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## Measuring the burden of taxation: An index number approach

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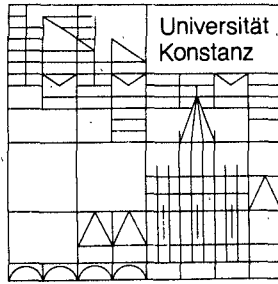
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**Fakultät für  
Wirtschaftswissenschaften  
und Statistik**

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Bernd Genser

**Measuring the Burden of Taxation:  
An Index Number Approach**

Diskussionsbeiträge

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MEASURING THE BURDEN OF TAXATION:

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Bernd Genser

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# Measuring the Burden of Taxation: An Index Number Approach

Bernd Genser

## Abstract

Quantitative indicators of the tax burden used in political discussions quite often do not prove very reliable for two reasons; either they are rather crude summary measures or they are rather selectively chosen benchmark tax load figures. A natural way to improve the situation is to introduce an index number concept.

The paper deals with index numbers for commodity taxes first. Fix weight commodity tax indices are defined and calculated numerically for Austria in close relation to official statistical price index analogues. To overcome the bias of Laspeyres indices economic tax burden indices are defined, which may be extended to include the deadweight loss of distorting consumer taxes as a burden element.

Direct taxes may be included in both statistical and economic household outlay indices, which also are closely related to the well known price indices. Nevertheless many convenient properties do not hold any more due to endogenous labour supply, distortions caused by the income tax code, and progressive tax schedules.

Although the economic approach tends to relate the tax index number concept to applied general equilibrium models, the empirical relevance of tax burden indices is demonstrated by calculating and interpreting monthly time series for Austria.

## Zusammenfassung

Quantitative Indikatoren der Steuerbelastung, die in der tagespolitischen Diskussion herangezogen werden, erweisen sich häufig aus zwei Gründen als wenig verlässlich; entweder sind sie aus groben und häufig periodenverzerrten Aggregatgrößen gewonnen oder sie sind recht willkürlich bis bewußt überzeichnend gewählte Belastungsdaten für spezielle Steuerpflichtige. Ein natürlicher Ausweg aus dieser unbefriedigenden Situation ist die Heranziehung eines Steuerindexkonzepts.

In der Arbeit werden zunächst Gütersteuerindizes mit einem intertemporal fixen Gewichtungsschema definiert und numerische Ergebnisse für Österreich angeführt, die größtenteils auf dem Datenmaterial der offiziellen Verbraucherpreisstatistik basieren. Um die systematische Verzerrung von Laspeyres-Indizes zu vermeiden, werden ökonomische Steuerlastindizes definiert, die derart erweitert werden können, daß sie auch den von verzerrenden Gütersteuern verursachten zusätzlichen Wohlfahrtsverlust als Belastungskomponente zu erfassen vermögen.

Auch direkte Steuern können in statistischen oder ökonomischen Haushaltsausgabenindizes berücksichtigt werden, die gleichfalls mit den bekannten Preisindizes eng zusammenhängen. Allerdings gelten für die verallgemeinerten Ausgabenfunktionen und die mit ihrer Hilfe gebildeten Indizes eine Reihe überaus angenehmer Eigenschaften nicht mehr, wofür die modellendogene Arbeitsangebotsentscheidung, allokativer Verzerrungen aufgrund der geltenden Einkommensbesteuerung und die Progressivität des Einkommensteuertarifs verantwortlich sind.

Während der ökonomische Ansatz das aufgeworfene Steuerlastindexkonzept eher als Ergänzung zu angewandten allgemeinen Gleichgewichtsmodellen ansieht, betont zum anderen die konkrete Berechnung und Interpretation von Monatsreihen einiger Steuerlastindizes für Österreich doch deutlich die empirische Relevanz der vorgestellten Konzepte.

MEASURING THE BURDEN OF TAXATION:  
AN INDEX NUMBER APPROACH\*

Bernd Genser

Measuring the burden which taxes impose upon a representative citizen has been both an issue of public finance theory and of fiscal policy analysis for a long time. Nevertheless official national and international statistics offer rather crude tax/income ratios (based on summary figures of budget data and/or national account statistics or on imputed figures of specific benchmark households) only. Standardized measures of the tax burden, similar to indices used in price, wage, or production statistics are still lacking although such measures should serve as more reliable quantitative indicators to judge tax policy programmes which dominate today's political discussions.

This paper offers a guideline to establish tax burden indices concentrating on commodity taxes in the first sections but tackles the measurement of direct household taxes too. Having shown elsewhere (Genser 1985, 1985a) that simple summary measures, viz. aggregate elasticities, may be rather misleading to illustrate changes in the burden of Austrian consumer taxes I introduce a Laspeyres consumer tax index and a related consumer tax quota in section 1. Both measures can be calculated from disaggregated price index data. To overcome the bias of a Laspeyres index economic tax burden indices are defined in section 2, which may be extended to include the tax burden caused by the deadweight loss of distorting consumer taxes. In section 3 the consumer tax index concept is modified further integrating income tax and social insurance contributions thus catching the burden of direct and indirect household taxes simultaneously. These direct taxes can also be included in an economic household outlay index (section 4). Some additional remarks on the usefulness of tax burden indices conclude the paper.

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## 1. The consumer tax index

Commodity taxes are a part of consumer prices and thus are included within the costs of living. Changing commodity tax rates give rise to economic adjustment processes and the resulting price changes can be measured by a cost of living index.

Usually the variety of influences on consumer prices will not allow to identify the tax induced component of price movements. Within an economic model the tax effects on consumer prices can be studied by splitting up consumer prices  $p$  into a producer price component  $\pi$  and a consumer tax component  $t$ .

$$p(x) = t + \pi(x) \quad (1)$$

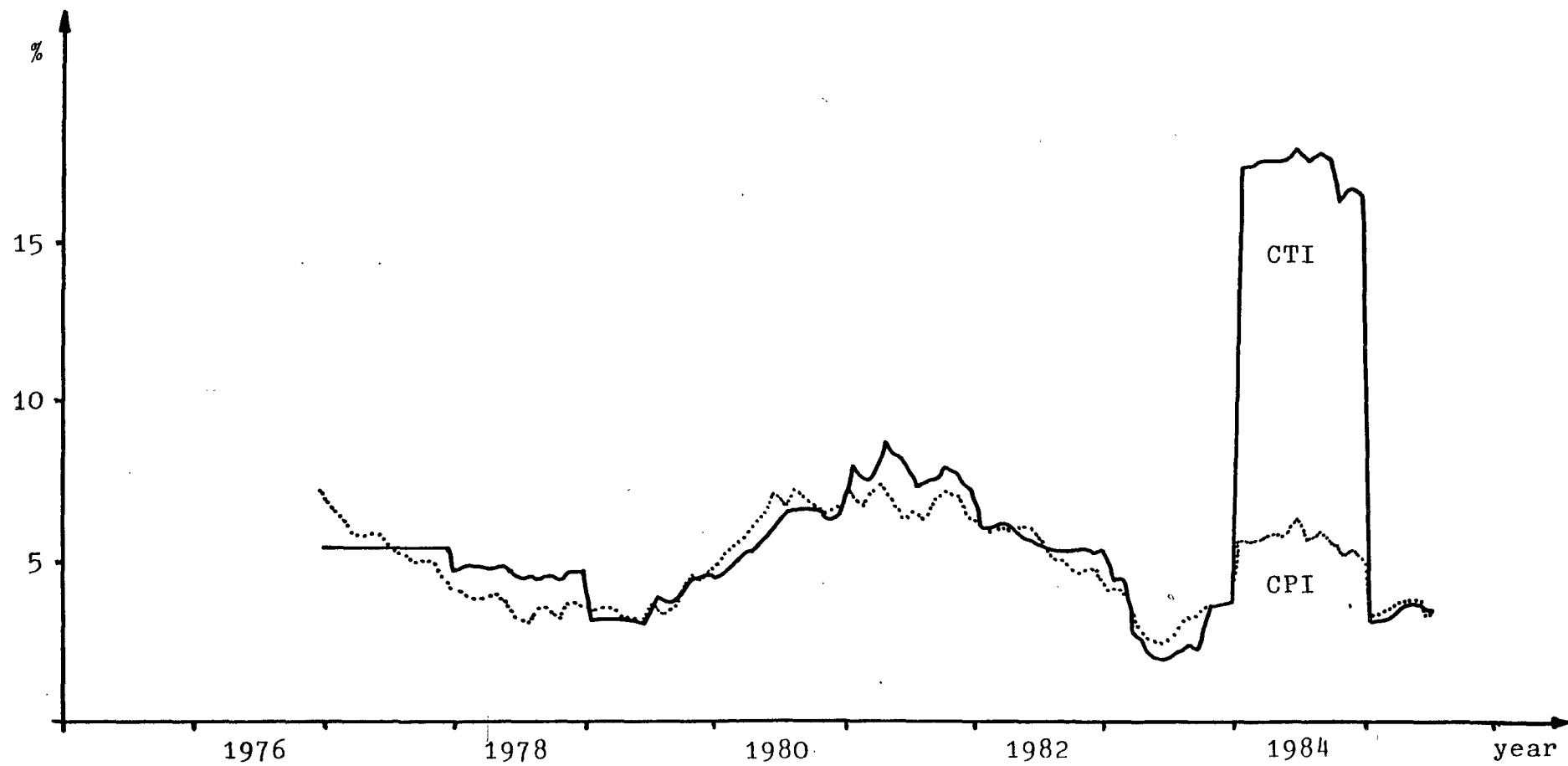
The structure of the underlying technology determines how tax rate changes influence consumer prices via producer price adjustments besides the direct tax effect.

Nevertheless it is always possible to identify the tax component of any consumer price equilibrium ex post and thus a broad variety of consumer tax indices might be defined in formal analogy to different types of consumer price indices. If the investigator is free to choose a certain index concept his selection will depend on a list of desirable properties which such an index should fulfill. For price indices the Fisher tests (Fisher 1922, Eichhorn/Voeller 1976) or recent attempts to base certain types of index numbers on the theory of functional equations (Eichhorn 1978) offer an apt survey. A catalogue of basic as well as desirable properties for consumer tax indices is listed in Genser (1985). Some of these properties, which should be helpful to judge a consumer tax index concept, are referred to below.

To calculate a consumer tax index for Austria numerically all potential candidates for indices had to be omitted which would have needed data not yet available. In this context a Laspeyres fix weight consumer tax index has proved most promising as this index is closely related to the official consumer price index and makes use of the data fund of monthly consumer price statistics.

The current Austrian consumer price index CPI is defined by means of prices  $p_i$  and quantities  $q_i$  of 582 commodity items, the superscript 0

Fig. 1: Consumer Tax Index and Consumer Price Index (annual growth rates)





refers to the base period 1976.

$$CPI = \sum p_i q_i^0 / \sum p_i^0 q_i^0 = \sum [p_i^0 q_i^0 / \sum p_j^0 q_j^0] \cdot [p_i / p_i^0] = \sum w_i RP_i \quad (2)$$

To catch the price movements across the whole nation relative price changes are reported instead of commodity prices and the CPI is calculated monthly as the weighted average of the national means of relative commodity price changes  $RP_i$ , the weights  $w_i$  being the expenditure shares of the commodity items in the base period consumer basket (ÖStZ 1976).

In analogy to (2) a consumer tax index CTI can be defined which making use of the same transformations reads as

$$\begin{aligned} CTI &= \sum t_i q_i^0 / \sum t_i^0 q_i^0 = \sum w_i [t_i / p_i^0] / \sum w_i [t_i^0 / p_i^0] = \\ &= \sum w_i RP_i [t_i / p_i] / \sum w_i [t_i^0 / p_i^0] \end{aligned} \quad (3)$$

with  $t_i$  the unit tax component included in the consumer price  $p_i$  of commodity  $i$ .

For a numerical calculation of the CTI according to (3) weights  $w_i$  and relative prices  $RP_i$  can be used from the monthly price statistics. As most of the commodity taxes are ad valorem taxes most tax ratios needed in (3) are determined directly by tax laws. Only for some unit taxes (petrol tax, beer tax, alcohol monopoly duty) actual tax and price data are necessary to calculate the consumer tax index. A monthly time series of an Austrian CTI has been calculated (Genser 1985a) which includes ten major commodity taxes (value-added tax, tobacco tax, petrol taxes, alcohol tax, motor vehicle tax, insurance tax, beverage tax, beer tax, alcohol monopoly duty). The time profile of annual growth rates is shown in fig. 1. The change in the relative tax burden can be illustrated by comparing the monthly figures of the CTI with the CPI, which measures the relative increase of the costs for the whole commodity basket.

The graphical analysis shows very clearly the "upward shift of the commodity tax burden in 1978, 1981 and 1984, when VAT rates were increased, but it also catches minor tax reliefs caused by inflationary erosion of unit taxes, especially the petrol tax and the motor vehicle excise tax in 1980 and 1981, and the consequences of inhomogeneous price

movements, which lower the tax burden if the prices of highly taxed "luxuries" rise less than those of lowly taxed "necessities" (or vice versa).

Besides this advantage of relying on official price data the consumer tax index (3) also fulfills important basic properties (monotonicity, homogeneity, identity, dimensionality and permutation invariance) and it satisfies the time reversal test, the circularity test; the determinateness test, and the decomposability test (cf. Genser 1985). The only well-known desideratum the CTI fails to pass is Fisher's factor reversal test or its less restrictive version, the product test. But opposite to the case of the price/quantity index symmetry the failure of the "quantity index of tax bases" defined by (3) implicitly

$$CTB = (\sum t_i q_i / \sum t_i^0 q_i^0) / CTI = \sum t_i q_i / \sum t_i q_i^0 \quad (4)$$

to pass the same tests as CTI, viz. the time reversal and the circularity test, thus violating the product test does not seem very harmful if one considers the minor economic relevance of such a tax base index.

Changes in the relative tax burden may be quantified by means of a consumer tax quota which measures consumer taxes as a share of overall commodity expenditures.

$$CTQ = \sum t_i q_i^0 / \sum p_i q_i^0 = CTI \cdot \sum t_i^0 q_i^0 / [CPI \cdot \sum p_i^0 q_i^0] = CTQ^0 \cdot CTI / CPI \quad (5)$$

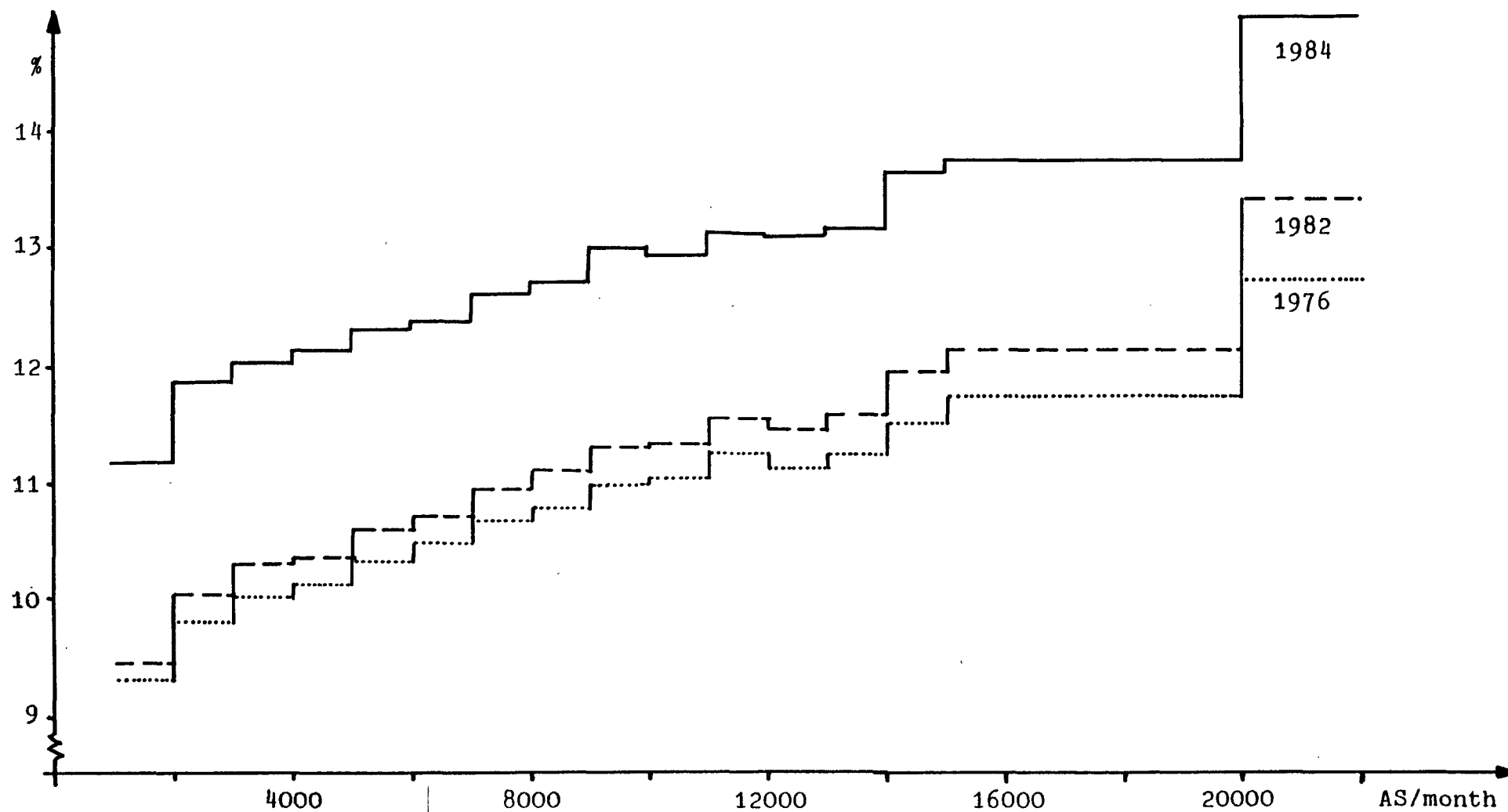
The standardized consumer tax quota turns out to be simply the base year quota adjusted by the ratio of consumer tax index and consumer price index. Formula (5) also offers a rather convenient factorization of the CTI

$$CTI = (CTQ / CTQ^0) \cdot CPI \quad (6)$$

splitting up the growth factor of standardized tax load into a "tax quota index" catching the change in the average tax rate between base and current period and into the pure inflationary trend CPI.

The quantitative analysis may be refined by disaggregating the consumer tax index into subindices of selected commodity subgroups or of single commodity taxes making use of the decomposition property of CTI. A further step of empirical analysis should dismiss the single represen-

Fig. 2: Value-added tax/expenditure ratio for different household expenditure levels



tative household concept and distinguish various household classes. A breakdown of households with respect to household income offers insight into the formal incidence of commodity taxes and allows an empirical test of the regressivity hypothesis of indirect taxes. A first quantitative analysis indicates that the graduation of the Austrian value-added tax is likely to produce a progressive structure of commodity taxes with respect to household budgets (fig. 2). Although reliable data on household saving are lacking, progressivity with respect to household income would only prevail if saving elasticities were rather close to one.

## 2. Economic consumer tax indices

Whereas fixed weight indices are very attractive from a statistical viewpoint, they suffer from a severe conceptional shortcoming by economic standards. By fixing weights, as for the Laspeyres price index, substitutional adjustments made by economic agents are ignored by definition. Thus also the information on changes in the tax burden mirrored in the CTI is biased. From an empirical point of view one might be tempted to shut one's eyes to this problem of the CTI, as long as the same faultiness arises with the CPI which is widely recognized as the economic indicator of inflation and one of the most important policy targets.

Nevertheless this pragmatic view of an empirical investigator does not render superfluous attempts to elaborate the concept of a "true consumer tax index". To catch economic behavior such an index must be based in the economic framework of rational household decisions. Again the formal analogy to economic price indices provides a useful guideline.

### 2.1 The Konyus consumer tax index

The Konyus price index, perhaps still better known as the "true cost of living index", is most conveniently defined by

$$CPI^K = e(p, u(q^*)) / e(p^0, u(q^*)) \quad (7)$$

$e(p, u(q^*))$  is the expenditure function which measures the minimal budget allowing the rational consumer to realize a welfare level  $u^*$  =

$u(q^*)$ , fixed by an arbitrarily chosen reference commodity bundle  $q^*$ , when the commodity price vector is  $p$ . The Konyus index  $CPI^K$  is the ratio of these two minimal budgets and by this compares the two price situations  $p^0$  and  $p$ .

Due to its definition the expenditure function has a lot of convenient properties, viz. monotonicity in  $p$  and in  $u$ , linear homogeneity in  $p$ , concavity in  $p$ , continuity in  $p$  (cf. Diewert 1982). If we additionally assume differentiability in  $p$  we can apply the Euler Theorem for homogeneous functions and we arrive at a representation of the expenditure function with the help of the Hicksian compensated demand functions  $q_i^c(p, u^*)$  (Shephard-Lemma)

$$\sum p_i [\partial e(p, u^*) / \partial p_i] = e(p, u^*) = \sum p_i \cdot q_i^c(p, u^*) \quad (8)$$

The Konyus price index can therefore be written equivalently as

$$CPI^K(p^0, p, u^*) = [\sum p q^c(p, u^*)] / [\sum p^0 q^c(p^0, u^*)] \quad (9)$$

where we have changed to vector notation and scalar products to simplify the formulae.

Concentrating on the consumer tax components included in consumer prices, given a commodity tax system, a Konyus consumer tax index may be defined

$$CTI^K(t^0, p^0, t, p, u^*) = [\sum t q^c(p, u^*)] / [\sum t^0 q^c(p^0, u^*)] \quad (10)$$

Index (10) allows one to compare the tax payments of a rational, utility maximizing consumer who adjusts his commodity bundle to changes in both the commodity tax and the commodity price vector and at the same time is fully income compensated to reach the reference utility level  $u^*$  at best.

Whereas the Konyus price index can be shown to fulfill a series of desirable properties, eg. the tests mentioned in the previous section (cf. Diewert 1981, Genser 1985), the Konyus commodity tax index seems less attractive. Major desiderata, eg. homogeneity, proportionality, dimensionality, do no longer hold for  $CTI^K$ . Nevertheless the Konyus consumer tax index may be proved to inherit the time reversibility and the circularity property from the Konyus price index, as well as the

invariance of utility level property for homothetic preferences.

As regards bounds for  $CTI^K$  the **mean** value property of the price index cannot be reestablished, but similar interval bounds may be shown to hold, shifting the extreme tax rate ratios by some "Laspeyres tax base index". For the lower bound of  $CTI^K$  we get

$$CTI^K = tq^c(p, u^*) / t^o q^c(p^o, u^*) = ( \sum (t_i / t_i^o) t_i^o q^c(p, u^*) ) / t^o q^c(p^o, u^*) \geq \min\{t_i / t_i^o\} \cdot t^o q^c(p, u^*) / t^o q^c(p^o, u^*) = \min\{t_i / t_i^o\} TBI \quad (11)$$

Combined with an analogous upper bound  $CTI$  is bounded by

$$\max\{t_i / t_i^o\} \cdot TBI \geq CTI^K \geq \min\{t_i / t_i^o\} TBI \quad (12)$$

In a similar way inequalities can be found which resemble the well known Laspeyres and Paasche bounds of the Konyus index, but modified by a factor catching the maximum variation of compensated commodity changes

$$t^1 q^c(p^1, u^o) / t^o q^c(p^o, u^o) \leq CTI^L \cdot \max\{q_i^c(p^1, u^o) / q_i^c(p^o, u^o)\}$$

$$t^1 q^c(p^1, u^1) / t^o q^c(p^o, u^1) \geq CTI^P \cdot \max\{q_i^c(p^1, u^1) / q_i^c(p^o, u^1)\}$$

For homothetic preferences the two inequalities can be combined yielding a transformed interval of Laspeyres and Paasche bounds for the Konyus commodity tax index.

The Konyus consumer tax index and the Konyus price index can be linked by a standardized tax quota which measures the share of consumer taxes included in the minimum budget for the utility compensated commodity basket.

$$CTQ^K(t, p, u^*) = [tq^c(p, u^*)] / [pq^c(p, u^*)] = CTQ^{K, o} \cdot CTI^K / CPI^K \quad (14)$$

Reshuffling (14) allows to split up the Konyus commodity tax index into a "Konyus tax quota index" and the true inflationary trend CPI in analogy to formula (6). If the expenditure function and thus also the compensated demands were known, then by imputation of consumer tax rates both the Konyus consumer tax index and the corresponding consumer tax quota may be calculated as well.

## 2.2 The price increment index

The Konyus consumer tax index catches the loss in private purchasing power due to the payment of taxes. But it is well known that commodity taxes usually do not only shift purchasing power to the public sector but additionally reduce it due to a deadweight efficiency loss. This welfare reduction of taxation imposes an excess burden on the consumer which should be taken into account if the total economic burden of a tax system is considered.

Formally commodity taxes cause an excess burden, if it is possible to withdraw a lump-sum tax which is larger than the amount of commodity taxes without reducing the consumer's welfare level, viz. if the strict inequality sign holds in the net expenditure relation.

$$e(p-t, u^*) \leq e(p, u^*) - tq^c(p, u^*) \quad (14)$$

In extending the burden consideration all the loss in purchasing power connected with the existence of commodity taxes can be regarded simultaneously. I have defined an absolute price increment of taxation  $d(t, p, u^*)$  as the rise of the minimal budget which is necessary to realize a reference welfare level  $u^*$  if commodity taxes  $t$  exist opposed to the fictitious no tax case.

$$d(t, p, u^*) = e(p, u^*) - e(p-t, u^*) \geq tq^c(p, u^*) \quad (15)$$

A comparison of two commodity tax structures differing because of a change in commodity taxes can be made by looking at their absolute price increments. The ratio of two price increments may be called price increment index

$$PII^k(t^0, t, p^0, p, u^*) = d(t, p, u^*) / d(t^0, p^0, u^*) \quad (16)$$

where the superscript  $k$  again refers to the embedding of (16) in the Konyus conceptional framework.  $PII^k$  measures the relative increase of additional financial means necessary to realize the fixed welfare level  $u^*$  if commodity taxes are changed. This incremental budget flows partly to the fiscal authority as commodity tax, the rest is an income compensation for the deadweight loss due to efficiency distortions. Evidently, the price increment index will coincide with the Konyus price index if no excess burden occurs, viz. the inequality sign in (15) holds thus

indicating that a commodity tax system is non-distorting and equivalent to a lump sum tax.

Again a burden quota can be defined connecting  $PII^K$  with the true cost of living index

$$PIQ^K(t, p, u^*) = d(t, p, u^*) / e(p, u^*) = PIQ^{K,0} \cdot PII^K / CPI^K \quad (17)$$

The relative price increment of commodity taxes  $PIQ^K$  indicates the share of the minimal budget which does not contribute to individual welfare but is absorbed by tax payments and efficiency losses.

To arrive at numerical calculations for economic consumer tax indices information on complete consumer demand systems is necessary. To demonstrate the differences between consumer tax indices Genser (1985) has considered the special case of fuel taxation in Austria. Fuel is subject to two mineral oil taxes (since 1982 integrated) based on the weight of the fuels and to two ad-valorem taxes, the value-added tax and a special duty on mineral-oil. Within a two commodity world it is possible to arrive at explicit solutions for the expenditure function by integrating the system of Roy-Ville identities if Marshallian demand functions are given (cf. Hausman 1981). Annual growth rates of the Laspeyres, the Konyus and the price increment index for Austrian fuel taxes are shown in Table 1.

Basically, the Laspeyres index shows smaller rates of growth in periods of stable petrol prices, when economic indices reflect an increase in compensated fuel demand, whereas it shows higher rates than the economic indices, when petrol prices were rising during the second oil shock. In quantitative terms the differences are rather small, as might have been suspected, because of the low price elasticity for fuel demand. Thus, the Laspeyres and Konyus indices do not deviate very much as compensated and Marshallian demands are only slightly different. The excess burden of major fuel tax changes remains small too and does not cause differences between the Konyus and the price increment index.



Table 1: Fuel tax indices and total fuel taxes (annual growth rates)

	CTI <sup>L</sup>	CTI <sup>K</sup>	PII	Fuel Taxes
1977	1.9	3.1	2.8	6.7
1978	0.0	0.8	0.6	4.2
1979	9.0	7.8	8.2	11.8
1980	8.7	6.7	6.8	2.7
1981	19.4	16.0	17.0	16.4
1982	2.7	3.0	2.8	5.0
1983	-0.8	0.5	0.2	3.2
average 76/83	5.7	5.3	5.35	7.0

Source: Genser (1985)

### 3. Integration of direct taxes in statistical tax and price indices

The structure of the Laspeyres index offers the possibility to extend a tax index stepwise if one wants to include further taxes. Thus also direct household taxes (income taxes, social insurance contributions) can be integrated into a generalized index covering both commodity and income taxes. A generalized tax index can be defined simply by extending the quantity vector  $q^0$  by factor supplies  $s^0$  covering the tax bases of direct taxes and the tax rate vector  $t$  by the rates for direct taxes  $r$ .

$$GTI = (tq^0 + rs^0)/(t^0q^0 + r^0s^0) \quad (18)$$

An index of this type is calculated in Canada. The Fraser Institute has been publishing an annual Canadian Consumer Tax Index since 1976 (Pipes/Walker 1982) to illustrate the growing tax burden on the average Canadian consumer.

If one is interested in direct taxes primarily then a comparison of tax burdens can also be made by using a generalized cost of living index, which includes household outlays on direct taxes in addition to consumption expenditures

$$TPI = (pq^0 + rs^0)/(p^0q^0 + r^0s^0) \quad (19)$$

The TPI quantifies the increase in household gross income necessary to buy the fixed commodity basket at current prices as well as to pay current direct taxes connected with a fixed factor supply. A monthly "Tax and Price Index" was first published in England in 1979 (CSO 1979). This index has been developed to deflate gross household income more properly than usual price deflators do, as real purchasing power is influenced by changes of both consumer prices and direct household taxes.

To calculate indices like (18) and (19) numerically requires a consistent data base on the relevant determinants of direct household taxes. Besides this statistical problem there is a methodical difficulty, too. Whereas the fixed commodity basket  $q^0$  reflects the commodity tax base of a non-growing economy, the fixed supply of labour is well in accordance with empirical evidence and thus  $s^0$  reflects an adequate wage tax base of a growing economy. The TPI will therefore overemphasize direct taxes as opposed to commodity taxes. A bias of that type as well as some problems of data collection can be avoided by integrating direct household taxes into a household outlay index.

### 3.1 Household outlay index

The household outlay index (HOI) does not look at actual bases of income taxes but postulates an immediate relation between consumer expenditures and gross income which allows a consumer to cover these expenditures after having paid his income taxes. The HOI compares two gross budget levels which are necessary to keep a reference consumption level, taking into account changing prices and payment of direct taxes. A Laspeyres-type HOI is defined by

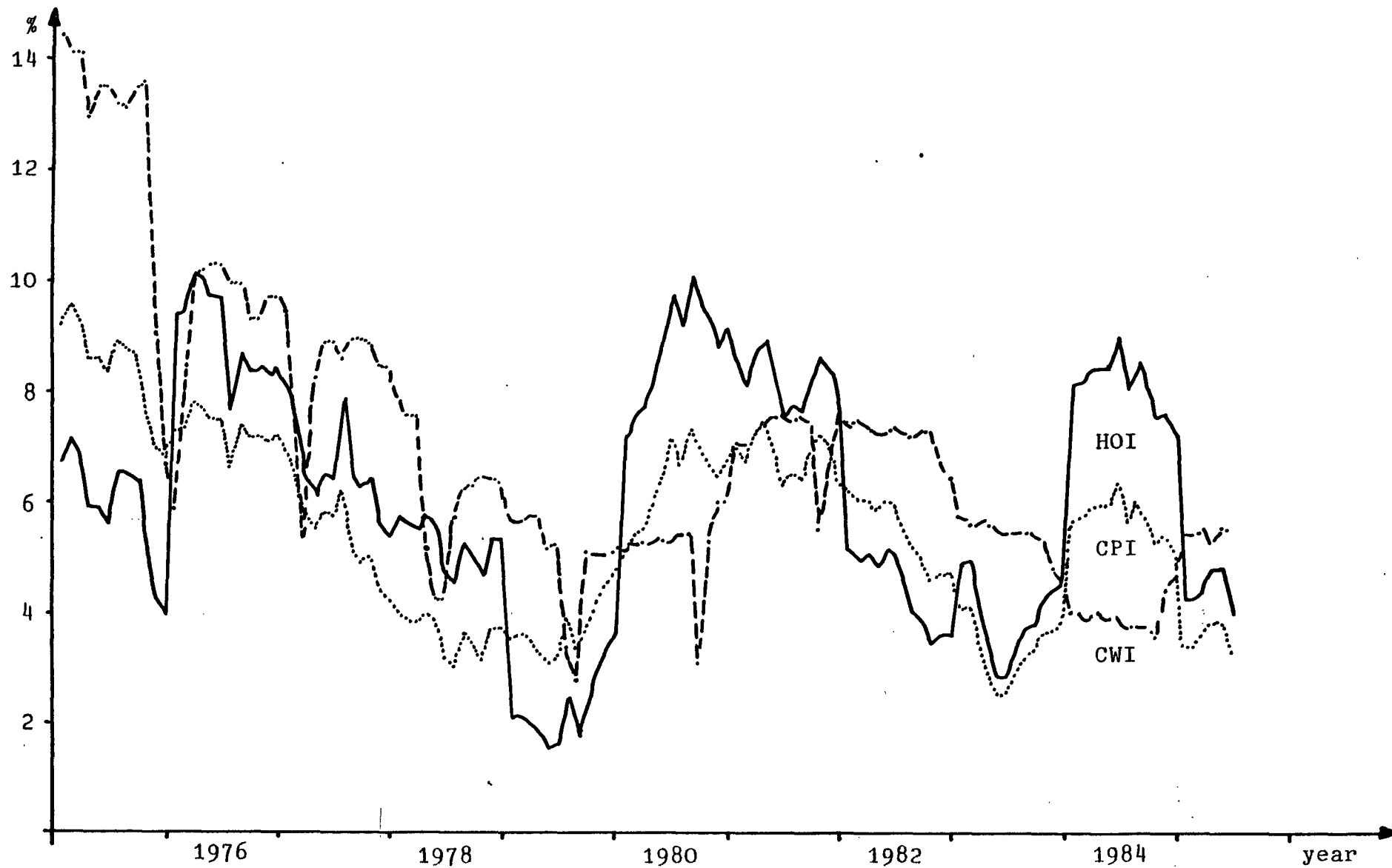
$$HOI = (pq^0 + t(y)) / (p^0q^0 + t^0(y^0)) \quad (20)$$

where  $y$  and  $y^0$  are gross income levels sufficient to buy consumer goods  $q^0$  as well as to pay direct household taxes

$$y = pq^0 + t(y) \quad (21)$$

For a numerical calculation of HOI one certainly needs information about relevant income tax characteristics of the average consumer but no wage rates and other income data. Imputation of monthly figures is

Fig. 3: Household Outlay Index, Consumer Price Index and Contractual Wage Index  
(annual growth rates)



not trivial as progressivity effects and the annualization of income tax liability must be considered to produce a monthly series of HOI for Austria (Genser 1983). A HOI series for white collar workers in Austria (fig. 3) shows quite markedly the fiscal drag of the Austrian income tax which in 1980, 1981, and again in 1984 has led to a loss in net purchasing power although real gross income has grown. On the other hand income tax adjustments show considerable purchasing power gains in 1975, 1979, and 1982. In these years the rise in real income falls short of actual consumer gains, whereas all the other years are characterized by net purchasing power gains less than real income gains.

Along similar lines a Gross Earnings Deflator has been developed for England (Kay/Morris 1984). Monthly data have been published since 1979. Gillingham/Greenlees (1983) have recently used a HOI concept for US data on an annual base.

#### 4. Integration of direct taxes in an economic household outlay index

Critical remarks on the neglect of commodity substitution are justified for the HOI as well as for any other Laspeyres index. By endogenizing commodity demand in (21) one arrives at a gross expenditure function which can be defined implicitly

$$g(p,u) = e(p,u) + t(g(p,u)) \quad (22)$$

Comparing the gross expenditure levels of two tax/price situations leads to the Konyus household outlay index which is defined as

$$HOI^K(t^0, t, p^0, p, u^*) = g(p, u^*) / g(p^0, u^*) \quad (23)$$

As regards index (23) it does not seem convincing to allow for substitution on the commodity side but at the same time to neglect possible consumer reactions on the labour supply side. If consumer decisions are considered within a model of endogenous work/leisure choice, the traditional view of the price index problem has to be changed. Disregarding direct taxes for the moment efficient allocations within a work/leisure model are characterized by an indirect utility function which includes the wage rate  $w$  as a further price parameter.

$$v(p, w, y) = \max_{q, f} \{u(q, f) \mid pq \leq w \cdot (1-f) \text{ with } s+f = 1\} \quad (24)$$

In (24) leisure  $f$  enters the utility function and is normalized as usual to add up to unity together with labour supply  $s$ . Solving the indirect utility function  $v(p, w, y)$  with respect to  $y$  for a given utility level  $u^*$  we may define a modified expenditure function as the solution of the dual minimum problem

$$e(p, w, u^*) = \min \{pq - w(1-f) \mid u(q, f) \geq u^*\} \quad (25)$$

But it is evident that defining a Konyus index with the help of expenditure function (25)

$$CPI^K(p^0, w^0, p, w, u^*) = e(p, w, u^*) / e(p^0, w^0, u^*) \quad (26)$$

will not result in a useful price index. By definition, expenditure function (25) catches only a residual component of the consumer budget, the lump-sum income in excess or short of compensated labour income to cover compensated consumer demand. This lump-sum income may well become zero or negative thus also the price index (26) may be undefined or negative.

Nevertheless a "useful" price index can be regained if the expenditure function is extended. We may define a "full expenditure function" which includes fictitious household outlays on leisure, the opportunity costs of reduced labour income,

$$e^F(p, w, u^*) = \min_{q, f} \{pq + wf \mid u(q, f) \geq u^*\} = w + e(p, w, u^*) > 0 \quad (27)$$

$e^F$  must be positive by definition and  $e^F$  and  $e$  emanate from the same optimal consumption plan because the minimum problems (25) and (27) only differ in a scale shifting of the object function. Thus  $e$  and  $e^F$  describe the same efficient household decision. A Konyus type price index

$$CPI^F(p^0, w^0, p, w, u^*) = e^F(p, w, u^*) / e^F(p^0, w^0, u^*) \quad (28)$$

is always well defined and it measures the growth of the minimal "full expenditure budget" that allows the rational household to realize the reference utility level  $u^*$  after price and wage rate changes.

Following the lines of the household outlay index integration of direct taxes is straightforward. Besides outlays on commodities and leisure also direct taxes have to be paid. Nevertheless we have to consider that leisure is untaxed within existing income tax schemes and a "gross full expenditure function" must take care of this tax rule.

$$g^F(p, w, u^*; t) = \min_{q, f} \{g | u(q, f) \geq u^* \wedge g = pq + wf + t(g - wf)\} \quad (29)$$

$t$  is the income tax function which is applied to any tax base "gross full expenditure less fictitious outlays on leisure" to calculate tax liability. The "full household outlay index"

$$HOI^F = g^F(p, w, u^*; t) / g^F(p^0, w^0, u^*; t^0) \quad (30)$$

compares the rise in gross budget needs of the representative household to cover expenditures on market commodities plus direct taxes to stay equally well off when prices, wages, and/or tax codes change.

Looking at the marginal conditions characterizing the Hicksian demands solving (29) we see immediately that  $g^F$  describes an efficient household decision which basically will deviate from that described by  $e^F$ , when income taxes are ignored. This is of course due to the fact that exempting leisure from taxation creates a distortion in relative prices which will cause excess burden losses if substitutional adjustments are made. However, the optimal plans yielding  $e^F$  and  $g^F$  will coincide, if leisure is taxed too and thus income taxation is lump-sum. It can be shown further that although taking into account income taxes changes some of the well known duality results (homogeneity and symmetry properties),  $g^F$  is still monotonically increasing in prices, wages, and utility (cf. Baye/Black 1986, 47f.).

The full household outlay function  $g^F$  includes both consumer outlays on commodity taxes paid as a share of market prices and on income taxes which have been introduced explicitly in (29). Following the lines of commodity tax measurement in section 2 now again "true economic tax burden indices" and "standardized tax quotas" can be defined consistently for any tax or group of taxes desired by imputation of the respective tax amounts contained in  $g^F$ .

As regards progressivity analyses of the income tax an approach may be adopted which compares the current standardized tax load with a ficti-

cious proportional income tax schedule determined by the average tax rate of some base period. For this special case of taxing income at a fixed tax rate  $\tau$  the full household outlay function can be shown to be linearly homogeneous in its price variables

$$\begin{aligned} g^F(\lambda p, \lambda w, u^*; \tau) &= \min\{g | u(q, f) \geq u^* \wedge g = \lambda p q + \lambda w f + \tau(g - \lambda w f)\} = \\ &= \min_{q, f} \{\lambda \cdot \tilde{g} | u(q, f) \geq u^* \wedge g/\lambda = \tilde{g} = p q + w f + \tau(\tilde{g} - w f)\} = \\ &= \lambda g^F(p, w, u^*; \tau) \end{aligned} \quad (31)$$

a condition which ceases to hold for progressive or regressive tax codes. Nevertheless the proportionality benchmark case does not rule out the distorting effects of the income tax. If one compares the full household outlay function with a suitable expenditure function for a fictitious no tax case one is able to catch the efficiency loss due to the excess burden and to quantify the total burden of a distorting tax or tax system.

## 5. Concluding remarks

A natural way to improve the situation of lacking standardized measures of tax burden has been the introduction of an index concept. Statistical tax indices may be defined and calculated numerically in close relation to statistical price indices.

Like their price analogues statistical tax burden indices are biased, which calls for the development of economic tax burden indices. Although making use of the duality approach allows to define rather attractive economic indices, which even allow to quantify the burden due to the deadweight loss of welfare caused by distortionary taxes, economic tax burden indices do not possess certain convenient properties of cost of living indices. The situation even worsens if a progressive income tax is considered within a work/leisure model besides commodity taxes. Modified expenditure functions lose some of their important and convenient properties, but still allow to arrive at consistent measures of the joint burden of direct and indirect household taxes.

The question how closely true economic burden measures are approximated by statistical indices, which can be calculated using the existing data base, cannot be solved analytically. Nevertheless the instruments used to define the indices fit into the framework of applied general

equilibrium models, which have recently been developed for some countries (cf. Shoven/Whalley 1984) and are being developed for some others. These models should offer further insight whether tax burden indices calculated from empirical data will be able to answer relevant questions, which can only be posed within a fully specified general equilibrium model at the moment.

## References

- Baye, Michael / Black, Dan (1986): Consumer Behavior, Cost of Living Measures, and the Income Tax. Springer, Berlin.
- Central Statistical Office (1979): The Tax and Price Index - Sources and Methods. Economic Trends 310 (August), 81-89.
- Diewert, Erwin (1981): The Economic Theory of Index Numbers: A Survey. In: A. Deaton (ed.): The Theory and Measurement of Consumer Behaviour. Cambridge University Press, 163-208.
- Diewert, Erwin (1982): Duality Approaches to Microeconomic Theory. In: K. Arrow / M. Intriligator (eds.): Handbook of Mathematical Economics. Vol. 2, North Holland, Amsterdam, 535-599.
- Eichhorn, Wolfgang (1978): Functional Equations in Economics. Addison-Wesley, Reading.
- Eichhorn, Wolfgang / Voeller, Joachim (1976): Theory of the Price Index. Fisher's Test Approach and Generalisations. Springer, Berlin.
- Fisher, Irving (1922): The Making of Index Numbers. Riverside Press, Cambridge (3rd ed.).
- Genser, Bernd (1983): Household Outlay Index. A Measure of Real Purchasing Power and Tax Progressivity. Empirica 10, 205-214.
- Genser, Bernd (1985): Steuerlastindizes. Theorie und empirische Evidenz. Springer, Berlin.
- Genser, Bernd (1985a): Die Messung der Konsumsteuerbelastung. Forschungsbericht 8501, L.Boltzmann Institut für ökonomische Analysen, Wien.
- Gillingham, Robert / Greenlees, John (1983): Incorporation of Direct Taxes into a Consumer Price Index. In: E. Diewert / C. Montmarquette (eds.): Price Level Measurement. Statistics Canada, Ottawa, 619-654.
- Hausman, Jerry (1981): Exact Consumer's Surplus and Deadweight Loss. American Economic Review 71, 662-676.
- Kay, John / Morris, Nick (1984): The Gross Earnings Deflator. Economic Journal 94, 357-369.



Pipes, Sally / Walker, Michael (1982): Tax Facts 3: The Canadian Consumer Tax Index and You. Fraser Institute, Vancouver.

Shoven, John / Whalley, John (1984): Applied General-Equilibrium Models of Taxation and International Trade: An Introduction and Survey. Journal of Economic Literature 22, 1007-1051.