresponding properties of $\Phi(a_0)$ and $H(a_0)$. As $\partial h(k,K)/\partial K > 0$, $K_1 > K_2$ implies $g(K_1) > g(K_2)$. □

Corollary 2 *(i) There is a positive capital stock $\underline{K}$ below which no capital is employed in human–capital production ($n = 0$). (ii) There exists $\overline{K} > 0$ above which all agents become skilled ($n = 1$). (iii) For $\underline{K} < K < \overline{K}$, the fraction of skilled workers n is strictly increasing in K.*

Proof. (i) $\underline{K}$ is given by $\underline{H}\,\underline{k}$. For all $K \leq \underline{K}$ we have $g(K) \leq \underline{k}$ which implies $a_0(w(k), r(k)) \geq \overline{a}$. Hence, all capital is used for production of goods and effective labor supply does not vary with K. (ii) $\overline{K}$ equals $\overline{H}\,\overline{k} + I$. For all $K \geq \overline{K}$ we have $g(K) \geq \overline{k}$ which implies $a_0(w(k), r(k)) \leq \underline{a}$. Hence, even the least talented individuals invest in education and effective labor supply does not vary with K. (iii) For all $K_1, K_2 \in (\underline{K}, \overline{K})$, $K_1 > K_2$ implies $g(K_1) > g(K_2)$. Since a_0 decreases as k rises, we immediately get that n increases with K. □

Corollary 2 is quite intuitive. It states that in early stages of development ($K < \underline{K}$) human–capital investment by the agents is not profitable because of the high marginal product of capital in production. The marginal product of labor in production is bounded from above, as effective labor always exceeds $\underline{H}$. If a capital stock above $\underline{K}$ is available, investment becomes attractive for the most talented among the individuals. Eventually, all human–capital–investment opportunities are exploited, and excess capital $K - I$ is directly applied in production.[9]

[9] The evolution of the production pattern which we obtain in our model is similar to that in Galor and Moav (2000b). In our model, however, this path is not a consequence of the interaction of capitalists and workers, but it follows from investments in physical capital and in human capital in a competitive environment.

Proposition 1 implies that the function $a_s : \mathbb{R}_+ \to \mathbb{R}_+$ which assigns the corresponding threshold ability to every capital stock K, is well defined. More precisely, it has the form

$$a_s = a_0 \circ (w, r) \circ g. \tag{7}$$

According to Corollary 2, $a_s(K) \geq \overline{a}$ for $K \leq \underline{K}$ and $a_s(K) \leq \underline{a}$ for $K \geq \overline{K}$.

We are now in the position to examine the relation between available capital and equilibrium output. The amount of capital used as a direct input in production amounts to total capital, K, minus capital used for production of human capital, $n(a_s(K))I$.

Lemma 4 *Output realized in the competitive equilibrium for given K can be described by an increasing and strictly concave function $Y : \mathbb{R}_+ \to \mathbb{R}_+$ with*

$$Y(K) := F\left[K - n(a_s(K))I, H(a_s(K))\right]. \tag{8}$$

Proof. See Appendix. □

In our frictionless economy the output effect of a marginally increased capital stock does not depend on the sector of application. This implies that the derivative of equilibrium output as a function of K is continuous and strictly decreasing. Concavity seems logical, as we have a decreasing returns to scale production function and additional capital for schooling makes individuals invest whose ability is strictly smaller than that of previously educated.[10] Plotting the graph of Y as a function of K yields a figure which is very similar to that of an ordinary production function. This is precisely the reason why we can determine the dynamic structure of the economy without further complications.

4 Transitional Dynamics

Without having to deal with issues concerning the wealth distribution, we can state that aggregate wealth at the end of a period consists of labor income and capital plus interest income. Due to full depreciation of capital as production

[10]Note that we do not refer to particular persons or dynasties. Independence across generations means that children from a dynasty which worked as skilled in the last period need not upgrade their abilities.

and human–capital input, its aggregate value equals total input in the economy. Together with a constant bequest share s resulting from homothetic utility, we obtain next generation's aggregate capital endowment as

$$K_{t+1} = sY_t\,.$$

The analysis of the static equilibrium derived a functional dependence of final output realized in the competitive environment on the available capital K, so we have $K_{t+1} = sY(K_t)$. The dynamic evolution of an economy starting with K_0 in period 0 can be fully described by the following first–order difference equation and boundary (initial) condition

$$\begin{aligned} &K_{t+1} = sF\left[K_t - n(a_s(K_t))I, H(a_s(K_t))\right] \quad \text{for} \quad t \geq 0 \\ &K_0 \quad \text{given}\,. \end{aligned} \tag{9}$$

Our economy is said to be in a steady state, iff it has some capital stock which once reached will never be left again without exogenous disturbance (i.e. for a steady state capital stock K^* the relation $K^* = \alpha Y(K^*)$ must be valid). The properties of Y as a function of available capital K inferred in Lemma 4 immediately yield the following characterization of the dynamic evolution:

Proposition 2 *(i) The dynamical system given in (9) has exactly two steady states: $K = 0$ and $K = K^* > 0$. (ii) The no–capital equilibrium is unstable and, unless an economy starts in $K_0 = 0$, it converges to K^* (i.e. K^* is globally stable in this sense). (iii) The convergence obtained for $K_0 \neq 0$ in (ii) is monotone: for $K_0 < K^*$ it is strictly increasing, while for $K_0 > K^*$ it is strictly decreasing.*

Proof. (i) The existence of the steady state $K = 0$ is obvious, as an economy that starts with no capital can never produce anything.[11] Existence of a steady state $K^* > 0$ is a consequence of the properties of Y as a function of K (Lemma 4), Corollary 2 and the intermediate value theorem. For $K < \underline{K}$ no human–capital investments take place. We have $Y(K) = F(K, \underline{H})$ with $\lim_{K\to 0} F_K(K, \underline{H}) = +\infty$ according to the first Inada–condition. For $K > \overline{K}$ we have $Y(K) = F(K - I, \overline{H})$, as all human–capital–investment opportunities are

[11]The fact that capital is essential for production for our neoclassical production function follows from a proof provided, e.g., in Barro and Sala–i–Martin (1995) and is an implication of the Inada–conditions.

exploited, with $\lim_{K \to +\infty} F_K(K - I, \overline{H}) = 0$ due to the second Inada–condition. This implies that for K close to zero, $sY(K)$ must exceed K, while for large K we have $sY(K) < K$. So an application of the intermediate value theorem guarantees the existence of $K^* > 0$. Strict concavity of Y (Lemma 4) makes sure that it is the only positive one.

(ii) and (iii) are implied by Lemma 4, the Inada–conditions and Corollary 2.

□

The shape of the phase diagram for the path of aggregate capital which follows from Proposition 2 is depicted in Figure 2.

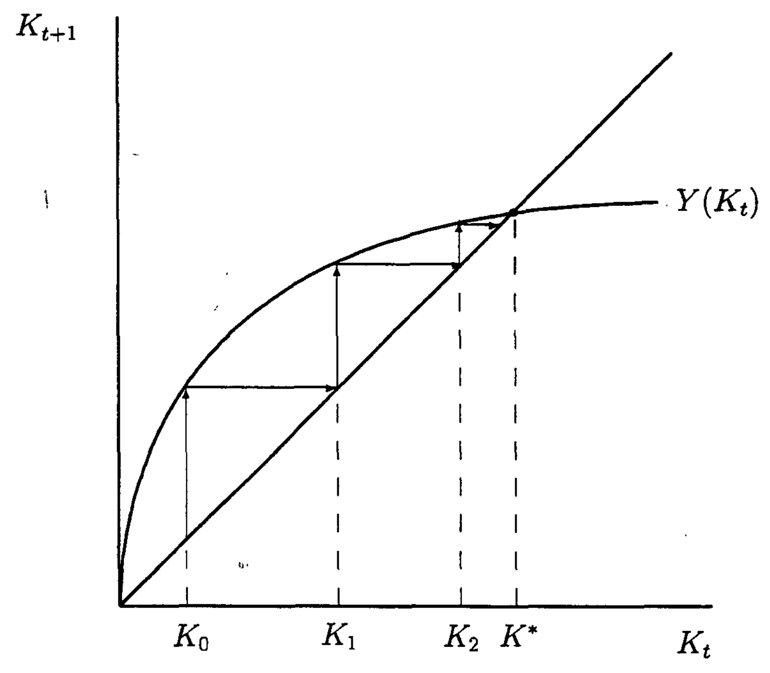

Figure 2: Phase diagram of the aggregate capital stock

The results of Proposition 2 do not come as a surprise, because our model uses a production technology with decreasing returns to scale with respect to both factors of production and for which the Inada–conditions hold. In this purely neoclassical setup, long–run growth from factor accumulation cannot be expected.

As the development of the economy is completely determined by the sequence of capital stocks $(K_t)_{t\geq 0}$, some results for the evolution of human capital and factor prices follow immediately from the properties of the functions g, a_s, and H (we will restrict attention to the case $K_0 < K^*$ in the sequel):

Corollary 3 *For an initial $K_0 < K^*$, the sequences $(r_t)_{t\geq 0}$ and $(a_{s,t})_{t\geq 0}$ are strictly decreasing, while $(w_t)_{t\geq 0}$ is strictly increasing. The human-capital sequence $(H_t)_{t\geq 0}$ is non-decreasing.*

An economy starting with a positive amount of capital which is smaller than the steady state value will experience a strictly increasing development of K over time. Whether the long-run equilibrium induces a 100% share of skilled workers $(K^* \geq \overline{K})$ or whether there are any highly qualified workers at all $(K^* \leq \underline{K})$ is just a matter of the parameters and the production function. To be able to study effects of wage inequality between the high-skilled and the low skilled group we postulate

Assumption 3 *Parameters and the production function are chosen such that $K^* \geq \overline{K}$.*

Having determined the dynamical behavior of the economy, we now turn to the evolution of wage inequality along the adjustment path.

5 The Development of Wage Inequality

We focus on two aspects of wage inequality: first, the wage differential by skills (between-skill-group inequality), and second, the wage dispersion within the groups (within-group inequality). With heterogeneous individuals earning different wage incomes, we have to choose some wage index for the groups to be able to compare wages of skilled and unskilled workers. Empirical studies use both the mean income and the median income as such a measure. In the following, we choose the mean income as wage index for skill groups, although our results are not dependent on this specific index.[12]

[12] Note that there is a difference between these measures due to the convexity of the skilled working hours in workers' abilities. In general, the median seems preferable because of its outlier resistance. In the present case, however, this advantage of the median does not pertain since we assumed abilities to be uniformly distributed over a bounded support. Cf. Meckl and Zink (2001) for the use of the median income with more general distributions of abilities.

We presuppose $K_0 < K^*$ and determine wage inequality for an increasing sequence $a_{s,t}$ (Corollary 3) of threshold abilities. We first analyze how relative inequality between the groups evolves along the adjustment path to the steady-state equilibrium. As there are no skilled workers for $K \leq \underline{K}$ and no unskilled workers for $K \geq \overline{K}$, we focus on $a_s \in (\underline{a}, \overline{a})$. In particular, we study inequality for decreasing $a_s \in (\underline{a}, \overline{a})$, where $a_s = a_s(K)$ denotes the threshold value above which education occurs. As a measure of between-group wage inequality we take the ratio of the average wages of the two groups, as the use of these relative wages guarantees scale invariance.

We define the function $w_l : (\underline{a}, \overline{a}) \to \mathbb{R}_+$, with $w_l(a_s)$ being the mean wage of the low-skilled group given that people with at least $a_s \in (\underline{a}, \overline{a})$ acquire education. Analogously, we define $w_h : (\underline{a}, \overline{a}) \to \mathbb{R}_+$, where $w_h(a_s)$ is the respective mean income of skilled workers. Relative inequality based on mean incomes is then given by

$$\omega(a_s) := \frac{w_h(a_s)}{w_l(a_s)}.$$

The unskilled workers have abilities which are distributed uniformly on $[\underline{a}, a_s]$, while uniform abilities of the qualified workers have support $[a_s, \overline{a}]$. Calculating the first moments delivers the functions w_l and w_h as[13]

$$\begin{aligned} w_l(a_s) &= \frac{a_s + \underline{a}}{2} w(a_s) \\ w_h(a_s) &= \frac{\overline{a}^2 + \overline{a}a_s + a_s^2}{3} w(a_s). \end{aligned}$$

So we obtain the relative wage

$$\omega(a_s) = \frac{2}{3} \frac{\overline{a}^2 + \overline{a}a_s + a_s^2}{\underline{a} + a_s}. \tag{10}$$

As a_s decreases, the mean wage falls for both skill groups: On the one hand, the most talented among the previously unskilled become skilled, thus reducing w_l. On the other hand, their ability is smaller than that of workers that already became skilled before, thus reducing w_h. We find that under some simple condition the relative wage $\omega(a_s)$ is not monotone in $(\underline{a}, \overline{a})$:

Lemma 5 *For $3\underline{a}^2 + \overline{a}\underline{a} - \overline{a}^2 < 0$ there exists $\widetilde{a} \in (\underline{a}, \overline{a})$ so that $\omega' > 0$ for $a_s > \widetilde{a}$ and $\omega' < 0$ for $a_s < \widetilde{a}$.*

[13] As the wage for an effective unit of labor changes for increasing human–capital–investment incentives, we let $w(a_s)$ denote this functional dependence.

Proof. The derivative of ω with respect to a_s can be calculated as

$$\omega'(a_s) = \frac{2}{3}\frac{\underline{a}\overline{a} + 2a_s\underline{a} + a_s^2 - \overline{a}^2}{\underline{a} + a_s}. \tag{11}$$

The sign of (11) is equal to the sign of the numerator which is a quadratic function m in $a_s \in (\underline{a}, \overline{a})$. Obviously, the quadratic function is strictly increasing in a_s. We observe that $\lim_{a_s \to \overline{a}} m(a_s) > 0$. The condition stated in the theorem is sufficient and necessary to guarantee $\lim_{a_s \to \underline{a}} m(a_s) < 0$, so that continuity and monotonicity of m prove the claim by an application of the intermediate value theorem. □

Note that the condition guaranteeing non–monotonicity will always be satisfied, if $\overline{a} - \underline{a}$ is sufficiently large. For $\underline{K} < K < \overline{K}$, we obtain a strictly decreasing threshold abilities. The preceding considerations show that the relative wage ω can behave non–monotonically under some circumstances. Sufficient conditions for this are stated in the following theorem which immediately follows from the above Lemma 5.

Proposition 3 *Let $a_{s,t}$ denote the sequence of threshold abilities implied by the sequence of capital stocks K_t. Let, furthermore, $N \subset \mathbb{N}$ ($M \subset \mathbb{N}$) denote the set of natural numbers for which $a_{s,t} \in (\underline{a}, \tilde{a}]$ ($a_{s,t} \in [\tilde{a}, \overline{a})$); i.e. $N := \{t \in \mathbb{N} | a_{s,t} \in (\underline{a}, \tilde{a}]\}$ and $M := \{t \in \mathbb{N} | a_{s,t} \in [\tilde{a}, \overline{a})\}$. If both of these sets have at least two elements, then the economy has a period of decreasing between–group inequality followed by an increasing between–group inequality.*[14]

A path of threshold abilities $a_{s,t}$ which is sufficient to generate non–monotone behavior of the relative wage is depicted in Figure 3.
So under relatively weak conditions, Proposition 3 shows that along adjustment paths to the steady–state equilibrium with an initial capital stock $K < K^*$, an economy can experience extended periods of decreasing between–group wage inequality followed by a phase of increasing inequality.[15] This is because the rise in the return to education (increasing w) and the simultaneous reduction in cost of education (decreasing r) make more people invest in human capital.

[14]Both sets N and M depend on the initial capital stock.

[15]Uniform abilities facilitate the analysis but they are by no means necessary for the non–monotonicity. As shown by Meckl and Zink (2001) in a partial–equilibrium context, more realistic distributions of genetic abilities generate similar results.

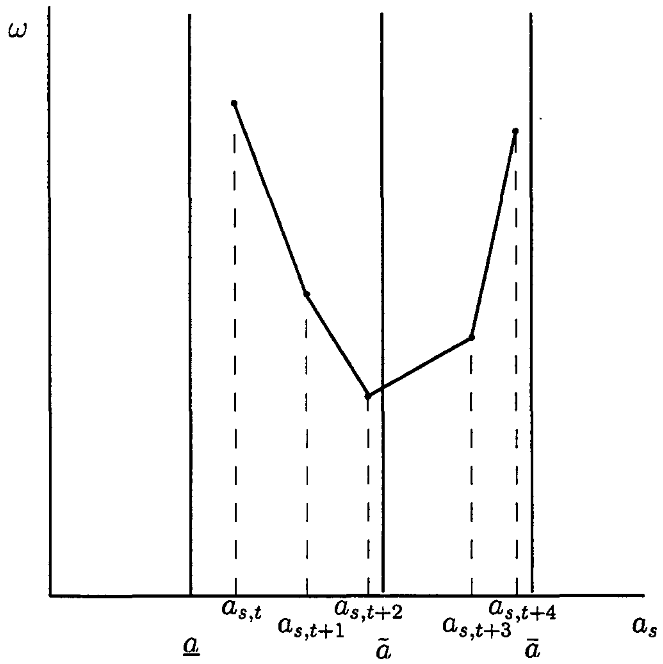

Figure 3: The U–shaped path of the relative wage

The behavior of between–group inequality displayed by this simple supply–side model can account for the observation of decreasing skill premium in the 70s and increasing wage inequality in the 80s and 90s, if the difference between the least able and the most talented individuals is large enough.

Apart from the non–monotonicity observed for the skill premium, empirical studies obtain a rise in the within–group inequality at least for the skilled, whereas evidence with respect to within–group wage inequality for unskilled labor is mixed (cf. Introduction). The rest of this section is devoted to showing that the model can account for increasing inequality for the skilled workers and decreasing inequality for the unskilled without any changes.

In order to assess the change of inequality, we calculate the Gini coefficient at first for the educated class. For $a_s \in (\underline{a}, \overline{a})$ the Gini–coefficient for the skilled workers is

$$G_h(a_s) = \frac{\overline{a}^2 - a_s^2}{2(a_s^2 + \overline{a}a_s + \overline{a}^2)}, \tag{12}$$

with $G_h'(a_s) < 0$. So along the adjustment path, the accumulation of capital is accompanied by a rise of inequality within the group of the skilled workers.[16]

Proposition 4 *For $K_0 < K^*$, within–group inequality of skilled workers rises along the adjustment path.*

This result seems quite intuitive, as a larger capital stock provides people with lower abilities with investment incentives.

To investigate the change of within–group inequality for the low talented people, we evaluate the Gini–coefficient for this group. After some manipulations we get

$$G_l(a_s) = \frac{5\,a_s - \underline{a}}{3\,a_s + \underline{a}}. \tag{13}$$

Of course, $G_l'(a_s) > 0$, i.e. lower a_s implies a lower within–group inequality for the unskilled.

Proposition 5 *For initial capital below the positive steady state value ($K_0 < K^*$), the wage dispersion in the unskilled group measured by the Gini–coefficient is declining.*

As the size of the unskilled group shrinks, an intertemporally decreasing degree of inequality does not come as a surprise. Relative inequality (measured by the Gini coefficient or some other scale invariant measure like the coefficient of variation which is merely a linear function of G) is a non–increasing function of the overall capital stock.

Hence, apart from consistence with the empirical evolution of the wage differential by skill, the model can account for the observed development of wage dispersion within the group of the skilled workers (Proposition 4). Furthermore, our theoretical results about inequality within the unskilled group is consistent with empirical findings of at least some countries.

6 Wage Floors and Within–Group Inequality

Although our result about wage compression in the low–skilled group obtained in Proposition 5 is compatible with observations in countries such as, e.g., Belgium

[16] Alternatively, one could have considered the evolution of various quantile coefficients, but the results are similar.

or Germany, it is at odds with the empirical facts for other countries—most prominently, the US and the UK. In order to bring our model to these facts, we integrate an idea put forward by Card (1996), DiNardo et al. (1996), and Freeman (1996) into our basic framework. These authors attribute the rise in within–group wage inequality to the non–adjustment of some institutionally set wages floors (either minimum wages or wages bargained between unions and firms). As wage incomes *in general* rise with increasing productivity of labor, the incomes of workers that just receive the minimum wage income remain unchanged. Consequently, within–group wage inequality rises if wage floors do not keep up with increases in labor productivity. Card (1996) and DiNardo et al. (1996) claim this to be the case especially for the US and the UK, either because minimum wages did not keep up with the productivity increase (US) or because of declining bargaining power of unions (UK).

Integrating such an argument into our model makes our results consistent with these country–specific observations. Moreover, our analysis sheds light on the differences in the evolution of unemployment rates between the US or UK on the one hand, and Germany on the other.

We concentrate on the basic mechanism and present the argument in a rather informal way rather than adapting the complete general–equilibrium structure of our model. Suppose there is an institutionally set wage floor below which wage incomes paid in the economy must not lie. For simplicity, we take this minimum wage income, $\overline{w}$, as given exogenously. As a result of a binding wage floor, there are individuals with a productivity that is too low to guarantee them a wage income of $\overline{w}$. Consequently, these individuals are not employed by any of the firms. For given effective wage w, the threshold ability to become employed is $\overline{w}/w$.[17]

As w rises along the adjustment path (we again assume $K_0 < K^*$), the threshold ability $\overline{w}/w$ falls over time if wage floors are not adjusted. In reaction to that, some individuals that were previously unemployed can now get a job. Additionally—as in the case without institutional wage rigidities—, more people acquire education, thus increasing the number of skilled workers. While these changes in the composition of both groups of labor do not affect inequal-

[17]We exclude for simplicity the possibility that the least able can alternatively acquire education thus raising their productivity above the threshold.

ity for the high–skilled, they have an important consequence for the low–skilled group. As the most able leave this group for skilled work, and individuals with lower abilities enter the group, it is now by no means clear that within–group wage dispersion must be decreasing. Hence, increasing inequality for the low–earnings group may result if wage floors are not adjusted.

The change in the threshold ability that enables individuals to become employed also affects the rate of unemployment. As $\overline{w}/w$ declines along the adjustment path, a greater range of abilities becomes employed. Therefore, the unemployment rate should fall due to the non–adjustment of wage floors. On the other hand, if wage floors are adjusted and keep up with the rise in labor productivity, the unemployment rate will not be affected by labor–productivity gains. Thus, there is a trade off between wage inequality and employment opportunities for unskilled labor. In the US and the UK, the extraordinary reductions in the unemployment rates (esp. for unskilled labor) came at cost of a considerable rise in residual wage inequality. In contrast, the success of German unions to keep up bargained wages to the rise in productivity reduced residual inequality at cost of persistent high unemployment there.

7 Conclusions

This paper analyzed the development of wage inequality by skills within a simple neoclassical growth model with heterogeneous labor. The change in incentives for human–capital investments combined with a convex education technology accounts for a non–monotonicity of the wage differential by skills, which is observed in the data. The development of wage dispersion within skill groups is also consistent with empirical facts from a number of countries, esp. if we additionally incorporate some non–adjustment of wage floors—a feature that has been claimed to be important for the US and the UK. A model extended along these lines also accounts for observed differences in the behavior of the unemployment rates over time. It emphasizes the trade off between wage inequality and employment opportunities.

We do not claim the model to be a comprehensive explanation of the wage structure. It is rather meant to be a complement than a substitute to popular explanations emphasizing the change in relative labor demand (due to globalization or skilled–biased technological change). Although the predictions are consistent with the empirical facts, we do not assert that our mechanism is solely responsible for the evolution of the wage structure over some longer time perspective. The paper was to show that a simple supply–side model where changing factor prices determine human–capital–accumulation incentives is compatible with many of the empirical observations. There is no absolute necessity to refer to exogenous ingredients to explain the data. Changes in the degree of market imperfection (used by Galor and Moav, 2000a) or a larger number of people attending college possibly to avoid the Vietnam war (Acemoglu, 1998) may account for the non–monotonicity of wage differential by skills in particular countries. However, one might object that the observed coherence of the movement of the relative wage in many countries is unlikely to be a consequence of those specific changes. In being more general in this sense and not very specific, the present model encompasses these problems, allows for new insights and extends the existing literature.

Appendix

This appendix contains the proof of Lemma 4. The proof makes use of the following lemma:

Lemma A.1 *The outcome of the competitive economy is equivalent to the outcome in a command economy where a social planner chooses an allocation that maximizes aggregate output.*

We proof this equivalence first.

Proof. A social planner that tries to maximize aggregate output has to decide how to allocate given initial K between production and human–capital acquisition (i.e. he has to choose how much capital K_P to apply as a direct input to production and how much capital K_I to use for education). His problem is

$$\max_{K_I, K_P} \left\{ F(K_P, H) \ : \ K \geq K_P + K_I \, , \, K_I = (\overline{a} - a_1)/(\overline{a} - \underline{a}) I \, , \right.$$
$$H = \int_{\underline{a}}^{a_1} \frac{a}{\overline{a} - \underline{a}} \, da + \int_{a_1}^{\overline{a}} \frac{a^2}{\overline{a} - \underline{a}} \, da \, ,$$
$$\left. K_P \geq 0 \, , \, 0 \leq K_I \leq I \right\} ,$$

where a_1 denotes the threshold ability chosen by the planner. That optimization problem can be recast as

$$\max_{a_1} \left\{ F \left(K - \frac{\overline{a} - a_1}{\overline{a} - \underline{a}} I, \frac{1}{2} \frac{a_1^2 - \underline{a}^2}{\overline{a} - \underline{a}} + \frac{1}{3} \frac{\overline{a}^3 - a_1^3}{\overline{a} - \underline{a}} \right) \ : \ \underline{a} \leq a_1 \leq \overline{a} \right\} .$$

Some simple calculations show that for $K < \underline{K}$ no physical capital is used for additional qualification of the workers, as in this case $F_K(K, \underline{H}) > F_H(K, \underline{H})(\overline{a}^2 - \overline{a})/I$ according to the definition of $\underline{K}$. This result is analogous to the corresponding competitive equilibria in which there are no skilled workers. For $K \geq \overline{K}$ all human–capital–investment opportunities are exploited and the dictator uses additional capital as a direct production input. For $K \in [\underline{K}, \overline{K}]$, the command economy has an interior solution with a_1 implicitly given by

$$F_K \left(K - \frac{\overline{a} - a_1}{\overline{a} - \underline{a}} I, \frac{1}{2} \frac{a_1^2 - \underline{a}^2}{\overline{a} - \underline{a}} + \frac{1}{3} \frac{\overline{a}^3 - a_1^3}{\overline{a} - \underline{a}} \right)$$
$$- F_H \left(\frac{\overline{a} - a_1}{\overline{a} - \underline{a}} I, \frac{1}{2} \frac{a_1^2 - \underline{a}^2}{\overline{a} - \underline{a}} + \frac{1}{3} \frac{\overline{a}^3 - a_1^3}{\overline{a} - \underline{a}} \right) (a_1^2 - a_1)/I = 0$$

Obviously this corresponds to a competitive equilibrium with

$$k = \frac{K - \frac{\overline{a} - a_1}{\overline{a} - \underline{a}} I}{\frac{1}{2} \frac{a_1^2 - \underline{a}^2}{\overline{a} - \underline{a}} + \frac{1}{3} \frac{\overline{a}^3 - a_1^3}{\overline{a} - \underline{a}}} ,$$

as $1+r = f'(k)$ and $w = f(k) - kf'(k)$ imply a threshold ability a_1. As competitive equilibria for given K are unique, the equivalence is proven. □

We are now able to prove Lemma 4.

Proof. As we have shown the equivalence of the competitive and the command equilibrium, the continuity between K and k (cf. Corollary 1) holds in the command economy, as well. For $K > 0$, the threshold ability above which the planner chooses to upgrade human capital is given by the $a_s(K)$ (the corresponding function a_s was introduced in (7)). An additional unit of capital yields more output according to $F_K(K - (\overline{a} - a_1)(\overline{a} - \underline{a})I, H(a_1))$ (the envelope theorem).[18] The equivalence between the environments implies that the marginal product equals $f'(g(K))$. Continuity of g yields continuity of dY/dK. Concavity of the per-capita production function f and the monotonicity of g prove the claim. □

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[18] For $a_1 \in [\underline{a}, \overline{a}]$ this is due to the fact that in the margin capital is equally useful in both applications.

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