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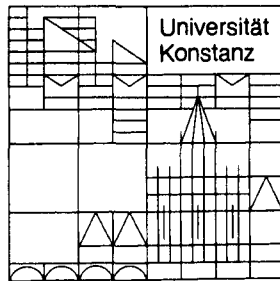
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**Fakultät für  
Wirtschaftswissenschaften  
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Helmut Zink

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Price Randomization,  
and Price Stickiness**

Diskussionsbeiträge

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**OVERHEAD COST, PRICE RANDOMIZATION, AND PRICE STICKINESS**

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# OVERHEAD COST, PRICE RANDOMIZATION, AND PRICE STICKINESS

by

Helmut Zink

March 1994

## Abstract

This paper investigates price setting when firms produce with increasing returns and customers are imperfectly informed about prices but customers can search. It is shown that there exists a unique randomization equilibrium where each customer is offered a random price by each firm known to him. If the number of firms is fixed then the expected price to be paid by a customer responds countercyclically to variations of demand per customer and it remains invariant with respect to variations of the number of customers. This price behavior corresponds well with new evidence on price behavior.

Keywords: increasing returns, price stickiness, search

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## 1. INTRODUCTION

There is a long tradition in economics to develop explanations for price stickiness. These questions are at the heart of many policy recommendations. Recent research has given empirical evidence for prices even moving countercyclically with production.<sup>1</sup> In this paper I present a joint explanation for two types of price behavior. In response to demand variations prices may either remain invariant or move countercyclically. The type of the price response depends on the nature of the underlying demand variation:

I apply two sets of assumptions. First I assume that firms produce with overhead cost per period. This assumption is a simplified description of production cost in case of underutilization of capacities.<sup>2</sup> Second I assume that customers are initially uninformed about offers but customers can search where each search step involves search cost.

To analyze pricing under these conditions I allow firms to offer randomly and such that different customers may get different prices. Any given firm, however, draws all its offer prices from the same probability distribution. I find that there exists a unique randomization equilibrium. That is, there is a unique probability distribution such that if all firms set prices according to this probability distribution then no firm has an incentive to deviate, given that all other firms stick to their behavior. If a firm increases its price within the support of the equilibrium probability distribution then there are two effects. First, the revenue per customer increases. Second, the expected number of customers buying from this firm decreases because the probability increases that competing firms can attract away customers. Thus, this firm's average production cost will increase. If all firms offer according to the equilibrium probability distribution then both effects balance and no firm can affect its expected profit by varying its price. If the firm sets its price above the support then customers prefer searching and the firm does not get any customers. If the firm sets its price below the support then its expected profit is smaller than that amount which it would get at the lower end of the support.<sup>3</sup>

This model can be utilized to study the effects of demand variations. If the number of customers varies while the number of firms is fixed then the price distribution is invariant. In response to an increase of the number of customers the profit level of all firms will increase but no firm has an

incentive to deviate because search behavior is unaffected.

If, instead, demand per customer varies while the number of customers and the number of firms remain fixed then search behavior of customers is affected. If demand per customer increases then the incentive to search increases and thus prices will fall. Prices will fall such that the profit level remains invariant.<sup>4</sup>

In the following, the assumptions of the model are presented in Section 2, equilibria are described in Section 3, proofs are provided in Section 4 and the Appendix, the effects of demand variations on prices are summarized in Section 5, and conclusions are drawn in Section 6.

## 2. ASSUMPTIONS

We assume that there are  $N$  firms and  $c$  customers. Each firm produces a homogeneous good with variable cost of  $a$  per unit of the good and overhead cost  $b$ .<sup>5</sup> Each customer wants to buy  $\theta$  units of the good but, initially, does not know the offer price of any firm. While in Zink (1991, 1993) the offer price of any single firm is deterministic and valid for all customers we now assume that each firm  $i \in \{1, 2, \dots, N\}$ ,  $N \geq 2$ , randomizes its price for each customer. We assume that all offers are stochastically independent. Hence a customer does not only get different offers from different firms, but each firm also presents different offers to different customers. We say a market is distributed according to  $(\phi_j)_{j=1, \dots, N}$  if for each customer each firm  $j$  randomizes its offer according to the probability distribution  $\phi_j$ .<sup>6</sup>

Each customer may search for offers. Each search step incurs search cost of  $\delta$  to the customer. With each search step each firm  $i \in \{1, 2, \dots, N\}$  gets known to him stochastically independent with probability  $\pi$ ,  $\pi \in (0, 1)$ . Customers know the model including the equilibrium concept and can infer the probability distributions  $\phi_i$  employed by firms in equilibrium. Each customer searches until he finds an offer such that the reduction of expenditure to be expected from any further search step is smaller than or equal to search cost  $\delta$ . Such an offer is called acceptable. He then buys  $\theta$  units of the good from the cheapest firm known to him.<sup>7</sup>

Suppose (1) firm  $i$  chooses  $p$  as offer price, (2) each firm  $j \neq i$  randomizes its offer according to  $\phi_j$ , and (3) each customer expects each firm  $j=1, 2, \dots, N$  to offer according to  $\phi_j$ . Then the expected number of customers buying from firm  $i$  is denoted by  $q_i(p) = q_i(p | (\phi_j)_{j=1, 2, \dots, N})$  and the (expected) profit of

firm is given by

$$(1) \quad g_i(p) = g_i(p | (\phi_j)_{j=1,2,\dots,N}) = \theta q_i(p)(p-a) - b.$$

We distinguish between short-run and long-run equilibria.

DEFINITION 1:  $(\phi_j)_{j=1,\dots,N}$  is said to form a short-run randomization equilibrium (with respect to the exogenous number of firms  $N \geq 2$ ) if for each firm  $i \in \{1, 2, \dots, N\}$  there is a profit level  $\gamma_i \in (-\infty, \infty)$  such that

$$(2) \quad g_i(p) = g_i(p | (\phi_j)_{j=1,2,\dots,N}) \begin{cases} = \gamma_i & \text{if } p \in S_i \\ \leq \gamma_i & \text{else,} \end{cases}$$

where  $S_i$  is the support of  $\phi_i$ .  $\square$  <sup>8 9 10</sup>

Hence, if all firms  $j=1, \dots, N$  offer according to a short-run randomization equilibrium, none of these firms has an incentive to apply a deviating offer distribution. Incentives for exit or entry of additional firms, though, may exist. In the following long-run equilibrium the number of firms, too, is in equilibrium.

DEFINITION 2:  $(\phi_j)_{j=1,\dots,N}$  is said to form a long-run randomization equilibrium if (1)  $N \geq 2$ , (2)  $(\phi_j)_{j=1,\dots,N}$  is a short-run randomization equilibrium with respect to  $N$  and with each firm  $j$  making non-negative expected profit  $\gamma_j$ , and (3) for any larger number of firms  $N' > N$  the corresponding short-run randomization equilibrium  $(\phi_j)_{j=1,\dots,N'}$  with respect to  $N'$  implies negative expected profit  $\gamma_i < 0$  for some firm  $i \in \{1, 2, \dots, N'\}$ .  $\square$

### 3. CONSTRUCTION AND UNIQUENESS OF EQUILIBRIA

First I construct a short-run randomization equilibrium with respect to an exogenous number of firms  $N$  and prove its uniqueness.

THEOREM 1: For each exogenous integer  $N \geq 2$  the vector of probability distributions  $(\phi_i)_{i=1}^N$  forms a short-run randomization equilibrium if for all firms  $i=1, \dots, N$ ,  $\phi_i$  is equal to  $\phi^*$  where

$$(3) \quad \phi^*(p) = \begin{cases} \frac{1}{\pi} \{1 - [(p-a)/(p-a)]^{[1/(N-1)]}\} & \text{if } p \in [p, \bar{p}] \\ 0 & \text{if } p \leq p \\ 1 & \text{if } p \geq \bar{p} \end{cases}$$

$$(4) \quad p = a + \frac{b+\gamma}{\theta c \pi} [1 - (1-\pi)^N]$$

$$(5) \quad \bar{p} = a + \frac{b+\gamma}{\theta c \pi} \frac{1-(1-\pi)^N}{(1-\pi)^{N-1}}$$

$$(6) \quad \gamma = \frac{\delta c \pi}{1-(1-\pi)^N} \frac{1}{[(1-\pi)^{-(N-1)} - 1] + (N-1) \ln(1-\pi)} - b.$$

In this equilibrium each firm offers with probability one, i.e.  $\phi^*(\bar{p})=1$ , all offers are acceptable, and each firm has expected profits of  $\gamma > -b$ . There is no short-run randomization equilibrium  $(\phi_j)_{j=1}^N$  other than that determined by (3)-(6).  $\square$

Next I describe long-run equilibria.

**THEOREM 2:** For large enough market size  $c$ , large enough search cost  $\delta$ , small enough market transparency  $\pi$ , or small enough overhead cost  $b$  there exists a unique long-run randomization equilibrium. This equilibrium is given by  $(\phi^*)^N$  where the probability distribution  $\phi^*$  is determined by (3) to (6) and the long-run number of firms is the smallest integer for which the corresponding short-run randomization equilibrium implies a non-negative profit level.  $\square$

#### 4. PROOFS

PROOF OF THEOREM 1:

1. First we show that under  $(\phi^*)^N$  all offers  $p \leq \bar{p}$  are acceptable and those with higher prices are not acceptable. Suppose the market is distributed according to  $(\phi^*)^N$ , a given customer is informed about exactly one firm - say firm  $N$  without loss of generality - , and this firm has offered to him price  $\bar{p}$ . Let  $p_S$  be the cheapest offer price this customer receives by an additional search step and let  $E p_S$  be the mathematical expectation of  $p_S$ . It suffices to show that the expected expenditure reduction  $R$  resulting at  $\bar{p}$  from one search step is equal to the cost of this search step,

$$(7) \quad R := \theta(\bar{p} - E p_S) = \delta.$$

To calculate  $E p_S$  we first determine the probability distribution of  $p_S$ . For each  $j \in \{1, 2, \dots, N-1\}$  let  $p_j$  denote the price which firm  $j$  offers to the considered customer. Then we get from (3) the following probabilities

$$(8) \quad \begin{aligned} \text{prob}(p_S \leq p) &= \text{prob} \left\{ \begin{array}{l} \text{there exists a firm } j \in \{1, 2, \dots, N-1\}: \\ \text{customer finds firm } j \text{ by one search step and } p_j \leq p \end{array} \right\} \\ &= 1 - \prod_{j=1}^{N-1} [1 - \text{prob}(\text{customer finds firm } j \text{ and } p_j \leq p)] = 1 - [1 - \pi \phi^*(p)]^{N-1} \end{aligned}$$



$$= 1 - \frac{[(p-a)/(p-a)]}{\bar{p}}$$

Hence knowing the density of  $p_S$  from (8), we get from (4) to (6)

$$(9) \quad \bar{p} - E p_S = \int_{\bar{p}}^{\bar{p}} (\bar{p} - x) \frac{(p-a)}{(x-a)^2} dx$$

$$= \frac{b+\gamma}{\theta c \pi} [1 - (1-\pi)^N] \left\{ \frac{1 - (1-\pi)^{N-1}}{(1-\pi)^{N-1}} + (N-1) \ln(1-\pi) \right\} = \delta/\theta$$

which proves (7).

2. Next we develop a formula to describe the expected number of customers any firm  $i \in \{1, 2, \dots, N\}$  gets for alternative prices and any  $N$ -tuple of probability distributions  $(\phi_j)_{j=1, 2, \dots, N}$ . First we note that with probability one each firm will pose acceptable offers only. Otherwise firm  $i$  could profitably attract additional customers at a price just above marginal cost  $a$  instead of offering unacceptable prices or not offering at all. Hence we may assume that under  $(\phi_j)_{j=1, 2, \dots, N}$  any price  $p$  within the union of all supports,  $p \in \bigcup_{j=1}^N S_j$ , is acceptable to customers. As we will see in the Appendix it also suffices to consider only atomless distributions. Then for any price  $p$  in the union of the supports of the distributions the expected number of customers of firm  $i$  is given by

$$(10) \quad q_i(p | (\phi_j)_{j=1, 2, \dots, N}) = c \sum_{t=1}^{\infty} (1-\pi)^{N(t-1)} \pi \prod_{\substack{j=1 \\ j \neq i}}^N [1 - \pi \phi_j(p)]$$

$$= \frac{c\pi}{1 - (1-\pi)^N} \prod_{\substack{j=1 \\ j \neq i}}^N [1 - \pi \phi_j(p)].$$

To give an interpretation of (10) we note that  $t$  refers to the possible number of search steps undertaken by a customer,  $(1-\pi)^{N(t-1)}$  is the probability that during his first  $t-1$  search steps the customer has not found any acceptable firm,  $\pi$  is the probability that the customer finds firm  $i$  during the  $t$ -th search step, and the product  $\prod_{j \neq i} [1 - \pi \phi_j(p)]$  describes the probability that in his  $t$ -th search step he does not find any cheaper offer than that of firm  $i$ .

3. Now we show that under  $(\phi^*)^N$  it is not profitable for any firm  $i$  to vary its offer price given that customers expect all firms to offer according to  $\phi^*$ .

Because of (1), (10) and  $\phi^*(\bar{p})=1$  we get

$$(11) \quad g_i(p | (\phi^*)^N) = \theta c \pi \frac{[1 - \pi \phi^*(p)]^{N-1}}{1 - [1 - \pi]^N} (p - a) - b.$$

Due to (3) and (4), (11) simplifies for  $p \in [p, \bar{p}]$  to

$$(12) \quad g_i(p | (\phi^*)^N) = \frac{\theta c \pi}{1 - (1 - \pi)^N} (p - a) - b = \gamma.$$

For  $p < p$  the expected profit of firm  $i$  is smaller than  $\gamma$  since  $q_i(p | (\phi^*)^N) = q_i(p | (\phi^*)^N)$ . For  $p > \bar{p}$  profits are equal to  $-b < 0$  since each customer prefers searching.

4. The proof of uniqueness is given in the Appendix. It is complicated by the fact that we prove uniqueness within the set of all distributions including those with atoms and singular continuous parts.

#### PROOF OF THEOREM 2:

We know that for each integer number of firms  $n$  there exists a unique short-run randomization equilibrium  $(\phi_n)^n$  with a profit level  $\gamma_n$  common to all  $n$  firms where  $\phi_n$  and  $\gamma_n$  are given by (3) to (6) and where  $\gamma_n$  decreases in  $n$ . Get  $N$  be the largest integer with a non-negative profit level,  $N := \max\{n \in \mathbb{N} : \gamma_n \geq 0\}$ . According to Definition 2  $(\phi_N)^N$  is a long-run randomization equilibrium if  $N \geq 2$ .

To establish conditions for  $N \geq 2$  we note that for each  $n$  the profit level  $\gamma = \gamma_n$  is determined by (6) which is equivalent to

$$(13) \quad \frac{\delta c}{b + \gamma} = \frac{1 - (1 - \pi)^n}{\pi} \left\{ (1 - \pi)^{-(n-1)} - 1 + (n-1) \ln(1 - \pi) \right\}.$$

We investigate how  $n$  depends on  $\gamma$ . For this we allow  $n$  to be real-valued. For  $n \geq 1$  the right side of (13) is monotonously increasing in  $n$ , for  $n=1$  it is zero and for  $n \rightarrow \infty$  it increases to infinity. Hence for any positive value of  $\delta c / (b + \gamma)$  and any  $\pi \in (0, 1)$  there is a unique real-valued  $n \in (1, \infty)$  which solves (13). As  $\delta c / (b + \gamma)$  increases from 0 to  $\infty$  the corresponding  $n$  increases continuously from 1 to  $\infty$ . As  $\pi$  increases from 0 to 1,  $n$  decreases from  $\infty$  to 1. Let  $n^\circ$  be that (real-valued) firm number for which  $\gamma=0$ . Then  $N = \max\{m \in \mathbb{N} : m \leq n^\circ\}$ . Since  $n^\circ$  is unique, so is  $N$ . Further, for large enough  $\delta c / b$  or small enough  $\pi$  we have both  $n^\circ \in (2, \infty)$  and  $N \geq 2$ . For too large  $\pi \in (0, 1)$  we have  $n^\circ < 2$ .  $\square$

#### 5. COMPARATIVE-STATIC PRICE BEHAVIOR

Let  $p^*$  be the expected price to be paid by a customer under the short-run randomization equilibrium with respect to  $N$  firms. We denote  $p^*$  as short-run

average price. To calculate  $p^*$  we make use of the following accounting identity: aggregate expected spending of customers is equal to aggregate expected production cost plus aggregate expected profit. Aggregate expected spending is given by  $\theta cp^*$ , aggregate expected production cost by  $Nb + \theta ca$ , and aggregate expected profit by  $N\gamma$ . Solving for  $p^*$  we get

$$(14) \quad p^* = a + \frac{N(b+\gamma)}{\theta c}.$$

Price equation (14) can be utilized to investigate how prices respond in the short run after a demand variation when the number of firms is held fixed on its previous long-run level. From profit equation (6) we see that the level of expected profit is independent of the demand intensity  $\theta$  and that the sum of overhead cost and profit per customer  $N(b+\gamma)/c$  is independent of the number of customers  $c$ . Hence price equation (14) yields the asserted price behavior.

**THEOREM 3:** In the short-run the average price  $p^*$  responds countercyclically with respect to variations of the demand intensity  $\theta$ , and  $p^*$  remains invariant with respect to variations of the number of customers  $c$ .  $\square$

## 6. CONCLUSIONS

I showed existence and uniqueness of an equilibrium in a market with price setting firms in face of increasing returns in production. The market share of each firm is bounded due to imperfect information of customers and offer randomization of firms. To study the effects of demand variations on prices, two types of demand variations have to be distinguished.

First, if the number of customers varies then prices are invariant since customers' search behavior is not affected. In this case expected profits respond procyclically. Second, if demand per customer varies while the number of customers is constant, then search is affected. With increasing demand per customer search intensifies and prices fall while the endogenous level of expected profit is invariant.

In real markets both the number of customers and demand per customer will vary procyclically over business cycles. The model then provides an explanation for two stylized facts of business cycles: prices are countercyclical and profits are procyclical.

## REFERENCES

- Allen, B. and M. Hellwig, (1986), "Bertrand-Edgeworth Oligopoly in Large Markets", *Review of Economic Studies*, Vol. 53, 175-204.
- Beckmann, M. (1965), "Edgeworth-Bertrand Duopoly Revisited", in R. Henn (ed.), *Operations-Research Verfahren*, Vol. 3, Meisenheim, 55-68.
- Burdett K. and K. Judd (1983), "Equilibrium Price Dispersion", *Econometrica*, Vol 51 (July), 955-969.
- Bils, M. (1987), "The Cyclical Behavior of Marginal Cost and Price", *American Economic Review*, Vol. 77 (December), 838-855.
- Cooley, Th.F. and L.E. Ohanian (1991), "The Cyclical Behavior of Prices", *Journal of Monetary Economics*, Vol 28, 25-60.
- Fiorito, R. and T. Kollintzas (1992): "Stylized Facts of Business Cycles in the G7 from a real Business Cycles Perspective", Discussion Paper No 681, Centre for Economic Policy Research, London.
- Harsanyi, J. (1973), "Games with Randomly Disturbed Payoffs: A New Rational for Mixed-Strategy Equilibrium Points", *International Journal of Game Theory*, Vol 2, 1-23.
- Levitan R.E. and M. Shubik (1972), "Price Duopoly and Capacity Constraints", *International Economic Review*, Vol 13., 111-142.
- Shilony, Y. (1977), "Mixed Pricing in Oligopoly", *Journal of Economic Theory*, Vol. 14, 373-388.
- Smith, R.T. (1992), "The Cyclical Behavior of Prices", *Journal of Money, Credit, and Banking*, Vol. 24, 413-430.
- Varian, H.R. (1980), "A Model of Sales", *American Economic Review*, Vol. 70 (September), 651-659.
- Zink, H. (1991), "Increasing Returns, Quality Uncertainty, Product Differentiation, and Countercyclical Pricing", Stanford Institute for Theoretical Economics, Technical Report No. 12.
- (1993), "Monopolistic Competition and the Cyclicalness of Pricing and Productivity - A Typology of Industries", *Journal of Economics*, Vol. 58, 109-152.

## APPENDIX

We prove uniqueness of the equilibrium described in Theorem 1. Suppose the market is distributed according to  $(\phi_j)_{j=1, \dots, N}$  and  $(\phi_j)_{j=1, \dots, N}$  is a short-run randomization equilibrium. For each  $i=1, 2, \dots, N$  let  $p_i := \inf S(\phi_i)$  and  $\bar{p}_i := \sup S(\phi_i)$  where  $S(\phi_i)$  is the support of the distribution  $\phi_i$ . In the following we make use of the reservation price  $p_R$  which is defined by the requirement that under  $(\phi_j)_{j=1, \dots, N}$  any customer stops searching once he gets informed about an offer with price  $p \leq p_R$ .<sup>12</sup>

1. First we show that all firms choose atomless offer distributions. Suppose

firm  $i$  has chosen a distribution  $\phi_i$  with an atom of positive mass  $\mu$  at some price  $p^\circ$ . First we consider the case of  $p^\circ < p_R$ . Let  $p' := \inf\{p \geq p^\circ : p \in S(\phi_j) \cup \{\infty\} \text{ for some } j \neq i\}$ . Then  $p' > p^\circ$  which can be seen as follows. Since  $\phi_i$  has an atom at  $p^\circ$  any firm  $j \neq i$  offering at  $p^\circ$  or slightly above could profitably increase its number of customers by decreasing its price slightly below  $p^\circ$ .<sup>13</sup> Because of  $p_R > p^\circ$ , however,  $p' > p^\circ$  would imply that firm  $i$  could profitably increase its price from  $p^\circ$  to  $[p^\circ + \min(p_R, p')]/2$ , for example. Hence we have shown that there can be no atom below  $p_R$ .

Now we consider the case of  $p^\circ = p_R$ . Let  $(p'_n)_{n \in \mathbb{N}}$  be a sequence of prices below  $p_R$  which converges to  $p_R$ . Then, on the one hand the atom of firm  $i$  implies for all  $k \neq i$   $\lim_{n \rightarrow \infty} g_k(p'_n) \leq \lim_{n \rightarrow \infty} g_i(p'_n)$  because we have

$$(a) \quad \lim_{n \rightarrow \infty} g_k(p'_n) \leq g_i(p_k) \text{ due to (2),}$$

$$(b) \quad g_k(p_k) \leq g_i(p_k) \text{ due to } \phi_k(p_k) = 0 \leq \phi_i(p_k) \text{ which implies}$$

$$\frac{q_i(p_k)}{q_k(p_k)} = \frac{\prod_{j \neq i} [1 - \pi \phi_j(p_k)]}{\prod_{j \neq k} [1 - \pi \phi_j(p_k)]} = \frac{[1 - \pi \phi_k(p_k)]}{[1 - \pi \phi_i(p_k)]} \geq 1,$$

$$(c) \quad g_i(p'_k) \leq g_i(p_R) \text{ due to (2), and}$$

$$(d) \quad g_i(p_R) = \lim_{n \rightarrow \infty} g_i(p'_n) \text{ due to the continuity of } \phi_k \text{ for all } k \neq i.$$

On the other hand the atom of firm  $i$  at  $p_R$  implies  $\lim_{n \rightarrow \infty} g_k(p_n) > \lim_{n \rightarrow \infty} g_i(p_n)$  because we have  $\lim_{n \rightarrow \infty} \phi_k(p_n) = 1 > \lim_{n \rightarrow \infty} \phi_i(p_n)$  which implies

$$\frac{\lim_{n \rightarrow \infty} g_k(p_n) + b}{\lim_{n \rightarrow \infty} g_i(p_n) + b} = \frac{\lim_{n \rightarrow \infty} q_k(p_n)}{\lim_{n \rightarrow \infty} q_i(p_n)} = \frac{[1 - \pi \lim_{n \rightarrow \infty} \phi_i(p_n)]}{[1 - \pi \lim_{n \rightarrow \infty} \phi_k(p_n)]} \geq 1.$$

2. Now we show by contradiction that under  $(\phi_j)_{j=1, \dots, N}$  all firms  $j$  will make the same expected profit  $\gamma$ . Suppose for two firms  $i$  and  $k$  we had  $\gamma_i < \gamma_k$ . Then with  $p_j := \inf S_j$  we have  $g_i(p_k) \leq \gamma_i < \gamma_k = g_k(p_k)$ . This implies  $q_i(p_k) < q_k(p_k)$ . Due to (10) we then get  $\phi_i(p_k) < \phi_k(p_k)$ . But this is a contradiction with the definition of  $p_k$  which implies  $\phi_k(p_k) = 0$ .

3. Next we show that all firms choose the same offer distribution  $\phi$ . Since under  $(\phi_j)_{j=1}^N$  all firms have the same profit level  $\gamma$ , equilibrium condition (2) can be rewritten with the help of (10) as

$$(15) \quad \sum_{\substack{j=1 \\ j \neq i}}^N -\ln[1 - \pi \phi_j(p)] \geq \ln \left[ \frac{\theta c \pi}{b + \gamma} \frac{p - a}{1 - (1 - \pi)^N} \right] \text{ for } p \leq \bar{p}_i$$

with equality for  $p \in S(\phi_i)$  where  $i=1,2,\dots,N$ . Thus, with  $\tilde{\phi}_i(p) := -\ln[1-\pi\phi_i(p)]$ ,  $\Phi(p) := \sum_{j=1}^N -\ln[1-\pi\phi_j(p)]$ , and  $\Psi(p) := \ln\{[\theta c\pi/(b+\gamma)](p-a)/[1-(1-\pi)^N]\}$  we have for  $p \leq \bar{p}_i$

$$(16) \quad \tilde{\phi}_i(p) \leq \Phi(p) - \Psi(p)$$

the right side of which is independent of  $i$ , with equality for  $p \in S(\phi_i)$ . Since all  $\phi_i$  are continuous (16) implies that all supports  $S(\phi_i)$  are equal to some interval  $S$ .<sup>14</sup>

4. Finally we can calculate the equilibrium distribution  $\phi$ . Summation of (15) over all  $i=1,2,\dots,N$  yields for  $p \leq \bar{p}' := \max S$

$$(17) \quad (N-1)\Phi(p) \geq N\Psi(p)$$

with equality for  $p \in S$ . Using the definitions of  $\Phi$  and  $\Psi$  we get from (17) for  $p \leq \bar{p}'$

$$(18) \quad \phi(p) \geq \frac{1}{\pi} \left\{ 1 - \left[ \frac{b+\gamma}{\theta c\pi} \frac{1-(1-\pi)^N}{p-a} \right]^{1/(N-1)} \right\}$$

with equality for  $p \in S$ . Thus on  $S$ ,  $\phi$  is equal to  $\phi^*$  as it is defined in (3) and (4). To calculate the endpoints of  $S$ ,  $p'$  and  $\bar{p}'$ , we use that for any  $i$  we have  $g_i(p' | (\phi)^N) = g_i(\bar{p}' | (\phi)^N) = \gamma$ ,  $\phi(p') = 0$  and  $\phi(\bar{p}') = 1$ . Then (10) implies immediately (4) and (5). The profit level  $\gamma$  is determined by (6) as can be seen from the last equation of (9). From the last equation of (9) we can also see that  $\gamma > -b$  if and only if  $N \geq 2$ .  $\square$

## NOTES

<sup>1</sup> M. Bills (1987) shows that in most U.S. manufacturing since 1956, the mark-ups of prices above marginal cost are strongly countercyclical, and even prices are countercyclical. Cooley and Ohanian (1991) give evidence for strong and consistently negative correlations between detrended prices and output for quarterly post 1948 U.S. data. Fiorito and Kollintzas (1992) confirm this negative correlation for all G7 countries. Similar evidence is also given by Smith (1992).

<sup>2</sup> If demand has increased so much that capacities are no longer underutilized then pricing becomes procyclical again. See Zink (1993) for an extension of a related approach to this case.

<sup>3</sup> In the literature there are related approaches to construct mixed strategy pricing equilibria under overhead cost in production. To my knowledge, however, none of them analyze how pricing is affected by demand variations. See for example Allen and Hellwig (1986), Beckmann (1965), Burdett and Judd (1983), Levitan and Shubik (1972), Shilony (1977), and Varian (1980). The noisy search model of Burdett and Judd (1983) comes closest to the model of the present paper but it assumes that each search step informs about at least one firm and it derives no explicit formulas for the equilibrium.

<sup>4</sup> In Zink (1991, 1993) I study Epsilon-Nash equilibria in pure strategies. These Epsilon-Nash equilibria are a suitable concept to describe markets if each firm incurs small switching cost any time it varies its price. With any positive switching cost, price variations become unprofitable if the market size  $c$  is large enough or the degree of market transparency  $\pi$  is small enough, respectively. Without switching cost firms would keep varying their prices in order to exploit profit possibilities of epsilon magnitude. In Zink (1991, 1993) price responses (with respect to demand variations) are of the same kind as in the present paper.

<sup>5</sup> The underlying view is that there are many periods in each of which overhead cost accrue and that in each period there are  $c$  new customers each of which lives for one period.

<sup>6</sup>  $\phi_j(p)$  is defined for all prices  $p$  as the probability that the offer of firm  $j$  is smaller or equal to  $p$ .

<sup>7</sup> Technically, offers are made to all customers simultaneously, then customers search and after completing search each customer buys from the cheapest firm known to him. (Hence, in particular, each customer can get at most one offer per firm). We note that these assumptions are consistent with the view that firms present offers only to those customers already known to them. Customers are not considered as players. A customer may not buy randomly from unknown firms. If he knows several acceptable offers with the same minimal price then he chooses randomly between them.

<sup>8</sup> Conventionally, the support of a distribution  $\phi$  is defined as  $\{p \in \mathbb{R} : \Phi(p-\varepsilon) < \Phi(p+\varepsilon) \text{ for all } \varepsilon > 0\}$ . With this definition the support is a closed set. All arguments in the paper, however, are also valid for a more general definition of a support which, for example, allows for firms offering at all prices below some threshold. For such a definition we would assume that each firm  $i$  chooses a tuple  $(\phi_i, S_i)$  where  $\phi_i$  is the probability distribution and  $S_i$  is a measurable set contained in the support of  $\phi_i$  with  $\int_{S_i} d\phi_i = 1$ .

9 For each firm  $i$  expected profit  $g_i(p | (\phi_j)_{j=1,2,\dots,N})$  depends on all equilibrium distributions  $(\phi_j)_{j=1,2,\dots,N}$  including  $\phi_i$  because we assumed customers to base their searching decisions not on actual probability distributions but on the equilibrium distributions which customers can infer from knowledge of the game.

We would get a different game if customers based their searching decisions on actual probability distributions. In that case any firm  $i$  could increase its expected profit by choosing its price at the lower end of the support with probability one,  $p_i = \min S_i$ . With this strategy each customer's expected return from search would increase, each customer's reservation price would decrease, each customer's expected number of search steps would increase, and thus the expected number of customers buying from firm  $i$  would increase. However, this effect becomes negligibly small if the number of firms  $N$  increases to infinity. This view of not considering the reaction of customers' reservation price corresponds with the short-run nature of the Nash-equilibrium concept where the profitability of a firm's deviation is calculated under the assumption that the behavior of competitors remains invariant.

10 See Harsanyi (1973) for a justification of mixed-strategy equilibria.

11 For parameters  $(c, \delta, \pi, b)$  not fulfilling the above requirement the long-run equilibrium number of firms would be smaller than 2. - One could establish existence and uniqueness of a randomization equilibrium for any parameter constellation if firms were allowed to offer with a probability smaller than one and overhead cost occurred only in cases of actual offers.

12 To give a more formal definition, let  $p_S(p')$  be the unknown (random) minimal price which a customer receives during one additional search step if he is informed about exactly one offer and the price of this offer is  $p'$ . Then the reservation price  $p_R$  is defined by  $\theta E[p_R - p_S(p_R)] = \delta$ . To avoid too tedious mathematics we take for granted here that  $p_R$  is uniquely defined by this equation and that  $\theta E[p - p_S(p)] \leq \delta$  if and only if  $p \leq p_R$ .

13 This argument can be made precise: For each  $j$  let  $\mu_j$  be the size of the atom  $\phi_j$  might have at  $p^\circ$ , and for each  $k$  define  $\nu_k := \prod_{j \neq k} [1 - \pi \phi_j(p^\circ)] / [1 - \pi \phi_j(p^\circ) - \pi \mu_j]$  and  $\epsilon_k := [(v_k - 1) / (1 + v_k)] (p^\circ - a)$  where  $v_k > 1$  and  $\epsilon_k > 0$  due to  $\mu_i > 0$ . Let  $p'' \in [p^\circ, p^\circ + \epsilon_k)$ . Now, according to (11) and (12), if firm  $k$  offered at  $p''$  then it would better offer at  $p^\circ - \epsilon_k$ ,

$$\frac{g_k(p^\circ - \epsilon_k) + b}{g_k(p'') + b} = \frac{p^\circ - \epsilon_k - a}{p'' - a} \prod_{j \neq k} \frac{1 - \pi \phi_j(p^\circ - \epsilon_k)}{1 - \pi \phi_j(p'')} > \frac{p^\circ - \epsilon_k - a}{p^\circ + \epsilon_k - a} \delta_k = 1.$$



14 For each  $i$  the endpoints of  $S_i$ ,  $p_i = \inf S_i$  and  $\bar{p}_i = \sup S_i$ , are given by  $\phi_i(p_i) = 0$  and  $\phi_i(\bar{p}_i) = 1$ . Hence they are given by  $\tilde{\phi}_i(p_i) = \Phi(p_i) - \Psi(p_i) = 0$  and  $\tilde{\phi}_i(\bar{p}_i) = \Phi(\bar{p}_i) - \Psi(\bar{p}_i) = -\ln(1-\pi)$  which implies that they are identical over all  $i$ . Next we show that each  $S_i$  is an interval. If for some  $i$   $S_i$  were not an interval then there would be an interval  $(p_{i1}, p_{i2})$  with  $p_{i1} \in S_i$  and  $p_{i2} \in S_i$  but  $(p_{i1}, p_{i2})$  lying in the complement of  $S_i$ . Hence (16) would imply that  $\phi_i$  has an atom at  $p_{i2}$  which contradicts with  $\phi_i$  being continuous.