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Appropriation of common access natural resources through exploration: A differential game of a claiming rush

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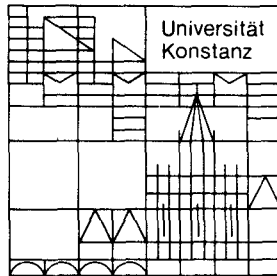
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**Fakultät für
Wirtschaftswissenschaften
und Statistik**

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**Appropriation of Common Access
Natural Resources
Through Exploration:
A Differential Game
of a Claiming Rush**

Diskussionsbeiträge

**APPROPRIATION OF COMMON ACCESS NATURAL
RESOURCES THROUGH EXPLORATION:
A DIFFERENTIAL GAME OF A
CLAIMING RUSH**

ERNST MOHR *

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Abstract

This paper provides a model of natural resource exploration, where the sole motivation to explore arises from a strategic incentive to preempt competitors. It is assumed that private ownership rights over a finite unexplored and commonly held resource stock can be established through a costly deterministic exploration process. The open-loop and the feedback Nash equilibrium is analysed. It is shown that in between intervals of soaring exploratory activities there may be an interval where exploration is declining over time.

Serie A: Volkswirtschaftliche Beiträge

Serie B: Finanzwissenschaftliche Arbeitspapiere

Serie C: Betriebswirtschaftliche Beiträge

1. INTRODUCTION

In the economic literature, two motives for natural resource exploration have been extensively discussed. The first is the incentive to obtain information, where exploration is seen as an input into the production of information on the uncertain size or quality of the resource stock. (For example see Gilbert (1977, 1979), Deshmukh and Pliska (1980), Arrow and Chang (1982), and Hartwick (1983).) The second is the incentive to save on extraction costs through exploration if costs vary inversely with the size of proven reserves. (For example see Peterson (1978), Pindyck (1978, 1980).)

In the model presented here the reason for engaging in a costly exploration activity is different. The abovementioned exploration incentives are absent because, by assumption, uncertainty does not exist and resource extraction is costless. However, a finite stock of unexplored natural resources is assumed to be the common property of a given number of firms, and private property rights over the as of yet unexplored resources can be established through a costly deterministic exploration process. Thus resources appropriated through exploration are lost to other firms. This gives resource exploration a strategic dimension since each firm has an incentive to preempt its competitors on the commons by building up a private stockpile of explored resources. The firms' decision problem consists of balancing this incentive to

preempt (in order not to be preempted) against the incentive to save on discounted exploration costs by keeping a low stock of claimed resources. In order to focus attention solely on the strategic interdependency of the firms on the commons, market interdependencies are eliminated by assuming perfectly competitive resource markets. ¹ The gold rushes in the 19th century provide an illustrative example for the situation discussed in this paper.

The model can also accommodate a number of contemporary situations including those listed below.

Under the U.S. Mining Act of 1872 and the Mineral Leasing Act of 1920, the federal lands are open to the public for certain types of mineral exploration. If private party prospecting is successful, the title to the minerals is transferred in fee simple to the prospector. (See Krutilla and Fisher (1975, p. 21) and Peterson (1978).)

Eckert (1974) and Lecomber (1979, p. 57f), amongst others, have argued that exploration of the deep sea floor establishes private property rights over previously undiscovered commonly held manganese nodules.

A widely discussed common property problem is the case of common pool oil extraction. More recent discussions on this topic can be found in McMillan and Sinn (1984), Eswaran and Lewis (1984) and Reinganum and Stokey (1985). The model to be presented in this paper can accommodate this case if the exploration rate is interpreted as the rate of oil extraction from the common pool, the stock of explored resources as the

stock of oil kept in private storage, and the extraction rate as the rate of sales drawn from those stores. ²

In section 2 the exploration model is developed as a differential game. In sections 3 and 4 the feedback and the open-loop Nash equilibrium is calculated. It is shown that in the former the rent on claimable reserves is completely dissipated while in the latter rent is strictly positive. A notable property of the constrained equilibria is derived; namely that in between intervals of growing exploration activity there is a phase of declining market exploration in the open-loop equilibrium while in the feedback equilibrium market exploration is stagnant in that intermediate interval.

While in sections 3 and 4 firms are exogenously restricted to play feedback or open-loop strategies respectively, in section 5 the Nash equilibrium is derived under the assumption that firms are free to choose between these different types of strategies. It is shown that, if the choice between open-loop and feedback strategies is endogenously determined, the equilibrium to the exploration game is the open-loop Nash equilibrium. In the remainder of section 5 some properties of the open-loop Nash equilibrium are given. Section 6 concludes the paper with some remarks on problems closely related to the exploration game discussed in this paper.

2. THE MODEL

There is a given number n , $n \geq 2$, of symmetrical firms in a competitive market for a nonrenewable natural resource of homogenous quality. Let $P = f(q^m)$ be the inverse demand function for the resource with $f'(q^m) < 0$, and where $q^m \equiv \sum_{i=1}^n q^i$ is the rate of market extraction and q^i is the extraction rate of firm i .³ There is a finite cutoff price $P_c = f(0) < \infty$. Exhaustibility of the resource implies that $P = P_c$ in finite time t_c . For simplicity, it is assumed that the firms' time horizon, t_3 , is finite and $t_3 \geq t_c$. The interest rate is fixed at r . Resource exploration is a deterministic process so that the rate of exploration, s^i , equals the rate of discoveries.

Resource extraction is costless. However, exploration costs for firm i , $C(s^i)$, are a function of firm i 's exploration rate with $C'(s^i) > 0$, $C''(s^i) > 0$. The model allows for positive flow fixed costs which are incurred until exploration is terminated for good. Thereafter no costs accrue in operating the firm. For simplicity, we define $t = 0$ as the date when exploration commences. Then necessarily $\inf[C(s^i)/s^i] \leq P$ at $t = 0$.

R^i is the stock of explored resources at t in the possession of firm i . This stock changes through time according to

$$\dot{R}^i = s^i - q^i, \quad i = 1, \dots, n. \quad (1)$$

Clearly R^i , s^i and q^i are constrained to be nonnegative at all times.

X is the common stock of unexplored resources at time t over which firms can establish private property rights through exploration. Therefore the change through time of X is given by

$$\dot{X} = - \sum_{i=1}^n s^i . \quad (2)$$

Let τ^i be the time when firm i terminates exploration for good. Then firm i , $i = 1, \dots, n$, is constrained by $X_{\tau^i} \geq 0$. The state of the system at $t = 0$ is given by $\langle R_0^1, \dots, R_0^n, X_0 \rangle \equiv z_0$, with $R_0^i \geq 0$, $i = 1, \dots, n$, and $X_0 > 0$.

The decision problem for each player then is

$$\begin{aligned} \max_{q^i, s^i} J = & \int_0^{\tau^i} [P \cdot q^i - C(s^i)] \cdot e^{-rt} dt \\ & + W^i(R_{\tau^i}^i, \tau^i) \cdot e^{-r\tau^i} \end{aligned}$$

subject to equations (1) and (2) and

$$R_{\tau^i}^i \geq 0, \quad X_{\tau^i} \geq 0, \quad s^i \geq 0, \quad q^i \geq 0,$$

X_0 given, and R_0^i given, where

$$W^i(R_{\tau^i}^i, \tau^i) = \max_{q^i} \int_{\tau^i}^{t_3} P \cdot q^i \cdot e^{-r(t-\tau^i)} dt$$

subject to

$$\dot{R}^i = -q^i, \quad q^i \geq 0, \quad R_{\tau^i}^i \text{ given, and } R_{t_3}^i \geq 0,$$

and where q^j, s^j , $j \neq i = 1, \dots, n$,

is taken as given by player i , $i = 1, \dots, n$.

(3)

The optimization problem is set up to explicitly allow for an eventual second phase $[\tau^i, t_3]$, where a firm continues to extract from its stock of claimed resources though exploration is terminated for good at τ^i . Each firm i can freely set τ^i in the interval $[0, t_3]$, given $X_{\tau^i} \geq 0$. At τ^i , firm i ceases to be a player in the exploration game.

We are interested in the properties of the optimal feedback and open-loop strategies along the equilibrium price path $\{P^*\}_0^{t_3}$. To that end, some properties of $\{P^*\}_0^{t_3}$ are given:

Lemma 1. The equilibrium price path $\{P^*\}_0^{t_3}$ has the following properties:

- (i) $\{P^*\}_0^{t_3}$ is continuous.
- (ii) $P^* = P_c$, all $t \geq t_c$.
- (iii) Whenever at least one firm i is unconstrained with respect to $R^i \geq 0$, $\dot{P}^*/P^* = r$.
- (iv) If all firms are constrained with respect to $R^i \geq 0$, $i = 1, \dots, n$, $\dot{P}^*/P^* < r$.

Proof. (i) and (ii): obvious. (iii): If at least one firm is unconstrained, then the market can be supplied at zero costs from private stockpiles. (iv): If all firms are constrained, the marginal unit can be put on the market only at the marginal 'extraction costs' $C'(s^i)$. Conversely, if $\dot{P}^*/P^* < r$ no firm will hold the asset 'claimed reserves'. ||

3. THE FEEDBACK NASH EQUILIBRIUM

Feedback strategies may depend on time and the current state of the system $z \equiv \langle R^1, \dots, R^n, X \rangle$.

Definition 1:

The set of admissible feedback strategies is

$$Q_F^i = \{q^i(t, z) \mid q^i \geq 0, \text{ all } (t, z) \in [0, t_3] \times B\}$$

$$S_F^i = \{s^i(t, z) \mid s^i \geq 0, \text{ all } (t, z) \in [0, \tau^i] \times B\}$$

where

$$B \equiv [0, R_0^1 + X_0] \times \dots \times [0, R_0^n + X_0] \times [0, X_0].$$

and where we require

- (i) $q^i(t, z)$ and $s^i(t, z)$ to be continuous in (t, z) , and
- (ii) $k_q(t) \cdot |z - \bar{z}| \geq |q^i(t, z) - q^i(t, \bar{z})|$ for some $k_q(t) \geq 0$
 $k_s(t) \cdot |z - \bar{z}| \geq |s^i(t, z) - s^i(t, \bar{z})|$ for some $k_s(t) \geq 0$,
 where $k_q(t)$, $k_s(t)$ are nonnegative integrable functions over $[0, t_3]$, $[0, \tau^i]$ respectively.

Conditions (i) and (ii) of Definition 1 ensure the existence of a unique state trajectory through every initial value of z_0 .

Optimal feedback strategies can be determined recursively applying the method of dynamic programming. To that end let $t_0 = 0$ and let

$$V^i(t_0, z_0) = \sup_{\substack{q^i(t, z) \\ s^i(t, z)}} \int_0^{\tau^i} [P \cdot q^i(t, z) - C(s^i(t, z))] \cdot e^{-rt} dt \\ + W^i(\tau^i, R_{\tau^i}^i) \cdot e^{-r\tau^i},$$

subject to the conditions in problem (3). Define the Hamiltonian

$$\begin{aligned}
 H^i &= [P \cdot q^i(t, z) - C(s^i(t, z))] \cdot e^{-rt} \\
 &+ \sum_{j=1}^n \partial V^i / \partial R^j \cdot [s^j(t, z) - q^j(t, z)] \\
 &- \partial V^i / \partial X \cdot \sum_{j=1}^n s^j(t, z) .
 \end{aligned}$$

Let Ω^i be the multiplier associated with the constraint $R^i \geq 0$.⁴ Then an optimal feedback strategy must satisfy

$$\begin{aligned}
 \partial V^i(t, z) / \partial t + \max_{\substack{q^i(t, z) \in Q_F^i \\ s^i(t, z) \in S_F^i}} (H^i + \Omega^i \cdot R^i) &= 0 \quad (4)
 \end{aligned}$$

at each point of differentiability of $V^i(t, z)$. By the strict convexity of the cost function these conditions are also sufficient. Carrying out the optimization in (4) entails

$$P \cdot e^{-rt} = \partial V^i / \partial R^i, \quad (5)$$

$$[P - C'(s^i(t, z))] \cdot e^{-rt} = \partial V^i / \partial X. \quad (6)$$

These conditions do not require $q^i(t, z)$ to take any specific value. It therefore can be assumed that $q^i(t, z)$ is some function of time $q^{*i}(t)$ being the solution to $\sum_{j=1}^n q^j(t, z) = f^{-1}(P^*)$. Letting $V_x^i \equiv \partial V^i / \partial X$, the candidate functions satisfying equation (6) can then be expressed as $s^i(t, z, V_x^i)$. The Hamilton-Jacobi equations are

$$\begin{aligned}
& \partial V^i / \partial t + [P^* \cdot q^{*i}(t) - C(s^i(t, z, V_x^i))] \cdot e^{-rt} \\
& + \sum_{j=1}^n \partial V^i / \partial R_t^j \cdot [s^j(t, z, V_x^j) - q^j(t, z)] \\
& - V_x^i \cdot \sum_{j=1}^n s^j(t, z, V_x^j) + \Omega^i \cdot R^i = 0, \quad (7)
\end{aligned}$$

where $\partial V^i / \partial R_t^i = P^* \cdot e^{-rt}$, and where

$$R^i \begin{cases} = \\ \geq \end{cases} 0 \quad \text{whenever} \quad \Omega^i \begin{cases} > \\ = \end{cases} 0, \quad i = 1, \dots, n.$$

The following lemma is proved in Appendix 1.

Lemma 2. *There exists an optimal feedback strategy for each firm i , $i = 1, \dots, n$, consisting of functions $q^{*i}(t)$ and $s^{*i}(t)$, dependent only on time, where $s^{*i}(t)$ is the solution to $C'(s^i(t)) = P^*$, all $t \in [0, \tau^i[$.*

Remark 1. The strategy $(q^{*i}(t), s^{*i}(t))$ is firm i 's optimal feedback strategy for any chosen strategies of the other firms, because for equations (A2) the Hamilton-Jacobi equation (7) is independent of s^j and q^j for all $j \neq i$.

Remark 2. Since $C'(s^{*i}(t)) = P^*$, the intertemporally optimal feedback exploration rate in each moment is the same as the optimal static strategy. The latter is the 'maximal' exploration rate a firm sensibly may consider under

intertemporal and static optimization, since the rent on unclaimed resources, V_x^i is completely dissipated under this policy. ⁵

By the value function (A1), $V^i(t, R^i) = R^i \cdot P^* \cdot e^{-rt}$, all $t \geq \tau^i$. Hence the stock of claimed resources must be exhausted in an optimal program once the cutoff price is reached. By (iii) of Lemma 1, and Lemma 2, $s^{*i}(t) > 0$ as $t \rightarrow \tau^i$. However, $q^m \rightarrow 0$ as $P^* \rightarrow P_c$ so that $R_{t_c}^i > 0$ if $\tau^i = t_c$. Therefore $\tau^i < t_c$ in an unconstrained optimal program. No one firm will terminate exploration for as long as claimable resources are available. Consistency of the individual programs in a Nash equilibrium then implies $\tau^i = \tau$, $i = 1, \dots, n$. The following proposition now follows directly from Lemmas 1 and 2.

Proposition 1. *There exists an unconstrained feedback Nash equilibrium in which firms have extraction and exploration strategies which are functions of time only such that*

(i) $s^{*i}(t)$ is the solution to $C'(s^i(t)) = P^*$, all $t \in [0, \tau[$, $s^{*i}(t) = 0$, all $t \geq \tau = \tau^i$, $i = 1, \dots, n$,

(ii) $q^{*i}(t)$ is a solution to $f[\sum_{j=1}^n q^j(t)] = P^*$, $i = 1, \dots, n$,

(iii) $P^* = P_c \cdot e^{-r(t_c - t)}$ all $t \in [0, t_c]$,
 $P^* = P_c$ all $t \geq t_c$, and

(iv) $t_c > \tau > 0$, where

τ is the solution to $X_0 = \int_0^{\tau} \sum_{i=1}^n s^{*i}(t) dt$,

and where t_c is the solution to

$$X_0 + \sum_{i=1}^n R_0^i = \int_0^{t_c} \sum_{i=1}^n q^{*i}(t) dt.$$

Remark 3. The unconstrained feedback Nash equilibrium consists of two phases. In the first, exploration increases over time such that discounted marginal exploration costs are constant, until the common property resource is completely privatized. In the second, firms supply the market exclusively from a stock of claimed resources.

For a ceteris paribus initial unexplored resource endowment of sufficient size, the feedback Nash equilibrium is constrained. In the unconstrained equilibrium, $dP_0^*/dX_0 < 0$. By Lemma 2, for a low enough P_0^* the initial market exploration rate, s_0^m , must be declining if X_0 is further increased. Since $dq_0^m/dP_0^* < 0$, initial market extraction must therefore exceed initial market exploration for large enough X_0 . Claimed market reserves, R^m , therefore must decline over an initial interval until $s^m \geq q^m$. Cumulative sales drawn from R_0^m must increase as P_0^* decreases. Therefore, given R_0^m , there must be a function $\bar{X}_0 = \bar{X}(R_0^m)$ such that the feedback Nash equilibrium is effectively constrained by $R^i > 0$, $i = 1, \dots, n$, if $X_0 > \bar{X}_0$.

Proposition 2. There is a number \bar{X}_0 such that for $X_0 > \bar{X}_0$ the feedback Nash equilibrium is constrained. A constrained feedback Nash equilibrium, in which the extraction and exploration strategies depend only on time, has properties (i) and (ii) of Proposition 1 and

$$\begin{aligned}
\text{(iii)} \quad P^* &= P_1 \cdot e^{-r(t_1-t)} && \text{all } t \in [0, t_1[, \\
P^* &= P_1 && \text{all } t \in [t_1, t_2[, \\
P^* &= P_1 \cdot e^{r(t-t_2)} && \text{all } t \in [t_2, t_c], \\
P^* &= P_c && \text{all } t \geq t_c, \\
P_1 &\text{ is the solution to } P^* = f[\sum_{i=1}^n s^{*i}(t)], && \text{and}
\end{aligned}$$

(iv) $t_c > \tau > t_2 > t_1 \geq 0$, where

$$t_c \text{ is the solution to } X_0 + \sum_{i=1}^n R_0^i = \int_0^{t_c} \sum_{i=1}^n q^{*i}(t) dt,$$

$$\tau \text{ is the solution to } X_0 = \int_0^{\tau} \sum_{i=1}^n s^{*i}(t) dt,$$

$$t_2 \text{ is the solution to } P_1 \cdot e^{r(t_c-t_2)} = P_c,$$

and where t_1 is the solution to

$$\sum_{i=1}^n R_0^i + \int_0^{t_1} \sum_{i=1}^n s^{*i}(t) dt = \int_0^{t_1} \sum_{i=1}^n q^{*i}(t) dt.$$

Proof. (iii): By Lemma 2, $\text{sign } \dot{s}^{*i}(t) = \text{sign } \dot{P}^*$,
 $i = 1, \dots, n$. Since $f'(q^m) < 0$, $\dot{P}^* = 0$ in the constrained
interval $[t_1, t_2[$ where $s^m = q^m$ by definition. (iv): Since
 $s^m = q^m$ in $[t_1, t_2[$, $P^* < P_c$ in $[t_1, t_2[$. Hence $t_2 < t_c$. By
(iii) of Lemma 1, and Lemma 2, it then follows that $\tau < t_c$ or
else $R_{t_c}^i > 0$ which would violate (A1). Therefore,
 $t_c > \tau > t_2 > t_1 \geq 0$ in a constrained equilibrium. The
remainder of (iii) and (iv) is straightforward. \parallel

Remark 4. In the constrained feedback Nash equilibrium
with a positive initial stock of claimed resources there is an
interval of stagnant exploration activities in between
intervals of growing exploratory efforts.

THE OPEN-LOOP NASH EQUILIBRIUM

Open-loop strategies depend on time and the initial state of the system.

Definition 2:

The set of admissible open-loop strategies is

$$Q_o^i = \{q^i(t) \mid q^i(t) \in Q_P^i\}$$

$$S_o^i = \{s^i(t) \mid s^i(t) \in Q_P^i\}$$

Optimal open-loop strategies can be calculated applying control theory. From the necessary conditions for an optimal open-loop strategy for problem (3) we obtain

$$\delta^i = P^* - C'(s^i(t)), \quad (8)$$

$$\dot{\delta}^i / \delta^i = r, \quad (9)$$

$$\dot{P}^* = r \cdot P^* - \Omega^i, \quad (10)$$

$$\Omega^i \geq 0, \quad R^i \geq 0, \quad \Omega^i \cdot R^i = 0, \quad (11)$$

and the transversality conditions

$$P_{\tau^i}^* \geq \partial W^i(R_{\tau^i}^i, \tau^i) / \partial R_{\tau^i}^i, \quad R_{\tau^i}^i \cdot [P_{\tau^i}^* - \partial W^i(R_{\tau^i}^i, \tau^i) / \partial R_{\tau^i}^i] = 0, \quad (12)$$

$$X_{\tau^i} \geq 0, \quad \delta_{\tau^i}^i \geq 0, \quad \delta_{\tau^i}^i \cdot X_{\tau^i} = 0, \quad (13)$$

$$[C'(s^i(\tau^i)) \cdot s^i(\tau^i) - C(s^i(\tau^i)) - \delta_o^i \cdot e^{r\tau^i} \cdot \sum_{j \neq i}^n s^j(\tau^i)] \cdot e^{-r\tau^i} + \partial W^i(R_{\tau^i}^i, \tau^i) \cdot e^{-r\tau^i} / \partial \tau^i \geq 0, \quad (14)$$

with strict equality if $t_3 > \tau^i$. δ^i is firm i 's resource rent associated with the stock of unclaimed resources X .⁶

Conditions (8) - (14) do not require $q^i(t)$ to take any particular form. Therefore we can set $q^i(t) = q^{*i}(t)$.

In case there is no second phase $\partial W^i / \partial R_{\tau^i}^i = 0$ in equation (12) so that in this case $R_{\tau^i}^i = 0$.

The optimization of the second-phase plan (if one exists) is a standard problem. By property (iii) of the equilibrium price path, $\dot{P}^*/P^* = r$ during firm i 's second phase. Hence $\partial W^i / \partial R_{\tau^i}^i = P_{\tau^i}^*$ in equation (12). Furthermore $R_{t_c}^i = 0$. But for a given cumulative extraction in the second phase and an exponentially growing price,

$$\partial W^i(R_{\tau^i}^i, \tau^i) \cdot e^{-r\tau^i} / \partial \tau^i = 0.$$

Hence condition (14) implies

$$C'(s^i(\tau^i)) \cdot s^i(\tau^i) - C(s^i(\tau^i)) - \delta_0^i \cdot e^{r\tau^i} \cdot \sum_{j \neq i}^n s^j(\tau^i) \geq 0, \quad (15)$$

with strict equality if $t_3 > \tau^i$. The proof for the following lemma is given in Appendix 2.

Lemma 3. *An optimal open-loop strategy consists of functions $q^{*i}(t)$ and $s^{**i}(t)$, where $s^{**i}(t)$ is the solution to $C'(s^i(t)) = P^* - \delta_0^i \cdot e^{rt}$, all $t \in [0, \tau^i[$, and where $\delta_0^i > 0$, $\tau^i < t_c$, and $X_{\tau^i} = 0$, $i = 1, \dots, n$.*

Remark 5. If players are symmetrical, a rational player must expect others to be rational the same way as he himself is. Since it is optimal for firm i itself to continue exploration until X is exhausted, it is rational for this firm

to expect $\sum_{j \neq i}^n s^j(\tau^i) > 0$ in case the exhaustion date is set at a marginally later date. That is, a rational firm will not conceive it possible to establish private property rights over unclaimed resources simply by restraining exploration in order to extend the exhaustion date past the date when all other firms will have terminated exploration.

Remark 6. The difference between the optimal feedback and the optimal open-loop strategy can be traced to the players' ability to influence the duration of the game. Calculating the optimal strategy recursively makes it optimal to claim as much of the common stock left for exploration in the last moment of the game, τ^i , as is optimal in a static game (compare Remark 2). This recursively accounts for the dissipation of rent at earlier moments of an optimal feedback program. But it also accounts for a reduction of the duration of the game (relative to the open-loop case). If the game were of fixed duration, then such a policy were not admissible, and a feedback optimum depending on time only would necessarily coincide with an open-loop optimum (Basar and Olsder (1982), section 6.2.2). ⁷

The open-loop Nash equilibrium can be characterized as follows:

Proposition 3. *The open-loop Nash equilibrium, unconstrained by $R^i \geq 0$, $i = 1, \dots, n$, has the properties:*

- (i) $\delta_0^i > 0$, and $s^{**i}(t)$ is the solution to
- $$C'(s^i(t)) = (P_0 - \delta_0^i) \cdot e^{rt}, \quad \text{all } t \in [0, \tau[,$$
- $$\tau = \tau^i, \quad i = 1, \dots, n,$$

(ii) $q^*(t)$ is a solution to $f[\sum_{i=1}^n q^i(t)] = P^*$,
 $i = 1, \dots, n$,

(iii) $P^* = P_c \cdot e^{-r(t_c-t)}$ all $t \in [0, t_c]$,
 $P^* = P_c$ all $t \geq t_c$,

(iv) $t_c > \tau > 0$, where

τ is the solution to $X_0 = \int_0^\tau \sum_{i=1}^n s^{**i}(t) dt$,

and where t_c is the solution to

$$X_0 + \sum_{i=1}^n R_0^i = \int_0^{t_c} \sum_{i=1}^n q^{*i}(t) dt .$$

Proof. Straightforward or else given in the proof to Lemma

3.

There is a number \bar{X}_0 , $\bar{X}_0 = \bar{X}_0(R_0^m)$, such that the open-loop Nash equilibrium is constrained if $X_0 > \bar{X}_0$. From Propositions 1 and 3 it follows that $\bar{X}_0 > \bar{X}_0$ for all R_0^m , that is to say, X_0 may take values for which the feedback equilibrium is unconstrained whereas the open-loop equilibrium is constrained.

Proposition 4. For $X_0 > \bar{X}_0$ the open-loop Nash equilibrium is constrained by $R^i > 0$, $i = 1, \dots, n$. In addition to (ii) of Proposition 3 the constrained equilibrium has the following properties:

(i) $\delta_0^i > 0$, and $s^{**i}(t)$ is the solution to
 $C'(s^i(t)) = P^* - \delta_0^i \cdot e^{rt}$, all $t \in [0, \tau[$ such that
 $s^{**i}(t)$ is increasing in t , all $t \in [0, t_1[$ and all
 $t \in [t_2, \tau[$, $s^{**i}(t)$ is decreasing in t , all
 $t \in [t_1, t_2[$, $\tau = \tau^i$, $i = 1, \dots, n$,

(iii) P^* is continuous such that

$$\begin{aligned} P^* &= P_0^* \cdot e^{rt} && \text{all } t \in [0, t_1[, \\ r > \dot{P}^*/P^* > 0 && \text{all } t \in [t_1, t_2[, \\ \dot{P}^*/P^* &= r && \text{all } t \in [t_2, t_c[, \\ P^* &= P_c && \text{all } t \geq t_c , \end{aligned}$$

where P_0^* is the solution to

$$\sum_{i=1}^n R_0^i + \int_0^{t_1} \sum_{i=1}^n s^{**i}(t) dt = \int_0^{t_1} f^{-1}(P_0 \cdot e^{rt}) dt ,$$

(iv) $t_c > \tau > t_2 > t_1 \geq 0$, where

$$t_c \text{ is the solution to } X_0 + \sum_{i=1}^n R_0^i = \int_0^{t_c} \sum_{i=1}^n q^{*i}(t) dt$$

$$\tau \text{ is the solution to } X_0 = \int_0^{\tau} \sum_{i=1}^n s^{**i}(t) dt ,$$

$$t_2 \text{ is the solution to } P_c \cdot e^{-r(t_c - t_2)} = f[\sum_{i=1}^n s^{**i}(t_2)] ,$$

$$t_1 \text{ is the solution to } P_0 \cdot e^{rt_1} = f[\sum_{i=1}^n s^{**i}(t_1)] .$$

Proof. (iii): Only $r > \dot{P}^*/P^* > 0$ in the constrained interval $[t_1, t_2[$ is not straightforward. In Appendix 2 it was shown that $r > \dot{P}^*/P^* \geq 0$. Suppose $\dot{P}^* = 0$ in $[t_1, t_2[$. Then $\dot{q}^m = 0$ in $[t_1, t_2[$, and by Lemma 3, $\dot{s}^i(t) < 0$, $i = 1, \dots, n$. However, $R^i = \dot{R}^i = 0$ so that $q^i(t) = s^i(t)$ in $[t_1, t_2[$, $i = 1, \dots, n$. Hence $r > \dot{P}^*/P^* > 0$ in $[t_1, t_2[$.

(i): Follows from Lemma 3 except that the market exploration rate is declining in $[t_1, t_2[$. By (iii), $\dot{P}^* > 0$ in $[t_1, t_2[$. Hence $\dot{q}^m = \dot{s}^m < 0$ in $[t_1, t_2[$.

(iv): By Lemma 3, $\tau^i < t_c$. $R_{t_2}^i = 0$, $i = 1, \dots, n$. Hence $t_2 < \tau^i = \tau$ or else $\{P^*\}_{\tau}^{t_c}$ could not be sustained. The remainder of (iv) is straightforward. || ^s

The constrained open-loop Nash equilibrium is illustrated in Figure 1.

Remark 7. With reference to Remark 5, the equilibrium is stable in the sense that a firm i can have no incentive to restrain its own exploration with an eye to being the sole explorer past the equilibrium τ .

Put Figure 1 about here

Remark 8. There can be only one constrained interval. After a constrained interval, market exploration is increasing and exceeds market extraction which is decreasing so that $\dot{R}^m > 0$. Hence the end of a constrained interval cannot precede the beginning of another constrained interval. ⁹

5. ENDOGENOUS CHOICE BETWEEN FEEDBACK AND OPEN-LOOP STRATEGIES

In the previous analysis the firms were exogenously restricted to either feedback or open-loop strategies. A natural question is what will be the Nash equilibrium for this game if firms are free to select from among feedback *and* open-loop strategies. Then the set of admissible strategies is given by Definition 1, because strategies which depend only on time are contained in the sets Q_F^i and S_F^i . However, contrary to the case discussed in section 3, if a firm restricts itself to the subset of strategies which depend only on time, it may choose a strategy which does not satisfy the criterium of recursive optimality, i.e. a strategy which is not perfect in the sense of Selten (1975). A Nash equilibrium for this game is defined as follows:

Definition 3: The strategy 2n-tuple

$(\tilde{q}^1(t,z), \tilde{s}^1(t,z), \dots, \tilde{q}^n(t,z), \tilde{s}^n(t,z))$ is a Nash equilibrium for the game where firms can choose open-loop and feedback strategies if and only if

- (i) $\tilde{q}^i \equiv \tilde{q}^i(t,z) \in Q_F^i, \tilde{s}^i \equiv \tilde{s}^i(t,z) \in S_F^i, i = 1, \dots, n,$
- (ii) $(\tilde{q}^i, \tilde{s}^i)$ may be imperfect if \tilde{q}^i and \tilde{s}^i are independent of $z, i = 1, \dots, n,$ and
- (iii) $J^i(\tilde{q}^1, \tilde{s}^1, \dots, \tilde{q}^n, \tilde{s}^n) \geq J^i(\tilde{q}^1, \tilde{s}^1, \dots,$

$$\dots, \tilde{q}^{i-1}, \tilde{s}^{i-1}, q^i(t,z), s^i(t,z), \tilde{q}^{i+1}, \tilde{s}^{i+1}, \dots, \tilde{q}^n, \tilde{s}^n)$$

for all $q^i(t,z) \in Q_F^i$ and all $s^i(t,z) \in S_F^i,$
 $i = 1, \dots, n.$

An equilibrium for this game can take one of three forms. All players choose their optimal open-loop strategy, all choose their optimal feedback strategy, or k players, $1 \leq k \leq n-1$, choose open-loop strategies and $n-k$ players choose a feedback strategy.

It will be shown that the Nash equilibrium for this game is the open-loop Nash equilibrium defined in Propositions 3 and 4. To that end it will be shown that Lemmas 2 and 3 pertain to the case where the choice between open-loop and feedback strategies is endogenously determined.

Suppose player i is considering to choose a feedback strategy. Then the optimal feedback strategy must satisfy equation (7), where s^j and q^j , $j \neq i$, depend only on time if firm i expects firm j to choose an open-loop strategy. By Remark 1, the value function (A1) is a solution to the Hamilton-Jacobi equation for any choice of other firms between open-loop and feedback strategies. Consequently Lemma 2 also applies to the game discussed in this section.

Now suppose firm i considers to choose an open-loop strategy. It must expect that all other firms j , $j \neq i = k + 1, \dots, n$, which are expected to choose a feedback strategy will select the dominant feedback strategy $(q^{*j}(t), s^{*j}(t))$. Firm i therefore must expect that the strategies of all n firms are functions of time only. Hence, firm i 's optimal open-loop strategy must satisfy conditions (8) - (14). Lemma 3 therefore also applies to the game where the

choice between open-loop and feedback strategies is endogenously determined. By Lemma 2, the optimal feedback strategy belongs to the set of admissible open-loop strategies. However, by Lemma 3, it is not an optimal open-loop strategy, and therefore $(q^{*i}(t), s^{*i}(t))$ must imply a lower payoff than $(q^{*i}(t), s^{**i}(t))$. Therefore we can establish:

Proposition 5. *The open-loop Nash equilibrium is the equilibrium for the exploration game where firms are free to select open-loop and feedback strategies and rank all strategies according to the associated stream of payoff.*

In the remainder of this section some properties of the open-loop Nash equilibrium are given.

Proposition 6. *The open-loop Nash equilibrium is inefficient compared to total surplus maximization.*

Proof. If $n = 1$, problem (3) characterizes the optimization problem of a social planner maximizing total surplus. All the optimality conditions for an optimal open-loop strategy also apply to the social planning problem except condition (15), which is replaced by $C'(s(\tau)) \geq C(s(\tau))/s(\tau)$. Hence $s(\tau) > 0$ if there are positive flow fixed costs, so that $\tau < t_c$ and the transversality condition must hold with strict equality. If there are zero flow fixed costs, $s(\tau) = 0$ is possible, also implying strict equality of marginal and average

costs. Comparing $C'(s(\tau)) = C(s(\tau))/s(\tau)$ with equality (15), and noting that $\delta_0^i > 0$ in the open-loop Nash equilibrium, reveals that the final exploration rate is excessive in the open-loop Nash equilibrium. By backward induction, the open-loop equilibrium exploration then is excessive in at least a final interval, which in the case of the unconstrained equilibrium extends until $t = 0$. \parallel ¹⁰

Proposition 7. *As $n \rightarrow \infty$, the open-loop Nash equilibrium has the properties:*

$$(i) \quad \lim_{n \rightarrow \infty} \delta_0^i = 0 \quad \text{and} \quad \lim_{n \rightarrow \infty} s^{**i}(0) = C'^{-1}(P_0^*), \\ i = 1, \dots, n,$$

$$(ii) \quad \lim_{n \rightarrow \infty} \tau = 0.$$

Proof. For $n \rightarrow \infty$, $\sum_{j \neq i}^n s^j(\tau^i) \rightarrow \infty$ so that equality (15) can only be fulfilled if $\delta_0^i \rightarrow 0$, $i = 1, \dots, n$. Then by equation (8), $C'(s^i(t)) \rightarrow P^*$, and by the finiteness of X_0 , $\tau \rightarrow 0$ as $n \rightarrow \infty$. \parallel

Remark 9. For $n \rightarrow \infty$ the open-loop Nash equilibrium is unconstrained and converges towards the feedback Nash equilibrium. With an infinite number of firms on the commons claimable resources are almost instantaneously exhausted, leaving an individual firm no freedom to influence the length of the game. For this game of 'fixed' duration, the open-loop and the feedback Nash equilibrium must coincide (compare Remark 6).

6. CONCLUSION.

If a cake becomes the common property of a number of decision makers, the problem of how to eat a cake of finite size then becomes a problem of how to appropriate a cake of dwindling size. In the context of resource exploration, this paper examines the decision problem in which a cake can either be instantaneously eaten as it is appropriated at costs, or else be costlessly conserved in private stores.

In addition to being relevant to the situations given in the introduction, the model may also shed some light on other related situations.

A sale in a department store can transform an ordinary reproduceable consumer good into an exhaustible common property commodity. Stockpiles of sales items in private households may be evidence of some of the considerations discussed in the present paper. Asymmetric information between housing developers and land owners in suburban growth areas may lead to a race between developers - for the appropriation of the Ricardian rent on land closer to the city center - well in advance of actual development. During the times of imperialism european powers were engaged in a race to get their share of the free cake 'Africa'. Today property rights over Antarctica are at best only vaguely defined. And in the future, outer

space may provide a plethora of other examples, where the commonality of resources may give rise to a claiming rush similar to the one discussed in this paper.

APPENDIX 1

Proof of Lemma 2. Consider the value functions

$$\begin{aligned}
 V^i(t, R^i) &= \int_t^{\tau^i} [P^*(T) \cdot q^{*i}(T) - C(s^{*i}(T))] \cdot e^{-rT} dT \\
 &+ \int_t^{\tau^i} \dot{R}^i(T) \cdot P^*(T) \cdot e^{-rT} dT \\
 &+ R_t^i \cdot P^*(t) \cdot e^{-rt}, \quad i = 1, \dots, n. \quad (A1)
 \end{aligned}$$

Upon differentiating this value function we obtain

$$v_x^i = \partial V^i / \partial R^j = 0 \quad \text{all } j \neq i, \quad i = 1, \dots, n, \quad (A2)$$

$$\partial V^i / \partial R_t^i = P^* \cdot e^{-rt}, \quad i = 1, \dots, n, \quad (A3)$$

and upon recalling that $R^i = \dot{R}^i = 0$ in a constrained interval and $\dot{P}^*/P^* = r$ in an unconstrained interval

$$\partial V^i / \partial t =$$

$$\left\{ \begin{array}{ll} - [P^* \cdot q^{*i}(t) - C(s^{*i}(t))] \cdot e^{-rt} & \text{if } \Omega^i > 0 \\ - [P^* \cdot q^{*i}(t) - C(s^{*i}(t))] \cdot e^{-rt} - \dot{R}_t^i \cdot P^* \cdot e^{-rt} & \text{otherwise} \end{array} \right\} \quad (A4)$$

$i = 1, \dots, n.$

Substituting equations (A2), (A3) and (A4) into equations (7) we obtain $\partial V^i / \partial t - \partial V^i / \partial t = 0, \quad i = 1, \dots, n. \quad \parallel$

APPENDIX 2

Proof of Lemma 3. Suppose $\tau^i = t_c$. By equation (10), if $\dot{P}^*/P^* = r$, all $t \in [0, \tau^i[$, then firm i is unconstrained so that by equations (8) - (10) $\dot{C}'(s^i(t))/C'(s^i(t)) = r$, all $t \in [0, \tau^i[$. For $t \rightarrow t_c$, $q^m \rightarrow 0$, $q^{*i}(t) \rightarrow 0$ and $s^i(t) > 0$. However, for $\tau^i = t_c$, $R_{\tau^i}^i = 0$. Hence $\tau^i < t_c$ if firm i is unconstrained.

Suppose firm i is constrained. Then we might have $t_2 = \tau^i = t_c$, where t_2 is the end of a constrained phase where $R^i = \dot{R}^i = 0$ and $s^i(t) = q^{*i}(t)$. Then $s^i(t) = q^{*i}(t) \rightarrow 0$ as $t \rightarrow t_c$ so that $R_{t_c}^i = 0$, if $t_2 = \tau^i = t_c$. However, for $s^i(t) \rightarrow 0$ as $t \rightarrow \tau^i$ condition (15) is violated given positive flow fixed costs.

If there are zero flow fixed costs, condition (15) is fulfilled with strict equality for $s^i(t) \rightarrow 0$ as $t \rightarrow \tau^i$ and $\delta_0^i = 0$, in case $t_c = \tau^i = t_2$. If $\delta_0^i = 0$, $\dot{s}^i(t) > 0$ whenever $\dot{P} > 0$, as must be the case for $t \rightarrow t_c$. But for $t \rightarrow t_c = \tau^i = t_2$, $\dot{s}^i(t) > 0$ and $q^i(t) \rightarrow 0$ are once again violating the condition $q^i(t) = s^i(t)$ for $t \rightarrow t_2$. Therefore $\tau^i < t_c$.

With $\tau^i < t_c \leq t_3$, firm i is unconstrained with respect to the terminal exploration date, and condition (15) must hold with strict equality.

Suppose $\delta_0^i = 0$. Then by equations (8) and (15),

$$P_{\tau^i}^* = C'(s^i(\tau^i)) = C(s^i(\tau^i))/s^i(\tau^i) = \inf[C(s^i)/s^i] .$$

Since $t = 0$ is defined as the date when exploration begins, we must have $P_0^* \geq \inf[C(s^i)/s^i]$. However by equation (10), P^* is growing exponentially if $\Omega^i = 0$,

all $t \in [0, \tau^i[$ so that $P_{\tau^i}^* > \inf[C(s^i)/s^i]$ if exploration takes place at all. Hence $\delta_0^i > 0$ if $R^i \geq 0$ is never binding.

If the firm is constrained, there is an interval where $\dot{P}^*/P^* < r$. The proof that $\delta_0^i > 0$ also in the constrained case then is completed if it can be shown that $0 \leq \dot{P}^*/P^* < r$.

Suppose $\dot{P}^* < 0$ in a constrained interval $[t_1, t_2[$. By equations (8) and (9), if $\dot{P}^* < 0$, $\dot{s}^i < 0$, $i = 1, \dots, n$, for $\delta_0^i \geq 0$. However, in $[t_1, t_2[$, $\dot{q}^m = \dot{s}^m > 0$, if $\dot{P}^* < 0$. Hence $\dot{P}^* \geq 0$ in $[0, t_c[$ and therefore $\delta_0^i > 0$, $i = 1, \dots, n$.

For $\delta_0^i > 0$, equations (9) and (13) imply that

$$X_{\tau^i} = 0. \quad \parallel$$

NOTES

* This paper is based on my Ph.D. thesis 'Common Access and Strategic Considerations in Natural Resource Exploration' (The London School of Economics (1985)). I would like to thank A. Ingham, J. Lane, C. Noebel, J. Rapport and O. Schneider for helpful discussions. My greatest personal debt, however, is to my supervisor David de Meza who imparted to me continuous encouragement, sharp criticism and stimulating discussions on the topic. Any remaining errors are of course solely my responsibility.

1. Another interdependency in common property resource exploration may occur if prospectors do not know which sites have already been appropriated. Then sequential exploration of an already claimed area may occur, making a firm's appropriation costs a function of cumulative discoveries of the entire industry. This externality is discussed by Peterson (1978). The strategic dimension of exploration, however, is neglected there due to the assumption of an infinite size of the unexplored resource stock. In contrast, we will concentrate here exclusively on the strategic aspect by assuming that a firm's exploration-cum-discovery costs depend only on that firm's exploration rate.

2. A common pool oil extraction model with private storage facilities is discussed by Sinn (1984). There, costs arise from keeping the oil in the private store but not from the appropriation process itself. Furthermore markets are assumed to be oligopolistic. Finally, firms are assumed to be able to influence each others extraction and sales rates but to take the timepath of the stock of common oil as exogenous. In contrast, here it is assumed that firms take each others extraction and sales rates as exogenous but can influence the rate of decline of the common oil stock. On this last point see Mohr (1985).

3. Time subscripts are suppressed whenever possible without ambiguity.

4. The nonnegativity constraint on q^i and s^i will be neglected. It will be seen that the optimal strategy satisfies these constraints.

5. McMillan and Sinn (1984), and Reinganum and Stokey (1985) provide other examples, where in a continuous time model of common property resource exploitation optimal feedback strategies require an exploitation effort which is 'maximal' in this sense.

6. Since claimed resources can be sold on the market at zero costs, the rent on explored resources is always equal to the equilibrium price P^* .

7. Notably, the proof that the feedback optimum differs from the open-loop optimum is based on the transversality condition (15) which is associated with a firm's ability to determine the duration of the game. With the length of the game fixed, equation (15) vanishes.

8. In Definition 2 only continuous functions are admitted. A more general approach would be to admit piecewise continuous strategies. Then at the moments of transition to or from a constrained interval the costate variable associated with the constrained state variable could be discontinuous. In the present model the costate variable associated with the stock of claimed resources is equal to the market price. From the 'Seierstad and Sydsaeter sufficiency conditions' it follows, however, that the resource price is continuous at t_1 and t_2 even if piecewise continuous strategies are admitted. Admitting only continuous strategies therefore does not distort the results.

9. For the same reasons this also must be true for the constrained feedback Nash equilibrium. Also, from Propositions 1 - 4 it readily follows that along the feedback and open-loop

equilibrium trajectories the constraints $q^i \geq 0$, $s^i \geq 0$ are never tight.

10. Comparing Lemma 2 with Lemma 3 one can conclude that the same is true for the feedback Nash equilibrium.

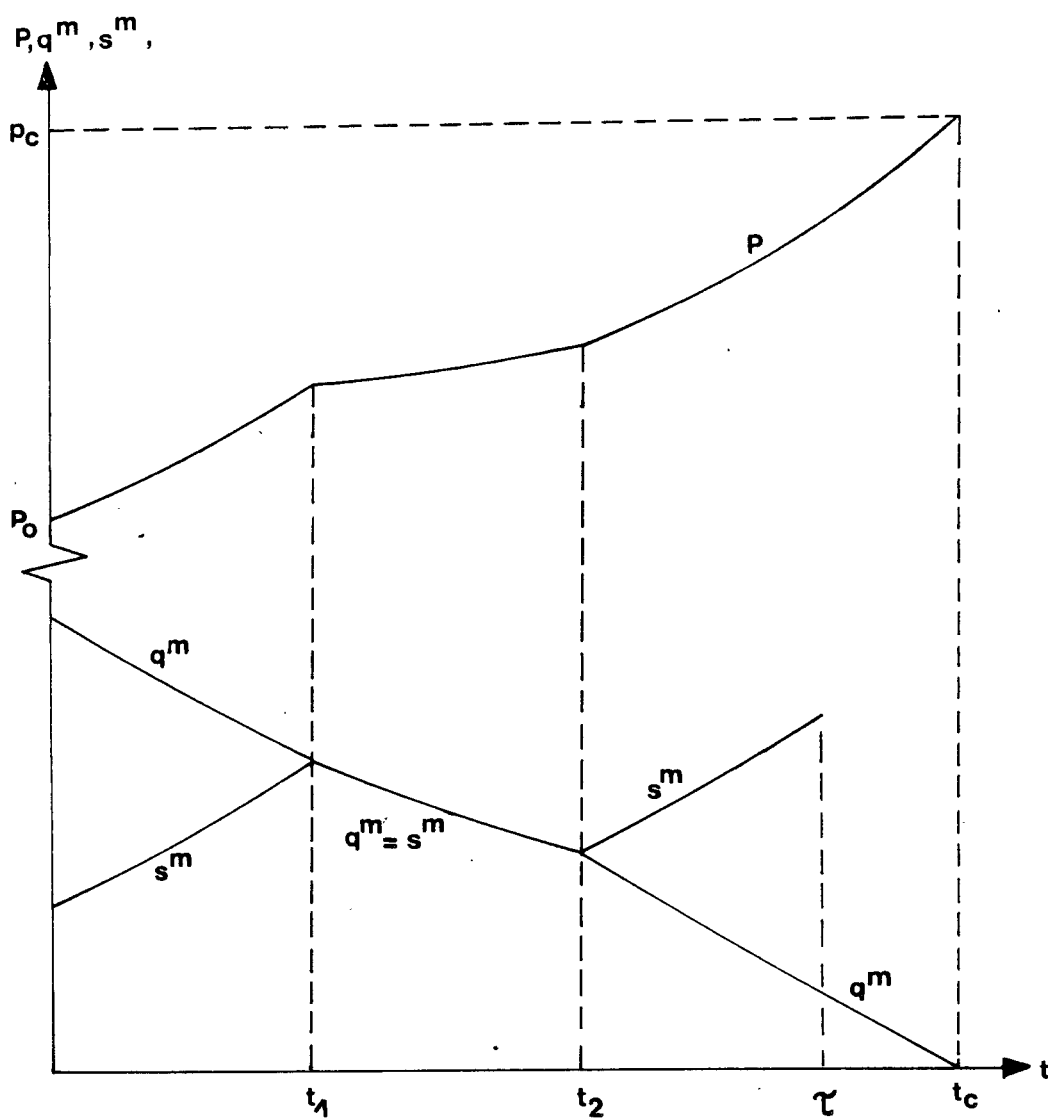


Figure 1

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