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Vertical product differentiation and social health systems in a market for prescription drugs:
Revised version of a talk given at the 2. World Congress on Health Economics in Zürich, Sept. 10 - 14, 1990

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Vertical Product Differentiation and Social Health Systems in a Market for Prescription Drugs

Diskussionsbeiträge
Vertical Product Differentiation and Social Health Systems in a Market for Prescription Drugs*

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Abstract

Following models of vertically differentiated duopoly this paper deals with the influence of a social health system on the sustainability of the duopoly and on the respective prices. Emphasis is laid on the effect of a reduction of the coinsurance rate for drugs on prices and on the market structure. The results depend on the size of the relative effect such a reduction has on the rate of the disposable income. The social health system is assumed to be financed by deductions from the income of the insured. Conclusions are drawn with respect to different health systems.
1 Introduction

Intuitively one would doubt the importance of competition among vertically differentiated products on a market for prescription drugs, where a social health system subsidizing the consumption of pharmaceuticals is present. The consumption subsidies of the social health system should create a situation where income differentials do not matter for the consumers' choice of quality, which are usually taken as a precondition for the possibility of competition among different qualities of the same product.¹

In the following section a simple duopoly model for a specific differentiated drug market is presented where the social health system is characterized by a price dependent subsidy on the consumption of drugs and by an income dependent contribution to the costs of the social health system, which covers also the costs of other medical treatments. A duopoly model is chosen, because it is the most simple form of oligopoly. Since there are different therapeutical requirements caused by particular diseases, the pharmaceutical industry is often assumed to consist of independent submarkets where only few firms compete with each other.² Hence, oligopoly models seem to be most adequate. Although the author considers horizontal product differentiation as even more important than vertical product differentiation in the market of prescription drugs, this paper is not concerned with the former.³ Besides determining the possible prices in the duopoly framework the condition which guarantees that at most two firms operate in the market is derived.

In the third section of this paper the effect of a reduction in the coinsurance rate on prices, profits as well as on the sustainability and viability is shown. It turns out that the results depend heavily on the value of the elasticity of the rate of the disposable income with respect to the coinsurance rate. Only if this elasticity is relatively small the above assumed effect that smaller coinsurance rates would provoke fewer quality levels in the market is confirmed by the analysis.

In the final section the results are summarized and conclusions are drawn from the analysis with regard to the effects of different institutional arran-

²See the empirical literature concerning the pharmaceutical industry in the European Communities and in the OECD countries, e.g. Burstall, Senior (1985), p. 76 ff. and Burstall, Dunning, Lake (1981), p. 59 f.
³Vertical and horizontal product differentiation was first distinguished by Lancaster (1979), p. 27 f. Vertical product differentiation means that all potential consumers unanimously agree in the quality ranking of the products in the market. In a market with horizontally differentiated products, however the consumers tastes' are different. Hence, consumers differ in their favourite product even if they are offered for the same price.
gements of the financing of a social health system.

2 A Model of Vertically Differentiated Duopoly

2.1 The Supply Side of the Market

There are two firms, each producing a product of the same character but on differing quality levels.\(^4\) \(u\) indicates the quality of the product of firm one, \(\bar{u}\) that of firm two with \(\bar{u} > u\). The same production technology is available for both firms and is represented by the following cost function:

\[
C = c \cdot x + F(u)
\]  

(1)

This cost function implies, that both firms have the same constant marginal costs \(c\) independent of their quality level \(u\) and the number of sold products \(x\) but differing fixed costs \(F(u)\), which are considered as sunk costs and rise with the quality. In the case of the pharmaceutical industry the assumption of constant marginal costs seems to be justified because the variable costs of production play a rather minor role in comparison with the fixed and sunk costs of research and development, which bear also the burden of quality improvements.\(^5\)

Both firms compete with each other in prices. For the sake of simplicity it is assumed that they have perfect information about their demand situation and the demand and cost situation of their competitor.

2.2 The Demand Side of the Market

The demand side is characterized by a continuum of consumers, who are identical in tastes, but differ in income. They are uniformly distributed with density 1 in the income interval \([a, b]\).

The individuals have a unit demand. This means that they buy one or none of the offered products according to the maximization of their utility, which is represented by the following function: \(^6\)

\[
U = \max\{u[y(1 - s) - kp], \quad u_0y(1 - s)\}
\]  

(2)

\(^4\)The model follows in principle Shaked, Sutton (1982) and (1983).

\(^5\)See Burstall, Dunning, Lake (1981), p. 47 for the cost structure of the pharmaceutical industry.

\(^6\)So far the analysis is only applicable to therapeutical submarkets where a useful medical dosage is well defined. This will be more probable in a market for prescription drugs than in a market for over-the-counter drugs. The doctor who prescribes a medicine is assumed to act only in the interest of the patient. Therefore he does not have to be especially addressed in the analysis. See for the following Shaked, Sutton (1982), p. 4 and (1983), p. 1473 f. This kind of utility function guarantees that the consumer's willingness to pay for quality improvements is an increasing function of income, see Sutton (1986), p. 394.
The first part of equation (2) indicates the utility the person in question can achieve if he or she buys one medicine of quality $u$ at a price $p$ and is subsidized with $(1 - k)p$ by the social health system with $0 \leq k \leq 1$. $k$ is supposed to be the general coinsurance rate for any kind of drug. The second part describes the level of utility which is reached, if no medicine is purchased on this particular market by the person in question. Since $y(1 - s)$ with $0 \leq s \leq 1$ represents the income of the individual after the deduction of his or her contribution to the social health system $u_0y(1 - s)$ is the utility which is realized, if the whole disposable income is spent on an outside good. It is postulated that the social health system is completely financed by the contributions of the insured and that it subsidizes not only the consumption of drugs but also the costs of other medical treatments.

The following assumption is made:

$$u_0 < u$$  \hspace{1cm} (3)

Without this assumption everybody would prefer to buy only the outside product instead of the low quality drug $u$.

The consumer who is just indifferent between buying the outside good and the medicine of quality $u$ has the income $y_0$ which can be calculated by the condition of indifference:

$$u_0y_0(1 - s) = u[y_0(1 - s) - kp_u]$$

and is given by:

$$y_0 = \frac{k}{1 - s}r_0p_u,$$  \hspace{1cm} (4)

where $r_0$ is defined by:

$$r_0 = \frac{u}{u - u_0} > 1$$

Consumers earning lower incomes would prefer the outside good. Those with higher incomes buy the drug of quality $u$. By analogy, the income of the consumer who is indifferent between buying the drug of quality $u$ and $\bar{u}$ is given by:

$$y_u = \frac{k}{1 - s} \left[ (1 - r_\bar{u})p_u + r_\bar{u}p_u \right],$$  \hspace{1cm} (5)

with:

$$r_\bar{u} = \frac{\bar{u}}{u - \bar{u}} > 1$$

Since individuals have a unit demand consumers can now be distinguished by their incomes into three groups: Those who buy only the outside good, those, who buy quality $u$, and those, who buy quality $\bar{u}$. From this the
market demands for the two firms can be derived. The demand $x_u$ for the drug of quality $u$ can be described by the following function:\(^7\)

$$x_u = \begin{cases} 
  y_u - a & \text{for } y_0 \leq a \\
  y_u - y_0 & \text{for } y_0 > a 
\end{cases}$$  
(6)

The demand $x_u$ for quality $u$ is given by:

$$x_u = b - y_u$$  
(7)

### 2.3 The Nash-Equilibrium in Prices

The Bertrand-Nash Equilibrium can now be analyzed.\(^8\) The reaction functions of the two firms are derived from profit maximization. Differentiating the profit of firm two with respect to its own price, setting it equal to zero and rearranging results for $p_u$ in:\(^9\)

$$p_u = \frac{r_u - 1)p_u}{2r_u} + \frac{b(1 - s)}{2r_u} + \frac{c}{2}$$  
(8)

From the first order necessary condition for a profit maximum in prices we obtain for firm one:

$$p_u = \frac{r_y p_u}{(r_y - 1)2} - \frac{\alpha(1 - s)}{(r_y - 1)2k} + \frac{c}{2} \quad \text{for } a > y_0$$  
(9)

$$p_u = \frac{r_y p_u}{(r_y - 1 + r_0)2} + \frac{c}{2} \quad \text{for } a < y_0$$

The existence of a Nash-Equilibrium in pure strategies is always guaranteed, if the pay-off functions, in this case the profit functions, are quasi-concave and continuous in $p_u$ and in $p_y$ respectively.\(^10\) Since the second order conditions for a profit maximum are fulfilled, existence problems do not originate in the quasi-concavity condition.\(^11\) Moreover the profit function is continuous, because the cost function (1) and the function of sales is continuous even for firm one, which has a discontinuous reaction function. The continuity of the cost function is guaranteed as soon as the demand is continuous, which is verified at first sight for firm two.\(^12\) The same is true for firm one.

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\(^7\) See Shaked, Sutton (1982), p. 4.

\(^8\) See for the interpretation and appropriateness of this equilibrium concept Johansen (1982).

\(^9\) See for the differentiation of the profit function Appendix A.


\(^11\) See Appendix A for the derivation of the second order conditions.

\(^12\) See equation (7). Inserting the definition (5) for $y_u$ implies:

$$x_u = b - \frac{k}{1 - s}(1 - r_y)p_u + r_y p_u,$$

which is a continuous linear function of $p_u$. 


the demand of which is continuous, but not differentiable, if \( p^*_a \) is set on that level, where \( a = y_0 \) is fulfilled.\(^{13}\) Taking into account the definition (4) of \( y_0 \), this level is given by:\(^{14}\)

\[
\hat{p}_m = \frac{a(1-s)}{r_0 k} \tag{10}
\]

Since the sales function is just a multiplication of the demand function by the respective price, its continuity follows from the former one’s.

Having checked the existence of a Nash-Equilibrium the question has to be answered, whether there is just one equilibrium or whether we have to expect several equilibria.\(^{15}\) Therefore the reaction functions (8) and (9) have to be analyzed. The reaction functions are linear and the one of the high quality firm is in any case steeper.\(^{16}\) Since the reaction function of firm two starts for \( p_u = 0 \) with a more negative value, the problem of multiple equilibria can only be caused by the discontinuous price setting function of firm one.

First of all one has to decide what happens, if the reaction function of firm two passes through the range of discontinuity of the reaction function of firm one. The discontinuity is a consequence of the kink in the demand function of the low quality firm at \( p^*_u \). If \( \hat{p}_m \) is chosen as an optimal strategy by firm one, it usually forms a corner solution. Therefore variations of \( p_a \) do not result in a variation of \( p^*_u \), because neither raising nor reducing its price would increase the profit of the low quality firm. Hence, at the range of discontinuity its reaction function is a horizontal line. The problem of multiple equilibria arises now, if firm one offers its products at the price \( \hat{p}_m \) at a higher level of \( p_a \) following the first part of its reaction function than following the second part. The two possible cases are illustrated in figure 1. In order to avoid the problem of multiple equilibria in the course of the argument it is assumed here that the following condition holds:\(^{17}\)

\[
a \geq \frac{cr_0 k}{1-s} \tag{11}
\]

The assumption ensures that even the poorest consumer in the market would prefer the drug with quality \( y \) to the outside good if it were available at marginal costs. The fulfillment of this condition is the more probable, the higher the consumers’ smallest net income in the market, the smaller the ratio between the coinsurance rate and the rate of income at one’s own

\(^{13}\)The demand function of firm one is kinked at this price for given \( p_a \). The reason for this kink is, that up to a particular limit raising its price only leads to a switch of consumers to the high quality product. If the price is set beyond this limit, consumers will switch to the outside good, too. See *Shaked, Sutton* (1982), p. 6.

\(^{14}\)The demand \( x_k \) given by equation (6) is linear and continuous in \( p_m \), which can be checked by substituting (5) and (4) in (6).

\(^{15}\)See Johansen (1982), p. 437 for the difficulties in interpreting multiple equilibria.

\(^{16}\)This is checked by rearranging (8) to \( p_a = \frac{3p_m x_k}{r_x^2(1-s)} - \frac{k(1-s)}{r_x^2} - \frac{cr_k}{r_x^2} \).

\(^{17}\)See Appendix B for the calculation and the sufficiency of this condition.
Figure 1: Possible Nash-Equilibria in the Price Setting Game
disposal, the smaller the difference between the quality $u$ and the quality parameter of the outside good $u_0$, and the smaller the marginal costs $c$.\textsuperscript{18} Although the question of multiple equilibria is sensitive to the parameters of the social health system and the regulation of a minimum quality, its main source is the assumption of positive marginal costs.\textsuperscript{19}

After having ensured that a unique equilibrium exists the concrete values of $p_u$ and $p_\bar{u}$ at the equilibrium remain to be calculated. Three cases have to be distinguished:

- The reaction functions (8) and (9) intersect in the region where $y_0 < a$ is fulfilled. Inserting (8) in the first part of (9) results in:

  \[ p_u^* = \frac{c}{3} \left( 2 + \frac{r_u}{r_u - 1} \right) + \frac{1 - s}{k} \left[ b - 2a \right] \frac{3}{3(r_u - 1)} \]  
  \[ p_\bar{u}^* = \frac{c}{3} \left( 2 + \frac{r_\bar{u} - 1}{r_\bar{u}} \right) + \frac{1 - s}{k} \left[ 2b - a \right] \frac{3}{3r_\bar{u}} \]  

From these equilibrium prices it is easily proved that the two firms can coexist with a positive price in any case, if $b > 2a$ is assumed.\textsuperscript{20} Otherwise a particular relation between $r_u$, $c$, $s$, $k$, $a$ and $b$ must be guaranteed in order to have a positive value of equation (12).

- The reaction function (8) cuts (9) at the range of discontinuity. As already explained above, firm one will set its price at $p_u$ which is given by equation (10). By substituting $p_u$ in equation (9) we obtain for $p_\bar{u}$:

  \[ p_\bar{u}^* = \frac{c}{2} \left( 1 - s \right) \left[ a + r_0 b \right] \frac{1}{2r_\bar{u}r_0} \]  

$p_u$ and $p_\bar{u}$ are always positive.

- The reaction functions (8) and (9) intersect in the region, where we have $y_0 > a$. By substituting (9) in (8) and (8) in (9) respectively the following equilibrium prices are calculated:

  \[ p_u^{***} = \frac{c(3r_u - 2 + 2r_0)}{3r_u - 3 + 4r_0} + \frac{b(1 - s)}{k(3r_u - 3 + 4r_0)} \]  

\textsuperscript{18}The parameter $u_0$ for the outside good can be interpreted as the quality of life in the case of no drug treatment of the considered kind.

\textsuperscript{19}For this statement see the models of Shaked, Sutton (1982), Jaskold Gabszewicz, Thisse (1980), and Jaskold Gabszewicz, Shaked, Sutton, Thisse (1986) where marginal costs are zero and the problem of multiple equilibria does not exist. Where positive marginal costs are assumed, the problem is often not addressed, see Shaked, Sutton (1983), and Sutton (1986).

\textsuperscript{20}In Shaked, Sutton (1982), p. 7, this condition also ensures that at least two firms are viable in the market. Since positive costs are assumed here, things are more complicated. For the questions of viability and sustainability of duopoly in this context see section 2.4.
\[ p_{\ddagger}^{***} = \frac{c(r_u - 1)(r_u - 1 + r_0)}{r_u(3r_u - 3 + 4r_0)} + \frac{2b(1 - s)(r_u - 1 + r_0)}{kr_u(3r_u - 3 + 4r_0)} \]  

(16)

The equilibrium actually realized depends on the value of the parameters \( a, b, u, \bar{u}, s, \) and \( k \) of the reaction functions (8) and (9).

2.4 The Sustainability of the Duopoly from an Income Dispersal Point of View

Up to now we have paid no attention to the question which of the circumstances could determine the existence of precisely two vertically differentiated firms in the market. Two distinct approaches are offered in the literature:21

1. The presence of sunk costs prevents other firms from entering the market with identical products, because they assume that incumbent firms are prepared to lower their prices up to the marginal costs, and further products on other quality levels are not viable, because the dispersal of the income is too small.

2. The presence of sunk costs prevents other firms from entering the market with identical products for the same reason as already explained, and the dispersion of the income allows for more than two quality levels, but the volume of the market demand is too small to cover the fixed costs of three firms.

This paper is only concerned with the first item although the fixed and sunk costs \( F(u) \) have been assumed in the cost function and could also be responsible for a special structure on such a market. But since the second item is not typical for a market with different quality levels and would restrict the number of firms in any market, whether it is differentiated or not, it is excluded from this analysis.

Two conditions have to be fulfilled if the duopoly is originated in the income dispersal. The first can be derived by assuming that the entry of a third firm in the market would lead to a situation where the equilibrium price for the lowest quality firm does not cover marginal costs.22 This situation is caused by the width of the income dispersal, the height of the marginal costs and the relation of the parameters of the social health insurance, but might also be caused by the relation of the offered qualities. The second condition is that the equilibrium prices of the duopoly have to exceed marginal costs.

Starting with the analysis of at most two products in the market we have to distinguish three cases: the entry of a firm with a lower quality drug as the

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21 See Waterson (1987) for a general review of these two distinct approaches.

two incumbants, the entry of a firm with an intermediate quality drug, and
the entry of a firm with a higher quality drug. The latter two cases need
not be considered in detail, since the condition which ensures prices below
marginal costs in the first case does not depend on the quality levels chosen
by the three firms in the market. If the condition is fulfilled and firms enter
with intermediate or higher qualities, the lowest quality firm which is in the
latter cases the firm with a drug of quality \( u \) will be threatened by its prices
to be below marginal costs and will leave the market.\(^{23}\)

Let us now assume a third firm with quality \( \tilde{u} \), \( \tilde{u} < u < \bar{u} \) using the same
technology as the two incumbants represented by equation (1). Its demand
is given by:

\[
x_{\tilde{u}} = \begin{cases} 
  y_{\tilde{u}} - a & \text{for } y'_{0} < a \\
  y_{\tilde{u}} - y'_{0} & \text{for } y'_{0} > a
\end{cases}
\]  

(17)

A consumer of income \( y_{\tilde{u}} \) is indifferent towards the drug of quality \( \tilde{u} \) and
quality \( u \). \( y_{\tilde{u}} \) is given by:\(^{24}\)

\[
y_{\tilde{u}} = \frac{k}{1 - s} [(1 - r_{\tilde{u}})p_{\tilde{u}} + r_{\tilde{u}}p_{u}]
\]

(18)

with:

\[
r_{\tilde{u}} = \frac{\bar{u}}{\bar{u} - \tilde{u}}
\]

\( y'_{0} \) is equivalent to \( y_{0} \) in (4), but has changed to:

\[
y'_{0} = \frac{k}{1 - s} r'_{0} p_{\tilde{u}}
\]

(19)

with:

\[
r'_{0} = \frac{\bar{u}}{\bar{u} - u_{0}}
\]

since \( \tilde{u} \) is now the lowest quality level in the market. The demand for the
products of firm one \( x'_{u} \) is now given by:

\[
x'_{u} = y_{u} - y_{\tilde{u}}
\]

(20)

The demand for the drugs of the highest quality level \( x_{u} \) is still represen-
ted by equation (7). As in the preceding section we assume that the three
firms set their prices in order to maximize profits in a non cooperative way,
knowing the demand and cost side of each of their competitors. The prices
can be calculated by solving the system of first order conditions.\(^{25}\) The entry
of the firm with quality \( \tilde{u} \) is blocked if the income dispersal does not allow
for its equilibrium price to exceed marginal costs.

\(^{23}\)See for a similar argumentation with respect to the order of leaving the market Jaskold

\(^{24}\)The derivation is analogous to that of equation (4) and (5).

\(^{25}\)See Appendix C where the second order conditions are also checked.
The demand function of the newcomer is also kinked at the price $p_u$ where $y'_0$ equals $a$. Therefore the problem of multiple equilibria is similar to the duopoly case present here if we do not assume the newcomer to choose $u$ in such a way that the following condition equivalent to (11) is fulfilled:

$$a \geq \frac{cr_0k}{1-s}$$  \hspace{1cm} (21)

Since we are not especially interested in the values of the equilibrium in the case of the three firms but in the conditions which ensure that three firms are not viable, the assumption of (21) is only used to distinguish different cases.

If (21) is fulfilled as an inequality $p_u$ is greater than the marginal costs $c$. Therefore in the equilibrium the price of firm $u$ which prevents it from entering the market must in any case be below $p_u$ and is thus defined by that part of the reaction function of firm $u$ where $a > y'_0$. In this case the condition of entry deterrence for any quality level below $y$ is given by:

$$p_u^* - c < 0,$$

where $p_u^*$ is the equilibrium price calculated in the case of $a > y'_0$. After inserting $p_u^*$ we get:

$$b < a - \frac{3r_u - 3 + 4r_u}{r_u} - \frac{3ck}{1-s} \frac{r_u - 1 + r_u}{r_u} = A$$  \hspace{1cm} (22)

$A$ is a decreasing function in $u$. In order to exclude entry on any level of $u$ we have to demand (22) for all $u < u^*$, which fulfill (21), meaning for all $u \in \left[\frac{a(1-s)}{a(1-s) - ck}, y\right]$. The lower bound of the interval is positive if its denominator is positive which is implied by assumption (11). A negative denominator would mean that the disposable income of the poorest individual in the market could not cover the subsidized marginal costs of the drugs in the market and would in any case contradict assumption (11). Since $A$ is a decreasing function in the relevant range of $u$ it is sufficient

26 The condition can be derived by the same analysis as (11) in the duopoly case because the reaction function of firm $u$ does not depend on the price $p_u$ of the highest quality firm and has the same structure as that of the low quality firm in the duopoly case. Since we have $r_0 < r'_0$ (21) is not implied by condition (11).

27 See figure 2 for the illustration of the two different cases.

28 See (68) in Appendix C for the calculation.

29 The derivative of $A$ with respect to $u$ is given by:

$$\frac{\partial A}{\partial u} = \left[a - \frac{ck}{1-s}\right] \frac{-3(r_u - 1)}{r_u} \frac{u}{(u - u)^2},$$

and is negative by the assumption of (21) which ensures $a > \frac{ck}{1-s}$.

30 The lower bound is derived by rearranging (21) for $u$. 

Figure 2: The two possible equilibria in the three firms’ case

to demand the fulfillment of (22) for the limit of $A$ if $u \to u_$. The limit of $A$ is given by:

$$\lim_{u \to u_} A = 4a - \frac{3ck}{1-s}$$  \hspace{1cm} (23)

If (21) is violated or fulfilled as an equality the marginal costs $c$ is equal or exceeds the level of $p_u$. In addition the second part of the reaction function given for $a < y_0$ lies above the first part if it is also considered in the range where we have $a > y'_0$. Therefore if the equilibrium price $p^*_u$ derived from the second part of the reaction function of firm $u$ given for the case $a < y'_0$ does not exceed the marginal costs, then even if the real equilibrium price is formed by the first part of the reaction function this will be even smaller. Hence the condition which prevents entry is developed from the second part of the reaction function if (21) is violated or just fulfilled as an equality.

$^{31}$See figure 2 where the two different cases for equilibrium are illustrated.
From demanding:

\[ p_u^{**} - c < 0 \]  

and inserting \( p_u^{**} \) from (72) in Appendix C and rearranging the following condition which must hold for all \( \tilde{u} \in \left] u_0, \frac{a(1-s)}{a(1-s) - ck} u_0 \right] \) can be derived:

\[
b < \frac{ck}{1-s} \left( 1 + \frac{r'_{\tilde{u}} - 1}{r_{\tilde{u}}} (3r_{\tilde{u}} - 3 + 4r_{\tilde{u}}) \right) = A' \tag{25}\]

Since \( A' \) is also a decreasing function of \( \tilde{u} \) we have to check the fulfillment of (25) only for the upper limit of \( \tilde{u} \) in this case, which is given by \( \tilde{u} = \frac{a(1-s)}{a(1-s) - ck} u_0 \).\(^{32}\) The condition which can be derived by this procedure is given by:

\[
\lim_{\tilde{u} \to \frac{a(1-s)}{a(1-s) - ck} u_0} A' = a \left( 3r_{\tilde{u}} + 1 - 3 \frac{u_0}{\tilde{u} - y} \right) - 3r_{\tilde{u}} \frac{ck}{1-s} > b \tag{26}\]

If we compare (23) with (26) we recognize that (23) is the more restrictive one because the following statement is true in any case where (11) holds:\(^{33}\)

\[
4a - \frac{3ck}{1-s} \leq a \left( 3r_{\tilde{u}} + 1 - 3 \frac{u_0}{\tilde{u} - y} \right) - 3r_{\tilde{u}} \frac{ck}{1-s} \tag{27}\]

Therefore it is sufficient to demand only the fulfillment of (23) if the entry of a lower quality firm than \( u \) should be prevented in any case because of a too narrow income dispersal.

The above developed argument can analogously be applied to the case of a market entry with an intermediate quality drug. This is the case because \( u \) and \( \bar{u} \) have been arbitrarily chosen for the incumbants and we could have alternatively stated that the low quality incumbent produces a drug with the quality \( \bar{u} \) and that there is a newcomer with quality \( y \). Condition (23) would ensure that the low quality incumbent will decide on leaving the market. The same is true if a newcomer offered a higher quality as the two incumbants. Then we could assume quality levels \( \bar{u} \) and \( y \) for the two incumbants and \( \bar{u} \) for the newcomer. In this case again (22) ensures prices below marginal costs for the low quality incumbent. The right-hand side \( A \) decreases in \( \bar{u} \), the quality level of the newcomer in this case, and, taking the limit of \( A \) with \( \bar{u} \to +\infty \), (22) again reduces to (23). Hence the fulfillment of (23) also ensures the exit of the lowest quality incumbent.

\(^{32}\) The derivative of \( A' \) with respect to \( \bar{u} \) is given by:

\[
\frac{\partial A'}{\partial \bar{u}} = \frac{ck}{1-s} \left[ \frac{-u_0}{(\bar{u} - u_0)^2} \left( \frac{3(r_{\bar{u}} - 1)}{r_{\bar{u}}} + 4 \right) - \frac{\bar{u}}{(y - \bar{u})^2} \frac{3(r_{\bar{u}} - 1)(r_{\bar{u}} - 1)}{(r_{\bar{u}})^2} \right]
\]

\(^{33}\) Equivalently rearranging (27) results in (11).

12
As a result of the preceding analysis we can summarize that not more than two firms exist in the market from an income dispersal point of view if condition (23) holds. But it does not have to be the initial firms because a newcomer improving the quality compared to \( u \) would force firm one out of the market.

Up to now we have assumed that there are at least two firms in the market and have discussed how far they are threatened by new competitors. What remains to be shown are the circumstances where two firms are viable from an income dispersal point of view. In order to have at least two firms in the market we have to ensure that in this case even the equilibrium price of the low quality firm covers marginal costs. If we distinguish the three cases from the preceding subsection we have to demand at all events:

\[
p_{ul} - c > 0
\]

By inserting \( p^u \), \( p^h \) and \( p^{**u} \) respectively given by (12), (10) and (15) respectively, the following condition can be derived:\(^{34}\)

\[
b > 2a - \frac{ck}{1 - s} = E
\]

\[
a > \frac{ckr_0}{1 - s}
\]

\[
b > \frac{c(2r_0 - 1)k}{1 - s}
\]

(29) is equivalent to (11) which has been assumed to hold here. If (28) is compared to (30) it is obvious that by the assumption of (11) the following relation is ensured:

\[
2a - \frac{ck}{1 - s} > \frac{c(2r_0 - 1)k}{1 - s}
\]

(31)

Therefore it is sufficient to demand (28) for the viability of the duopoly from an income dispersal point of view.

In the preceding subsection we worked out conditions ensuring a duopoly. Following our results (23) and (28) we have to demand for the income dispersal, as well as for the parameters of the social health system and the marginal costs that:\(^{35}\)

\[
4a - \frac{3ck}{1 - s} > b > 2a - \frac{ck}{1 - s}
\]

(32)

If the sustainability of the initial firms should be guaranteed we have in addition to (32) to demand that none of the potential competitors is able to offer a better quality than that of the worst incumbent.

---

\(^{34}\)The argument is similar to that of Shaked, Sutton (1982), p. 7.

\(^{35}\)The sufficiency of (23) and (28) has been shown in (27) and (31).
3 Effects of a Fall in the Coinsurance Rate for Drugs on a Vertically Differentiated Duopoly

This section deals with the influence a change in the social health system might have on the above analyzed market for vertically differentiated drugs. It should be emphasized here that a unique coinsurance rate for all the drugs available is assumed. Therefore a variation of this coinsurance rate will not leave the rate of the disposable income unchanged if the analysis is based on a social health system which is financed by income dependent contributions.\(^{36}\) If the coinsurance rate \(k\) decreases the rate of the contribution \(s\) is expected to increase in the following analysis.

3.1 The Effects on Prices and Profits in the Duopoly

Starting the analysis while neglecting for the moment that a variation of the coinsurance rate might shift the Nash equilibrium from one part of the reaction function (9) to the other. Its effects on the prices can be derived by differentiating the relevant equilibrium prices from section 2.3 with respect to \(k\). Distinguishing the same three cases as in section 2.3 the following results concerning the influence of \(k\) on the duopoly prices are derived:

- Starting with the case where \(y_0 < a\) is fulfilled the differentiation of (12) and (13) with respect to \(k\) yields:
  \[
  \frac{\partial p^*_u}{\partial k} = \left( -\frac{\partial s}{\partial k} k - (1 - s) \right) \frac{b - 2a}{3(r_u - 1)} \]
  \[ (33) \]
  \[
  \frac{\partial p^*_n}{\partial k} = \left( -\frac{\partial s}{\partial k} k - (1 - s) \right) \frac{2b - a}{3r_u} \]
  \[ (34) \]

- In the case of \(y_0 = a\) in equilibrium the differentiation of (10) and (14) with respect to \(k\) is given by:
  \[
  \frac{\partial p^*_u}{\partial k} = \left( -\frac{\partial s}{\partial k} k - (1 - s) \right) \frac{a}{r_0} \]
  \[ (35) \]
  \[
  \frac{\partial p^*_n}{\partial k} = \left( -\frac{\partial s}{\partial k} k - (1 - s) \right) \frac{a + r_0b}{2r_u r_0} \]
  \[ (36) \]

- For the last case of \(y_0 > a\) the respective differentiations of (15) and (16) result in:
  \[
  \frac{\partial p^{***}_u}{\partial k} = \left( -\frac{\partial s}{\partial k} k - (1 - s) \right) \frac{b}{3r_u - 3 + 4r_0} \]
  \[ (37) \]

\(^{36}\)This is also true for systems which are financed by general income taxes.
\[
\frac{\partial p_u}{\partial k} = \left( \frac{-\frac{\partial a}{\partial k} k - (1-s)}{k^2} \right) \frac{2b(r_u - 1 + r_0)}{r_u(3r_u - 3 + 4r_0)} \tag{38}
\]

The derivatives are negative or positive in any of the three cases depending on the sign of the expressions in the parentheses. A negative value of the expressions in parentheses is equivalent to the elasticity of the rate of the disposable income with respect to the coinsurance rate being smaller than one which is given by:

\[
\frac{\partial a}{\partial k} \frac{1-s}{k} < 1 \tag{39}
\]

(39) means that in the case of a decreasing coinsurance rate the decrease in the rate of the disposable income is relatively smaller. This is the more probable the more services are subsidized by the social health system with a different coinsurance rate than \(k\). If (39) holds the prices will rise in response to a reduced coinsurance rate.

The same is true if by a decrease of \(k\) the equilibrium is shifted from one part of the reaction function (9) to the other. From figure 1 and the reaction functions (8) and (9) it is noticed that in the case that (39) holds (8) and the first part of (9) are shifted downwards, whereas the range of discontinuity is shifted upwards. The second part of (9) is with the exception of its starting point not touched by the variation. Therefore a shift from the range of discontinuity to the second part of (9) or the other way round caused by a decrease of \(k\) if (39) is fulfilled would result in a rise of \(p_u\) and \(p_u\) as in the above discussed cases. Shifts of the equilibrium from the first part of (9) to the range of discontinuity would yield the same results. This is also true for shifts from the range of discontinuity to the first part of (9) but by looking at figure 1 one can only state an increase of \(p_u\). The increase of \(p_u\) is proved by differentiating (8) with respect to \(k\), which yields:

\[
\frac{\partial p_u}{\partial k} = \frac{b}{r_u - 1} \left( \frac{-\frac{\partial a}{\partial k} k - (1-s)}{k^2} \right), \tag{40}
\]

and by rearranging the second part of (9) for \(p_u\) and differentiating with respect to \(k\), which yields:

\[
\frac{\partial p_u}{\partial k} = \frac{a}{2(r_u - 1)} \left( \frac{-\frac{\partial a}{\partial k} k - (1-s)}{k^2} \right) \tag{41}
\]

Since we consider a decrease of the coinsurance rate under the condition of (39), both reaction functions are shifted to the right, but the shift of (8) is greater.\(^{37}\) Therefore starting the analysis from the original price equilibrium at the range of discontinuity the two reaction functions will even in this case

\(^{37}\)This follows from \(b > \frac{a}{2}\).
intersect at a higher level of $p_u$. If (39) does not hold the argument can be reversed and even if there is a jump of the equilibrium from one part of (9) to the other, the equilibrium prices will fall if the coverage of the social health insurance increases.

Summarizing the preceding analysis, the expected result that the prices would rise in equilibrium if the coinsurance rate decreased, depends on the elasticity of the rate of the disposable income with respect to a variation of the special coinsurance rate for drugs. If this elasticity is greater than one this means that the effect of a reduced disposable income is stronger than that of a more subsidized price and the incumbants are forced to lower their prices in response to a decrease of $k$.

After having derived the optimal price response to a reduction of the coinsurance rate the effects on the profits of the duopoly firms follow immediately. We have to distinguish the same three cases:

- If the equilibrium lies in the range where $a > y_0$ the effect of a reduction of the coinsurance rate on the profit of firm one is given by:

$$\frac{\partial \pi_u}{\partial k} = \frac{\partial p^*_u}{\partial k} (y_u - a) + (p^*_u - c) \frac{\partial y_u}{\partial k}$$

with:

$$\frac{\partial y_u}{\partial k} = \frac{(1 - s) + \frac{\partial p_u}{\partial k} k}{(1 - s)^2} \cdot \frac{c}{3}$$

Throwing a glance on (33) and on (43) it is obvious that the effect on the profit of firm one depends not only on the elasticity of $1 - s$ with respect to $k$ but also on the relative price effect compared to the relative effect on the numbers of sold products. If (39) holds and the following condition is fulfilled:

$$\frac{\partial y_u}{\partial k} < \frac{\partial p^*_u}{\partial k}$$

the profit of the low quality firm will rise if the coinsurance rate is reduced. If in this case (44) is not fulfilled the profit will decrease. The argument is reversed if (39) does not hold. The effects on the profit of firm two is given by:

$$\frac{\partial \pi_u}{\partial k} = \frac{\partial p^*_u}{\partial k} (b - y_u) - (p^*_u - c) \frac{\partial y_u}{\partial k}$$

Considering (34) and (43) we can derive a rising profit for the high quality firm as a response to a decreasing coinsurance rate in any case in which (39) is fulfilled.
In the case \( y_0 = a \) in equilibrium the effects of a reduction of the coinsurance rate on the profit of firm one is given by:

\[
\frac{\partial \pi_u}{\partial k} = \frac{\partial p_u}{\partial k} (y_u - a) + (p_u - c) \frac{\partial y_u}{\partial k}
\]

with:

\[
\frac{\partial y_u}{\partial k} = \left( \frac{1 - s}{(1 - s)^2} \right) \cdot \frac{r_u c}{2}
\]

Since (47) is positive and (35) is negative in any case where (39) is fulfilled the effect on the profit again depends on the relative effect on the price compared to the relative effect on the number of sold drugs. The profit will rise if the condition equivalent to (44) holds. Again, the argument has to be reversed if (39) does not hold. The effects on the profit of the high quality firm is given by:

\[
\frac{\partial \pi_\alpha}{\partial k} = \frac{\partial p^{**}_\alpha}{\partial k} (b - y_\alpha) - (p^{**}_\alpha - c) \frac{\partial y_\alpha}{\partial k}
\]

From (47) and (36) it follows immediately that the profit of the high quality firm will rise if (39) is fulfilled and if not it will fall.

In the case of \( y_0 > a \) in equilibrium the effect of a decreasing coinsurance rate on the profit of firm one is given by:

\[
\frac{\partial \pi_u}{\partial k} = \frac{\partial p^{***}_u}{\partial k} (y_u - y_0) + (p^{***}_u - c) \left( \frac{\partial y_u}{\partial k} - \frac{\partial y_0}{\partial k} \right)
\]

with:

\[
\frac{\partial y_u}{\partial k} - \frac{\partial y_0}{\partial k} = \frac{1 - s}{(1 - s)^2} \left[ \frac{c(2r_0 - 1)(r_u - 1 + r_0)}{3r_u - 3 + 4r_0} \right]
\]

Since (50) and (37) are both negative in the case where (39) holds we have to expect a rising profit for the low quality firm if the coinsurance rate is reduced. If (39) does not hold the contrary is true. The effect of a decreasing coinsurance rate on the profit of firm two is given by:

\[
\frac{\partial \pi_\alpha}{\partial k} = \frac{\partial p^{***}_\alpha}{\partial k} (b - y_\alpha) - (p^{***}_\alpha - c) \frac{\partial y_\alpha}{\partial k}
\]

with:

\[
\frac{\partial y_\alpha}{\partial k} = \frac{(1 - s + \frac{\partial k}{\partial k}) \frac{c[(r_u - 1 + r_0) + r_u r_0]}{3r_u - 3 + 4r_0}}{(1 - s)^2}
\]

The profit of the high quality firm is also rising if (39) is fulfilled and if \( k \) is reduced. It decreases if (39) does not hold.
It is not possible here to make any statements about the effects on the profits if the equilibrium is shifted from one range of the reaction function (9) to the other. In all the other cases the high quality firm will profit from a reduction of the coinsurance rate for drugs whereas the low quality firm might possibly lose if the rise in prices is not high enough to compensate for the loss of consumers who switch to the high quality drug.

Since the opportunities to earn profits and the prices are changed on the considered market the market structure can also be influenced by the discussed variation of the coinsurance rate which is discussed in the following subsection.

3.2 The Effects on the Market Structure

As we have already emphasized in section 2.4 the duopoly in our model can be based on two reasons, the income dispersal and the height of the fixed costs which might differently be affected by a reduction of the coinsurance rate. In this paper we will only consider the former.

The criterion which ensures the duopoly from an income dispersal point of view is given by (32). We start with the left-hand side identical to (23) which guarantees that only drugs with a higher quality than \( u \) can be successfully introduced in the market. Differentiating \( A \) with respect to \( k \) yields the following result:

\[
\frac{\partial A}{\partial k} = - \left[ \frac{(1 - s) + \frac{\partial s}{\partial k} k}{(1 - s)^2} \right] \cdot 3c, \tag{53}
\]

which is negative if (39) holds which means that the relative change of the rate of the disposable income is smaller compared with the relative change of the coinsurance rate from which it results. In this case \( A \) will rise if the coinsurance rate is reduced and the entry of a low quality firm will still be deterred. If (39) does not hold the entry might become possible because the higher price subsidies cause a great loss in the disposable income thus making low quality drugs more attractive.

The right-hand side of (32) guarantees the viability of the duopoly, and is equivalent to (28). It is influenced because the variation of \( k \) would change the value of \( E \). Differentiating \( E \) with respect to \( k \) results in:

\[
\frac{\partial E}{\partial k} = - \left[ \frac{(1 - s) + \frac{\partial s}{\partial k} k}{(1 - s)^2} \right] \cdot c, \tag{54}
\]

which is negative if (39) holds and positive otherwise. This means that in the case of (39) the viability of the duopoly is threatened by a reduction of

\[38\] In order to analyse the effect on \( y_k \) and \( y_0 \) one can repeat the procedure of Shaked, Sutton (1982), p. 5f., but where it depends on the concrete relations of the qualities of the two incumbants whether they are rising or falling in the case of a shift of the equilibrium.
the coinsurance rate and strengthened otherwise. In the former case the low quality firm might be forced out of the market whereas in the latter even a lower quality firm has a greater chance to enter the market successfully.

It should be emphasized here that the tendency to fewer or more firms is completely independent of the variation of profits caused by the reduction of the coinsurance rate. Both incumbants might realize higher profits and at the same time a third firm might have even more problems to enter the market.\footnote{If the market structure were determined by the height of the fixed costs any increase of the profits of the two incumbants would make the entry of a third firm more probable.}

4 Conclusion

The main result of the preceding analysis is that the number of qualities is only influenced by the social health system in the expected way if the elasticity of the rate of the disposable income with respect to the coinsurance rate is relatively small. The preference for a certain quality level is determined by the height of one’s income and the connected ability to pay for quality. In this context the reduced coinsurance rate has two effects. On the one hand, the price of a certain quality drug is more subsidized thus lowering the necessary ability to pay for it. On the other hand, the higher subsidies have to be financed by higher contributions to the social health system which make the income and the ability to pay for quality decrease. Only if the first effect dominates there is a tendency to fewer quality levels in the market.

Since the firms adjust their prices to the changed ability to pay for quality caused by a reduced coinsurance rate the results concerning their price responses are not surprising. If the elasticity of the disposable income with respect to the coinsurance rate is relatively low their prices will rise. If it is relatively high they have to lower their prices.

Provided the presented model is not a too far reaching simplification of the reality one can draw some general conclusions from the results concerning the effects of a reduction in the coinsurance rate. Since all these results depend heavily on the value of the elasticity of the rate of the disposable income with respect to the coinsurance rate one can expect higher prices and fewer quality levels in those social health systems where this elasticity can be assumed to be lower. This is the case if apart from pharmaceuticals more other medical services are covered with a different coinsurance rate by the social health system or if the social health system is not financed by special contributions but by general income dependent taxes. If the social health system has a very broad coverage of medical treatments, a variation of the coinsurance rate for drugs will have a relatively smaller influence of the rate of the disposable income. The same is true for a national health
system which is financed by taxes, because the variation of the coinsurance rate for drugs will have a relative minor influence on the general tax rate.

Since in the case of a relative minor elasticity the profits of the high quality firm will in any case rise if the coinsurance rate is reduced and the low quality competitor might be forced out of the market, one might even expect the high quality firm to influence the political process in this respect. Perhaps they may even try to change the institutional arrangements concerning the social health system in the political process in order to lower the elasticity. Whether this rent seeking behaviour is a successful strategy cannot be decided on the basis of the model presented here because the firms with lower qualities might have the opposite interest and the high quality suppliers of other medical services might have the same interest with respect to their own specific coinsurance rate. In the last case they might succeed in influencing their respective coinsurance rate but they will certainly not be able to earn higher profits because the overall effect on the rate of the disposable income will be too high.

Of course, the conclusions which have been drawn here for the expected behaviour of the firms are based on the assumption that the simplifications of the model are not responsible for the results of the analysis.

From a normative welfare-point of view the model presented here is not very meaningful because it is a partial equilibrium analysis. Only one market for specific drugs is considered. On this market the welfare might rise by a reduction of the coinsurance rate if the profits of both firms rise and if more people are able to buy the higher quality drug, but it is assumed that the price of the outside good is unaffected as well as the levels of gross income of the considered patients. Moreover healthy people who do not demand this specific drug will have to pay higher contributions to the social health system without having any benefit from it.

On the other hand one might doubt whether welfare is the right criterion to judge a social health system since the introduction of a social health system has rarely been motivated by welfare considerations. Considerations of equity seem to be at least of the same importance. A dominating criterion used in the literature for evaluating equity with regard to health systems is equal access for equal need. Within the framework of the model presented above the access to the analysed drug market, especially to the high quality products, becomes more equal if the above described elasticity is not too high and the coinsurance rate for drugs is reduced. But, since the model is just a partial equilibrium model, one should also regard the effect of a

\[^{40}\]See the results concerning the profits in 3.1.

\[^{41}\]This is the well known prisoners' dilemma situation which is usually the result of rent-seeking behaviour. See, e.g., Tullock (1980).

reduction of the coinsurance rate for drugs on the equal access to other medical services.
Appendix

A First and Second Order Conditions for Profit Maximization

The profit functions of firm one are:

\[ \pi_u = \begin{cases} 
(p_u - c)(y_u - a) - F(y_u) & \text{for } y_0 < a \\
(p_u - c)(y_u - y_0) - F(y_u) & \text{for } y_0 > a 
\end{cases} \]  

(55)

After inserting the definitions (4) and (5) of \( y_u \) and \( y_0 \) the differentiation of (55) with respect to \( p_u \) yields:

\[ \frac{\partial \pi_u}{\partial p_u} = k \frac{1}{1-s} \left[(1 - r_u)p_u + r_up_a\right] - a \]

(56)

\[ + (p_u - c) \frac{k}{1-s} \left[1 - r_u\right] \text{ for } y_0 < a \]

\[ \frac{\partial \pi_u}{\partial p_u} = \frac{k}{1-s} \left[(1 - r_u)p_u + r_up_a - r_0p_a\right] \]

(57)

\[ + (p_u - c) \frac{k}{1-s} \left[1 - r_u - r_0\right] \text{ for } y_0 > a \]

The second derivative of the profit function is given by:

\[ \frac{\partial^2 \pi_u}{\partial p_u^2} = \begin{cases} 
2k(1-r_u) & \text{for } y_0 < a \\
\frac{2k(1-r_u-r_0)}{1-s} & \text{for } y_0 > a 
\end{cases} \]

(58)

Since we have by definition: \(^{43} r_u > 1\), the second derivative of the profit function is always negative and the quasi-concavity of the profit function of firm one is guaranteed.

The profit function of firm two is:

\[ \pi_\bar{u} = (p_a - c)(b - y_u) - F(\bar{u}) \]

(59)

By inserting the definition (5) and differentiating with respect to \( p_a \) we obtain:

\[ \frac{\partial \pi_\bar{u}}{\partial p_a} = b - \frac{k}{1-s} \left[(1 - r_{\bar{u}})p_a + r_up_a\right] \]

(60)

\[ - (p_a - c) \frac{k r_{\bar{u}}}{1-s} \]

The second derivative of the profit function yields:

\[ \frac{\partial^2 \pi_\bar{u}}{\partial p_a^2} = \frac{-2k r_{\bar{u}}}{1-s}, \]

which is negative. Hence both pay-off functions are quasi-concave.

\(^{43}\)See for the definition of \( r_{\bar{u}} \) (5).
B Analysis of the Possibility of Multiple Equilibria

From figure 1 it is obvious that the problem of multiple equilibria can be avoided if \( p_u^* \) is reached at a lower level of \( p_u \) for the first part of the reaction function (9) than for the second. In order to check this definition (10) is inserted in both parts of the reaction function for \( p_u \) and then rearranged for \( p_u^* \) resulting in:

\[
p_u' = \frac{a(1-s)[2(r_u - 1) + r_0]}{kr_0r_u} - \frac{c(r_u - 1)}{r_u}
\]

for \( p_u^* \) following the first part of (9) and

\[
p_u'' = \frac{a(1-s)2(r_u - 1 + r_0)}{kr_0r_u} - \frac{c(r_u - 1 + r_0)}{r_u}
\]

following its second part. Comparing \( p_u' \) and \( p_u'' \) it can easily be seen that the condition \( p_u' < p_u'' \) is not fulfilled in any case, but can be assured by assumption (11).

C The Derivation of the Necessary Conditions for Deterring the Entry of a Third Firm in the Market

For the moment we assume three products of differing quality levels like it is done in section 2.4. Differentiating the three profit functions with respect to their own prices respectively and setting them equal to zero yields:45

\[
\frac{\partial \pi_u}{\partial p_u} = y_u - a - (p_u - c)(r_u - 1)\frac{k}{1-s} = 0 \text{ for } y_0 \leq a \quad (61)
\]

\[
\frac{\partial \pi_u}{\partial p_u} = y_u - y_0' - (p_u - c)[(r_u - 1) + r_0']\frac{k}{1-s} = 0 \text{ for } y_0' > a
\]

\[
\frac{\partial \pi_u}{\partial p_u} = y_u - y_u' - (p_u - c)[(r_u - 1) + r_u']\frac{k}{1-s} = 0 \quad (62)
\]

\[
\frac{\partial \pi_u}{\partial p_u} = b - y_u - (p_u - c)r_u\frac{k}{1-s} = 0 \quad (63)
\]

44For definition see equation (10).

45The argumentation is the same as in Shaked, Sutton (1982), p. 5. The profit functions are continuous which can be proved by the same considerations as in section 2.3 and quasi-concave. Therefore a Bertrand-Nash-Equilibrium exists.
The second order conditions are given by:

\[
\frac{\partial^2 \pi_u}{\partial p_u^2} = \begin{cases} 
-\frac{k}{1-s}2(r_u - 1) & \text{for } y'_0 < a \\
-\frac{k}{1-s}2(r_u - 1 + r'_0) & \text{for } y'_0 > a 
\end{cases} \quad (64)
\]

\[
\frac{\partial^2 \pi_{\tilde{u}}}{\partial p_{\tilde{u}}^2} = -\frac{k}{1-s}2(r_{\tilde{u}} - 1 + r_{\tilde{u}}) \quad (65)
\]

\[
\frac{\partial^2 \pi_u}{\partial p_u^2} = -\frac{k}{1-s}2r_u \quad (66)
\]

and are fulfilled, which ensures that the profit functions are concave.

By inserting the definitions (5) for \(y_u\) and (19) for \(y'_0\) in (61), (62), and (63), and by rearranging two systems of equations can be derived which can be solved for the prices in equilibrium. ⁴⁶

In the case where the first part of (61) is relevant we get the following system:

\[
\begin{pmatrix}
-2(r_u - 1) & r_u \\
(r_u - 1) & -2(r_{\tilde{u}} - 1 + r_{\tilde{u}}) \\
0 & (r_{\tilde{u}} - 1)
\end{pmatrix}
\begin{pmatrix}
p_u^* \\
p_{\tilde{u}}^* \\
p_u^*
\end{pmatrix}
= \begin{pmatrix}
0 \\
-2r_{\tilde{u}} \\
0
\end{pmatrix}
\quad (67)
\]

From solving the system (67) for \(p_u^*\) we get:

\[
p_u^* = \frac{1 - s}{k} \left[br_u - a(3r_{\tilde{u}} - 3 + 4r_u)\right] + c[(6r_u - 3)(r_{\tilde{u}} - 1 + r_u)] \quad (68)
\]

\[
p_u^* = \frac{1}{6(r_u - 1)(r_{\tilde{u}} - 1 + r_u)}
\]

\(p_u^*\) and \(p_{\tilde{u}}^*\) can be derived analogously and are given by:

\[
p_{\tilde{u}}^* = \frac{b - a}{k} \left[3r_{\tilde{u}} - 1 + r_u\right] + c \quad (69)
\]

\[
p_u^* = \left[\frac{1 - s}{k} \left[b(4r_{\tilde{u}} - 4 + 3r_u) - a(3r_{\tilde{u}} - 1)\right]
\right] + c[(6r_u - 3)(r_{\tilde{u}} - 1 + r_u)] \quad (70)
\]

\[
+ \frac{1}{6r_u(3r_{\tilde{u}} - 1 + r_u)}
\]

⁴⁶A third system could be derived if the equilibrium lied in the range where the profit function of the firm with quality \(u\) could not be differentiated with respect to \(p_u\) because it was chosen in such a way that \(y'_0 = a\) held. Instead of the first order condition for profit maximization of the firm with quality \(u\) the fact \(y'_0 = a\) is used and rearranged for \(p_u\). Since we are especially interested in the price of the newcomer and this price is already known by \(y'_0 = a\) a further analysis of the third system can be neglected and is therefore omitted here.
For the case that the second part of (61) is relevant the same analysis is carried out. The equation system is then given by:

\[
\begin{pmatrix}
-2(r_u - 1 + r_0) & r_u & 0 \\
-(r_u - 1) & -2(r_u - 1 + r_u) & r_u \\
0 & -(r_u - 1) & -2r_u \\
\end{pmatrix}
\begin{pmatrix}
p_u^{**} \\
p_m^{**} \\
p_u^{**} \\
\end{pmatrix}
= 
\begin{pmatrix}
-c(r_u - 1 + r_0) \\
-c(r_u - 1 + r_u) \\
-b(1-s) - cr_u \\
\end{pmatrix}
\]

Solving for \(p_u^{**}\) yields:

\[
p_u^{**} = \left[ c[(r_u - 1 + r_0)(3r_u - 3 + 4r_u) + r_u(3r_u - 2 + 2r_u)] + b(r_u - 1 + r_0) \frac{1-s}{k} \right] \frac{1}{2[3r_u(r_u - 1) + (r_0 - 1)(3r_u - 3 + 4r_u)]}
\]

Calculating \(p_u^{**}\) and \(p_m^{**}\) analogously yields:

\[
p_u^{**} = \left[ c[(4r_u - 3 + 3r_u)(r_u - 1) + r'_0(3r_u - 2 + 2r_u)] + b(r_u - 1 + r'_0) \frac{1-s}{k} \right] \frac{1}{3r_u(r_u - 1) + (r'_0 - 1)(3r_u - 3 + 4r_u)}
\]

\[
p_u^{**} = \left[ c[(r_u - 1 + r'_0)(3r_u - 1)(r_u - 1 + r_u) + r_u(r_u - 1)] - r_u r_u (r_u - 1) + b[4(r_u - 1 + r'_0)(r_u - 1 + r_u) - r_u (r_u - 1)] \frac{(1-s)}{k} \right] \\
\frac{1}{2r_u[3r_u(r_u - 1) + (r'_0 - 1)(3r_u - 3 + 4r_u)]}
\]
References


